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Socially Optimal Districting: An Empirical Investigation
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ABSTRACT

This paper provides an empirical exploration of the potential gains from socially optimal districting. As emphasized in the political science literature, districting matters because it determines the seat-vote curve, which relates the fraction of seats parties obtain to their share of the aggregate vote. Building on the theoretical work of Coate and Knight (2006), which develops and analyzes the optimal seat-vote curve, this paper develops a methodology for computing actual and optimal seat-vote curves and for measuring the potential welfare gains that would emerge from implementing optimal seat-vote curves. This method is then applied to analyze districting plans in place during the 1990s to elect U.S. State legislators. The analysis shows that the plans used by the states in our data set generate seat-vote curves that are overly responsive to changes in voters' preferences. While there is significant variation across states, the potential welfare gains from implementing optimal seat-vote curves are on average small relative to the overall surplus generated by legislatures. This appears to be because seat-vote curves are reasonably close to optimal rather than because aggregate welfare is insensitive to varying districting plans. Interestingly, implementing proportional representation would produce welfare levels quite close to those achieved by implementing optimal seat-vote curves.

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1 Introduction

Districting, the process of creating political districts to elect legislators, is a key determinant of the representation of parties in legislatures. To illustrate, consider a legislature with 10 seats and suppose that 50% of the population are Democrats and 50% are Republicans. Then, while the "unbiased" seat share for the Democrats is 5, the actual share can vary from 1 to 9 depending upon the districting. For example, the Democrats can get 9 seats by creating 9 districts that are 51% Democrat. In the U.S., political districts at federal, state, and local levels are typically redrawn following the decennial release of Census population data. As legislatures have become more polarized and districting plans more refined, the stakes associated with districting have risen. These increased stakes have in turn led to heightened interest among voters, politicians and parties alike.

The considerable public interest in districting has given rise to a large empirical literature in political science on the subject.[1] In this literature, districting plans are evaluated by the properties of the *seat-vote curve* that they generate. The seat-vote curve is a function $S(V)$ where V is the aggregate fraction of votes received by (say) the Democrats and S is the fraction of seats in the legislature that they hold. Key properties of a seat-vote curve are its *partisan bias* - defined most simply as $S(1/2)-1/2$ - and its *responsiveness* - defined as $\Delta S/\Delta V$. The literature develops statistical methods to estimate seat-vote curves and analyzes how bias and responsiveness are altered following redistrictings.

While this literature is very interesting, the normative lessons to be drawn from its findings are unclear. Is bias bad and if so why? What is the optimal level of responsiveness? More generally, how should voters be allocated across districts? In Coate and Knight (2006), we provide a welfare economic analysis of these questions. We begin by developing a simple micro-founded model that provides a framework for thinking about the problem consistent with the concerns of the existing literature. In this model, districting determines the relationship between seats and votes and

[1] Important papers in this literature include Gelman and King (1990), (1994), King (1989), King and Browning (1987) and Tufte (1973). For general discussions of districting see Galderisi (2005) and Mann and Cain (2005). While the topic has attracted less theoretical attention, there is a small body of work on the *partisan gerrymandering problem*: that is, how to craft political districts with the aim of maximizing a party's expected seat share. This literature seeks to shed light on how partisan redistricting committees might further their political objectives. Friedman and Holden (2006) provide an elegant and comprehensive analysis of the problem. Earlier papers include Owen and Grofman (1988), Gilligan and Matsusaka (1999) and Sherstyuk (1998). In an interesting application of the theory, Shotts (2001) and (2002) uses models of partisan gerrymandering to understand the policy implications of mandating that districting authorities form so-called *majority-minority districts.*

social welfare depends upon the properties of this relationship. We then characterize the ideal relationship between seats and votes - the *optimal seat-vote curve*. Finally, we show that, under a seemingly permissive condition, the optimal seat-vote curve is *implementable* in the sense that there exist districtings that generate this optimal relationship between seats and votes. These underlying districtings are *socially optimal districtings*.

These theoretical findings raise several natural empirical questions. How close are the seat-vote curves associated with real world districting plans to optimal seat-vote curves? Are optimal seat-vote curves implementable in practice? If so, how large are the associated welfare gains? This paper builds on the theoretical model in Coate and Knight (2006) to develop an empirical methodology for answering these questions. First, we provide a method for estimating actual seat-vote curves. We then show how to compute optimal seat-vote curves and how to check the condition for implementability. Finally, we develop expressions for the welfare gains from implementing optimal seat-vote curves.

The paper then applies this methodology to analyze the districting plans used to elect U.S. State legislators during the 1990s. We find that the seat-vote curves generated by these districting plans are overly responsive to changes in voter preferences. In addition, we find that the condition for implementability is indeed satisfied in practice. While there is significant variation across states, our measures of the potential welfare gains from socially optimal districting are quite small, at least relative to the overall surplus generated by state legislatures. We show that this is because states' districting plans were reasonably close to optimal rather than because aggregate welfare was insensitive to varying districting plans. Finally, we show that implementing a proportional representation system would lead to welfare gains in all states in our data set and would produce welfare levels quite close to those achieved with socially optimal districting.

Despite its importance, the problem of optimal districting has attracted scant attention.[2] In

[2] The few normative papers that have been written on districting have worked with very different underlying political models and objective functions than used in Coate and Knight (2006). Gilligan and Matsusaka (2005) look at the optimal districting problem from a median voter perspective. In Downsian fashion, they assume that candidates adopt the ideology of the median voter in their districts and that policy outcomes depend upon the ideology of the median legislator. Their social objective is to minimize the distance between the median legislator's ideology and the median voter's ideology and this is achieved by creating identical districts. Epstein and O'Hallaran (2004) focus on the *racial gerrymandering problem*, under which the planner attempts to maximize the welfare of minority groups. Their model formalizes the intuition that there maybe a trade-off between descriptive and substantive representation. Descriptive representation is achieved by having districts elect black representatives, while substantive representation is achieved when the legislature chooses policies that favor black voters. Maximizing descriptive representation may require concentrating black voters into majority-minority districts, while maximizing substantive representation may require a more even spreading of black voters.

particular, as far as we are aware, there have been no previous attempts to empirically evaluate the welfare gains from socially optimal districting.[3] While we will delay a full discussion of exactly how our approach to estimating seat-vote curves differs from the existing literature, its distinctive feature is that it begins with an underlying micro-founded model of the political process. The only other work we are aware of that explores the micro foundations of seat-vote curves is the independent work of Besley and Preston (2006). They develop a model (similar to Coate and Knight (2006)) that generates an equilibrium relationship between seats and votes. Their main theoretical point is to show that the shape of the seat-vote curve is a key determinant of parties' electoral platforms. Supporting empirical evidence demonstrates that local government policy choice in the U.K. is related to the shape of the seat-vote curve in the way the theory suggests.

The paper proceeds as follows. Section 2 summarizes the relevant theoretical background material from Coate and Knight (2006). Section 3 develops our empirical methodology for measuring seat-vote curves and evaluating the potential gains from socially optimal districting. Section 4 applies this methodology to U.S. State legislative elections. Section 5 briefly discusses how our method of measuring seat-vote curves compares with existing approaches. Section 6 concludes with a summary of the main lessons of the analysis.

2 Theoretical background

To keep the paper a reasonable length, our exposition of the theoretical background material must be concise. Readers are referred to Coate and Knight (2006) for more detailed discussion of the model's assumptions and explanation of the key theoretical results.

2.1 The model

Consider a state in which there are three types of voters - Democrats, Republicans, and Independents. Voters differ in their political ideologies which are measured on a 0 to 1 scale. Democrats and Republicans have ideologies 0 and 1, respectively. Independents have ideologies that are uniformly distributed on the interval $[m - \tau, m + \tau]$ where $\tau > 0$. These voters are "swing voters" and so the ideology of the median Independent may vary across elections. Specifically, m is

[3] In an interesting paper, Cameron, Epstein and O'Halloran (1996) develop a methodology for assessing the effect of different districting schemes on the substantive representation of minority interests as measured by U.S. House members' roll-call voting scores on minority issues. This methodology is then applied to calculate the districting strategy that would maximize substantive black representation in the U.S. House.

the realization of a random variable uniformly distributed on $[1/2 - \varepsilon, 1/2 + \varepsilon]$, where $\varepsilon \in (0, \tau)$ and $\varepsilon + \tau \leq 1/2$. The latter assumption guarantees that Independents are always between Democrats and Republicans, while the former guarantees that some Independents lean Democrat and some lean Republican. The fractions of voters statewide who are Democrats, Republicans, and Independents are, respectively, π_D, π_R, and π_I.

Policy choices in the state are determined by an n-seat legislature. Legislators' policy choices are influenced by their ideologies and, hence, citizens care about the ideological make up of their legislature. Specifically, if the average ideology of the legislature is x', a citizen with ideology x experiences a quadratic payoff given by $\beta - \gamma(x - x')^2$. The parameter β is the surplus a citizen would obtain from having a legislature that is perfectly congruent with his own ideology, while the parameter γ measures the rate at which this surplus is dissipated as the ideology of the legislature diverges from that of the citizen. The ratio γ/β will play an important role in the welfare analysis and can be interpreted as the fraction of the surplus a partisan (i.e., a Democrat or Republican) obtains from having a perfectly congruent legislature that is dissipated by having a legislature composed entirely of the opposing ideology. This ratio is assumed to be bounded between zero and one ($\gamma/\beta \in [0, 1]$).

To select legislators, the state is divided into n equally-sized political districts indexed by $i \in \{1/n, 2/n, ..., 1\}$. Each district then elects a representative to the legislature. Candidates are put forward by two political parties: the Democrats and Republicans. These parties are made up of collections of citizens who share the same political ideology, so that the membership of the Democrat Party are Democrats and the membership of the Republican Party are Republicans. Candidates are party members and, following the citizen-candidate approach (Besley and Coate (1997) and Osborne and Slivinski (1996)), candidates cannot credibly promise to run on an ideology different from their true ideology. Accordingly, Democratic candidates are associated with ideology 0 and Republican candidates with ideology 1. Elections are held simultaneously in each of the n districts and the candidate with the most votes wins. In each district, every citizen votes sincerely for the representative whose ideology is closest to his own.[4]

---

[4] Note that an Independent voter who leans Democrat may be better off when his district elects a Republican if other districts disproportionately elect Democrats. For if his district elected a Republican, the average legislator ideology would be closer to his ideal point. The decision to nonetheless assume sincere voting (i.e., voting for the ideologically closest candidate) is motivated by two main considerations. First, assuming sophisticated voting would substantially complicate the analysis because voters' optimal decisions would be strategic and determined as part of a statewide voting equilibrium. This equilibrium (which need not be unique) would of course be influenced by the

2.2 Districtings

A *districting* is a division of the population into n districts. Formally, a districting is described by $(\pi_D(i), \pi_R(i), \pi_I(i))_{i=1/n}^{1}$ where $\pi_D(i)$ represents the fraction of Democrats in district i, $\pi_R(i)$ the fraction of Republicans, and $\pi_I(i)$ the fraction of Independents. The districting is chosen by a districting authority that knows the group membership of citizens and faces no geographic constraints in terms of how it can group citizens.[5] Thus, any districting $(\pi_D(i), \pi_R(i), \pi_I(i))_{i=1/n}^{1}$ such that the average fractions of voter types equal the actual is feasible. Let Δ denote the set of feasible districtings and, to simplify notation, let $\delta \in \Delta$ denote a generic feasible districting.

