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ABSTRACT

Regulations that restrict pollution by firms also affect decisions about use of labor and capital. They thus affect relative factor prices, total production, and output prices. For non-revenue-raising environmental mandates, what are the general equilibrium impacts on the wage, the return to capital, and relative output prices? Perhaps surprisingly, we cannot find any existing analytical literature addressing that question. This paper starts with the standard two-sector tax incidence model and modifies one sector to include pollution as a factor of production that can be a complement or substitute for labor or for capital. We then look not at taxes but at four types of mandates, and for each mandate determine conditions that place more of the burden on labor or on capital. Stricter regulation does not always place less burden on the factor that is a better substitute for pollution. Also, a relative restriction on the amount of pollution per unit of output creates an "output-subsidy effect" on factor prices that can offset and reverse the traditional output effect and substitution effect. An analogous effect is found for a relative restriction on pollution per unit of capital.

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Much literature compares the efficiency properties of environmental policies, generally finding that incentives like taxes or permits are more cost-effective than mandates – at least in the case where firms are heterogeneous and government cannot tailor mandates to each firm. In contrast, the literature on the distributional effects of such policies is limited. Some papers identify demographic characteristics or locations of households in jurisdictions that are differentially affected by environmental protection, while others look at the burdens on households that buy products made more expensive by those regulations.[1] All of these papers ignore effects of environmental policies on the wage rate and the return to capital – both of which also affect real incomes. Yet, restrictive command and control (CAC) regulations can simultaneously affect both the product prices and factor prices.

Of course, the public economics literature since Harberger (1962) is replete with analytical general equilibrium studies of the incidence of taxation. A few papers look at the incidence of environmental taxes, where the question is about how the burden of collecting the revenue is distributed.[2] Mandates do not have revenue whose burden can be distributed. Yet CAC mandates clearly interfere with firms' decisions about use of labor, use of capital, the amount to produce, and the price to charge. We therefore find it surprising that we cannot find any analytical general equilibrium model of the incidence of non-revenue-raising environmental regulations, with simultaneous effects on the sources side of income (relative factor prices) and the uses side (product prices).[3]

To begin such a literature, this paper starts with rudimentary models in the style of Harberger (1962), with two competitive sectors and constant returns to scale, but we add the important complication that the "dirty" sector uses three inputs to production: labor, capital, and pollution. Thus, any two of these inputs can be complements or substitutes. The "clean" sector uses only labor and capital, which are in fixed supply but

[1] Examples in the first category include Becker (2004) or Sieg et al (2004). Those in the second category include Gianessi and Peskin (1980), Robison (1985), Metcalf (1999), or West and Williams (2004).

[2] The incidence of a pollution tax is studied by e.g. Yohe (1979), Bovenberg and Goulder (1997), Chua (2003), and Fullerton and Heutel (2007).

[3] In a trade model with fixed output prices, Das and Das (2007) show the effect on factor prices from one type of mandate (a pollution quota). Here, we find both factor prices and output prices in a closed economy for four types of mandate. Yet results would differ in open economy. Computable general equilibrium (CGE) models calculate effects of regulations on factor prices as well as goods prices, including Hazilla and Kopp (1990), Jorgenson and Wilcoxen (1990), Goulder et al (1997), Burtraw et. al. (2001), and Fischer and Fox (2004). These CGE studies can provide elaborate simulations with numerical magnitudes. Some include simple analytical models. But none provide general analytical models with closed form solutions for any parameter values that show effects of mandates on the wage, return to capital, and output prices.

perfectly mobile between sectors. We then solve models representing four types of non-revenue-raising policies: the handout of pollution permits, a restriction on the "absolute" quantity of pollution, a "relative" standard on pollution per unit of output, and a relative standard on pollution per unit of an input (such as capital).

We start with results for permits and quotas, because they are easy to understand. First, they restrict the amount of pollution, so they create scarcity rents. The incidence of these scarcity rents may be fairly obvious: gains accrue to whatever entity is handed the permits or the rights to the restricted quantity of pollution. And the incidence on labor or capital may be understood using two effects already identified by Mieszkowski (1967). First, the "substitution effect" raises the return to whichever factor is a better substitute for pollution (and injures the relative complement to pollution). Second, the "output effect" reduces the return to whichever factor is used intensively in the regulated sector.

We then extend the simple model to consider relative standards. Often regulators limit the ratio of emissions to output, a restriction that firms can meet partly by reducing emissions in the numerator and partly by increasing output in the denominator. We derive closed form solutions for each price change, and we identify an additional term we call the "output-subsidy effect". This effect *helps* whichever factor is intensively used in the dirty sector, and it dominates the usual output effect under plausible conditions we identify. The important point is that government-imposed costs on the dirty sector may place *less* burden on the factor intensively used there. Finally, we model a restriction on the ratio of emissions to capital, where solutions are used to identify another new term we call the "capital-subsidy effect." Since this restriction can be satisfied partly by increased use of capital, this effect tends to raise the relative return to capital (or, more generally, the return to any input in the denominator of the restricted ratio). We also find effects on the output prices to show burdens on the uses side of income.

Thus, we see standard principles of tax incidence at play, but we introduce other effects specific to mandates. And this analysis can be applied beyond environmental policy to any restriction on use of inputs. For example, our three inputs could be interpreted as labor, capital, and land. Then agricultural policy may restrict use of land per unit output, or urban zoning rules may restrict building heights (capital per unit land).

Section 1 reviews mandates in actual policymaking, and it reviews some of the literature analyzing them. Section 2 then introduces our model. Section 3 looks at permits, while Section 4 looks at absolute quotas. We then turn to the main contributions of our paper. Section 5 models a restriction on pollution per unit of output, and Section 6 restricts pollution per unit input. Section 7 offers concluding remarks.

1. Review of Environmental Mandates and Modeling

Since the earlier years of their existence, environmental regulations have most often been command-and-control (CAC) mandates rather than incentives like pollution taxes or tradable permits. Those mandates, though, can take many forms for different industries. Historically, the Water Quality Act of 1965 was the first national policy requiring that states set water quality standards, determine maximum discharge limits, and allocate non-tradable quotas. Thus, it is an absolute quantity restriction. Then the Federal Water Pollution Control Act of 1972 authorized the EPA to set effluent standards that are based on technological factors. In this case, each facility faced restrictions based on the type of facility and perhaps on capital use or output.

The 1970 Clean Air Act Amendments (CAAA) set national ambient air quality standards, and states that do not meet them are forced to create specific implementation plans. These plans often differ greatly from each other. Modeling the Clean Air Act as a single limit on emissions is difficult, except perhaps for the national emissions standards for new facilities under the New Source Performance Standards of the 1970 CAAA. Like the water standards, these air pollution regulations are technology-based, that is, determined by the current state of abatement technology.

Tradable emissions permits are becoming more popular, including the 1990 CAAA national market for sulfur dioxide (SO_2) emissions, the market in the northeast for nitrogen oxide (NO_X), the South Coast Air Quality Management District permit system for SO_2 and NO_X in the Los Angeles area launched in January 1994, and the seven states that have established emissions credit programs for NO_X and volatile organic compounds (VOC) since 1989 under the EPA's emissions trading framework. These tradable permits may achieve the same net level of environmental benefits as technology mandates, and perhaps more cheaply, but they have important distributional differences.

While quotas and permits set the absolute quantity of emissions, other mandates set relative standards such emissions per unit of output or per unit of some input. These policies may be seen as more reasonable than absolute limits per firm, especially when applied to firms of various sizes. A regulator would not expect a large firm to reach the same level of emissions as a small firm. By enacting a relative policy, the regulator can avoid deciding on a specific allocation of allowed emissions levels. Because of the variety of mandates under different state implementation plans, it is difficult to pinpoint what policies have this relative form. In a 1982 survey of regulators administered by Resources for the Future, however, 97% of air pollution regulating agencies and 100% of water pollution regulating agencies said they use limits on emissions per unit of some input, and 70% of air and 50% of water agencies said they use limits on emissions per unit output (Russell et al, 1986, p. 19). Such large proportions suggest modeling some environmental policies as limits on ratios of pollution to output or to an input.

Current environmental regulations are quite complex. Title 40 of the Code of Federal Regulations lists hundreds of rules that apply to various emitters and industries, and most states have their own regulations. Some requirements are described in terms of emissions per unit output, such as the VOC standard for automobile refinish coatings that is stated in grams per liter of coating. For new producers of sulfuric acid, the emissions standard for SO_2 is 2 kg per metric ton of acid produced.[4] The Texas Commission for Environmental Quality sets standards for hazardous waste generators that are based on the amount of output produced.[5] The state of New York sets limits on fluoride emissions per unit output from aluminum reduction plants.[6] Other countries also use relative standards. In the European carbon permit market, some allocation mechanisms that have been proposed are based on the historical output of firms (Bohringer and Lange 2005).

Mandates also take the form of emissions per unit of some input. The federal standard for particulate matter emissions for fossil-fuel-fired steam generators is 43 nanograms per joule of heat input derived from fossil fuel or wood residue.[7] Other standards are stated in terms of emissions per unit heat input in Texas for electric generators and solid fossil-fuel fired steam generators.[8] Even for a particular industry, emissions standards can differ based on the technology. Emissions rates for iron and steel processes in New York depend on the technology of the plant.[9] Limits on SO_2 emissions for oil and gas producers in Texas are 25 tons/year *per facility*.[10] Standards

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[4] Code of Federal Regulations, Title 40, Chapter 1, §60.82.
[5] Texas Administrative Code, Title 30, Section 1, §335.69.
[6] New York Environmental Conservation Rules and Regulations §209.2.
[7] Code of Federal Regulations, Title 40, Chapter 1, §60.42.
[8] Texas Administrative Code, Title 30, Section 1, §117.105 and §112.8.
[9] New York Environmental Conservation Rules and Regulations, §216.3.
[10] Texas Administrative Code, Title 30, Section 1, §106.352.

per facility also apply in New York for petroleum refineries.[11] Finally, we note that agricultural policies often limit the amount of nitrogen per hectare of land.

