East Asia vs. Latin America: TFP and Human Capital Policies

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October, 2011

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April, 2010

Abstract

1   Introduction

In any empirical analysis of cross country economic performance it is easy to find a few episodes of fast growth, as well as many instances of economic stagnation. A major challenge for economic theory is to identify what are the driving forces behind the successes and failures. Ultimately, the objective of the theory is to come up with a recipe that a country can use to produce economic miracles as well as disasters. No amount of atheoretical empirical work can discover the engines (or the brakes) of growth. Since there are plenty of theoretical models that can, on paper, produce economic miracles,

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it is necessary to better understand the implications of each model for how economic variables respond to exogenous shocks.

In this paper we make a (very) preliminary assessment of the ability of a version of the neoclassical growth model to explain episodes of fast growth, as well as instances of economic stagnation. The model that we study features individuals who are finitely lived but care about their descendants, albeit in an imperfect way. There are two capital stocks: physical and human capital. The key difference between them is that the latter completely depreciates when an individual dies. This, in turn, implies that our setup is not a special case of the infinitely lived version. We follow Lucas' (1993) suggestion (but not his model) and allow human capital to have both a quality and a quantity dimension, and we go beyond the schooling decision and recognize that on-the-job training is an important component of aggregate human capital. The human capital accumulation technology that we use is a version of the Ben-Porath (1967) model, which has the advantage of predicting the shape of the age earnings profile. This, in turn, allows us to pin down the key parameters.

We calibrate the model to match (mostly) U.S. data, and then we use the calibrated model to explore the roles that total factor productivity (TFP) and demographic shocks have in explaining the stellar economic performance of several East Asian countries over the last 40 years, as well the as the relatively poor performance of many Latin American economies.

There is a large literature that has studied both cases. Prescott and Hayashi (2002) argued that low TFP growth explains Japan's poor performance since the early 1990s. More recently, Chen, Ïmrohoroglu and Ïmrohoroglu (2006) find that a simple neoclassical model can account for the large differences in saving rates between U.S. and Japan. Other studies, e.g.

Chang and Hornstein (2007) and Papageorgiu and Perez-Sebastian (2005), reach the opposite conclusion: they claim that models that deviate from the standard neoclassical model are needed to explain the economic performance of East Asian countries. In the case of Latin America, DeGregorio (2006) presents a good summary of the region's economic performance. Bergoing et.al (2002) used the neoclassical model to compare the performance of Chile and Mexico and concluded that the timing and nature of economic reforms in Chile account for its better performance. Kydland and Zarazaga (2002), using a similar model, concluded that economic performance in Argentina was significantly influenced by changes in TFP. Cole et. al. (2005) conclude that TFP (but not human capital) is the dominant force behind the economic performance of Latin America.

By construction, the model in this paper is driven by TFP shocks and (exogenous) demographic shocks. However, the human capital sector plays a central role in understanding economic growth. Moreover, policies that affect the allocation of resources in this sector can have a significant impact on output.

For the East Asian countries, we show that the model is consistent with their stellar economic performance in the sense that it predicts changes in the average level of schooling of the labor force in line with the evidence. Moreover, the model is also consistent with the observed increase in the investment-output ratio. We briefly explore the transitional dynamics in this case, and we argue that the dynamic responses of schooling and investment to a one time TFP shock are plausible.

Our findings for Latin America using the same basic model are not encouraging. We find that, in general, the model underpredicts schooling in many countries. Thus, the standard version of the model reproduces the "ex-

cess education puzzle" for Latin America: significant increases in schooling have not resulted in substantial increases in output. To further understand the role of resource allocation (across individuals and levels of schooling), we modify the model along two dimensions: we allow for heterogeneity in ability and we add a public education system. With this change the "excess education" puzzle is much smaller for the three countries that we study in some detail (Argentina, Brazil and Chile). We then introduce an imperfection in capital markets: individuals are restricted to a non-negative asset position. In this case the interaction of a suboptimal allocation of resources in the public sector (even when we allow for a private educational sector) and the inability of poor individuals to borrow against their future income makes the prediction of the model consistent with the data. Thus, two restrictions that distort investment in the human capital sector suffice to "solve" the excess education puzzle. We conduct several experiments that try to quantify the long run gains of reallocating educational resources, and we find that the potential benefits are large.

In this paper we do not provide a "recipe for growth." Rather, we view our results as suggesting the possibilities that the neoclassical model, augmented with a reasonable demographic structure and a realistic technology for the accumulation of human capital, has to explain growth and stagnation. We also show that frictions that result in misallocation of resources in the human capital investment sector can have a large impact on output. Overall, we find that human capital matters.

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2 Model

The model uses the same technology as in Manuelli and Seshadri (2007).
We view the economy as being populated by overlapping generations of
individuals who live for T periods. The time line is the following: After
birth, say at time t_0, an individual remains attached to his parent until he
is I years old (at time $t_0 + I$); at that point he creates his own family and
has, at age B (i.e. at time $t_0 + B$), $e^{f(t_0+B)}$ children that, at time $t_0 + B + I$,
leave the parent's home to be become independent.

The utility functional of a parent who has h units of human capital, and
initial wealth (a bequest from his parents) equal to b, at age I, in period t
is given by

$$
\begin{aligned}
V^P(h,b,t) \; = \; & \int_I^T e^{-\rho(a-I)} u[c(a,t+a-I)]da + e^{-\alpha_0+\alpha_1 f(t+B-I)} \quad (1) \\
& \int_0^I e^{-\rho(a+B-I)} u[c_k(a,t+B-I+a)]da \\
& +e^{-\alpha_0+\alpha_1 f(t+B-I)} e^{-\rho B} V^k(h_k(I),b_k,t+B),
\end{aligned}
$$

where $c(a,t)$ $(c_k(a,t))$ is consumption of a parent (child) of age a at time t.
The term $f(t)$ denotes the log of the number of children born at time t.

We assume that parents are imperfectly altruistic: The contribution
to the parent's utility of a unit of utility allocated to an a year old child
attached to him is $e^{-\alpha_0+\alpha_1 f(t+B-I)} e^{-\rho(a+B-I)}$, since at that time the parent
is $a + B$ years old. In this formulation, $e^{-\alpha_0+\alpha_1 f(t+B-I)}$ captures the degree
of altruism. If $\alpha_0 = 0$, and $\alpha_1 = 1$, the preference structure is similar
to that in the infinitively-lived agent model. Positive values of α_0, and
values of α_1 less than 1 capture the degree of imperfect altruism. The term
$V^k(h_k(I),b_k,t+B)$ stands for the utility of the child at the time he becomes
independent.