2.3 Seat-vote curves

Any feasible districting $\delta = (\pi_D(i), \pi_R(i), \pi_I(i))_{i=1/n}^{1}$ implies a relationship between the Democratic seat share in the legislature and their statewide vote share. Note first that if the median independent has ideology m, the fraction of voters in district i voting for the Democratic candidate is

$$V(i; m) = \pi_D(i) + \pi_I(i)[\frac{1/2 - (m - \tau)}{2\tau}]. \tag{1}$$

This group consists of all the Democrats and those Independents whose ideologies are less than 1/2. The statewide vote share of the Democrat Party is therefore

$$V(m) = \pi_D + \pi_I[\frac{1/2 - (m - \tau)}{2\tau}]. \tag{2}$$

Let $\overline{V}$ and $\underline{V}$ denote, respectively, the maximum and minimum statewide Democrat vote shares; i.e., $\overline{V} = V(1/2 - \varepsilon)$ and $\underline{V} = V(1/2 + \varepsilon)$.

Now, for any feasible statewide vote share $V \in [\underline{V}, \overline{V}]$, let $m(V)$ denote the ideology of the median Independent that would generate the vote share V; i.e., $m(V) = V^{-1}(V)$. From (2), we have that

$$m(V) = \frac{1}{2} + \tau[\frac{\pi_I + 2\pi_D - 2V}{\pi_I}]. \tag{3}$$

districting. Second, as an empirical matter, it is not clear that most voters are this sophisticated. Similar incentives to diverge from voting for the candidate closest to one's own ideology arise when voters are electing Congressional and Presidential candidates and policy outcomes depend upon the ideologies of both Congress and the President (Alesina and Rosenthal (1995) and Fiorina (1992)). However, using a data set on voting behavior in these elections, Degan and Merlo (2006) show that sincere voting can explain virtually all individual-level observations.

[5] The former assumption seems reasonable as a first approximation, since information about voters' ideological attachments is available through voter registration data or the study of past voting patterns (see the discussion in Altman, Mac Donald and McDonald (2005)). The latter assumption is more difficult to defend on realism grounds. However, as shown in Coate and Knight (2006) and discussed further below, when the optimal seat-vote curve is implementable it can typically be implemented by a large class of districtings, some of which look quite "straightforward". Hence geographic constraints may actually be easily accommodated.

Substituting this into (1), we obtain

$$V(i;m(V)) = \pi_D(i) + \pi_I(i)[\frac{V-\pi_D}{\pi_I}]. \tag{4}$$

District i elects a Democrat if $V(i;m(V)) \geq 1/2$, or, equivalently, if

$$V \geq V^*(i) = \pi_D + \pi_I[\frac{1/2-\pi_D(i)}{\pi_I(i)}], \tag{5}$$

where $V^*(i)$ is the critical statewide vote threshold above which district i elects a Democrat. District i is a *safe Democrat* (*safe Republican*) seat if $V^*(i) \leq \underline{V}$ ($V^*(i) \geq \overline{V}$). A seat which is not safe is *competitive*.

Without loss of generality, order the districts so that $V^*(1/n) \leq V^*(2/n) \leq .... \leq V^*(1)$. Then, the fraction of seats the Democrats receive when they have vote share V is

$$S(V\,|\delta\,) = \max\{i : V^*(i) \leq V\}. \tag{6}$$

This is the *seat-vote curve* associated with the districting δ.

2.4 The optimal seat-vote curve

The *optimal seat-vote curve* $S^o(V)$ describes the ideal relationship between Democratic seats and aggregate votes ignoring the constraint that this relationship be generated by some feasible districting. When the Democrats have seat share S, the average legislator ideology is $1-S$.[6] Thus, aggregate utility when the median Independent has ideology m is given by:

$$W(S,m) = \beta - \gamma[\pi_D(1-S)^2 + \pi_R S^2 + \pi_I \int_{m-\tau}^{m+\tau}(1-S-x)^2\frac{dx}{2\tau}]. \tag{7}$$

Expected aggregate utility under the seat-vote curve $S(V)$ is therefore given by

$$EW(S(V)) = \int_{\underline{V}}^{\overline{V}} W(S(V),m(V))\frac{dV}{\overline{V}-\underline{V}}. \tag{8}$$

[6] The choice of the average legislator ideology rather than the median as a summary statistic of the distribution of legislator ideology is motivated by two main considerations. First, the median assumption corresponds to the idea that the party with the majority of seats is completely decisive and minority party members have no influence at all. This seems unrealistic. A real world state legislature makes numerous decisions on many different areas of policy. Many of these decisions will be made by small sub-committees of legislators. This gives a legislator influence even if he is not in the majority party. Reflecting this, we seriously doubt that voters would be indifferent between a state legislature that is 51% Democrat and one that is 100% Democrat. Besley and Case (2003) provide evidence in support of this general view; conditional on the Democrats controlling the state legislature, an increase in seats for Democrats leads to an increase in state government spending. Second, the implicit assumption of the empirical districting literature is that something like average legislator ideology matters. After all, this literature focuses on the responsiveness and bias of the seat-vote curve over its entire range. But, under the median assumption, the properties of the seat-vote curve are irrelevant for citizens' welfare over almost all of its range. All that matters for welfare is the vote share at which the Democrats become the majority party.

To avoid tedious integer concerns, the number of districts is assumed to be large enough to justify treating S as a continuous variable defined on the unit interval $[0, 1]$. This yields the following simple characterization of the optimal seat-vote curve:

Proposition 1: *The optimal seat-vote curve* $S^o : [\underline{V}, \overline{V}] \rightarrow [0, 1]$ *is given by*

$$S^o(V) = 1/2 + (\pi_D - \pi_R)(1/2 - \tau) + 2\tau(V - 1/2). \tag{9}$$

Proof: See Coate and Knight (2006).

Thus, the optimal seat-vote curve is linear.[7] Its *partisan bias* - defined as $S^o(1/2) - 1/2$ - is $(\pi_D - \pi_R)(1/2 - \tau)$. The system is therefore optimally biased towards the party with the largest partisan base. This optimal bias reflects the fact that when the legislature is equally divided between Democrats and Republicans, a marginal change in the composition of the legislature has a greater impact on partisans' surplus than on Independents. This in turn reflects the assumption that the loss experienced by citizens from having a legislature that diverges from their own ideology is a convex function of the extent of this divergence.

The *responsiveness* of the optimal seat–vote curve - defined as $\Delta S^o/\Delta V$ - is 2τ. Thus, as the policy preferences of Independent voters become more diverse, the Democrats' seat share becomes more responsive to changes in their aggregate vote. Notice, however, that since $\tau < 1/2$, optimal responsiveness is always less than 1 meaning that if the Democratic vote share goes up by ΔV, the Democrats share of seats in the legislature should optimally go up by less than ΔV.[8]

2.5 The conditions for implementation

The optimal seat-vote curve is *implementable* when there exist feasible districtings under which the seat-vote curve is optimal; that is, if there exists $\delta \in \Delta$ such that $S(V \,|\delta\,) = S^o(V)$. The

[7] As explained in Coate and Knight (2006), this linearity is a consequence of the assumptions that citizens have quadratic loss functions and that the distribution of Independents' ideologies is uniform across its support.

[8] To understand this result, note first that the Democratic seat share will optimally be such as to make average legislator ideology equal to average ideology in the population at large. Then observe that changes in vote share always over-estimate changes in average population ideology; that is, if the Democratic vote share goes up by ΔV, the decrease in average population ideology will be less than ΔV. This is because changes in vote share are driven by changes in the voting behavior of Independents and such changes can arise from fairly small shifts in preferences. To illustrate, consider the case in which τ is very small so that Independents all have basically the same ideology - namely m. Now suppose that m changes from 0.51 to 0.49 so that all the Independents switch from voting Republican to voting Democrat. Then there is a massive change in the Democratic vote share but virtually no change in the average ideology of the population and hence virtually no change in the optimal share of Democratic seats in the legislature.

underlying districtings are referred to as *socially optimal districtings*. Whether the optimal seat-vote curve is implementable turns out to depend upon the fraction of Independents. Two special cases provide intuition for this. First, if there were no Independents, then the optimal seat-vote curve would be a single point [$S^o(V) = \pi_D$] and could be implemented by creating a fraction π_R of majority Republican districts and a fraction π_D of majority Democrat districts. On the other hand, if the entire population were Independents, then all districts would necessarily be identical and the equilibrium seat-vote curve would jump from zero to one at $V = 1/2$. The optimal seat-vote curve, by contrast, is linear in this case [$S^o(V) = 1/2 + 2\tau(V - 1/2)$] and is thus clearly not implementable. This motivates:

Proposition 2: *The optimal seat-vote curve is implementable if and only if*

$$\pi_I[\frac{\varepsilon}{2\tau} + \varepsilon - (\tau + \varepsilon)\ln(1 + \frac{\varepsilon}{\tau})] \leq \min(\pi_D, \pi_R). \tag{10}$$

Proof: See Coate and Knight (2006).

Thus, the fraction of Independents must be sufficiently small. This condition appears permissive as the coefficient multiplying the fraction of Independents is less than 1/2. As shown in Coate and Knight (2006), when the condition is satisfied, there will typically exist a large class of socially optimal districtings. These typically include some that look quite "straightforward" in the sense that they do not require patterns of concentration of voter types that would seem likely to be infeasible when account is taken of the geographic constraints that real world districters face.[9] This suggests that the optimal seat-vote curve may be an attainable benchmark.

2.6 The welfare gain to socially optimal districting

Assuming that the optimal seat-vote curve is implementable, what would be the welfare gains associated with implementing the optimal seat-vote curve? Conveniently, the welfare gain turns out to be proportional to the squared distance between the baseline and optimal seat-vote curves. To see this, note first that expected citizen welfare can be expressed as a function of the baseline and optimal seat-vote curves:

Lemma: *It is the case that*

$$E[W(S(V))] = \beta - \gamma\{c + E[S(V)^2] - 2E[S(V)S^o(V)]\} \tag{11}$$

[9] Coate and Knight (2006) show that the optimal seat-vote curve can be implemented with a districting that involves a constant fraction of Independents in each district under a condition that is only slightly more stringent than the condition in Proposition 2.

where c is a constant given by $c = \pi_D + \pi_I[1/4 + \varepsilon^2/3 + \tau^2/3]$.

