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Individual income, incomplete information, and aggregate consumption

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Discussion Paper

Discussion Paper No 91-07

Individual Income, Incomplete Information, and Aggregate Consumption

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Individual Income, Incomplete Information, and Aggregate Consumption

by

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Abstract

In this paper I study a model of life-cycle consumption in which individuals react optimally to their own income process but ignore economy wide information. Since individual income is less persistent than aggregate income consumers will react too little to aggregate income variation. Aggregate consumption will be excessively smooth. Since aggregate information is slowly incorporated into consumption, aggregate consumption will be autocorrelated and correlated with lagged income. The second part of the paper provides empirical evidence on individual and aggregate income processes and calibrates the model using the estimated parameters. The model predictions roughly correspond to the empirical findings for aggregate consumption data. Allowing for the existence of measurement error in micro income, durables, finite lifetimes of consumers, and advance information improves the predictions of the model.

JEL-classification: E21

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1. Introduction

Contrary to the predictions of the modern version of the permanent income hypothesis (Hall, 1978), aggregate consumption changes in the U.S. are correlated with lagged income changes (see Flavin, 1981, and Nelson, 1987). Moreover, Deaton (1987) and Campbell and Deaton (1989) demonstrated that consumption is smoother than predicted by the model if income follows a highly persistent process. In individual data, on the other hand, the permanent income model is much harder to reject. One of the more careful micro study on the PSID seems to be Altonji and Siow (1987), who account for measurement error and reconcile their positive findings with previous work. Zeldes (1989) finds, however, that the permanent income model fails for low wealth consumers. If it is true that the model holds for individual data but not for aggregate data¹ there are only two possibilities for this failure: First, finite lifetimes will introduce a dependence of consumption on cohort characteristics at the aggregate level and the martingale result found by Hall will not hold. Galí (1990) has developed this point in a recent paper and shown that it is not important enough empirically to explain aggregate income and consumption data. On the other hand, if the model of infinitely lived consumers is not a bad approximation² then consumption changes should be unpredictable at the aggregate level using any aggregate variable that is contemporaneously in the consumers' information sets. This is a powerful implication of the law of iterated expectations and is true under rather general conditions (see e.g. Grossman and Shiller, 1982). Only by restricting consumers contemporaneous aggregate information can the predictions of the model diverge in micro and aggregate data. Building on previous work by Goodfriend (1991), I explore a model where individuals lack aggregate information and investigate its empirical implications.

It is not unlikely that information on aggregate income plays little role in household decisions since the economic environment in which individuals operate differs sharply from the economy as it is described by aggregate data. Most importantly, individual income is much more variable than aggregate income: Below, I estimate that the standard deviation of quarterly individual income changes is about sixty times larger than that for aggregate per capita income. While some of this variation will be attributable to measurement problems, a large part may reflect idiosyncratic income shocks. Therefore, individuals may make little effort to gather information on the behavior of the economy, but rather watch only their own prospective fortunes. Furthermore, individual income processes are much less persistent than aggregate income. The optimal consumption response calculated on the basis of individual income processes

¹ The inability to reject the model in micro data may of course also stem from problems related to measurement error, inexact variable definitions, etc. that make these tests less powerful.

² Attanasio and Weber (1990) have addressed the impact of finite lifetimes on the estimated intertemporal elasticity of substitution. They have found a substantially different estimates between aggregate and average cohort data.

differs substantially from the predictions of a representative agent model calibrated with aggregate data. Using these facts, I construct a simple model in which agents react optimally to their individual income innovations but do not incorporate information on economy wide variables. The model correctly predicts what we observe in aggregate data: the correlation of consumption changes with lagged income and excess smoothness.

A simple example makes clear how the model works. Suppose a worker gets laid off from his job; he does not know immediately whether this is due to specific conditions at his firm or because of the onset of a general recession. If the layoff is due to highly individual factors then it will be easy for the worker to find new employment and the income reduction associated with the unemployment spell does not call for a major revision in consumption expenditures. Should the unemployment be due to aggregate factors, employment will be depressed at other firms as well and lead to a much longer expected unemployment spell. The necessary revision in consumption will be much larger than in the former case. The worker adjusts consumption in a way that will be correct on average given his overall experience with unemployment. Looking at aggregate data, an econometrician will find *ex post* that everybody revised consumption downward too little at the onset of a recession. Subsequently, there will be further revisions once workers learn about the true scope and persistence of the shock. Consumption will appear correlated with lagged income and will appear smoother than predicted by a model where agents know the cause and length of their unemployment spell immediately.

There are a number of well known expositions of the idea that individual agents may have incomplete aggregate information. Robert Lucas (1973) suggested a model in which suppliers confuse aggregate and relative price movements. This yields an observable Phillips curve relationship in aggregate data which is not predicted by a full information representative agent model. Altonji and Ashenfelter (1980) use the same feature in a life-cycle model of labor supply to generate an intertemporal substitution effect. If the aggregate wage follows a random walk and agents have full information there is no room for intertemporal substitution. If workers only know the lagged aggregate wage and their own wage, consisting of an individual and an aggregate component, then the model yields aggregate employment fluctuations even if the aggregate wage is a random walk. Froot and Perold (1990) have recently suggested a model where securities market specialists observe only information on their own stock contemporaneously but not aggregate information. Their model yields correlated aggregate stock returns.

In all of these models agents observe the aggregate variable with a one period lag. An analogous model in which agents learn about aggregate income with a one quarter delay has been suggested for consumption behavior by Goodfriend (1991). His model yields an MA(1) process for consumption changes. Therefore, no variable lagged at least twice should be able to predict consumption changes. The hypothesis of lagged information on income has been considered informally by Holden and Peel (1985). They reject this model on U.K. data by regressing consumption changes on income

and consumption lagged twice. Campbell and Mankiw (1989, 1990) use information variables lagged at least two periods and find the same result for the U.S. and other countries.

This paper examines a model in which agents know only their own income processes but never observe the aggregate component in their income. I will also present results for Goodfriend's model with lagged information on aggregate income. Flavin's (1981) model with quadratic instantaneous utility is a convenient tool for this analysis. There are two virtues to this model; firstly, it allows explicit solutions for the consumption process. Given the joint behavior of income and consumption it is then possible to calculate the regression coefficient of consumption changes on lagged income changes and the ratio of the variability in consumption to the variability in the income innovation. These predictions are easily compared to the sample statistics for aggregate data. Furthermore, while not the most realistic model, the quadratic utility model serves as a useful benchmark. It allows me to assess how much incomplete information and aggregation by themselves can contribute toward explaining the aggregate consumption puzzles.

To calibrate the model it is necessary to have information on aggregate and individual income processes. While some estimates for individual earnings are available in the literature they are not well suited for the present purpose. In particular, no estimates are available that utilize quarterly income information comparable to the sampling frequency of aggregate data. I use the 1984 Panel of the Survey of Income and Program Participation which contains monthly information on family income to construct the appropriate quarterly micro data. I obtain estimates for the micro income process that are quite similar to the findings for annual earnings.

Using these results, I find that the model yields predictions that are in the correct direction and deviate substantially from the full information case. Quantitatively, they do not match the results for U.S. aggregate data exactly, however. The model generally tends to predict too high a correlation of consumption with lagged income but not smooth enough consumption. Notice, however, that my procedure, using actual micro parameters to calibrate the model, subjects the model to a much more stringent test than is usually adopted in the macro consumption literature. Furthermore, the predictions are somewhat closer to the aggregate results than they are for Goodfriend's model. I argue that rational consumers would not concern themselves with acquiring aggregate information as assumed by Goodfriend because the gain only amounts to a few cents every quarter.

Given the simplicity of the model we cannot expect it to explain aggregate consumption perfectly. Allowing for measurement error in the micro income data, the existence of durable goods, finite lifetimes of consumers, and advance information about income tends to improve the predictions of the model.