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Each parent maximizes $V^P(h, b, t)$ subject to two types of constraints: the budget constraint, and the production function for human capital. The former is given by

$$\int_I^T e^{-\int_t^{t+a-I} r(s)ds} c(a, t + a - I) da + \tag{2}$$

$$e^{f(t+B-I)} \int_0^I e^{-\int_t^{t+a+B-I} r(s)ds} c_k(a, t + B - I + a) da$$

$$+ \int_I^R e^{-\int_t^{t+a-I} r(s)ds} x(a, t + a - I) da +$$

$$e^{f(t+B-I)} \int_0^I e^{-\int_t^{t+a+B-I} r(s)ds} x_k(a, t + B - I + a) da +$$

$$e^{f(t+B-I)} e^{-\int_t^{t+B} r(s)ds} b_k + e^{f(t+B-I)} e^{-\int_t^{t+B+6-I} r(s)ds} x_E$$

$$\leq \int_I^R e^{-\int_t^{t+a-I} r(s)ds} w(t + a - I) h(a, t + a - I)(1 - n(a, t + a - I)) da$$

$$+ e^{f(t+B-I)} \int_6^I e^{-\int_t^{t+a+B-I} r(s)ds} [w(t + a + B - I)$$

$$h_k(a, t + B - I + a)(1 - n_k(a, t + B - I + a))] da + b.$$

Since we are interested in understanding transition effects, we allow the interest rate and the wage rate to vary over time.

We adopt Ben-Porath's (1967) formulation of the human capital production technology, augmented with an early childhood period. Specifically, we assume that (ignoring the temporal dependence to simplify notation)

$$\dot{h}(a) = z_h[n(a)h(a)]^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a), \qquad a \in [I, R) \tag{3}$$

$$\dot{h}_k(a) = z_h[n_k(a)h_k(a)]^{\gamma_1} x_k(a)^{\gamma_2} - \delta_h h_k(a), \qquad a \in [6, I) \tag{4}$$

$$h_k(6) = h_B x_E^{\upsilon}, \tag{5}$$

$$h(I) \quad given, \qquad 0 < \gamma_i < 1, \qquad \gamma = \gamma_1 + \gamma_2 < 1,$$

Even if there is perfect altruism, we assume that when an individual dies, his human capital dies with him. Thus, the depreciation rate is 100%

at age T.

If asset transfers are not constrained, the income maximization and utility maximization problems can be solved independently. In this case, it is optimal for an individual to maximize the present discounted value of net income. We assume that each agent retires at age $R \leq T$. The maximization problem for an agent born at time t is

$$\max_{h,n,x} \int_6^R e^{-\int_{t+6}^{t+a-6} r(s)ds} [w(t+a-6)h(a)(1-n(a)) - x(a)]da - x_E \qquad (6)$$

subject to

$$\dot{h}(a) = z_h[n(a)h(a)]^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a), \qquad a \in [6, R), \qquad (7)$$

and

$$h(6) = h_E = h_B x_E^{\upsilon} \qquad (8)$$

with h_B given. Equations (7) and (8) correspond to the standard human capital accumulation model initially developed by Ben-Porath (1967). This formulation allows for both market goods, $x(a)$, and a fraction $n(a)$ of the individual's human capital, to be inputs in the production of human capital. Investments in early childhood, which we denote by x_E (e.g. medical care, nutrition and development of learning skills), determine the level of each individual's human capital at age 6, $h(6)$, or h_E for short.[1] This formulation captures the idea that nutrition and health care are important determinants of early levels of human capital, and those inputs are, basically, market goods.[2]

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[1] It should be made clear that market goods ($x(a)$ and x_E) are produced using the same technology as the final goods production function. Hence the production function for human capital is more labor intensive than the final goods technology.

[2] It is clear that parents' time is also important. However, given exogenous fertility, it

The solution to the problem is such that $n(a) = 1$, for $a \le 6 + s(t)$. Thus, we identify $s(t)$ as *years of schooling* of the cohort born at time t. In the stationary case, i.e. $r(s) = r$ and $w(s) = w$, Manuelli and Seshadri (2008)) characterize s and $h(s + 6)$.

An important property of the solution from the point of view of the exercise in this paper is the role played by the real wage. Imagine that technological improvements (or other shocks) results in a higher level of equilibrium wages. This —given $\gamma_2 - \upsilon(1 - \gamma_1) > 0$ which is satisfied in our specification— induces individuals to stay in school longer (i.e. s increases) and to acquire more human capital per unit of schooling.

In the stationary case, if $h(s + 6)$ is the amount of human capital that an individual has at age $6 + s$ (i.e. at the end of the schooling period), ti follows that

$$\frac{dh(s+6)}{dw} = \frac{\partial h(s+6)}{\partial s}\frac{ds}{dw} + \frac{\partial h(s+6)}{\partial w}.$$

The first term on the right hand side can be interpreted as the effect of changes in the wage rate on the *quantity* of human capital (years of schooling), while the second term captures the impact on the level of human capital per year of schooling, a measure of *quality*. Direct calculations (see Manuelli and Seshadri (2008)) show that the elasticity of quality with respect to the wage rate is $\gamma_2/(1 - \gamma)$, which is fairly large in our preferred parameterization.[3] This result illustrates one of the major implications of the approach that we take in measuring human capital in this paper: differences in years of schooling are not perfect (or even good in some cases) measures of differ-

seems best to ignore this dimension. For a full discussion of the endogneous fertility case see Manuelli and Seshadri (2009a).

[3]To be precise, we find that $\gamma_2 = 0.33$, and $\gamma = 0.93$. Thus the elasticity of the quality of human capital with respect to wages is 4.71.

ences in the stock of human capital. Cross-country differences in the quality of schooling can be large, and depend on the level of development. If the human capital production technology is 'close' to constant returns, then the model will predict large cross country differences in human capital even if TFP differences are small.[4]

It is possible to show that, in the steady state, the interest rate must satisfy

$$r = \rho + [\alpha_0 + (1 - \alpha_1)f]/B,$$

which implies that decreases in fertility are associated with lower interest rates. This has three effects. First, it lowers the cost of capital inducing increases in the capital-human capital ratio which, in general, results in higher levels of output per worker. Second, it lowers the opportunity cost of staying in school. As a result, individuals choose to invest more in schooling and to allocate more resources to on the job training. This implies that the effective amount of human capital in the economy increases. Finally, negative fertility shocks have an impact on the age structure of the population. The relevant effect is that the fraction of high human capital individuals —i.e. those in the peak earning years— increases and this, in turn, contributes to an overall increase in the amount of effective labor available in the economy

The last shock that we study is a change in the (relative) price of capital. In the steady state, the condition that pins down the capital-human capital ratio requires that the cost of capital equal its marginal product. In symbols, this corresponds to

$$p_k(t)[r(t) + \delta_k] = z(t)F_k(\kappa(t), 1), \tag{9}$$

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[4]It can be shown that the elasticity of quality with respect to TFP is $\gamma_2/[(1-\theta)(1-\gamma)]$, where θ is capital share.

where $\kappa(t)$ is the physical capital - human capital ratio. Thus, a decrease in the price of capital has a direct impact on the physical capital - human capital ratio. This, in turn, increases the wage rate per unit of human capital and induces more investment in human capital. Even though during the transition the interest rate can respond to the changes in price of capital, in the steady state it is pinned down by demographic factors and, as such, does not add to the effect of p_k

2.1   Equilibrium

Given the interest rate, standard profit maximization pins down the equilibrium capital-human capital ratio. However to determine output per worker, it is necessary to compute 'average' human capital in the economy. Since we are dealing with finite lifetimes —and full depreciation of human capital— there is no aggregate version of the law of motion of human capital since the amount of human capital supplied to the market depends on an individual's age (see the expressions in the Appendix). Thus, to compute average 'effective' human capital we need to determine the age structure of the population.

