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WHY ARE BUYOUTS LEVERED:
THE FINANCIAL STRUCTURE OF PRIVATE EQUITY FUNDS

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ABSTRACT

This paper presents a model of the financial structure of private equity firms. In the model, the general partner of the firm encounters a sequence of deals over time where the exact quality of each deal cannot be credibly communicated to investors. We show that the optimal financing arrangement is consistent with a number of characteristics of the private equity industry. First, the firm should be financed by a combination of fund capital raised before deals are encountered, and capital that is raised to finance a specific deal. Second, the fund investors' claim on fund cash flow is a combination of debt and levered equity, while the general partner receives a claim similar to the carry contracts received by real-world practitioners. Third, the fund will be set up in a manner similar to that observed in practice, with investments pooled within a fund, decision rights over investments held by the general partner, and limits set in partnership agreements on the size of particular investments. Fourth, the model suggests that incentives will lead to overinvestment in good states of the world and underinvestment in bad states, so that the natural industry cycles will be multiplied. Fifth, investments made in recessions will on average outperform investments made in booms.

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Practitioner: “Things are really tough because the banks are only lending 4 times cash flow, when they used to lend 6 times cash flow. We can’t make our deals profitable anymore.”

Academic: “Why do you care if banks will not lend you as much as they used to? If you are unable to lever up as much as before, your limited partners will receive lower expected returns on any given deal, but the risk to them will have gone down proportionately.”

Practitioner: “Ah yes, the Modigliani-Miller theorem. I learned about that in business school. We don’t think that way at our firm. Our philosophy is to lever our deals as much as we can, to give the highest returns to our limited partners.”

1. Introduction

Private equity funds are responsible for a large and increasing quantity of investment in the economy. According to a July 2006 estimate by Private Equity Intelligence, investors have allocated more than \$1.3 trillion globally for investments in private equity funds.¹ These private equity funds are active in a variety of different types of investments, from small startups to buyouts of large conglomerates to investments in real estate and infrastructure. Private equity investments are now of major importance not just in the United States, but internationally as well; for example, the Wall Street Journal recently reported that private equity firms are responsible for 40% of M&A activity in Germany (WSJ, Sept. 28, 2004, p. C1). Yet while a massive literature has developed with the goal of understanding the financing of corporate investments, very little work has been done studying the financing of the increasingly important investments of private equity funds.

Private equity investments are generally made by funds that share a common organizational structure (see Sahlman (1990), or Fenn, Liang and Prowse (1997) for more discussion). Typically, these funds raise equity at the time they are formed, and raise additional capital when investments are made. This additional capital usually takes the form of debt when the investment is collateralizable, such as in buyouts, or equity from syndication partners when it is not, as in a startup. The funds are usually organized as limited partnerships, with the limited partners (LPs) providing most of the capital and the general partners (GPs) making investment decisions and receiving a substantial share of the profits (most often 20%). While the literature has spent much effort understanding some aspects of the private equity market, it is very surprising that there is no clear answers to the basic questions of how funds are structured financially, and what the impact of this structure is on the funds’ choices of investments and their performance. Why is most private equity activity undertaken by funds where LPs commit capital for a number of investments over the fund’s life? Why are the equity investments of these funds complemented by deal-level financing from third parties? Why do GP compensation contracts have the nonlinear incentive structure commonly observed in practice? What should we expect to observe about the relation between industry cycles,

¹Ass reported by Financial Times, July 6 2006.

bank lending practices, and the prices and returns of private equity investments? Why are booms and busts in the private equity industry so prevalent?

In this paper, we propose a new explanation for the financial structure of private equity firms. Private equity firms rely on the ability of their general partners to make value-increasing investments. To do so, these managers must have sufficient freedom to be able to negotiate deals when the GP becomes aware of them. Yet, this very freedom creates a fundamental governance problem; limited partners commit capital to private equity funds with no right to sell their position or an ability to vote out the fund's managers.² As such, governance issues in private equity funds are potentially even more problematic than in public corporations. We argue in this paper that one reason why a number of institutions commonly observed in private equity contracts arise is as partial solutions to this fundamental governance problem.

We present a model based on this idea in which a number of features of private equity markets arise as equilibrium outcomes. First, the model suggests that private equity investments should be done through funds that pool investments across the fund. Second, funds should raise some capital at the fund level, prior to discovering individual deals, and supplement fund-level capital with additional, deal-specific capital. This additional capital takes the form of highly risky debt, and should be raised from different investors than the once who supply fund capital. Third, the payoffs to GPs should be a nonlinear profit-sharing arrangement similar to those observed in practice. Fourth, somewhat paradoxically, the optimal fund structure involves giving complete discretion to the GPs to undertake investments, without LPs being able to veto or otherwise interfere with investment decisions. Fifth, the model predicts that the commonly-observed pattern of investments made during busts outperforming investments made during booms on average is a natural consequence of the contracting inefficiencies between GPs and LPs.

The model is in a sense a dynamic extension of the standard adverse selection model of Myers and Majluf (1984) and Nachman and Noe (1994), in which informed firms raising capital from uninformed investors have an incentive to overstate the quality of potential investments and therefore cannot credibly communicate their information to the market. We assume that the GP faces two potential investment opportunities over time which require financing. The intertemporal element of this problem leads to a new financing decision for the GP relative to the static case considered by the standard adverse selection model. We consider regimes when the GP raises capital on a deal by deal basis (ex post financing), raises a fund of capital to be used for several future projects (ex ante financing), or uses a combination of the two types of financing.

With ex post financing, the solution is the same as in the static adverse selection model. Debt is the optimal security, and GPs will choose to undertake all investments they can get financing for, even if those investments are value-decreasing. Whether deals will be financed at all depends

²Limited partners often do have the right to terminate the partnership; however it typically takes 80% of the value-weighted claims of the limited partners to do so. Sales of partnership interests require the approval of the GP.

on the state of the economy – in good times, where the average project is positive NPV, there is overinvestment, and in bad times there is underinvestment.

Ex ante financing, however, can alleviate some of these problems. By tying the compensation of the GP to the collective performance of a fund, the GP has less of an incentive to invest in bad deals, since bad deals dilute his returns from the good deals. Tying pay-offs of past and future investments together is in a sense a way to create inside wealth endogenously and to circumvent the problems created by limited liability. Thus, a fund structure often dominates deal-by-deal capital raising. Furthermore, debt is typically not the optimal security for a fund. Since the capital is raised before the GP has learned the quality of the deals he will have an opportunity to invest in, there is no such thing as a “good” GP who tries to minimize underpricing by issuing debt. Indeed, issuing debt will maximize the risk shifting tendencies of a GP since it leaves him with a call option on the fund. We show that instead it is optimal to issue a security giving investors a debt contract plus a levered equity stake, leaving the GP with a “carry” at the fund level that resembles contracts observed in practice.

The downside of pure ex ante capital raising is that it leaves the GP with substantial freedom. Once the fund is raised, he does not have to go back to the capital markets, and so can fund deals even in bad times. If the GP has not encountered enough good projects and is approaching the end of the investment horizon, or if economic conditions shift so that not many good deals are expected to arrive in the future, a GP with untapped funds has the incentive to “go for broke” and take bad deals.

We show that it is therefore typically optimal to use a mix of ex ante and ex post capital. Giving the GP funds ex ante preserves his incentives to avoid bad deals in good times, but the ex post component has the effect of preventing the GP from being able to invest in bad deals in bad times. This financing structure turns out to be optimal in the sense that it is the one that maximizes the value of investments by minimizing the expected value of negative NPV investments undertaken and good investments ignored. In addition, the structure of the securities in the optimal financing structure mirrors common practice; ex post deal funding is done with highly risky debt that has to be raised from third parties such as banks, the LP’s claim is senior to the GP’s, and the GP’s claim is a fraction of the profits.

Even with this optimal financing structure, investment nonetheless deviates from its first-best level. In particular, during good states of the world, firms are prone to overinvestment, meaning that some negative net present value investments will be undertaken. In addition, during bad states of the world, there will be underinvestment, i.e., valuable projects that cannot be financed. During recessions, there not only will not be as many valuable investment opportunities, but those that do exist will have difficulty being financed. Similarly, during boom times, not only will there be more good projects than in bad times, but bad projects will be financed in addition to the good ones. The implication of this pattern is that the informational imperfections we model are likely

to exacerbate normal business cycle patterns of investment, creating a cyclical multiplier. Thus, the investment distortions described by our model are a potential explanation for the common observation that the private equity investment process is extremely procyclical (see Gompers and Lerner (1999b)). This logic also suggests that there is some validity to the common complaint from GPs that during tough times it is difficult to get financing for even very good projects, but during good times many poor projects get financed.

An empirical implication of this result is that returns to investments made during booms will be lower on average than the returns to investments made during poor times. Consistent with this implication is anecdotal evidence about poor investments made during the internet and biotech bubbles, as well as some of the most successful deals being initiated during busts. More formally, academic studies have found evidence of such countercyclical investment performance in both the buyout (Kaplan and Stein, 1993) and the venture capital market (Gompers and Lerner, 2000).

Our paper relates to a theoretical literature that analyzes the effect of pooling on investment incentives and optimal contracting. Diamond (1984) shows that by changing the cash flow distribution, investment pooling makes it possible to design contracts that incentivizes the agent to monitor the investments properly. Bolton and Scharfstein (1990) and Laux (2001) show that tying investment decisions together can create “inside wealth” for the agent undertaking the investments, which reduces the limited liability constraint and helps design more efficient contracts. Unlike our model, neither of these papers consider project choice under adverse selection, or have any role for outside equity in the optimal contract. Our paper also relates to an emerging literature analyzing private equity fund structures.³ Jones and Rhodes-Kropf (2003) and Kandel, Leshchinskii, and Yulea (2006) also argue that fund structures can lead GPs to make inefficient investments in risky projects. Unlike our paper, however, these papers take fund structures as given and do not derive investment incentives resulting from an optimal contract. Inderst and Muennich (2004) argue that pooling private equity investments together in a fund helps the GP commit to efficient liquidation decisions in a manner similar to the winner-picking model of Stein (1997). However, the Inderst and Muennich mechanism relies on always making the fund capital constrained, which we show is not optimal in our model. Most importantly, none of the previous theoretical papers analyze the interplay of ex ante pooled financing and ex post deal-by-deal financing, which lies at the heart of our model.

The next section presents the model and its implications. There is a discussion and conclusion following the model.

³Lerner and Schoar (2003) also model private equity fund structures, but focus on explaining the transfer restrictions of limited partnership shares.

2. Model

There are three types of agents in the model: General partners (GPs), limited partners (LPs) and fly-by-night operators. All agents are risk-neutral, and have access to a storage technology yielding the risk-free rate, which we assume to be zero.

The timing of the model is summarized in Figure 2.1. There are two periods. Each period a candidate firm arrives. We assume it costs I to invest in a firm. Firms are of two kinds: good (G) and bad (B). The quality of the firm is only observed by the GP. A good firm has cash flow $Z > 0$ for sure, and a bad firm has cash flow 0 with probability $1 - p$, and cash flow Z with probability p , where:

$$Z > I > pZ$$

Good firms therefore have positive net present values, while bad firms have a negative NPV. All cash flows are realized at the end of the second period.

Each period a good firm arrives with probability α , and a bad firm with probability $1 - \alpha$.⁴ We think of α as representing the common perception of the quality of the type of deals associated with the specialty of the GP that are available at a point in time. To facilitate the analysis, we assume there are only two possible values for α , α_H which occurs with probability q each period, and α_L which occurs with probability $1 - q$ each period. Also, we assume $\alpha_H > \alpha_L$. Since we would like α to reflect possibly unmeasurable perceptions in the marketplace, we assume it is observable but not verifiable, so it cannot be contracted on directly.

Furthermore, we assume that there is an infinite supply of unserious fly-by-night operators that investors cannot distinguish from a serious GP. Fly-by-night operators can only find useless firms with a maximum payoff less than capital invested, or store money at the riskless rate.

2.1. Securities

We assume the GP has no money of his own and finances his investments by issuing a security $w_I(x)$ backed by the cash flow x from the investments, and keeps the residual security $w_{GP}(x) = x - w_I(x)$.⁵ The securities have to satisfy the following monotonicity condition:

Monotonicity $w_I(x), w_{GP}(x)$ are non-decreasing.

⁴Equivalently, we can assume that there are always bad firms available, and a good firm arrives with probability α .

⁵If the GP had sufficient capital, the agency problems would be alleviated if he were to finance a sufficiently large part of the investments himself. In practice, GPs typically contribute 1% of the partnership's capital personally. However, so long as the GP cannot finance such a large part of investments that the agency problems completely disappear, allowing for GP wealth does not change the qualitative nature of our results.

This assumption is standard in the security design literature and can be formally justified on grounds of moral hazard.⁶ An equivalent way of expressing the monotonicity condition is

$$x - x' \geq w_{GP}(x) - w_{GP}(x') \geq 0 \quad \forall x, x' \text{ s.t. } x > x'$$

However, if the security issued pays off less than the total cash flow whenever the cash flow is below the invested capital K , the fly-by-night operators can store the money and earn rents. Since the supply of fly-by-night operators is potentially infinite, there cannot be an equilibrium where fly-by-night operators earn positive rents and investors break even. Any candidate equilibrium security design therefore has to satisfy:

Fly-by-night For invested capital K , $w_{GP}(x) = 0$ whenever $x \leq K$.

The existence of fly-by-night operators also implies that GPs should be contractually prohibited from investing in any public capital market securities, such as stocks or options. Otherwise, there would always be some chance for a fly-by-night operator to earn a positive surplus by gambling in securities markets, so that limited partners could never break even.⁷

2.2. Forms of Capital Raising

In a first best world, the GP will invest in all good firms and no bad firms. Because the GP has private information about firm type, this investment policy will not be achievable - there will typically be overinvestment in bad projects and underinvestment in good projects. Our objective is to find a method of capital raising that minimizes these inefficiencies. We consider three methods of capital raising:

- *Pure ex post* capital raising is done in each period after the GP encounters a firm. The securities investors get are backed by each individual investment's cash flow.
- *Pure ex ante* capital raising is done in period zero before the GP encounters any firm. The security investors get is backed by the sum of the cash flows from the investments in both periods.