We cannot incorporate all of these different types of mandates in a single model with clear analytical results. We can, however, model a few types of mandates and compare results, to see their differential impacts. For example, we model technology mandates and per facility standards as limits on the amount of pollution per unit capital. Our model can also be applied to other policies such as the limit on fertilizer per hectare, by reinterpreting our input of "capital" as input of "land".

The economics literature often uses a tax on pollution to summarize the effects of all environmental policies. Some papers look at quotas or permits that restrict absolute amounts of pollution. Yet actual policy rarely employs a pollution tax, and mandates typically restrict emissions per unit output or per unit of an input. The few studies of relative standards are focused on economic efficiency, not distributional effects.[12]

The most exhaustive theoretical analysis of different environmental mandates is in Helfand (1991). Her model contains a single consumption good produced using a "dirty" input that causes pollution and a "clean" one that abates pollution. The various mandates considered are: a limit on emissions, a limit on output, an upper limit on the dirty input, a lower limit on the clean input, and limits on the ratio of emissions to output or the ratio of emissions to either input. By normalizing all of these standards so that they result in the same abatement, she can compare their effects on output, each input, and firm profits. She finds that the output restriction most reduces input and output levels. The restriction on pollution itself yields the highest firm profits. In most cases, however, the signs of these changes depend on the form of the production function. Some counterintuitive results are reached as well. For instance, a standard per unit output may actually increase total emissions; the same result may occur from a standard limiting total output.[13]

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[11] New York Environmental Conservation Rules and Regulations §223.3.

[12] Hochman and Zilberman (1978) model standards as limits on emissions per unit output or per unit input. Harford and Karp (1983) compare the two policies and find that a standard per unit output is more efficient than a standard per unit input. Similarly, Thomas (1980) compares the welfare costs of different policies. Fullerton and Metcalf (2001) model a technology restriction as a limit per unit output. Fredriksson et al (2004) model environmental policy as a limit on the energy-capital ratio, citing the Corporate Average Fuel Economy Standards. None of these studies investigate distributional impacts.

[13] More recently, Jou (2004) compares quotas with emissions/output standards and finds that the former leads to less pollution. Also, Goulder et al (1999) compare efficiency effects of environmental policies in the presence of distortionary taxes. McKitrick (2001) solves for the efficiency costs of ratio standards. Aidt and Dutta (2004) develop a political economy model and find that the increasing use of incentives

While Helfand provides a number of valuable insights regarding the differences among various mandates, she does not address incidence.[14] In fact, her input supply curves are horizontal, so no policy can affect factor prices. Furthermore, the two inputs are a clean and dirty input. Even as these two input prices change, the implications are unclear for returns to labor and capital. Here, we model production as using capital, labor, and pollution. These inputs have endogenous prices, so we can capture the differential effects of environmental standards on their relative returns. Which factors gain or lose can have a large effect on what policies are chosen (Keohane et al 1998).

2. The Basic Model

Our model is similar to that in Fullerton and Heutel (2007), where we analyze a tax on emissions. For ease of exposition, we start here with the simplest version where emissions, Z, have a price, p_Z. This version is directly applicable to the first policy below (tradable permits). We then explain changes necessary to model each mandate. In this model, as in Harberger (1962), we assume a closed economy with many identical firms, perfect competition, fixed factor supplies, perfect mobility, and no uncertainty. We compare two equilibria rather than the transition period between them.[15]

One sector produces a clean good X, with price p_X, in a constant returns to scale (CRS) production function using labor and capital, $X = X(K_X, L_X)$. Totally differentiate this production function and the zero-profits condition, to get:

$$\hat{X} = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X \tag{1}$$

$$\hat{p}_X + \hat{X} = \theta_{XK}(\hat{r} + \hat{K}_X) + \theta_{XL}(\hat{w} + \hat{L}_X) \tag{2}$$

where a hat over any variable represents a proportional change (e.g. $\hat{X} \equiv dX/X$). Also, w is the wage, r is the rate of return, and θ_{Xi} is the share of production for factor i in sector X (e.g., $\theta_{XK} \equiv rK_X/Xp_X$). Then we use the definition of σ_X, the elasticity of substitution in production between capital and labor to get:

follows from increasingly high environmental goals. See also Keohane et al (1998) for a similar model of policy choice. Montero (2002) compares effects on R&D incentives, while Requate and Unold (2003) compare incentives to adopt abatement technology. Bovenberg et al (2005) see how efficiency costs of mandates and taxes are affected by a constraint to avoid adverse industry-distributional effects.

[14] Helfand and House (1995) empirically estimate the costs of different environmental policies for lettuce growers in California's Salinas Valley. They find that mandates reduce farm profits less than do taxes.

[15] Using a CGE model, Goulder and Summers (1989) analyze transitions due to imperfect factor mobility. Adding imperfect mobility or adjustment costs can alter the equilibrium outcome, not just the transition.

$$\hat{K}_X - \hat{L}_X = \sigma_X(\hat{w} - \hat{r}) \tag{3}$$

The other sector produces a dirty good Y, with price p_Y, in a CRS production function, $Y = Y(K_Y, L_Y, Z)$. Here, we use input demand equations for each of the three inputs based on all three input prices (r, w, and p_Z). Differentiate the three input demand equations and use the fact that only two of the three are independent to get:[16]

$$\hat{K}_Y = a_{KK}\hat{r} + a_{KL}\hat{w} + a_{KZ}\hat{p}_Z + \hat{Y} \tag{4}$$

$$\hat{L}_Y = a_{LK}\hat{r} + a_{LL}\hat{w} + a_{LZ}\hat{p}_Z + \hat{Y} \tag{5}$$

where a_{ij} is the elasticity of demand for factor i with respect to the price of factor j. Allen (1938) shows that $a_{ij} = \theta_{Yi}e_{ij}$, where e_{ij} is the Allen elasticity of substitution between inputs i and j. For this sector, differentiation of production and of the zero-profits condition yield:

$$\hat{Y} = \theta_{YK}\hat{K}_Y + \theta_{YL}\hat{L}_Y + \theta_{YZ}\hat{Z}\,. \tag{6}$$

$$\hat{p}_Y + \hat{Y} = \theta_{YK}(\hat{r} + \hat{K}_Y) + \theta_{YL}(\hat{w} + \hat{L}_Y) + \theta_{YZ}(\hat{p}_Z + \hat{Z}) \tag{7}$$

The resource constraint for capital is $K_Y + K_Y = \overline{K}$, where $\overline{K}$ is the fixed total capital stock. Differentiation of that and the analogous labor constraint yields:

$$\hat{K}_X\lambda_{KX} + \hat{K}_Y\lambda_{KY} = 0 \tag{8}$$

$$\hat{L}_X\lambda_{LX} + \hat{L}_Y\lambda_{LY} = 0 \tag{9}$$

where λ_{ij} is sector j's share of input i (e.g. $\lambda_{KX} \equiv K_X/\overline{K}$). Finally, preferences are modeled using the definition of σ_u, the elasticity of substitution in utility:

$$\hat{X} - \hat{Y} = \sigma_u(\hat{p}_Y - \hat{p}_X) \tag{10}$$

The clean good is chosen as numeraire, so $\hat{p}_X$ is fixed at zero, and we have ten equations for eleven unknown changes: $\hat{K}_X, \hat{K}_Y, \hat{L}_X, \hat{L}_Y, \hat{w}, \hat{r}, \hat{X}, \hat{p}_Y, \hat{Y}, \hat{p}_Z, \hat{Z}$. For each policy below, we specify one more equation or exogenous policy change. For example, an existing restriction on the number of permits implies an initial $p_Z > 0$. Then the authorities can reduce the number of permits by a small amount (e.g., $\hat{Z} = -0.01$). The

[16] See Mieszkowski (1972) for an early use of this method. Stability conditions for the input demand system are satisfied so long as the own price elasticities a_{ii} are negative, as assumed here.

system (1)-(10) can then be used to solve for the ten remaining unknowns, by successive substitution. The steps are omitted but may be requested from the authors. Here, we report only the solutions for $\hat{r}$, $\hat{w}$, $\hat{p}_Z$, and $\hat{p}_Y$. The first three of these determine the sources-side incidence of the policy, and the last determines the uses-side.

Because X is produced with no excess profit using only labor and capital, and its output price is fixed by assumption, r and w cannot both move in the same direction. If $\hat{r} = \hat{w} = 0$, the implication is not that factors bear no burdens. Rather, since p_Y may rise, $\hat{r} = \hat{w} = 0$ means that labor and capital bear real burdens in proportion to their shares of national income. Hence, a positive value for $\hat{r}$ just means that capital bears a burden that is proportionally less than that of labor.

3. Tradable Pollution Permits

Results for permits illustrate a few properties of our basic model, to set the stage for our main results regarding the incidence of relative standards. The permit policy imposes costs by forcing firms to reduce emissions or to buy permits. The mandated overall limit on pollution creates scarcity rents, however, and the distribution of those rents must be considered as part of the incidence.[17] If the permits are grandfathered to the firms, then their owners capture those scarcity rents. In addition to evaluating changes in returns to capital and labor, our model solves for changes in permit-created scarcity rents. All three of these price changes contribute to the sources-side incidence.

The general solutions are presented in Table 1, where we assume the denominator D is positive.[18] The factor price equations demonstrate effects first identified by Mieszkowski (1967). The first term in the curly brackets, $\sigma_u(\gamma_K - \gamma_L)$, is his "output effect": the policy $\hat{Z} < 0$ raises the cost of production and thus reduces output in a way that depends on consumer preferences σ_u. Then if Y is capital intensive, $(\gamma_K - \gamma_L)>0$, the output effect reduces r and raises w. The other terms represent a "substitution effect"

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[17] Parry (2004) uses a partial equilibrium model to calculate incidence, including the distribution of scarcity rents created by emissions permits for carbon, SO_X, and NO_X.

