Dynamic Stability and Reform of Political Institutions

Roger Lagunoff*

November, 2008

Abstract

When are political institutions stable? When do they tend toward reform? This paper examines a model of dynamic, endogenous institutional change. I introduce a class of dynamic political games in which the political aggregation rules used at date $t+1$ are instrumental choices under rules at date t. A political rule is *stable* if it selects itself. A *reform* occurs when an alternative rule is selected. It turns out that the stability of a political rule depends on whether its choices are *dynamically consistent*. For instance, simple majority rules can be shown to be dynamically consistent in many common environments where wealth-weighted voting rules are not. More generally, the result applies to an extended class of political rules that incorporate private activities such as extra-legal protests, threats, or private investment. The model makes use of an interpretation of rules as "players" who can strategically delegate future policy-making authority to different institutional types. The approach can be viewed as a comprehensive way of understanding various explanations of institutional change proposed in the literature. A parametric model of dynamic public goods provision gives an illustration.

JEL Codes: C73,D72, D74
Key Words and Phrases: Endogenous political institutions, recursive, dynamic political games, reform, stability, dynamically consistent rules, forward and backward consistency, inessential.

*Department of Economics, Georgetown University, Washington DC 20057 USA. 202 687-1510. lagunofr@georgetown.edu. http://www.georgetown.edu/lagunoff/lagunoff.htm . This project began while the author visited Johns Hopkins University and the W. Allen Wallis Institute of Political Economy at the University of Rochester. I am grateful for their hospitality. I also thank two anonymous referees, and Daron Acemoglu, Luca Anderlini, Jimmy Chan, Eddie Dekel, William Jack, Jim Jordan, Mark Stegeman, and seminar participants at Berkeley, Cal Tech, Georgetown, Penn State, Toronto, VPI, Yale, the Penn PIER Conference, and the Latin American and North American Winter Meetings of the Econometric Society for their helpful comments and suggestions on earlier drafts of this paper.

1 Introduction

Reforms of political institutions are common throughout history. They come in many varieties. For example, periodic expansions of voting rights occurred in governments of ancient Athens (700- 338BC), the Roman Republic (509BC-25AD), and most of Western Europe in the 19th and early 20th centuries.[1] Medieval Venice (1032-1300) altered its voting rule by gradually lowering the required voting threshold from unanimity to a simple majority in its Citizens' Council. Nineteenth century Prussia, where votes were initially weighted by one's wealth, eventually equalized the weights across all citizens. England and France changed the scope of their decision authority by privatizing common land during the 16th and 17th century enclosure movement.[2] The U.S. government, by contrast, increased its scope under the 16th Amendment by legalizing federal income tax in 1913.

The objective of this study is to understand what underlies a change in political institutions over time. Specifically, which environments tend toward institutional stability? Which environments admit change? What are the relevant forces that drive this change?

These questions are addressed in an infinite horizon setting in which political institutions are endogenous. The model has three critical features. First, political institutions arise in a recursive social choice framework: a *political rule* in date t is a social choice correspondence that aggregates preferences of the citizens at each date t to produce both a policy in date t and a future political rule for date $t+1$. Second, institutional choices are instrumental. That is, the political rules do not enter payoffs or technology directly. Third, institutional choice is wide and varied: the types of rules that can be chosen are not limited.

Admittedly, the social choice approach is abstract, far more so than, say, any particular game-theoretic model of institutions. Its virtue is its flexibility. Social choice rules can generally capture the most elemental attributes of a polity without being mired in or overly sensitive to the fine details of political mechanisms. Questions of who participates; how are votes ultimately counted; what types of policy choices are admissible, can all be accommodated in the present framework. This study demonstrates how the approach can offer a comprehensive way of understanding various explanations of institutional change proposed in the literature.

The model is developed in two stages. First, a benchmark model posits that all decisions occur in the public sector. The model is then extended to account for private decisions that are not explicitly part of the polity. The main results concern the stability and reform of *political rules*. A political rule is a social choice correspondence indexed by some parameter θ_t that summarizes the formal procedures for governance, e.g., election laws, voting eligibility, fiscal constitutions, and so on, that are in use at that date. A *politically feasible* strategy is an endogenous law of motion that maps the prevailing political rule θ_t in date t to a possibly new rule θ_{t+1} for use in date $t+1$. Politically feasible strategies are the "equilibria" of this model.

[1] See Fine (1983), Finer (1997), and Fleck and Hanssen (2003).
[2] See MacFarlane (1978) and Dahlman (1980).

They are analogous to Subgame Perfect equilibria in the sense that the strategies must yield implementable social choices after any history.

By definition, a political rule θ_t *admits reform* whenever the politically feasible strategy maps θ_t to a different rule $\theta_{t+1} \neq \theta_t$. That is, next period's political rule is chosen to be different than the present one. A rule θ_t is *stable* whenever $\theta_{t+1} = \theta_t$, i.e., no reform occurs. The main concern of this study is therefore whether and when a rule is stable, or alternatively, when does it admit reform?

To address this question, the present paper adopts an approach taken in a number of politico-economic models of policy dating back at least to Krusell, Quadrini, and Ríos-Rull (1997).[3] In these models, policy outcomes are obtained by maximizing a time consistent social welfare function or *aggregator*. Because the political rule is identified by an aggregator at each date, a political rule can be re-interpreted as a "player" in a dynamic stochastic game. This player's "preferences" are those of the aggregator. In the present model, the political rule at $t+1$ is also a choice variable, and so a reform entails a strategic delegation of authority from one type of "rule-as-player" to another. Rules are therefore stable when the current type chooses not to delegate.

It turns out that the stability of such a rule can be characterized by its *dynamic consistency* or lack thereof. A rule θ_t is said to satisfy *forward dynamic consistency (FDC)* if it is rationalized by an aggregator in which the optimal choice of θ_{t+1} in date t would remain optimal if the decision were hypothetically made at the beginning of $t+1$. In other words, the current rule would not "change its mind" if called upon to make the decision in the future. The rule θ_t satisfies *backward dynamic consistency (BDC)* if the hypothetical $t+1$ choices are, in fact, optimal at t. A *dynamically consistent rule* is one that is both FDC and BDC. Theorem 1 asserts that any dynamically consistent rule is stable, and any stable rule is forwardly dynamically consistent (FDC). Because FDC and BDC coincide when the choice of political rule is unique, these notions provide an approximately complete characterization of stable political institutions.

In the paper, this notion is referred to as "rule-based dynamic consistency" in order to distinguish it from consistency of an individual's preferences. Rule-based dynamic consistency has some similarities. Like individual consistency, rule based consistency concerns the conflict between a decision maker's current and his future "self" in evaluating tradeoffs.

However, unlike individual consistency, rule-based consistency comes from the internal workings of a political system. A rule may be inconsistent (in the rule-based sense) even if all individual preferences are perfectly consistent. For example, wealth-weighted voting rules, i.e., rules in which an individual's vote is weighted by his wealth, are usually dynamically *in*consistent in environments where wealth distribution changes through time. Hence, they admit reform. Why? The reason is straightforward. The identity of a pivotal voter under wealth weighted voting is sensitive to natural changes in the wealth distribution over time. But when the iden-

[3]See the Literature Review in Section 5.

tity of the pivotal voter changes, a conflict arises in the preferences of the current and future pivotal voters. So why does this translate into institutional change? Notice that the conflict between present and future pivotal voter all occurs under a *fixed* political rule θ_t. Hence, *nominal* political authority remains the same, while *de facto* authority changes. Consequently, the nominal delegation of authority from θ_t to θ_{t+1} may be viewed as an attempt to limit a *de facto* change due to a change in the wealth distribution.

By contrast, under standard formulations of technical change, the simple Majority Voting rule is dynamically consistent, hence stable. The reason is that in these formulations, the order statistics of the wealth distribution are preserved in expectation even as its higher order moments change. In other words, if one individual is currently richer than another, then on average he is expected to be richer tomorrow. Since simple majority voting typically depends only on the median (an order statistic), the identity of the pivotal voter is not expected to change.

Hence, while both individual and rule based consistency concern the conflicts between present and future "self," the latter also depends on subtleties of political aggregation. In particular, as the wealth-weighted voting example demonstrates, the stability of a political rule may depend on the internal structure of the rule itself, *even in the absence of private sector influences*. This contrasts with the current literature on dynamically endogenous institutions. That literature tends to focus on extra-legal protests, usurpations of power, or other activities that are not directly part of the polity.[4] For example, in an influential paper Acemoglu and Robinson (2000) argue that expansions of the voting franchise were used by the 19th century European elite to "buy off" the threat of a peasant revolt. The rationale for reform then comes from the fact that institutional change solves a commitment or credibility problem when a government's policies can be undone by private behavior - in this case the peasants' threat. Similar rationales for change are found in Justman and Gradstein (1999), Acemoglu and Robinson (2001, 2005), Lagunoff (2001), Gradstein (2003), Greif and Laitin (2004), Jack and Lagunoff (2006a,b), Cervellati, et. al. (2006), and Gradstein (2007).[5]

Even if the various "external threat" theories of institutional change are correct, rule-based consistency offers a comprehensive way of understanding them. To show this is so, the full model incorporates arbitrary private activities such as those examined in the literature. An *equilibrium* of this full model consists of politically feasible public sector strategies and Subgame Perfect private sector ones. Despite the addition of private sector activities, our main result shows that the previous result extends to this environment. That is, when political aggregation encompasses both public and private decisions, dynamically consistent "extended rules" are shown to be stable, and stable rules are forwardly dynamically consistent (Theorem 2).