Proof: See Appendix.

Using this formula to compute the welfare gain immediately establishes:

Proposition 3: *The welfare gain from socially optimal districting can be written as*

$$G = E[W(S^o(V))] - E[W(S(V))] = \gamma E[(S^o(V) - S(V))^2]. \tag{12}$$

Intuitively, the smaller the distance from the optimal seat-vote curves, the larger are the welfare gains associated with socially optimal districting.

3 Empirical methodology

From an empirical perspective, we would like to know how seat-vote curves generated by legislative districting plans differ from optimal seat-vote curves. We would also like to know if the condition for implementability is satisfied. Finally, if this condition is satisfied, we would like to know the magnitude of the welfare gains from implementing the optimal seat-vote curve. This section presents our method for doing all this. In developing this method, we assume that the researcher has estimates at the district level of the mean and standard deviation of the Democratic vote share under the districting plan in question. In addition, we assume that the analyst knows the statewide fraction of voters who identify as Independents.[10]

3.1 Estimating seat-vote curves

As explained in Section 2.3, seat-vote curves are determined by the range of possible statewide Democratic vote shares $[\underline{V}, \overline{V}]$ and the pattern of district-specific threshold vote levels (V_i^*). We will show that both the range of vote levels and the vote thresholds can be expressed solely as a function of the means and standard deviations of the district-level and statewide Democratic vote share. The first step in establishing this is to provide expressions for these moments. Beginning with the district-specific moments, the mean and standard deviation of Democratic votes in district

[10] In the empirical application to follow, we estimate the moments by running panel data regressions that relate district voting returns to voter characteristics. Data on the statewide fraction of independents is taken from survey data.

i, as expressed in equation (1) are

$$\begin{array}{c} \mu_i = E(V_i) = \pi_D^i + \frac{1}{2}\pi_I^i = \frac{1}{2}[1 + \pi_D^i - \pi_R^i] \\ \sigma_i = \sqrt{Var(V_i)} = \sqrt{\frac{\pi_I^{i2} var(m)}{4\tau^2}} = \frac{\pi_I^i \varepsilon}{2\sqrt{3}\tau} \end{array}. \tag{13}$$

Observe that the standard deviation of the Democratic vote share is proportional to the fraction of Independents. It is increasing in the degree to which the median Independent shifts support between the two candidates from election to election (ε) but is decreasing in the diversity of preferences among Independents (τ).

Turning to the statewide moments, we take cross-district averages of the district-specific means and standard deviations to obtain:

$$\begin{array}{c} \mu = E(V) = \pi_D + \frac{1}{2}\pi_I = \frac{1}{2}[1 + \pi_D - \pi_R] \\ \sigma = \sqrt{Var(V)} = \sqrt{\frac{\pi_I^2 var(m)}{4\tau^2}} = \frac{\pi_I \varepsilon}{2\sqrt{3}\tau} \end{array}. \tag{14}$$

Using these state-wide moments, we can now write the maximum and minimum statewide Democratic vote shares as

$$\begin{array}{c} \underline{V} = V(1/2 + \varepsilon) = \mu - \sqrt{3}\sigma \\ \overline{V} = V(1/2 - \varepsilon) = \mu + \sqrt{3}\sigma \end{array}. \tag{15}$$

Moreover, using the district-specific and statewide moments and the definition of V_i^* in equation (3), we can write the vote threshold for electing a Democratic candidate in district i as follows:

$$V_i^* = \mu + \frac{\sigma}{\sigma_i}(1/2 - \mu_i). \tag{16}$$

Relabelling the districts so that

$$\mu + \frac{\sigma}{\sigma_{1/n}}(1/2 - \mu_{1/n}) \leq \mu + \frac{\sigma}{\sigma_{2/n}}(1/2 - \mu_{2/n}) \leq .... \leq \mu + \frac{\sigma}{\sigma_1}(1/2 - \mu_1),$$

the seat-vote curve is the function defined on the interval $[\mu - \sqrt{3}\sigma, \mu + \sqrt{3}\sigma]$ given by:

$$S(V) = \max\{i : \mu + \frac{\sigma}{\sigma_i}(1/2 - \mu_i) \leq V\}. \tag{17}$$

3.2 Estimating the optimal seat-vote curve

Using the fact that $\pi_D - \pi_R = 2\mu - 1$, we can re-write the optimal seat-vote curve described in Proposition 1 as follows:

$$S^o(V) = 1/2 + (2\mu - 1)(1/2 - \tau) + 2\tau(V - 1/2). \tag{18}$$

Observe that the optimal seat-vote curve and, in particular, its responsiveness and partisan bias parameters depend critically upon the diversity of preferences among Independents (τ). Even with information on the statewide standard deviation, this parameter τ cannot be identified separately from the underlying parameters π_I and ε (see equation (14)).[11] However, it is possible to identify the ratio $\alpha = \varepsilon/\tau$, with data on the statewide fraction of Independents (π_I) and the statewide standard deviation of the Democratic vote share (σ) as follows:

$$\alpha = \frac{\varepsilon}{\tau} = \frac{2\sqrt{3}\sigma}{\pi_I}. \tag{19}$$

Further, using the theoretical restriction on the sum of these preference parameters $\epsilon + \tau \leq 1/2$, we can place an upper bound on optimal responsiveness:

$$2\tau \leq \frac{1}{(1+\alpha)}. \tag{20}$$

In the baseline analysis of the empirical application to follow, we assume that optimal responsiveness equals this upper bound. In addition, as a robustness check, we allow the optimal responsiveness to fall in a range below this upper bound.

3.3 Verifying the condition for implementation

The condition for implementability presented in Proposition 2 cannot be verified directly without information on the underlying preference parameters (ε, τ). As just noted however, with outside information on the fraction of Independents, we can identify the ratio $\alpha = \varepsilon/\tau$. We can use information on this ratio to place an upper bound on the coefficient associated with the implementability of the optimal seat-vote curve. In particular, we can show that

$$\left(\frac{\varepsilon}{2\tau} + \varepsilon - (\tau + \varepsilon)\ln(1 + \frac{\varepsilon}{\tau})\right) \leq \alpha/2, \tag{21}$$

and this implies that a sufficient condition for implementability is:

$$\pi_I \leq \frac{2\min(\pi_D, \pi_R)}{\alpha}. \tag{22}$$

[11] Intuitively, large swings in the Democratic vote share within a state could be due to a large fraction of Independents (π_I), large swings in the preferences of the median Independent (ε), or a tight distribution of ideology among Independents (τ), in which case relatively small swings in the preferences of the median Independent translate into relatively large swings in voting outcomes.

As the ratio α approaches unity, this sufficient condition converges to $\pi_I \leq 2\min(\pi_D, \pi_R)$. For any given $\alpha < 1$, however, this ratio provides additional information to the researcher attempting to verify the condition for implementation. Substituting in the expression for α from equation (19) and using the fact that $\mu = \pi_D + \pi_I/2$, this sufficient condition can be re-written as

$$\pi_I \leq 2\min(\mu - \sqrt{3}\sigma, 1 - \mu - \sqrt{3}\sigma). \tag{23}$$

Thus, the fraction of Independents must be below a critical value, the calculation of which only requires information on the statewide mean and standard deviation of the Democratic vote share.

3.4 Estimating the welfare gains from socially optimal districting

Given that we only observe voting outcomes, which do not reveal the intensity of voter preferences for one party over another, the parameters of the surplus expression (β, γ) in equation (7) are not identified in the empirical analysis. However, we can use the theoretical restriction on the ratio of these parameters in order to calculate the range of proportionate welfare gains. To this end, first note that the percentage increase in aggregate welfare from socially optimal districting can be written as follows:

$$\Delta G = \frac{E[W(S^o(V))] - E[W(S(V))]}{E[W(S(V))]}. \tag{24}$$

Then, using the expressions from equations (11) and (12) and dividing through by β, we have that:

$$\Delta G(\frac{\gamma}{\beta}) = \frac{\frac{\gamma}{\beta}E[(S^o(V) - S(V))^2]}{1 - \frac{\gamma}{\beta}\{c + E[S(V)^2] - 2E[S(V)S^o(V)]\}}. \tag{25}$$

Recall that the ratio γ/β is the fraction of the surplus a partisan obtains from having a perfectly congruent legislature that is dissipated by having a legislature composed entirely of the opposition party. When parties are not that polarized in terms of their underlying ideologies or when the legislature is responsible for choosing only policies on which there is little disagreement across ideologies (for example, spending on public safety and highway maintenance) this ratio may be close to zero. When parties are polarized and are choosing policies on which there is strong ideological disagreement (such as the level of transfer payments for the poor or the regulation of abortion), this ratio may be close to one. In the former case, districting is not very important, while in the latter case it is crucial to citizen welfare. Using the restriction that $\gamma/\beta \in [0, 1]$, we can thus bound these proportionate welfare gains as follows:

$$0 \leq \Delta G(\frac{\gamma}{\beta}) \leq \Delta G(1). \tag{26}$$

Because the upper bound will only be relevant for legislatures in states in which parties are polarized and which choose policies on which there is strong disagreement, we will provide welfare calculations for different values of this key ratio (γ/β) in the empirical application to follow.

We next turn to the measurement of this welfare gain. Inserting equation (18) into equation (25), we have that

$$\Delta G(\frac{\gamma}{\beta}) = \frac{(\frac{\gamma}{\beta})\{(\xi^o)^2 + 2\xi^o r^o \mu + (r^o)^2(\mu^2 + \sigma^2) - 2E[(\xi^o + r^o V)S(V)] + E[S(V)^2]\}}{1 - (\frac{\gamma}{\beta})\{c + E[S(V)^2] - 2\xi^o E[S(V)] - 2r^o E[VS(V)]\}} \tag{27}$$

where $r^o = 2\tau$ represents optimal responsiveness and $\xi^o = \mu(1 - 2\tau)$ represents the vertical intercept of the optimal seat-vote curve. Notice that we can express the constant c as $c = \mu + \frac{2\sqrt{3}\tau\sigma}{\varepsilon}[\varepsilon^2/3 + \tau^2/3 - 1/4]$, and hence, given a particular value of the ratio γ/β and the parameter τ, this expression can be evaluated by computing three moments of the baseline seat-vote curve: $E[S(V)]$, $E[S(V)^2]$, and $E[VS(V)]$. In the Appendix, we develop expressions for these moments and show that they are related to the district-specific vote thresholds (V_i^*), which, as noted above, can in turn be related to the moments of the Democratic vote share.