The rest of the paper is organized as follows. In the next section, I review the basic full information model, its implications, and the empirical failures it has generated. Using a simple income process as an example, section 3 analyzes the model with no observability of aggregate income and describes its implications. In section 4, I do the

same exercise for the model of Goodfriend where aggregate information becomes available with a one period lag. Section 5 presents estimates of a simple model for individual income. In section 6, I estimate aggregate income processes and summarize the stylized facts on the consumption puzzles. Section 7 uses the estimates on the income process to predict features of aggregate consumption and compares the results to the findings in the previous section. Some extensions of the model are suggested in section 8 and section 9 provides conclusions. Appendices contain derivations of the consumption process resulting for general income processes, details on the samples used in the empirical work, and a derivation of the utility loss from ignoring aggregate information.

2. The Full Information Representative Agent Model

In this section I will review the representative agent model of consumption with full information and its empirical implications. The consumer solves the life-cycle maximization problem:

$$\text{Max}_{\{c_t\}} E_t \sum_{s=t}^{\infty} \left(\frac{1}{1+\delta} \right)^{s-t} u(c_s) \quad (1)$$

$$\text{s.t. } W_{t+1} = (1+r)[W_t + y_t - c_t]$$

$$\lim_{t \rightarrow \infty} (1+r)^{-t} W_t = 0 \quad \text{a.s.}$$

c_t is consumption, y_t is non-interest income, and W_t is non-human wealth at the beginning of period t . Income is paid and consumption takes place before interest accrues on wealth. r and δ are the interest rate and the time discount rate, respectively. Both are assumed to be constant.

Flavin (1981) has shown that a quadratic instantaneous utility function and $r = \delta$ yields the following relation for the change in consumption

$$\Delta c_t = r \sum_{s=0}^{\infty} \frac{(E_t - E_{t-1})y_{t+s}}{(1+r)^{s+1}} \quad (2)$$

This relationship just says that the change in consumption equals the present value of the news about future income.

If income follows a univariate time series process known to the consumer then (2) can be used to relate changes in consumption to the innovations in the income process directly. Let income be a stationary ARMA process given by

$$\Phi(L)y_t = \Psi(L)\varepsilon_t \quad (3)$$

Then (2) can be rewritten as

$$\Delta c_t = \frac{r}{1+r} \frac{\Psi\left(\frac{1}{1+r}\right)}{\Phi\left(\frac{1}{1+r}\right)} \varepsilon_t \equiv v_t \quad (4)$$

where the lag polynomials are the same as in (3).

Equation (4) exemplifies the first implication of the representative agent model: changes in individual consumption are only related to the innovations in labor income and cannot be forecasted by any other variable known at time $t-1$. If agents know aggregate income then this relationship has to be true for aggregate data. In particular, no coefficients in a regression of the change in consumption on variables dated $t-1$ and earlier should be significant. This martingale property has been tested by Hall (1978) by regressing consumption changes on lags of consumption, income, and stock prices. Hall found little explanatory power for income but rejected nonpredictability for stock prices. I will call this rejection of the representative agent model the *orthogonality failure*.

Flavin (1981) has taken a different route for testing the model. Instead of testing the martingale property like Hall she imposed the cross equation restrictions implied by (3) and (4) and tested whether consumption reacts to income beyond the predictions of the model. She called this failure *excess sensitivity* of consumption to income. Her test is directly related to Hall's orthogonality test; the orthogonality failure is just a reduced form implication of excess sensitivity.

The model for income in (3) assumes stationarity while empirical income processes are nonstationary. Deaton (1987) has argued that aggregate income is well described by a stationary process in first differences. The model in (3) and (4) can easily accommodate this aspect. Hansen and Sargent (1981) have shown that the formula in (4) is still valid even in the presence of a unit root in $\Phi(L)$ as long as there is discounting.³

Whether income has a unit root or not is quite important for the consumption response to income innovations. For example, say income follows a random walk. Then all innovations in income are permanent and consumption will respond one-to-one to an income innovation. Compare this to an income process that is an AR(1) in levels with a coefficient near one, say 0.95. If the interest rate is 0.01, then the response of consumption to an income innovation is only 0.167. In the infinite horizon model, modelling innovations as a unit root process or as a serially correlated but stationary process around trend gives rise to a sizable difference in the consumption response.

Deaton (1987) first pointed out these implications. He estimated an AR(1) in first differences for quarterly aggregate income and obtained a coefficient of about 0.4. Taking variances on both sides of (4) implies

3 Although you cannot invert $\Phi(L)$ you can invert $\Phi(1/(1+r))$ if $r > 0$.

$$\frac{\sigma_{\Delta c}}{\sigma_{\varepsilon}} = \frac{r}{1+r} \frac{\Psi\left(\frac{1}{1+r}\right)}{\Phi\left(\frac{1}{1+r}\right)} \quad (5)$$

The ratio of the standard deviation of consumption changes to the standard deviation of income innovations should equal the consumption response predicted by the model, 1.65 for an interest rate of 0.01. Using σ_{ε} from an AR(1) in first differences, Deaton found that this ratio is only one half in the data. So consumption is much smoother than the representative agent model predicts. This failure of the model is often referred to as the Deaton paradox, I will call it the *excess smoothness result*. Compared to the orthogonality failure or excess sensitivity this failure of the model rests on the additional assumption that the income process is highly persistent.

3. Heterogeneity in Income and Unobservable Aggregate Shocks: An Example

I present a simple example to describe the implications of the model where the individual income process differs from the time series structure of aggregate income. The income process in this example is not necessarily representative of actual data but it serves to illuminate how the model works.⁴ Results for general income processes are presented in Appendix A. The example will show that both the orthogonality failure and the smoothness result may arise in aggregate data in this model. I will use subscripts i to denote individual variables while no subscript refers to aggregate variables.

Let individual income be composed of two parts, an aggregate component, which follows a random walk and an individual specific component, which is white noise. The first difference of income then obeys the following process

$$\Delta y_{it} = \varepsilon_t + u_{it} - u_{it-1} \quad (6)$$

where ε_t is the aggregate income innovation and u_{it} is the individual income shock. The innovations are assumed to be uncorrelated.

This model where income consists of a permanent component and a purely transitory component has been studied originally by Muth (1960). If the individual cannot distinguish the aggregate and the individual component, as I will assume throughout this section, then this process to her looks just like an MA(1) process for the first differences in income. The income process the individual observes can thus be written as

$$\Delta y_{it} = \eta_{it} - \theta \eta_{it-1} \quad (7)$$

⁴ I work with a more accurate model in the empirical estimation below.

The innovation η_{it} is a compound of the aggregate and the individual components. Muth has shown that $(1 - \theta)\eta_{it}$ is the optimal predictor of the innovation to the random walk component of income. Note that $\{\eta_{it}\}$, though not a fundamental driving process of the model, is an innovation sequence with respect to the history of individual income changes. The MA parameter θ in (7) depends on the relative variances of the aggregate and individual income shocks and $0 \leq \theta \leq 1$.

Equation (4) is still true for the individual. Changes in individual consumption therefore follow

$$\Delta c_{it} = \frac{r}{1+r} \frac{1 - \frac{\theta}{1+r}}{1 - \frac{1}{1+r}} \eta_{it} = \frac{1+r-\theta}{1+r} \eta_{it} \equiv A \eta_{it} = A \frac{\Delta y_{it}}{1-\theta L} \quad (8)$$

Individual consumption changes are a martingale with respect to the history of individual consumption and income. A researcher doing Hall's (1978) analysis on panel data for individuals should not reject the permanent income model.⁵ This type of testing procedure has been carried out by Altonji and Siow (1987) who do not reject the model. Estimating a structural model in which consumers can distinguish innovations to the individual and aggregate income components in (6) as in Hall and Mishkin (1982) would not be correct.⁶ The correct structural model would use the income process in (7) instead. This has been pointed out by Speight (no date) who finds support for the model with incomplete information on Austrian panel data while the Hall and Mishkin model is rejected.

