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PUBLIC POLICY TOWARD LIFE SAVING: MAXIMIZE LIVES SAVED VS. CONSUMER SPENDING

William Gould

Richard Thaler

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Public Policy Toward Life Saving: Maximize Lives Saved vs. Consumer Sovereignty

ABSTRACT

This paper is a theoretical analysis of individual and societal demands for life saving. We begin by demonstrating that the allocation of health expenditures to maximize lives saved may be inconsistent with the willingness-to-pay criterion and consumer sovereignty. We further investigate the effects of information on aggregate willingness to pay. This discussion is related to the concepts of statistical and identified lives. Methods of financing health expenditures are considered. We show that risk averse individuals may reject actuarially fair insurance for treatments of fatal diseases even if they plan to pay for the treatment if they get sick. This result has implications regarding the choice of treatment or prevention. Finally, we examine the importance of the timing of life-saving decisions. A conflict arises between society's preferences before it is known <u>who</u> will be sick and after, even if it is known in advance how <u>many</u> people will be sick.

William Gould
Unicon Research Corporation
1100 Glendon Avenue, Suite 805
Los Angeles, CA 90024
(213) 478-7737

Richard Thaler
Graduate School of
Business and Public
Administration
Cornell University
Ithica, NY 14853
(607) 256-5002

INTRODUCTION

Economists who have worked in the area of health economics have generally offered policy-makers two pieces of advice. First, policy-makers have been urged to equate the productivity of various expenditures at the margin. In this spirit economists have frequently pointed to large discrepencies in costs per life saved among different government programs as an example of serious misallocations of resources. A reallocation, it is argued, could increase lives saved while holding expenditures constant.

A second frequent bit of advice is to try to allocate resources in a way which approximates the allocation which would occur in a market setting. Thus consumers' willingness to pay should be used as an indicator of their valuation of government programs.

In recent years, economists have forged a link between these two pieces of advice. They show that there is a well-defined concept of the "value of life" based on the willingness to pay approach and that the use of this concept in project evaluation leads to an allocation of expenditures across programs so as to maximize the number of lives saved.[1]

This value of life concept is based on the amount of money an individual would be willing to pay to reduce his chance of death by a small amount. For example, suppose that an individual is willing to pay $500 for an increase in his survival chances of 1/1000. A thousand such individuals would be willing, collectively, to pay $500,000 for a project that increased the survival chances of each person

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by 1/1000. If implemented, the project would be expected (in a statistical sense) to reduce the number of deaths by one among the group of one thousand people. In this sense, the value of a "statistical life" is said to be $500,000. The willingness to pay approach would then suggest that the project be adopted if its cost is less than $500,000 and be rejected if its cost is more than $500,000. More broadly, this approach suggests that expenditures on each health or safety program be increased or decreased until the marginal cost of an additional (statistical) life saved is equal to the value of a (statistical) life. Such a policy would maximize the expected number of lives saved, given the level of aggregate expenditures on health and safety.

There is considerable appeal to this approach both because of its theoretical basis and the relative simplicity of using it in practice. For example, a glance at Table 1 shows radically different costs per life saved for various governmental health and safety programs. Ignoring possible discrepancies between average and marginal costs and certain other qualifications, it is clear that survival rates could be increased by reallocating resources away from cancer diagnosis toward safety belts and other safety devices. Moreover, this approach seems to be gaining acceptance among noneconomists. For example, Dr. Robert Grossman, acting director of the dialysis program at the University of Pennsylvania Hospital, has stated:

> I can see us in the next few years having programs for people with heart disease, people with cancer, and it's something we can't afford. I don't even know if we can afford this [the dialysis program]. The question is, how much is a life worth? We're seeing a resurgence of polio (and other diseases) because children are not getting their shots. Can we afford this at a time when we could be spending the money on basic health care to keep the general population healthy?[2]

Table 1. "Cost" of a life.

Program	Cost of program per death avoided (Thousands of dollars)
Safety belts	87
Other safety devices	103
Pedestrian education	666
Motorcycle helmets	3,300
Cancer of the uterus treatment	3,500
Campaign against drunken driving	5,800
Lung cancer treatment	6,400
Breast cancer treatment	7,700
Physical checkup upon application for license	13,800
Syphilis treatment	22,300
Tuberculosis treatment	22,800
Head and neck cancer treatment	29,100
Cancer of the colon and rectum treatment	42,900

Source: W. Gorman, "Deux années d'expérience dans l'application du PPBS on comment a méliorer le partage du gâteau public?" Analyse et Prèvision, Vol. V, No. 6, June 1968, pp. 403-416. Cited in The Price of Health, Jean-Luc Miguè and Gérard Bèlanger, Macmillan Company, Canada, 1974.

In this paper, we argue that the link between the willingness to pay criterion and the goal of maximizing lives saved is neither so secure nor so simple as much of the recent economic literature has portrayed it to be. Is it really true that we can tell at a glance that the governmental health and safety programs listed in Table 1 are grossly inefficient because they fail to equate the cost per life saved across programs? If so, how can we rationalize the fact that many individuals in their private decisions appear to display large variations in the marginal cost per unit increase in survival probability across different activities? Our impression is that many of the same people who refuse to purchase or use seat belts are also willing to pay large sums for apparently ineffective treatments such as laetrile should they get cancer. Indeed, the private preference for treatment over prevention is frequently decried by physicians and policy-makers.

The main thrust of our paper is to re-examine the value that private individuals place on their own lives in various kinds of situations. We then attempt to use this analysis to shed light on the uses of the willingness to pay criteria for public policy problems and the circumstances under which maximization of lives saved is a rational policy goal.

The plan of the paper is as follows: Section I examines individuals' demand for life-saving programs. We show that those individuals with low survival chances will pay more for a given increment in probability of life than an otherwise identical but healthy individual. This is shown to be a result of the inability to trade life-saving chances among different people.

Section II investigates the effects of information on willingness to pay. We find that certain types of information will increase the divergence between willingness to pay and maximizing lives saved.

Section III considers the financing of health expenditures. We see that risk averse individuals may reject fair insurance for treatments of fatal diseases even if they plan to pay for the treatment if they get sick. This result has important implications regarding the choice of prevention or treatment.

Section IV looks at the timing of life-saving decisions. We show that while willingness-to-pay may be inconsistent with maximizing lives saved after some people are sick, if all decisions were made before it was known who would get sick the two criteria would be identical. We then briefly discuss how such a policy could be implemented.

I. INDIVIDUAL DEMAND FOR LIFE-SAVING

Before beginning our analysis a few caveats and disclaimers are in order. First, the term "life-saving" is really a misnomer. Lives cannot really be saved; they can only be prolonged. Nevertheless, we will maintain the usual nomenclature. Dealing with lives saved allows us to keep our formal analyses reasonably simple, without excessive loss of generality. Essentially we collapse an intertemporal stochastic choice problem into a single period problem.[3]

Another issue we will ignore for the most part is variation in the quality of lives saved. Saving young people may be more valuable than saving old ones, saving the healthy more valuable than the crippled, etc., but we will model societies with identical people, and thus avoid such issues. These issues have been discussed at length by Zeckhauser and Shepard [1976].