⁶See, for example, Innes (1990) or Nachman and Noe (1994). Suppose an investor claim $w(x)$ is decreasing on a region $a < x < b$, and that the underlying cash flow turns out to be a . The GP then has an incentive to secretly borrow money from a third party and add it on to the aggregate cash flow to push it into the decreasing region, thereby reducing the payment to the security holder while still being able to pay back the third party. Similarly, if the GP's retained claim is decreasing over some region $a < x \leq b$ and the realized cash flow is b , the GP has an incentive to decrease the observed cash flow by burning money.

⁷This assumption also distinguishes our results from the model of Myers and Majluf (1984). In their model, a firm would never raise financing and invest in a negative net present value project, because they implicitly assume that there is also the possibility of investing in zero net present value assets with similar risk as the investment being considered, such as stocks of publicly traded companies.

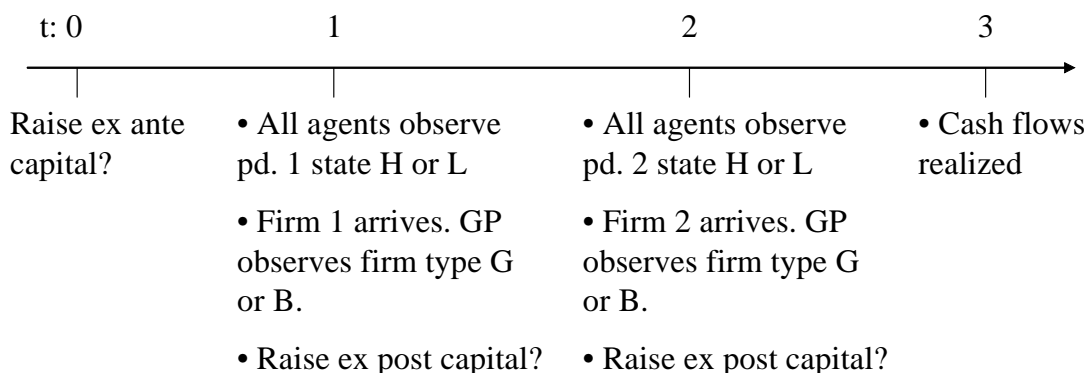


Figure 2.1: Timeline

- *Ex ante and ex post* capital raising uses a combination of the two approaches. Investors supplying ex post capital in a period receive a security backed by the cash flow from the investment in that period only. Investors supplying ex ante capital receive a security backed by the cash flows from both investments combined.

We now analyze and compare each of these financing arrangements.⁸

3. Pure ex post capital raising

We now characterize the pure ex post capital raising solution. We start by analyzing the simpler static problem in which the world ends after one period, and then show that the one period solution is also an equilibrium period by period in the dynamic problem.

In a one-period problem, the timing is as follows: After observing the firm's quality, the GP decides whether to seek financing. After raising capital, he decides whether to invest in the firm or in the riskless asset.

Given these assumptions, the GP has an incentive to seek financing regardless of the firm's quality, since he receives nothing otherwise. To invest in a firm, the GP must raise I by issuing a security $w_I(x)$, where $x \in \{0, I, Z\}$. Also, in any equilibrium where the GP receives financing and investors break even, the GP cannot get anything if the cash flow from his investment is below I . Otherwise, there will be an infinite supply of fly-by-night operators who can earn a positive return by raising money and investing in the riskfree asset. Therefore, the security design has $w_I(I) = I$. But this in turn implies that the GP will invest both in bad and good firms whenever he can raise

⁸This is not an exhaustive list of financing methods. We briefly discuss slightly different forms below as well, such as raising ex ante capital for only one period, raising only one unit of capital for the two periods, and allowing for ex post securities to be backed by more than one deal. None of these other methods improve over the once we analyze in more detail.

capital, since his payoff is zero if he invests in the riskless asset. A GP with a good firm cannot separate himself from a GP with a bad firm, so the only equilibrium is a pooling one in which all GPs issue the same security.

The security pays off only if $x = Z$, so the break even condition for investors after learning the expected fraction of good firms α in the period is

$$(\alpha + (1 - \alpha)p)w_I(Z) \geq I$$

Thus, financing is feasible as long as

$$(\alpha + (1 - \alpha)p)Z \geq I$$

and in that case, the GP will invest in all firms. The payoff $w_I(Z)$ will be set so that investors just break even, and the security can be thought of as debt with face value $w_I(Z)$. When it is impossible to satisfy the break even condition, the GP cannot invest in any firms.

We assume that the unconditional probability of success is too low for investors to break even:

Condition 3.1.

$$(E(\alpha) + (1 - E(\alpha))p)Z < I$$

Condition 3.1 implies that ex post financing is not possible in the low state. Whether pure ex post financing is possible in the high state depends on whether $(\alpha_H + (1 - \alpha_H)p)Z \geq I$ holds.

The two-period problem is somewhat more complicated, as the observed investment behavior in the first period may change investors' belief about whether a GP is a fly-by-night operator, which in turn affects the financing equilibrium in the second period. We show in the appendix, however, that a repeated version of the one-period problem is still an equilibrium:^{9,10}

Proposition 1. *Pure ex post financing is never feasible in the low state. If*

$$(\alpha_H + (1 - \alpha_H)p)Z \geq I$$

⁹The equilibrium concept we use is Bayesian Nash, together with the requirement that the equilibrium satisfies the "Intuitive Criterion" of Cho and Kreps (1987).

¹⁰The result only holds if we stick to the assumption that the GP is not allowed to invest in zero net present value public market securities, such as the S&P500. Above, we argued that it is optimal to disallow such investments in the presence of fly-by-night operators. However, this will no longer be true in the second period if it is assumed that fly-by-night operators do not raise money and invest in the first period, since they are then screened out. But if it was anticipated that such investments would be allowed in the second period for GPs who invested in the first period, there would be no way to screen out fly-by-night operators. One can show that the whole market for financing would therefore break down in period 1.

	$\underline{\underline{(\alpha_H + (1 - \alpha_H) p) Z > I}}$		$\underline{\underline{(\alpha_H + (1 - \alpha_H) p) Z < I}}$	
	<u>State</u>		<u>State</u>	
	High	Low	High	Low
<u>Firm</u>	Good	X	O	O
	Bad	X	O	O

Figure 3.1: Investment behavior in the pure ex post financing case. X denotes that an investment is made, O that no investment is made.

it is feasible in the high state, where the GP issues debt with face value F given by

$$F = \frac{I}{\alpha_H + (1 - \alpha_H) p}$$

In the solution above, we assume that fly-by-night operators do not try to raise financing, or if they do raise financing, that they invest in the risk free asset since they gain nothing regardless of their investment strategy.¹¹

3.1. Efficiency

The investment behavior with pure ex post financing is illustrated in Figure 3.1. Investment is inefficient in both high and low states. There is always underinvestment in the low state since good deals cannot get financed. In the high state, there is underinvestment if the break even condition of investors cannot be met, and overinvestment if it can, since then bad deals get financed.

4. Pure Ex Ante Financing

We now study the polar case in which the GP raises all the capital to be used over the two periods for investment ex ante, before the state of the economy is realized. Suppose the GP raises $2I$ of

¹¹We could also have imagined period-by-period financing where the security is issued after the state of the economy is realized, but *before* the GP knows what type of firm he will encounter in the period. In a one-period problem, the solution would be the same as for the pure ex post case analyzed above. However, one can show that if there is more than one period, the market for financing would completely break down except for the last period. This is because if there is a financing equilibrium where fly-by-night operators are screened out in early periods, there would be an incentive to issue straight equity and avoid risk shifting in later periods. (As we show in the proof of Proposition 1, straight equity does not survive the Cho and Kreps (1987) intuitive criterion when GPs know the type of their project at the time of issuance, but this is no longer true when the security is issued ex ante.) But straight equity leaves rents to fly-by-night operators, who therefore would profit from mimicking serious GPs in earlier periods by investing in wasteful projects. Therefore, it is impossible to screen them out of the market in early periods, so there can be no financing at all.

ex ante capital in period zero, implying that the GP is not capital constrained and can potentially invest in both periods.¹²

We solve for the GP's security $w_{GP}(x) = x - w_I(x)$ that maximizes investment efficiency. For all monotonic stakes, the GP will invest in all good firms he encounters over the two periods. However, if no investment was made in period 1, it is impossible to motivate him to avoid investing in a bad firm in period 2. This inefficiency follows from the fly-by-night condition, since the GP's payoff has to be zero when fund cash flows are less than or equal to the capital invested.

We show that it is possible to design $w_{GP}(x)$ so that the GP avoids all other inefficiencies. Under this second-best contract, he avoids bad firms in period 1, and avoids bad firms in period 2 as long as an investment took place in period 1.

To solve for the optimal security, we maximize the GP payoff subject to the monotonicity, fly-by-night, and investor break even conditions, and make sure that the second-best investment behavior is incentive compatible. The security payoffs $w_{GP}(x)$ must be defined over the following potential fund cash flows: $x \in \{0, I, 2I, Z, Z + I, 2Z\}$. Note that under a second-best contract, $x \in \{0, 2I, Z\}$ will never occur. These cash flows would result from the cases of two failed investments, no investment, and one failed and one successful investment respectively, neither of which can result from the GP's optimal investment strategy. Nonetheless, we still need to define security payoffs for these cash flow outcomes to ensure that the contract is incentive compatible.

The fly-by-night condition immediately implies that $w_{GP}(x) = 0$ for $x \leq 2I$. The following lemma shows that only one inequality has to be satisfied to induce the GP to follow the described investment behavior above:

Lemma 1. *For the pure ex ante case, a necessary and sufficient condition for a contract $w_{GP}(x)$ to induce the GP to only invest in good firms in period 1 and, if an investment was made in period 1, to pass up a bad firm in period 2 is given by:*

$$\begin{aligned} & (E(\alpha) + (1 - E(\alpha))p) w_{GP}(Z + I) \\ \geq & ((1 - p)E(\alpha) + 2p(1 - p)(1 - E(\alpha))) w_{GP}(Z) \\ & + p(E(\alpha) + (1 - E(\alpha))p) w_{GP}(2Z) \end{aligned} \tag{4.1}$$

Proof. In Appendix. ■

The left hand side is the expected payoff for a GP who encounters a bad firm in period 1, passes it up, and then invests in any firm that appears in period 2. The right hand side is the expected payoff if he invests in the bad firm in period 1, and then invests in any firm in period

¹²Below we show that in the pure ex ante case, it is never optimal to make the GP capital constrained by giving him less than $2I$.

2. Therefore, when Condition 4.1 holds, the GP will never invest in a bad firm in period 1.¹³ For incentive compatibility, we also must ensure that the GP does not invest in a bad firm in period 2 after investing in a good firm in period 1. It turns out that this incentive compatibility constraint holds whenever Condition 4.1 is satisfied.

The full maximization problem can now be expressed as:

$$\begin{aligned} & \max_{w_{GP}(x)} E(w_{GP}(x)) \\ = & E(\alpha)^2 w_{GP}(2Z) + \left(2E(\alpha)(1 - E(\alpha)) + (1 - E(\alpha))^2 p\right) w_{GP}(Z + I) \end{aligned}$$

such that

$$E(x - w_{GP}(x)) \geq 2I \quad (BE)$$

$$\begin{aligned} & (E(\alpha) + (1 - E(\alpha))p) w_{GP}(Z + I) \geq \quad (IC) \\ & ((1 - p)E(\alpha) + 2p(1 - p)(1 - E(\alpha))) w_{GP}(Z) \\ & + p(E(\alpha) + (1 - E(\alpha))p) w_{GP}(2Z) \end{aligned}$$

$$x - x' \geq w_{GP}(x) - w_{GP}(x') \geq 0 \quad \forall x, x' \text{ s.t. } x > x' \quad (M)$$

$$w_{GP}(x) = 0 \quad \forall x \text{ s.t. } x \leq 2I \quad (FBN)$$

There are two possible payoffs to the GP in the maximand. The first payoff, $w_{GP}(2Z)$, occurs only when good firms are encountered in both periods. The second payoff, $w_{GP}(Z + I)$, will occur either (1) when one good firm is encountered in the first or the second period, or (2) when no good firm is encountered in any of the two periods, and the GP invests in a bad firm in period 2 that turns out to be successful. Condition *BE* is the investor's break-even condition. Finally, the maximization has to satisfy the monotonicity (*M*) and the fly-by-night condition (*FBN*). The feasible set and the optimal security design which solves this program is characterized in the following proposition:

Proposition 2. *Pure ex ante financing is feasible if and only if it creates social surplus. An optimal investor security $w_I(x)$ (which is not always unique) is given by*

$$w_I(x) = \begin{cases} \min(x, F) & x \leq Z + I \\ F + k(x - (Z + I)) & x > Z + I \end{cases}$$

¹³It could be that if the GP invests in a bad firm in period 1, he would prefer to pass up a bad firm encountered in period 2. For incentive compatibility, it is necessary to ensure that the GP gets a higher pay off when avoiding a bad period 1 firm also in this case. We show in the proof, however, that if 4.1 holds, this additional condition must hold as well.

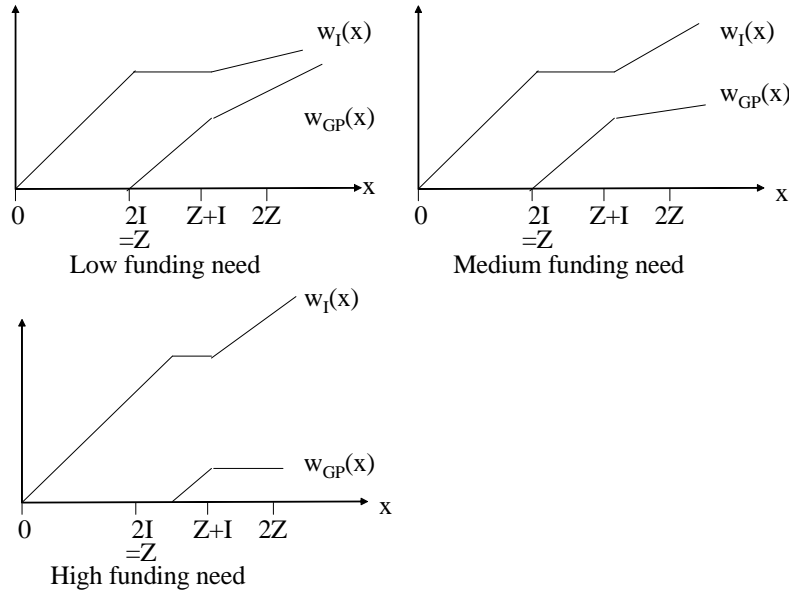


Figure 4.1: GP securities ($w_{GP}(x)$) and investor securities ($w_I(x)$) as a function of fund cash flow x in the pure ex ante case. The three graphs depict contracts under high (top left graph), medium (top right graph), and low (bottom graph) levels of $E(\alpha)$. A high level of $E(\alpha)$ corresponds to high social surplus created, which in turn means that a lower fraction of fund cash flows have to be pledged to investors.

where $F \geq 2I$ and $k \in (0, 1)$.