Returning to the "external threat" explanation of reform, the idea of rule-based consistency adds to main two insights. First, it provides a concise characterization of the differing incentives

[4]One exception is Messner and Polborn (2004). See Section 5 for a further discussion.
[5]This literature and related work is discussed in detail in Section 5.

of government at various points in time. For example, the Acemoglu and Robinson (2000) argument in which the franchise expansion ameliorates the threat of a peasant revolt may be re-interpreted as a failure of dynamic consistency of the extended aggregator. In the absence of institutional change, the elite responds to the private sector threat under the assumption that the future elite does not. Hence, a conflict between present and future objectives arises.

Second, rule-based consistency shows that seemingly distinct explanation for reform are, in fact, *the same underlying explanation*. For example, Acemoglu and Robinson (2000) explanation for franchise expansion is sometimes contrasted with that of Lizzeri and Persico's (2004) who posit an "internal conflict" rationale for reform. In their model, political competition induces over-investment in redistributive transfers relative to public goods. This over-investment is more severe in a restricted voting franchise, and so franchise expansion is in some cases unanimously preferred among the elite. Though the Lizzeri-Persico model is static, there is, nevertheless, a tension between the extended rule's "objectives" before and after the results of the political competition. The inefficiency might be mitigated if, for instance, citizens could commit to contingent votes before the political competition begins. Hence, in both papers, private sector activities account for institutional reform only insofar as they induce dynamically inconsistent political aggregation. We illustrate this phenomenon as well in an Example in Section 4.

Section 2 introduces the benchmark model. Section 3 contains the results of that model and goes on to illustrate the main trade-offs involved in keeping or changing a political rule in a parametric example of public goods provision. Section 4 extends the model to private sector decisions and revisits the parametric example with private decisions included. Section 5 discusses the related literature in more detail, placing the present model in the context of that literature. Section 6 is an Appendix with the proofs of all the results.

2 The Benchmark Model

This Section introduces the benchmark model of a pure public sector economy. For now, we abstract away from all private sector behavior such as individuals' savings, investment, or labor decisions. Hence, in this benchmark model, all relevant decisions are carried out in the public sector. Though not realistic, this benchmark isolates some of the relevant features of a political institution in order to determine its stability. Private sector behavior is later introduced into the model in Section 4.

To lay the groundwork for the model, an illustration of the social choice approach is introduced below. The approach is then extended to an infinite horizon dynamic game which admits the potential for continual changes in the political rule itself. Though the model developed here is abstract, Section 3 illustrates the ideas in a concrete parametric model.

2.1 A Social Choice Illustration

Before proceeding with the model, consider first a simple static social choice "illustration" to fix ideas. Let $I = \{1, \ldots, n\}$ denote a society of individuals, with n odd, and let P denote a set of possible collective decisions or *policies.* These policies may be tax rates, allocations of public goods, or pork. Each citizen $i \in I$ has payoff function $v_i : P \rightarrow \mathbb{R}$ defined on the policy space. A standard social choice approach entails that policies are chosen from a social choice correspondence, C, that maps each preference profile $(v_1(\cdot), \ldots, v_n(\cdot))$ (from a set of such profiles) to a subset of policies $C(v_1(\cdot), \ldots, v_n(\cdot)) \subseteq P$.

This standard model can, in fact, be extended to allow for choice of the correspondence itself. Let Θ be an index set. Each parameter $\theta \in \Theta$ is identified with a distinct social choice correspondence $C(\cdot, \theta)$. Hence, given an index θ and a profile $(v_1(\cdot), \ldots, v_n(\cdot))$, a policy $p \in P$ is *politically feasible* if $p \in C(v_1(\cdot), \ldots, v_n(\cdot),\ \theta)$.

The index θ is, in a sense, a *political rule* since it summarizes society's aggregation process as defined by the correspondence $C(\cdot, \theta)$. The classic choice between "dictatorship" and "majoritarian democracy," for instance, may now be represented as a choice between two parameters, θ^D and θ^M. Parameter θ^D is defined by: $p \in C(v_1(\cdot), \ldots, v_n(\cdot),\ \theta^D)$ if p maximizes v_i for some individual i (the "dictator"). θ^M is defined as: $p \in C(v_1(\cdot), \ldots, v_n(\cdot),\ \theta^M)$ if p is undominated by any alternative $\hat{p}$ in a simple majority vote. The problem in this static setup is: what rule determines the choice over these two rules? A dynamic model solves this problem by supposing that the choice between θ^D and θ^M for date t always takes place at a prior date in accordance with the rule in use at that time.

2.2 A Dynamic Extension of the Social Choice Model

From here on, all defined sets are assumed to be Borel measurable subsets of Euclidian spaces, and all feasible choice spaces (e.g., the space of feasible policies) are assumed to be compact. At each date $t = 0, 1, 2, \ldots$ society must collectively choose a policy $p_t \in P$. Each citizen i has a payoff in each date given by $u_i(\omega_t, p_t)$ where ω_t is a state variable drawn from a set, Ω. The state can include a citizen's asset holdings or any stock of physical or human capital that affects the citizen's payoffs. u_i is assumed to be continuous and nonnegative. The state evolves according to a Markov transition technology q with $q(B|\ \omega_t,\ p_t)$ the probability that ω_{t+1} belongs to the (Borel) set $B \subseteq \Omega$ given the current state ω_t and current policy p_t. For each such B, $q(B|\cdot)$ is continuous on $\Omega \times P$. A citizen's discounted payoff over the course of his lifetime is:

$$E_q\left[\sum_{t=0}^{\infty} \delta^t (1-\delta)\ u_i(\omega_t,\ \ p_t\ )\Big|\ \omega_0\right] \tag{1}$$

where δ is the common discount factor, and the expectation $E_q[\cdot]$ is taken with respect to Markov transition q. The expectation in (1) is assumed to be finite for any feasible sequence of states and policy choices.

Extending the notation of the static model, let $\theta_t \in \Theta$ denote the prevailing *political rule* at date t. The collective decision faced by a society at date t under rule θ_t is the choice pair (p_t, θ_{t+1}), i.e., the choice of current policy and subsequent political rule. The composite state, $s_t = (\omega_t, \theta_t)$, consists of the economic state and the political rule. The set of states is $S = \Omega \times \Theta$. A history at date t is the list $h^t = ((s_0, p_0), \ldots, (s_{t-1}, p_{t-1}), s_t)$ which includes all past data up to that point in time as well as the current state $s_t = (\omega_t, \theta_t)$. An initial history is given by $h^0 = (s_0, \emptyset)$. Let H denote the set of all histories at all dates.

Society's endogenous choices are summarized by a *public strategy*, a pair of decision functions ψ and μ, defined as follows. Given a history $h^t \in H$, ψ determines the chosen policy $p_t = \psi(h^t)$ while μ determines the political rule $\theta_{t+1} = \mu(h^t)$ chosen for next period.[6] The institutional strategy μ is of particular interest since it describes an endogenous dynamic process of institutional change from current to future rules. Denote the sets of each type of strategy by Ψ and M, respectively. After any history $h^t \in H$, a public strategy $(\psi, \mu) \in \Psi \times M$ produces a *public decision*, $(\psi(h^t), \mu(h^t)) = (p_t, \theta_{t+1})$ which determines today's policy and tomorrow's political rule for this society. Each citizen's continuation payoff is expressed recursively as

$$V_i(h^t;\ \psi, \mu) \ = \ (1-\delta)\ u_i(\omega_t, \psi(h^t)\ ) \ + \ \delta\ E_q \left[V_i(h^{t+1};\ \psi, \mu) \ \middle|\ h^t, \psi(h^t), \mu(h^t) \right] \qquad (2)$$

In (2), μ matters only indirectly through the policy strategy ψ since ψ may vary with the current and past θs.

Notice that $V_i(h^t;\ \psi, \mu)$ is a payoff, i.e., a scalar. However, social choice rules are defined on profiles of *preference orderings* rather than payoffs, and so we use the notation $V_i(h^t;\ \cdot)$, with the "dot" used as a placeholder for a public strategy (ψ, μ), and connoting citizen i's preference ordering over all $(\psi, \mu) \in \Psi \times M$.

We now have all the tools to define social choice in the dynamic model. As in the static model, a political rule θ_t is an index for a social choice correspondence C. The difference is that now C maps to strategies rather than outcomes. Formally, $(\psi, \mu) \in C(V(h^t;\ \cdot),\ s_t\ )$ expresses the idea that the strategy (ψ, μ) is a feasible social choice of the correspondence $C(\cdot, s_t)$ under the preference profile $V(h^t;\ \cdot) \equiv (V_1(h^t;\ \cdot), \ldots V_n(h^t;\ \cdot)\ )$ following history h^t.[7] Notice that C varies with "economic" state ω_t as well as the political rule θ_t. This is natural if the aggregation procedure makes use of economic information such as wealth. However, C is assumed to have no *structural* dependence on past history. Indirect dependence can arise through citizens' preferences. The pair (C, Θ) is referred to as the *class* of political rules.