Finally, we ignore the bequest motive. If bequests are treated in models, the usual way is to introduce an additional bequest function; thus expected utility is given by

$$(1) \qquad pU(X) + (1-p)\,\psi(B),$$

where p is the probability of survival, X is consumption conditional upon the individual living, B is the bequest left to one's heirs. This is a very special form of additive separability in which the weights applied to the utility of consumption and bequests are just the survival and death probabilities. In particular, this specification assures that the contribution to expected utility from bequests is proportional to the probability of death. We know of no empirical support for this hypothesis. One alternative would be to adopt a more general formulation such as

(2) $EU = U(p, X, F)$

(3) $F = f(p, X, B)$,

where f is the group utility function of other family members and depends on the probability of survival, the individual's family wealth (X) conditional upon living, and bequests (B). One might additionally assume that f is additively separable of the form $F = pf_1(X) + (1-p)\ f_2(B)$, where f_1 and f_2 are the family's utility functions in the states in which the individual is alive or dead respectively. Without empirical evidence to suggest constraints, the above specification yields few theoretical results. Furthermore, there is no theoretical rationalization for bequests being a one-way street. While a sick individual may obtain utility from increasing bequests, his heirs may simultaneously obtain utility from increasing either his consumption or chances for survival. Without empirical evidence it is impossible to predict which effect will dominate.[4] Thus we adopt the simplification of ignoring bequests and expected utility is simply $EU = pU(X)$.

I.1 The Value of Life Saving

We now investigate the individual's demand for life-saving. The rate at which the individual will trade X for p (holding utility constant) is

$$(4) \qquad -\frac{dX}{dp} = \frac{U(X)}{pU'(X)} = V.$$

Thaler and Rosen [1976] empirically measured V by examining wage differentials in hazardous occupations and found V to be roughly $200,000 in 1967 dollars.

Given a single measure of the value of a life, the implications are that a government that wishes to maximize the benefits of adopting life-saving programs should maximize lives saved. In addition, if we can agree that V is roughly $200,000 then we have a convenient cut-off point: All life-saving projects that are undertaken should be operated at the level where the last life saved cost just $200,000.

But what if some lives are worth more than others, either to their owners or to society? Then saving 50 more valuable lives may be considerably more efficient than saving 100 less valuable lives. Why might the value of lives differ? First, we've said nothing about the income distribution in society. Assuming that some individuals are wealthier than others, and that life-saving is a normal good, the more wealthy will clearly be desirous of purchasing more life-saving for themselves, just as they purchase more of everything else.

Even in an egalitarian society where all members have identical wealth, some lives may still be more valuable than others. Much like a wealth distribution, there exists a distribution of survival probabilities--some people are more likely to die than others. Again assuming that life-saving is like any other normal economic good, individuals with lower survival probabilities should be willing to pay more than individuals with higher survival probabilities for a life-saving project.

A game of Russian roulette illustrates this point.[5] Suppose an individual is forced to play a single game of Russian roulette. Before the game is "played" he is allowed to purchase the removal of one bullet. How does his willingness to pay for this bullet removal depend on the proportion of the chambers that are full?

Proposition 1: The amount an individual will pay for a given increase in the probability of survival, Δp, is inversely related to the level of p.[6]

Proof: Since

$$\frac{dV}{dp} = -\frac{U(X)}{p^2 U'(X)} < 0,$$

this proposition follows simply from the assumption that both consumption and probability of survival are normal goods.

The importance of Proposition 1 can be illustrated by a simple numerical example.

Example 1: Consider a society of 25 people where everyone has the utility function $p \ln(X+1)$, where specifically $X = 10$. Assume that the society is divided into two groups. Group I has 10 members each with $p = .2$, while Group II has 15 members each with $p = .7$.

A treatment can be provided to either group at the same total cost, and will reduce the fatality rate of either group by 0.1. Which treatment should be provided?

Table 2 displays the amount an individual with the posited utility function will pay for a .1 increase in p as p varies from 0 to .9.

The division of society into two groups can be thought of as two different diseases, with the Group I disease more likely to be fatal. The specification that treatment costs are identical is mainly a specification of extreme economies of scale; for instance, all the

Table 2. Willingness to pay in Numerical Example 1.

Probability of survival	Maximum amount individual would pay for .1 increase in probability of survival
0	10.00
.1	7.68
.2	6.05
.3	4.96
.4	4.19
.5	3.62
.6	3.19
.7	2.85
.8	2.57
.9	2.35

Assumptions: Individuals maximize $pU(X)$, $U(X) = \ln(x+1)$ over an initial endowment of $10.00.

costs may be in research. Finally, since the treatment reduces the fatality rate of either group by .1 (i.e., yielding a Group I and Group II survival rate of 0.3 and 0.8 respectively), the implied cure rates are different. The treatment produces a cure only one-eighth of the time for Group I, but one-third of the time for Group II.

If the willingness to pay criterion is used, then the treatment should be provided to Group I since collectively they would be willing to pay up to $60.50 while Group II would pay at most $42.75. The point of interest in this example is that this choice does not maximize the number of lives expected to be saved. Providing treatment for Group I would save an expected 1 life, while providing treatment for Group II would save an expected 1.5 lives. We do not save the maximum number of lives because Group I values an increase in their survival chances more than Group II.

I.2 Inability to Trade

The reason why we fail to maximize lives saved in the previous example, and the reason why Proposition 1 is important, is that unlike most goods, there are very sharp limitations on the extent to which survival probabilities can be traded in the market place. For freely traded goods we know that everyone who consumes a positive quantity of the good values an extra unit of the good equally. In life saving, marginal valuations differ among individuals because they cannot trade to equate those valuations. If a man in our numerical example gets very sick, reducing his p from .9 to .2, he would pay up to $6.05 to increase his survival probability to .3. If it were possible to do so, others whose p were still .9 would be willing to give up .1 of their survival chances for anything over $3.84 (not shown in Table 1). Of course, current technologies (aside from some transplant operations) do not permit such trades. Once the inability to trade is recognized, failure to maximize lives saved seems less paradoxical.[7]

The reader may be troubled by our assumption that bullets cannot be traded. Can't individuals, in fact, adjust their lifestyles to alter their survival probability? We wish to argue that while there are in fact many ways an individual can regulate the safety level he faces, our no trade assumption is reasonable on two grounds.

First, when faced with a substantial probability of death from a disease any adjustments the individual might make would be small relative to the risk from the disease. A healthy man aged 35 faces a total risk of death in one year of about .002. Even if he works in a very risky occupation his risk of dying is unlikely to increase more than .005. (See Thaler and Rosen [1976].) Thus, compared to a risk of death of even .05, any changes in lifestyle would be of a trivial level.