4 Application to U.S. state legislatures

In this section, we apply our methodology to analyze the districting plans used to elect U.S. State legislators during the 1990s districting period.[12] For consistency with the theoretical framework, we focus on states with single-member districts.[13] In addition, given the bicameral nature of state legislatures, we follow the existing empirical literature on redistricting and focus on elections to the lower house. As shown in Table 1, we have complete data for 28 states, most of which adopted redistricting plans in 1992 and then again in 2002. For these states, there were five elections held under the 1990s districting plan: 1992, 1994, 1996, 1998, and 2000.[14] To

[12] We prefer this setting over the U.S. House because, in federal redistricting, state officials control the redistricting process and each redistricting plan thus only partially contributes to the resulting allocation of national seats across parties in Congress. Redistricting plans for state legislatures are also controlled by state officials, and redistricting plans thus perfectly correspond to changes in seats in state legislatures.

[13] Some states elect multiple members from each district to state legislatures.

[14] States deviating from this pattern of elections include Virginia, which has elections in odd years and adopted redistricting plans in 1991 and 2001, and Colorado, whose district lines were redrawn in 1998 following litigation over the representation of minority groups in the state legislature.

estimate the moments of the Democratic vote shares under these districting plans, we use data from Ansolabehere and Snyder (2002) on state legislative election returns together with Census data on voter characteristics by state legislative district.[15] We also use state-level estimates of the fraction of Independent voters derived from annual New York Times surveys in which voters are asked to self-identify as Republican, Democrat, or Independent.[16]

4.1 Estimation of moments

There are a number of possible methods for estimating the moments of the voting distribution under a districting plan, the appropriateness of which may vary from application to application. With a sufficiently long panel, for example, moments in the key expressions above could simply be replaced with their analogous sample moments. However, because redistricting typically occurs every ten years in the U.S. and elections every two years, in our application we have five observations per district at most. In addition, this approach is problematic given that sample moments cannot be calculated for districts with uncontested elections, which occur frequently at the state-level. As an alternative, we use an econometric model for estimating the moments as a function of the characteristics of voters residing in the district. In addition to circumventing the short panel problem and naturally handling the issue of uncontested elections, this econometric model allows the researcher to compute confidence intervals around key measures, such as the welfare gains to socially optimal districting.

Before providing a specific econometric formulation for these moments, it is instructive to note that, using equations (1) and (13), the Democratic vote share in district i can be written as a linearly separable function of the district-specific mean and variance along with a shock to the preferences of Independent voters:

$$V_i = \mu_i + \sigma_i w \tag{28}$$

where $w = \frac{\sqrt{3}}{\epsilon}(1/2 - m)$ is distributed uniformly with mean zero and variance equal to 1. This formulation then naturally leads to the following specification, in which the two moments are

[15] These data, which are published in Barone et al (1998), include the fraction of residents living in urban areas, the fraction living in suburban areas, household income, percent of residents with a college degree, percent over age 65, percent African-American, and percent Hispanic.

[16] These data were downloaded from the website http://php.indiana.edu/~wright1/cbs7603_pct.zip. In order to compute the time-invariant fraction of Independents for each state, we take averages across the years listed in Table 1.

related to voter characteristics:

$$\begin{array}{c} \mu_i = X_i'\theta + \sigma_\eta \eta_i \\ \sigma_i^2 = \exp(X_i'\delta + \sigma_\nu \upsilon_i) \end{array}, \tag{29}$$

where X_i denotes a vector of observed voter characteristics, (θ, δ) denote parameters to be estimated, and (η_i, υ_i) denote district-specific random effects, assumed to be normally distributed with zero mean and standard deviations of σ_η and σ_ν, respectively.

Reflecting our use of panel data, we next introduce a time dimension (t) and, inserting the above parameterizations for the two moments into equation (28), we obtain the following random effects model with heteroskedasticity:

$$\begin{array}{c} V_{it} = X_i'\theta + \sigma_\eta \eta_i + u_{it} \\ \ln(u_{it}^2) = X_i'\delta + \sigma_\nu \upsilon_i + \ln(w_t^2) \end{array}. \tag{30}$$

Estimation of the parameters of equation (30) follows a standard two-step approach. First, we estimate a subset of the parameters (θ, σ_η) using a random-effects panel data regression of votes in district i (V_{it}) on observed voter characteristics in district i (X_i). In the second step, we regress the log of the squared residual obtained from the first step on observed district characteristics and obtains estimates of the remaining parameters of interest (δ, σ_ν).

Table 2 provides the results from estimating the random effects econometric model as outlined above. In order to increase the power associated with this prediction approach, we run this regression across all states in our sample, implicitly assuming that the mapping from observed characteristics into partisan affiliation is homogeneous across states. While this assumption is somewhat restrictive, we do include a set of state dummy variables in both equations, thereby allowing two districts with identical observable voter characteristics but in different states to have different voting patterns. As shown in the first column, the mean vote share for the Democratic party (μ_i), is increasing in the percent urban and suburban (both of these are relative to the omitted category - percent rural), percent with a college degree, percent over age 65, percent African-American, and percent Hispanic (both of these are relative to the omitted category - percent white) but is decreasing in household income. As shown in the second column, the variance is decreasing in household income and in the percent African-American.

These coefficients in Table 2 are then used in order to compute the key moments of the voting distribution. Even with the parameter estimates $(\widehat{\theta},\widehat{\sigma}_{\eta},\widehat{\delta},\widehat{\sigma}_{\nu})$ and data on district characteristics (X_i), we do not learn the district-specific mean and variance of the voting distribution, as expressed in equation (29), because the random effects (η_i,υ_i) are unobserved. One possible solution to this problem is to simply "shut down" this unobserved component and use only the observed component as our estimate of the moments. This is, after all, the central tendency (the mean and median) of the distribution of moments. The difficulty with this approach, however, is that it will tend to understate the degree of cross-district heterogeneity, which in turn may tend to overstate the responsiveness of the seat-vote curve.[17]

As an alternative, we use a simulation approach, which incorporates both the observed and unobserved sources of cross-district heterogeneity, in calculating these moments. In particular, for each replication $r = 1, 2, ..., R$ and for each district i, random effects (η_i^r,υ_i^r) are drawn from the standard normal distribution. Combining these realized random effects with observed district characteristics and the estimated parameters, the district-specific moments (μ_i^r,σ_i^r) can be calculated as follows for each replication r:

$$\begin{aligned} \mu_i^r &= X_i'\widehat{\theta} + \widehat{\sigma}_\eta \eta_i^r \\ \sigma_i^r &= \sqrt{\exp(X_i'\widehat{\delta} + \widehat{\sigma}_\nu \upsilon_i^r)} \end{aligned} \quad . \tag{31}$$

Statewide moments associated with replication r can then be calculated by averaging across the district-specific moments. With these district-specific and statewide moments in hand, key objects of interest, such as the vote threshold for electing a Democratic candidate, as expressed in equation (16), can be calculated for each replication r. Finally, aggregating across all replications $r = 1, 2, .., R$, key aspects of the distribution of seat-vote curves, such as average responsiveness and bias, average optimal responsiveness and bias, and the probability of implementability, can be calculated.

[17] To see this, consider the extreme case in which the true moments are heterogeneous across districts but in which the observed voter characteristics have no explanatory power in the regressions. Then, if the unobserved component is shut down, the estimated seat-vote curve will be that associated with identical districting and will thus jump from 0 to 1 at V=1/2, and all districts will thus be considered competitive. Of course, our voter characteristics do have explanatory power but the more general lesson still holds: ignoring the unobserved component tends to understate the degree of heterogeneity across districts.

4.2 Seat-vote curves

Given that a plot of all replications would be cumbersome, we begin by presenting the seat-vote curves from a particular replication, that associated with the median welfare gain from socially optimal districting (as expressed in equation (27)). As shown in Figure 1, the range of possible statewide support for the Democratic Party seems reasonable. For example, in New York, a heavily Democratic state, the party receives support on the range [0.56,0.69], while in Utah, a heavily Republican state, the Democrats receive support on the range [0.35,0.50]. Notice also that the seat-vote curve is close to linear in some states, such as Alabama, Connecticut, Montana, and Pennsylvania, while it has important non-linearities in other states, such as Delaware and Wisconsin.

As shown in column 1 of Table 3, the responsiveness associated with the estimated seat-vote curve exceeds two in every state, and, in some states, exceeds three.[18] Findings of significant responsiveness are quite common in the existing literature, which has focused on a responsiveness of three, a finding that has become known as the "Cube Law" (King and Browning, 1987). Column 4 of Table 3 reports the partisan bias associated with seat-vote curves, measured by $S(1/2)-1/2$. Notice that in the five states for which $V = 1/2$ does not lie in the range of possible Democratic vote shares, this measure cannot be computed. The interesting thing to note is that the seat-vote curve is biased towards the Republicans in 16 states but towards the Democrats in only 6 states. The cross-state average bias of -3% implies that, when voters are equally split, Republicans would secure 53% of the seats on average, relative to 47% for Democrats, and would thus hold a significant advantage of 6% in the legislature.

4.3 Optimal seat-vote curves

For comparison purposes, Figure 1 also includes the optimal seat-vote curves from the replication associated with the median welfare gain for each state. These are plotted under the assumption that optimal responsiveness, 2τ, is at its maximal level $1/(1+\alpha)$ in each state. It is apparent from Figure 1, that the actual seat-vote curves are overly responsive in all cases, suggesting that

[18] This responsiveness measure is obtained by computing the slope of the linear seat-vote curve that best approximates the estimated seat-vote curve. To be more precise, define the *linearized estimated seat-vote curve* to be $S^l(V) = 1/2 + b + r(V - 1/2)$, where b and r are chosen in order to minimize the expected square distance between the estimated and the linearized estimated seat-vote curves. That is: $r = \text{cov}[V, S(V)]/\text{var}(V)$ and $b = E[S(V)] - 1/2 - r[E(V) - 1/2]$. Then our measure of responsiveness is r.

districting plans used to elect U.S. State legislators during the 1990s created too few safe seats. Column 2 of Table 3 reports the responsiveness of the optimal seat-vote curve. As noted in the theoretical section, optimal responsiveness is always below one, while actual responsiveness substantially exceeds one in all cases. That is, as shown in column 3, the difference between actual and optimal responsiveness is close to 2 in most states.

Column 5 of Table 3 reports the partisan bias associated with the optimal seat-vote curve, which is defined as $S^o(1/2) - 1/2$. It is notable that in all but five states actual bias is less than optimal bias. Thus, the partisan bias estimated towards the Republicans cannot be justified as "optimal partisan bias". Indeed, in twelve states, the actual seat-vote curve is biased towards the Republicans, when it should optimally be biased towards the Democrats!

A reasonable objection to this comparison of partisan bias is that it just tells us about the properties of the curves at the vote share $V = 1/2$. For a more global comparison, we computed the expected Democratic seat share under the estimated and optimal seat-vote curves ($E(S(V))$ and $E(S^o(V))$). Column 7 of Table 3 reports the difference in this expected seat-share. When this difference is positive, the expected Democratic seat share is higher than optimal. The interesting thing to note is that, in this expected seat sense, there appears to be no obvious bias towards Republicans. If anything, this alternative bias measure suggests that districting systems are overly biased towards Democrats as this measure is positive in over one-half of the states. This makes us hesitant to draw any strong conclusions concerning the general direction of bias.