Proof: See appendix.

Figure 4.1 shows the form of the optimal securities for different levels of social surplus created, where a lower surplus will imply that a higher fraction of fund cash flow has to be pledged to investors. This structure resembles the structure of actual securities used by private equity funds, in which investors get all cash flows below their invested amount and a proportion of the cash flows above that. Moreover, as shown in the proof, the contracts tend to have an intermediate region, where all the additional cash flows are given to the GP. This region is similar to a provision referred to in practice as "Carried Interest Catch Up," which is commonly used in private equity partnership agreements.

The intuition for the pure ex ante contract is as follows. If the GP were to receive a straight equity claim, he would make the first-best investments, i.e., take all positive net present value investments and otherwise invests in the risk-free asset. However, the problem with straight equity is that the GP receives a positive payoff even if no capital is invested, allowing fly-by-night operators to make money. To avoid this problem, GPs can be paid only if the fund cash flows are sufficiently high, introducing a risk-shifting incentive. The risk-shifting problem is most severe if investors hold debt and the GP holds a levered equity claim on the fund cash flow. The optimal contract

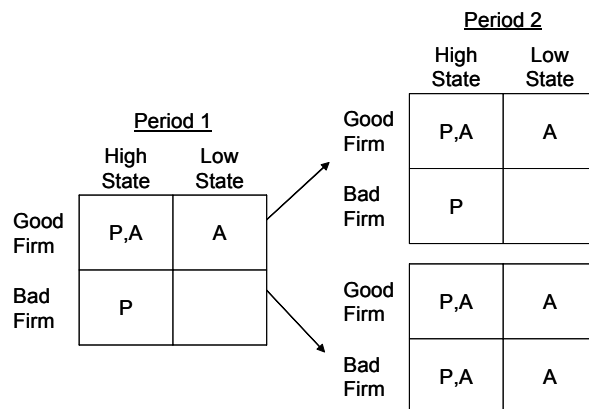


Figure 4.2: Investment behavior in the pure ex ante (A) compared to the pure ex post (P) case when ex post financing is possible in the high state.

minimizes the losses to risk shifting by reducing the levered equity claim of the GP and giving a fraction of the high cash flows to investors.¹⁴

Another way to why it is efficient for investors to receive a fraction of high cash flows (and hence make their payoffs more "equity-like") is by examining the IC constraint of the GP. When $Z \leq 2I$, the IC constraint simplifies to $w_{GP}(Z + I) \geq pw_{GP}(2Z)$, implying an upper bound on the fraction that the GP can receive of the highest fund cash flows.

As the investors have to be given more rents (to satisfy their break-even constraint), it is optimal to increase the payoff to investors for the highest cash flow states ($2Z$) first, while keeping the payoffs to GPs for the intermediate cash flow states ($Z + I$) as high as possible to reduce risk-shifting incentives. While our model set up delivers an intermediate region where investor payoffs are flat, we believe that this is not a generic feature of more general models. In particular, if we were to allow good projects to also have some risk, this flat region will likely disappear in favor of a more smooth equity piece given to investors.

4.1. Efficiency

The investment behavior in the pure ex ante relative to the pure ex post case is illustrated in Figure 4.2. In the ex ante case, the GP invests efficiently in period 1, meaning that he will accept good projects and reject bad ones. If he has access to and invests in a good project in the first period, then the investment will be efficient in period 2 as well. The only inefficiency is that the GP will invest in the bad firm in period 2 in the case where he encounters bad firms in each period.

The ex ante fund structure can improve incentives relative to the ex post deal-by-deal structure

¹⁴This is similar to the classic intuition of Jensen and Meckling (1976).

by tying the payoff of several investments together and structuring the GP incentives appropriately. In the ex post case, the investment inefficiency is caused by the inability to reward the GP for avoiding bad investments, since any compensation system that did so would violate the fly-by-night condition. In the ex ante case, the GP can be motivated to avoid bad firms as long as there is a possibility of finding a good firm in the second period. By giving the GP a stake that resembles straight equity for cash flows above the invested amount, he will make efficient investment decisions as long as he anticipates being “in the money”. Tying payoffs of past and future investments together is in a sense a way to improve incentives to invest in only good firms. When investment profits are tied together this way, bad investments dilute the returns from good investments, motivating managers to avoid making bad investments.

This logic suggests that one reason why investments are comingled within funds is that by doing so, managers are motivated to pick better investments. The one time when these incentives break down is when the firm faces a series of bad investments. The real-world counterpart to this case is when a partnership approaches the end of the ‘commitment period’ with a large pool of still-uninvested capital. Our model formalizes the concern voiced by practitioners today that the large overhang of uninvested capital can lead to partnerships overpaying for assets.

So far we have restricted the analysis of the ex ante case to a situation where the GP raises enough funds to invest in all firms. It turns out that this financing strategy dominates an ex ante structure in which the GP is capital constrained. To see why, suppose the GP only raises enough funds to invest in one firm over the two periods. He will then pass up bad firms in the first period in the hope of finding a good firm in the second period. Just as in the previous case, there is no way of preventing him from investing in a bad firm in the second period. However, there is an additional inefficiency in the constrained case, however, in that good firms have to be passed up in period 2 whenever an investment was made in period 1.¹⁵ Thus, investment efficiency is improved if private equity funds are *not* constrained in the amount of equity capital they have access to. This argument potentially explains the empirical finding of Ljungquist and Richardson (2003), who document that private equity funds seldom use up all their capital before raising a new fund.

Although the ex ante fund structure can improve efficiency over the pure ex post case, it is clear from Figure 4.2 that it need not always be the case. Clearly, pure ex ante financing always dominates when pure ex post financing is not even feasible in the high state, i.e. when $(\alpha_H + (1 - \alpha_H)p)Z < I$. Ex ante financing is feasible whenever it creates any positive surplus, which occurs as long as investors break even for the contract $w_{GP}(x) = 0$ for all x . When ex post

¹⁵This result is in contrast with the winner picking models in Stein (1997) and Inderst and Muennich (2004).

financing is feasible in the high state, ex ante financing will still be more efficient whenever:

$$\begin{aligned} & (1 - E(\alpha))^2 (I - pZ) \\ \leq & 2(q(1 - \alpha_H)(I - pZ) + (1 - q)\alpha_L(Z - I)) \end{aligned}$$

The left hand side is the NPV loss from investing in a bad project the second period, $I - pZ$, times the likelihood of this happening (probability of two bad firms in a row), $(1 - E(\alpha))^2$. The right hand side is the efficiency loss from ex post raising, which is that some bad firms are financed in the high state (which happens with probability $q(1 - \alpha_H)$ in each period) and some good firms are not financed in the low state (which happens with probability $(1 - q)\alpha_L$ in each period). Intuitively, ex post financing has the disadvantage that the GP will always invest in any firm he encounters in high states and cannot be motivated to make use of his information about investment. However, ex post financing also has the advantage that it is dependent on the realized value of α , which ex ante financing cannot be, since α is not known when funds are raised and is not verifiable, so contracts cannot be written contingent on its value.

The relative efficiency of ex post and ex ante financing depends on how informative α is about project quality. If low states are very unlikely to have good projects (α_L close to zero) and high states have almost only good projects (α_H close to one) the inefficiency with ex post fund raising is small. When the correlation between states and project quality is not so strong, pure ex ante financing will dominate.

However, even when pure ex ante financing is more efficient, it still may not be privately optimal for the GP to use. The ex ante financing contract must be structured so that the LPs get some of the upside for the GP to follow the right investment strategy, which sometimes will leave the LPs with strictly positive rents. So, there are cases in which total rents are higher under ex ante financing than under ex post financing, but the GP prefers ex post financing because he does not have to share the rents with the LPs. The following proposition characterizes the circumstances under which the GP leaves rents for the LP.

Proposition 3. *If $p > \frac{1}{2}$ and*

$$\left(\frac{E(\alpha)}{1 - E(\alpha)} \right)^2 > \frac{(1 - p) \frac{I}{Z}}{\left(1 - \frac{I}{Z}\right) \left(2 - \frac{1}{p}\right)}$$

then the LP gets a strictly positive rent in equilibrium with pure ex ante financing. Otherwise, the GP captures all the rent.

Proof. See appendix. ■

This result may shed some light on the seemingly puzzling finding in Kaplan and Schoar (2004) that succesful GPs seem not to increase their fees in follow-up funds enough to force LPs down to

a competitive rent, but rather ration the number of LPs they let into the fund.

5. Mixed ex ante and ex post financing

We now examine the model when managers can use a combination of ex post and ex ante capital raising. In the case when managers raise sufficient funding ex ante so they can potentially take all investments, the resulting equilibrium includes bad as well as good investments. This overinvestment occurs because when the GP does not invest in a bad firm in period 1, he will then invest in any firm that comes along in period 2, regardless of its quality. The possibility of using a combination of ex post and ex ante capital raising can limit this overinvestment in the second state without destroying period 1 incentives. It does so by making the GP *somewhat* capital constrained by limiting the funds that can be used for ex ante financing, and requiring him to go back to the market for additional capital to be able to make an investment.

To consider this possibility, we now assume that the GP raises $2K < 2I$ of ex ante fund capital in period 0, and is only allowed to use K for investments each period.¹⁶ The remaining $I - K$ has to be raised ex post, after the investments are discovered. As we show below, it is critical that ex post investors are distinct from ex ante investors.

Ex post investors in period i get security $w_{P,i}(x_i)$ backed by the cash flow x_i from the investment in period i . Ex ante investors and the GP get securities $w_I(x)$ and $w_{GP}(x) = x - w_I(x)$ respectively, backed by the fund cash flow $x = x_1 - w_{P,1}(x_1) + x_2 - w_{P,2}(x_2)$ (where $w_{P,i}$ is zero if no ex post financing is raised). The fly-by-night condition is now that $w_{GP}(x) = 0$ for all $x \leq 2K$. Finally, we also assume that whether the GP invests in the risk-free asset or a firm is observable by market participants, but it is infeasible to write contracts contingent upon this observation.

We characterize the contracts that lead to the most efficient equilibrium. Given these assumptions, it is sometimes possible to implement an equilibrium in which the GP invests only in good firms in period 1, only in good firms in period 2 if the GP invested in a firm in period 1, and only in the high state if there was no investment in period 1.¹⁷ As is seen in Figure 5.1, this equilibrium is more efficient than the one arising from pure ex ante financing since it avoids investment in the low state in period 2 after no investment has been done in period 1. It is also more efficient than the equilibrium in the pure ex post case, since pure ex post capital raising has the added inefficiencies that no good investments are undertaken in low states, and bad investments are undertaken in high states (if ex post capital raising is feasible).

¹⁶It is common in private equity contracts to restrict the amount the GP is allowed to invest in any one deal.

¹⁷Note that it is impossible to implement an equilibrium where the GP only invests in good firms over both periods, since if there is no investment in period 1, he will always have an incentive to invest in period 2 whether he finds a good or a bad firm.

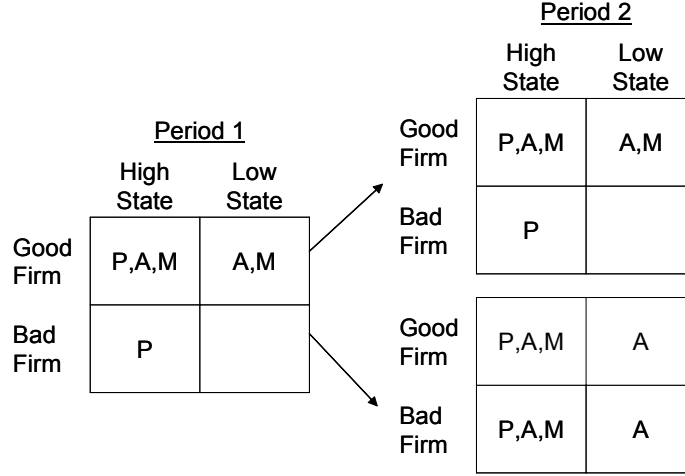


Figure 5.1: Investment behavior in the pure ex ante (A), pure ex post (P), and the postulated mixed (M) case when ex post financing is possible in the high state.

5.1. Ex Post Securities

We first show that to implement the most efficient outcome described above, the optimal ex post security is debt. Furthermore, the required leverage to finance each deal should be sufficiently high so that ex post investors are unwilling to lend in circumstances where the risk-shifting problem is severe.

If the GP raises ex post capital in period i , the cash flow x_i can potentially take on values in $\{0, I, Z\}$, corresponding to a failed investment, a risk-free investment, and a successful investment. If the GP does not raise any ex post capital, he cannot invest in a firm, and saves the ex ante capital K for that period, so that $x_i = K$. The security $w_{P,1}$ issued to ex post investors in period 1 in exchange for supplying the needed capital $I - K$ must satisfy a fly-by-night constraint and a break-even constraint:

$$w_{P,1}(I) - (I - K) \geq 0 \quad (5.1)$$

$$w_{P,1}(Z) \geq I - K \quad (5.2)$$

The fly-by-night constraint 5.1 ensures that a fly-by-night operator in coalition with an LP cannot raise financing from ex post investors, invest in the risk-free security, and make a strictly positive profit. The break even constraint 5.2 presumes that in equilibrium, only good investments are made in period 1, so that the cash-flow will be Z for sure. For ex post investors to break even, they require a payout of at least $I - K$ when $x_i = Z$. The ex post security that satisfies these two conditions and leaves no surplus to ex post investors is risk-free debt with face value $I - K$.

A parallel argument establishes debt as optimal for ex post financing in the second period in the case when no investment was made in the first. The fly-by-night condition stays unchanged, but the break even-condition becomes

$$w_{P,2}(Z) \geq \frac{I - K}{\alpha + (1 - \alpha)p} \quad (5.3)$$

The face value of the debt increases relative to the face value in the first period because when no investment has been made in the first period, the GP will have an incentive to raise money and invest even when he encounters a bad firm in period 2. To break even given this expected investment behavior, the cheapest security to issue is debt with face value of $\frac{I-K}{\alpha+(1-\alpha)p}$.