Demographics   We assume that, at time t, each B year old individual has $e^{f(t)}$ children at age B. Thus, the total mass of individuals of age a at time t satisfies

$$
\begin{aligned}
N(a;t) &= e^{f(t-a)}N(B;t-a), \\
N(t',t) &= 0, \qquad t' > T.
\end{aligned}
$$

If the economy converges to a steady state (as we assume), the birth rate, $f(t)$, converges to f. In this case, the steady state measure of the populations

satisfies

$$N(a,t) = \phi(a)e^{\eta t}, \qquad (10)$$

where

$$\phi(a) = \eta \frac{e^{-\eta a}}{1 - e^{-\eta T}}, \qquad (11)$$

and $\eta = f/B$ is the (long run) growth rate of population.

Aggregation To compute total output it is necessary to estimate the aggregate amount of human capital effectively supplied to the market, and the physical capital - human capital ratio. Effective human capital, $H^e(t)$ is

$$H^e(t) = \int_{6+s}^{R} h(a,t)(1 - n(a,t))dN(a;t).$$

This formulation shows that, even if R —the retirement age— is constant, changes in the fertility rate can have an impact on the average stock of human capital.

Equilibrium Optimization on the part of firms implies that

$$p_k(r(t) + \delta_k) = z(t)F_k(\kappa(t), 1), \qquad (12)$$

where $\kappa(t)$ is the physical capital - human capital ratio. The wage rate per unit of human capital, w, is,

$$w(t) = z(t)F_h(\kappa(t), 1). \qquad (13)$$

Then, feasibility requires

$$\int_0^T [c(a,t) + x(a,t)]dN(a;t) + \dot{K}(t) \leq [z(t)F_h(\kappa(t), 1) - \delta_k]H^e(t),$$

where, given the specification of age, it is no longer necessary to distinguish between parent and children variables.

3 Calibration

We use standard functional forms. The production function is assumed to be Cobb-Douglas

$$F(k,h) = zk^\theta h^{1-\theta},$$

and the utility function is given by

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

Our calibration strategy involves choosing the parameters so that the steady state implications of the model economy presented above are *consistent with observations for the United States* (circa 2000). When we apply the model to the study of other economies the only technological parameter that we vary is z, which we identify as TFP.

Following Cooley and Prescott (1995), the depreciation rate is set at $\delta_k = .06$. We set $\sigma = 3$. Not much information is available on the fraction of job training expenditures that are not reflected in wages. There are several reasons why earnings ought not to be equated with $wh(1-n) - x$. First, some part of the training is off the job and directly paid for by the individual. Second, firms typically obtain a tax break on the expenditures incurred on training. Consequently, the government (and indirectly, the individual through higher taxes) pays for the training and this component is not reflected in wages. Third, some of the training may be firm specific, in which case the employer is likely to bear the cost of the training, since the employer benefits more than the individual does through the incidence of such training. Finally, there is probably some smoothing of wage receipts in the data and consequently, the individual's marginal productivity profile is likely to be steeper than the individual's wage profile. For all these reasons,

we set $\pi = 0.5$.[5] We also assume that the same fraction π is not measured in GDP.

Our theory implies that it is only the ratio $h_B^{1-\gamma}/(z_h^{1-\upsilon} w^{\gamma_2 - \upsilon(1-\gamma_1)})$ that matters for all the moments of interest. Consequently, we can choose z, p_k (which determine w) and h_B arbitrarily and calibrate z_h to match a desired moment. The calibrated values of z_h and h_B are common to all countries. Thus, the model does not assume any cross-country differences in an individual's 'ability to learn,' or initial endowment of human capital. We set $B = 25$, $R = \min\{64, T\}$ and a fertility rate of 2.1. We also assume that $\rho = 0.04$. This leaves us with 9 parameters, $\theta, r, \delta_h, z_h, \gamma_1, \gamma_2, \upsilon, \alpha_0$ and α_1. The moments we seek in order to pin down these parameters are:

1. Capital's share of income of 0.33. Source: NIPA

2. Capital output ratio of 2.52. Source: NIPA

3. Earnings at age R/Earnings at age 55 of 0.8. Source: SSA

4. Earnings at age 50/Earnings at age 25 of 2.17. Source: SSA

5. Years of schooling of 12.08. Source: Barro and Lee

6. Schooling expenditures as a fraction of GDP of 3.77. Source: OECD, Education at a Glance.

7. Pre-primary expenditures per pupil relative to GDP per capita of 0.14. Source: OECD, Education at a Glance.

8. Interest rate of 7%.

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[5] If we were to take the view that $\pi = 1$, our estimate of the returns to scale, $\gamma = \gamma_1 + \gamma_2$ increases to 0.96 thereby further increasing the elasticity of output with respect to TFP. In a sense, choosing $\pi = 0.5$ understates our case.

9. Lifetime Intergenerational Transfers/GDP of 4.5%. Gale and Scholz, 1994

The previous equations correspond to moments of the model when evaluated at the steady state. This, calibration requires us to solve a system of 9 equations in 9 unknowns. The resulting parameter values are

Parameter	θ	r	δ_h	z_h	γ_1	γ_2	ν	α_0	α_1
Value	0.315	0.07	0.018	0.361	0.63	0.3	0.55	0.728	0.55

4   Fast Growers: East Asia

In this section (mostly drawn from Manuelli and Seshadri (2009b)) we use the model to get a sense of the role that TFP and demographic change had in accounting for the economic performance of some of the fast growing countries in East Asia. We study the economic performance of Hong Kong, Korea, Malaysia, Singapore and Taiwan, as well as a synthetic "average" of those economies. In this first (preliminary) stage we concentrate on steady states. Specifically, we pick TFP in 1960 and 2000 so that the model's implications for output per worker for each of the countries coincide with the data. In comparing 1960 and 2000, we let the structure of the population change according to the experience of each country.

Since, we want to understand the role of TFP, we report the change in "true" productivity (labeled z) as well as an alternative measure, which we denote "Measured TFP" (and we label $\hat{z}$) that differs from the true measure by the way that human capital is computed. To be precise, define —in the

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context of this paper— the "Mincerian" stock of human capital, h^m, by

$$h^m(t) = h_0^m e^{\phi s(t)},$$

where ϕ is often associated with the return to education as estimated in a Mincer regression. For this calculation we chose $\phi = 0.10$, but the results are very similar for slightly higher and lower values. We take s to the the average years of schooling of the individuals in the work force. Finally, "Measured TFP" is the productivity measure that one would recover had one used the Mincerian measure of human capital without adjusting for on the job training and quality changes (i.e. changes in $h(6+s)$ and the impact of on-the-job training).