[18] The denominator is likely to be positive, except in perverse cases that are not the subject of this paper. In the case with equal factor intensities ($\gamma_K = \gamma_L = \gamma$), for example, a sufficient condition is that all of the Allen cross-price elasticities are positive. In fact, that condition is stronger than necessary; D is positive unless

$$\text{"Condition 1":} \quad e_{KL} < \frac{-\sigma_X[\theta_{YZ}\sigma_u + (\theta_{XL}\gamma + \theta_{YL})e_{LZ} + (\theta_{XK}\gamma + \theta_{YK})e_{KZ}] - A\theta_{XL}\theta_{YK}(Ge_{KK} + Fe_{LL})}{A\theta_{XL}\theta_{YK}(F+G)}.$$

The right side of this inequality must be negative, so Condition 1 says that e_{KL} is even more negative. Thus, D>0 does not require that K and L are substitutes, but only that they not be "too complementary".

involving the Allen elasticities. If capital is a better substitute for pollution than is labor ($e_{KZ} > e_{LZ}$), then the restriction $\hat{Z} < 0$ is more likely to help capital.

Table 1: Incidence of Absolute Quantity Restriction

$$\hat{r} = \frac{\theta_{YZ}\theta_{XL}}{D}\{\sigma_u(\gamma_K - \gamma_L) - e_{KZ}\gamma_K(1+\gamma_L) + e_{LZ}\gamma_L(1+\gamma_K)\}\hat{Z},$$

$$\hat{w} = \frac{\theta_{YZ}\theta_{XK}}{D}\{-\sigma_u(\gamma_K - \gamma_L) + e_{KZ}\gamma_K(1+\gamma_L) - e_{LZ}\gamma_L(1+\gamma_K)\}\hat{Z},$$

$$\hat{p}_Z = F^{-1}\{(\frac{1}{D})(G\gamma_L(1+\gamma_K) - F\gamma_K(1+\gamma_L))(\frac{\sigma_X}{A}(C+\beta_L) + \theta_{YL}\theta_{XK}(e_{KL} - e_{KK})) - \frac{\gamma_L(1+\gamma_K)}{A}\}\hat{Z}$$

$$\hat{p}_Y = \{[G\gamma_L(1+\gamma_K) - F\gamma_K(1+\gamma_L)][\frac{\theta_{YZ}}{D}(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK}) + \frac{\sigma_X(C+\beta_L) + A\theta_{YL}\theta_{XK}(e_{KL} - e_{KK})}{AFD}] - \frac{\theta_{YZ}\gamma_L(1+\gamma_K)}{FA}\}\hat{Z}$$

where: $\gamma_L \equiv \frac{\lambda_{LY}}{\lambda_{LX}} = \frac{L_Y}{L_X} > 0$, $\gamma_K \equiv \frac{\lambda_{KY}}{\lambda_{KX}} = \frac{K_Y}{K_X} > 0$, $\beta_K \equiv \theta_{XK}\gamma_K + \theta_{YK} > 0$,

$\beta_L \equiv \theta_{XL}\gamma_L + \theta_{YL} > 0$, $A \equiv \gamma_L\beta_K + \gamma_K\beta_L > 0$, $C \equiv \beta_K\theta_{YL} - \beta_L\theta_{YK}$,

$F \equiv \sigma_u(\frac{\gamma_L(1-\theta_{YK}) - \gamma_K\theta_{YL}}{A}) + e_{KZ}$, $G \equiv \sigma_u(\frac{\gamma_K(1-\theta_{YL}) - \gamma_L\theta_{YK}}{A}) + e_{LZ}$, and

$$D \equiv \sigma_u[\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK}][G(A - \gamma_L(1+\gamma_K)) - F(A - \gamma_K(1+\gamma_L))] + \sigma_X[G(C+\beta_L) + F(\beta_K - C)] + A\theta_{XL}\theta_{YK}[Fe_{KL} - Ge_{KK}] - A\theta_{YL}\theta_{XK}[Fe_{LL} - Ge_{KL}]$$

Next, in Table 1, the expression for $\hat{p}_Z$ seems quite complicated, but the final term inside the curly brackets, $\gamma_L(1+\gamma_K)/A$, is unambiguously positive and could be called a "direct effect": it reflects a downward-sloping demand for emissions permits, so the leftward shift of the vertical supply curve tends to raise the equilibrium permit price. Then the long first term could be called the "indirect effect," but it need not be positive. If it is sufficiently negative, then a decrease in the total permit allocation may actually *decrease* the permit price. The conditions under which this counterintuitive effect occurs are quite cumbersome and difficult to interpret, and hence they are not presented here.

Yet the effect is analogous to previous findings that an increase in the pollution tax can lead to an *increase* in emissions.[19]

Yet, unlike the incidence on labor and capital owners, the incidence on permit holders is not determined solely by the change in their factor price. Labor and capital are in fixed total supply and earn net returns determined by w and r, but the supply of permits has just been restricted by the policy ($\hat{Z} < 0$). The total return to permit holders is $p_Z \cdot Z$, and the proportional change in this product is $\hat{p}_Z + \hat{Z}$. Even if the policy raises the price p_Z, then permit holders are still not necessarily better off.

Furthermore, even the uses-side incidence result ($\hat{p}_Y$) is ambiguous. The final term in the curly brackets is a "direct effect" on the cost of production, indicating that a decrease in the number of permits tends to increase the price of the dirty good. However, the long previous term is an "indirect effect" that cannot be signed. It allows for another counterintuitive result: reducing the number of emissions permits may hurt consumers of the clean good more than consumers of the dirty good. To see if these ambiguities can be resolved, we next look at two special cases: equal factor intensities (to isolate the substitution effect), and no substitution in the dirty sector (to see the output effect).

3.1 Equal Factor Intensities

Suppose that γ_L and γ_K are equal to each other, and let their common value be γ. Note that this condition implies $L_Y/L_X = K_Y/K_X$. The output effect then disappears, and the substitution effect simplifies. Unfortunately, this special case does little to simplify the long expressions for $\hat{p}_Z$ and $\hat{p}_Y$, but for factor prices we have:

$$\hat{r} = -\frac{\theta_{YZ}\theta_{XL}}{D}\gamma(1+\gamma)(e_{KZ} - e_{LZ})\hat{Z}$$

$$\hat{w} = \frac{\theta_{YZ}\theta_{XK}}{D}\gamma(1+\gamma)(e_{KZ} - e_{LZ})\hat{Z}$$

The denominator in the general solution reduces a bit, but $D>0$ still requires Condition 1 above. If so, we reach a definitive conclusion about the effect of the regulation on r and

[19] See DeMooij and Bovenberg (1998) or Fullerton and Heutel (2007). This example is comparable to the "Edgeworth Taxation Paradox" studied in Hotelling (1932), where the imposition of a tax on a good can reduce its price to consumers and increase their purchases. Though Hotelling's model is not perfectly analogous to the one here, his inequalities (25) and (26) present conditions when the paradox holds; they similarly involve cross-price demand elasticities.

w. When emissions must be reduced, the dirty sector wants to substitute into both labor and capital, but if labor is a better substitute for pollution ($e_{LZ} > e_{KZ}$), then labor is hurt relatively less (i.e. $\hat{r} < 0$ and $\hat{w} > 0$).

3.2 No Substitution in Dirty Sector

We now let the factor intensities of the two sectors differ, but we assume the dirty sector cannot substitute among its inputs ($e_{ij} = 0$ for all i, j). While this assumption is clearly restrictive, it allows us to isolate the impact of factor intensities. The denominator D then simplifies to $\theta_{YZ}\sigma_X\sigma_u$, and the substitution effects in $\hat{r}$ and $\hat{w}$ disappear. Again $\hat{p}_Z$ and $\hat{p}_Y$ are not much simplified, but the changes in factor prices become:

$$\hat{r} = \frac{\theta_{XL}}{\sigma_X}(\gamma_K - \gamma_L)\hat{Z}$$

$$\hat{w} = -\frac{\theta_{XK}}{\sigma_X}(\gamma_K - \gamma_L)\hat{Z}$$

Here, the denominator is always positive. The sources side incidence includes only an output effect, determined by the sign of $\gamma_K - \gamma_L$. If the dirty sector is capital-intensive, this term is positive. Since $\hat{Z} < 0$, the rental rate falls and the wage rises. The magnitude of this effect is mediated by σ_X: if the clean industry can easily substitute between capital and labor, then these effects on input prices become smaller, since the clean sector can more easily accommodate the additional labor or capital.

4. Command and Control Restrictions on Firm-Specific Pollution Quantities

We started with tradable pollution permits above, because the permit market is easy to comprehend with a vertical supply, a downward-sloping demand, and many identical firms that each can buy as many permits as desired at the equilibrium market price p_Z. Then all firms in the dirty industry have symmetric demands for the three inputs (K, L, Z) based on the three input prices (r, w, p_Z).

We next consider briefly the case where *each* firm faces a restriction on its use of Z. Pollution has no market clearing price p_Z, but each firm with a restriction on Z can be said to face a shadow price p_Z. Each firm gets an allocation of permits that are not tradable. In our model with many identical firms, however, the firms cannot gain from trade. With constant returns to scale, each firm's labor and capital can adjust to its allocation of nontradable permits in a way that is equivalent to the transfer of permits to some other firm using that same labor and capital. In other words, firm-specific restrictions on pollution levels in this model yield the same results as we just derived for tradable permits. Equations above can be used for effects on total dirty-industry use of labor and capital and for consequent economy-wide returns to labor and capital.

5. "Performance Standard": Emissions per unit Output

An alternative form of environmental policy is to limit the ratio of emissions to output, a policy we call a "performance standard". With heterogeneous firm sizes, at least some consideration of this ratio seems necessary for a plausible policy. A large producer cannot reasonably be expected to achieve the same limit on emissions as a small firm. Considerations like these are also taken into account in other policies, such as a fixed number of tradable permits that are initially allocated according to market share. If firm-specific emission limits are tied to the firm's output level, even implicitly, then the policy may have no absolute limit on total emissions. Instead, total emissions vary with total output in a way that affects incentives and prices.