The last and trickiest case to analyze is the situation in period 2 when there has been an investment in period 1. The postulated equilibrium requires that no bad investments are then made in period 2. Furthermore, since fly-by-night operators are not supposed to have invested in period 1, ex post investors know that fly-by-night operators have been screened out. Therefore, we cannot use the fly-by-night constraint in our argument for debt. Nevertheless, as we show in the appendix, an application of the Cho and Kreps refinement used in the proof of Proposition 1 implies that $w_{P,2}(I) \geq I - K$. To see why, if $w_{P,2}(I) < I - K$, GPs finding bad firms will raise money and invest in the risk-free security. This in turn will drive up the cost of capital for GPs finding good firms, who therefore have an incentive to issue a more debt-like security. Therefore, risk-free debt is the only possible equilibrium security.

To sum up, debt is the optimal ex post security, and it can be made risk free with face value $F = I - K$ in period 1, and in period 2 if an investment was made earlier. When no investment has been made in period 1, optimal investment requires conditions on the quantity of ex post capital. In particular, the amount of capital $I - K$ the GP raises must be low enough so that the GP can invest in the high state, but high enough such that the GP cannot invest in the low state. Using the break even condition 5.3, the condition for this is:

$$(\alpha_H + (1 - \alpha_H)p)Z \geq I - K \geq (\alpha_L + (1 - \alpha_L)p)Z \quad (5.4)$$

We summarize our results on ex post securities in the following proposition:

Proposition 4. *With mixed financing, the optimal ex post security is debt in each period. The debt is risk-free with face value $I - K$ in period 1 and in period 2 if an investment was made in period 1. If no investment was made in period 1, and the period 2 state is high, the face value of debt is equal to $\frac{I-K}{\alpha_H+(1-\alpha_H)p}$. The external capital $I - K$ raised each period satisfies*

$$(\alpha_H + (1 - \alpha_H)p)Z \geq I - K \geq (\alpha_L + (1 - \alpha_L)p)Z$$

If no investment was made in period 1 and the period 2 state is low the GP cannot raise any ex post debt.

Proof. See appendix. ■

5.2. Ex Ante Securities

We now solve for the ex ante securities $w_I(x)$ and $w_{GP}(x) = x - w_I(x)$, as well as the amount of per period ex ante capital K . The security payoffs must be defined over the following potential fund cash flows, which are net of payments to ex post investors:

Fund cash flow x	Investments
0	2 failed investments.
$Z - (I - K)$	1 failed and 1 successful investment.
K	1 failed investment.
$2K$	No investment.
$Z - \frac{I-K}{\alpha_H + (1-\alpha_H)p} + K$	1 successful investment in period 2.
$Z - (I - K) + K$	1 successful investment in period 1.
$2(Z - (I - K))$	2 successful investments.

Note that the first two cash flows cannot happen in the proposed equilibrium and that the last three cash flows are in strictly increasing order. In particular, as opposed to the pure ex ante case, the expected fund cash flow now differs for the case where there is only one successful investment depending on whether the firm is encountered in the first or second period. This difference occurs because if the good firm is encountered in the second period, the GP is pooled with other GPs who encounter bad firms, so that ex post investors will demand a higher face value before they are willing to finance the investment.

The following lemma provides a necessary and sufficient condition on the GP payoffs to implement the desired equilibrium investment behavior. Just as in the pure ex ante case, it is sufficient to ensure that the GP does not invest in bad firms in period 1.

Lemma 2. *A necessary and sufficient condition for a contract $w_{GP}(x)$ to be incentive compatible in the mixed ex ante and ex post case is*

$$\begin{aligned}
& q(\alpha_H + (1 - \alpha_H)p)w_{GP}\left(Z - \frac{I - K}{\alpha_H + (1 - \alpha_H)p} + K\right) \\
> & E(\alpha)(pw_{GP}(2(Z - (I - K))) + (1 - p)w_{GP}(Z - (I - K))) + (1 - E(\alpha)) * \\
& *p \max[w_{GP}(Z - (I - K) + K), pw_{GP}(2(Z - (I - K))) + 2(1 - p)w_{GP}(Z - (I - K))]
\end{aligned} \tag{5.5}$$

Proof. In appendix. ■

The left hand side of the inequality in Lemma 1 is the expected payoff to the GP if he passes up a bad firm in period 1. He will then be able to invest in period 2 if the state is high (probability q), and will be rewarded if the second period firm is successful (probability $\alpha_H + (1 - \alpha_H)p$). If the state in period 2 is low, he cannot invest, and will get a zero payoff because of the fly-by-night constraint. The right hand side is the expected payoff if the GP deviates and invests in a bad firm in period 1. In this case, he will be able to raise debt at face value $F = I - K$ in both periods, since the market assumes that he is investing efficiently. The first line on the right hand side is his payoff if he finds a good firm in period 2. The last line is his payoff when he finds a bad firm in period 2, in which case his investment decision will depend on the relative payoffs.

Just as in the pure ex ante case, the incentive compatibility condition 5.5 shows that it is necessary to give part of the upside to investors to avoid risk-shifting by the GP. In particular, the GP stake after two successful investments ($w_{GP}(2(Z - (I - K)))$) cannot be too high relative to his stake if he passes up a period 1 bad firm ($w_{GP}\left(Z - \frac{I-K}{\alpha_H + (1-\alpha_H)p} + K\right)$).

To solve for the optimal contract, we maximize GP expected payoff subject to the investor break-even constraint, the incentive compatibility condition 5.5, the fly-by-night condition, the monotonicity condition, and Condition 5.4 on the required amount of per period ex ante capital K . The full maximization problem is given in the Appendix. The optimal security design is characterized in the following proposition.

Proposition 5. *The ex ante capital K per period should be set maximal at $K^* = I - (\alpha_L + (1 - \alpha_L)p)Z$. An optimal contract (which is not always unique) is given by*

$$w_I(x) = \min(x, F) + k(\max(x - S, 0))$$

where $F \in \left[2K^*, Z - \frac{I-K^*}{\alpha_H + (1-\alpha_H)p} + K^*\right)$, $S \in \left[Z - \frac{I-K^*}{\alpha_H + (1-\alpha_H)p} + K^*, Z - (I - K^*) + K^*\right]$ and $k \in (0, 1]$.

Proof. In appendix. ■

The mixed financing contracts are similar to the pure ex ante contracts. As in the pure ex ante case, it is essential to give the ex ante investors an equity component to avoid risk-shifting by the GP, so that he does not pick bad firms whenever he has invested in good firms or has the chance to do so in the future. At the same time, a debt component is necessary to screen out fly-by-night operators.

The intuition for why fund capital K per period should be set as high as possible is the following: The higher GP payoffs are if he passes up bad firms in period 1, the easier it is to implement the equilibrium. The GP only gets a positive payoff if he reaches the good state in period 2 and succeeds with the period 2 investment, so it would help to transfer some of his expected profits to this state from states where he has two successful investments. This is possible to do by changing the ex

ante securities, since ex ante investors only have to break even unconditionally. However, ex post investors break even state by state, so the more ex post capital the GP has to rely on, the less room there is for this type of transfer, and the harder it is to satisfy the GP incentive compatibility condition.

5.3. Optimality of third party financing

We now show that it is essential in the mixed financing solution that ex post and ex ante investors be different parties. One could have imagined that instead of going to a third party for ex post capital, the GP can go back to the ex ante investor and ask for more capital. However, it will often be ex post optimal for the limited partner to refuse financing in the second period if no investment was made in the first period. This in turn undermines the GP's incentive to pass up a bad firm in period 1, so that the mixed financing equilibrium cannot be upheld.

To show this result formally, suppose that the average project in the high state does not break even:

$$(\alpha_H + (1 - \alpha_H)p)Z < I$$

Now suppose we consider a possible contract between the GP and the LP, in which the GP has to ask the LP additional financing each period if he wants to invest in a firm. Any such contract would specify a split of the final fund cash flows $w_{GP}(x), w_I(x)$ such that $w_{GP}(x) + w_I(x) = x$, where possibly the structure of the securities are renegotiated depending on the outcome of the bargaining between the GP and the LP when the GP asks for extra financing. In keeping with the contracting limitations we have assumed before, the ex ante contract cannot be contingent on the state of the economy. Therefore, in period 2, the contract would either specify that the LP is forced to provide the extra financing regardless of state, or that the LP can choose not to provide extra financing.

Suppose no investment has been made in period 1, that the high state is realized in period 2, and that the GP asks the LP for extra financing. Because of the fly-by-night condition, the GP will ask the LP for extra financing regardless of the quality of the period 2 firm, since otherwise he will earn nothing. If the LP refuses to finance the investment, whatever amount $2K$ that was invested initially into the fund will have to revert back to the LP so as not to violate the fly-by-night condition. If the LP agrees and allows an investment, the maximum expected pay off for the LP is

$$(\alpha_H + (1 - \alpha_H)p)Z - I + 2K < 2K$$

Therefore, the LP will veto the investment. Clearly, he will also veto it in the low state since his returns will be even lower. Thus, there can be no investment in period 2 if there was none in period 1. But then, the GP has no incentive to pass up a bad firm encountered in period 1, so the mixed

financing equilibrium breaks down.

Proposition 6. *The mixed financing equilibrium cannot be implemented without a third party financier.*

In light of this reasoning, we can see the benefit of using banks as a second source of finance more clearly. We need ex post investors to make the contract more state contingent so that we can cut off financing in the low state in the second period while preserving financing in the high state. However, it may be necessary to subsidise the ex post investors in the high state if the average project does not break even. This is not possible unless we have two sets of investors where the ex ante investors commit to use some of the surplus they gain in other states to subsidise ex post investors.

This result separates our theory of leverage from previous theories, such as those relying on tax or incentive benefits of debt. Those benefits can be achieved without two sets of investors. Also, it shows that it will typically be inefficient to give LPs the right to approve individual deals. If LPs did have this right, their optimal strategy would be to veto any investment occurring in period 2 if an investment had not already been made in period 1. Of course, their ability to pursue this strategy would cause the mixed strategy equilibrium to break down, and the result would be less efficient investment. The model therefore also provides an explanation for why GPs are typically given complete control over their funds' investment policies.

5.4. Feasibility

The equilibrium which combines ex ante and ex post financing is strictly more efficient than either the pure ex post or pure ex ante cases considered above. A shortcoming of the mixed financing equilibrium is that, as opposed to the other cases, it is not always implementable, even when it creates surplus. The difficulty in implementing the equilibrium occurs because it is now harder than in the ex ante case to provide the GP with incentives to avoid investing in bad firms in the first period. If he deviates and invests, not only will he be allowed to invest also in the low state in period 2, but he will be perceived as being good in the high state, meaning that he can raise ex post capital more cheaply.

The following proposition gives the conditions under which the equilibrium is implementable:

Proposition 7. *Necessary and sufficient conditions for the equilibrium to be implementable are that it creates social surplus, that*

$$q(\alpha_H + (1 - \alpha_H)p) \geq p \tag{5.6}$$

and that

$$\frac{\alpha_L + (1 - \alpha_L)p}{\alpha_H + (1 - \alpha_H)p} < \min\left(\frac{I}{Z}, 1 - \frac{I}{Z} + \alpha_L + (1 - \alpha_L)p\right)$$

Proof: See appendix.

This proposition implies that the equilibrium can be implemented if the average project quality in high states (i.e. $\alpha_H + (1 - \alpha_H)p$) is sufficiently good, compared both to the overall quality of bad projects (p) and the average in project quality in low states ($\alpha_L + (1 - \alpha_L)p$). In other words, if the project quality does not improve sufficiently in high states, it will not be possible to implement this equilibrium. If project quality does not improve much in the high state, however, the efficiency gain from combining ex ante and ex post financing will be small compared to pure ex ante financing. Hence, when the efficiency gain from this equilibrium is large, it will also be feasible to implement.

It should also be noted that when the mixed financing equilibrium is not feasible, there may be other less efficient mixed financing equilibria that are. For example, there are equilibria in which the GP uses a mixed strategy in the first period and sometimes invests even when he has a bad project. These types of equilibria can still dominate pure ex ante and pure ex post financing. In the interest of brevity we do not fully characterize these equilibria here, but the qualitative point is the same: Mixed financing is likely to dominate because it combines the internal incentives of the pure ex ante case with the external screening that ex post financing provides.

5.5. Features of the equilibrium and robustness

The mixed financing equilibrium has a number of features that are worth highlighting. First, even though the solution is the most efficient that can be implemented, there are still investment distortions. As is seen in Figure 5.1, there is overinvestment in the good state since some bad investments are made, and there is underinvestment in bad states since some good investments get passed up. As a result, the natural industry cycles get multiplied, and private equity investment will exhibit particularly large cyclicalities.

Second, this investment pattern will affect the returns on the investments. The model predicts that in bad times, some good investments are ignored and in good times, some bad investments are undertaken. Thus, the average quality of investments taken in bad times will exceed that of those taken in good times. This prediction is consistent with industry folklore, as well as with the evidence of Gompers and Lerner (2000) and Kaplan and Stein (1993) that hot private equity markets are associated with increased transaction prices and depressed subsequent investment performance.¹⁸

Third, another essential feature of our equilibrium is that the GP is not allowed to invest more than K of the fund's capital in any given investment. In the case when the GP did not invest in

¹⁸Also, Kaplan and Schoar (2005) show that private equity funds raised in periods with high fundraising tend to underperform funds raised in periods with low fundraising. Although this finding seems consistent with our model, they do not explicitly look at the performance of individual investments undertaken in hot versus cold markets.

period 1, the GP would otherwise have an incentive to use the whole fund capital, $2K$, to finance a deal in the second period, and the equilibrium would break down. Similarly, if the GPs were allowed to back the ex post securities with total fund cash flows, rather than just the cash flows from an individual deal, this would be equivalent to using the first period capital to back the second period ex post debt. Hence, the model implies that contracts should impose restrictions on the amount of fund capital that can be used in a given deal, and prohibit GPs from using total fund cash flows to back financing for particular deals. In fact, both these restrictions are commonplace in real world private equity partnership agreements, as has been shown in Gompers and Lerner (1996).

Fourth, just as in the pure ex ante financing case, with mixed financing the ex ante investors cannot always be held to their break-even constraint and will sometimes be left with some rents in equilibrium. These rents should occur even if there is a competitive fund-raising market. Thus, the model provides a potential explanation of the Kaplan and Schoar (2004) finding that limited partners sometimes appear to earn predictable excess returns.