The results from this experiment are in Table 1

Table 1: East Asia

Country	$\Delta(Y/L)$ Data	Years of Schooling Data 1960	2000	Model 1960	2000	Δ(I/Y) Data	Model	$\Delta(z)$ Model	$\Delta(\hat{z})$ Model
Sing.	6.6	3.14	8.12	3.21	8.66	1.65	1.77	1.15	2.02
H.K.	9.09	4.74	9.47	4.36	8.87	0.89	1.74	1.20	2.62
Mal.	4.49	2.34	7.88	3.08	6.26	1.62	1.53	1.13	1.88
Taiwan	10.14	3.32	8.53	2.52	7.85	1.68	1.64	1.22	2.63
Korea	8.05	3.23	10.46	2.87	8.87	2.67	2.01	1.17	2.08

The model is relatively successful in the sense that associated with the required change in TFP, it predicts substantial increases in the level of schooling in all four countries. In this open economy experiment, changes in demographic structure affect the interest rate, and this results in a predicted

increase in the investment-output ratio that comes very close —with one exception— to the evidence. The model, however fails to reproduce Hong Kong's flat investment share of output, which is a puzzle for a variety of models (see Young (1992)).

The predictions of the model for Measured TFP suggest that productivity differences account for a good share of the increase in output per worker, typically around 40%. Our measures of TFP (z) show much smaller variation and, directly, account for about 8% of the change in output per worker.

Overall, and ignoring dynamics, it seems that the simple model that we have specified can explain fast growth as a result of changes in demographic structure and changes in productivity.

In a related paper (Manuelli and Seshadri (2009b)) we have studied more carefully the dynamics associated with this model. There, we show that if we calibrate the level of TFP (z) so that the predictions of the model for output per worker relative to the U.S. matches the *average* of the East Asian fast growing countries, then the adjustment path implied by the model is consistent with fast growth in response to a one time productivity shock.

Consider the following experiment: a once and for all TFP shock coupled with the actual changes in fertility. We pick the size of the TFP shock so that, in the long run, output per worker displays a 7.7 fold increase (which corresponds to the average increase in output per worker across the four East Asian economies that we study). We also subject this artificial economy to a demographic shocks that is such that the population growth rate decreases from 3.83% in 1960 to 1.68% in 2000, which is an average of the years surrounding the beginning and the end of the period under study.

The results of this experiment are presented in Figure 1. Output per

worker displays an S-shaped path, which implies a delayed response of the growth rate to the shocks. The reason is that it takes time to accumulate human capital: After the economy is hit with a TFP shock, agents find it optimal to increase their stock of human capital. New cohorts go to school longer than did their older counterparts. Individuals who are already working now engage in more on the job training. This implies that the stock of human capital takes time to respond, and this slows down the transition to the new steady state. The model is able to capture the rise in schooling, as well as the dramatic rise in the investment to GDP ratio.
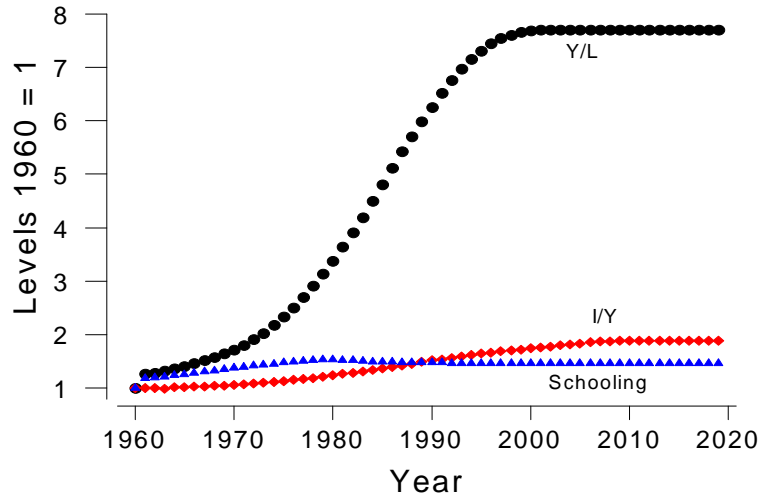

Figure 1: TFP and Fertility Shocks: Transitional Dynamics

Figure 2 reports the evolution of effective human capital per worker, the Mincerian level of human capital and measured TFP. For this experiment,

the increase in TFP induces a decrease in effective human capital. It takes
more than 10 years for h^e to get back to its pre-shock level. In the mean time,
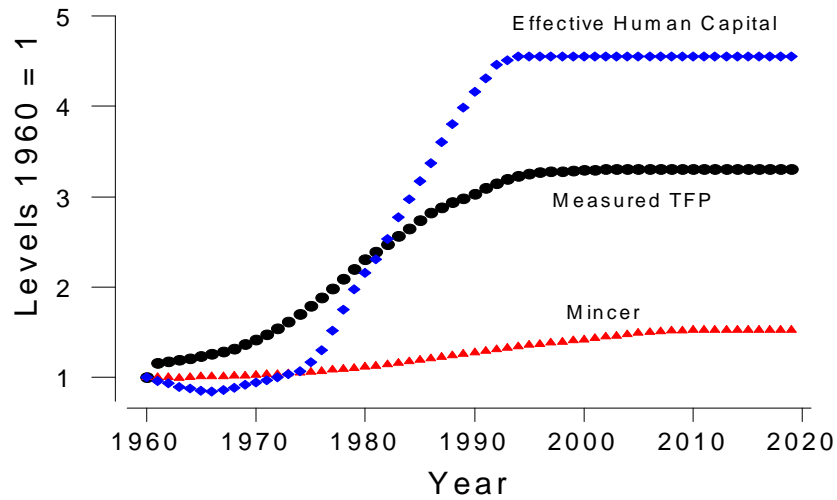the increase in output is due to the TFP shock and capital accumulation.

Figure 2: Fast Growers: Measured TFP, Mincerian and Effective Human
Capital

It is possible to determine (see Manuelli and Seshadri (2009b)) the rela-
tive importance of TFP and demographic shocks in accounting for the large
average increase in output per worker in this synthetic fast grower. For our
base case, approximately 30% of the increase in output per worker is due
to the change in population, 44% to the change in TFP, and the rest to
the interaction between those two shocks. Overall, the results indicate that
transitional dynamics cannot be ruled out as explanations for the growth

performance of the average East Asian fast grower.

5 Slow Growers: Latin America

To evaluate the implications of the model for Latin America we first study the steady state implications. Thus, we choose TFP in 1960 and 2000 so as to match output per worker. The results are in Table 2.

Table 2: Latin America

| Country | $\Delta(Y/L)$ Data | Years of Schooling | | | | X/Y | |
| | | Data | | Model | | Model | |
	1960	1960	2000	1960	2000	1960	2000
Argentina	1.37	4.99	8.49	4.32	6.52	2.1	3.6
Brazil	2.61	2.83	4.56	2.42	3.74	2.3	3.1
Chile	2.14	4.99	7.89	3.87	6.11	2.3	3.5
Colombia	1.39	2.97	5.01	3.11	3.92	2.4	2.9
Costa Rica	1.31	3.86	6.01	3.24	4.86	2.8	3.2
Ecuador	1.78	2.95	6.52	2.69	4.73	1.9	2.6
Mexico	1.84	2.41	6.73	2.43	5.12	2.0	3.5
Paraguay	1.42	3.35	5.74	3.51	4.01	2.0	2.7
Peru	1.00	3.02	7.33	2.94	3.07	2.1	2.1
Uruguay	1.46	5.03	7.25	5.27	7.02	3.0	3.4
Venezuela	0.70	2.53	5.61	2.31	1.43	2.5	2.2

The success of the private education model at replicating years of schooling (the appropriate test since we are matching output per worker) is, at best mixed. In general, the predictions for 1960 are reasonably close. However, as a general rule, the model underpredicts schooling in 2000. In the case of

economies that have been subject to large shocks (Peru and Venezuela in this group) the distance between the average years of education in the labor force and the predictions of the model is more than twice the level implied by the model.