[6] Strictly speaking ψ and μ need not be "strategic" choices. They could arise, for instance, from a purely mechanical process. For much of the analysis, however, these will be explicit strategic choices of some "Player" and so I just label them as "strategies" from the start.

[7] Formally, $C(\ V(h^t;\ \cdot),\ s_t\ ) \subset \Psi \times M$ is a cylinder set. What this means is that $C(\ V(h^t;\ \cdot),\ s_t\ )$ is a set of (ψ, μ) such that $(\psi(h^\tau), \mu(h^\tau))$ can take on any value in $P \times \Theta$ if $\tau < t$. In other words, there are no restrictions on public decisions prior to date t.

Suppose initially that this society could commit permanently to a fixed law of motion for political rules and policies. In that case, the social decision occurs once at $t = 0$. The solution, a public strategy (ψ, μ), then induces a distribution over paths $(p_0, \theta_1), (p_1, \theta_2), \ldots$, fully determined in advance. A "politically feasible strategy" in this sense would satisfy

$$(\psi, \mu) \ \in \ C(\ V(h^0;\ \cdot),\ s_0\ ) \quad \forall h^0 \tag{3}$$

At this point the dynamic model extends the static model in a straightforward way. Of course, a commitment of this type is arguably impossible in any real society. The Expression (3) is necessary but not sufficient for "political feasibility" in the recursive, "time consistent" sense. In order to satisfy the lack of strategic commitment, feasible strategies at date t must be consistent with feasible strategies at date $t + 1$. That is,

$$(\psi, \mu) \ \in \ C(\ V(h^t;\ \cdot),\ s_t\ ) \quad \forall h^t \quad \forall t = 0, 1, \ldots, \tag{4}$$

A *politically feasible strategy* is therefore a strategy (ψ, μ) which satisfies (4). On occasion we will look at politically feasible strategies (ψ, μ) that vary only with the current (Markov) state $s_t = (\omega_t, \theta_t)$. We refer to these as *politically feasible Markov strategies* (or just "Markov-feasible strategies" for short).

Political feasibility is analogous to Subgame Perfection since strategies must be politically feasible at each date and after any history. To emphasize the link with Subgame Perfection, consider the special case of a "Libertarian" political rule in which public decisions mimic privately chosen activities. Let $P \subset \mathbb{R}^n$ where p_i is chosen directly by the corresponding citizen i each period. Now let $\Theta = \{\theta\}$ where $C(\cdot,\ \omega, \theta)$ is the set of Nash equilibria in the subgame starting from t. The set of politically feasible strategies in this case is precisely the set of Subgame Perfect strategies of the n-player stochastic game. This "Libertarian" example also shows that the social choice approach is not at odds with a more traditional noncooperative game approach. In the latter, each political rule could be modeled explicitly as a noncooperative game, with the choice being over noncooperative games in future periods. The problem is: how should these game(s) be modeled? While there are agreed upon canonical social choice representations of voting, there are relatively few such games, perhaps because strategic models of politics are notoriously sensitive to minor details.[8] Robust results in noncooperative political game models are therefore hard to obtain. Consequently, this paper exploits a "detail free" approach that models basic properties of polities without specifying the fine details.

3 Stability versus Reform

Fix a class of political rules, (C, Θ). Recall that a history h^t includes the current state $s_t = (\omega_t, \theta_t)$ as well as all past policies and states. Denote by $h^t(\theta_t)$ a history in which the political

[8]Even in the more canonical endogenous candidate models of Besley and Coate (1997) and Osborne and Slivinsky (1996) there are reasonable and numerous, alternative specifications of how candidates could emerge.

rule at date t is $\theta_t \in \Theta$. Given an institutional strategy μ, a political rule $\theta_t \in \Theta$ *admits institutional reform in* μ if there exists a date t and a set of histories $h^t(\theta_t)$ of positive measure such that $\mu(h^t(\theta_t)) \neq \theta_t$.[9] Alternatively, *a rule* θ_t *is stable in* μ if it does not admit reform, i.e, if $\mu(h^t(\theta_t)) = \theta_t$ for almost every $h^t(\theta_t)$.

This study poses the question: when do political rules admit reform? When are they stable? Our main result shows that this question can be reduced to one checking whether the aggregator satisfies a certain "dynamic consistency" property. To motivate the result, we first examine the issue in a parametric special case of the model.

3.1 A Simple Parametric Model

In this section, we explore a parametric special case of the model. We assume a political institution determines the level of taxes in order to fund a durable public good such as public literacy. The economic state is $\omega_t = (G_t, y_t)$, where $y_t = (y_{1t}, \ldots, y_{nt})$ with y_{it} the wealth of Citizen i at date t (e.g, a parcel of land). Wealth y_{it} produces a one-to-one return each period of y_{it}. G_t is the stock of a durable public good at t. The public good in period $t+1$ depends on tax revenues in period t. A tax p_t is imposed on the yield from one's wealth/land. There is no depreciation. The transition law for the public good is deterministic and is given by $G_{t+1} = G_t + (p_t \sum_j y_{jt})^\gamma$, $0 < \gamma < 1$. Finally, each citizen's stage payoff is a linear function of his after-tax returns and his use of the public good: $u_i = (1 - p_t)y_{it} + G_t$.

There are two possible institutional choices: θ^M and θ^W. The political rule θ^M is the Majority Rule, whereas θ^W is the "Wealth-as-Power Rule" in which each individual is allocated a number of votes proportional to his wealth. According to our definition in (4), a politically feasible strategy is one that satisfies

$$(\psi, \mu) \in C(\ V(h^t(\theta^k);\ \cdot),\ \omega_t, \theta^k\ ) \quad \forall h^t(\theta^k)\ \ \forall k = M, W,\ \ \forall t$$

Formally, what this means is that under the rule θ^M, for any alternative strategy $(\hat{\psi}, \hat{\mu})$, the set of individuals that prefer $(\hat{\psi}, \hat{\mu})$ are not a majority, i.e,

$$J \equiv \{i \in I : V_i(h^t;\ \hat{\psi}, \hat{\mu}) > V_i(h^t;\ \psi, \mu)\}$$

satisfies $|J| \leq n/2$. Whereas under θ^W, the same set J of individuals have lower aggregate wealth: $\sum_{i \in J} y_i \ < \ \sum_{i \notin J} y_i$.[10]

[9]Positive measure here refers to the joint distribution on t-period histories induced by q, ψ and μ. Fixing initial history h^0 and rewriting history h^t as $h^t = (h^0, (\omega_1, \ldots \omega_t), (\theta_1, \ldots, \theta_t), (p_1, \ldots, p_{t-1})$, the measure on histories h^t is defined by the product measure $q^t \otimes 1_{\{\mu^t\}} \otimes 1_{\{\psi^{t-1}\}}$ where $1_{\{.\}}$ denotes the dirac measures on Θ^t and P^t respectively.

[10]The theoretical properties of wealth-weighted rules are typically difficult to characterize. In a purely redistributive environment, Jordan (2006) shows that outcomes of the Wealth-as-Power rule correspond to the core of a cooperative game in which a blocking coalition's feasible allocations are determined endogenously by the status quo allocation.

Case A. Fixed Wealth Distribution. Suppose first that the wealth distribution $y_t = (y_{1t}, \ldots, y_{nt})$ is fixed for all time, so that $y_t = y$ for all t. Order individual wealth holdings from poorest to richest: $0 \leq y_1 \leq y_2 \leq \cdots \leq y_n$. Denote aggregate wealth by $Y = \sum_j y_j$. In this case, it is straightforward to show that the most preferred policy of a citizen is invariant to time and states: $p_i^* = \min\{1, Ay_i^{-1/(1-\gamma)}\}$ where $A = (\frac{\gamma\delta}{1-\delta} Y^\gamma)^{1/(1-\gamma)}$ is a constant. Evidently, when preferred tax rates are interior, wealthier citizens prefer lower taxes.

Politically feasible strategies in each rule are as follows. Given θ^M, let m_t denote the identity of the Majority-pivotal voter at date t. Voter m_t corresponds to the citizen with the median wealth endowment, i.e., $m_t = m = (n+1)/2$. The identity of this individual never changes over time. Consequently, the resulting policy is $\psi(\omega, \theta^M) = \min\{1, Ay_m^{-1/(1-\gamma)}\}$.

Under the Wealth-as-Power rule, the pivotal decision maker is also time invariant. A policy is feasible under θ^W if it is the ideal policy of a voter, denoted by w, with the "median wealth" in an economy with y_i identical voters of type $i \in I$. More precisely, this voter may be found as follows: Citizen w satisfies $y_w = \max\{y_j : \sum_{k=j}^n y_k \geq \sum_i y_i/2\}$. The preferred policy for this individual is $p_w^* = \min\{1, Ay_w^{-1/(1-\gamma)}\}$. The resulting policy then is $\psi(\omega, \theta^W) = \min\{1, Ay_w^{-1/(1-\gamma)}\}$.