I want to focus here on the aggregate implications of the incomplete information case. To find the change in average per capita consumption use the last equality in (8) and equation (6) and sum over individuals.

$$\frac{1}{n} \sum \Delta c_{it} = \frac{A}{n} \sum \frac{\Delta y_{it}}{1-\theta L} = \frac{A}{n} \sum \frac{\varepsilon_t + u_{it} - u_{it-1}}{1-\theta L} \quad (9)$$

Because the individual shocks are mutually uncorrelated they will sum to zero in a large population so that we obtain

5 The martingale property only holds with respect to variables that are in individuals' information sets. Many researchers using panel data control for macroeconomic shocks. Goodfriend (1991) first pointed out that such controls also invalidate the Hall procedure. I show below that the variance of individual income innovations is far larger than the variance of the aggregate component; this will therefore not be very important in practice.

6 This is not literally true. Hall and Mishkin (1982) only distinguish a permanent and a transitory income component. These are not identified with aggregate and individual income processes as in the example in the text. Furthermore, Hall and Mishkin find nonzero correlations between consumption changes and lagged income changes or lagged consumption changes in their data. Apart from the appropriateness of the structural income process it is these correlations that lead to a rejection of the model in their sample.

$$\frac{1}{n} \sum \Delta c_{it} = \Delta c_t = A \frac{\varepsilon_t}{1 - \theta L}$$

$$\Delta c_t (1 - \theta L) = A \varepsilon_t \quad (10)$$

Equation (10) has a number of interesting implications. Unlike individual consumption, the per capita series of consumption in (10) is not a random walk as the representative agent model predicts. Consumption now follows an AR(1) in first differences. The intuition for this is rather simple. Suppose an aggregate shock hits the economy. All the individual consumers see their income changing but they assume that a part of the shock is idiosyncratic and therefore transitory. They will change their consumption but not by as much as the permanence of the shock calls for. Because the shock is persistent, in the following period they will be surprised again that their income is higher than expected, they will increase their consumption further and so on.

All this implies that an econometrician working with the representative agent model will find both the orthogonality failure and the smoothness result in aggregate data. Suppose the econometrician estimates the following model

$$\Delta c_t = \alpha + \beta \Delta y_{t-1} + e_t \quad (11)$$

If the data are generated by (10) the expected value of β would be

$$\begin{aligned} \beta &= \frac{\text{cov}(\Delta c_t, \Delta y_{t-1})}{\text{var}(\Delta y_{t-1})} \\ &= \frac{E\left\{A\left(\frac{\varepsilon_t}{1 - \theta L}\right)\varepsilon_{t-1}\right\}}{\sigma_\varepsilon^2} = \frac{A\theta\sigma_\varepsilon^2}{\sigma_\varepsilon^2} = A\theta \end{aligned} \quad (12)$$

Figure 1 below plots numerical results for the expected regression coefficient for various relative variances of the aggregate and the idiosyncratic shock.

Because individuals do not recognize an aggregate shock to be permanent they will not adjust their consumption by as much as they would if it were the only type of shock to occur. This will lead to more smoothness in aggregate data than predicted by the representative agent model. For the example, the representative agent model with random walk income implies that the standard deviation of consumption changes equals the standard deviation in aggregate income innovations. For the model with heterogeneous agents and incomplete information we get instead from (10)

$$\frac{\sigma_{\Delta c}}{\sigma_\varepsilon} = \frac{A}{\sqrt{1 - \theta^2}} \quad (13)$$

If idiosyncratic shocks are present and the interest rate is small enough the ratio of the standard deviations of the change in consumption and the aggregate income innovation will always be less than one.⁷

It is easy to see which features of the example drive the result. The representative agent model would hold for aggregate data if the aggregate and the individual income processes had the same persistence properties so that consumers would want to react in the same way to each type of shock. In this example, consumers do not want to increase consumption enough in response to an aggregate shock because they confuse it with the individual income innovation which is less persistent. Furthermore, to get both the smoothness result and a positive correlation of consumption changes with lagged income changes it is crucial that the aggregate income component is more permanent than the individual components. I will argue below that this is consistent with findings on individual income data.

The results also hinge on the assumption that individuals cannot (or do not find it profitable to) distinguish aggregate and idiosyncratic shocks. Otherwise they would react differently according to the persistence properties of the specific shock observed. This is the model Goodfriend (1991) has originally proposed, where information on aggregate income becomes available with a one period lag. For comparison, I will analyze the implications of his model with lagged information on aggregate income in the following section.

4. Lagged Information about Aggregate Shocks

Suppose aggregate data are published with a one period lag. In period t individual i will observe y_{it} and the aggregate shock ϵ_{t-1} . I also assume that the consumer has access to the infinite history of shocks and can therefore infer u_{it-1} as well once the aggregate shock is known. Write the income process (6) for the individual as

$$\Delta y_{it} = v_{it} - u_{it-1} \quad \text{where} \quad v_{it} = \epsilon_t + u_{it} \quad (14)$$

We can decompose the information the consumer gets every period into two parts. The first part is v_{it} , the current period innovation which is contained in current individual income y_{it} . The consumer does not know how the innovation in a particular period is composed of the permanent (aggregate) component and the transitory (individual) component. She will therefore attribute part of the current period innovation to each component given the relative variances. For every particular innovation there will be

⁷ If the interest rate is zero $A = 1 - \theta$. As long as $0 < \theta < 1$ we have $1 - \theta < \sqrt{1 - \theta^2}$.

errors, of course. Secondly, the consumer gets information from the lagged aggregate shock. Once this information arrives she will be able to correct the error made last period in attributing the innovation to its components.

The optimal consumption response will have two parts corresponding to the two pieces of information: a response to the new innovation and a term that corrects for the error made in the previous period. The first part of the consumption response, the reaction to the current period innovation can be written as

$$\omega v_{it} + (1 - \omega) \frac{r}{1+r} v_{it} = \frac{\omega+r}{1+r} v_{it} \quad (15)$$

where $\omega = \sigma_\varepsilon^2 / (\sigma_\varepsilon^2 + \sigma_u^2)$, the relative variance of the aggregate shock.⁸ The first term is the proportion of the new innovation expected to be permanent, the consumption response to that part is one. The second term is the part expected to be transitory, the response is $r/(1+r)$.

Consider the correction for errors made last period. Define the negative of the error in the aggregate component as

$$\begin{aligned} \xi_{it-1} &= \varepsilon_{t-1} - \omega v_{it-1} = \varepsilon_{t-1} - \omega(\varepsilon_{t-1} + u_{it-1}) \\ &= (1 - \omega)\varepsilon_{t-1} - \omega u_{it-1} \end{aligned} \quad (16)$$

The errors in the individual component and in the aggregate component have to sum to zero since the signal extraction problem the individual solved in $t-1$ yielded unbiased predictors of the two components. The response of consumption in period t to errors made in $t-1$ is therefore

$$(1+r) \left[\xi_{it-1} + \frac{r}{1+r} (-\xi_{it-1}) \right] = \xi_{it-1} \quad (17)$$

The first term in the square bracket is the correction of the error in the aggregate component, the second term the correction for the error in the individual component. Notice that interest accrued on the portions of the shocks that had not been consumed in the last period.

Putting together the two parts of the total consumption response from (15) and (17) we obtain

$$\Delta c_{it} = \frac{\omega+r}{1+r} v_{it} + (1 - \omega)\varepsilon_{t-1} - \omega u_{it-1} \quad (18)$$

⁸ There is a correspondence between ω in this section and θ in the previous section. Both capture the relative variances of the individual and the aggregate income shocks. It is much easier to work with ω here.