Even in countries that have performed reasonably well (e.g. Brazil and Chile), the model underpredicts schooling levels 20% lower than those observed in 2000. In addition, the model predicts that the share of educational expenditures in output is significantly lower than the observed values. Thus, our results can be interpreted as supporting the view (e.g. Cole et. al. (2005)) that human capital is not a key determinant of economic performance in the region. It is left to TFP to account for changes in output.

In what follows we will present more detailed results for three countries: Argentina, Brazil and Chile. For this subset of countries, the model implies a slight increase in investment ratios (measured in domestic prices) which is consistent with the Argentine experience but misses both the Brazilian and the Chilean observations.

Table 3: Population Growth Rates and $\Delta(I/Y)$

Country	Growth Rate 1960	Growth Rate 2000	$\Delta(I/Y)$ Model	$\Delta(I/Y)$ Data
Argentina	1.71	1.15	1.02	1.03
Brazil	2.91	1.49	1.05	.94
Chile	2.44	1.37	1.04	.90

Source: Population growth rates from GLOBALIS

5.1 Adding Heterogeneity

In order to better ascertain the role of education and the impact of the allocation of resources within the educational sector, we studied an extended

model that allows for heterogeneity across agents. We assume that each dynasty is characterized by an ability to learn parameter, z_h. Even with complete markets, differences z_h result in differences in asset holdings at a given point in the lifetime of an individual.

To determine the quantitative effect of adding heterogeneity and the potential for misallocating resources, we need to estimate the distribution of abilities in the population. Since we view differences in z_h as independent of economic status and nationality, we use U.S. data to calibrate the distribution. To be precise, we studied a version of the model with no distortions and chose the distribution of skills so that the predictions of the model match, for the year 2000, the observed distribution of schooling.[6] We assume that this distribution remains constant over time and across countries. Once we introduce heterogeneity, it is necessary to recalibrate the model. As before, we choose parameters to match U.S. data which in addition to the moments described earlier include 9 more levels of schooling. (Essentially we compute the average years of schooling for each decile based on data from NLSY). We continue to pin down the parameters of the human capital production function using the earnings profile of the average American (using the Social Security Administration data). Consequently, the estimated parameters of the human capital production function are the same (since we match the same interest rate) as before. However, the demand for physical capital does not equal the supply at the very same preference parameters. Consequently we adjust α_0 and α_1 so that the demand for capital equals supply. The new

---

[6]As a minimal quality control check, we used these estimates to create artificial age-earnings profiles for the U.S and used those, in turn, to estimate the implications of the artificial economy for the coefficients of the Mincer regression. The articial data are very succesful at generating predictions that come very close to the available estimates.

parameter values are $\alpha_0 = 0.739$ and $\alpha_1 = 0.79$. In addition we also have 9 more values for z_h that correspond to 9 additional schooling levels.

As before, we concentrate on the steady state implications of the model and we restrict attention to Argentina, Brazil and Chile. The experiment we conduct is the analog of the analysis in the previous section: we pick TFP so that, given the observed demographic structure, the models predictions agree with the observations of output per worker in each country relative to the U.S. The results are in Table 4.

Table 4: Private Education: Selected Countries

Country	Schooling				x/y				Δz	$\Delta \hat{z}$
	Data		Model		Data		Model			
	1960	2000	1960	2000	1960	2000	1960	2000		
Argentina	4.99	8.49	4.59	6.72	2.5	4.5	2.2	3.7	1.04	1.11
Brazil	2.83	4.56	2.57	3.49	3.0	5.1	2.5	3.2	1.11	1.76
Chile	4.99	7.89	3.58	6.47	2.8	4.6	2.4	3.6	1.10	1.67

Bringing in heterogeneity does not significantly change the predictions for 1960. As in the case of homogeneous agents, this version underpredicts the level of schooling of the Chilean workforce in 1960. For the more recent period, this more general model produces slightly higher estimates of schooling for Brazil and Chile, and slightly lower for Argentina. The message is unchanged: relative to its performance, the three countries had too much education. According to the model the "excess schooling" ranges from over one year in Brazil to over 1.8 years in Argentina. The "excess education" (measured as the ratio of actual to predicted level of schooling) is about 26%. An implication of this excess schooling is that the model underpredicts the

amount spent on education in 2000.

As expected, actual TFP changes (Δz) are relatively modest, but our approximation to conventionally measured TFP tells a different story: in both the relatively successful countries (Brazil and Chile), the model requires large increases in conventional TFP to account for the evidence on output per worker. On the other hand, measured TFP growth is very small in slow growing Argentina.

5.2   Educational and Early Childhood Policies

A major conclusion of the previous section is that increases in schooling have had relatively large payoffs in the fast growing countries of East Asia, but have produced meager returns in Latin America. This section describes a version of the model in which education is publicly provided (and free). Our approach is related to Erosa et. al. (2007) with two major differences: We retain the same Ben-Porath technology as in the private (or efficient) education case and we still allow for (privately financed) post-schooling investments in human capital.[7]

There are two reasons why a public education system can possibly account for the low effective return to education: it may allocate resources inefficiently across individuals of different ability, or it may allocate resources inefficiently across schooling levels.

In the first case, efficiency requires that high ability individuals be assigned to better schools. In the second case, it is possible to derive the implications of the model for the (privately) optimal amount of resources across schooling levels and compare that with the evidence. After we dis-

---

[7]Erosa et. al. (2007) also consider stochastic ability (across generations) and mortality risks; two features that we ignore.

cuss some theoretical considerations associated with public education, we analyze the impact of alternative allocation of resources

5.2.1 Basic Model

In this section we study the effects that changes in the quality of publicly provided schooling and in the cost of early childhood capital have on economic performance. We consider the same basic Ben-Porath model except that, during the schooling period, we assume that market inputs are publicly provided. As a first pass, we incorporate different degrees of financial market imperfections.

The representative agent solves the following problem

$$
\begin{aligned}
W(h_B, s) &= \max[-(1 + \tau^e)x_e + \\
&\quad e^{-r(6+s)} \int_{s+6}^{R} e^{-r(a-6-s)}[wh(a)(1 - n(a)) - x(a)]da], \\
&= \max_{x_e, s}[-(1 + \tau^e)x_e + e^{-r(6+s)}V(h(6 + s), s)],
\end{aligned}
$$

where $V(h(6 + s), s)$ is the present discounted value of net labor income in the post-schooling period. The constraints on the problem are

$$
\begin{aligned}
\dot{h}(a) &= z_h h(a)^{\gamma_1} x_G^{\gamma_2}, \qquad a \in [6, s + 6), \qquad h(6) = h_E, \\
h_E &= h(6) = z_e x_e^{\upsilon}.
\end{aligned}
$$

The first equation is just the natural extension of the Ben-Porath technology to the public education case. In this context, x_G is the amount of resources per student. The second, as before, captures the link between resources allocated to early childhood human capital and the level of that capital at age 6 —the beginning of the schooling period. The term $(1 + \tau^e)$ captures all the distortions that can potentially reduce the amount of early childhood

human capital. This wedge is intended to capture, in a simple way, all the factors that can effectively reduce the equilibrium level of childhood human capital. An example of a policy that could be captured with that specification is any change in the ease with which parents have access to publicly provided health care. If there is a private component (e.g. time and travel costs) policies that make access more difficult can, as a first pass, be captured by increases in τ^e.