Second, even if individuals could make significant changes in their lifestyles to lower the risk of dying when they contracted the disease, they would not choose to do so. Assume that the individual faces two independent sources of death risk which we will call discretionary and non-discretionary. The non-discretionary risk might be a disease and the discretionary risk might stem from an occupational choice. Let ϕ_1 be the probability that the individual will die from the nondiscretionary risk and ϕ_2 be the probability he will die from the discretionary risk. The probability of death, ϕ, is then

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2.$$

Will an individual's choice of ϕ_2 depend on ϕ_1? Will a man with a disease which might kill him take any more or less risk in his everyday life? One is tempted to argue both ways. On the one hand, a person with a disease seems to have less to lose by taking a risky job. On the other hand, he might want to "compensate" for his disease risk by being very careful in every way he can. As we show in Proposition 2, given our assumptions, the arguments cancel.

Proposition 2: The amount of money necessary to compensate an individual for taking some risk ϕ_2 does not depend on the level of other independent risks he faces, ϕ_1.

Proof: $(1-\phi_1)U(x)$ = expected utility without discretionary risk

$(1-\phi)U(x+c)$ = expected utility with discretionary risk,

where c is chosen such that

$$(1-\phi_1)U(x) = (1-\phi)U(x+c) = (1-\phi_1)(1-\phi_2)U(x+c).$$

Hence,

$$U(x) = (1-\phi_2)U(x+c),$$

which is independent of ϕ_1. Q.E.D.

This proposition simply states that independent events are treated independently. The policy implication in evaluating a project which alters the probability of survival for some population is that policymakers should ignore other sources of risk so long as those other risks are independent of the risk associated with the project. Thus in comparing sites for nuclear plants, it would be unnecessary to consider the occupational risks faced by the nearby population as long as they are independent of risks from the power plant. However, risk of earthquake would have to be considered since an earthquake could increase the risks from the power plant, thus violating the independence assumption.

II. IDENTIFIED LIVES

Thomas Schelling [1968] was the first writer to make the important distinction between identified and statistical lives:

> Let a six-year-old girl with brown hair need thousands of dollars for an operation that will prolong her life until Christmas, and the post office will be swamped with nickels and dimes to save her. But let it be reported that without sales tax the hospital facilities of Massachusetts will deteriorate and cause a barely perceptible increase in preventable deaths--not many will drop a tear or reach for their checkbooks.

The girl in the above example is an "identified" life, whereas the lives lost due to hospital deterioration are "statistical." There are three senses in which the girl's life differs from a statistical life. She is "identified" in the sense that (1) other people are willing to help her; (2) she has poorer than average survival chances; and (3) a program can be named that will help her and only her. The interesting point Schelling makes is that it is often easier to gather popular support to save identified rather than statistical lives.

Evidently Schelling intended the term "identified life" to be defined in the first of these senses.[8] The girl's life is identified because her personal plight provokes sympathy and a willingness to pay <u>by others</u> that is not stimulated by the anonymous and impersonal statistical life.[9]

In this paper we wish to concentrate on individuals' willingness to pay for improvements in their <u>own</u> survival probabilities. Thus the last two senses of the term "identified life" will be used in this section.

In particular, we consider identification as synonymous with information.[10] Ultimately, some individuals will get cancer, will get heart disease, will be involved in automobile accidents, etc. Information about the assignment of individuals to these health status cells will be labeled "individual identification." Lives are "unidentified" if each individual attributes the same probability of assignment to each health status cell as all other individuals. Simultaneously, programs can be listed that will increase survival rates of cancer, heart disease, and accident victims. Information about the distribution of (expected) benefits across health status cells will be labeled "project identification." A project is "unidentified" if all health status cell survival rates are expected to be affected equally.

The framework used here centers on these health status cells.[11] Each cell has associated with it a survival rate. Individuals are assigned to cells. Prior to assignment each individual has his own estimate of the probability he will be assigned to each cell. Individuals form estimated probability of survival by taking cell assignment probability weighted averages of cell survival rates. Projects can be named that affect the survival rate in various cells. Individuals form estimates of the benefits (to themselves) of a given project by taking cell assignment probability weighted averages of project induced cell survival probability increments.

We basically wish to ask one question using this framework: What kinds of information will alter the value of a life? In section II.1 we address this question with respect to individual identification. In section II.2 the analysis is repeated for project identification. Section II.3 discusses the results.

II.1. Individual Identification

Let there be K health status cells which we will denote by k, k=1,...,K. In addition, let there be N individuals denoted by i, i=1,...,N. There may be more or fewer cells than individuals. The "aggregate information" available to individuals is denoted by $\{f_k\}$, where f_k is the probability that a randomly drawn individual will be assigned to cell k. Clearly, f_k is nothing more than the population rate. If the only information available to individuals is the population rates, then each individual will view his probability of survival as being the same, given by

$$\bar{P} = \sum_k f_k p_k \tag{2.1}$$

where p_k is the probability of survival in cell k. We will refer to this case as the no individual identification case, I=0. In this case the value of a life is given by

$$V(I=0) = \frac{U(X)}{\bar{P}U'(X)} . \tag{2.2}$$

Now consider any other state of information denoted by $\{f_{ik}\}$, where f_{ik} is the probability individual i attributes to his assignment to cell k. We require a special form of coherence: If some individual believes that he is more likely than average to be assigned to cell k, then other individuals taken as a group must believe that they are less likely than average to be assigned to the cell. I.e.,

$$\frac{\sum_i f_{ik}}{N} = f_k . \tag{2.3}$$

Thus, we consider changes in distributional information holding aggregate information constant. Individual i now attributes probability P_i,

(2.4) $$P_i = \sum_k f_{ik} P_k ,$$

to his survival, and $\bar{P}$ may be interpreted as the average survival rate.

The value of a life given this information is given by

(2.5) $$V = \frac{1}{N} \sum_i \frac{U(X)}{P_i U'(X)}$$

$$= V(0) + \frac{U(X)}{U'(X)} \left[\frac{1}{N} \sum_i \left(\frac{1}{P_i} - \frac{1}{\bar{P}}\right)\right].$$

If we define individual identification, I, as the bracketed term, Equation (2.5) may be rewritten

(2.6) $$V(I) = V(0) + \frac{U(X)}{U'(X)} I .$$

Thus, I is a sufficient statistic for this problem. Information that increases I increases the value of a life.

It is quite simple to show that, given coherence, I is bounded from below by zero and that any divergence from no distributional information must increase I.[12] Further, I is additive in the sense that moving from any state of information I_1 to I_2, the value of a life may be rewritten as

(2.7) $$V(I_2) = V(I_1) + \frac{U(X)}{U'(X)} \Delta I$$

where $$\Delta I = I_2 - I_1$$

$$= \frac{1}{N} \sum_i \left(\frac{1}{P_{2i}} - \frac{1}{P_{1i}}\right) .$$

Other properties of I are most easily shown by example, which is done in Table 3. We summarize the results in the following proposition.