Proposition 1 *In Case A, both θ^M and θ^W are stable.*

Intuitively, each rule can be associated with a single, dynamically consistent decision maker - the pivotal citizen. The question becomes: would this individual ever delegate decision authority to a different individual whose preference over policy is different than his own? By the Principle of Unimprovability (e.g., Howard (1960)) the answer is no.

B. Differential Wealth Accumulation. Suppose that at $t = 0$ the initial wealth distribution is $y_{i0} = y_i$ where y_i is the same as before. Recall that $y_1 \leq \cdots \leq y_n$. Rather than remaining fixed, suppose that for $t \geq 1$, $y_{i\,t+1} = y_{it} + \beta_i$ with $\beta > 0$, $\sum_j \beta_j = 1$, and $\beta_1 < \cdots < \beta_n$. In this example, wealthier citizens accumulate wealth faster than poorer ones. One can check that each citizen's most preferred policy is again of the form: $p_{it}^* = \min\{1, A_t y_{it}^{-1/(1-\gamma)}\}$ where $A_t = (\frac{\gamma\delta}{1-\delta} Y_t^\gamma)^{1/(1-\gamma)}$ which now varies exogenously over time.

Under the differential accumulation rates, however, the two rules generally operate quite differently. Under the Majority Rule, the ordering of the wealth endowments never changes. Consequently, as before, the identity of the Majority-pivotal voter m never changes. This voter's wealth at date t is given by y_{mt}. Hence, while the policy itself may change over time according to $\psi(\omega_t, \theta^M) = \min\{1, A_t y_{mt}^{-1/(1-\gamma)}\}$, the Majority-pivotal citizen has no incentive to delegate decision authority to another. Therefore, the Majority rule remains stable.

By contrast, under the Wealth-as-Power rule, the identity of the pivotal voter may change over time. This gives a potential rationale for institutional reform: a current decision maker whose wealth is close to y_{mt} may prefer the Majority-pivotal voter as a "lesser of two evils." To illustrate this starkly, suppose that the parameters of the model are such that each citizen's preferred tax rate is always interior (strictly between zero and one). Suppose also that $\beta_n > 1/2$. This inequality implies

$$\frac{y_{nt}}{Y_t} \;=\; \frac{y_n + t\beta_n}{Y + t\sum_j \beta_j} \;\rightarrow\; \beta_n \;>\; 1/2$$

as $t \rightarrow \infty$. In other words, the richest citizen eventually accumulates over half the aggregate wealth. Letting $\{w_t\}$ denote the sequence of pivotal decision makers under θ^W, the policy strategy is given by $\psi(\omega_t, \theta^W) = A_t y_{w_t t}^{-1/(1-\gamma)}$. It is clear that at some date t^*,

$$w_0 \leq w_1 \leq w_2 \leq \cdots \leq w_{t^*} = n$$

Now consider a sufficiently egalitarian initial wealth distribution. Specifically, suppose $\sum_{i=1}^{(n+1)/2} y_i > Y/2$. This implies that $w_0 = m$. That is, the initial pivotal decision maker under θ^W is precisely the Majority-pivotal (median) voter.

In this case, the institutional choice is made easy. Starting from θ^W, the Citizen $w_0 = m$ will choose θ^M. Clearly, by remaining in θ^W, decision making authority is eventually transferred from the median citizen to the richest one. By choosing θ^M, however, this median citizen retains power. In the more general case where $w_0 \neq m$, this Citizen w_0 may choose θ^M if his preferences more closely resemble m's over time than n's. To summarize:

Proposition 2 *In Case B, suppose an initial state $\omega_0 = (y, G_0)$ such that $\sum_{i=1}^{(n+1)/2} y_i > Y/2$. Then θ^M is stable, while θ^W admits reform. In particular, there exists some date t such that $\mu(h^t(\theta^W)) = \theta^M$.*

Since the preference of type m is strict, the Majority rule is preferred in all states in a neighborhood of ω_τ. Hence, θ^W admits a reform toward θ^M.

The instability of θ^W owes to the fact that it varies in the economic state ω_t. This leads to change in the "identity" of the policy maker. If the shift is substantial enough, the current decision maker under θ^W may select an alternative political rule for self protection. Because the Majority Rule retains the same pivotal decision maker over time, no such self-protection is necessary.[11]

3.2 A Baseline Result

The intuition of the parametric model demonstrates that a rule's stability is closely aligned with its tendency to "preserve a public decision maker's identity" over time. Some parts of this statement need to be clarified. First, the idea of a rule's "identity" must be defined. We associate a political rule with a social objective function or *aggregator*, whose maximal elements

[11] A related result is found in Messner and Polborn (2004) who use dynamic inconsistencies in the preferences of the individual citizens. See Literature Review (Section 5) for more details.

are the outcomes of the rule. Formally, a class of rules (C, Θ) is *rationalized by an aggregator* $F : \mathbb{R}^n_+ \times S \to \mathbb{R}_+$ if for any politically feasible strategy (ψ, μ),

$$(\psi(s_t), \mu(s_t)) \in \arg\max_{p_t, \theta_{t+1}} F\Big((1-\delta)u(\omega_t, p_t\ ) + \delta E_q[V(h^{t+1};\ \psi, \mu) \mid h^t, p_t, \theta_{t+1}],\ \ s_t\Big) \tag{5}$$

An aggregator is a social welfare function that identifies a rule with a sequentially rational "player" whose payoff over public decisions each period is maximized by politically feasible strategies. It should be emphasized that this definition is unrestrictive: any class (C, Θ) is rationalized by a constant function.[12]

Having defined an aggregator F that rationalizes (C, Θ), we also need to be clear about what "preserving the aggregator's identity" really means.

Definition 1 Let (C, Θ) be a class of political rules. Let F be an aggregator that rationalizes (C, Θ), and let (ψ, μ) be any public strategy. A rule θ_t is then said to be *forwardly dynamically consistent (FDC)* if for all t, and every $h^t(\theta_t)$,

$$\begin{aligned} &\arg\max_{\theta_{t+1}} F\Big((1-\delta)u(\omega_t, \psi(h^t(\theta_t))\ ) + \delta E_q\Big[V(h^{t+1};\ \psi, \mu) \mid h^t(\theta_t), \psi(h^t(\theta_t)), \theta_{t+1}\Big],\ \omega_t, \theta_t\Big) \\ \subseteq\ &\arg\max_{\theta_{t+1}} E_q\Big[F\Big(V(h^{t+1};\ \psi, \mu),\ \omega_{t+1}, \theta_t\Big) \mid h^t(\theta_t), \psi(h^t(\theta_t)), \theta_{t+1}\Big] \end{aligned} \tag{6}$$

A rule is *backwardly dynamically consistent (BDC)* if (6) holds under the reverse inclusion, "$\supseteq$." A rule is *dynamically consistent* if it is both FDC and BDC.

A rule θ_t is therefore FDC if any choice of rule θ_{t+1} that maximizes $F(\cdot, \omega_t, \theta_t)$ also maximizes the expected continuation $E[F(\cdot, \omega_{t+1}, \theta_t)]$. A rule is BDC if the hypothetical maximizers of $E[F(\cdot, \omega_{t+1}, \theta_t)]$ also maximize $F(\cdot, \omega_t, \theta_t)$. Significantly, this definition does not say anything about what the solution to either maximization problem should be.

Generally speaking, both forward and backward consistency capture the idea that, in the absence of new information, such an individual should not "change his mind" about about a decision simply because the date at which the decision is made has changed. Alternatively, consistency can be formulated in terms of the value of commitment — a dynamically consistent decision maker is one for whom there is no value of commitment.

Clearly, this is not a new idea. Individual consistency dates back at least to Howard (1960). Here, however, consistency is property of a rule rather than an individual. To make sense of

[12] One could therefore strengthen the definition in (5) so that (C, Θ) is *fully rationalized* by F means the politically feasible strategies *coincide* with (rather than contained in) the set of maximizers of F. This eliminates the degenerate cases, but it entails some loss of generality. Not every class of rules is fully rationalized by an aggregator. In any case, since the results do not depend on which definition is used, I opt for the weaker definition.

this, we interpret a rule θ_t as a "player" who delegates authority to some possibly different "player" θ_{t+1} for next period. Hence, FDC states that, whatever political rule θ_{t+1} happens to be chosen at date t, the aggregator F will not "change its mind" once the stage payoffs from period t have been realized. Similarly, BDC means if the aggregator were hypothetically allowed to choose θ_{t+1} at the beginning of date $t+1$ (before the new state is realized), this aggregator would not "change its mind" if instead it could commit to a decision at date t. While neither FDC and BDC implies the other, in the important case where the maximizing sets in (6) are singletons (or, alternatively, both sets of maximizers are empty), then BDC and FDC coincide. We emphasize that while rule based consistency appears similar to individual consistency, it ultimately comes from characteristics of political aggregation.