⁹ This is a special case of equation (11) in Goodfriend (1991).

Like in the model of the previous section, individual consumption changes still follow a martingale with respect to the history of individual income and consumption.¹⁰ This can easily be seen by calculating the autocovariance $cov(\Delta c_{it}, \Delta c_{it-1})$. It will contain a term $(1 - \omega)\sigma_\varepsilon - \omega\sigma_u$ which is zero. The lagged income innovations in (18) arise from the fact that errors are corrected after one period. However, optimal choice of the weight ω implies that these errors contain no information correlated with lagged income or consumption changes.

Sum the individual consumption responses in (18) for a large population to get the per capita consumption response

$$\Delta c_t = \frac{1}{n} \sum \Delta c_{it} = \frac{\omega+r}{1+r} \varepsilon_t + (1-\omega)\varepsilon_{t-1} \quad (19)$$

The change in aggregate consumption follows an MA(1) process. The impact response to an aggregate shock is smaller in the lagged information model than in the no information model. The intertemporal budget constraint is responsible for this feature. Since agents only asymptotically gather enough information in the no information model to accurately categorize the shock as permanent their consumption response will always be below the response in the lagged information model after the second period. To satisfy the budget constraint they therefore have to respond more in the first period.

In the lagged information model, on the other hand, the full response occurs after the second period when the true shock is known. Both the orthogonality failure and the smoothness result will still arise in the lagged information model, but their quantitative importance will differ.¹¹

Consider the regression of the change in consumption on the lagged income change in (11) again. The coefficient on lagged income will be

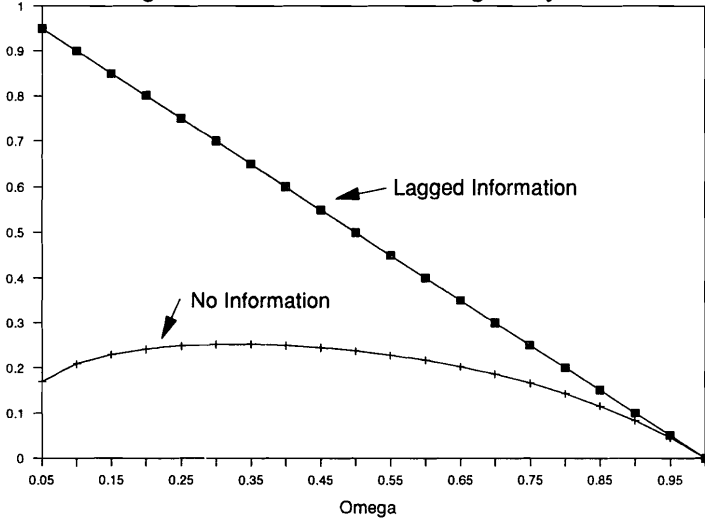
$$\begin{aligned} \beta &= \frac{cov(\Delta c_t, \Delta y_{t-1})}{var(\Delta y_{t-1})} \\ &= \frac{E\left\{\left[\frac{\omega+r}{1+r}\varepsilon_t + (1-\omega)\varepsilon_{t-1}\right]\varepsilon_{t-1}\right\}}{\sigma_\varepsilon^2} = 1 - \omega \end{aligned} \quad (20)$$

Figure 1 plots the expected regression coefficients from equations (12) and (20) for the two models with and without lagged information for various values of ω and an interest rate of 0.01. In the model with lagged information the coefficient on lagged income changes can be large when the variance of the aggregate component is small. In this case consumers attribute little of a shock to the aggregate component initially

¹⁰ I thank Steve Zeldes for pointing out an error in a previous draft.

¹¹ The test carried out by Campbell and Mankiw (1989, 1990) should not reject the model since their test only relies on information lagged at least two periods. Their rejection therefore implies a failure of the model with lagged information.

Figure 1
Regression Coefficient for Orthogonality Test



but adjust fully to the true shock in the next period, so the response to lagged income is large. This feature results from the somewhat artificial fact that there is full information after one period.

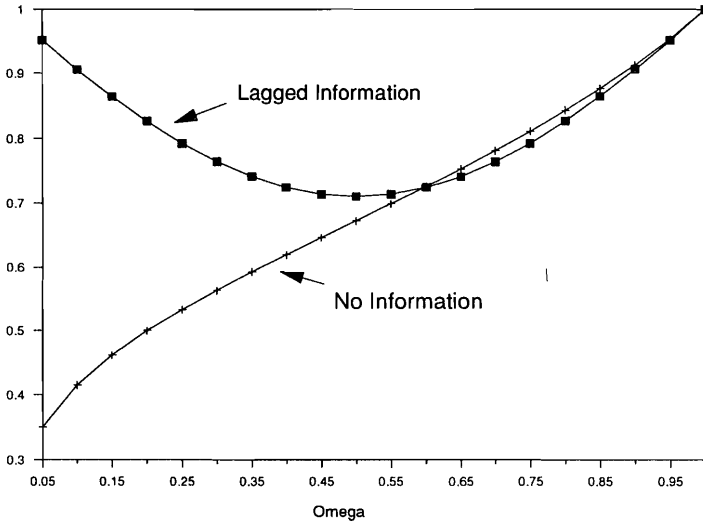
For low values of ω , i.e. values in the empirically relevant range, the models differ substantially. In the model with no information the coefficient will be small if the variance of the aggregate shock is very small. Two things happen. Agents expect the true process to be very transitory, so after an aggregate shock they are surprised period after period. This will raise the correlation between consumption changes and lagged income changes (captured by θ in (12)). But because the composite process consists mostly of transitory innovations agents react very little to shocks in general. Therefore consumption changes vary very little compared to income changes (reflected in A in (12)), lowering the regression coefficient. In the limit the coefficient is $r/(1+r)$.

With respect to the smoothness result the models are related in the opposite way. Taking variances in (19) yields

$$\frac{\sigma_{\Delta c}}{\sigma_e} = \sqrt{\left(\frac{\omega+r}{1+r}\right)^2 + (1-\omega)^2} \quad (21)$$

Figure 2 is an analogous plot to the previous figure for the excess smoothness ratios with no information (from equation 13) and with lagged information (from 21). In the representative agent model with random walk income the ratio is one. In both models with individual income the variability of consumption will be lower. For the no

Figure 2
Excess Smoothness Ratio



information case the response is monotonically increasing with the importance of the aggregate shock. In the lagged information model the response is lowest if the aggregate and individual shock are of equal importance. In this case the total response is split between two adjacent periods. If the variance of the aggregate component is large almost all the response is immediate; if it is small almost all the response is in the second period. In either case the total variance is close to one.¹²

The models above are easily extended to more general processes for income; Appendix A contains the necessary algebra. The implications remain the same as in the simple examples above. The more general models allow us to examine an additional implication due to Campbell and Mankiw (1989, 1990). They found that the conditional expectations of income and consumption changes are proportional. They interpret the coefficient of proportionality λ as the proportion of consumers who consume their current instead of their permanent income. This implication will only hold in the no information model under very special circumstances. If the individual and the aggregate income process are unconstrained there is no reason for the conditional expectations to be equal. The appendix contains a demonstration of this fact. This does not mean

¹² The two models can be thought of as special cases of a model where agents receive a noisy signal of the aggregate shock with a one period lag. This model generates an ARMA(1,1) process for consumption changes. The predictions for β and the ratio of the variability of consumption to the variability of the income innovation lie in between the predictions for the two polar cases considered above.

that the incomplete information model is empirically invalid. For empirically relevant income processes the proportionality result may be approximately true. In particular, aggregate income changes follow a positively autocorrelated process while the models generate positively autocorrelated time series for consumption changes.¹³

Which of the two models presented above is more reasonable? It seems that the model with lagged information is better suited to explain the behavior of rational decision makers who form expectations on the basis of all available information since basic aggregate statistics are provided virtually for free by the news media. However, a rational agent will not only consider the costs, which are admittedly small, but also the benefits. Cochrane (1989) has shown that it is possible to calculate the loss from nonmaximizing behavior and found that these losses are generally small for small deviations from the optimal path. The same should be true here. Since these calculations are rather sensitive to the parameters used I do not present any exemplary results here but relegate them to section 7 after I have shown what reasonable estimates for the individual and the aggregate income processes are. I will turn to these estimates next.