Table 3. Examples of I values.

Distributional information	I
1. $P_i = \bar{P} = .9$	0
2. P_i = 1 for 50 persons = .8 for 50 persons	.01389
3. P_i = 1 for 80 persons = .5 for 20 persons	.08889
4. P_i = 1 for 89 persons = 1/11 for 11 persons	.98889
5. P_i = 1 for 90 persons = 0 for 10 persons	∞

Assumptions: 100 persons with 90 expected to survive.

<u>Proposition 3</u>: More information, in the special sense of increasing I,

$$I = \frac{1}{N} \sum_i \left(\frac{1}{P_i} - \frac{1}{\bar{P}}\right)$$

always increases the value of a life. In particular, given any level of information I, the value of a life is given by

$$V(I) = V(0) + \frac{U(X)}{U'(X)} I$$

where

$$V(0) = \frac{U(X)}{\bar{P}U'(X)} .$$

II.2. Project Identification

In Proposition 3 the value of a life is determined by taking a very specific weighted average of each individual's value of a life. Namely, individuals gain an increase in the probability of survival of 1/N so that one life is expected to be saved. One way this could operationally be accomplished is to increase the probability of survival by 1/N in each health status cell. Thus the value of a life really refers to the value of a specific project, the unidentified project $I_p = 0$, and we adopt the notation $\tilde{V}(I, I_p = 0) = V(I)$. We now wish to consider the value of any other projects that also expect to save one life.

Let $\{\delta_k\}$ represent the distribution of a project's benefits across health status cells, defined so that $\frac{\delta_k}{N}$ is the absolute increment in cell k's associated survival probability. The unidentified project corresponds to $\delta_k = 1$. All other projects can be summarized by $\{\delta_k\}$ subject to the constant

$$\sum_k f_k \delta_k = 1 , \tag{2.8}$$

which arbitrarily scales all projects so that one life is expected to be saved.

It is apparent that so long as there is no individual identification then the value of all projects must be the same. No matter how various projects distribute their benefits across health status cells, there can be no distributional effect across individuals. All individuals have identical cell assignment probabilities and therefore must expect a gain of 1/N.

If there is individual identification, however, then there may be distributional effects. The value of the program, $\tilde{V}$, is given by the summation of individuals' value of a life multiplied by the expected benefits attributed to the program by the individual, or,

$$(2.9) \qquad \tilde{V} = \sum_i \frac{U(X)}{P_i U'(X)} \sum_k f_{ik} \frac{\delta_k}{N} .$$

Algebraic manipulation of equation (2.9) yields,

$$(2.10) \qquad \tilde{V} = \tilde{V}(I,0) + \frac{U(X)}{U'(X)} [\frac{1}{N} \sum_i \frac{1}{P_i} ((\sum_k f_{ik}\delta_k) - 1)]$$

or, defining the bracketed term as I_p and expanding $\tilde{V}(I,0)$ back into V,

$$(2.11) \; \tilde{V}(I,I_p) = V(0) + \frac{U(X)}{U'(X)} (I + I_p) .$$

I_p in this equation may be interpreted as the covariance between the (reciprocal of) initial survival probability and the distribution of the expected gain. I_p is positive if those with the lowest initial survival probabilities expect to gain relatively the most.[13]

We summarize our findings in the following proposition:

Proposition 4: Given any state of individual identification, which can be summarized by

$$I = \frac{1}{N} \sum_i (\frac{1}{P_i} - \frac{1}{P}) ,$$

then the value of a project expected to save one life is given by

$$V(I,I_p) = V(0) + \frac{U(V)}{U'(X)} (I + I_p)$$

where I_p is the covariance of the reciprocal of initial survival probabilities and relative expected survival probability gain. Ceteris paribus, programs that concentrate their benefits on those with the lowest survival probability will have greater value.

II.3. Discussion

Projects to save lives have value, and merely because two projects are expected to save an equal number of lives is not sufficient to insure the projects have equal value. While it is sensible to refer to the value of a life for a single individual, it is not sensible to refer to the value of a life for a group of individuals. The criteria of maximizing lives saved and maximizing ability to pay can clearly be at odds with each other.

Propositions 3 and 4 provide simple rules for determining the value of projects. As populations become better informed about the ultimate heterogeneity of survival probabilities, the value of the unidentified project will increase. In addition, projects that concentrate their benefits on those with the lowest survival probabilities will have greater value than those projects that do not.

As an example of two such projects, consider safety belt utilization and laetrile treatment for cancer. Safety belts are likely to be the nearest thing to the unidentified project that one can imagine. It is well known that safety belts generate low willingness to pay. Laetrile demand, on the other hand, concentrates its perceived benefits on a population with known low survival probability. Apparently, willingness to pay is quite high. Treatment is not covered by insurance and often requires the expense of travel to a foreign country in addition to the direct treatment cost.

To the extent that government programs attempt to mimic the allocation that would occur in a free market, one should not be surprised to find variations in expenditures per life saved. Criticism of government programs along these lines should be based upon careful analysis.

III. TREATMENT VERSUS PREVENTION

It is commonly believed that today's health industry shows an inordinate preference for treatment over prevention. In this section we demonstrate that this result could be generated by individual maximizing behavior. In particular, the existence of a relatively ineffective treatment may discourage individuals from purchasing a relatively effective prevention.

By way of arriving at this result, we will also show that, contrary to Bergstrom's [1974] result, individuals facing the prospect of becoming very sick, with a resulting high demand for treatment, will not necessarily purchase insurance.

We begin with Bergstrom's example. A man faces a probability λ that he has a certain fatal disease. A painless treatment is available at a cost c, but it succeeds with only probability θ. If the treatment is not purchased he will die quickly and painlessly. Health insurance is available which will pay for the treatment if the disease is contracted. It is priced at the actuarially fair price of λc. The man must decide whether to buy the insurance before it is known whether or not he has the disease. Three options are given to him:

1) Buy insurance.

2) Buy no insurance and die if he gets the disease.

3) Buy the treatment if he gets the disease.

Bergstrom assumes as we do that the man has no bequest motive. Further, the cost of the treatment is assumed to be less than the man's wealth, so he can afford to pay for the treatment himself if he chooses to do so.

Bergstrom compared the options pairwise. First consider options 1 and 2. Purchase of the insurance adds $\lambda\theta$ to his probability of life at a cost λc. If $\lambda\theta$ is "small" then the insurance will be purchased as long as the ratio of the gain in probability to the cost (c/θ) is greater than V. Let's assume the contrary $(\frac{c}{\theta} < V)$ so that the insurance is rejected, and option 2 is preferred to option 1.