Fifth, the model we have analyzed is a two-period model, and it is not clear to what extent our results would hold up in a multi-period setup. Nonetheless, we can make some conjectures about this case. First, it is clear that if we were to let the fund life go towards an infinite number of periods, we would approach first best investment with pure ex ante financing. As the fund life goes to infinity, the GP will be certain that he will eventually encounter enough good investments to provide sufficient incentives to avoid all bad ones. Clearly infinitely-lived private equity funds are not observed in practice; there are a number of reasons outside of the model such as LP liquidity constraints, that are likely explain this observation.¹⁹ If we take a finite fund life as given, it is likely that mixed ex ante and ex post financing will still be optimal, as long as the state of the world is sufficiently persistent. For example, suppose that the GP encounters a low state early in the fund's life, where good deals are very scarce, and that this low state is expected to last for a long time. If such a GP had sufficient financing to invest without going to the capital market for additional financing, then even in a multi-period setup, the GP potentially could have incentives to take bad investments. Ex post financing, which implicitly takes account of the state of the economy, would prevent such GPs from acting on these incentives.

6. Discussion and Conclusions

A voluminous literature in corporate finance concerns the capital structure of public firms and the manner in which firms decide to finance investments. Yet, much financing today is done through private capital markets, by private equity firms who receive funding from limited partners and use this money to finance investments, including both new ventures and buyouts of existing companies.

¹⁹One potential explanation for limited fund life is provided by Stein (2005), who develops a model where funds are open-end rather than closed-end because of asymmetric information about fund manager ability.

These firms generally have a common financial structure: They are finite-lived limited partnerships who raise equity capital from limited partners before any investments are made (or even discovered) and then supplement this equity financing with third party outside financing at the individual deal level whenever possible. General partners have most decision rights, and receive a percentage of the profits (usually 20%), which is junior to all other securities. Yet, while this financial structure is responsible for a very large quantity of investment, we have no theory explaining why it should be so prevalent.

This paper presents a model of the financial structure of a private equity firm. In the model, a firm can finance its investments either *ex ante*, by pooling capital across future deals, or *ex post*, by financing deals when the GP finds out about them. The financial structure chosen is the one that maximizes the value of the fund. Financial structure matters because managers have better information about deal quality than potential investors, leading to both underinvestment and overinvestment in equilibrium. The value maximizing financial structure of the firm minimizes the losses both from expected bad investments that are undertaken and good investments that are ignored.

Underlying the model is the notion that the governance problems inside private equity funds are fundamentally problematic, even more so than those of a public corporation. Once funds are committed, there is very little that limited partners can do if they become dissatisfied with fund management. Unlike shareholders in public firms, limited partners in private equity funds do not have the right to sell their partnership interests, nor do they have the right to vote out the general partners. Our model suggests that a number of contractual features common to private equity funds potentially arise as ways of partially alleviating these governance problems.

First, the model suggests that fund managers will be compensated using a profit sharing arrangement that balances the desire to pay the GP for performance with the sharing of profits with investors to mitigate excessive risk-taking. The optimal profit sharing arrangements are likely to be somewhat nonlinear, as is illustrated in Figure 4.1. This prediction mimics common practice, in which fund managers receive carried interest, or ‘carry’, usually of 20% (see Gompers and Lerner (1999a)). In fact, most partnership contracts give managers a nonlinear profit-sharing schedule similar to the one that is optimal in the model. In a typical scheme, limited partners receive all the cash flows until they reach a specified level (usually the value of the equity originally committed, sometimes with a ‘preferred return’ on top of the return of capital), then a ‘General Partner’s Carried Interest Catch Up’ region, in which general partners receive 100% of the profits, with the profits split 80-20 between the limited and general partners above that region.

Second, the model suggests that funds will be designed so that investments are pooled within a fund. By pooling investments, fund managers will have incentives to avoid bad investments because they will dilute the returns to the other investments in the fund. Probably because of this reason, most private equity funds do pool investments within funds, and base the GPs carry on

the combined profits from the pooled investments rather than having an individual carry based on the profits of each deal, a practice known as "aggregation". In fact, according to Schell (2006), it was common for private equity funds in the 1970's and early 1980's to calculate carried interest on a deal by deal basis. This practice was gradually replaced by aggregation, however, and today the deal by deal approach is virtually non-existent. The reason for the disappearance of the deal by deal approach was that it "...is fundamentally dysfunctional from an alignment of interest perspective. It tends to create a bias in favor of higher risk and potentially higher return investments. The only cost to a General Partner if losses are realized on a particular investment are reputational and the General Partner's share of the capital applied to the particular investment." (Schell, 2006, pp. 2.12-2.13). This observation is very much in line with the intuition of our theoretical model.

Third, the model suggests that financial structure of private equity fund will be such that most investments require a combination of ex ante financing, that is raised at the time the fund is formed, and ex post financing, that is raised deal by deal. The advantage of ex ante financing is that it improves incentives by for pooling across deals, while ex post financing implicitly relies on the capital markets to take account of public information about the current state of the economy as a whole, or even of a particular industry. In fact, investments financed by the private equity industry typically do rely on both kinds of financing. Buyouts are typically leveraged to a substantial degree, receiving debt from banks and other sources. Venture deals are often syndicated, with a lead venture capitalist raising funds from partners, who presumably take account of, at a minimum, information on the state of the economy and industry in the same way that banks providing financing to buyouts do. Our model also implies that the third party financing has to be sufficiently risky for the availability of financing to be sufficiently state-contingent. This provides an explanation for some standard features of limited partnership agreements (see Gompers and Lerner, 1996). In particular, standard covenants include restrictions on any one investment to no more than a prespecified fraction of the fund's capital (usually 20%), which forces the fund to seek third-party financing for at least all investments larger than this level. Similarly, partnership agreements typically prohibit taking on leverage at the fund level, which would make debt less risky and less state-contingent relative to leveraging up each individual deal.

Fourth, the model also provides an explanation of why GPs are left with so much discretion over the investment decisions, a practice that at first may appear to be one that exacerbates potential agency problems. In fact, we show that on the contrary the discretion is an important ingredient of the fund incentive scheme, and that removing it by giving limited partners decision rights over individual deals would lower the expected quality of investments that are undertaken.

The model predicts that while these provisions will serve to mitigate governance problems to some degree, investments nonetheless will deviate from the first best. In particular, observed investments in the private equity market should be more cyclical than the first-best investments, with the already procyclical nature of investment opportunities augmented by overinvestment in

good times and underinvestment in bad times. In addition, consistent with both casual observation (the internet and biotech bubbles) as well as more formal empirical evidence, this overinvestment and underinvestment predicts that average returns to investments made during booms will be worse than returns to investments made during recessions.

The intuitions coming from our model are also consistent with other common observations about the private equity industry. First, general partners almost always are required to contribute personally 1% of the ex ante capital raised by the funds. The incentives arising from this ownership stake serve to align the interests of general and limited partners. In the context of our model, to the extent the GPs have some wealth of their own, its investment in the fund would potentially mitigate some agency problems. Still, as long as GP wealth is limited so that the fund has to rely largely on the external capital from LPs, agency problems will remain and the fund structures derived above will still be optimal. Second, there are indeed circumstances where investors do provide financing for individual deals. Sellers sometimes provide partial financing of their firms, and GPs approach LPs for coinvestment opportunities. Each of these types of financing can be thought of in terms of our model in that they all occur in circumstances where the degree of information asymmetry is likely to be low. For example, when a seller helps to finance a deal, it typically supplements bank financing and is likely to occur when the seller has better information about his firm than the bank. When funds ask LPs to coinvest, our model suggests that they should be more sophisticated LPs, who can evaluate the deal themselves and be assured that it is a good investment. Finally, in those circumstances where specific funds are raised to finance particular deals, there should be a good reason why the initiating GP did not do the entire investment by himself. One potential reason is that the fund could be constrained in the size of its investment by its charter; an example of such a situation is Exxel's acquisitions of Argencard and Norte (see Hoye and Lerner (1995), Ballve and Lerner (2001)).

However, our model falls short in that it fails to address a number of important features of private equity funds. First, private equity funds tend to be finitely-lived; we provide no rationale for such a finite life. Second, our model does not incorporate the role of general partners' personal reputations. Undoubtedly these reputations, which provide the ability for GPs to raise future funds, are a very important consideration in private equity investment decisions. Third, while one might expect much of our analysis to apply equally to hedge funds, it is not clear that it does. Hedge funds are financed predominately by levered equity and we have no explanation for this phenomenon. Fourth, we still do not fully understand the different investment incentives of private equity funds and regular firms. Indeed, if we relabel the GP as the CEO, and replace the private equity investments with internal firm projects, it seems that we would have a model of internal capital markets. Still, this analogy is limited by some important features of firms, such as the fact that firms have infinite lives and that payoffs of individual projects within a firm are difficult to disentangle and contract on. Finally, while we identify potential investment distortions arising

even when funds use the optimal financial structure, we do not have a clear understanding of what practitioners and policy-makers could conceivably do to minimize these distortions. Knowing about any conceivable such policies clearly is a potentially valuable contribution to the study of, as well as the practice of, private equity.

7. Appendix

7.1. Proof of Proposition 1

We show that the solution postulated in the proposition is Perfect Bayesian and also satisfies the Intuitive Criterion of Cho and Kreps (1987) (defined below for our particular application). The one-period problem is straight-forward and solved in the text. The repeated problem has the added feature that investor beliefs about the type of the GP may change after observing financing and investment behavior in the first period, which in turn gives an extra signalling incentive in the first period.

Therefore, we have to show that the static equilibrium solution is also an equilibrium in the second period after fly-by-night operators have been screened out. In each period and state, the GP decides whether to not seek financing, or seek financing with some security $w = \{w_I(I), w_I(Z)\}$ satisfying monotonicity and limited liability. If the GP seeks financing, the investor then chooses whether to accept and supply financing I in exchange for security w , or deny financing in which case the game ends. If the investor accepts, the GP then decides whether to invest in a firm or the risk-free asset.

We start by analyzing the set of continuation equilibria in the second period. First, it is easy to see that there can never be a separating equilibrium where different types of GPs seek financing with different securities w . This is so since the investor never breaks even on a security issued by a fly-by-nighter or a GP with a bad project, so those types will always have an incentive to mimic a good type. In a financing equilibrium where GPs issue security w , the investor will have some set of beliefs over the type of GPs that seek financing. Denote by μ the probability the investor attaches to the GP being a fly-by-night operator, where we leave μ arbitrary for now. Given μ , however, we require that in an equilibrium involving financing with security w the investor attaches probability $(1 - \mu)\alpha$ and $(1 - \mu)(1 - \alpha)$ with $\alpha \in \{\alpha_H, \alpha_L\}$ for the probabilities that a GP seeking financing has access to a good and a bad firm, respectively. We also use the following tie-breaking rules: If the GP finds a bad firm and is indifferent between investing in it or the riskfree security, he invests in the bad firm. A fly-by-night operator always invests in the riskfree security, as he earns nothing in the second period by investing in a wasteful project.

We now state the Intuitive Criterion that a financing equilibrium must satisfy in the second period. (The general definition can be found in Cho and Kreps (1987); We state the particular version that applies to our setting). Suppose security w satisfies monotonicity and limited liability,

and that investors break even under the equilibrium beliefs:

$$\begin{aligned}
& \mu w_I(I) \\
& + (1 - \mu)(1 - \alpha) (w_I(I) * 1_{I - w_I(I) > p(Z - w_I(Z))} + p w_I(Z) * 1_{I - w_I(I) \leq p(Z - w_I(Z))}) \\
& + (1 - \mu) \alpha w_I(Z) \\
\geq & I
\end{aligned}$$

where the second row reflects the fact that GPs who find bad firms risk-shift unless $I - w_I(I) > p(Z - w_I(Z))$. The security design w is a financing equilibrium satisfying the intuitive criterion if and only if there is *no* security design w' satisfying monotonicity and limited liability such that:

1. Investors would be willing to finance the deal in exchange for w' if they believe the issuing GP is good:

$$w'_I(Z) \geq I$$

2. Fly-by-night operators and GPs finding bad firms are *strictly worse off* issuing w' than they are in the postulated equilibrium, even if investors are willing to finance the deal in exchange for w' :

$$\begin{aligned}
w'_I(I) & > w_I(I) \\
\max(I - w'_I(I), p(Z - w'_I(Z))) & < \max(I - w_I(I), p(Z - w_I(Z)))
\end{aligned}$$

3. GPs finding good firms are *strictly better off* issuing w' than they are in the postulated equilibrium if investors are willing to finance the deal in exchange for w' :

$$w'_I(Z) < w_I(Z)$$

If there were such a security w' , and it was issued out of equilibrium, we assume that investors would conclude that the issuing GP must be good. If investors have that belief, good GPs would indeed be better off issuing security w' , so w cannot be an equilibrium. (To rule out w as an equilibrium, it is essential that there is a w' that is only preferred by GPs finding good firms. If we cannot rule out that GPs finding bad firms might also be better off if financed by w' , investors could rationally believe that anyone offering w' out of equilibrium is bad, so that a best response could be to not supply financing for w' .)

The following Lemma shows that it is impossible to have a financing equilibrium where GPs who find bad projects pass it up in favor of a risk-free investment.

Lemma 3. *Given beliefs μ about the set of fly-by-night operators who seek financing and invest in*

the first period, necessary and sufficient conditions for a security design w satisfying monotonicity and limited liability to be a financing equilibrium in the second period are:

$$p(Z - w_I(Z)) \geq I - w_I(I)$$

(GP's who find bad firms prefer to risk-shift) and

$$\mu w_I(I) + (1 - \mu)(\alpha + p(1 - \alpha))w_I(Z) \geq I$$

(Investors break even).

Proof: To show necessity of the first condition, suppose, contrary to the claim in the lemma, that there is an equilibrium security w such that

$$p(Z - w_I(Z)) < I - w_I(I)$$

Then, GPs who find a bad firm prefer to pass it up and invest in the risk free asset. Also, suppose that w satisfies the break even condition:

$$(\mu + (1 - \mu)(1 - \alpha))w_I(I) + (1 - \mu)\alpha w_I(Z) \geq I$$

These two conditions together with limited liability and monotonicity imply that $w_I(I) < I < w_I(Z)$. Then, there is always another security w' such that $w'_I(I) > w_I(I)$ and $I < w'_I(Z) < w_I(Z)$. It is easy to check that both fly-by-night operators and GPs who find bad firms are strictly better off in the equilibrium than if they issue security w' , while GPs who find good firms are strictly better off with security w' if investors provide financing for it. But the intuitive criterion then says that investors should attach probability 1 to a GP having a good firm if he deviates and issues security w' , and would therefore provide financing for it. That in turn means that w cannot be an equilibrium. Thus, in any financing equilibrium, GPs who find bad firms will risk-shift, and this gives the necessary break even condition as in the lemma. To show that the two conditions in the lemma are also sufficient, note that any security w' such that GPs who find good firms would strictly prefer to deviate to it must have $w'_I(Z) < w_I(Z)$. But if $p(Z - w_I(Z)) \geq I - w_I(I)$, that means that any GP who finds a bad firm would also have an incentive to deviate to w' . End proof.