5. Empirical Results on Micro Income Processes

The remainder of the paper explores whether the data bear out the implications of the models studied above. I start in this section by presenting results on individual income processes. Previous studies in this area reveal that the main feature necessary for the model to work is present in micro data: i.e. income innovations for individuals are less persistent than shocks to aggregate income.

MaCurdy (1982) and Abowd and Card (1989) have analyzed the time series structure of earnings in micro data. They find that the log of earnings changes for male household heads in the U.S. is well described by an MA(2). Both MA coefficients are negative, with the first one between -0.25 and -0.4 and the second one closer to zero. The variance of log earnings changes is substantial. The standard deviations range from about 0.25 to a high of 0.45 for certain years. This means that a one standard deviation change in earnings is 25 percent to 45 percent of the previous level. Individual income risk is clearly the main source of income uncertainty individuals face.

MaCurdy only analyzes data from the Panel Study of Income Dynamics which is conducted annually. Abowd and Card also present results for data from the control groups of the Denver and Seattle Income Maintenance Experiments which correspond

¹³ Other models that yield autocorrelated consumption processes may not in general conform any better to the proportionality result. Not much research has been done on this issue. One exception is Flavin (1990) who has found that her excess sensitivity model yields the same restriction as the λ -model of Campbell and Mankiw. λ in her case has the interpretation of the excess sensitivity parameter.

to semiannual income. They find generally first autocorrelations that are even more negative for these data. However, this may not result from the different sampling frequency but from the fact that the experiment oversampled relatively poor households.

Results for the (annual) family income process are provided by Hall and Mishkin (1982) in their study of consumption behavior. They estimate a restricted MA(3) for income changes with results very similar to the studies mentioned above. Family income apparently follows a process very similar to individual earnings.

None of these results are directly suited for the present purpose. The stylized facts on aggregate consumption have all been established on quarterly series. In order to have analogous results for individual income I estimated restricted covariance models with quarterly data that I constructed from the 1984 Survey of Income and Program Participation (SIPP). This panel survey was conducted three times a year from late 1983 to the beginning of 1986 in about 20,000 households and collected monthly income information. The interviews took place on a rolling basis, with one fourth of the sample being interviewed each month. From these data I constructed a panel of quarterly income from the fourth quarter of 1983 to the first quarter of 1986, the longest span for which information on the entire sample is available.

Consumption decisions are most likely made on the family level. I therefore selected families that can be followed continuously throughout the sample period and did not change head or spouse. Most likely, events that change household composition in a major way will also lead to large income changes. The sample selection will therefore tend to understate the variance of income changes. Furthermore, I limited the sample to households whose head did not go to school during the sample period. These latter groups may have large movements in income which may be anticipated by the individuals but would appear as random elements in the estimation. For example, an individual just finishing school will have a large increase in income. But this jump will have been foreseen and has therefore, according to the model, already been incorporated in previous consumption decisions. I also eliminated non-family households since I cannot judge whether they make joint or individual consumption decisions. Finally, I limited the sample to families with heads between the ages of 16 and 70 during the survey period. Appendix B contains further details on the construction of the sample.

The correct income concept is net family income from all sources excluding capital income. Variables on total family income and income from capital are provided on the SIPP user tapes; these are aggregated from an array of detailed questions on various income categories for each family member. I use these variables although there are some problems associated with them. First, tax information is only collected infrequently and cannot be apportioned to single months. This is probably the most severe shortcoming of the data because gross income will have a higher variance and (in a progressive tax system) exhibit more transitory fluctuations. Furthermore, the individual variables that make up family income can have imputations. Since the imputations occur at the disaggregated level it would be rather arbitrary to decide which observations to delete because of the imputations. I decided to use all the data.

Table 1
Basic Sample Statistics

	SIPP Sample		CPS Sample	
	Mean	Std. Dev.	Mean	Std. Dev.
Age	43.7	12.9	42.5	13.4
Years of Schooling	12.6	3.25	12.5	3.22
Non-White	0.12	0.32	0.13	0.34
Male	0.77	0.42	0.73	0.44
Never Married	0.09	0.29	0.14	0.35
Family Size	3.04	1.55	2.82	1.56
Per capita Income 1984 [quarterly]	2,634	2,162	2,821	2,522
Sample Size	8,170		25,033	

Imputations should lower the variance of income changes, presumably largely at the cost of the transitory income component. Finally, all disaggregated income items are topcoded at \$8,333 per month. It is impossible to decide from the aggregated income items which variables have been topcoded. The topcoding should only affect a small portion of the sample and will also reduce the income variance. Because I use per capita data from the National Income Accounts below, I divided family income by family size each period. Income is deflated by the monthly CPI for urban consumers (base 1982-84). The final sample I used has 8,170 families. I provide some basic characteristics in table 1.

Table 1 also presents results from the March 1985 Current Population Survey. In most respects the SIPP sample matches the general population very closely. Families are slightly larger in the SIPP sample, this explains why per capita income is slightly less than in the CPS.

The estimation proceeds in three further stages. In a first step, I regressed changes in family income on a constant, changes in total family size, changes in the number of children, and age of the head to eliminate deterministic components of income dynamics; these regressors are similar to the ones used by Hall and Mishkin (1982). Separate regressions were run for each quarter. Thus the data will be purged of all common seasonal and aggregate components as well. None of the regressors explains income changes very well; as is usual in such regressions the R^2 s range from only 0.002 to 0.008! Adding lagged labor market indicators, like number of earners in the household, weeks worked by the head, weekly hours, etc., as additional regressors hardly changes the results at all.

The second step was to estimate the unrestricted covariance matrix of residual income changes. Table 2 displays this 9 x 9 matrix. The standard deviations of quarterly income changes range from \$1,173 to \$1,316. The mean level of per capita family income is \$2,646. The standard deviations are between 44% and 50% of the income level, somewhat higher than MaCurdy's and Abowd and Card's findings on annual data. The first order autocorrelations are negative and range from -0.230 to -0.331, surprisingly similar to the estimates for annual data. Time aggregation of ARMA processes does not have this feature. Measurement error in the income data may be responsible for this finding; I comment on the implications of measurement error below. Autocorrelations beyond the second are small and generally insignificant. An MA(2) seems to be an appropriate model for these quarterly income changes as well.