This model uses the same production functions as the previous model. Behavior in the clean sector is unchanged, with equations (1), (2), and (3). Total capital and labor are still fixed by equations (8) and (9), and consumer preferences are unchanged in (10). The only changes involve incentives facing firms in the dirty sector. Their behavior is:

$$\max_{K_Y, L_Y, Z} \quad p_Y Y(K_Y, L_Y, Z) - rK_Y - wL_Y$$

subject to the constraint $Z/Y \leq \delta$. The firms pay no explicit price for the input Z. Instead, their use of that input is limited by their output. The constraint must bind, since the production function is monotone increasing in all inputs. Solving the firms' first order conditions and rearranging terms yields $r = \dfrac{p_Y Y_K}{1-\delta Y_Z}$ and $w = \dfrac{p_Y Y_L}{1-\delta Y_Z}$, where subscripts on Y denote marginal products. The firm does not set the marginal value of an input equal to the input price, as it would without the performance standard, because of the denominator in these two equations. This denominator is less than one, so the marginal value of the factor is set lower than its input price. In other words, the firm wants to proceed further down its factor demand curves, using more labor and capital in order to increase output and qualify for an increase in valuable emission rights.

Tighter regulation means a decrease in δ. Totally differentiating the production function yields an equation analogous to (6) in the previous model:

$$\hat{Y} = \theta_{YK}(1-\nu)\hat{K}_Y + \theta_{YL}(1-\nu)\hat{L}_Y + \nu\hat{Z}\,, \tag{6'}$$

where $\nu \equiv \delta Y_Z = Y_Z Z/Y$. In the prior model with (6), an increase in a factor would raise output in proportion to its factor share. Now, since the marginal product of each factor is reduced by $(1 - \nu)$, its marginal contribution to output is reduced by $(1 - \nu)$. An increase in emissions Z raises output in proportion to $\nu = \delta Y_Z$, to reflect its marginal product Y_Z and its factor share $\delta = Z/Y$. Emission rights are valuable, of course, but firms do not pay for them through an explicit price. Instead, they pay for emission rights by paying factors more than their marginal products.

The assumptions of perfect competition and free entry/exit lead to a zero profit condition in the previous model. This condition remains under the policy specified here, though it takes a different form. Since costs no longer include the price of emission permits, the final term in equation (7) is dropped. The zero profit condition thus implies

$$\hat{p}_Y + \hat{Y} = \theta_{YK}(\hat{r} + \hat{K}_Y) + \theta_{YL}(\hat{w} + \hat{L}_Y)\,. \tag{7'}$$

The constraint may impose a "shadow price" on the factor Z, but since no explicit price is paid for that input, it is not included in the profits equation.[20]

Finally, we must replace equations (4) and (5) with their counterparts under the new policy. Input demand equations can no longer be functions of output and three explicit input prices (r, w, p_Z). Instead, we write input demand equations as functions of r, w, δ, and Y. Then totally differentiate these equations to get:

$$\hat{K}_Y = b_{KK}\hat{r} + b_{KL}\hat{w} + b_{KZ}\hat{\delta} + \hat{Y}$$

$$\hat{L}_Y = b_{LK}\hat{r} + b_{LL}\hat{w} + b_{LZ}\hat{\delta} + \hat{Y}$$

$$\hat{Z} = b_{ZK}\hat{r} + b_{ZL}\hat{w} + b_{ZZ}\hat{\delta} + \hat{Y}\,.$$

Notice that the b_{ij} appear in a form similar to the a_{ij} parameters in (4) and (5). They both represent input demand elasticities. For example, either a_{KL} or b_{KL} is the percent change in capital for a one percent change in the wage. However, a_{KL} is that response in the

[20] This alters the dynamic of firm entry and exit, but since our concern is general equilibrium effects and not the transition periods leading up to them, it does not affect our results.

first model holding p_Z and Y constant (so Z can change), while b_{KL} is that response in the model holding δ and Y constant (so $Z = \delta Y$ cannot change).[21]

The third equation giving the input demand for Z can be simplified greatly. We know that the constraint binds, so $Z = \delta Y$. Then total differentiation yields:

$$\hat{Z} = \hat{\delta} + \hat{Y}\,, \tag{5'}$$

which implies that $b_{ZK} = b_{ZL} = 0$, and $b_{ZZ} = 1$. Since only two of the three equations are independent, we subtract the second equation from the first and get

$$\hat{K}_Y - \hat{L}_Y = b_r\hat{r} + b_w\hat{w} + b_\delta\hat{\delta}\,, \tag{4'}$$

where $b_r \equiv b_{KK} - b_{LK}$, $b_w \equiv b_{KL} - b_{LL}$, and $b_\delta \equiv b_{KZ} - b_{LZ}$. In Appendix A1, we derive expressions for the b_{ij} elasticities in terms of e_{ij} and other parameters. Both b_{KK} and b_{LL} are negative, since increasing the price of a factor decreases its demand, even with the constraint on $\delta = Z/Y$. The Appendix also shows that the cross-price b_{ij} are positive ($i,j = K,L$), whether or not capital and labor are substitutes as defined by the sign of the Allen cross-price elasticity.[22] Thus, in (4'), a higher wage increases the capital/labor ratio ($b_w > 0$), and higher price of capital reduces it ($b_r < 0$). In fact, the Appendix shows that $b_r = -b_w$. Finally, it shows that $b_\delta \equiv b_{KZ} - b_{LZ}$ has the opposite sign of $e_{KZ} - e_{LZ}$. A tighter regulation means δ is decreased, and less pollution is allowed per unit output. If capital is a better substitute for pollution than is labor, that is, if $e_{KZ} > e_{LZ}$, then more capital must be used relative to labor ($b_{KZ} < b_{LZ}$, and hence b_δ is negative).

For this model we now have ten equations: (1), (2), (3), (4'), (5'), (6'), (7'), (8), (9), and (10). As before, we set $\hat{p}_X = 0$ and solve for the changes in returns to capital and labor attributable to a small change in the policy variable (δ). The solutions are presented in Table 2. Compared to the general solutions in Table 1, these equations are not as complicated. First, the denominator D is positive-definite. Second, expressions for $\hat{r}$ and $\hat{w}$ can be decomposed into three terms, each corresponding to a single effect. The

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[21] The b_{ij} elasticities (and the c_{ij} elasticities from the following section) are conceptually similar to the direct and indirect substitution effects of Ogaki (1990).

[22] Why is complementarity ruled out in this case? The Allen elasticities are defined for the input demand functions where all inputs are allowed to vary. Raising the price of labor w may then decrease the demand for capital, if the two inputs are complements, but the firm would be forced to increase its other input, pollution. Here, however, the third input demand equation ($\hat{Z} = \hat{\delta} + \hat{Y}$) indicates that a change in w, with no change in δ or Y, cannot change Z. Only labor and capital can vary, so they must be substitutes.

last term is the "substitution effect", since it involves $b_\delta \equiv b_{KZ} - b_{LZ}$ (which has the opposite sign of $e_{KZ} - e_{LZ}$). A reduction in $\delta = Z/Y$ raises r if capital is better than labor as a substitute for emissions ($e_{KZ} > e_{LZ}$). The second term is the "output effect" including $\sigma_u(\gamma_K - \gamma_L)$. This effect hurts capital if Y is capital intensive. Here, however, the first term is a new effect we call an "output-subsidy effect": since the policy mandates a lower *ratio* of pollution to output, it can be satisfied partially by increasing output – which *helps* the factor used intensively.[23]

Table 2: Performance Standard (Restriction on *Z/Y*)
$\hat{r} = [-\frac{\theta_{XL}\nu}{D}(\gamma_K - \gamma_L) + \frac{\theta_{XL}\nu\sigma_u}{D}(\gamma_K - \gamma_L) + \frac{\theta_{XL}\eta}{D}b_\delta]\hat{\delta}$
$\hat{w} = [\frac{\theta_{XK}\nu}{D}(\gamma_K - \gamma_L) - \frac{\theta_{XK}\nu\sigma_u}{D}(\gamma_K - \gamma_L) - \frac{\theta_{XK}\eta}{D}b_\delta]\hat{\delta}$
$\hat{p}_Y = \{\frac{1}{D}(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK})(-\nu(1-\sigma_u)(\gamma_K - \gamma_L) + \eta b_\delta) - \frac{\nu}{1-\nu}\}\hat{\delta}$
where $\eta \equiv (\theta_{YK}\gamma_L + \theta_{YL}\gamma_K + 1)(1-\nu) > 0$ and
$D \equiv (1-\nu)\sigma_u(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK})(\gamma_K - \gamma_L) + \eta[b_w + \sigma_X(\theta_{XL}\gamma_L + \theta_{XK}\gamma_K)] > 0$

Note that the first two terms have opposite signs and differ only by the appearance of σ_u in the output effect. Thus, the usual output effect dominates only when $\sigma_u > 1$, which raises an important question about the likely size of σ_u. This parameter has never been estimated for this particular aggregation, where the "dirty good" could represent a composite of gasoline, heating fuel, electricity, and all goods that make intensive use of fossil fuels. Yet all of these goods are usually found to have relatively low demand elasticities, which would imply that σ_u is less than one. If so, then the new "output subsidy effect" found here dominates the usual output effect, and tighter environmental policy places *less* burden on the factor that is used intensively in the dirty sector.

[23] In fact, the restriction on Z/Y could be *modeled* as a combination of a tax on emissions plus subsidy to output, which might be more consistent with the usual studies of incentive policies. We choose to model mandates directly as quantity constraints, however, for several reasons. First, they *are* quantity constraints. Second, these mandates do not raise revenue, so the equivalent incentive policy combination would require a complicated calculation of the endogenous output subsidy rate necessary to return all revenue from the emissions tax. Third, we want specifically to see what insights can be obtained by tackling the problem of quantity constraints in a way that is different from the usual model of incentive policies.