The properties of seat-vote curves reported in Table 3 are based upon a single replication, that associated with the median welfare gain. Table 4 reports more general findings of our analysis via the properties of the entire distribution of replicated seat-vote curves. The first three columns report the mean responsiveness across replications, the median responsiveness across replications, and the 90 percent confidence interval for responsiveness across replications. This confidence interval is computed by ranking our measures of responsiveness across replications and then choosing the 5th and 95th percentile of that distribution. As shown, the finding that the estimated seat-vote curve is overly responsive in Table 3 is a robust finding as the 90 percent confidence interval for the difference in responsiveness is positive in all states. The findings regarding partisan bias, by contrast, are more mixed. As shown in columns 4 and 5, the mean and median difference in bias is negative in most cases, reflecting the previous finding that estimated seat-vote curves tend to be overly biased in favor of the Republicans. The confidence interval, however, includes zero for all except eight states, and thus our finding that the actual seat-vote curve is overly biased in favor of Republicans should be interpreted with caution. Similarly, as shown in the final three columns, no definitive conclusions can be drawn in a statistical sense regarding the difference in expected seats.

4.4 Verifying the condition for implementation

As shown in Table 5, the condition for implementation is satisfied with probability one in every state. That is, in every replication, the fraction of Independents was below the maximal level described in equation (23). The state closest to not satisfying the requirements is Rhode Island, which is reported to have 51% Independents, just slightly below the cross-replication average maximal level of 58%. In no replications, however, did this maximum level fall below the reported 51% share of Independents. In summary, these results demonstrate that the condition for implementability of the optimal seat-vote curve is indeed permissive, being satisfied in all of 28 states included in our analysis and by a large margin in all cases except for the state of Rhode Island.[19]

4.5 Welfare gains

Given that the optimal seat-vote curve is implementable in all states and across all replications, it is interesting to measure the welfare gains associated with socially optimal districting. To begin with, we compute the percentage welfare gains under the assumptions that the ratio γ/β is at its maximal level (i.e., 1) and that optimal responsiveness is at its maximal level in each state (i.e., $1/(1+\alpha)$). As shown in Table 6, the percentage welfare gains to socially optimal districting are relatively small; averaged across all states, these median and average gains are reported at 1.76% and 1.81%, respectively. There is, however, considerable variation across states with Rhode Island and South Carolina at the opposite extremes. A visual comparison of the seat-vote curves in Figure 1 supports these welfare calculations as the seat-vote curves are similar in states with

[19] Note that in evaluating the conditions for implementation, we have not used external information on the statewide fraction of Democrats and Republican. It is comforting to note, however, that the implied fraction of partisans from voting behavior are highly correlated with the fraction of voters self-reporting as partisans. Using the fact that the implied statewide fraction of Democrats and Republicans are given by $\pi_D = \mu - \pi_I/2$ and $\pi_R = 1 - \mu - \pi_I/2$, we calculate correlation between the implied and reported fraction of voters of roughly 0.8 for both Republicans and Democrats.

low potential welfare gains but quite different in states with large potential welfare gains.[20] On average, states could increase welfare by under 2%, suggesting that the gains from socially optimal districting are typically small relative to the surplus that is generated by state legislatures. Regarding the precision of these estimates, the confidence intervals demonstrate that the upper bounds on these welfare gains are also quite low, ranging from 1.32% in South Carolina to 5.68% in Rhode Island.[21]

For a sense of how these results depend on the specific assumptions about the parameters γ/β and 2τ, Table 7 reports the average welfare gains, where the average is taken across both states and replications, associated with socially optimal districting arising under different parameter values. We allow the ratio γ/β to vary from 0.25 to 1 and allow 2τ to vary from $0.25/(1+\alpha)$ to $1/(1+\alpha)$ in each state. The notation η refers to the numerator in the ratio $\eta/(1+\alpha)$, so that the case in which 2τ is set equal to $0.25/(1+\alpha)$ in each state corresponds to $\eta = 0.25$; the case in which 2τ is set equal to $0.5/(1+\alpha)$ in each state corresponds to $\eta = 0.5$; etc. As shown, holding the ratio γ/β constant, the welfare gains to socially optimal districting, averaged across all states, are uniformly increasing as the parameter η is reduced. This pattern reflects the fact that reductions in the parameter η are associated with reductions in optimal responsiveness, which, as shown previously, was already below the responsiveness associated with the estimated seat-vote curves. Holding the parameter η constant and reducing the ratio γ/β, we have that welfare is uniformly decreasing. As explained above, districting matters less for welfare as this ratio decreases, reflecting the fact that the conflict between citizens over the available policy choices is less severe.

The lesson to be drawn from Tables 6 and 7 is that the welfare gains from socially optimal districting are relatively small as a proportion of the total surplus generated by state legislatures. In principle, there may be two reasons for this. The first is that the districting plans that states

[20] While a welfare comparison of the continuous optimal seat-vote curve and the discrete measured seat-vote curve is somewhat artificial, we use the continuous optimal seat-vote curve in order to apply the implementability condition from Coate and Knight (2006). To provide a sense of the error associated with this approximation, we have derived a discrete optimal seat-vote curve, which is a step-function approximation of the continuous optimal seat-vote curve. Welfare associated with this discrete optimal seat-vote curve is very similar to welfare under the continuous optimal seat-vote curve, and the approximation error associated with the use of a continuous optimal seat-vote curve is thus small in practice. The small size of this error should not be surprising given that the discrete and continuous optimal seat-vote curve converge as the number of districts grows large. As shown in Table 1, states have a large number of districts, averaging over 100 districts and ranging from 41 in Delaware to 203 in Pennsylvania.

[21] Of course, if this surplus is itself very large, then these gains could be quite large in monetary terms. But without further assumptions on the underlying welfare parameters β and γ, these percentage gains in welfare cannot be converted into monetary terms.

actually implement are relatively close to optimal plans. The second is that, because of the diverse ideological make up of the U.S. States, aggregate welfare is relatively insensitive to varying districting plans. To get a feel for which of these views is correct, we computed the proportionate welfare gains that would arise from implementing the optimal seat-vote curve over the case of identical districting, a natural benchmark in which each district is a microcosm of the whole.[22] The idea is that if these gains are large, then the second view cannot be correct. Table 8 reports the results under the same parametric assumptions that underlie Table 6 and they strongly suggest that the second view is not correct. Thus, it seems that the states are doing districting in a way that is generating seat-vote curves that are relatively close to optimal.

A further interesting benchmark is the seat-vote curve that would be generated by a *proportional representation* electoral system (PR); that is, $S(V) = V$. As shown in Table 9, introducing PR would raise welfare in all states. Moreover, the gains from a movement from PR to the optimal seat-vote curve are very small, ranging from an average of 0.01% in Michigan to 0.08% in Oklahoma under our baseline parameter values. These findings suggest that almost all the welfare gains associated with optimal districting could be achieved via PR. This result reflects several features of the seat-vote curves underlying these welfare calculations. First, the PR seat-vote curve is linear, a feature shared by the optimal, but not actual, seat-vote curves. Second, the PR seat-vote curve has responsiveness of 1, while responsiveness associated with the estimated seat-vote exceeds 1 in all cases. As noted previously, optimal responsiveness is less than 1.[23]

5 Comparison with existing approaches

As noted in the introduction, our analysis is related to a long-standing literature on the estimation of seat-vote curves, which, broadly speaking, has used three different approaches to estimation. Early studies employed the *uniform partisan swing model* (Butler, 1951). This begins with district-

[22] Note that the seat-vote curve associated with identical districting would equal 0 for $V < 1/2$ and 1 for $V \geq 1/2$. Interestingly, in the context of our model, this is also the seat-vote curve generated by at-large voting systems, providing an additional motivation for this benchmark.

[23] The reader may be concerned that our assumption that optimal responsiveness is at its upper bound may contribute in part to these small welfare gains associated with a movement from PR to optimal districting. Indeed, as the diversity parameter τ approaches 1/2, the optimal seat-vote curve converges to the PR seat-vote curve, as can be seen in equation (9). So there would obviously be no welfare gains to socially optimal districting in this case. To examine the role of this upper bound assumption, we experimented with alternative values of the parameter η. For example, when $\eta = 0.25$, optimal responsiveness equals $0.25/(1+\alpha)$ and is thus less than 1/4; even in this case, however, the welfare gains associated with a movement from PR to the optimal seat-vote curve are small, averaging 0.18% across states and replications. Thus, these results are robust to alternative parameter values and are not driven by the assumption that optimal responsiveness is at its upper bound.

specific voting returns and creates a seat-vote curve by simulating how the distribution of seats changes as support for the Democrats changes, implicitly assuming that district-specific votes move in tandem.[24] The second approach specifies a parametric form for the seat-vote curve, such as the linear seat-vote curve of Tufte (1973) and the bi-logit model of King and Browning (1987), and then estimates the underlying bias and responsiveness parameters by running time-series regressions of seats on votes.

Our method is most closely related to the third approach, which does not rely on parametric assumptions over the seat-vote curve and instead begins by specifying a statistical model of the vote generating process. Gelman and King (1990) posit a normal distribution for underlying support for the Democratic Party ($u_i \sim N(\alpha_i, \sigma^2)$) in district i, where this underlying support is related to votes (V_i) according to $V_i = \exp(u_i)/(1+\exp(u_i))$. The authors then make assumptions on the distribution of the district effect α_i and estimate the parameters of that distribution. Importantly, however, the variance of underlying support is assumed to be constant across districts. With these parameter estimates in hand, the authors then simulate the model, and the implied seat-vote curve, by drawing from the distribution of underlying support. In a follow-up paper, Gelman and King (1994) incorporate observable candidate characteristics (X_i), such as incumbency status and party control of the district, and estimate the parameters of the following regression equation:

$$V_i = X_i\beta + u_i. \tag{32}$$

where $u_i \sim N(0, \sigma^2)$. Again, the variance in support for the Democratic Party is assumed to be constant across districts.

Given our goal of evaluating districting plans from a welfare perspective, our approach begins with underlying citizen preferences rather than a functional form for the seat-vote curve or an assumed vote-generating process. These preferences and associated voting behavior in turn lead to a vote-generating process and ultimately a seat-vote curve. Relative to the most closely related approach, that of Gelman and King (1990, 1994), there are several key differences. First, while our approach allows for the variance of the vote-generating process to vary across districts, their approach assumes that this variance is constant across districts; in the context of our model,

[24] For example, suppose statewide support for Democratic candidates was recorded at 55 percent and consider an increase in support to 60 percent. The approach assumes that support for Democratic candidates in each district would increase by 5 percentage points so that seats won by Republicans with between 45 and 50 percent of the recorded vote would be predicted to flip to the Democrats.

this assumption would require that all districts have an equal fraction of Independent voters. Second, in explaining differences in voting patterns across districts, Gelman and King (1994) rely on observable candidate characteristics; this approach is inappropriate for our framework, in which candidates within a party are homogenous across districts and thus differences in voting outcomes across districts can be generated only by differences in voter preferences. Developing an approach that incorporates both candidate and voter characteristics would be desirable, and while beyond the scope of the current analysis, we hope to incorporate candidate heterogeneity, such as incumbency, in future extensions of this methodology.