Before proceeding with the Theorem below, we have one final caveat: in order to exclude answers that depend only on "ties," define a *conservative* feasible strategy to be one in which $\mu(h^t(\theta)) = \theta$ whenever θ maximizes the aggregator F. In other words, θ avoids reform unless there is a strict incentive to change. "Ties" favor the status quo. Conservative strategies arise naturally in any environment in which there is a small cost to changing the status quo.

Theorem 1 *Let (C, Θ) be a class of political rules, and let F be an aggregator that rationalizes (C, Θ). For any conservative politically feasible strategy (ψ, μ), every dynamically consistent political rule θ_t is stable, and every stable rule θ_t is forwardly dynamically consistent.*

Heuristically, dynamic consistency $\Rightarrow$ stability $\Rightarrow$ FDC. Here, FDC is clearly the key concept. The proof in the Appendix is not deep. It extends the intuition of the Parametric model in a straightforward way. In Case A of that model, both θ^M and θ^W are dynamically consistent due to the constant wealth distribution. However, in Case B the instability of the Wealth-is-Power Rule arises from a natural violation of FDC. Namely, θ^W varies in the economic state ω_t. This leads to change in the "identity" of the public decision maker. Reform therefore occurs as a *de jure* response to a change in *de facto* power. Indeed, any political rule which depends on cardinal properties of a distribution of tastes or income is susceptible to nonlinear changes in this distribution.

Consider the wealth-weighted voting rules such as θ^W. Though they have been widely used historically, they are often short-lived. An example is 19th century Prussia. In 1849, voting rights were extended to most citizens in Prussia, but not in an even-handed way. Rights were accorded proportionately to the percentage of taxes paid. The electorate was divided into thirds, each third given equal weight in the voting. The wealthiest individuals who accounted for the first third of taxes paid accounted for 3.5% of the population. The next wealthiest group that accounted for the middle third accounted for 10-12% of the population. The remainder accounted for the remaining third. Following a growth spurt in the late 19th century, the wealth-weights were eliminated and a majority rule was adopted (see Finer (1997)).

Some final remarks conclude this Section. First, the model does not establish existence of a politically feasible strategy. This issue is omitted in this paper, however, it is the subject of a companion paper (Lagunoff 2006b).

Second, the Parametric Model illustrates only one source of inconsistency. There are others. A rule could, for instance, fail to be dynamic consistent if a particular citizen's preference exhibits an inconsistency.[13] It must be emphasized, however, that rule-based inconsistencies are not dependent on individual inconsistencies.[14] The particular source of inconsistency in the Wealth-as-Power rule is a natural one arising from the rule's reliance on a current economic status quo and from its over-sensitivity to subsequent changes in that status quo.

Third, in standard macro models featuring a social planner, the types of tradeoffs illustrated in the Parametric Model do not arise. An even stronger form of dynamic consistency is typically presumed. Namely:

$$\begin{aligned} & F\left((1-\delta)u(\omega_t,p_t) \;+\; \delta E_q\left[V(h^{t+1};\ \psi,\mu)\mid \omega_t,p_t,\theta_{t+1}\right],\ s_t\right) \\ =\;\; & (1-\delta)F\left(u(\omega_t,p_t),\ \theta_t\right) \;+\; \delta\ E_q\left[F\left(V(h^{t+1};\ \psi,\mu),\ \theta_t\right)\ \mid \omega_t,p_t,\theta_{t+1}\right] \end{aligned} \tag{7}$$

The expression (7) corresponds to the standard social planner's problem according to which F is linearly separable and does not directly vary in the economic states ω_t. The Parametric Model illustrates how simple and common political rules violate the standard assumptions, and, consequently, will not last if the economic state changes in a natural way.

Finally, whether or not a particular rule is dynamically consistent can depend on Θ. For instance, if $\Theta = \{\theta^M\}$ in the Parametric Model, then F is dynamically consistent, but expanding to $\Theta = \{\theta^M, \theta^W\}$ destroys consistency, as illustrated in the Model.[15] Along the same lines, dynamic consistency can also depend on the class, though not necessarily the precise functional form, of strategies under consideration. For example, a rule that is *in*consistent under the Markov strategies may well be consistent in a class of trigger strategies that makes full use of the history of the game. The latter can create consistent choices by means of self enforcing punishments that sustain the desired outcomes.[16] The link between consistency and the strategy will become more apparent when the polity must confront private actions of the citizens.

4 Private Sector Decisions

In the benchmark model, there are no private decisions of individuals; all decisions are made through the polity whose form depends on the political rule. One reason for considering private

[13]See, for instance, Krusell, Kuruscu, and Smith (2002) or Messner and Polborn (2004).

[14]See Lagunoff (2006a) or Amador (2006) for models of dynamic inconsistencies arising from (exogenous) electoral or dynastic changes.

[15]I thank a referee for pointing this out.

[16]See, for example, Dutta (1995) for a Folk Theorem for stochastic games.

or external decisions is that they are thought to be a driving force for institutional change in the literature. Section 5 explores this literature in more detail. Whether or not this view is correct, this Section demonstrates that private decisions such as extra-legal protests, usurpations, labor and investments decisions, and so on, can be incorporated into an "extended political rule" to which main result still applies.

Let e_{it} denote i's private decision at date t, chosen from a (compact) feasible set E. A profile of private decisions is $e_t = (e_{1t}, \ldots, e_{nt}\ )$. These decisions may capture any number of activities, including labor effort, savings, or investment activities. They may also include "non-economic" activities such as religious worship or one's participation in a social revolt. The distinction between e_{it} and p_t is that while the latter is collectively determined, the former is necessarily chosen individually. Yet, in the larger sense described below, the individual decisions just reflect constraints on the overall social choice correspondence.

In keeping with the standard definition of a stochastic game, both private decisions e_t and the public decision p_t are made simultaneously. Clearly, there are sequential move alternatives, but none are clearly more compelling. In real time, private and public decisions are on-going. It seems natural then to define the dates as those intervals of time in which no agent is able to publicly pre-empt another.

Let $u_i(\omega_t, e_t, p_t)$ and $q(B|\ \omega_t,\ e_t,\ p_t)$ denote the citizen stage payoff and transition probability, respectively. Both now depend on the profile of private actions as well as the public ones. All these components are summarized by a dynamic political game G given by:

$$G \equiv \langle\ \overbrace{(u_i)_{i\in I}, q, E, P, \Omega}^{\text{economic primitives}}\ ,\quad \overbrace{C, \Theta}^{\text{political rules}}\quad ,\quad \overbrace{s_0}^{\text{initial state}}\quad \rangle.$$

4.1 Equilibrium and Extended Rules

A history at date t now includes the private action profiles, $e_\tau,\ \tau < t$. A *private strategy* for individual i is a history-contingent action, i.e., a function $\sigma_i : H \to E_i$ that prescribes action $e_{it} = \sigma_i(h^t)$ after history h^t. Let $\sigma = (\sigma_1, \ldots, \sigma_n)$. A strategy profile is a list (σ, ψ, μ), which includes both private and public strategies. A citizen's dynamic payoff may now be expressed as $V_i(h^t;\ \sigma, \psi, \mu)$. As before, $V(h^t;\ \sigma, \psi, \mu) = (V_i(h^t;\ \sigma, \psi, \mu)\ )_{i\in I}$.

Definition 2 An *Equilibrium* of a dynamic political game G is a profile (σ, ψ, μ) of strategies such that for all histories $h^t \in H$,

(a) *Private rationality*: For each citizen i, and each private strategy $\hat{\sigma}_i$,

$$V_i(h^t;\ \sigma, \psi, \mu)\ \geq\ V_i(h^t;\ \hat{\sigma}_i, \sigma_{-i}, \psi, \mu\ ) \tag{8}$$

(b) *Political feasibility*: Public strategies are politically feasible given σ:

$$(\psi, \mu\ ) \ \in \ C\Big(V(h^t;\ \sigma, \cdot),\ s_t\ \Big) \quad \forall h^t \tag{9}$$

A *Markov equilibrium* is an equilibrium in Markov strategies. As before, equilibria, Markov or otherwise, satisfy a "subgame perfection" requirement.

Because the political rule (C, Θ) does not generally coincide with the interests of any private decision maker, it will not be the case that the public decision can be coordinated with any citizen's private one. Rather than carving out an exception, Definition 2 does not allow for coordinated deviations between private and public decision makers even if they are one of the same. While this restriction enlarges the equilibrium set, there is an obvious way to circumvent this issue by an appropriate reclassification of private decisions as public ones (along the lines of the "Libertarian" rule) whenever the public decision maker is one of the citizens. Indeed, even when this is not the case, the map C can be extended to include private decisions. Formally, given any class of rules (C, Θ), an *extended* social choice map C^* may be defined by

$$(\sigma, \psi, \mu) \ \in \ C^*(V(h^t, \cdot),\ s_t) \quad \Leftrightarrow \quad \begin{cases} (\psi, \mu) \in C(V(h^t;\ \sigma, \cdot),\ s_t) \\ \\ \sigma_i \in \arg\max_{\hat{\sigma}_i} V_i(h^t;\ \hat{\sigma}_i, \sigma_{-i}, \psi, \mu) \ \ \forall i \end{cases}$$

We refer to a political rule θ^* associated with the extended class (C^*, Θ) as an *extended political rule*. Here, extended political rules incorporate private as well as public decisions.[17] The aggregator F^* is constructed so that it gives the same outcome as a "centralized" political system that would have occurred under in equilibrium under the "decentralized" political system in which public and private decisions are kept separate. F^* is therefore a "player" whose payoff is maximized by an equilibrium (σ, ψ, μ). In this sense, we extend the rules-as-players approach developed in the previous Sections extends in a straightforward way to the extended class (C^*, Θ).