For three reasons, this paper omits a section to assign numerical parameter values. First, Fullerton and Heutel (2007) already provide plausible parameter values that readers can insert into formulas here.[24] Second, however, parameter values are unnecessary. The relative impact of these two effects depends entirely on the size of σ_u, which is plausibly less than one. Third, numerical values would sidetrack readers from our main point, which is purely conceptual. We have identified a new effect that likely reverses the usual output effect, and it indicates the importance of further research to estimate σ_u.

In the equation for $\hat{p}_Y$ in Table 2, the last term is a "direct effect" that raises the price of the dirty good. Again, however, the "indirect effect" is ambiguous. It is complicated by the fact that producers have incentive to sell *more* of this good, to qualify for more pollution rights. In this case, the ambiguity can be resolved in special cases.

5.1 Equal Factor Intensities

Since the output effect and output-subsidy effect operate through differential factor intensities, the assumption $\gamma_K = \gamma_L = \gamma$ makes them both disappear. Then only the third term for the substitution effect remains in $\hat{r}$ and $\hat{w}$. The solutions reduce to:

$$\hat{r} = \frac{\theta_{XL}\gamma b_\delta}{\sigma_X + \gamma b_w}\hat{\delta}$$

$$\hat{w} = \frac{-\theta_{XK}\gamma b_\delta}{\sigma_X + \gamma b_w}\hat{\delta}$$

$$\hat{p}_Y = \frac{-\nu}{1-\nu}\hat{\delta}\,.$$

In this case, the factor that is a relative substitute for pollution is burdened less by a strengthening of environmental policy ($\hat{\delta} < 0$). If labor is the better substitute for pollution ($b_\delta > 0$), then the wage rises and the return to capital falls.[25] This case also provides unambiguous results for incidence on the uses side of income. Only the "direct effect" remains in the expression for $\hat{p}_Y$. A tightening of environmental policy increases the price of the dirty good, hurting those who buy more than average amounts of it.

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[24] For example, industries that pollute more tend to be capital-intensive (Antweiler et. al. 2001, p. 879). Also, some estimates of production functions suggest that capital may be a better substitute for pollution than is labor (DeMooij and Bovenberg 1998, Considine and Larson 2006).

[25] This simple intuition could fail in the permits case, but here the denominator cannot be negative.

5.2 No Substitution Effect in Dirty Sector

As we did with the previous policy, we can isolate the effect of factor intensities by assuming away differential substitution. Before, we set all a_{ij} to zero, but here we set only b_δ to zero.[26] Hence, the substitution effect is eliminated. Then solutions reduce to:

$$\hat{r} = -\frac{1}{D}\theta_{XL}\nu(1-\sigma_u)(\gamma_K - \gamma_L)\hat{\delta}$$

$$\hat{w} = \frac{1}{D}\theta_{XK}\nu(1-\sigma_u)(\gamma_K - \gamma_L)\hat{\delta}$$

$$\hat{p}_Y = [-(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK})\nu(1-\sigma_u)(\gamma_K - \gamma_L)\frac{1}{D} - \frac{\nu}{1-\nu}]\hat{\delta},$$

where $D \equiv (1-\nu)\sigma_u(\theta_{XL}\theta_{YK} - \theta_{YL}\theta_{XK})(\gamma_K - \gamma_K) + \eta\sigma_X(\theta_{XL}\gamma_L + \theta_{XK}\gamma_K) > 0$.

In the first two expressions, we combine the "output effect" and the "output-subsidy effect" from Table 2. If the dirty sector is capital intensive, so that $(\gamma_K - \gamma_L) > 0$, then capital is hurt more than labor only if σ_u is greater than one. This special case does not remove the ambiguity in $\hat{p}_Y$ however.

Table 3: The Sign of Each Effect on the Rate of Return to Capital [a]

Type of Restriction	Substitution Effect	Output Effect	Output-Subsidy Effect	Capital-Subsidy Effect
Quantity Z	$e_{KZ} - e_{LZ}$	$(\gamma_L - \gamma_K)$		
Ratio Z/Y	$e_{KZ} - e_{LZ}$	$(\gamma_L - \gamma_K)$	$(\gamma_K - \gamma_L)$	
Ratio Z/K_Y	$e_{KZ} - e_{LZ}$	$(\gamma_L - \gamma_K)$	$(\gamma_K - \gamma_L)$	$e_{KL} - e_{KK}$

[a] The effect on the wage rate always has the opposite sign.

All of the effects in this paper are summarized in Table 3. For the policy in each row, the sign of the term in each cell indicates the sign of that column's effect on the rate of return to capital, r. Because all of these policies restrict Z in some fashion, the substitution effect in the first column always raises r if capital is better than labor as a substitute for emissions ($e_{KZ} > e_{LZ}$). The output effect always raises r if the impacted sector is labor intensive ($\gamma_L > \gamma_K$). Those two effects follow Harberger (1962) and

[26] We cannot set all b_{ij} elasticities to zero, since Appendix A1 shows that some of them are of definite sign.

Mieszkowski (1967), and they still pertain to all mandates here. Yet this section has analyzed a restriction in emissions per unit of output (Z/Y) and found an "output-subsidy effect". It raises r if the impacted sector is capital intensive. The next section analyzes a restriction of emissions per unit of capital (Z/K_Y), and it identifies another new effect.

6. "Technology Mandate": Emissions per unit Input

Whereas the previous section examines a limit on emissions per unit output, we now examine a regulation that limits emissions per unit of an input. Such limits are common, as described in our first section above. We have only two clean inputs in our model, so we capture the nature of a limit on emissions per unit input by modeling a limit on emissions per unit of capital. We refer to this policy as a "technology mandate", since forcing the adoption of a particular technology in production may effectively fix the emissions/capital ratio. Capital and labor are each in fixed supply and mobile between sectors, so they are perfectly symmetric in this model. Thus, the results for a limit per unit labor can be obtained directly from results below by interchanging every K and L (as well as every w and r).

As with other policies considered earlier, the equations that describe the behavior of consumers and of producers of the clean good do not change here. Equations (1), (2), (3), (8), (9), and (10) fall into this category and are applicable to this section. The only aspect of the model that requires revision is the behavior of producers of the dirty good. Consider their maximization problem. As in the previous policy considered, firms pay no explicit price for the pollution input. Instead, they face an exogenous ceiling on their ratio of emissions to capital. Formally, this problem is

$$\max_{K_Y, L_Y, Z} \quad p_Y Y(K_Y, L_Y, Z) - rK_Y - wL_Y$$

subject to the constraint $Z/K_Y \leq \zeta$. A tightening of environmental policy is defined as a decrease in ζ. It is clear that the policy constraint binds: since firms pay no price per unit of pollution, and this input is productive, they will employ as much of it as possible, an amount $Z = \zeta K_Y$. Thus, we use below the fact that $\partial Z/\partial K_Y = \zeta$. The first order conditions for the maximization problem are

$$r = p_Y(Y_K + \zeta Y_Z)$$

$$w = p_Y Y_L.$$

The second of these equations is identical to the first order condition in the original problem where firms face a price for all three inputs and no other constraint: the marginal value of labor is equal to the wage. The first equation differs from the standard condition. For the choice of capital input demanded, the marginal value of capital Y_K is *lower* than the rental rate (since ζY_Z is positive). The intuition here is that each unit of capital employed gives value to the firm in two ways. First, it increases their output directly (since $Y_K > 0$). Second, it allows more pollution, which also increases output. The second term represents this effect, since Y_Z is the marginal product of pollution and $\zeta = \partial Z/\partial K_Y$ is the pollution increase made possible by the increased capital. The value of investing in a marginal unit of capital is composed of these two terms and at the optimum is set equal to the cost of that investment, the rental rate r.

Totally differentiate the production function and substitute in these first order conditions. After dividing through by Y, we have:

$$\hat{Y} = (\theta_{YK} - \nu)\hat{K}_Y + \theta_{YL}\hat{L}_Y + \nu\hat{Z} \qquad (6'')$$

The constant ν is still equal to $Y_Z{\cdot}Z/Y$, as in the previous section. Also, as before, an increase in any one input does not generally increase output by a proportion equal to its factor share. This condition does hold for labor in (6"), since that input choice is not distorted, but the constraint does distort the choice of capital. Yet, from (6"), we see that a one percent increase in all three inputs yields a one percent increase in output, from the assumption of constant returns to scale. The zero profit condition still holds as well, even though firms do not pay for pollution, because entry and exit are still allowed. Thus equation (7') from the prior model also applies to this one.

Finally, the dirty sector's chosen amount of each input (K_Y, L_Y, and Z) depends on input prices, the policy parameter, and output (r, w, ζ, and Y). We totally differentiate these input demand equations to get:

$$\hat{K}_Y = c_{KK}\hat{r} + c_{KL}\hat{w} + c_{KZ}\hat{\zeta} + \hat{Y}$$

$$\hat{L}_Y = c_{LK}\hat{r} + c_{LL}\hat{w} + c_{LZ}\hat{\zeta} + \hat{Y}$$

$$\hat{Z} = c_{ZK}\hat{r} + c_{ZL}\hat{w} + c_{ZZ}\hat{\zeta} + \hat{Y} .$$

The elasticity of demand for input i with respect to price j is defined here as c_{ij} (but this response depends on the nature of the constraint, so the c_{ij} elasticities are not the same as the a_{ij} or b_{ij} elasticities). Only two of these equations are independent of each other, so we subtract each of the bottom two equations from the top one to get two equations to use in our solution. The first of these equations is

$$\hat{K}_Y - \hat{L}_Y = c_r \hat{r} + c_w \hat{w} + c_\zeta \hat{\zeta} \,, \tag{4''}$$

where $c_r \equiv c_{KK} - c_{LK}$, $c_w \equiv c_{KL} - c_{LL}$, and $c_\zeta \equiv c_{KZ} - c_{LZ}$. The second resulting equation can be simplified using the policy constraint $Z/K_Y = \zeta$, since total differentiation gives:

$$\hat{K}_Y - \hat{Z} = -\hat{\zeta} \,. \tag{5''}$$

Substituting this into the equations above implies that $c_{KK} - c_{ZK} = 0$, $c_{KL} - c_{ZL} = 0$, and $c_{KZ} - c_{ZZ} = -1$. These relationships are verified in Appendix A2.