The lemma implies that there can be no financing in the low state in the second period, regardless of what happened in the first period, since the unconditional project in the low state does not break even. In the high state, debt as in the proposition is a financing equilibrium for any μ as long as the unconditional project in the high state breaks even. Thus, the static solution is an equilibrium whatever happened in the first period. But then, the static solution is also an equilibrium in the first period.

7.2. Proof of Lemma 1

To implement the investment behavior in Figure 4.2, we first check that the GP always invests in good firms regardless of what other investments he has made. For any random variable $y \in \{0, I, Z\}$ resulting from investment behavior in a period, the condition for this is:

$$E(w_{GP}(Z + y)) \geq E(w_{GP}(I + y))$$

This holds automatically from the monotonicity condition. It remains to check that the GP does not invest in bad firms in period 2 after investing in a good firm in period 1, and that the GP does not invest in bad firms in period 1. Using that $w_{GP}(0) = w_{GP}(I) = w_{GP}(2I) = 0$ from the fly-by-night constraint, the incentive compatibility conditions are:

$$w_{GP}(Z + I) \geq (1 - p) w_{GP}(Z) + p w_{GP}(2Z) \quad (7.1)$$

$$\begin{aligned} & (E(\alpha) + (1 - E(\alpha))p) w_{GP}(Z + I) \quad (7.2) \\ \geq & (1 - p) E(\alpha) w_{GP}(Z) \\ & + p(1 - E(\alpha)) w_{GP}(Z + I) + pE(\alpha) w_{GP}(2Z) \end{aligned}$$

$$\begin{aligned} & (E(\alpha) + (1 - E(\alpha))p) w_{GP}(Z + I) \quad (7.3) \\ \geq & ((1 - p) E(\alpha) + 2p(1 - p)(1 - E(\alpha))) w_{GP}(Z) \\ & + (pE(\alpha) + p^2(1 - E(\alpha))) w_{GP}(2Z) \end{aligned}$$

The first condition assures that the GP does not invest in a bad firm in period 2 after investing in a good firm in period 1. The two last conditions assure that the GP does not invest in a bad firm in period 1. The two conditions differ only on the right hand side, corresponding to the two possible off-equilibrium investment decisions in period 2: Only investing in good firms in period 2 after making a bad investment in period 1 (Condition 7.2), or investing in all firms in period 2 (Condition 7.3).

Deducting $(1 - E(\alpha))p w_{GP}(Z + I)$ from both sides of Condition 7.2 and dividing by $E(\alpha)$, we see that it is identical to Condition 7.1.

Rearranging Condition 7.3, we get

$$w_{GP}(Z + I) \geq \frac{(E(\alpha) + (1 - E(\alpha))p) + (1 - E(\alpha))p}{(E(\alpha) + (1 - E(\alpha))p)} (1 - p) w_{GP}(Z) + p w_{GP}(2Z) \quad (7.4)$$

Note that this implies Condition 7.1, and is therefore a necessary and sufficient condition for

incentive compatibility.

7.3. Proof of Proposition 2:

We need to solve for optimal values of $w_{GP}(Z)$, $w_{GP}(Z + I)$, and $w_{GP}(2Z)$. We start by establishing the following lemma:

Lemma 4. *Holding $w_{GP}(Z + I)$ fixed, we should set $w_{GP}(Z)$ as low as possible in an optimal contract: $w_{GP}(Z) = \max(0, w_{GP}(Z + I) - I)$*

Proof: First note that we must have $w_{GP}(Z) \geq \max(0, w_{GP}(Z + I) - I)$ from monotonicity and limited liability. Suppose contrary to the claim that $w_{GP}(Z) > \max(0, w_{GP}(Z + I) - I)$ in an optimal contract. Then, we can relax the *IC* constraint by decreasing $w_{GP}(Z)$ without violating *M* or *FBN*. The maximand and the break even constraint are unaffected by this, since $x = Z$ does not happen in equilibrium so that $w_{GP}(Z)$ does not enter the maximand or the break even constraint. QED.

Given this, the program now becomes

$$\max_{w_{GP}(x)} E(\alpha)^2 w_{GP}(2Z) + \left(2E(\alpha)(1 - E(\alpha)) + (1 - E(\alpha))^2 p\right) w_{GP}(Z + I)$$

such that

$$\begin{aligned} & E(\alpha)^2 (2Z - w_{GP}(2Z)) && (BE) \\ & + \left(2E(\alpha)(1 - E(\alpha)) + (1 - E(\alpha))^2 p\right) (Z + I - w_{GP}(Z + I)) \\ & + (1 - E(\alpha))^2 (1 - p) I \geq 2I \end{aligned}$$

$$\begin{aligned} & (E(\alpha) + (1 - E(\alpha))p) w_{GP}(Z + I) \geq && (IC) \\ & ((1 - p)E(\alpha) + 2p(1 - p)(1 - E(\alpha))) \max(0, w_{GP}(Z + I) - I) \\ & + p(E(\alpha) + (1 - E(\alpha))p) w_{GP}(2Z) \end{aligned}$$

$$x - x' \geq w_{GP}(x) - w_{GP}(x') \geq 0 \quad \forall x, x' \text{ s.t. } x > x' \quad (M)$$

$$w_{GP}(x) = 0 \quad \forall x \text{ s.t. } x \leq 2I \quad (FBN)$$

Lemma 5. *In the optimal contract under pure ex ante financing, we will either have (1) $w_{GP}(Z + I) = Z - I$ and $w_{GP}(2Z) = Z - I + (1 - k)(Z - I)$, where $0 < k < 1$ or (2) $w_{GP}(Z + I) < Z - I$ and $w_{GP}(2Z) = w_{GP}(Z + I)$.*

Proof: Suppose not. Then, we will show that you can relax the *IC* constraint by increasing $w_{GP}(Z + I)$ and decreasing $w_{GP}(2Z)$ without violating *FBN*, *M* or *BE*. Note that if $w_{GP}(Z + I) = Z - I$ or $w_{GP}(2Z) = w_{GP}(Z + I)$, $w_{GP}(Z + I)$ cannot be increased without violating monotonicity.

Case 1: Suppose $w_{GP}(Z) = 0 > w_{GP}(Z + I) - I$. Then, increase $w_{GP}(Z + I)$ and decrease $w_{GP}(2Z)$ to keep the break even constraint and the maximand constant:

$$-dw_{GP}(2Z) = \frac{2E(\alpha)(1 - E(\alpha)) + (1 - E(\alpha))^2 p}{E(\alpha)^2} dw_{GP}(Z + I)$$

This relaxes *IC*.

Case 2: $w_{GP}(Z) = w_{GP}(Z + I) - I$. Doing the same perturbation, we show that *IC* is relaxed. Moving all terms to the LHS of *IC*, the change in the LHS is equal to

$$1 - \frac{E(\alpha) + (1 - E(\alpha))2p}{E(\alpha) + (1 - E(\alpha))p} (1 - p) + \frac{2E(\alpha)(1 - E(\alpha)) + (1 - E(\alpha))^2 p}{E(\alpha)^2} p$$

We show that this is positive. The derivative w.r.t. to p of the expression above is equal to

$$\begin{aligned} & 1 - (1 - E(\alpha)) \frac{(1 - 2p)E(\alpha) - p^2(1 - E(\alpha))}{(E(\alpha) + (1 - E(\alpha))p)^2} + \frac{2E(\alpha)(1 - E(\alpha)) + 2(1 - E(\alpha))(1 - E(\alpha))p}{E(\alpha)^2} \\ & > - (1 - E(\alpha)) \frac{(1 - 2p)E(\alpha)}{(E(\alpha) + (1 - E(\alpha))p)^2} + \frac{2E(\alpha)(1 - E(\alpha))}{E(\alpha)^2} \\ & > 0 \end{aligned}$$

This follows since $E(\alpha)^2 \leq (E(\alpha) + (1 - E(\alpha))p)^2$, and since $(1 - 2p) < 2$. Thus, if the change is non-negative for $p = 0$, *IC* is relaxed. Substituting for $p = 0$, the change becomes zero. *QED*

Using the above results and the fact that $w_I(x) = x - w_{GP}(x)$, we see that the optimal investor security w_I is a combination of debt with face value $w_I(Z + I) \geq 2I$, and an equity piece $w_I(2Z) - w_I(Z + I)$ given by

$$\begin{aligned} w_I(2Z) - w_I(Z + I) &= Z - I \quad \text{if } w_I(Z + I) > 2I \\ w_I(2Z) - w_I(Z + I) &\in [0, Z - I] \quad \text{if } w_I(Z + I) = 2I \end{aligned}$$

, so that $w_I(x)$ is as in the proposition.

We now show that the equilibrium is always implementable as long as it generates social surplus. Suppose you give the GP the following contract:

$$\begin{aligned} w_{GP}(Z) &= 0 \\ w_{GP}(Z + I) &= \varepsilon \\ w_{GP}(2Z) &= \varepsilon \end{aligned}$$

For $\varepsilon > 0$, the *IC* condition holds strictly. Making ε small, an arbitrarily large fraction of cash flows can be given to investors, and *M* and *FBN* hold. Therefore, the *BE* condition can always be made to hold as long as the equilibrium creates social surplus.

End proof.

7.4. Proof of Proposition 3:

First, we show that if $p > \frac{1}{2}$, the condition in the proposition is necessary and sufficient for the LP to get positive rents. Then, we show that when $p < \frac{1}{2}$, the GP captures all the rents.

Suppose $p > \frac{1}{2}$. Then, we must have $Z < 2I$, or else investing in the bad firm is a positive NPV project. This also implies that $w_{GP}(Z) = 0$ from the fly-by-night condition, so that the IC constraint becomes

$$w_{GP}(Z + I) \geq pw_{GP}(2Z)$$

Suppose we set

$$\begin{aligned} w_{GP}(Z + I) &= Z - I \\ w_{GP}(2Z) &= \frac{Z - I}{p} \end{aligned}$$

Note that this is the maximal values of $w_{GP}(Z + I)$ and $w_{GP}(2Z)$ such that the IC constraint is satisfied. In turn, that means that if the break even constraint of the LP is slack at this contract, LPs will earn strictly positive rents for any incentive compatible contract. Plugging in the contract above into the break even constraint gives

$$E(\alpha)^2 \left(2Z - \frac{Z - I}{p} \right) + \left(2E(\alpha)(1 - E(\alpha)) + (1 - E(\alpha))^2 p \right) 2I + (1 - E(\alpha))^2 (1 - p) I \geq 2I$$

which can be rewritten as

$$E(\alpha)^2 \left(2Z - \frac{Z - I}{p} \right) + (1 - E(\alpha))^2 (1 - p) I \geq \left(E(\alpha)^2 + (1 - E(\alpha))^2 (1 - p) \right) 2I$$

Dividing by $(1 - E(\alpha))^2$ and gathering terms gives the condition in the proposition. If this condition is not satisfied, decreasing $w_{GP}(Z + I)$ and $w_{GP}(2Z)$ while keeping the IC constraint constant can be made until the LP just breaks even, so in that case the GP captures all the rent.

We now show that for $p < \frac{1}{2}$, the GP captures all the surplus.

Case 1: $Z \leq 2I$. Again, $w_{GP}(Z) = 0$ from the fly-by-night condition, so that the IC constraint is

$$w_{GP}(Z + I) \geq pw_{GP}(2Z)$$

Note that if we set

$$\begin{aligned} w_{GP}(Z + I) &= k(Z - I) \\ w_{GP}(2Z) &= k2(Z - I) \end{aligned}$$

for $k \in [0, 1]$ the IC constraint is satisfied since $p < \frac{1}{2}$. Then, there is always a k such that LPs just break even if the social surplus is positive, since at $k = 1$ they do not break even and at $k = 0$ they get the whole social surplus. Thus, the GP captures all the surplus.

Case 2: $Z > 2I$. For this case, suppose we set $w_{GP}(Z + I) = Z - I$ and, according to Lemma 4, $w_{GP}(Z) = Z - 2I$. The break even constraint of the LP then becomes

$$E(\alpha)^2(2Z - w_{GP}(2Z)) + \left(2E(\alpha)(1 - E(\alpha)) + (1 - E(\alpha))^2 p\right) 2I + (1 - E(\alpha))^2(1 - p)I \geq 2I$$

Suppose we force this to hold with equality and solve for $w_{GP}(2Z)$:

$$E(\alpha)^2(2Z - w_{GP}(2Z)) + (1 - E(\alpha))^2(1 - p)I = \left(E(\alpha)^2 + (1 - E(\alpha))^2(1 - p)\right) 2I$$

\Leftrightarrow

$$w_{GP}(2Z) = 2(Z - I) - \left(\frac{1 - E(\alpha)}{E(\alpha)}\right)^2 (1 - p)I \quad (7.5)$$

First, suppose $w_{GP}(2Z)$ as defined above is lower than $Z - I$, in which case monotonicity is violated. Then, it is easy to verify that there is always an $x < Z - I$ such that we can set

$$\begin{aligned} w_{GP}(2Z) &= w_{GP}(Z + I) = x \\ w_{GP}(Z) &= \max(0, x - I) \end{aligned}$$

and such that the break even constraint is satisfied with equality and the IC constraint is slack. Suppose instead that $w_{GP}(2Z)$ as defined in Equation 7.5 is bigger than or equal to $Z - I$, so that monotonicity is not violated. We now show that the IC constraint is satisfied for this contract, so that the GP captures all the surplus. Plugging in for $w_{GP}(2Z)$ from above, the IC constraint is slack if

$$\begin{aligned} Z - I &\geq \frac{1 + \frac{1 - E(\alpha)}{E(\alpha)} 2p}{1 + \frac{1 - E(\alpha)}{E(\alpha)} p} (1 - p)(Z - 2I) \\ &\quad + p \left(2(Z - I) - \left(\frac{1 - E(\alpha)}{E(\alpha)}\right)^2 (1 - p)I \right) \end{aligned}$$

Taking the derivative of the right hand side with respect to $x \equiv \frac{1 - E(\alpha)}{E(\alpha)}$ gives

$$\frac{2p(1 + xp) - p(1 + 2xp)}{(1 + xp)^2} (1 - p)(Z - 2I) - 2px(1 - p)I \quad (7.6)$$

which has the same sign as

$$\frac{Z - 2I}{(1 + xp)^2} - 2xI$$

This is decreasing in x . Thus, if it is negative for the lowest possible x , it is always negative. The lowest possible $x \equiv \frac{1-E(\alpha)}{E(\alpha)}$ is derived from Condition 3.1 as

$$x = \frac{1 - \frac{I}{Z}}{\frac{I}{Z} - p}$$

Plugging this into Expression 7.6 gives

$$\begin{aligned} & \frac{Z - 2I}{(1 + xp)^2} - 2xI \\ &= \frac{Z - 2I}{\left(1 + \frac{1 - \frac{I}{Z}}{\frac{I}{Z} - p} p\right)^2} - 2 \frac{1 - \frac{I}{Z}}{\frac{I}{Z} - p} I \\ &= \frac{Z - 2I}{\left(1 + \frac{1 - \frac{I}{Z}}{\frac{I}{Z} - p} p\right)^2} - 2 \frac{Z - I}{1 - \frac{Z}{I} p} < 0 \end{aligned}$$

Thus, the derivative w.r.t. to $\frac{1-E(\alpha)}{E(\alpha)}$ is everywhere negative, and we should set $\frac{1-E(\alpha)}{E(\alpha)}$ as low as possible to make it hard to satisfy the *IC* constraint.