Table 2
Covariance Matrix of Income Changes
(Income / 1000)
(standard errors in parentheses)

	84:1	84:2	84:3	84:4	85:1	85:2	85:3	85:4	86:1
84:1	1.730 (0.172)	-0.279	-0.049	-0.064	-0.032	-0.029	0.001	-0.019	0.050
84:2	-0.440 (0.109)	1.439 (0.143)	-0.292	-0.164	0.025	0.014	-0.002	-0.030	-0.021
84:3	-0.075 (0.077)	-0.410 (0.091)	1.375 (0.119)	-0.279	-0.181	-0.015	0.057	-0.022	-0.020
84:4	-0.102 (0.051)	-0.240 (0.049)	-0.398 (0.084)	1.479 (0.126)	-0.323	-0.092	-0.091	0.091	-0.017
85:1	-0.056 (0.065)	0.040 (0.068)	-0.278 (0.055)	-0.517 (0.091)	1.731 (0.132)	-0.357	-0.156	0.027	0.039
85:2	-0.045 (0.056)	0.020 (0.046)	-0.021 (0.036)	-0.134 (0.040)	-0.561 (0.071)	1.427 (0.104)	-0.230	-0.207	-0.006
85:3	0.002 (0.031)	-0.002 (0.039)	0.080 (0.040)	-0.132 (0.044)	-0.245 (0.050)	-0.328 (0.056)	1.430 (0.100)	-0.292	-0.174
85:4	-0.032 (0.064)	-0.046 (0.037)	-0.033 (0.043)	0.143 (0.060)	0.046 (0.068)	-0.320 (0.058)	-0.450 (0.079)	1.667 (0.148)	-0.331
86:1	0.082 (0.047)	-0.031 (0.034)	-0.029 (0.037)	-0.025 (0.045)	0.064 (0.047)	-0.009 (0.052)	-0.258 (0.047)	-0.529 (0.096)	1.534 (0.126)

Covariances below the diagonal, correlations above the diagonal

An MA(2) with constant coefficients has three parameters that restrict the 45 covariances above. I impose these restrictions using minimum distance estimators.¹⁴ I proceed by estimating the restricted variances and autocorrelations. The MA(2) coefficients can then be easily derived. Table 3 reports the results.¹⁵ The first column gives estimates using the empirical fourth moments as weights. The estimate of the standard deviation of (the stochastic part of) income changes is about \$1,100 per family member per quarter. Notice that the optimally weighted estimator yields lower estimates of the variance than the unweighted estimator (second column). In fact the weighted estimate is below the standard deviations in all years. Optimal weighting seems to lead to underfitting of the variances whenever off-diagonal terms are restricted to zero. The last column, where I present results for an MA(1) shows that this problem becomes more severe as more restrictions are imposed.

The minimand of optimal minimum distance estimator multiplied by the sample size yields a goodness-of-fit test for the model. This specification test does not reject the stationary MA(2) model at the 5 percent level. An analogous test can be constructed for arbitrary weighting matrices (Newey, 1985); the unweighted model is rejected by this test. The MA(1) also clearly fails the specification test; the data prefer a nonzero second order autocorrelation. In the last row I also present a test of the stationarity restrictions for the MA(2) model. Chamberlain (1984) shows that the difference of the test-statistics for two models has also a χ^2 -distribution. The unrestricted model allows different covariances for every year but still restricts the higher order correlations to zero. Unlike the findings of Abowd and Card, the stationarity assumption cannot be rejected for the SIPP data. This may be due to the fact that only a short time period (two years) is involved.

The optimally weighted estimates imply a standard deviation of the income innovation of \$1,026 for the MA(2). The MA coefficients are -0.447 and -0.205. According to these estimates income surprises are large and contain a substantial transitory component.¹⁶ One might object that the high variance I obtained may be due to heterogeneity of the individual income processes rather than uncertainty, pertain to variation anticipated by individual households, and may result from measurement error. I will take up these issues in turn.

14 See Chamberlain (1984) and Abowd and Card (1989).

15 I initially estimated covariances. The standard errors on the reported results are obtained by the delta method.

16 Guiso, Jappelli, and Terlizzese (1991) have obtained direct survey evidence on individual earnings uncertainty for Italy. Their results indicate that two thirds of the individuals in the sample believe their earnings uncertainty for next year to be 1.5 percent of the level of earnings or less. This would correspond to an innovation standard deviation of only \$40 in the SIPP sample. While this indicates that income uncertainty may be lower than estimated from panel data their results seem extremely low.

Table 3			
Stationary MA(2) and MA(1) Models for Income Changes			
(standard errors in parentheses)			
Coefficient	MA(2)		MA(1)
	Optimally Weighted Estimates	Unweighted Estimates	Optimally Weighted Estimates
Standard Deviation	1143 (22.5)	1239 (28.3)	1068 (23.6)
1st autocorrelation	-0.286 (0.011)	-0.296 (0.014)	-0.428 (0.005)
2nd autocorrelation	-0.165 (0.011)	-0.144 (0.016)	---
Specification test			
χ^2 -statistic [dof]	57.7 [42]	95.9 [42]	294.6 [43]
p-value	0.054	0.000	0.000
Test for Stationarity			
χ^2 -statistic [dof]	16.9 [21]	---	---
p-value	0.720		

Deterministic differences in income changes among households, for example, due to different positions on the age-earnings profile, would show up as variance in the income innovation. However, pure individual fixed effects or differential slopes of the age-income relationship would yield positive values for the higher order autocovariances in table 2. If these effects were important then the MA(2) model ought to be rejected. There is no evidence that this is the case. Yet, there may be numerous events that are deterministic or anticipated by the individual households that I cannot capture in the estimation. While such events seem reasonable they have been notoriously hard to find in micro earnings data. As an illustration, to make an extreme adjustment, I reestimated the covariance matrix after first removing household specific seasonal effects. Given that I only have nine periods of data this will also spuriously eliminate

a lot of "true" variation. The estimated standard deviations of the income changes are still in the order of \$800. Furthermore, the first order autocorrelations are still negative and in the order of -0.15.

Finally, in the estimates presented here no allowance has been made for the possible existence of measurement error in these data. Duncan and Hill (1985), Bound et.al. (1989) and Bound and Krueger (1991) document the importance of measurement error in micro income data. For data sets with annual recall these studies find a reliability ratio, i.e. the ratio of signal to total variance, of about 0.65 for first differences of income. This would imply that the standard deviation of individual income changes is only 80 percent of the estimates above or about \$900 which is still quite substantial. However, the SIPP only relies on recall of income over the four most recent months and may be more accurate than datasets that ask respondents to recall income of the previous year. On the other hand, annual income surveys are usually conducted during spring when individuals prepare their tax returns and may therefore have more accurate income records available. Another source of error in the estimation of the variance arises from the inexact definition of some of the variables. For example, remember that I have not subtracted tax payments from gross income.

Measurement error may also obscure the estimates of the time series structure of income. If measurement error is white noise in the level of income then the first difference of the measurement error will follow an MA(1) with a negative coefficient. Thus measurement error may account for some of the transitory component of income and true income may follow a much more permanent process. However, Bound and Krueger (1991) and Bound et.al. (1989) found that measurement error is positively correlated over time. Depending on the source of this autocorrelation, this may ameliorate the degree to which measurement error is responsible for the negative MA coefficients found above. If measurement error consists of an individual specific effect and noise then the first differences of the data will only contain the part of the measurement error due to noise which introduces the negative autocorrelation alluded to above. On the other hand, if measurement error follows an AR(1) process with a positive coefficient then the autocorrelation of the measurement error in first differences will be less than it is in the case of pure noise. In the limit, if the measurement error follows a random walk then it will add pure noise in first differences. From the studies mentioned above it is impossible to tell which model for the measurement error is the more relevant here.

Another complication arises in the present case because the monthly responses for a quarter can originate from the same or from two successive interviews. Because of the way in which the different rotation groups are interviewed in the SIPP this will also differ for different households. The fact that some of the data for adjacent quarters come from the same interview will introduce additional autocorrelation in the measurement error.

As a final comment on the issue let me note that measurement error may not bias the estimates in this case. To the extent that measurement error is response error rather than coding error survey responses may actually be what individuals believe about their income. In this case the mismeasured rather than the true income should enter the estimation.

All the forms of mismeasurement alluded to here will overstate the variance and the amount of transitory variation in income. I deal with these issues below by checking the robustness of the model predictions to a lower variance and higher persistence of individual income.

6. Aggregate Stylized Facts on Income and Consumption

In this section I report the stylized facts pertaining to income and consumption processes in aggregate data. This has two purposes. First, I will try to establish some simple time series model for the aggregate income process. Together with the results of the previous section this will allow me to calculate predictions from the model with heterogeneous agents for aggregate consumption. I will therefore also report results on consumption here to compare them to the predictions in the following section.