The relevant first order conditions are

$$\dot{q}(a) = rq(a) - q(a)z_h h(a)^{\gamma_1 - 1} x_G^{\gamma_2},$$

$$\dot{h}(a) = z_h h(a)^{\gamma_1} x_G^{\gamma_2},$$

and,

$$h(6) = \left(\frac{q(6)v}{1 + \tau^e} \right)^{\frac{v}{1-v}} z_e^{\frac{1}{1-v}}.$$

In addition, the two transversality conditions are

$$q(6+s) = \frac{w}{r+\delta} \left(1 - e^{-(r+\delta)(R-6-s)} \right),$$

and

$$q(6+s)z_h h(6+s)^{\gamma_1} x_G^{\gamma_2} = -\frac{\partial V(h(6+s),s)}{\partial s}.$$

Simple calculations show that human capital as a function of age (during the schooling period) is given by

$$h(a) = [h(6)^{1-\gamma_1} + (1-\gamma_1)z_h x_G^{\gamma_2}(a-6)]^{\frac{1}{1-\gamma_1}}, \qquad a \in [6, s+6),$$

and that the shadow price of capital evolves according to

$$q(a) = \frac{De^{r(a-6)}}{[h(6)^{1-\gamma_1} + (1-\gamma_1)z_h x_G^{\gamma_2}(a-6)]^{\frac{1}{1-\gamma_1}}}, \qquad a \in [6, s+6),$$

where D is a constant to be determined.

Computation of $\partial V/\partial s$ requires knowledge of the equilibrium path of human capital in the post-schooling period. The formulas in Manuelli and Seshadri (2008) imply that the optimal choice of schooling satisfies

$$
\begin{aligned}
q(6+s)z_h h(6+s)^{\gamma_1} x_G^{\gamma_2} = \ & wh(6+s) - \frac{w\gamma}{\gamma_1}q(6+s)^{\frac{1}{1-\gamma}}[\frac{z_h}{w^{1-\gamma_2}}(\frac{\gamma_2}{\gamma_1})^{\gamma_2}]^{\frac{1}{1-\gamma}} \\
& -q(6+s)\delta h(6+s) + \\
& \left(\frac{r+\delta}{w}\right)^{\frac{\gamma}{1-\gamma}}[\frac{\gamma_2^{\gamma_2}\gamma_1^{\gamma_1}z_h w^{\gamma_2}}{(r+\delta)^{\gamma}}]^{\frac{1}{1-\gamma}}q(6+s)^{\frac{1}{1-\gamma}},
\end{aligned}
$$

Putting together all the conditions, a system of two equations suffice to determine the two unknowns D and s.

The two equations are

$$
\frac{De^{rs}}{[\left(D^\upsilon\left(\frac{\upsilon}{1+\tau^e}\right)^\upsilon z_e\right)^{1-\gamma_1} + (1-\gamma_1)z_h x_G^{\gamma_2}s]^{\frac{1}{1-\gamma_1}}} = \frac{w}{r+\delta}m(6+s) \qquad (14)
$$

and

$$
\begin{aligned}
D = \ & z_h x_G^{\gamma_2}\left(\frac{w}{r+\delta}\right)^{1-\gamma_1}\left(\frac{m(6+s)}{e^{rs}}\right)^{1-\gamma_1}\frac{m(6+s)}{r+\delta-\delta m(6)}D^{\gamma_1} \qquad (15) \\
& + \left[\frac{w^{1-\gamma_1}\gamma_2^{\gamma_2}\gamma_1^{\gamma_1}z_h}{r+\delta}\right]^{\frac{1}{1-\gamma}}\left(\gamma\gamma_1^{-\frac{1}{1-\gamma}}-1\right)\frac{m(6+s)}{e^{rs}}\frac{m(6+s)^{\frac{1}{1-\gamma}}}{r+\delta-\delta m(6+s)}
\end{aligned}
$$

where

$$
m(6+s) = 1 - e^{-(r+\delta)(R-6-s)},
$$

For the solution to be well defined, it is necessary that, after leaving the public system, the individual does not choose to "go back to school"[8]. This requires that

$$
n(6+s) = \frac{m(6+s)^{\frac{2-\gamma}{1-\gamma}}\left[\frac{z_h}{(r+\delta)}(\frac{\gamma_2}{\gamma_1})^{\gamma_2}\right]^{\frac{1}{1-\gamma}}w^{\frac{1-\gamma_1}{1-\gamma}}}{(r+\delta)De^{rs}} \leq 1.
$$

---

[8] This case can easily be handled, but the formulae is different.

It is possible to show that equation (14) defines a downward sloping locus in (s, D) space. Let $D^1(s; \tau^e, x_G)$ be the level of D that satisfies the first equation and $D^2(s; x_G)$ the function associated with the second condition (i.e. defined by equation (15)). It can be shown that the D^i are indeed functions and that

$$D^1(0; \tau^e, x_G) > 0, \ D^1(R-6; \tau^e, x_G) = 0, \ \frac{\partial D^1(s; \tau^e, x_G)}{\partial \tau^e} < 0, \ \frac{\partial D^1(s; \tau^e, x_G)}{\partial x_G} > 0,$$

and

$$D^2(0; x_G) > 0, \ D^2(R - 6; x_G) = 0, \ \frac{\partial D^2(s; x_G)}{\partial x_G} > 0,$$

It also follows that, given our preferred parameterization,

$$D^2(0; x_G) > D^1(0; \tau^e, x_G).$$

This inequality is more likely to be satisfied the higher is the basic wage rate, w, and the higher are expenditures per student, x_G. At our preferred values, the inequality holds [details missing].

We can show (details are in the Appendix) that D^1 and D^2 are downward sloping (as a function of s) and that D^1 intersects D^2 from below. In this case, the solution is unique, and it is such that increases in the cost of acquiring early childhood capital (τ^e) will result in increases in schooling. The effect of x_G on the equilibrium level of schooling is theoretically ambiguous but, for our parameter values, increases in the quality of public education increase schooling.