6 Conclusion

Building on the theoretical model presented in Coate and Knight (2006), this paper has developed a methodology for comparing actual and optimal seat-vote curves and for evaluating the welfare gains from socially optimal districting. It starts by showing that seat-vote curves can be expressed as a function of the district-specific and statewide mean and variance of the Democratic vote share. It then develops empirical expressions for the optimal seat-vote curve, which can be uncovered using the statewide moments of the Democratic vote share along with data on the statewide fraction of Independents. In addition, the condition for implementing the optimal seat-vote curve can be expressed as an upper bound on the fraction of Independents and can thus be verified using similar data on these statewide moments and the fraction of Independents. Finally, the paper develops expressions for the welfare gains from socially optimal districting.

The paper then applies this methodology to analyze the districting plans in place in the 1990s to elect U.S. State legislators. The analysis shows that responsiveness tends to be too large, suggesting that U.S. State legislative elections were too competitive from a social welfare perspective. Encouragingly, the conditions for implementing the optimal seat-vote curve are permissive and are satisfied in all states and by a large margin in most states. While there is significant variation across states, the welfare gains to socially optimal districting are on average quite small. This appears to be because the state districting plans are reasonably close to optimal rather than because aggregate welfare is unresponsive to varying districting plans. The analysis also reveals that implementing proportional representation would produce welfare levels quite close to those achieved with socially optimal districting.

It will be clear to the reader that the theory of socially optimal districting is in its infancy. The theoretical model in this paper ignores many factors that are important in real world discussions of districting. Nonetheless, the model delivers some sharp prescriptions and this paper shows that these prescriptions can be investigated empirically. As the theory of socially optimal districting develops, we hope that the development of the empirical methodology presented here will be pursued in tandem.

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7 Appendix

Proof of Lemma: From (7), aggregate welfare when the median Independent has ideology m and the Democrats have seat share S is given by:

$$W(S,m) = \beta - \gamma[\pi_D(1-S)^2 + \pi_R S^2 + \pi_I \int_{m-\tau}^{m+\tau} (1-S-x)^2 \frac{dx}{2\tau}].$$

Using the fact that $\int_{m-\tau}^{m+\tau}(1-S-x)^2 \frac{dx}{2\tau} = (1-S-m)^2 + \tau^2/3$, we can re-write welfare as follows:

$$W(S,m) = \beta - \gamma\{\pi_D(1-S)^2 + \pi_R S^2 + \pi_I[(1-S-m)^2 + \tau^2/3]\}.$$

Reorganizing terms, we have that:

$$W(S,m) = \beta - \gamma\{S^2 - 2S[\pi_D + \pi_I(1-m)] + \pi_D + \pi_I[(1-m)^2 + \tau^2/3]\}.$$

Using the fact that $\pi_D + \pi_I(1-m) = S^o$, we have:

$$W(S,m) = \beta - \gamma\{S^2 - 2SS^o + \pi_D + \pi_I[(1-m)^2 + \tau^2/3]\}.$$

Finally, taking expected values with respect to V and using the fact that $E[(1-m)^2] = E(m^2) = 1/4 + \varepsilon^2/3$, we have that:

$$E[W(S)] = \beta - \gamma\{c + E(S^2) - 2E(SS^o)\},$$

where the constant is given by $c = \pi_D + \pi_I[1/4 + \varepsilon^2/3 + \tau^2/3]$. ■

Derivation of the moments of the seats distribution: Let the associated range of average votes be $[\underline{V}, \overline{V}]$ and the district-specific vote thresholds be (V_i^*), and let $\underline{i}$ be the smallest district such that $V_i^* \geq \underline{V}$ and $\overline{i}$ be the largest district such that $V_i^* \leq \underline{V}$.[25] Then, using straightforward area calculations, we can convert the three key expectations into functions of the district-specific vote thresholds as follows:

$$E[S(V)] = \sum_{i=\underline{i}}^{\overline{i}+1/n} (i - 1/n) \frac{\Delta V_i^*}{\overline{V} - \underline{V}},$$

[25] As pointed out in Coate and Knight (2006), any seat-vote curve $S(V)$ may be equivalently described by a triple $\{\underline{i}, \overline{i}, (V_i^*)_{i=\underline{i}}^{\overline{i}}\}$ - the so-called *inverse seat-vote curve*. The interpretation is that districts $i = 1/n, .., \underline{i} - 1/n$ are safe Democrat; $i = \overline{i} + 1/n, .., 1$ are safe Republican and V_i^* is the statewide vote threshold for competitive district $i \in \{\underline{i}, .., \overline{i}\}$.

$$E[S(V)^2] = \sum_{i=\underline{i}}^{\bar{i}+1/n} (i - 1/n)^2 \frac{\Delta V_i^*}{\overline{V} - \underline{V}},$$

and

$$E[VS(V)] = \sum_{i=\underline{i}}^{\bar{i}+1/n} \frac{1}{2}(i - 1/n) \frac{\Delta V_i^{*2}}{\overline{V} - \underline{V}},$$

where $\Delta V_{\underline{i}}^* = V_{\underline{i}}^* - \underline{V}$, $\Delta V_{\bar{i}+1/n}^* = \overline{V} - V_{\bar{i}}^*$, and $\Delta V_i^* = V_{i+1/n}^* - V_i^*$ for $i = \underline{i} + 1/n, ..., \bar{i}$.

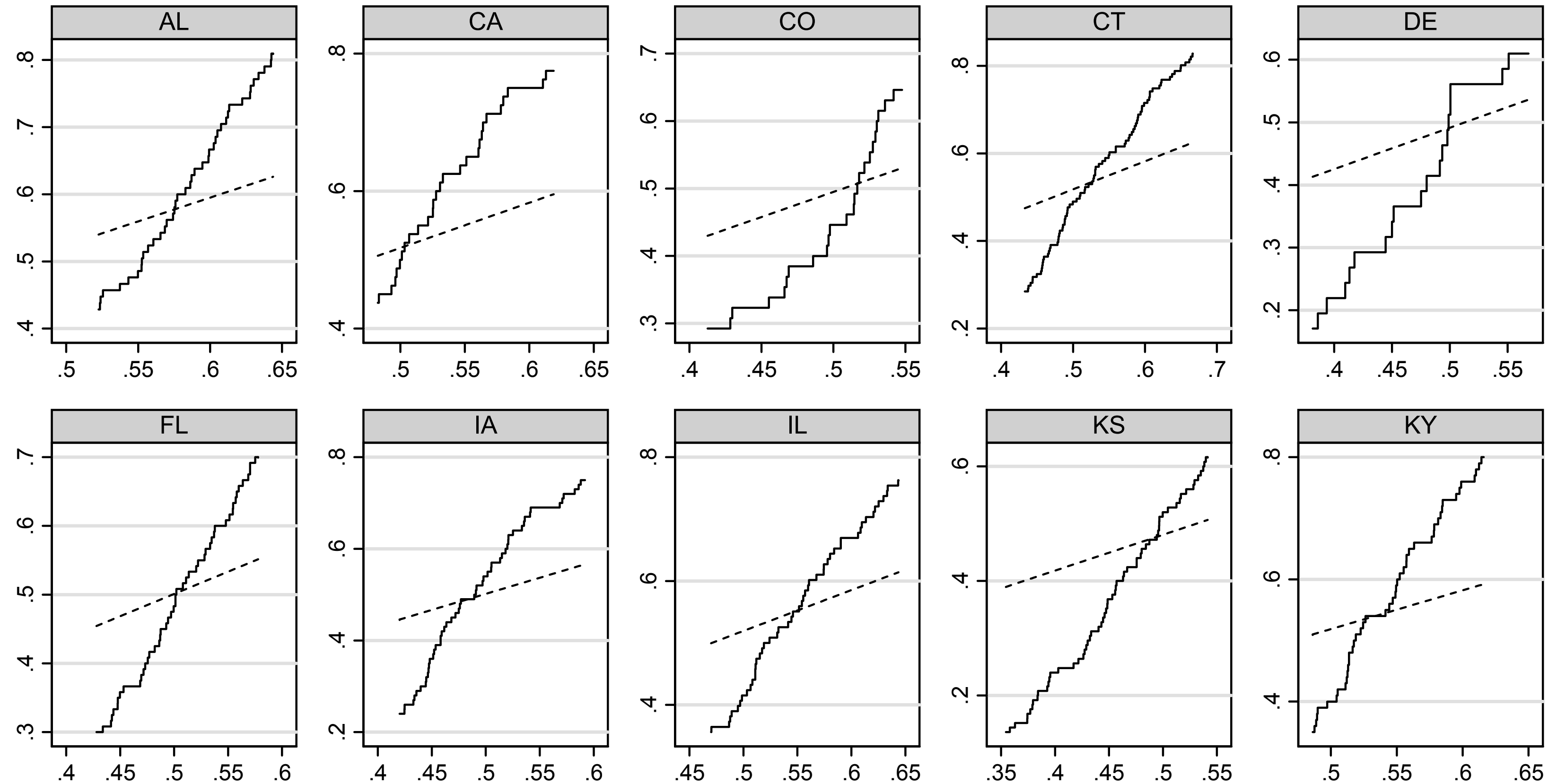

Figure 1a: Seat-Vote Curves

S(V) Optimal S(V)