Plugging $\frac{1-E(\alpha)}{E(\alpha)} = \frac{1 - \frac{I}{Z}}{\frac{I}{Z} - p}$ into the *IC* constraint gives

$$\begin{aligned} Z - I &\geq \frac{1 + \frac{1 - \frac{I}{Z}}{\frac{I}{Z} - p} 2p}{1 + \frac{1 - \frac{I}{Z}}{\frac{I}{Z} - p} p} (1 - p) (Z - 2I) \\ &\quad + p \left(2(Z - I) - \left(\frac{1 - \frac{I}{Z}}{\frac{I}{Z} - p} \right)^2 (1 - p) I \right) \end{aligned}$$

which can be rewritten as

$$\left(\frac{Z - I}{I - Zp} \right)^2 (1 - p) I + \frac{I - Zp}{p} > (Z - 2I) \left(\frac{Z - I}{I} \right)$$

Noting that

$$\frac{Z - I}{I - Zp} > \frac{Z - I}{I}$$

it is harder to satisfy the constraint if we divide the LHS by $\frac{Z-I}{I-Zp}$ and the RHS with $\frac{Z-I}{I}$, which

gives

$$\begin{aligned} \left(\frac{Z - I}{I - Zp} \right) (1 - p) I + \frac{I - Zp}{p} &> Z - 2I \iff \\ (Z - I) \frac{1 - p}{1 - \frac{Z}{I}p} + \frac{I - Zp}{p} &> Z - 2I \end{aligned}$$

This always holds, since

$$\frac{1 - p}{1 - \frac{Z}{I}p} > 1$$

. Thus, the *IC* constraint is always satisfied when the investor just breaks even, which shows that the GP captures the whole surplus.

7.5. Proof of Proposition 4:

In the first period, and after no investment in the second, a necessary condition for the ex post security is that $w_{P,i}(I) \geq I - K$. Otherwise, a fly-by-night operator can collude with LP's, raise money from ex post investors, and invest it in the risk-free asset so that the coalition between the LPs and the fly-by-night operator gets strictly positive surplus. Thus, such a contract would attract an infinite set of fly-by-night operators so that ex post investors cannot break even. This pins down debt as the only feasible security in the first period under the postulated equilibrium investment behavior, where $w_{P,1}(Z) = I - K$. For the second period, if no investment was made, any ex post security with $w_{P,2}(Z) = \frac{I-K}{\alpha_H + (1-\alpha_H)p}$, $w_{P,2}(I) \in \left[I - K, \min \left(\frac{I-K}{\alpha_H + (1-\alpha_H)p}, I \right) \right]$ is equivalent, so we might as well restrict attention to debt.

It remains to analyze the situation in the second period where an investment was made in period 1. Under the equilibrium investment behavior, the period 1 investment should have been in a good firm, and all fly-by-night operators should be screened out. Also, if the GP finds a bad firm, he should either not raise financing, or raise financing and invest in the risk-free asset. For this to be incentive compatible, it has to be the case that either

$$w_{GP}(Z - (I - K) + K) > 0 \tag{7.7}$$

or

$$w_{GP}(Z - (I - K) + I - w_{P,2}(I)) > 0 \tag{7.8}$$

Otherwise, the GP is strictly better off investing in the bad project. Suppose first that $w_{P,2}(I) < I - K$. Then, condition 7.8 holds automatically from monotonicity if condition 7.7 holds, and so

must always hold. Furthermore, we have to have

$$\begin{aligned} w_{GP}(Z - (I - K) + I - w_{P,2}(I)) &\geq w_{GP}(Z - (I - K) + K) \\ w_I(Z - (I - K) + I - w_{P,2}(I)) &\geq w_I(Z - (I - K) + K) \end{aligned}$$

with at least one of these inequalities strict. Thus, the GP and LP individually are weakly better off, and seen as a coalition are strictly better off raising capital $I - K$ from the ex post investor and investing it in the risk-free security than not raising any money. We therefore assume that the GP will raise money in this situation. Assume that the GP does raise money and invests in the risk-free asset by issuing security $w_{P,2}$. Then, we have to have $w_{P,2}(Z) > I - K$ for ex post investors to break even. But then we can apply the logic in the proof of Proposition 1 and show that this security does not satisfy the Cho and Kreps intuitive criterion, because a GP finding a good firm always has an incentive to deviate and issue a security with $w'_{P,2}(Z) = w_{P,2}(Z) - \varepsilon_1$, $w'_{P,2}(I) = w_{P,2}(Z) + \varepsilon_2$ for some $\varepsilon_1, \varepsilon_2 > 0$. Thus, we have to have $w_{P,2}(I) = I - K$. Since only GPs finding good firms are supposed to invest, we have to have $w_{P,2}(Z) = I - K$. Thus, debt is the only possible security.

7.6. Proof of Lemma 2:

If the GP invested in a good firm in period 1, he will pass up a bad firm if:

$$\begin{aligned} &w_{GP}(Z - (I - K) + K) \\ &> pw_{GP}(2(Z - (I - K))) + (1 - p)w_{GP}(Z - (I - K)) \end{aligned} \tag{7.9}$$

The last term is the case where the bad firm does not pay off, and the fund defaults on its period 2 ex post debt.

We also have to check the off-equilibrium behavior where the GP invested in a bad firm in period 1. If the GP invested in a bad firm in period 1 he will pass up a bad firm in period 2 if:

$$\begin{aligned} &pw_{GP}(Z - (I - K) + K) + (1 - p)w_{GP}(K) \\ &> p^2w_{GP}(2(Z - (I - K))) + p(1 - p)w_{GP}(Z - (I - K)) \\ &\quad + (1 - p)pw_{GP}(Z - (I - K)) \end{aligned}$$

The two last terms are, respectively, the case where the first bad firm pays off and the second does not, and the case where the first bad firm does not pay off and the second does. Since

$w_{GP}(K) = 0$ from the fly by night condition, this can be rewritten as

$$\begin{aligned} & w_{GP}(Z - (I - K) + K) \\ & > pw_{GP}(2(Z - (I - K))) + 2(1 - p)w_{GP}(Z - (I - K)) \end{aligned} \quad (7.10)$$

Note that this is a stricter condition than condition 7.9.

Given the period two incentive compatibility constraints, we can now consider the GP's investment incentives in the first period. In period 1, it is always optimal to invest in a good project. We must check that the GP does not want to invest in a bad project to sustain the separating equilibrium.

The condition for not investing in a bad project in period 1 becomes

$$\begin{aligned} & q(\alpha_H + (1 - \alpha_H)p)w_{GP}\left(Z - \frac{I - K}{\alpha_H + (1 - \alpha_H)p} + K\right) \\ & > E(\alpha)(pw_{GP}(2(Z - (I - K))) + (1 - p)w_{GP}(Z - (I - K))) + (1 - E(\alpha)) * \\ & \quad * p \max(w_{GP}(Z - (I - K) + K), pw_{GP}(2(Z - (I - K))) + 2(1 - p)w_{GP}(Z - (I - K))) \end{aligned}$$

The last line is the GP pay off when he has invested in a bad firm in period 1 and encounters another bad firm in period 2, in which case he will either invest in it or not, depending on whether Condition 7.10 holds or not. Note that this condition implies condition 7.9, since

$$w_{GP}\left(Z - \frac{I - K}{\alpha_H + (1 - \alpha_H)p} + K\right) \leq w_{GP}(Z - (I - K) + K)$$

and

$$\begin{aligned} & \frac{E(\alpha)}{q(\alpha_H + (1 - \alpha_H)p)}(pw_{GP}(2(Z - (I - K))) + (1 - p)w_{GP}(Z - (I - K))) \\ & + \frac{(1 - E(\alpha))p}{q(\alpha_H + (1 - \alpha_H)p)} \max\left(\begin{array}{l} w_{GP}(Z - (I - K) + K), pw_{GP}(2(Z - (I - K))) \\ + 2(1 - p)w_{GP}(Z - (I - K)) \end{array}\right) \\ & \geq \frac{(E(\alpha) + (1 - E(\alpha))p)}{q(\alpha_H + (1 - \alpha_H)p)}(pw_{GP}(2(Z - (I - K))) + (1 - p)w_{GP}(Z - (I - K))) \\ & \geq pw_{GP}(2(Z - (I - K))) + (1 - p)w_{GP}(Z - (I - K)) \end{aligned}$$

Thus, the only relevant incentive constraint is the period 1 IC constraint.

7.7. Proof of Proposition 5:

The full maximization problem can now be expressed as

$$\begin{aligned}
& \max E(w_{GP}(x)) \\
&= E(\alpha)^2 w_{GP}(2(Z - (I - K))) + E(\alpha)(1 - E(\alpha)) w_{GP}(Z - (I - K) + K) \\
&\quad + (1 - E(\alpha)) q(\alpha_H + (1 - \alpha_H)p) w_{GP}\left(Z - \frac{I - K}{\alpha_H + (1 - \alpha_H)p} + K\right)
\end{aligned}$$

such that

$$E(x - w_{GP}(x)) \geq 2K \quad (BE)$$

$$\begin{aligned}
& q(\alpha_H + (1 - \alpha_H)p) w_{GP}\left(Z - \frac{I - K}{\alpha_H + (1 - \alpha_H)p} + K\right) \\
> & E(\alpha)(p w_{GP}(2(Z - (I - K))) + (1 - p) w_{GP}(Z - (I - K))) + (1 - E(\alpha)) * \\
& * p \max[w_{GP}(Z - (I - K) + K), p w_{GP}(2(Z - (I - K))) + 2(1 - p) w_{GP}(Z - (I - K))] \quad (IC)
\end{aligned}$$

$$x - x' \geq w_{GP}(x) - w_{GP}(x') \geq 0 \quad \forall x, x' \text{ s.t. } x > x' \quad (M)$$

$$w_{GP}(x) = 0 \quad \forall x \text{ s.t. } x \leq 2K, \quad (FBN)$$

and

$$I - (\alpha_H + (1 - \alpha_H)p)Z \leq K \leq I - (\alpha_L + (1 - \alpha_L)p)Z$$

7.7.1. Proof that $K = I - (\alpha_L + (1 - \alpha_L)p)Z$:

We want to show that the ex ante capital K should be set maximal at $K^* = I - (\alpha_L + (1 - \alpha_L)p)Z$. First, we have to have $Z - \frac{I-K}{\alpha_H + (1 - \alpha_H)p} + K > Z - (I - K)$ and $Z - \frac{I-K}{\alpha_H + (1 - \alpha_H)p} + K > 2K$ for the equilibrium to be feasible, or else the IC condition will not be satisfied. Suppose this is true, so that cash-flow states are ordered by

$$\begin{aligned}
& 2(Z - (I - K)) \\
> & Z - (I - K) + K \\
> & Z - \frac{I - K}{\alpha_H + (1 - \alpha_H)p} + K \\
> & \max(Z - (I - K), 2K) \\
> & K
\end{aligned}$$

Suppose contrary to the claim in the proposition that $K < K^*$ at some candidate optimal contract w_I satisfying monotonicity and limited liability.

Now suppose we increase K by Δ arbitrarily small, increase $w_I(K)$ by Δ , increase $w_I(2K)$ by 2Δ , increase $w_I(Z - (I - K))$ by

$$\begin{aligned} \Delta & \text{ if } w_I(Z - (I - K)) = Z - (I - K) \\ 2\Delta & \text{ if } w_I(Z - (I - K)) < Z - (I - K) \end{aligned}$$

and increase $w_I\left(Z - \frac{I-K}{\alpha_H + (1-\alpha_H)p} + K\right)$, $w_I(Z - (I - K) + K)$, and $w_I(2(Z - (I - K)))$ by $B \in \left(2\Delta, \Delta + \frac{\Delta}{\alpha_H + (1-\alpha_H)p}\right)$ such that the break even constraint and the maximand are unchanged:

$$\begin{aligned} & (B - 2\Delta) \left(E(\alpha)^2 + E(\alpha)(1 - E(\alpha)) + (1 - E(\alpha))q(\alpha_H + (1 - \alpha_H)p) \right) \\ & = \Delta(1 - E(\alpha))q(1 - \alpha_H)(1 - p) \end{aligned}$$

Note that for small Δ , these changes do not violate monotonicity or the fly-by-night condition. However, the IC constraint is weakly relaxed, since $w_{GP}\left(Z - \frac{I-K}{\alpha_H + (1-\alpha_H)p} + K\right)$ goes up weakly and $w_{GP}(Z - (I - K) + K)$ and $w_{GP}(2(Z - (I - K)))$ go down weakly. Hence, the problem is relaxed, and we can increase K without loss of generality. Thus, there is no loss of generality from setting $K = K^*$ in an optimal contract. End Proof.

7.7.2. Proof of optimal contract:

The second issue is how the investor and GP securities should be designed. We will derive the securities under two different cases.