4.2 A Theorem

For the remainder of the Section, fix a class (C, Θ) of political rules, and let (C^*, Θ) be the extended class. Let F^* be an aggregator which rationalizes (C^*, Θ).

Theorem 2 *Let (σ, ψ, μ) be any conservative equilibrium. Then any dynamically consistent extended political rule $\theta^* \in \Theta$ is stable, and any stable extended rule θ^* is forwardly dynamically consistent.*

[17] Although the extended rule θ^* comes from the same index set Θ as before.

The proof has two steps. First, an appropriate relabeling of variables is required in order to apply the proof of Theorem 1. This involves taking private sector actions and appropriating them to the government or public sector. Private sector or individually chosen actions are therefore part of a general social choice problem in which the social choice rule delegates decision authority over certain variables to the individual citizens. The second step repeats the steps of the Theorem 1 proof. We omit the details.

Despite the straightforward proof, the consequences are different from that of Theorem 1 because the tension is between different agents rather than between different incarnations of a political institution at different points in time. These consequences are illustrated in a parametric example below. The parametric structure is the same as before except that private decisions are included.

4.3 Adding a Private Sector to the Parametric Model

We revisit the parametric example, adding a private sector to the production of a public good. As before, the economic state is $\omega_t = (G_t, y)$ with G_t a durable public good and y the wealth distribution assumed here to be exogenous. Let $0 = y_1 < \cdots < y_n$. Given $\epsilon > 0$ small, assume n is sufficiently large and the wealth distribution y "evenly distributed" in the sense that $y_1 < \epsilon$ and $y_j - y_{j-1} < \epsilon$. Now assume that each citizen can contribute labor effort e_{it} at time t toward the production of the public good. The transition is given by $G_{t+1} = G_t + (p_t \sum_j y_j)^\gamma \sum_j e_{jt}$ with $0 < \gamma < 1$. Increased public literacy, for example, requires both public capital as given by $(p_t \sum_j y_j)^\gamma$ and the sum of individual effort, $\sum_j e_{jt}$. Payoffs each period are defined by $u_i = y_i(1 - p_t) + G_t - e_{it}^2$, now reflecting the cost or disutility of effort.[18]

Let (C, Θ) be a class of *pivotal rules* defined as those in which one of the citizens, say, citizen i with wealth y_{it} is the dictator/pivotal voter. In that case, $\theta_t = y_{i_t}$. Given a state summarized by $s_t = (G_t, y, \theta_t)$, a strategy (σ, ψ, μ) defines a law of motion over future states, e.g.,

$$s_{t+1} = (G_t + (\psi(s_t) \sum_j y_j)^\gamma \sum_j \sigma_{jt}(s_t),\ y,\ \mu(s_t)\ ).$$

To find a closed form solution for an equilibrium, we restrict attention in the example to Markov equilibria — equilibria in which strategies vary only with the state s_t. A citizen's payoff may then be expressed recursively by

$$V_i(s_t; \sigma, \psi, \mu) = (1 - \delta)[y_i(1 - \psi(s_t)) + G_t - (\sigma_i(s_t))^2] + \delta V_i(s_{t+1}; \sigma, \psi, \mu)$$

Since each political rule is dictatorial, the class of rules is alternatively rationalized by a simple aggregator corresponding to the dictator's payoff, .i.e.,

$$F(V(s_t;\ \sigma, \psi, \mu), s_t)\ =\ (1-\delta)[\theta_t(1-\psi(s_t)) + G_t - (\sigma_{i_t}(s_t))^2] + \delta\ F(V(s_{t+1};\ \sigma, \psi, \mu), s_{t+1}) \quad (10)$$

[18] A similar example with ideologically heterogeneous agents is developed in Jack and Lagunoff (2006b).

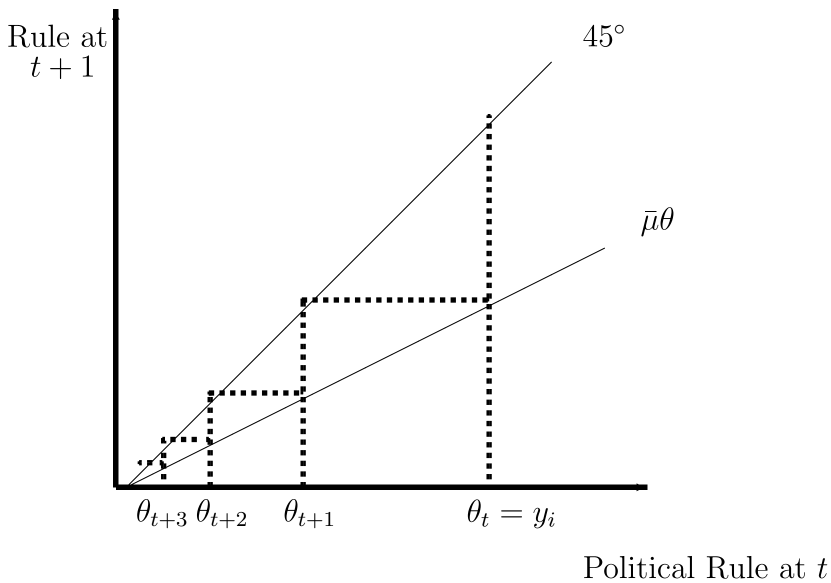

Figure 1: A Linear Rule in Wealth Endowment

Using this aggregator, the Proposition below characterizes an approximation to a Markov equilibrium.

Proposition 3 *A Markov equilibrium (σ, ψ, μ) exists, and it is given by*

$$
\begin{aligned}
\psi(\theta_t) &= \left(\frac{2(1-\delta)^2}{n\gamma\delta^2(\sum_j y_j)^{2\gamma}}\right)^{1/(2\gamma-1)} \theta_t^{1/(2\gamma-1)} \equiv B\ \theta_t^{1/(2\gamma-1)}, \\
\sigma_j(\theta_t) &= \frac{\delta(\sum_j y_j)^\gamma}{2(1-\delta)} B^\gamma\ \theta_t^{\gamma/(2\gamma-1)}, \quad \textit{and}
\end{aligned}
\tag{11}
$$

and $\mu(\theta_t)$ is the wealth level in $\{y_i\}_{i\in I}$ satisfying $||\mu(\theta_t) - \bar{\mu}\theta_t|| < \epsilon$ where $0 < \bar{\mu} < 1$ a constant defined by the implicit solution to

$$\bar{\mu} = \frac{n}{2n-1} + \frac{\delta(n-1)}{2n-1}\bar{\mu}^{\frac{2\gamma}{2\gamma-1}} \tag{12}$$

Figure 1 illustrates the simple dynamics of the equilibrium path of rules. Every type except y_1 (the lowest) admits a reform downward. The unique stable political rule therefore is $\theta = y_1$. The intuition is as follows. The degree of complementarity between the tax rate and effort is determined by γ. When $\gamma > 1/2$, taxes (i.e., spending on the public good) and effort are highly complementary and so effort is inefficiently high in equilibrium. On the other hand, when $\gamma < 1/2$, the degree of complementarity is small, and so the free rider problem in private effort predominates: effort is too low. In either case, the current dictator has an incentive to delegate future authority to an individual with lower wealth. When $\gamma < 1/2$, effort that is too low in equilibrium can be increased by transferring decision authority to a dictator from the lower wealth strata. The transfer of power represents a commitment to a higher tax rate. Similarly, when $\gamma > 1/2$, excessively high effort can be reduced by the same such transfer, in this case representing a commitment to a lower tax rate. In either case, the rich type cedes authority to the poor type to gain more favorable private sector effort from the rest of the citizenry.

This private sector rationale *seems* quite different from the internal problems that created instability in the wealth-as-power rule in the previous example. Theorem 2, however, suggests that it is not. Consider what the extended aggregator would look like in this case. Let θ_t^* denote an extended rule, and let $s_t^* = (G_t, y, \theta_t^*)$. By reverse-engineering the equilibrium in Proposition 3, the extended aggregator is derived as[19]

$$F^*(V(s_t^*;\ \sigma,\psi,\mu), s_t^*) \ = \ (1-\delta)[\frac{\theta_t^*}{n}(1-\psi(s_t^*)) + G_t - \sum_j (\sigma_j(s_t^*))^2] + \delta\ F(V(s_{t+1};\ \sigma,\psi,\mu), s_{t+1}) \tag{13}$$

Compare this F^* to F defined in (10). There are two main differences. First, the current stage payoff in F^* eliminates the decentralization of, hence the free rider problem in, labor decisions. All citizens' labor decisions are included, not just those of the dictator. Second, in order to "compensate" for the lack of a free rider problem, the current dictator's wealth is now deflated by $\frac{1}{n}$ (by assuming a type θ^*/n rather than θ^*). Intuitively, a wealthy dictator would like to commit to act as if he were a poorer agent.