In order to replicate the results often cited in the literature I make the same adjustments to the NIPA data as Blinder and Deaton (1985) did.¹⁷ My sample ranges from the first quarter of 1954 to the fourth quarter of 1990, the data are taken from the 1991 Citibase tape. A detailed description of the adjustments I make is given in Appendix B.

Table 4 presents results on the income process. The income series refers to "labor" income, i.e. disposable income excluding capital income. There is a slight conceptual difference to the micro estimates since the aggregate income series excludes taxes. However, whether taxes are excluded or not makes little difference for the aggregate estimates. I therefore use the series commonly used in the literature. As for individual income I will use an MA(2) model for the first differences of aggregate income but I also present results for an AR(1). The MA coefficients are estimated by conditional least squares, the AR model is estimated by OLS. I report results for two different sample periods. 1954 to 1984 is the period of the Binder and Deaton (1985) dataset that has been used extensively by various researchers. Notice that extending the sample to 1990 reduces the autocorrelation in the income changes slightly. Both the AR(1) and the MA(2) fit the data well. The quarterly standard deviation for aggregate per capita income is only around \$15, compared to the \$1,000 I found for the individual income component above!

¹⁷ Unlike Blinder and Deaton (1985) I did not adjust income and consumption for nontax payments to state and local governments since the series on Citibase is only available starting in 1958. For the post-1958 sample the difference is completely inconsequential.

Table 4				
Aggregate Stylized Facts on Income				
(standard errors in parentheses)				
Sample Period	AR(1)	MA(2)		
		First coefficient	Second coefficient	Std. Dev. of Income Innovations
NIPA 1954-1984	0.368 (0.083)	0.392 (0.090)	0.022 (0.090)	16.1 (1.02)
NIPA 1954-1990	0.307 (0.079)	0.309 (0.083)	0.023 (0.083)	17.0 (0.99)

Table 5 reports some results on aggregate consumption for similar sample periods as the previous table. It has been customary in the macro literature to use consumer expenditure on nondurables and services as consumption measure. Like Blinder and Deaton I eliminated expenditures on clothing and shoes from the nondurable consumption series. To make units comparable to total income I multiplied these expenditures by the sample average of the ratio of total expenditures to expenditures on nondurables and services.

Table 5				
Aggregate Stylized Facts on Consumption				
(standard errors in parentheses)				
Sample Period	Coef. of Consumption Changes on Income Lag	AR (1) coefficient	MA (1) coefficient	Excess Smoothness Ratio
1954-1984	0.138 (0.047)	0.225 (0.087)	0.220 (0.088)	0.583 (0.060)
1954-1990	0.131 (0.043)	0.230 (0.081)	0.249 (0.081)	0.562 (0.052)

The table reports the regression coefficient of consumption changes on lagged income changes which is in the order of 0.13 and clearly significant. Consumption changes are positively autocorrelated as measured by an AR(1) or MA(1) parameter. The last column gives the excess smoothness ratio of about 0.6. All these estimates are in line with previous findings in the literature.

7. Predictions from the Model

I am now ready to present predictions from the models using the empirical estimates for the individual and aggregate parts of the income process. Since the estimates vary slightly for different sample periods I will present a number of results.

I assume that both the individual income process and the aggregate income process are described by an MA(2) in first differences.

$$\begin{aligned} \Delta y_{it} &= (1 + \phi_1 L + \phi_2 L^2) \varepsilon_t + (1 - \alpha_1 L - \alpha_2 L^2) u_{it} \\ &= (1 - \theta_1 L - \theta_2 L^2) \eta_{it} \end{aligned} \tag{22}$$

Appendix A shows how to derive the consumption processes for the two models. In the case of the no information model aggregate consumption follows an ARIMA(2,1,2) process. For the lagged information model, consumption changes are always an MA(1). The Appendix also presents the formulae for β , the coefficient for a regression of consumption changes on lagged income changes, and the excess smoothness ratio $\sigma_{\Delta c} / \sigma_e$.

Predictions for these parameters are shown in table 6 and compared to the aggregate stylized facts about consumption from table 5. The base case uses the weighted estimates for the individual income process and the 1954 - 1990 results for aggregate income. The no information model predicts both parameters qualitatively correctly but β is about twice its empirical value while $\sigma_{\Delta c} / \sigma_e$ is predicted about correctly. Thus, for these parameters the model predicts an even more striking failure of the representative agent life-cycle model than the aggregated data reveal! The results for the lagged information model are also qualitatively correct but the numerical results are far off. β is almost seven times its empirical value, indicating that the lagged information model implies much too fast incorporation of aggregate information into households' consumption decisions. The excess smoothness ratio is also too high but not as far out of line with the aggregate estimates. The last column shows what the per capita utility loss is for a household that uses no aggregate information compared

to the full information case.¹⁸ The loss is expressed in Dollars per quarter and calculated for a coefficient of relative risk aversion of two. It amounts to 45 cents or 0.02 percent of total utility. This is slightly above the findings by Cochrane (1989) who estimated the utility loss for a representative consumer exhibiting excess sensitivity. This is expected since excess sensitivity is the aggregate implication of the model discussed here and the model predicts more excess sensitivity than the data exhibit. The loss for higher risk aversion is easily obtained by dividing by two and multiplying by the new coefficient. Even for a risk aversion coefficient of 10 the loss would still be minor. This provides some evidence that the assumptions of the no information model seem to be quite reasonable: it does not pay to collect aggregate information to improve consumption decisions.

Table 6
Comparison of Model Predictions and Aggregate Estimates

Case	Aggregate Estimates		No Information Model		Lagged Information Model		Utility Loss [\$/quarter]
	β	$\frac{\sigma_{\Delta c}}{\sigma_\epsilon}$	β	$\frac{\sigma_{\Delta c}}{\sigma_\epsilon}$	β	$\frac{\sigma_{\Delta c}}{\sigma_\epsilon}$	
base	0.131	0.565	0.306	0.535	0.896	1.044	0.448
2	0.138	0.583	0.335	0.562	0.922	1.122	0.468
3	0.131	0.565	0.310	0.542	0.892	1.042	0.766
4	0.131	0.565	0.288	1.096	0.383	1.054	0.024

Base case: $\sigma_u = \$1,026$, $\alpha_1 = 0.047$, $\alpha_2 = 0.205$, $\sigma_\epsilon = \$16,99$, $\phi_1 = 0.309$, $\phi_2 = 0.023$,
interest rate = 0.01, mean income = \$2,646,
coef. of rel. risk aversion = 2

Case 2: As base case but $\sigma_\epsilon = \$16.10$, $\phi_1 = 0.392$, $\phi_2 = 0.022$

Case 3: As base case but $\sigma_u = \$256$

Case 4: As base case but $\alpha_1 = 0$, $\alpha_2 = 0$

¹⁸ Instead of comparing the model with no information to the Goodfriend model I use a model with full contemporaneous information on aggregate variables as benchmark. Utility for this model is calculated much more easily than for the lagged information model. The utility differences I present are therefore upper bounds for the differences between the two models in the paper.

The next rows present slight changes to the base case. Case 2 uses the aggregate estimates for the 1954 - 1984 period; the results are very similar. Cases 3 and 4 investigate the possible implications for the model if the variance and transitory nature of the individual income process is overstated, which is likely to be the case. For illustrative purposes I use cases that deviate far from the estimates although I do not believe that mismeasurement alter the results to such a large degree. Case 3 uses only a quarter of the standard deviation for individual income innovations. Despite this drastic change the predictions of the model are practically unaltered. The variance of individual income changes still completely dominates the total variance so that the change has little impact. The implications of a change in the persistence properties are much more severe. Case 4 presents the results under the assumption that the mean reversion in individual income is due to measurement error or estimation bias and the true process is a random walk. This lowers the predictions of β slightly and doubles the excess smoothness ratio for the no information model. Both coefficients are roughly twice their empirical value now. For the lagged information model β is now much more reasonable, about three times its empirical value.