Now consider alternatives 2 and 3. If he selects option 2 he will die for sure if he gets the disease and thus his expected utility is $E_2 = (1-\lambda)\ U(X)$. On the other hand, if he buys the treatment upon discovery of the disease, he will get an extra survival chance $\lambda\theta$, although with reduced wealth. Thus his expected utility will be

$$E_3 = (1 - \lambda)\ U(x) + \lambda\theta\ U(x - c).$$

Option 3 clearly dominates option 2.

Now compare 1 and 3. Bergstrom argues that 1 should be preferred to 3:

> . . . he realizes that he would go ahead and try the cure even if he bought no insurance. Knowing that this is the case he realizes that whether or not he buys insurance he will attempt a cure if he has the disease. Whether or not he buys the insurance, his probability of dying from the disease is $\lambda(1-\theta)$. In either case his expected cost is λc, but if he buys insurance he pays λc with certainty and if he buys no insurance he pays c with probability λ and 0 with probability $1-\lambda$. If he is a risk averter he will prefer to buy insurance. . . .

While this argument seems compelling, it is wrong.[14] Even if the individual is risk-averse, he may reject the insurance, as we state in the following proposition:

<u>Proposition 5</u>: If an individual has a probability λ of contracting a fatal (if untreated) disease, for which the cost of treatment is c (less than his current assets) and the probability of cure is θ, the individual may refuse to buy the insurance for the treatment at the actuarially fair price λc, even though he plans to purchase the treatment if he gets the disease.

<u>Proof</u>: Labeling the three options as above, the expected utility of each is:

$$E_1 = (1-\lambda)U(x-\lambda c) + \lambda\theta U(x-\lambda c)$$
$$E_2 = (1-\lambda)U(x)$$
$$E_3 = (1-\lambda)U(x) + \lambda\theta U(x-c).$$

Notice that these expected utilities are just numbers so an intransitivity is impossible. As we pointed out above, E_3 must be preferred to E_2 since $E_3 = E_2 + \lambda\theta U(x-c)$ and the second term is positive as long as $c < x$, which has been assumed. Now compare E_1 and E_2. Notice that if $\theta = 0$ then $E_1 = (1-\lambda)U(x-\lambda c) < (1-\lambda)U(x) = E_2$. If the cure is completely ineffective, then the purchase of insurance lowers the individual's wealth without raising the chances of survival. Clearly, there will be some θ^* such that $E_1 = E_2$, but for any $\theta < \theta^*$, $E_1 < E_2$. In this case we shall have $E_3 > E_2 > E_1$. Q.E.D.

The key role of θ is illustrated in Figure 1.[15] If $\theta < \theta^*$ insurance is actually worse than the "die if sick" option 2. Not until $\theta > \theta^{**}$ will insurance be preferred to paying for the care himself.

This result is generated by the assumption that money is worthless if the individual dies. Consider an analgous example. The probability of an earthquake strong enough to destroy a man's house is λ. Given such an earthquake, the probability the man will survive is θ. Will he purchase actuarially fair earthquake insurance for his house if θ is small? Clearly not; since he doesn't care about his asset

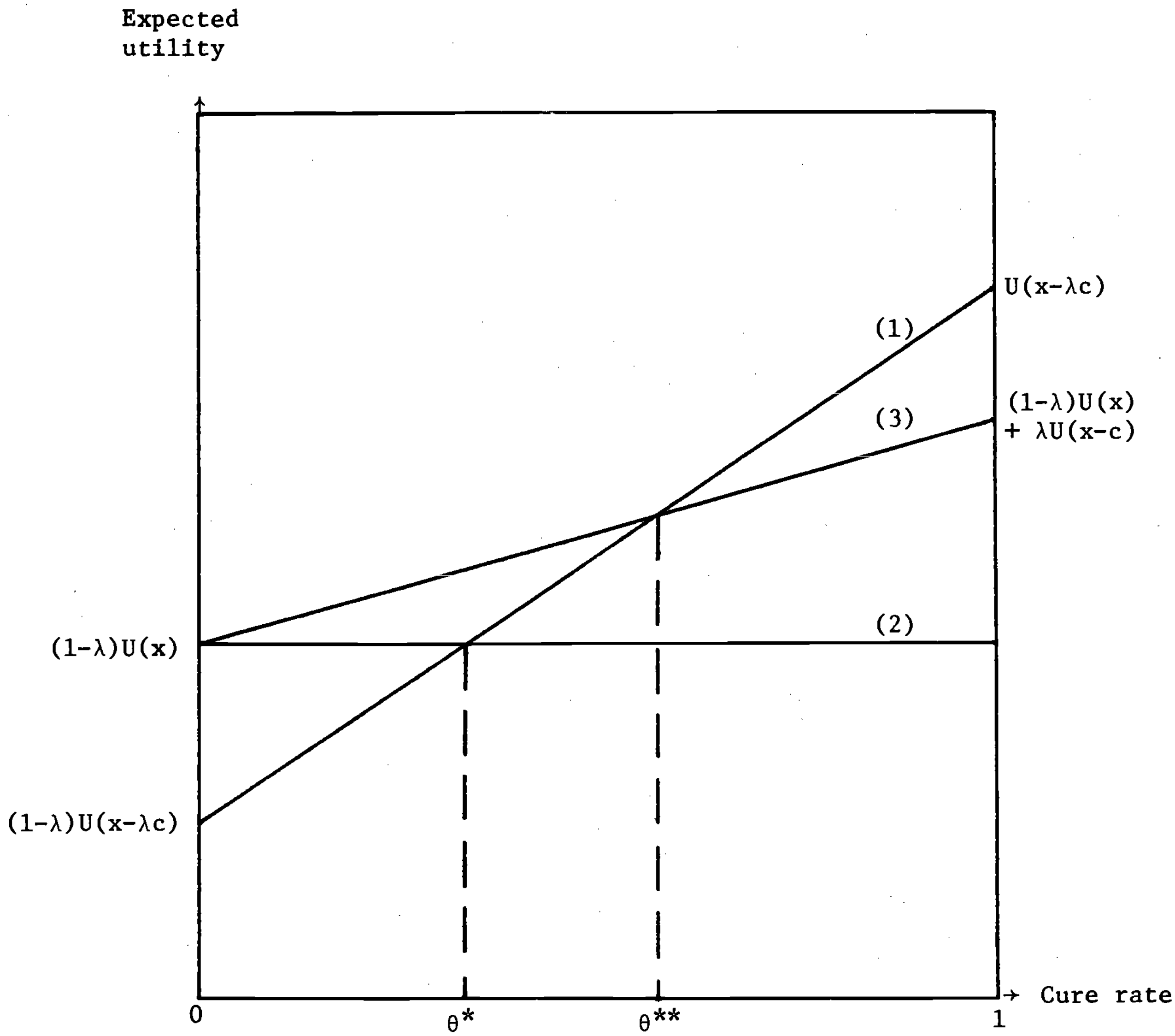

Figure 1. Illustration of Proposition 5.

position when dead, he won't be interested in insurance which pays off mainly in states of the world in which he is dead.[16]

Proposition 5 has implications which go beyond the question of financing health expenditures. The results for insurance apply with equal force to prevention activities. Consider a disease which has two strains, A and B. The probability of getting the disease is λ. Given the disease has been contracted, the probability that it is strain A is θ while the probability it is strain B is $1-\theta$. A cure exists at a cost c which is 100 percent successful for strain A, but completely unsuccessful for strain B. Similarly, an innoculation is available at a price π which prevents the individual from getting strain A but not strain B. Straightforward application of Proposition 5 implies that people may purchase the cure for c dollars rather than the innoculation, even if $\pi < \lambda c$. This implies that individuals' purchases of cures will yield smaller increases in survival rates than will equivalent purchases of prevention activity.