Also in that Appendix, we evaluate the elasticities of input demand. An important condition for their signs relates to the relative complementarity of capital and pollution:

"Condition 2": $e_{KZ} > (e_{KK} + e_{ZZ})/2$.

The right hand side of this inequality must be negative, since all own-price elasticities are negative. This condition always holds, then, when capital and pollution are substitutes ($e_{KZ} > 0$). It also holds when capital and pollution are not "too complementary". With this condition, the Appendix shows that $c_r < 0$ and $c_w > 0$. That is, an increase in the capital rental rate must reduce the ratio K_Y/L_Y demanded, and an increase in the wage must increase it. The ratio of Z to K_Y is fixed, and so producers really have only two inputs between which they can substitute; once they choose K_Y and L_Y, then Z is given by the constraint. With only two inputs K_Y and L_Y, they must be substitutes.[27]

The system of equations containing (1), (2), (3), (4"), (5"), (6"), (7'), (8), (9), and (10) are ten equations in ten unknowns. In Table 4, these equations are solved for the proportional change in each price from an exogenous change in ζ. When Condition 2 holds, the denominator D must be positive.

These equations are strikingly similar to their counterparts for the previous policy. As before, the second term in either factor price equation is an "output effect": this policy impinges on the dirty sector, which tends to raise the output price and discourage

[27] If condition 2 fails, then $e_{KZ} < (e_{KK} + e_{ZZ})/2 < 0$, so $c_r > 0$ and $c_w < 0$. Counterintuitively, an increase in r/w then *raises* the desired K_Y/L_Y ratio. Because capital and pollution are highly complementary, the increase in r makes firms want less K *and* less Z. Wanting less Z reduces the pressure of the constraint ($Z/K_Y \leq \zeta$), which reduces the shadow price on Z (i.e., the right to emit is not so valuable). The reduced shadow price on Z by itself would mean more demand for Z and more K_Y, since they are complements. If they are *sufficiently* complementary, then the result is a net increase in capital relative to labor.

purchases. By itself, this effect would hurt capital if the sector is capital intensive. Again the first term is an "output-subsidy effect" with the opposite sign. Again it is larger than the output effect if $\sigma_u < 1$. In the prior case, however, the output-subsidy effect arises because the mandate to reduce Z/Y provides an implicit subsidy to output. Why is output subsidized here? As we show in a moment, this mandate to reduce Z/K_Y provides an implicit subsidy to the use of capital K_Y, but this subsidy itself also reduces the cost of production, and it therefore also has an output-subsidy effect in the first term.

Table 4: Technology Mandate (Restriction on Z/K_Y)
$$\hat{r} = [-\frac{\theta_{XL}\nu}{D}(\gamma_K - \gamma_L) + \frac{\theta_{XL}\nu\sigma_u}{D}(\gamma_K - \gamma_L) + \frac{\theta_{XL}\pi}{D}c_\zeta]\hat{\zeta}$$
$$\hat{w} = [\frac{\theta_{XK}\nu}{D}(\gamma_K - \gamma_L) - \frac{\theta_{XK}\nu\sigma_u}{D}(\gamma_K - \gamma_L) - \frac{\theta_{XK}\pi}{D}c_\zeta]\hat{\zeta}$$
$$\hat{p}_Y = \{\frac{1}{D}(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK})(-\nu(1-\sigma_u)(\gamma_K - \gamma_L) + \pi c_\zeta) - \nu\}\hat{\zeta}$$
where $\pi = \gamma_L(1+\gamma_K) + \theta_{YL}(\gamma_K - \gamma_L) > 0$, and
$$D \equiv \sigma_u(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK})(\gamma_K - \gamma_L) - \pi(\theta_{XL}c_r - \theta_{XK}c_w) + \sigma_X(\theta_{XK}\gamma_K + \theta_{XL}\gamma_L + 1)$$

The third term in these factor price equations depends on c_ζ, which Appendix A2 shows can itself be subdivided. In particular, it shows that c_ζ can be written as:

$$c_\zeta = M + H(e_{LZ} - e_{KZ}) + H(e_{KK} - e_{KL})$$

where M and H are both defined in the Appendix and are both positive under Condition 2. In this expression, the last term is the promised "capital-subsidy effect": it includes e_{KK}, which is always negative, so the policy ($\hat{\zeta} < 0$) has a positive effect on the use of capital and its return r. It also includes $-e_{KL}$, which operates in the same direction if labor and capital are substitutes ($e_{KL}>0$). The extent to which dirty firms can substitute away from labor and into capital helps drive up demand for K, and thus the return r. As shown in Table 3, the capital-subsidy effect raises r if $(e_{KK} - e_{KL}) < 0$.

The second term in the c_ζ expression represents the usual "substitution effect": the mandate to reduce the ratio ζ can also be satisfied partly by reducing Z in the numerator, which means substituting from Z into other inputs (K or L). If labor is a better substitute for pollution than is capital ($e_{LZ} > e_{KZ}$), then this policy induces more demand for labor than capital from the substitution effect. Then this term is positive, so multiplication by $\hat{\zeta}<0$ means that it decreases r and increases w.

In summary, the forced reduction in Z/K_Y can be satisfied partly by reducing emissions in the numerator (the substitution effect) but also partly by increasing capital in the denominator (the capital-subsidy effect). This implicit subsidy itself reduces the cost of production (the output-subsidy effect), which offsets the usual way in which regulations raise costs (the output effect). All four effects appear in Table 3.

6.1 Equal Factor Intensities

The assumption $\gamma_K = \gamma_L = \gamma$ eliminates the two output effects in Table 4. The factor price equations do not need to be repeated here, as they then contain only the third term with c_ζ (including both the substitution effect and the capital-subsidy effect). However, the output price equation reduces to:

$$\hat{p}_Y = -\nu\hat{\zeta} \ .$$

On the uses side, incidence is unambiguous. A tighter environmental policy must increase the price of the dirty good relative to the price of the clean good – due to the direct effect of the policy on the cost of production in the Y sector only.

6.2 No Substitution Effect in Dirty Sector

Here we assume that $c_r = c_w = c_\zeta = 0$, which is not quite as strong as saying that the dirty sector cannot substitute at all.[28] Instead, this assumption eliminates the capital-subsidy effect and the substitution effect. The remaining factor price equations are straight from Table 4, but without the last term (c_ζ). Thus, they still include the output effect, and the output-subsidy effect.

The output price equation also looks much like the one in Table 4, but without the c_ζ term. The last term ($-\nu$) is definitely negative, so this "direct effect" raises the cost of production and thus raises the breakeven price p_Y. The long first term is an indirect effect. Since $\gamma_K - \gamma_L$ has the same sign as $(\theta_{YK}\theta_{XL} - \theta_{YL}\theta_{XK})$, this term has the opposite sign of $(1 - \sigma_u)$. When σ_u is smaller than one, then a tighter mandate must increase the

[28] In fact, all c_{ij} cannot be zero, since we showed earlier that $c_{KZ} - c_{ZZ} = -1$.

price of good Y. When σ_u is large, however, the indirect effect can dominate the direct effect, so that a tighter mandate *decreases* the price of the dirty good.[29]

7. Conclusion

Just like taxes, regulations that restrict emissions affect producer decisions about use of labor and capital, and they thus affect relative factor prices, total production, and output prices. Existing models analyze the distribution of burdens from taxes, but this paper points out that non-revenue raising restrictions also have burdens on the sources side of income through changes in factor prices as well as burdens on the uses side through changes in output prices. Our model is based on the standard two-sector tax incidence model, but with two important modifications. First, we allow one sector to include pollution as a factor of production that can be a complement or substitute for labor or for capital. Second, we look not at taxes but at four types of mandates.

The model in this paper can be applied beyond environmental policy to analyze any regulation that restricts use of inputs. Alternatively, the model could be extended in any of the many ways that the Harberger model has been extended, for example to consider increasing returns to scale, imperfect competition, international trade, or capital mobility. Future research could consider capital formation, endogenous technology, and uncertainty. Also important is the interaction of environmental mandates with other types of regulation, especially in the highly regulated electric utility sector.

With any of those extensions, the model would become more complicated, and the price change equations might have more terms. But the effects we have uncovered here would still pertain. With no existing research on this topic at all, we believe that this simple model is the right place to begin. And even in this simple model, we get some interesting results. First, a mandate may hurt consumers of the clean good more than consumers of the dirty good. Second, we show how a mandate may burden either the factor that is a better substitute for pollution or the factor that is a relative complement to pollution. Third, restrictions on the absolute level of emissions differ from restrictions on emissions per unit output or per unit of an input. A restriction on pollution per unit of output has not only an "output effect" that burdens any factor used intensively in

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[29] In the $\hat{p}_Y$ equation, for a large indirect effect, suppose σ_u and $(\gamma_K - \gamma_L)$ are large. The sector is highly capital intensive. The output effect dominates the output-subsidy effect, so the tighter mandate means less demand for capital. Thus r falls. As seen in the $\hat{r}$ equation, large σ_u and $(\gamma_K - \gamma_L)$ mean r falls a lot. The dirty sector is highly capital intensive, so its cost of production and p_Y fall.

production, but also an "output-subsidy effect" that encourages output to help satisfy the mandated ratio. Similarly, a restriction on pollution per unit capital creates a "capital-subsidy effect" that increases demand for capital and thus raises the rental rate.

An implication is that researchers need to be careful about the nature of an environmental restriction before concluding that it injures the factor used intensively or the factor that is a better substitute for pollution. Those usual effects can be completely offset by other effects we identify in this paper.

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Appendix A1: Finding the Substitution Elasticities b_{ij}

The b_{ij} elasticities are evaluated from the production function in the dirty sector in a manner analogous to Allen (1938, p. 505-508). We are solving for the derivatives of input demands with respect to changes in either input prices or δ, the policy parameter. These input demand equations come from the firm's cost minimization problem, where the total quantity to be produced is exogenous. First consider a small change in the price of capital, dr. If we differentiate the production function with respect to r we get

$$Y_K \frac{dK_Y}{dr} + Y_L \frac{dL_Y}{dr} + Y_Z \frac{dZ}{dr} = \frac{dY}{dr} = 0,$$

where the last equation comes from the fact that total output demanded is exogenous and not a function of the rental rate.