The key observation is that extended aggregator is adjusted so as to yield the same outcome that would have been obtained in the decentralized model with private decisions. It does so by reformulating the decentralized problem as a centralized one with a dynamically inconsistent aggregator. Because the aggregator F^* has the same continuation as F but a different stage payoff, it exhibits a dynamic inconsistency between objectives at date t and those of $t+1$. This inconsistency is not unlike that of hyperbolic models. The conflict is induced by the decision maker's lack of direct control over private sector activities.

Consequently, Theorem 2 may be applied as follows to show that in a Markov equilibrium, the extended rule $\theta^* = y_1$ is dynamically consistent (hence is stable) while every other extended political rule fails FDC (hence admits a reform). To see this, consider the extended class as given by (13), and substitute the Markov equilibrium in the Proposition in the definition of F^*. Then dynamic consistency would imply that the solution set

$$\arg\max_{\theta_{t+1}}\ (1-\delta)\left[\frac{\theta_t^*}{n}(1-\psi(\theta_t^*)) + G_t - \sum_j (\sigma_j(\theta_t^*))^2\right]\ +\ \delta F(V(\omega_{t+1},\theta_{t+1};\ \sigma,\psi,\mu) \tag{14}$$

[19]For a derivation, see the Appendix Proof to Proposition 3.

is equivalent to $\arg\max_{\theta_{t+1}} F^*(V(\omega_{t+1}, \theta_{t+1};\ \sigma, \psi, \mu)$. In fact, this equivalence does *not* hold for all extended rules except at the lower bound, $\theta^* = y_1$.

5 Summary and Review of Literature

The idea of identifying a political institution with a time consistent social objective function is not new. Krusell, Quadrini, and Ríos-Rull (1997), Hassler, et. al. (2005), and many others have used it to model endogenous policy.[20] The present paper builds on that idea by examining the choice of institution itself as a part of the aggregator's objective. The model blends ideas from both social choice and stochastic games to produce a law of motion for political rules in which current rules select future ones. A stable rule is one that selects itself for use in the future. Stable rules can be understood in terms of a rule based dynamic consistency concept that identifies a rule with an aggregator.

This notion of self-selection bears some relation to "self-selected rules" in the static social choice models of Koray (2000) and Barbera and Jackson (2000), and also to the infinite regress model of endogenous choice of rules by Lagunoff (1992). These all posit social orderings on the rules themselves based on the outcomes that these rules prescribe. Rules that "select themselves" do so on basis of selecting the same outcome as the original rule. The present model has key two differences. First, institutional choice is explicitly dynamic. Second, the present model trades off economic fundamentals in the public and private sectors.

Apparently, the literature on explicitly dynamic models of endogenous political institutions is sparse. There is a larger literature discussing the historical basis for modeling institutional change - see, for example, North (1981), Ostrom (1990), Grief and Laitin (2004), and the extensive political science literature discussed in Acemoglu and Robinson (2005). There is also a larger literature on static models of endogenous political rules. Examples include Conley and Temimi (2001), Aghion, Alesina, and Trebbi (2002), and Lizzeri and Persico (2004).

Among the dynamic models, a number of them concern the dynamic evolution of institutions under various exogenous rationales for reform. Roberts (1998, 1999) is an early example of this. He posits a dynamic delegation model as a way to understand progressive changes in a society's demographic composition. A pivotal voter in period t chooses a pivotal voter at date $t+1$. In Robert's model the attributes of a future voter appear directly in the preferences of the current voter. A single crossing condition on the preferences guarantees that process of change is monotone or *progressive* from lower medians to higher ones. Because rules are direct arguments of utility functions, the model is better suited to explain *how*, rather than *why*, political institutions change.

Similarly, Justman and Gradstein (1999) examine endogenous voting rights under exogenous

[20] See Persson and Tabellini (2001) for other references.

costs of disenfranchisement. Barbera, Maschler, and Shalev (2001) examine club formation games in which players have exogenous preferences over the size or composition of the group. As in Robert's work, this "first wave" is ideally designed to study the process of, rather than the underlying rationale for, change.

The present paper bears a closer relation to a "second wave" of studies that attempt to put micro-foundations on the dynamics of institutional change. In this literature, institutions are instrumental choices that arise from fundamental preferences over consumption and/or technological progress in economic tangibles. The additional step of obtaining robust and convincing micro-foundations has proved difficult. Perhaps the most influential of these second wave contributions are a series of papers by Acemoglu and Robinson (2000, 2001, 2005). They propose an instrumental theory to explain the process of democratization such as that which occurred in 19th century Western Europe. The 2000 paper posits a dynamic model in which an elite must choose each period whether or not to expand the voting franchise to the rest of the citizenry. The elite possesses the authority to tax but cannot commit to limit future taxes on the citizens. Because tax concessions alone cannot "buy off" the threat of an uprising, a more permanent commitment — namely an expansion of the voting franchise is required. The voting franchise is therefore expanded to head off social unrest. The timing of the reform depends on, among other things, the likely success of an uprising in the absence of institutional change. Related ideas are found in Gradstein (2007) who models an elite's choice of current property protection as a commitment to future property rights, and in Cervellati, et. al. (2006) who model institutional choice in a game between competing social groups. Predatory conduct determines the nature of the game — coordination or Prisoner's dilemma — which, in turn, determines how successful the institutional move toward democracy will ultimately be. Jack and Lagunoff (2006a,b) go a step further by incorporating gradualism (as found, for instance, in Robert's work) in an instrumental model of franchise expansion. Monotone rules of succession are derived when individuals differ by income, ideology, or both.

Though the following are not models of endogenous institutions per se, Powell (2004), Egorov and Sonin (2005), and Gomes and Jehiel (2005) all construct interesting dynamic games in this spirit. Powell constructs a dynamic game in which a temporarily weak government may lack credibility to induce another government to restrain its inefficient use of power such as launching a coup or attacking. Egorov and Sonin construct a game in which a ruler's power may be usurped by a rival. The ruler can choose to kill a potential rival who constitutes a likely threat. Finally, Gomes and Jehiel (2005) construct a institutional dynamic in which coalitions form and unform over time, creating persistent inefficiencies along the way.

A somewhat different type of "instrumental institution" is modeled by Lagunoff (2001) who formulates a dynamic game of endogenously chosen civil liberties. There, tolerant civil liberties are chosen by a majority faction today in order to prevent a legal precedent for intolerance that may be used against it by hostile majorities in the future. This self-reinforcing idea is also found in Grief and Laitin (2004) in which institutions are self enforcing outcomes in a repeated game.

The common thread in all these papers is that the impetus for change comes from the inability of a current polity to commit in advance to a future rule and thereby internalize the social externalities created by *private* actions. A notable exception to this is the model of Messner and Polborn (2004). In their model, institutional change is not driven by private actions, but rather by dynamic inconsistencies in citizens' preferences. They posit an OLG model in which current voting rules determine future ones. It is assumed that a citizen's policy preference changes as he ages. However, the age distribution is assumed constant, and so a pivotal voter today will be too old to remain pivotal tomorrow. For this reason, current rules tend to select more conservative (i.e., supermajority) rules in the future. As in the benchmark model here, change is driven by a conflict between the present and future pivotal voters.

The present study is a step toward unifying the literature in the following sense. It re-interprets existing results as instances of a general phenomenon which can be understood without many stylized assumptions. Namely, that all reforms arise from some form of dynamic inconsistency in the extended aggregation process. Hence, the idea that progressive change in the voting franchise occurs when a rule-by-oligarchic-elite is threatened by an external revolt can be interpreted as an inconsistency between the present and future extended aggregation process. While the current extended payoff internalizes the peasants' threat of uprising, the extended continuation does not. Consequently, delegation of authority occurs in order to internalize the future threat.

Yet, the results are not specific to enfranchisement or any other particular type of institutional reform. Nor is it the case that a reform requires an external threat of this type. As the Wealth-as-Power rule demonstrates, a reform can occur because common changes in the economic state can change the de facto distribution of power in a way that leads to institutional change.

It seems that the general link between an institution's stability and its dynamic consistency has not, to my knowledge, been broadly understood. However, considerable caution is warranted. As the last section (Section 4.3) suggests, the mapping between primitives and the extended rules is far from complete. Nor is it obvious that in all such cases, the re-interpretation by means of extended aggregation will be a useful one. These issues are focal points for future work.

6 Appendix

To ease notation, we adopt the usual convention of using primes, e.g., θ' to denote subsequent period's variables, θ_{t+1}, with double primes, θ'' for θ_{t+2}, and so on.

Proof of Theorem 1 Fix a politically feasible strategy (ψ, μ). Since this strategy is fixed for the remainder of the proof we simplify notation by letting h denote a history consistent with

θ, i.e., $h = h(\theta)$. Also, define $V^*(h) \equiv V(h; \psi, \mu)$.