7.7.3. Case 1: $Z - (I - K^*) \leq 2K^*$

This is the case when the GP gets no payoff if he fails with one project, so $w_{GP}(Z - (I - K^*)) = 0$. For this case, the IC condition reduces to

$$\begin{aligned} & q(\alpha_H + (1 - \alpha_H)p)w_{GP}\left(Z - \frac{I - K^*}{\alpha_H + (1 - \alpha_H)p} + K^*\right) \\ & > E(\alpha)pw_{GP}(2(Z - (I - K^*))) \\ & \quad + (1 - E(\alpha))p \max(w_{GP}(Z - (I - K^*) + K^*), pw_{GP}(2(Z - (I - K^*)))) \end{aligned}$$

Given a certain expected payoff $E(x - w_{GP}(x))$ to investors, the optimal contract should relax the IC condition maximally without violating the fly-by-night condition or the monotonicity constraints. Any decrease of $w_{GP}(2(Z - (I - K^*)))$ or $w_{GP}(Z - (I - K^*) + K^*)$ and increase of $w_{GP}\left(Z - \frac{I-K^*}{\alpha_H + (1-\alpha_H)p} + K^*\right)$ that keeps the expected value of the security constant relaxes the constraint. The optimal contract is given in the following proposition:

Proposition 8. : Suppose $Z - (I - K^*) \leq 2K^*$. The optimal investor security $w_I(x)$ is debt with face value $F = w_I\left(Z - \frac{I-K^*}{\alpha_H+(1-\alpha_H)p} + K^*\right) \in \left[2K^*, Z - \frac{I-K^*}{\alpha_H+(1-\alpha_H)p} + K^*\right)$ plus a carry $k(\max(x - S, 0))$ starting at $S \in \left[Z - \frac{I-K^*}{\alpha_H+(1-\alpha_H)p} + K^*, Z - (I - K^*) + K^*\right]$. For $F = 2K^*$, we have $k \in (0, 1)$, $S \in \left[Z - \frac{I-K^*}{\alpha_H+(1-\alpha_H)p} + K^*, Z - (I - K^*) + K^*\right]$ and for $F > 2K^*$, we have $k = 1$ (call option) and $S = Z - \frac{I-K^*}{\alpha_H+(1-\alpha_H)p} + K^*$. For a fixed expected value $E(w_I(x))$ given to investors, F is set minimal.

Proof: First, suppose $w_I\left(Z - \frac{I-K^*}{\alpha_H+(1-\alpha_H)p} + K^*\right) > 2K^*$. The optimal contract in the proposition then claims that

$$\begin{aligned} w_I(Z - (I - K^*) + K^*) &= w_I\left(Z - \frac{I - K^*}{\alpha_H + (1 - \alpha_H)p} + K^*\right) + (I - K^*) - \frac{I - K^*}{\alpha_H + (1 - \alpha_H)p} \\ w_I(2(Z - (I - K^*))) &= w_I(Z - (I - K^*) + K^*) + Z - I \end{aligned}$$

Suppose this is not true. First, suppose

$$\begin{aligned} w_I(Z - (I - K^*) + K^*) &< w_I\left(Z - \frac{I - K^*}{\alpha_H + (1 - \alpha_H)p} + K^*\right) + (I - K^*) - \frac{I - K^*}{\alpha_H + (1 - \alpha_H)p} \\ w_I(2(Z - (I - K^*))) &\leq w_I(Z - (I - K^*) + K^*) + Z - I \end{aligned}$$

Then, we can increase $w_I(Z - (I - K^*) + K^*)$ and decrease $w_I\left(Z - \frac{I-K^*}{\alpha_H+(1-\alpha_H)p} + K^*\right)$ (which means we decrease $w_{GP}(Z - (I - K^*) + K^*)$ and increase $w_{GP}\left(Z - \frac{I-K^*}{\alpha_H+(1-\alpha_H)p} + K^*\right)$) to keep the break even constraint and the maximand constant without violating monotonicity. This relaxes the IC constraint and so improves the contract.

Now, suppose

$$\begin{aligned} w_I(Z - (I - K^*) + K^*) &= w_I\left(Z - \frac{I - K^*}{\alpha_H + (1 - \alpha_H)p} + K^*\right) + (I - K^*) - \frac{I - K^*}{\alpha_H + (1 - \alpha_H)p} \\ w_I(2(Z - (I - K^*))) &< w_I(Z - (I - K^*) + K^*) + Z - I \end{aligned}$$

Then, we can increase $w_I(2(Z - (I - K^*)))$ by ε and decrease $w_I(Z - (I - K^*) + K^*)$ and $w_I\left(Z - \frac{I-K^*}{\alpha_H+(1-\alpha_H)p} + K^*\right)$ by

$$\frac{\varepsilon E(\alpha)^2}{E(\alpha)(1 - E(\alpha)) + (1 - E(\alpha))q(\alpha_H + (1 - \alpha_H)p)}$$

to keep the break even constraint and the maximand constant without violating monotonicity. This relaxes the IC constraint and so improves the contract.

Next suppose $w_I\left(Z - \frac{I-K^*}{\alpha_H+(1-\alpha_H)p} + K^*\right) = 2K^*$. Then, $w_I\left(Z - \frac{I-K^*}{\alpha_H+(1-\alpha_H)p} + K^*\right)$ cannot be lowered without violating the fly by night condition.

First, note that increasing $w_I (2(Z - (I - K^*)))$ by ε and reducing $w_I (Z - (I - K^*) + K^*)$ by

$$\varepsilon \frac{E(\alpha)}{(1 - E(\alpha))}$$

to keep the break even constraint constant leaves the IC constraint unchanged if

$$w_{GP} (Z - (I - K^*) + K^*) > pw_{GP} (2(Z - (I - K^*)))$$

and relaxes it if

$$w_{GP} (Z - (I - K^*) + K^*) < pw_{GP} (2(Z - (I - K^*)))$$

. Therefore, if such a transfer does not violate monotonicity, it (weakly) relaxes the IC constraint. Thus, a contract that maximally relaxes the IC constraint keeping the expected value $E(w)$ constant should have

$$w_I (2(Z - (I - K^*))) = w_I (Z - (I - K^*) + K^*) + Z - I$$

if $w(X(Z, K^*)) > 2K^*$. However, for such a contract we have

$$\begin{aligned} pw_{GP} (2(Z - (I - K^*))) &= p[2(Z - (I - K^*)) - (w_I (Z - (I - K^*) + K^*)) + Z - I] \\ &= pw_{GP} (Z - (I - K^*) + K^*) \\ &< w_{GP} (Z - (I - K^*) + K^*) \end{aligned}$$

and therefore the IC constraint is unchanged if we lower $w_I (2(Z - (I - K^*)))$ and increase $w_I (Z - (I - K^*) + K^*)$ slightly so that

$$w_I (2(Z - (I - K^*))) = w_I (Z - (I - K^*) + K^*) + k(Z - I)$$

where $k < 1$. Thus, this contract can be expressed as a carry. End proof.

7.7.4. Case 2: $Z - (I - K^*) > 2K^*$

This is the case when the GP can get some payoff even if he fails with one project, so it is possible to have $w_{GP} (Z - (I - K^*)) > 0$. It is always optimal to set $w_I (Z - (I - K^*))$ as high as possible at $\min \left(Z - (I - K^*), w_I \left(Z - \frac{I - K^*}{\alpha_H + (1 - \alpha_H)p} + K^* \right) \right)$, so the contract will have a debt piece as before with face value $w_I \left(Z - \frac{I - K^*}{\alpha_H + (1 - \alpha_H)p} + K^* \right)$. However, it is no longer true that we want to set this face value as low as possible given a fixed $E(w_I)$ by increasing the higher pay offs. This is because when we reduce the face value, we also increase the pay off to the GP if he fails with one and succeeds with one firm, which can worsen incentives. The following proposition characterizes the optimal contract.

Proposition 9. Suppose $Z - (I - K^*) > 2K^*$. The optimal investor security $w_I(x)$ is debt with face value $F = w_I \left(Z - \frac{I-K^*}{\alpha_H + (1-\alpha_H)p} + K^* \right) \in \left[2K^*, Z - \frac{I-K^*}{\alpha_H + (1-\alpha_H)p} + K^* \right)$ plus a carry $k(\max(x - S, 0))$ starting at $S \in \left[Z - \frac{I-K^*}{\alpha_H + (1-\alpha_H)p} + K^*, Z - (I - K^*) + K^* \right]$. For $S < Z - (I - K^*) + K^*$, we have $k = 1$ (call option), and for $S = Z - (I - K^*) + K^*$, we have $k \in (0, 1)$.

Proof: We start with the following Lemma:

Lemma 6. $w_I(Z - (I - K^*)) = \min \left(Z - (I - K^*), w_I \left(Z - \frac{I-K^*}{\alpha_H + (1-\alpha_H)p} + K^* \right) \right)$.

Proof. First, note that given $w_A \left(Z - \frac{I-K^*}{\alpha_H + (1-\alpha_H)p} + K^* \right)$, the highest we can set $w_A(Z - (I - K^*))$ is the expression in the lemma from monotonicity and the fact that $Z - \frac{I-K^*}{\alpha_H + (1-\alpha_H)p} + K^* > Z - (I - K^*)$ in feasible contracts. Suppose $w_A(Z - (I - K^*))$ is lower than this upper bound. Then, we can increase it without changing the break even constraint and the maximand, since the outcome $Z - (I - K^*)$ does not happen in equilibrium. This relaxes the IC constraint and so improves the contract. ■

This proves that the first piece is debt with face value $w_I \left(Z - \frac{I-K^*}{\alpha_H + (1-\alpha_H)p} + K^* \right)$.

Next, suppose $w_I(Z - (I - K^*) + K^*) > w_I \left(Z - \frac{I-K^*}{\alpha_H + (1-\alpha_H)p} + K^* \right)$. Then, the proposition states that

$$w(2(Z - (I - K^*))) = w_I(Z - (I - K^*) + K^*) + Z - I$$

which is the highest possible value for $w_I(2(Z - (I - K^*)))$ given $w_I(Z - (I - K^*) + K^*)$. Suppose this is not the case. Then, we can lower $w_I(Z - (I - K^*) + K^*)$ and increase $w_I(2(Z - (I - K^*)))$ to keep the break even constraint and the maximand constant without violating monotonicity. If

$$w_{GP}(Z - (I - K^*) + K^*) > pw_{GP}(2(Z - (I - K^*))) + 2p(1-p)w_{GP}(Z - (I - K^*))$$

this does not change the IC constraint, but if

$$w_{GP}(Z - (I - K^*) + K^*) < pw_{GP}(2(Z - (I - K^*))) + 2p(1-p)w_{GP}(Z - (I - K^*))$$

the IC constraint is relaxed and so this improves the contract. End Proof.

7.8. Proof of Proposition 7:

Proof: First, it is necessary that

$$Z + K - \frac{I - K}{\alpha_H + (1 - \alpha_H)p} > 2K$$

or else the left hand side of the IC condition 5.5 is zero from monotonicity. Second, it is necessary that

$$Z + K - \frac{I - K}{\alpha_H + (1 - \alpha_H)p} > Z - (I - K)$$

since otherwise

$$w_{GP} \left(Z - \frac{I - K}{\alpha_H + (1 - \alpha_H)p} + K \right) \leq w_{GP} (Z - (I - K))$$

This would violate the IC condition 5.5, since in that case the right hand side of the IC condition becomes

$$\begin{aligned} & E(\alpha) [pw_{GP} (2(Z - (I - K))) + (1 - p)w_{GP} (Z - (I - K))] + (1 - E(\alpha))p * \\ & \max(w_{GP} (Z - (I - K) + K), pw_{GP} (2(Z - (I - K))) + 2(1 - p)w_{GP} (Z - (I - K))) \\ & \geq (E(\alpha) + (1 - E(\alpha))p) [pw_{GP} (2(Z - (I - K))) + (1 - p)w_{GP} (Z - (I - K))] \\ & \geq (E(\alpha) + (1 - E(\alpha))p) w_{GP} \left(Z - \frac{I - K}{\alpha_H + (1 - \alpha_H)p} + K \right) \end{aligned}$$

Since

$$E(\alpha) + (1 - E(\alpha))p > q(\alpha_H + (1 - \alpha_H)p)$$

this is larger than the left hand side of the IC condition.

The two necessary conditions above can be rewritten as

$$\frac{I - K}{\alpha_H + (1 - \alpha_H)p} < Z - K$$

and

$$\frac{I - K}{\alpha_H + (1 - \alpha_H)p} < I$$

Note that both these are easier to satisfy for higher K , and by setting K maximal at K^* from Proposition 5, the conditions become

$$\frac{\alpha_L + (1 - \alpha_L)p}{\alpha_H + (1 - \alpha_H)p} Z < Z - (I - (\alpha_L + (1 - \alpha_L)p)Z)$$

and

$$\frac{\alpha_L + (1 - \alpha_L)p}{\alpha_H + (1 - \alpha_H)p} Z < I$$

These conditions together give the last expression in the proposition.

The first part from the proposition is proved as follows. The right hand side of the IC condition

5.5 is given by

$$\begin{aligned}
& E(\alpha) (pw_{GP} (2(Z - (I - K))) + (1 - p) w_{GP} (Z - (I - K))) + (1 - E(\alpha)) p * \\
& * \max (w_{GP} (Z - (I - K) + K), pw_{GP} (2(Z - (I - K))) + 2(1 - p) w_{GP} (Z - (I - K))) \\
& \geq E(\alpha) pw_{GP} (2(Z - (I - K))) + (1 - E(\alpha)) pw_{GP} (Z - (I - K) + K) \\
& \geq pw_{GP} \left(Z - \frac{I - K}{\alpha_H + (1 - \alpha_H) p} + K \right)
\end{aligned}$$

where the last step follows from monotonicity. Therefore, the IC condition can only be satisfied if

$$q(\alpha_H + (1 - \alpha_H) p) \geq p$$

Thus, this is a necessary condition for the equilibrium to be implementable. To show that it together with the other conditions are sufficient, suppose they are satisfied. Then, for ε small enough, it is always possible to set

$$\begin{aligned}
w_{GP} (Z - (I - K)) &= 0 \\
w_{GP} (2(Z - (I - K))) &= \varepsilon \\
w_{GP} \left(Z - \frac{I - K}{\alpha_H + (1 - \alpha_H) p} + K \right) &= \varepsilon \\
w_{GP} (Z - (I - K) + K) &= \varepsilon
\end{aligned}$$

For this contract, the IC condition reduces to

$$q(\alpha_H + (1 - \alpha_H) p) \geq p$$

For ε small enough, investors always break even as long as social surplus is created. End Proof.

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