The above example was constructed so that the cure would never be purchased if the prevention had already been purchased, since it would be ineffective. This was necessary to apply Proposition 5 directly. However, Proposition 5 can be generalized to the case where both the prevention and the cure may be purchased.

Proposition 6: Given the choice of buying prevention in advance or treatment once the disease is discovered, or both (or nothing), an individual may choose to purchase only the treatment, even though the prevention is more cost effective.

Proof: See Appendix.

An interesting implication of Proposition 6 is that the introduction of an ineffective treatment can induce people to cease prevention activities even though their expected outlays will go up and their expected survival chances go down. An extreme case of such behavior can be illustrated with another numerical example.

<u>Example 3</u>: Consider a single individual with utility function $p \ln(x+1)$, with $x = 10$. Assume he faces a probability $\lambda = 0.01$ of contracting a fatal disease. An innoculation exists which reduces this probability by 35 percent and costs \$.09. A treatment is available at \$9, which is effective only five percent of the time.

Even at these odds (with the prevention seven times more effective) the individual will choose to buy just the treatment if sick, yielding a survival probability of .9905. Yet if the treatment were not available, he would choose the prevention and have a survival chance of .9935.

It is interesting to compare these theoretical results with the data presented in Table 1. The table presents estimates of the cost of various life-saving programs per life saved. The programs are ranked from most cost effective to least. While we are not sure exactly what each program entailed, it appears true that prevention programs such as safety belts are much more effective than treatment programs such as the cancer programs. This is, of course, consistent with the behavior characterized in Proposition 6.

IV. THE TIMING OF LIFE-SAVING DECISIONS

IV.1. Ex Ante and Ex Post

In section I we showed that the goal of maximizing lives saved might be inconsistent with consumer sovereignty and the corresponding willingness to pay criterion. In section II we examined how value of a life varied with "information." We now examine a related question: How do society's preferences about life saving vary with the timing of information? We return to Example 1 to illustrate the potential importance of the timing of life-saving decisions.

A society of 25 people was divided into two groups. Group 1 contained 10 people with survival probability $p = .2$, while Group 2 contained 15 people with $p = .7$. A program to raise p by .1 could be adopted for only one group. We saw that while giving the program to Group 2 saved more lives, it would not be the program adopted under willingness to pay. Stated in the language of Section II, the covariance of reciprocal initial survival probabilities and the distribution of program benefits is higher for Program 1. Obviously, if individuals did not know into which group they would be allotted there can be no project identification, and using ability to pay, Program 2 would be chosen.

Example 4: Assume that the 25 individuals learn of the risks described in Example 1 today, but will not learn of the composition of the groups until the following week.

Ex ante, Project 2 would be chosen. Ex post, Project 1 would be chosen. When will the decision be made?[17] The optimal decision, by assumption, is the one that maximizes willingness to pay. Ex ante

Project 1 can attract \$44.75; Project 2, \$67.31. Ex post the corresponding amounts are \$60.50 and \$42.75. Project 2, chosen ex ante, will be the decision made. A profit maximizing firm would choose to market Project 2 ex ante and collect from the whole population. It is quite simple to alter the numbers above so that Project 1, chosen ex post, becomes optimal. A profit-maximizing firm would wait and market Project 1 ex post, collecting only from the members of Group 1.

The assumption that no one knows which group he will be in corresponds to Rawls' [1971] "original position" where decisions are made "behind a veil of ignorance." Since an individual's preferences in the original position simply reflect in which society he would prefer to be a random citizen, ex ante preferences have some normative significance.

The choice made using the ex ante willingness to pay criterion has one peculiar feature. Program 1 is selected even though the members of Group 2 will clearly be worse off. Another criterion such as Rawls' maximin would choose Program 2.

Example 5: Consider a society that will have two groups. Let both groups be of the same size, have the same initial survival probabilities, but have different wealth levels. Assume Group H will have high wealth while Group L will have low wealth. A program can be adopted to save one life from either group, but it must be selected before the assignment of individuals to groups is known.

In this example individuals will prefer the program that saves one member of Group H. Society will choose to save the rich! This is not due to the greater ability of the rich to pay. It is due to the life-or-death nature of the problem. Since there is no way to make the quality of Group L life better through the choice of either program, individuals will choose the program which improves their survival chances in the richer state. This result is even more strikingly inegalitarian than the result in Example 4.

What is the normative significance of these ex ante choices? A logical case can be made both for and against the ex ante choice. First consider the arguments against.

1) The choices have "bad" distributional outcomes.

2) They ignore ex post willingness to pay.

3) Later decisions are based on more information and thus should be "better."

But the arguments in favor are:

1) All choices made in the original position are in some sense egalitarian. In this case the individuals know about the forthcoming distributions, but prefer to act in a way which just appears inegalitarian.

2) Ex post willingness to pay by Group 2 (with the low survival probability) <u>should</u> be ignored (or, rather, discounted) because their willingness is based, in part, on their slim chances of survival. Since they are likely to die, the dollars they are offering are in some sense worth less to them. It is only their inability to trade with Group 1 (and thus purchase some p directly) which allows them to outbid Group 1.

3) The "information" is purely distributional--who will lose rather than how much will be lost. When choosing in the original position this information is ignored <u>by intent</u>. Its knowledge cannot improve the decisions.

Three among the many possible criteria are: 1) ex ante willingness to pay; 2) ex post willingness to pay; 3) maximum willingness to pay (as would be practiced by a profit-maximizing firm). We leave to the reader to evaluate the normative merits of each of these criteria.

IV.2. How to Maximize Lives Saved

While we do not necessarily advocate maximization of lives saved as a goal for government policy, it is interesting to consider how it could be accomplished. We conclude that given reasonable assumptions about insurance and annuity markets, it could only be accomplished through extensive government intervention.