We first show: if θ is dynamically consistent then θ is stable. Suppose, by contradiction that θ is not stable. Then there is a set of h with positive measure such that for each such h, $\mu(h) = \tilde{\theta} \neq \theta$. In that case,

$$\tilde{\theta} \in \arg\max_{\theta'} F\left((1-\delta)u(\omega, \psi(h))\ ) + \delta \int V^*(h'(\theta'))dq(\omega'|\, \omega, \psi(h)\ ),\ \omega, \theta\right),$$

and, because (ψ, μ) is conservative, θ unstable implies,

$$\begin{aligned} & F\left((1-\delta)u(\omega, \psi(h))\ ) + \delta \int V^*(h'(\tilde{\theta}))dq(\omega'|\, \omega, \psi(h)\ ),\ \omega, \theta\right) \\ > \ & F\left((1-\delta)u(\omega, \psi(h))\ ) + \delta \int V^*(h'(\theta))dq(\omega'|\, \omega, \psi(h)\ ),\ \omega, \theta\right) \end{aligned}$$

By FDC,

$$\int F\left(V^*(h'(\tilde{\theta})),\ \omega', \theta\right) dq(\omega'|\, \omega, \psi(h)\ ) \geq \int F\left(V^*(h'(\theta)),\ \omega', \theta\right) dq(\omega'|\, \omega, \psi(h)\ ) \tag{15}$$

However, by BDC the inequality in (15) must be strict. Consequently, there is a set of ω' with positive measure such that

$$F\left(V^*(h'(\tilde{\theta})),\ \omega', \theta\right) > F\left(V^*(h'(\theta)),\ \omega', \theta\right) \tag{16}$$

But by definition,

$$\begin{aligned} F(V^*(h'(\theta)), \omega', \theta) \ &= \ \max_{p', \theta''} F\left((1-\delta)u(\omega', p')\ ) + \delta \int V^*(h''(\theta''))dq(\omega''|\, \omega', p'\ ),\ \omega', \theta\right) \\ &\geq \ F\left((1-\delta)u(\omega', \psi(h'(\tilde{\theta})))\ ) + \delta \int V^*(h''(\mu(\omega', \tilde{\theta}))\ )\ dq(\omega''|\, \omega', \psi(h'(\tilde{\theta}))\ ),\ \omega', \theta\right) \\ &= \ F(V^*(h'(\tilde{\theta})), \omega', \theta) \end{aligned}$$

which contradicts the inequality in (16).

Next, we show: if θ is stable then θ is FDC. By stability, $\mu(h) = \theta$ a.e. By political feasibility of (ψ, μ) it follows that

$$\theta \in \arg\max_{\tilde{\theta}} F\left((1-\delta)u(\omega, \psi(h)\ ) + \delta \int V^*(h'(\tilde{\theta}))dq(\omega'|\, \omega, \psi(h)\ ), \omega, \theta\right)$$

Suppose by way of contradiction, that θ is not FDC. Then

$$\theta \notin \arg\max_{\tilde{\theta}} \int F\left(V^*(h'(\tilde{\theta})), \omega', \theta\right) dq(\omega'|\, \omega, \psi(h)\ ) \tag{17}$$

In that case there exists a $\hat{\theta} \neq \theta$ and a set of ω' with positive measure such that

$$F\left(V^*(h'(\hat{\theta})), \omega', \theta\right) \; > \; F\left(V^*(h'(\theta)), \omega', \theta\right) \tag{18}$$

Notice that this does not automatically imply a contradiction because in (18) F is evaluated under *next* period's state ω' whereas one must show that stability is violated in the current state ω.

By the definition of feasibility, for all $h'(\theta)$,

$$\begin{aligned} F\left(V^*(h'(\theta)),\ \omega', \theta\right) \;&=\; \max_{\tilde{p},\tilde{\theta}}\ F\left((1-\delta)u(\omega',\tilde{p})\ ) + \delta \int V^*(h''(\tilde{\theta}))dq(\omega''|\ \omega', \tilde{p}\ ),\ \omega', \theta\right) \\ \geq\ & F\left((1-\delta)u(\omega', \psi(h'(\hat{\theta}))\ ) + \delta \textstyle\int V^*(h''(\mu(\omega',\hat{\theta}))\ )\ dq(\omega''|\ \omega', \psi(h'(\hat{\theta}))\ ),\ \omega', \theta\right) \\ =\ & F\left(V^*(h'(\hat{\theta})),\ \omega', \theta\right) \end{aligned} \tag{19}$$

∎

Proof of Proposition 3.

We prove the second part of the Proposition — the explicit construction of a Markov equilibrium — first. Consider any Markov strategy (σ, ψ, μ) and any state $s_t = (G_t, y, \theta_t)$. Observe first that the state enters linearly in the stage payoff. This means that a more convenient expression of the recursive payoff function for a citizen can be derived by grouping the subsequent period's state with the current stage payoff. Define $W_i(\theta_t;\ \sigma, \psi, \mu) \equiv V_i(s_t; \sigma, \psi, \mu) - G_t$. One can show that the state variable in the right hand side of this equation cancels out. Consequently, W_i does not vary with G_t and can be expressed a function of θ_t alone. The aggregators F and F^* become

$$\begin{aligned} F(W(\theta_t;\ \sigma,\psi,\mu), \theta_t) \quad &= \quad (1-\delta)[\theta_t(1-\psi(\theta_t)) + (\psi(\theta_t)\sum_j y_j)^\gamma \sum_j \sigma_{jt}(\theta_t) - (\sigma_{i_t}(\theta_t))^2] \\ &\quad + \ \delta\ F(W(\theta_{t+1};\ \sigma,\psi,\mu), \theta_{t+1}), \quad \text{and} \\ F^*(W(\theta_t^*;\ \sigma,\psi,\mu), \theta_t^*) \quad &= \quad (1-\delta)[\frac{\theta_t^*}{n}(1-\psi(\theta_t^*)) + (\psi(\theta_t^*)\sum_j y_j)^\gamma \sum_j \sigma_{jt}(\theta_t^*) - \sum_j(\sigma_j(\theta_t^*))^2] \\ &\quad + \ \delta\ F(W(\theta_{t+1};\ \sigma,\psi,\mu), \theta_{t+1}) \end{aligned} \tag{20}$$

respectively. Notice that the first order conditions under F with respect to p, and the first order conditions of the $V_j,\ j \in I$ with respect to e_j all coincide with the joint first order conditions in p and $e_j,\ j \in I$ under F^*. In particular, Fixing $\theta = y_i$, the first order conditions in p and $e_i,\ i \in I$ imply the policy and private sector strategies of the form in (11).

Since W_i does not depend on the economic state, then with strategies in (11), the recursive payoff to citizen i is

$$W_i(\theta_t;\ \sigma, \psi, \mu) \ = \ (1-\delta)\left[y_i(1 - B\ \theta_t^{1/(2\gamma-1)}) + A\theta_t^{2\gamma/(2\gamma-1)}\right] \ + \ \delta W_i(\theta_{t+1};\ \sigma, \psi, \mu) \qquad (21)$$

where $A \equiv \frac{1}{4}(2n-1)\frac{\delta^2}{(1-\delta)^2}B^{2\gamma}(\sum_j y_j)^{2\gamma}$, a positive constant.

To find the approximate equilibrium institutional rule, μ, we guess and verify a linear approximation $\theta_{t+1} = \bar{\mu}\theta_t$ to the discrete solution. Using the equation for W in (21), the equilibrium continuation payoff is of the form

$$\begin{aligned} F^*(W(\theta^*_{t+1};\ \sigma, \psi, \mu) &= W_{i_{t+1}}(\theta_{t+1};\ \sigma, \psi, \mu) \\ &= \sum_{\tau=t+1}^{\infty}(1-\delta)\delta^{\tau-t-1}\left[y_i(1 - B\ (\bar{\mu}^\tau\theta_{t+1})^{1/(2\gamma-1)}) + A(\bar{\mu}^\tau\theta_{t+1})^{2\gamma/(2\gamma-1)}\right] \end{aligned} \qquad (22)$$

Clearly the set of θ^*_{t+1} that maximize (22) is nonempty since they come from the finite set, $\{y_1, \ldots, y_n\}$. We obtain an approximation by taking first order conditions with respect to θ_{t+1}. Substituting for B and A, we verify that $\theta' = \bar{\mu}\theta$ is an equilibrium for $\bar{\mu}$ that satisfies (12). It remains to show that a solution to (12) exists. To verify this final step, observe that as $\bar{\mu}$ varies from 0 to 1, the left side (12) is continuously increasing from 0 to 1. Meanwhile, the right side of (12) is continuously decreasing from $n/(2n-1)$ and approaches 0 asymptotically if $\gamma < 1/2$. If $\gamma > 1/2$, the right side of (12) is continuously increasing from $n/(2n-1)$ but has value less than one at $\bar{\mu} = 1$. In either case, Brouwer's Theorem and the Intermediate Value Theorem imply a solution $\bar{\mu} \in (0, 1)$ exists. Given that $y_j - y_{j-1} < \epsilon$, it follows that $||\mu(\theta) - \bar{\mu}\theta|| < \epsilon$. ∎

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