The model can easily be extended to the case in which different levels of schooling are allocated differential levels of resources. Let x_i, $i \in \{P, S, T\}$ be the level of expenditure per student in primary, secondary and tertiary education. Let s_i be the length (in years) of each schooling level. Then, the

human capital of a student with s years of schooling is

$$h(6+s) = [h(6)^{1-\gamma_1} + (1-\gamma_1)z_h\bar{x}_E(s)]^{\frac{1}{1-\gamma_1}},$$

where $\bar{x}_E(s)$ is a measure of the amount of effective resources available to a student with s years of schooling, and it is given by

$$\bar{x}_E(s) \begin{cases} x_P^{\gamma_2}s & 0 \leq s \leq s_P \\ x_P^{\gamma_2}s_P + x_S^{\gamma_2}(s - s_P) & s_P \leq s \leq s_P + s_S \\ x_P^{\gamma_2}s_P + x_S^{\gamma_2}s_P + x_T^{\gamma_2}(s - s_S - s_P) & s_P + s_S \leq s \end{cases}.$$

The optimal choice of schooling and early childhood human capital satisfy the analog of conditions (14) and (15). We do not present the details here.

5.2.2 Quantitative Results

Throughout this section we study the version of the model with heterogeneity in ability.

Public Education with Efficient Capital Markets Since public education plays such a large role in Latin America, our first experiment introduces public schools an assumes that resources, across schooling levels are allocated in accordance with the observations.[9] The public educational system is financed through taxes on capital and labor. In addition to picking TFP to match output per worker, we choose taxes so that revenue could finance a total level of expenditures equal to the sum of educational expenditures and a non-educational component that we set equal to 20% of output.

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[9]Public education is also very important in the U.S. However, the fact that local financing comprises a large fraction of educational expenditures makes it more likely, using Tybout like arguments, that the allocation of resources in the U.S. resemble the privately optimal distribution.

Unlike the private education case, educational expenditures are exogenous in this case. To determine the appropriate values we use data on relative expenditures per student at different schooling level. The following table presents the data that we used.

Table 5: Relative Expenditures per Student

Country	x_S/x_P		x_T/x_P	
	1960	2000	1960	2000
Argentina	1.46	1.46	3.53	3.53
Brazil	4	1.14	11	13.9
Chile	1.93	1.1	4.74	4.0

Source: UNESCO (****)[10]

All three countries show significant differences in the amount of resources allocated to the three levels. Brazil funds its tertiary sector at much higher levels than Argentina or Chile. As a reference, the relevant ratios for the U.S. are 1.32 and 2.20 in the year 2000. This suggests that the three Latin American countries that we are studying might be overinvesting in higher education.

To complete the calibration of the model, we chose the level of expenditures in primary schooling both in 1960 and 2000 so the implications of the model match the data for the share of educational expenditures in GDP. Thus, in this exercise, the free variable is years of schooling. The results of the experiment are in Table 6.

---

[10] We could not find data for Argentina in 1960 and we assume that the ratios are the same as in 2000. For Chile, our 1960 ratios correspond to 1970.

Table 6: Public Education

Country	Schooling				x/y (in %)		Δz	$\Delta \hat{z}$
	Data		Model		Data=Model			
	1960	2000	1960	2000	1960	2000		
Argentina	4.99	8.49	5.07	7.83	2.5	4.5	1.05	1.08
Brazil	2.83	4.56	2.89	4.15	3.0	5.0	1.13	1.65
Chile	4.99	7.89	4.34	6.65	2.8	4.6	1.12	1.54

Bringing in public education helps to partially solve the "education puzzle." The predictions for 1960 are much closer to the data (the model still underpredicts Chile), and the difference between the data and the predictions of the model for 2000 are significantly smaller. Even though, according to our model the three countries have overinvested in education, the excess years of schooling now range from 0.4 years in Brazil to 1.25 years in Chile. In relative terms, the private education economy underpredicted actual years of schooling by 26%, while the public education economy misses by less than half of that percentage, 12%.

The results suggest that the relatively low payoff associated with the observed increase in average years of schooling in Latin America may be due to a combination of two factors. Thus, the two potential sources of misallocation can partially explain the inefficiency of education in these three countries. In addition to this effect, increases in the educational budget have to be financed with distortionary taxes. This, in turn, lowers the payoff from additional schooling.

As before, relatively modest increases in productivity appear (except in Argentina) as large increases in measured TFP.

To ascertain the impact that reallocating educational resources could

have on output we conducted the two experiments. First, holding the *share* of educational expenditures to output constant, we eliminated all funding to the tertiary level, and the freed up resources were reallocated to primary education. We find that this reallocation has a large effect of output per worker. In the year 2000, it results in 12% increase in output per worker in Argentina (in the long run), a 19% increase in Brazil and a 17% increase in Chile.[11]

The second experiment holds constant the amount of resources devoted to the tertiary level and reallocates them to primary schooling. In this case we find that the impact on output per worker is smaller but still sizeable: 4% in Argentina, 9% increase in Brazil, and 5% increase in Chile.[12] Thus, the long run effect of the composition of educational expenditures has a large impact on output. We are now currently studying the optimal allocation.

Public and Private Education with Borrowing Constraints So far we have imposed no restrictions on borrowing. However, one of the justifications for publicly financed education is that due to financial market imperfections, some individuals would be forced to underinvest in education. If this is the case, any misallocation of educational resources has an even larger impact on output.

As a first step we seek to understand the interaction between borrowing constraints and TFP we modify the previous setup in two ways. First, dynasties are required to hold at all times a nonnegative level of assets.

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[11] Since the differences were much larger in 1960, we find that the gains could have been much larger as well. For Argentina the potential increase was 22%, for Brazil we estimate it to be 46% and for Chile the increase is 43%.

[12] Since the absolute levels were much lower in 1960, the gains in this case are smaller. The corresponding values are: 1.2% in Argentina, 0.8% in Brazil, and 0.6% in Chile.

Second, we allow individuals to supplement the (freely) provided public educational resources with private expenditures. Thus, we allow for private education[13]. We assume that dynasties cannot borrow. Since there are many steady states depending on the initial level of assets, we study the long run distribution associated with an initial condition in which all individuals have an equal share of a (small) level of capital. This determines a joint distribution of ability and assets in the long run.

The results corresponding to this specification are in Table 7.

Table 7: Borrowing Constrained Economy

Country	Schooling				x/y (in %)				Δz	$\Delta \hat{z}$
	Data		Model		Data-public		Model-total			
	1960	2000	1960	2000	1960	2000	1960	2000		
Argentina	4.99	8.49	4.78	8.40	2.3	3.6	2.5	4.5	1.03	1.06
Brazil	2.83	4.56	2.82	4.61	2.8	4.0	2.9	4.9	1.1	1.53
Chile	4.99	7.89	4.38	7.66	2.7	4.0	2.8	4.5	1.09	1.41

The remarkable finding is that, with the added friction, the model comes very close to perfectly predicting schooling in both periods for all three countries. Years of schooling are close to the evidence and the share of private resources allocated to education appears close to the evidence in the case of Argentina and Brazil (1% of GDP), but a little low in the case of Chile (0.5% of GDP). Even though our results are extremely preliminary and ignore other forms of allocative inefficiency, we are led to conclude that a simple variant of the neoclassical model can account for the "education

---

[13] We have not imposed the non-convexity associated with sending a child to a public or a private school. At this point, private education complements public resources. We plan to look at the more realistic case in future research.

puzzle." Thus, limited access to schools, and allocative inefficiencies seem to account for the low impact of the increase in schooling. As before measured TFP increases substantially in Brazil and Chile (much less in Argentina), while true productivity does not change much.