We have shown that a pure market system will lead those who are fatally ill to spend large amounts on ineffective, expensive treatments. Yet in the original position everyone would prefer to spend the money in other more productive ways. To avoid the low productivity expenditures society will have to precommit everyone to the prescribed decisions. This seems to require enforced prohibitions against ineffective treatments. Private health insurance companies or prepaid health care plans could exclude such treatments from their coverage. However, this would not prevent those enrollees who get very sick from spending their own (more limited) funds elsewhere. Furthermore, such contracts would be very complex and probably too expensive to be feasible. In fact, the existence of these institutions probably exaggerates the effect since they provide the very sick with the means to satisfy their nearly unlimited desire for treatment.

It is interesting to note that one way insurance companies have limited such expenditures is by placing an upper limit on total claims. This type of "shallow" coverage has been severely criticized by most economists writing in this area (see for example Arrow [1963] or Feldstein [1971]). Yet we can see that a case could be made for these limits being socially desirable. In fact, if the government provides "catastrophic" insurance for health care, then there may well be a sizeable increase in expenditures on ineffective treatments.

Banning ineffective treatments, however, is a policy of dubious value. Practically, it may be unenforceable (as the recent laetrile experience has suggested) and politically it appears to be both cold-blooded and meddlesome--a combination likely to anger both liberals and conservatives.

If one is intent upon maximizing lives saved, intermediate steps may be possible. For example, developmental research leading to new treatments is often funded at government expense. Using expected cost per life saved as one of many criteria by which research priorities are established would be one such step.

FOOTNOTES

1. See Mishan [1971], Schelling [1968], Thaler and Rosen [1976], and Zeckhauser [1975].

2. As quoted in Steven E. Rhoades [1978].

3. For analyses of "once and for all" risks this is quite reasonable. For a formal justification, see Bergstrom [1974].

4. This point was made to us by Victor R. Fuchs.

5. This example, which we believe is due to Zeckhauser, is also discussed by Bergstrom.

6. This, and all following propositions, assume individuals maximize the expected utility function $pU(x)$.

7. An economy will not, for example, maximize the number of gallons of drinking water it could produce for a given expenditure. Since water is costly to transport, a gallon of water in Palo Alto, California, will be valued more than a gallon of water in Rochester, New York. A plan to provide an extra million gallons of water to Palo Alto would not seem unreasonable just because five million gallons could be provided to Rochester for the same price!

8. This intent was confirmed by Schelling in personal conversation with one of the authors.

9. The importance of this effect is illustrated by the attempt of charities to "identify" otherwise statistical lives in their fund-raising activities. A well-known example was the annual selection of a "poster child" in the March of Dimes drive against polio.

10. This presentation was stimulated by H. M. Shefrin.

11. The use of health status as the basis for cell definitions is arbitrary. Alternative bases could be age, sex, race, etc.

12. Note that I may be rewritten as the difference between the inverse of the harmonic and arithmetic means of P_i. Since the harmonic mean is always less than the arithmetic mean except in the special case where all P_i's are equal, the result follows.

13. The term relative here has little meaning since the covariance may be rewritten in terms of absolute gains $(\sum_k f_{ik} \frac{\delta_k}{N})$. The interpretation of I_p is then N multiplied by the covariance in absolute terms.

14. Jack Hirschleifer has also pointed out the same error in personal communication with Bergstrom.

15. The diagram in Figure 1 is due to Robert Willis.

16. The result may be eliminated if a strong bequest motive exists. Also, in the absence of a bequest motive it will be eliminated by the introduction of perfect annuity markets. If the individual can purchase a fair annuity his valuation of money becomes independent of the probability of survival.

17. A similar formal problem has been discussed in different contexts by Hirshleifer (1971) and Arrow (1978).

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APPENDIX A: PROOF OF PROPOSITION 6

Notation:

λ: probability has disease

c: cost of treatment

θ_c: probability treatment produces cure.

p: cost of innoculation

θ_p: probability innoculation produces cure.

Numbering the four options:

1. Purchase neither the prevention nor the treatment,
2. Purchase the treatment if sick, but not the prevention,
3. Purchase the prevention but not the treatment if sick,
4. Purchase the prevention and the treatment if sick,

the survival rates (π) the individual faces are:

$$\pi_1 = 1 - \lambda$$

$$\pi_2 = 1-\lambda + \lambda\theta_c$$

$$\pi_3^3 = 1-\lambda + \lambda\theta_p$$

$$\pi_4 = 1-\lambda + \lambda\theta_p -1 \ \lambda(1-\theta_p)\theta_c .$$

The expected utilities are:

$$E_1 = (1-\lambda)U(x)$$

$$E_2 = (1-\lambda)U(x) + \lambda\theta_c U(x-c)$$

$$E_3 = (1-\lambda + \lambda\theta_p)U(x-p)$$

$$E_4 = (1-\lambda + \lambda\theta_p)U(x-p) + \lambda(1-\theta_p)\theta_c U(x-p-c).$$

Clearly option 2 dominates option 1 ($E_2 > E_1$), and option 4 dominates option 3 ($E_4 > E_3$). Define θ_p^* as the value of θ_p that equates E_3 and E_1.

Then,

$$\theta_p^* = (\frac{1-\lambda}{\lambda}) (\frac{U(x)}{U(x-p)} - 1) > 0.$$

If $0 \leq \theta_p^* \leq 1$, then θ_p^* has the interpretation as that prevention effectiveness that, given the cost of prevention p, would make the individual indifferent between the two alternatives: dying if sick and purchasing the prevention. θ_p^* need not be so bounded, however.

Now choose $\theta_p \leq \theta_p^*$, $0 < \theta_p \leq 1$, and choose

$$\theta_c \leq \theta_p, \quad 0 \leq \theta_c < 1.$$

Given this value θ_p, $E_3 \leq E_1$.

Define $E_2' = E_3 + \lambda\theta_c U(x-c)$ and note that E_2 can be written

$$E_2 = E_1 + \lambda\theta_c U(x-c).$$

Hence, $E_2' \leq E_2$. Proving that $E_2' > E_4$ is then sufficient to prove $E_2 > E_4$. Clearly,

$$U(x-c) > (1-\theta_p)U(x-p-c).$$

Multiplying through by $\lambda\theta_c$ and then adding E_3 to both sides, we obtain

$$E_3 + \lambda\theta_c U(x-c) > E_3 + \lambda(1-\theta_p)\theta_c U(x-p-c)$$

or

$$E_2' > E_4. \qquad \text{Q.E.D.}$$

APPENDIX B: ADDITIONAL PROOFS

Generalizations of Proposition 1

Proposition 1 stated and proved that the "value of a life" V, measured by willingness to pay, falls as the probability of survival increased for all utility functions of the form $E = pU(x)$, $U'(x) > 0$. We shall now show the conditions under which this result may be generalized both over utility functions that include an explicit bequest branch and over perfect life insurance markets. To clearly differentiate between the effects of assuming perfect markets and the effects of assuming bequest dependent utility, we will first generalize Proposition 1 to allow for perfect annuity markets.