The first order condition of the minimization problem with respect to the choice of K_Y is $\frac{p_Y}{1-\delta Y_Z} Y_K = r$. Differentiate this equation with respect to r, multiply through by $\frac{1-\delta Y_Z}{p_Y}$, and collect terms to get

$$\frac{Y_K}{p_Y}\frac{dp_Y}{dr} + [Y_{KK} + Y_K Y_{ZK}\xi]\frac{dK_Y}{dr} + [Y_{KL} + Y_K Y_{ZL}\xi]\frac{dL_Y}{dr} + [Y_{KZ} + Y_K Y_{ZK}\xi]\frac{dZ}{dr} = \frac{1-\delta Y_Z}{p_Y},$$

where $\xi \equiv \frac{\delta}{1-\delta Y_Z}$. Similarly, differentiate the next first order condition, $\frac{p_Y}{1-\delta Y_Z} Y_L = w$, with respect to r and rearrange to get

$$\frac{Y_L}{p_Y}\frac{dp_Y}{dr} + [Y_{LK} + Y_L Y_{ZK}\xi]\frac{dK_Y}{dr} + [Y_{LL} + Y_L Y_{ZL}\xi]\frac{dL_Y}{dr} + [Y_{LZ} + Y_L Y_{ZK}\xi]\frac{dZ}{dr} = 0.$$

Note that the right hand side of this equation is zero, since a change in r has no effect on w, which is exogenous to this input demand system. Finally, the policy constraint binds, so $Z = \delta Y$. Since Y and δ are both exogenous variables in the input demand system, a change in r has no effect on their values. Hence, differentiating this equation with respect to r yields $\frac{dZ}{dr} = 0$.

Writing these four equations in matrix form allows use of Cramer's rule to evaluate the derivatives. This equation is

$$\begin{bmatrix} 0 & Y_K & Y_L & Y_Z \\ Y_K & Y_{KK}+Y_K Y_{ZK}\xi & Y_{KL}+Y_K Y_{ZL}\xi & Y_{KZ}+Y_K Y_{ZZ}\xi \\ Y_L & Y_{LK}+Y_L Y_{ZK}\xi & Y_{LL}+Y_L Y_{ZL}\xi & Y_{LZ}+Y_L Y_{ZZ}\xi \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{p_Y}\frac{dp_Y}{dr} \\ dK_Y/dr \\ dL_Y/dr \\ dZ/dr \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1-\delta Y_Z}{p_Y} \\ 0 \\ 0 \end{bmatrix}$$

Follow the notation of Allen (1938) and use F to denote the determinant of the bordered Hessian of the production function, and use F_{ij} to denote the cofactor of element i,j of that matrix. The determinant of the matrix of coefficients in the above equation simplifies to F_{ZZ} (the terms with ξ all cancel each other out). With an odd number of inputs, the assumption of constant returns to scale (linear homogeneity) implies that $F < 0$ and $F_{ZZ} > 0$. Using Cramer's rule, we solve for the derivatives of interest:

$$\frac{dK_Y}{dr} = \frac{-(Y_L)^2(1-\delta Y_Z)}{p_Y F_{ZZ}} < 0 \ , \quad \frac{dL_Y}{dr} = \frac{Y_L Y_K(1-\delta Y_Z)}{p_Y F_{ZZ}} > 0 \, .$$

These sign indicate that $b_{KK} < 0$ and $b_{LK} > 0$, as we now show. The term $1 - \delta Y_Z$ is strictly positive for the following reason. The policy parameter $\delta = Z/Y$ is the inverse of *average* output per unit of Z. It is multiplied by Y_Z, the *marginal* output per unit of Z. Since production is constant returns to scale, the average output must exceed the marginal output, and hence $\delta Y_Z < 1$. Furthermore, both first derivatives of Y are positive, and $F_{ZZ} < 0$ as mentioned before. Thus $b_{KK} < 0$ and $b_{LK} > 0$.

We take the production function, the first order conditions for the cost minimization problem, and the binding constraint, and then we differentiate all, this time with respect to w. Writing these four equations in matrix form yields a similar system of equations:

$$\begin{bmatrix} 0 & Y_K & Y_L & Y_Z \\ Y_K & Y_{KK}+Y_K Y_{ZK}\xi & Y_{KL}+Y_K Y_{ZL}\xi & Y_{KZ}+Y_K Y_{ZZ}\xi \\ Y_L & Y_{LK}+Y_L Y_{ZK}\xi & Y_{LL}+Y_L Y_{ZL}\xi & Y_{LZ}+Y_L Y_{ZZ}\xi \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{p_Y}\frac{dp_Y}{dw} \\ dK_Y/dw \\ dL_Y/dw \\ dZ/dw \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{1-\delta Y_Z}{p_Y} \\ 0 \end{bmatrix} .$$

The matrix of coefficients is the same as for dr above; the only difference is in which element of the vector of constants is nonzero. Here it is the element corresponding to the differentiation of the first order condition for labor input, since w is changing. Solving this system yields

$$\frac{dK_Y}{dw} = \frac{Y_K Y_L(1-\delta Y_Z)}{p_Y F_{ZZ}} > 0 \ , \ \frac{dL_Y}{dw} = \frac{-(Y_K)^2(1-\delta Y_Z)}{p_Y F_{ZZ}} < 0 .$$

These solutions can be used to evaluate the input demand elasticities.

$$b_r = b_{KK} - b_{LK} = \frac{r}{K_Y}\frac{dK_Y}{dr} - \frac{r}{L_Y}\frac{dL_Y}{dr} = \frac{r(1-\delta Y_Z)Y_L}{p_Y F_{ZZ}}(-\frac{Y_L}{K_Y} - \frac{Y_K}{L_Y}) < 0$$

$$b_w = b_{KL} - b_{LL} = \frac{w}{K_Y}\frac{dK_Y}{dw} - \frac{w}{L_Y}\frac{dL_Y}{dw} = \frac{w(1-\delta Y_Z)Y_K}{p_Y F_{ZZ}}(\frac{Y_L}{K_Y} + \frac{Y_K}{L_Y}) > 0 .$$

We can substitute in the first order conditions $p_Y Y_K = r(1 - \delta Y_Z)$ and $p_Y Y_L = w(1 - \delta Y_Z)$ to simplify these expressions.

$$b_r = -\frac{Y_K Y_L}{F_{ZZ}}(\frac{Y_L}{K_Y} + \frac{Y_K}{L_Y}) \ , \ b_w = \frac{Y_K Y_L}{F_{ZZ}}(\frac{Y_L}{K_Y} + \frac{Y_K}{L_Y}) .$$

This substitution demonstrates that $b_r = -b_w$.

Lastly, we want to find the derivatives of factor demands with respect to a change in the policy parameter δ. Again, differentiate the production function and the first order conditions, here with respect to δ. The policy constraint $(Z = \delta Y)$ differentiated with respect to δ yields $dZ/d\delta = Y$. The matrix form of this system of equations is

$$\begin{bmatrix} 0 & Y_K & Y_L & Y_Z \\ Y_K & Y_{KK} + Y_K Y_{ZK}\xi & Y_{KL} + Y_K Y_{ZL}\xi & Y_{KZ} + Y_K Y_{ZZ}\xi \\ Y_L & Y_{LK} + Y_L Y_{ZK}\xi & Y_{LL} + Y_L Y_{ZL}\xi & Y_{LZ} + Y_L Y_{ZZ}\xi \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{p_Y}\frac{dp_Y}{d\delta} \\ dK_Y/d\delta \\ dL_Y/d\delta \\ dZ/d\delta \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{Y_Z Y_K}{1-\delta Y_Z} \\ -\frac{Y_Z Y_L}{1-\delta Y_Z} \\ Y \end{bmatrix} .$$

Again the matrix of coefficients is the same, with determinant F_{ZZ}. Solving for the derivatives of interest yields:

$$\frac{dK_Y}{d\delta} = Y\frac{F_{KZ}}{F_{ZZ}} \ , \ \frac{dL_Y}{d\delta} = Y\frac{F_{LZ}}{F_{ZZ}} ,$$

where again F_{ij} denotes the cofactor of element i,j in the bordered Hessian of the production function. These cofactors are not immediately interpretable, but they are an integral part of the definition of the Allen elasticities. They are defined as:

$e_{ij} \equiv \frac{p_Y Y}{i_Y j_Y} \cdot \frac{F_{ij}}{F}$, where i_Y is the quantity of input i used. With these definitions we can calculate the remaining input demand elasticities:

$$b_\delta = b_{KZ} - b_{LZ} = \frac{\delta}{K_Y}(Y\frac{F_{KZ}}{F_{ZZ}}) - \frac{\delta}{L_Y}(Y\frac{F_{LZ}}{F_{ZZ}}) = \frac{\delta Z}{p_Y}\frac{F}{F_{ZZ}}(e_{KZ} - e_{LZ}),$$

where e_{ij} is the Allen elasticity of substitution between inputs i and j. Since $F/F_{ZZ} < 0$, the sign of b_δ is opposite the sign of $e_{KZ} - e_{LZ}$; if capital is a better substitute for pollution than is labor, then b_δ is negative.