In this version of the model, measured TFP ($\hat{z}$) accounts for about 45% of the increase in output in Brazil and Chile, and close to 20% in Argentina. On the other hand, actual TFP's share hovers around 10%. Not surprisingly, the empirical multiplier effect of productivity in the distorted Latin American economies is smaller than in the East Asian countries.

As a test of the model we report the implications for the Gini coefficient and the fraction of the population with zero schooling. The income inequality measures are a little lower than the available evidence but that is to be expected since our model includes no shocks and hence it will tend to underestimate income dispersion.

Table 8: Inequality

	Gini		No Schooling (%)	
	Model		Model	
Country	1960	2000	1960	2000
Argentina	0.36	0.39	22%	8%
Brazil	0.42	0.47	48%	18%
Chile	0.39	0.44	27%	13%

We do not (at this point) have evidence on the fraction of the population with no schooling and, hence, we cannot evaluate the predictions of the model. However, if one uses the illiteracy rate as a measure of no schooling, the model seems to overpredict the level, although it does a much better job of forecasting relative levels (e.g. Brazil's illiteracy rate in 1960 was much

higher than Argentina's or Chile's).

As in the case of perfect capital markets, we studied the output gains associated with redistributing resources. Holding the *share* of educational expenditures constant, the policy that redistributes public educational resources from higher education to primary schooling has a much larger effect: In Argentina it is predicted that it increases output (in the long run) by 28%, while in Brazil the comparable number is 56%. The reason for this much larger impact is that with imperfect capital markets some high skilled students do not have access to higher education due to lack of resources. In that case, redistribution toward primary schooling has the impact of providing more resources to some high skill individuals —with a large payoff in terms of human capital— that do not go to college because of financial constraints.

6 Concluding Comments

We find that a model in which educational resources are allocated efficiently can account for the fast growth episodes that characterized some East Asian countries. However, the same model creates an "education puzzle" when applied to Latin American countries: Relative to the model (that matches by design the changes in output per worker), Latin American countries have "too much" education.

We show that inefficiently provided public education and imperfect capital markets —that, effectively, prevent able individuals from acquiring the optimal amount of education— can result in "low quality" schooling from a macroeconomic perspective. Simple calculations suggest large potential gains from reallocating resources across educational levels.

7   Appendix

In this appendix we discuss the property of the equilibrium with public education. Let,

$$\hat{m}(s) = m(6+s)e^{rs}.$$

Note that

$$D^1(0; \tau^e, x_G) = [\frac{wz_e}{r+\delta}\hat{m}(s)(\frac{\upsilon}{1+\tau^e})^{\upsilon}z_e]^{\frac{1}{1-\upsilon}}.$$

Thus, $D^1(0; \tau^e, x_G)$ is independent of x_G and it has a wage elasticity equal to $1/(1-\upsilon)$. $D^2(0; x_G)$ is the solution to

$$
\begin{aligned}
D^2 &= z_h x_G^{\gamma_2} \left(\frac{w}{r+\delta}\right)^{1-\gamma_1} \frac{m(6)^{2-\gamma_1}}{r+\delta-\delta m(6)} (D^2)^{\gamma_1} \\
&+ \frac{w}{r+\delta}\left[\frac{\gamma_2^{\gamma_2}\gamma_1^{\gamma_1}z_h w^{\gamma_2}}{(r+\delta)^{\gamma}}\right]^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{\gamma_1^{\frac{1}{1-\gamma}}}-1\right) m(6+s)^{\frac{2-\gamma}{1-\gamma}} \frac{1}{r+\delta-\delta m(6+s)}.
\end{aligned}
$$

Since $D^2(0; x_G)$ is increasing in x_G, it follows that higher expenditures make it more likely to have an interior solution. Note that

$$D^2 > w^{\frac{1-\gamma_1}{1-\gamma}}\left[\frac{\gamma_2^{\gamma_2}\gamma_1^{\gamma_1}z_h}{(r+\delta)}\right]^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{\gamma_1^{\frac{1}{1-\gamma}}}-1\right)m(6+s)^{\frac{2-\gamma}{1-\gamma}}\frac{1}{r+\delta-\delta m(6+s)}.$$

If

$$\frac{1-\gamma_1}{1-\gamma} > \frac{1}{1-\upsilon}$$

(which is satisfied in our calibrated experiment) then $D^2(0; x_G) > D^1(0; \tau^e, x_G)$. We next argue that for s close to $R-6$, $D^2(s; x_G) < D^1(s; \tau^e, x_G)$. Actually we can show that

$$\lim_{s \to R-6} \frac{D^2(s; x_G)}{D^1(s; \tau^e, x_G)} = 0.$$

To see this, let's rewrite the two equations as

$$\frac{D^1}{\hat{m}(s)} = \frac{w}{r+\delta}[\left((\frac{D^1}{\hat{m}(s)})^{\upsilon}\hat{m}(s)^{\upsilon}(\frac{\upsilon}{1+\tau^e})^{\upsilon}z_e\right)^{1-\gamma_1} + (1-\gamma_1)z_h x_G^{\gamma_2}s]^{\frac{1}{1-\gamma_1}},$$

and

$$\frac{D^2}{\hat{m}(s)} = z_h x_G^{\gamma_2} \left(\frac{w}{r+\delta}\right)^{1-\gamma_1} \frac{m(6+s)}{r+\delta-\delta m(6+s)} \left(\frac{D^2}{\hat{m}(s)}\right)^{\gamma_1}$$
$$+ \frac{w}{r+\delta} \left[\frac{\gamma_2^{\gamma_2}\gamma_1^{\gamma_1} z_h w^{\gamma_2}}{(r+\delta)^\gamma}\right]^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{\gamma_1^{\frac{1}{1-\gamma}}} - 1\right) \hat{m}(s)^{\frac{1}{1-\gamma}} \frac{e^{\frac{\gamma}{1-\gamma}rs}}{r+\delta-\delta m(6+s)}.$$

Simple inspection shows that

$$\lim_{s\to R-6} \frac{D^1(s;\tau^e,x_G)}{\hat{m}(s)} = \frac{w}{r+\delta}[(1-\gamma_1)z_h x_G^{\gamma_2} s]^{\frac{1}{1-\gamma_1}} > 0,$$

and

$$\lim_{s\to R-6} \frac{D^2(s;x_G)}{\hat{m}(s)} = 0,$$

since $\hat{m}(R-6) = 0$. Thus, even though both D^1 and D^2 converge to 0 as s goes to $R-6$, the convergence of D^2 is faster and, hence, for s near $R-6$, $D^2(s;x_G) < D^1(s;\tau^e,x_G)$. Since, given our parameter values $D^2(0;x_G) > D^1(0;\tau^e,x_G)$, and since the functions are continuous, this establishes the existence of a solution. Moreover, it shows that there is at least one solution, such that $D^2(s;x_G)$ intersects $D^1(s;\tau^e,x_G)$ from above. Since both functions are downward sloping and $\partial D^1(s;\tau^e,x_G)/\partial\tau^e < 0$, this implies that increases in τ^e increase schooling, even though human capital decreases. Finally, this is a concave programming problem and, as such, it has a unique solution.

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