Assuming perfect annuity markets, the expected utility function is of the form

(B-1) $$E = pU(x/p), \qquad U' > 0$$

where p = the probability of survival and x = initial wealth. The term "perfect annuity markets" simply means that should the individual survive he consumes x/p, while if he dies his endowed wealth x is given over to some third party. We draw no distinction here between human and physical wealth, but assume that all wealth is transferable. It is worth noting that a utility function like (B-1) may in some sense be a more reasonable specification than the more restrictive specification used in the text. So long as some wealth is transferable, and society does not adopt the policy of burying physical wealth with corpses, some form of annuity "market" must exist. The form of the distribution rule, however, need not be as postulated here, where the wealth of the dead is divided among the survivors according to an inverse probability rule.

Theorem B-1

Given the utility function (B-1), the "value of a life," $V = -\frac{dx}{dp}$ is strictly positive, zero, or negative, according to whether u is strictly concave, strictly quasi-concave, or strictly convex, respectively.

Proof: Taking the total derivative of (B-1) we obtain

$$\text{(B-2)} \qquad -\frac{dx}{dp} = \frac{U(\frac{x}{p}) - \frac{x}{p} U'(\frac{x}{p})}{U'(\frac{x}{p})}$$

and $U(\frac{x}{p}) \gtreqless (\frac{x}{p}) U'(\frac{x}{p})$ according to the concavity of U. Q.E.D.

Theorem B-2 (Proposition 1 variant)

Given the utility function (B-1), and assuming U to be strictly concave, the "value of a life" V decreases with p.

Proof: Taking the derivative of equation (B-2) with respect to p, we obtain

$$\text{(B-3)} \qquad \frac{dV}{dp} = \frac{x}{p^2} \frac{U''(\frac{x}{p})}{U'(\frac{x}{p})} [\frac{1}{p} + V],$$

which is strictly negative since $U'' < 0$ by the definition of concavity, and $V > 0$ by Theorem A.1. Q.E.D.

Thus, we have established Proposition 1 under the assumption of perfect annuity markets for strictly concave utility functions. More interesting, we have proven in Theorem B-1 that the probability of survival is not even a good for non-strictly concave utility functions.

We now turn to utility functions that include a bequest motive. In particular, let

$$\text{(B-4)} \qquad E = pU(x_L) + (1-p)\psi(x_D),$$

$$U' > 0$$

$$\psi' \geq 0$$

where x_L = wealth conditional upon living, and x_D = (bequest motivated) wealth conditional upon dying. For the moment we treat x_L and x_D as exogenous, i.e., we assume the non-existence of insurance markets that allow transferring wealth from one state to the other.

Theorem B-3

Given a utility function of the form (B-4), a sufficient condition to insure that the "value of a life" V is positive is that $\psi(x_D) < U(x_L)$ for all values of x_D and x_L.

Proof:

$$\text{(B-5)} \qquad V = -\frac{dx}{dp} = \frac{U(x_L) - \psi(x_D)}{pU'(x_L)} .$$

Thus, the sign of V hinges upon the sign of $U(x_L) - V(x_D)$. Q.E.D.

Theorem B-4 (Proposition 1 variant)

Given a utility function of the form (B-4), and assuming $\psi(x_D) < U(x_L)$ for all values of x_D and x_L, the "value of a life" V decreases with p.

Proof: Taking the derivative of equation (B-5) with respect to p, we obtain

$$\text{(B-6)} \qquad \frac{dV}{dp} = -\frac{U(x_L) - \psi(x_D)}{p^2 U'(x_L)} < 0.$$

Q.E.D.

Thus we have established Proposition 1 under the assumption of no insurance markets and a bequest utility function. Sufficient, although not necessary for the result, is that ψ lie everywhere below U, i.e., suicide is never motivated by economic reasons. Ignoring the existence of insurance markets, however, does not seem very reasonable. But now imagine that x_L and x_D are the result of a previous maximization--namely the purchase of insurance. Further assume, as is eminently reasonable, that the insurance contract is not contingent upon future behavior, i.e., x_L and x_D are not affected by changes in p. Then this result would take on particular relevance if it could be proved that the ability to re-enter the insurance market, i.e., sell back some of the previous contract or buy more of a new contract at currently fair prices, in no way affects the above results. In fact, it can be proved that the individual would never re-enter the insurance market at all--i.e., insurance is a once-and-for-all decision that once made does not affect future behavior except in that it affected x_L and x_D.

To demonstrate this result, let us rewrite equation (B-4) as

(B-7) $$E = pU(x - \pi I) + (1-p)\psi(x_N + I)$$

$$U', \psi' > 0; \qquad U'', \psi'' < 0$$

where I is the insurance sold at price π, x is total consumable wealth, human and non-human, conditional upon living, and x_N is that part of total wealth that is transferable.

Theorem B-5:

Given a utility function of the form (B-7) and assuming life insurance is available at "fair" prices with multiplicative loading factor k, i.e.,

(B-8) $$\pi = \left(\frac{1-p}{p}\right)k, \qquad k \geq 1.$$

Then the optimal purchase of the insurance, I*, is a positive function of the survival probability p, $0 < p < 1$.

Proof: We first derive I* by

(B-9) $$\max_{\{I\}} E = pU(x - \pi I) + (1-p)\psi(x_N + I)$$

The optimal I* is given by

(B-10) $$U'(x - \pi I) = \frac{1}{k}\psi'(x_N + I).$$

And the sufficient condition for I* to be a solution is that $U'', \psi'' < 0$. Totally differentiating (B-9) with respect to I and p yields

(B-11) $$\frac{dI}{dp} = \frac{U''k}{p^2(U''\pi + \frac{1}{k}\psi'')} > 0$$

where the arguments for U and V have been suppressed for notational clarity. Q.E.D.

Theorem B-6

Under the assumptions of Theorem B-5 above, if after solving (B-9) and obtaining I* consistent with (B-10), the probability of survival changes from p to p', and the individual is allowed to change his insurance position at the new "fair" prices with the same multiplicative loading factor, i.e.,

(B-12) $$\pi' = (\frac{1 - p'}{p'})k.$$

He will neither purchase nor sell insurance. I.e., I* is a solution to any survival probability after it has been purchased.

Proof: Given I* consistent with (B-10), the problem facing the individual is given by

(B-13) $$\max_{\{\Delta I\}} p'U(x - \pi I^* - \pi'\Delta I) + (1 - p')\psi(x_N + I^* + \Delta I).$$

Thus, I must be chosen consistent with

(B-14) $$U'(x - \pi I^* - \pi'\Delta I) = \frac{1}{k}\psi'(x_N + I^* + \Delta I).$$

Clearly one possible ΔI is $\Delta I^* = 0$, since in that case (B-14) reduces to (B-10). Indeed, given the convexity of E, the solution must be unique and so $\Delta I^* = 0$ is the only solution.

Q.E.D.

The final question, then, is does Proposition 1 hold if the life insurance and increments in survival probability are purchased simultaneously? Unfortunately, there is no simple solution to this problem.