Graphs by state

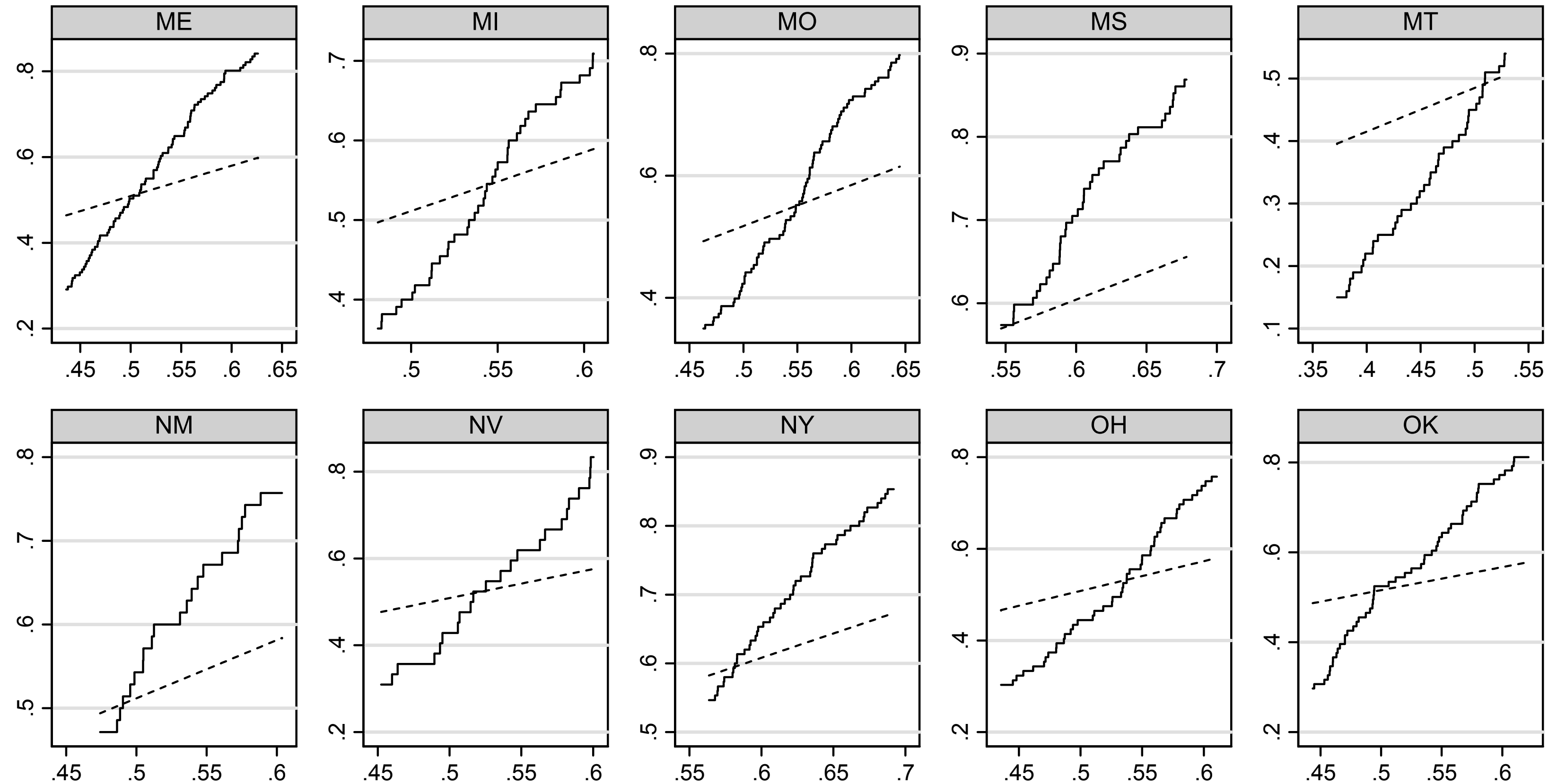

Figure 1b: Seat-Vote Curves

S(V)   Optimal S(V)

Graphs by state

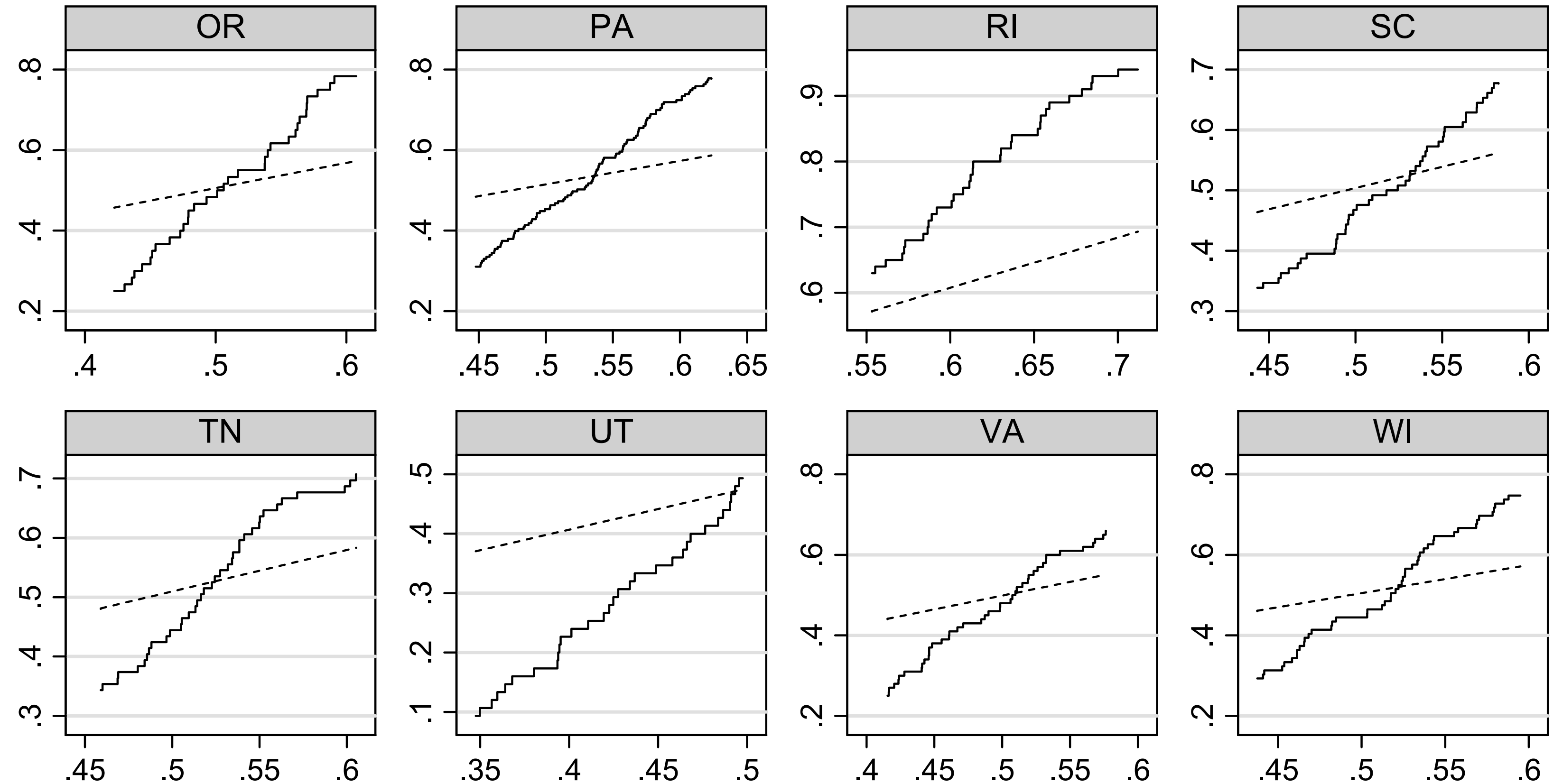

Figure 1c: Seat-Vote Curves

S(V) Optimal S(V)

Graphs by state

Table 1: States and Years Included in Analysis

state	*first redistricting*	*subsequent redistricting*	*number districts*
AL	1994	2002	105
CA	1992	2002	80
CO	1992	1998	65
CT	1992	2002	151
DE	1992	2002	41
FL	1994	2002	120
IA	1992	2002	100
IL	1992	2002	118
KS	1992	2002	125
KY	1996	2002	100
ME	1994	2002	151
MI	1992	2002	110
MO	1992	2002	163
MS	1995	2002	122
MT	1994	2002	100
NM	1992	2002	70
NV	1992	2002	42
NY	1992	2002	150
OH	1992	2002	99
OK	1992	2002	101
OR	1992	2002	60
PA	1992	2002	203
RI	1992	2002	100
SC	1992	1998	124
TN	1994	2002	99
UT	1992	2002	75
VA	1991	2001	100
WI	1992	2002	99

Table 2: Random Effects Regression Results

(all regressions include state-specific constant terms, standard errors in parentheses)

Moment	mean	variance
percent urban	0.0666**	-0.0426
	(0.0078)	(0.0912)
percent suburban	0.0374**	-0.1058
	(0.0083)	(0.0955)
household income (thousands)	-0.0036**	-0.0099**
	(0.0003)	(0.0040)
percent with college degree	0.1424**	-0.4446
	(0.0358)	(0.4143)
percent over age 65	0.3550**	-0.0073
	(0.0429)	(0.4997)
percent African American	0.4785**	-2.7376**
	(0.0183)	(0.2192)
percent Hispanic	0.2945**	-0.1505
	(0.0295)	(0.3494)
R-squared	0.4324	0.0501
Number of observations	8697	8697

Table 3: Properties of Estimated and Optimal Seat-Vote Curves
(for replication associated with median welfare loss by state)

state	*r*	*optimal r*	*difference*	*b*	*optimal b*	*difference*	*expected seats difference*
AL	3.2472	0.7081	2.5391				0.0265
CA	2.5449	0.6559	1.8890	0.0000	0.0172	-0.0172	0.0856
CO	2.6446	0.7431	1.9014	-0.0538	-0.0069	-0.0469	-0.0643
CT	2.3504	0.6422	1.7082	-0.0099	0.0176	-0.0275	0.0308
DE	2.5227	0.6598	1.8629	0.0122	-0.0093	0.0215	-0.0677
FL	2.7820	0.6482	2.1338	-0.0167	0.0007	-0.0174	-0.0138
IA	3.0523	0.7019	2.3504	0.0400	0.0003	0.0397	0.0321
IL	2.4497	0.6597	1.7900	-0.0847	0.0184	-0.1032	0.0096
KS	2.6567	0.6251	2.0316	0.0200	-0.0196	0.0396	-0.0891
KY	3.4178	0.6348	2.7831	-0.1000	0.0169	-0.1169	0.0394
ME	3.0375	0.7065	2.3310	0.0033	0.0082	-0.0049	0.0595
MI	2.9170	0.7381	2.1789	-0.1000	0.0072	-0.1072	-0.0079
MO	2.6964	0.6712	2.0252	-0.0767	0.0170	-0.0937	0.0221
MS	2.3714	0.6547	1.7167				0.1123
MT	2.4715	0.6994	1.7722	-0.0500	-0.0185	-0.0315	-0.1198
NM	2.3251	0.6942	1.6308	0.0429	0.0110	0.0319	0.0892
NV	3.3145	0.6678	2.6467	-0.0714	0.0054	-0.0768	0.0009
NY	2.4279	0.7064	1.7215				0.0861
OH	2.8277	0.6493	2.1783	-0.0556	0.0066	-0.0622	-0.0140
OK	2.8837	0.5135	2.3702	0.0248	0.0130	0.0117	0.0499
OR	2.9938	0.6260	2.3678	-0.0167	0.0012	-0.0179	0.0118
PA	2.7588	0.5833	2.1755	-0.0468	0.0144	-0.0612	0.0130
RI	2.1169	0.7635	1.3534				0.1737
SC	2.4976	0.7032	1.7944	-0.0323	0.0029	-0.0352	-0.0225
TN	2.7715	0.6987	2.0728	-0.0556	0.0088	-0.0644	0.0116
UT	2.4767	0.6932	1.7835				-0.1453
VA	2.4602	0.6815	1.7787	-0.0200	-0.0023	-0.0177	-0.0223
WI	3.0384	0.6981	2.3403	-0.0556	-0.0058	-0.0498	0.0062
Average	2.7162	0.6724	2.0438	-0.0305	0.0046	-0.0351	0.0105