Appendix A2: Finding the Substitution Elasticities c_{ij}

We calculate these elasticities using a method similar to the one in Appendix A1. First, consider the effect of small changes in the capital rental rate. If we differentiate the production function with respect to r we get, as before:

$$Y_K \frac{dK_Y}{dr} + Y_L \frac{dL_Y}{dr} + Y_Z \frac{dZ}{dr} = \frac{dY}{dr} = 0$$

The first order condition from the maximization problem with respect to capital is $r = p_Y(Y_K + \zeta Y_Z)$. Differentiate this with respect to r, divide through by p_Y, and rearrange terms to get:

$$\frac{Y_K + \zeta Y_Z}{p_Y}\frac{dp_Y}{dr} + [Y_{KK} + \zeta Y_{ZK}]\frac{dK_Y}{dr} + [Y_{KL} + \zeta Y_{ZL}]\frac{dL_Y}{dr} + [Y_{KZ} + \zeta Y_{ZK}]\frac{dZ}{dr} = \frac{1}{p_Y}.$$

The first order condition for labor is $w = p_Y Y_L$. Differentiating this equation by r and similarly rearranging yields

$$\frac{Y_L}{p_Y}\frac{dp_Y}{dr} + Y_{LK}\frac{dK_Y}{dr} + Y_{LL}\frac{dL_Y}{dr} + Y_{LZ}\frac{dZ}{dr} = 0.$$

Finally, differentiate the policy constraint $Z = \zeta K_Y$ by r to obtain

$$\frac{dZ}{dr} = \zeta\frac{dK_Y}{dr}.$$

Combining these four equations into matrix form allows us to solve for any of the derivatives. This matrix equation is

$$\begin{bmatrix} 0 & Y_K & Y_L & Y_Z \\ Y_K+\zeta Y_Z & Y_{KK}+\zeta Y_{ZK} & Y_{KL}+\zeta Y_{ZL} & Y_{KZ}+\zeta Y_{ZZ} \\ Y_L & Y_{LK} & Y_{LL} & Y_{LZ} \\ 0 & \zeta & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{p_Y}\frac{dp_Y}{dr} \\ dK_Y/dr \\ dL_Y/dr \\ dZ/dr \end{bmatrix} = \begin{bmatrix} 0 \\ 1/p_Y \\ 0 \\ 0 \end{bmatrix}.$$

We solve for these derivatives using Cramer's Rule, where the denominator is the determinant of the matrix of coefficients. Call this denominator D. Solving along the bottom row, and using known properties of determinants, we get:

$$D = \zeta(F_{KZ} + \zeta(-F_{KK})) - (F_{ZZ} + \zeta(-F_{KZ})) = -\zeta^2 F_{KK} - F_{ZZ} + 2\zeta F_{KZ},$$

where the F_{ij} notation is from Allen (1938), just as in the previous section. We can solve for this denominator in terms of the Allen elasticities using their definitions:

$$D = -\zeta^2 \frac{Fe_{KK}K_Y{}^2}{p_Y Y} - \frac{Fe_{ZZ}Z^2}{p_Y Y} + 2\zeta \frac{Fe_{KZ}K_Y Z}{p_Y Y}.$$

And, since $\zeta = Z/K_Y$,

$$D = \frac{FZ^2}{p_Y Y}(-e_{KK} - e_{ZZ} + 2e_{KZ}).$$

We can sign the denominator with information about these three Allen elasticities. The ratio in the front of this expression is negative, since $F < 0$ and all of the other constants are positive. The own-price elasticities e_{KK} and e_{ZZ} must be negative. Hence, D is negative if and only if e_{KZ} is not too negative:

$$\text{Condition 2:} \quad e_{KZ} > \frac{e_{KK} + e_{ZZ}}{2}.$$

Since the right hand side of this inequality is strictly negative, a sufficient condition for D to be negative is capital and pollution are substitutes in production ($e_{KZ} > 0$). However, D is still negative if K and Z are not too complementary.

We now use Cramer's Rule to solve for the derivatives.

$$\frac{dK_Y}{dr} = \frac{1}{D}\frac{Y_L{}^2}{p_Y}, \quad \frac{dL_Y}{dr} = -\frac{1}{D}\frac{Y_L(Y_K + \zeta Y_Z)}{p_Y}.$$

When $D < 0$, then $dK_Y/dr < 0$ and $dL_Y/dr > 0$. We can also use Cramer's rule to solve for dZ/dr, but differentiation of the policy constraint provides it as a function of dK_Y/dr.

Now, we solve for the elasticities c_{KK} and c_{LK}, and the difference (which is defined as c_r in the text):

$$c_r \equiv c_{KK} - c_{LK} \equiv \frac{r}{K_Y}\frac{dK_Y}{dr} - \frac{r}{L_Y}\frac{dL_Y}{dr} = \frac{r}{K_Y}\frac{{Y_L}^2}{Dp_Y} + \frac{r}{L_Y}\frac{Y_L(Y_K + \zeta Y_Z)}{Dp_Y} = \frac{rY_L}{Dp_Y}(\frac{Y_L}{K_Y} + \frac{Y_K + \zeta Y_Z}{L_Y})$$

The sign of c_r is thus equal to the sign of D.

The same method is used to solve for the derivatives with respect to w and ζ. Differentiating the four equations with respect to w yields:

$$\begin{bmatrix} 0 & Y_K & Y_L & Y_Z \\ Y_K + \zeta Y_Z & Y_{KK} + \zeta Y_{ZK} & Y_{KL} + \zeta Y_{ZL} & Y_{KZ} + \zeta Y_{ZZ} \\ Y_L & Y_{LK} & Y_{LL} & Y_{LZ} \\ 0 & \zeta & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{p_Y}\frac{dp_Y}{dw} \\ dK_Y/dw \\ dL_Y/dw \\ dZ/dw \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1/p_Y \\ 0 \end{bmatrix}.$$

The denominator again is D. Solving for the derivatives gives:

$$\frac{dK_Y}{dw} = -\frac{1}{D}\frac{Y_L(Y_K + \zeta Y_Z)}{p_Y} \quad , \quad \frac{dL_Y}{dw} = \frac{1}{D}\frac{(Y_K + \zeta Y_Z)^2}{p_Y}.$$

So if $D < 0$, then $dK_Y/dw > 0$ and $dL_Y/dw < 0$. This gives us an expression for c_w:

$$c_w \equiv c_{LK} - c_{LL} = \frac{w}{K_Y}\frac{-Y_L(Y_K + \zeta Y_Z)}{Dp_Y} - \frac{w}{L_Y}\frac{(Y_K + \zeta Y_Z)^2}{Dp_Y} = -\frac{w(Y_K + \zeta Y_Z)}{Dp_Y}(\frac{Y_L}{K_Y} + \frac{(Y_K + \zeta Y_Z)}{L_Y})$$

The sign of c_w is the opposite of the sign of D.

Finally, we differentiate the four equations with respect to ζ to generate:

$$\begin{bmatrix} 0 & Y_K & Y_L & Y_Z \\ Y_K + \zeta Y_Z & Y_{KK} + \zeta Y_{ZK} & Y_{KL} + \zeta Y_{ZL} & Y_{KZ} + \zeta Y_{ZZ} \\ Y_L & Y_{LK} & Y_{LL} & Y_{LZ} \\ 0 & \zeta & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{p_Y}\frac{dp_Y}{d\zeta} \\ dK_Y/d\zeta \\ dL_Y/d\zeta \\ dZ/d\zeta \end{bmatrix} = \begin{bmatrix} 0 \\ -Y_Z \\ 0 \\ -K_Y \end{bmatrix}.$$

The difference on the right hand side comes from the fact that, when differentiating with respect to ζ, the term ζ can no longer be treated as a constant. For example, the policy constraint $Z = \zeta K_Y$ when differentiated yields $\frac{dZ}{d\zeta} = K_Y + \zeta \frac{dK_Y}{d\zeta}$, the bottom row of the matrix equation.

The denominator is the same as in earlier cases. Solving for the derivatives gives:

$$\frac{dK_Y}{d\zeta} = \frac{1}{D}(-Y_L{}^2 Y_Z - K_Y(F_{KZ} - \zeta F_{KK})),$$

$$\frac{dL_Y}{d\zeta} = \frac{1}{D}(Y_L Y_Z (Y_K + \zeta Y_Z) - K_Y(F_{LZ} - \zeta F_{KL})).$$

The first derivative above consists of two offsetting terms whenever capital and pollution are substitutes, since $D < 0$, $F_{KZ} < 0$, and $F_{KK} > 0$. Therefore, when policy is tightened and ζ falls, then demand for capital may fall or rise. The sign of the derivative of labor demand with respect to ζ is also ambiguous. It depends on both D and the relative magnitude of F_{KZ} and F_{LZ}, or e_{KZ} and e_{LZ}.

Solving for the elasticity $c_\zeta \equiv c_{ZK} - c_{ZL} \equiv \frac{\zeta}{K_Y}\frac{dK_Y}{d\zeta} - \frac{\zeta}{L_Y}\frac{dL_Y}{d\zeta}$, we get:

$$\begin{aligned}
c_\zeta &= \frac{\zeta}{K_Y}\frac{-Y_L{}^2 Y_Z - K_Y(F_{KZ} - \zeta F_{KK})}{D} - \frac{\zeta}{L_Y}\frac{Y_L Y_Z(Y_K + \zeta Y_Z) - K_Y(F_{LZ} - \zeta F_{KL})}{D} \\
&= \frac{\zeta}{D}(-Y_L Y_Z(\frac{Y_L}{K_Y} + \frac{Y_K + \zeta Y_Z}{L_Y}) + \frac{FK_Y Z}{p_Y Y}(-e_{KZ} + e_{KK} + e_{LZ} - e_{KL})) \\
&= M + H(e_{LZ} - e_{KZ}) + H(e_{KK} - e_{KL})
\end{aligned}$$

The constants $M \equiv -\frac{\zeta}{D}(Y_L Y_Z(\frac{Y_L}{K_Y} + \frac{Y_K + \zeta Y_Z}{L_Y}))$ and $H \equiv \frac{\zeta F K_Y Z}{D p_Y Y}$ are both positive when Condition 2 holds ($D < 0$), because $F < 0$ and all first derivatives of the production function Y_K, Y_L, and Y_Z are positive.

Finally, the text uses three relationships $c_{KK} - c_{ZK} = 0$, $c_{KL} - c_{ZL} = 0$, and $c_{KZ} - c_{ZZ} = -1$. These can be verified using the derivations of the appropriate elasticities.