Table 4: Properties of Estimated and Optimal Seat-Vote Curves
(properties of distribution across all replications)

	difference in responsiveness			*difference in bias*			*difference in expected seats*		
	mean	*median*	*confidence interval*	*mean*	*median*	*confidence interval*	*mean*	*median*	*confidence interval*
AL	1.9047	1.9220	1.2926 , 2.5643	-0.0803	-0.0879	-0.1382 , -0.0224	0.0547	0.0540	0.0125 , 0.0963
CA	1.9153	1.8863	1.2423 , 2.5759	-0.0343	-0.0316	-0.0952 , 0.0201	0.0770	0.0794	0.0303 , 0.1285
CO	1.9236	1.8969	1.2446 , 2.6502	0.0018	-0.0001	-0.0597 , 0.0668	-0.0567	-0.0553	-0.1132 , -0.0040
CT	1.7036	1.7063	1.3678 , 1.9834	-0.0564	-0.0554	-0.0952 , -0.0139	0.0282	0.0281	0.0054 , 0.0542
DE	1.8842	1.8448	1.3237 , 2.5197	-0.0305	-0.0325	-0.0939 , 0.0347	-0.0591	-0.0588	-0.1182 , -0.0051
FL	2.0569	2.0639	1.5929 , 2.5579	-0.0291	-0.0260	-0.0870 , 0.0169	-0.0222	-0.0200	-0.0610 , 0.0152
IA	2.4048	2.4075	1.8460 , 2.9415	0.0004	0.0042	-0.0508 , 0.0414	-0.0200	-0.0225	-0.0658 , 0.0270
IL	1.7964	1.7878	1.3589 , 2.2360	-0.0909	-0.0954	-0.1330 , -0.0451	0.0108	0.0097	-0.0261 , 0.0491
KS	2.0509	2.0372	1.6528 , 2.5019	-0.0040	-0.0072	-0.0433 , 0.0407	-0.0839	-0.0839	-0.1149 , -0.0418
KY	2.4485	2.4668	1.7533 , 3.1465	-0.0478	-0.0494	-0.0935 , -0.0035	0.0513	0.0482	0.0060 , 0.0938
ME	2.3549	2.3409	1.9842 , 2.7348	-0.0050	-0.0072	-0.0465 , 0.0360	0.0433	0.0411	0.0142 , 0.0771
MI	1.9166	1.9070	1.2888 , 2.5739	-0.0900	-0.0927	-0.1300 , -0.0428	-0.0211	-0.0208	-0.0685 , 0.0187
MO	2.1380	2.1285	1.7532 , 2.5359	-0.0809	-0.0813	-0.1195 , -0.0469	0.0173	0.0189	-0.0296 , 0.0546
MS	1.6907	1.6806	1.1752 , 2.1914				0.1108	0.1140	0.0644 , 0.1487
MT	2.2402	2.2607	1.6136 , 2.9551	0.0172	0.0189	-0.0431 , 0.0699	-0.1016	-0.1026	-0.1424 , -0.0480
NM	1.9229	1.9202	1.1982 , 2.6886	-0.0041	-0.0093	-0.0701 , 0.0578	0.0729	0.0756	0.0197 , 0.1168
NV	2.2118	2.2450	1.2400 , 3.0578	-0.0207	-0.0252	-0.0928 , 0.0579	0.0401	0.0374	-0.0256 , 0.1074
NY	1.5800	1.6199	1.0665 , 1.9876				0.0865	0.0869	0.0518 , 0.1215
OH	2.0700	2.0838	1.4821 , 2.6220	-0.0560	-0.0605	-0.0967 , -0.0045	-0.0351	-0.0332	-0.0728 , 0.0067
OK	2.3986	2.3928	1.8103 , 2.9489	-0.0384	-0.0404	-0.0853 , 0.0176	0.0396	0.0391	-0.0005 , 0.0953
OR	2.4100	2.3817	1.7184 , 3.0779	-0.0078	-0.0061	-0.0718 , 0.0538	-0.0040	-0.0009	-0.0637 , 0.0473
PA	2.1437	2.1445	1.8033 , 2.5313	-0.0640	-0.0623	-0.0964 , -0.0326	-0.0180	-0.0181	-0.0552 , 0.0085
RI	1.2748	1.2550	0.6928 , 2.0177				0.1738	0.1714	0.1355 , 0.2098
SC	1.7945	1.8336	1.2013 , 2.4776	-0.0288	-0.0315	-0.0735 , 0.0171	-0.0101	-0.0080	-0.0457 , 0.0247
TN	1.9940	2.0130	1.4438 , 2.5939	-0.0728	-0.0738	-0.1119 , -0.0255	0.0023	0.0021	-0.0349 , 0.0445
UT	1.8558	1.8156	1.1018 , 2.6942	0.0274	0.0345	-0.0322 , 0.0749	-0.1462	-0.1462	-0.1909 , -0.1009
VA	1.7624	1.7582	1.2550 , 2.3586	-0.0309	-0.0350	-0.0710 , 0.0176	-0.0093	-0.0092	-0.0445 , 0.0252
WI	2.2230	2.2397	1.6035 , 2.8915	-0.0339	-0.0331	-0.0903 , 0.0177	-0.0239	-0.0246	-0.0601 , 0.0161

Table 5: Conditions for Implementability

state	*% independents*	*average maximum % independents*	*Pr(implementable)*
AL	29.44%	70.28%	1
CA	26.01%	73.61%	1
CO	39.12%	80.64%	1
CT	41.89%	67.09%	1
DE	36.20%	77.13%	1
FL	27.59%	83.59%	1
IA	40.46%	80.45%	1
IL	33.64%	71.74%	1
KS	31.18%	73.72%	1
KY	22.55%	78.76%	1
ME	45.71%	76.01%	1
MI	35.16%	79.94%	1
MO	37.15%	74.07%	1
MS	24.94%	63.62%	1
MT	36.36%	75.17%	1
NM	29.43%	78.40%	1
NV	29.78%	78.85%	1
NY	30.92%	60.12%	1
OH	32.41%	80.78%	1
OK	18.79%	75.74%	1
OR	30.99%	79.99%	1
PA	24.59%	78.77%	1
RI	51.19%	58.02%	1
SC	33.02%	84.22%	1
TN	33.94%	77.63%	1
UT	33.91%	70.45%	1
VA	34.42%	81.77%	1
WI	36.43%	82.49%	1

Table 6: Welfare Gains to Optimal Districting

state	*median % welfare gains*	*average % welfare gains*	*confidence interval*	
AL	1.09%	1.16%	0.52% ,	1.92%
CA	1.67%	1.80%	0.85% ,	3.29%
CO	1.32%	1.35%	0.54% ,	2.50%
CT	1.76%	1.81%	1.21% ,	2.42%
DE	1.91%	2.07%	0.98% ,	3.49%
FL	1.10%	1.14%	0.74% ,	1.64%
IA	1.91%	1.98%	1.16% ,	3.18%
IL	1.02%	1.07%	0.62% ,	1.63%
KS	2.54%	2.50%	1.60% ,	3.58%
KY	1.63%	1.60%	0.81% ,	2.46%
ME	2.44%	2.48%	1.69% ,	3.28%
MI	0.79%	0.82%	0.32% ,	1.47%
MO	1.48%	1.52%	1.04% ,	2.15%
MS	2.15%	2.19%	1.10% ,	3.29%
MT	2.61%	2.65%	1.53% ,	3.86%
NM	1.53%	1.56%	0.64% ,	2.61%
NV	1.64%	1.73%	0.66% ,	3.12%
NY	1.42%	1.46%	0.80% ,	2.33%
OH	1.57%	1.63%	0.82% ,	2.63%
OK	2.28%	2.40%	1.49% ,	3.56%
OR	2.06%	2.14%	1.02% ,	3.38%
PA	1.57%	1.58%	1.12% ,	2.12%
RI	4.14%	4.22%	2.78% ,	5.68%
SC	0.72%	0.76%	0.33% ,	1.32%
TN	1.03%	1.03%	0.57% ,	1.57%
UT	3.41%	3.56%	1.97% ,	5.47%
VA	0.93%	0.96%	0.56% ,	1.53%
WI	1.44%	1.46%	0.72% ,	2.34%
Average	1.76%	1.81%		

Table 7: Average Welfare Gains Under Alternative Parameter Values

	η=*1*	η=*0.75*	η=*0.50*	η=*0.25*
γ/β=*1*	1.81%	1.98%	2.17%	2.38%
γ/β=*0.75*	1.28%	1.40%	1.54%	1.69%
γ/β=*0.50*	0.81%	0.89%	0.98%	1.07%
γ/β=*0.25*	0.38%	0.42%	0.46%	0.51%

Table 8: Average Welfare Gains From Identical Districting

state	*% welfare gains (relative to identical districting)*
AL	27.28%
CA	32.72%
CO	35.63%
CT	31.97%
DE	35.59%
FL	38.83%
IA	35.83%
IL	32.04%
KS	34.92%
KY	36.52%
ME	34.00%
MI	35.37%
MO	33.57%
MS	22.28%
MT	33.01%
NM	35.19%
NV	36.32%
NY	19.54%
OH	37.26%
OK	38.42%
OR	37.35%
PA	38.33%
RI	19.24%
SC	38.02%
TN	35.11%
UT	27.65%
VA	37.13%
WI	37.01%
Average	33.43%

Table 9: Average Welfare Gains from PR

state	*% welfare gains to optimal districting from PR*	*% welfare gains to implementing PR*
AL	0.02%	1.15%
CA	0.03%	1.77%
CO	0.02%	1.34%
CT	0.07%	1.74%
DE	0.06%	2.02%
FL	0.03%	1.12%
IA	0.03%	1.95%
IL	0.03%	1.04%
KS	0.05%	2.45%
KY	0.03%	1.58%
ME	0.03%	2.44%
MI	0.01%	0.81%
MO	0.03%	1.49%
MS	0.02%	2.16%
MT	0.02%	2.63%
NM	0.03%	1.55%
NV	0.05%	1.71%
NY	0.02%	1.44%
OH	0.04%	1.60%
OK	0.08%	2.32%
OR	0.05%	2.10%
PA	0.05%	1.53%
RI	0.02%	4.20%
SC	0.02%	0.74%
TN	0.02%	1.01%
UT	0.02%	3.54%
VA	0.03%	0.93%
WI	0.03%	1.44%
Average	0.03%	1.78%