The changes presented here are somewhat extreme. It seems quite reasonable that the model predictions should lie somewhere inbetween cases 1 and 4. I conclude therefore from this exercise that both coefficients are predicted to be too high with β being overpredicted to a larger degree. Furthermore, the no information model seems to do slightly better in matching the aggregate results.

The results above only pertain to the most simple minded version of a life-cycle consumption model with an information structure different from the typical representative agent model. Other aspects that are potentially important have recently been incorporated into this basic model. I will take up a few of those extensions in the next section and discuss how the model with heterogeneous agents and incomplete information behaves under these modifications.

8. Extensions of the Model

Some of the assumptions of the model of life cycle consumption that I used throughout this paper are clearly not satisfied in reality. The model only explains the behavior of nondurable consumption and neglects the presence of durable expenditures of individuals. A further restriction is the assumption of infinite lifetimes. Finally, recent literature has allowed for superior information of consumers about their income. I will take up these features in turn and discuss briefly how relaxing these assumptions may change the results in the previous section.

The Presence of Durable Goods. In the empirical results above durable goods were treated as absent and the measure of nondurable consumption was scaled accordingly. When some goods are durable the time series process of nondurable consumption will be altered as well. This is true even if durable consumption is additively separable because durables introduce an additional intertemporal link. It would be much more

satisfactory to incorporate the presence of durables directly into the model. Unfortunately, the quadratic utility framework does not allow enough flexibility to add durables in a simple but reasonable way. Therefore, I only explore a crude way to allow for differential reactions of durable and nondurable goods to changes in income by postulating a linear Engel curve relation between total expenditures and expenditures on nondurables and services:

$$c_{it}^{nd} = \pi + \tau c_{it} \quad (23)$$

Take first differences in (23) and apply the result to (10) and (14). In the model with a linear Engel curve, τ multiplies both the regression coefficient $\hat{\beta}$ and the excess smoothness ratio.

Using a sample of 3728 households from the 1985 Consumer Expenditure Survey¹⁹ I obtained a slope coefficient of 0.615 with a standard error of 0.016. The linear Engel curve specification of nondurables fits remarkably well for the bulk of the sample. Under these circumstances the coefficients in table 6 (base case) would change as follows: $\hat{\beta}$ is 0.188 from the no information model and 0.551 from the lagged information model while the unscaled empirical value is 0.107. The empirical value of $\sigma_{\Delta c} / \sigma_{\epsilon}$ is 0.465 while the no information model predicts 0.346 and the lagged information model 0.641. Both models do much better in matching the empirical moments. Nevertheless, more careful modeling of the interaction of durable and nondurable purchases is definitely desirable since the Engel curve approach can at most serve as an approximation.

Finite Lifetimes and Retirement. As the work of Clarida (1988) and Galí (1990) shows, finite lifetimes alone can generate excess sensitivity and excess smoothness. It would therefore seem that introducing finite lifetimes into the model would only move the excess smoothness parameter in the desired direction but increase the regression coefficient on lagged income even more. It turns out that this is not the case. Excess sensitivity in Galí's model results from the fact that his overlapping generations structure introduces autocorrelation in the aggregate consumption process while the consumption changes of individuals follow a random walk. However, Galí finds that this autocorrelation is very small for reasonable life spans. It is therefore safe to neglect this feature.

Both Galí and Clarida assume that income declines in the later part of the life cycle so that consumers have an incentive to save for retirement purposes. Therefore, consumers will have a lower propensity to consume out of income shocks. To illuminate how Galí's results change the implications of my model return to the example in section 3. The feature of retirement is captured by geometrically declining labor income in Galí's

¹⁹ This sample eliminated about the 2.5 % of the households with the highest total expenditures. The remaining sample should give a better estimate of the slope of the Engel curve in its mid-range that is most relevant for the bulk of the sample. The measure of nondurables and services I used does not correspond precisely to the NIPA definition. I thank Annamaria Lusardi for making an extract of the CES data available to me.

paper. He finds that the (aggregate) marginal propensity to consume out of labor income is equal to the ratio of the gross return on wealth and the sum of this return and the rate of decline of labor income (see his equation (9)). This MPC will multiply A in my equation (10). From equations (12) and (13) above it is therefore clear that the MPC will multiply the regression coefficient of consumption changes on lagged income changes and the excess smoothness ratio. Galí estimates the aggregate MPC to be between 0.8 and 0.9 so that both coefficients will be slightly lower.

Advance Information about Income. The model in this paper assumes that consumers face a univariate income process and observe individual income when it is realized. Campbell (1987) has devised a framework to test the permanent income model when consumers have more information than the econometrician. His evidence is consistent with the interpretation that consumers learn about movements in income some time in advance.

It is not straightforward to build the feature of extraneous information into the model of this paper since the decomposition into aggregate and individual income is done most easily with a univariate income process. As a simple way to allow for advance information, assume that consumers learn about the income shock one period in advance but still do not decompose it into its individual and aggregate component. In terms of the example in section 3, equation (8) changes to

$$\Delta c_{it} = \frac{1+r-\theta}{(1+r)^2} \eta_{it+1} \equiv A^* \eta_{it+1} \quad (24)$$

For small interest rates A^* does not differ much from A . Therefore the change in the timing will affect the regression coefficient on lagged income but not excess smoothness. Aggregate (24)

$$\Delta c_t(1-\theta L) = A^* \varepsilon_{t+1} \quad (25)$$

This yields a regression coefficient of

$$\beta = \frac{E\left\{A^* \left(\frac{\varepsilon_{t+1}}{1-\theta L}\right) \varepsilon_{t-1}\right\}}{\sigma_\varepsilon^2} = \frac{A^* \theta^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2} = A^* \theta^2 \quad (26)$$

The regression coefficient is multiplied by the AR coefficient θ . The excess smoothness ratio is

$$\frac{\sigma_{\Delta c}}{\sigma_\varepsilon} = \frac{A^*}{\sqrt{1-\theta^2}} \quad (27)$$

which is virtually unchanged. For more complicated income process similar derivations can be carried out. In general β will be lower because the income innovations used to make consumption decisions are further removed in time from the income changes they are correlated with in the regression. If income changes are positively autocorrelated there is an offsetting effect, however, because more innovations will contribute

to the correlation of the two processes although each innovation will enter with less weight. For the processes in the base case of table 6 β would be 0.225 instead of 0.306.

Various conclusions emerge from the exercises in this section. First, these extensions change the predicted coefficients to some degree, but these changes seem to be relatively minor compared to the impact that a change in the information structure has. It seems that incomplete information and aggregation bias may be an empirically more important contributor to the failure of the permanent income hypothesis in aggregate data than other proposed extensions of the model.

Secondly, both models seem to perform better with some of these extensions. They all yield lower coefficients for β and the excess smoothness ratio, except for advance information which only lowers the former. Adding all these adjustments together seems to yield predictions very close to the empirical values for aggregate data. Since I only discussed each of these extensions in terms of a simple example, it is impossible to assess the exact empirical importance of these alterations and their joint impact. This should be a topic of future research.

9. Concluding Comments

In this paper I have analyzed the implications of heterogeneity in income and incomplete information on the source of income shocks for the form of the aggregate consumption process and its relation to observed income. The failures of the full information life-cycle consumption model usually found in aggregate data clearly arise if individual consumers adjust their consumption correctly to individual income innovations but do not care to distinguish aggregate and idiosyncratic income variation. Using estimated parameter values for individual and aggregate income processes, the model gives predictions that deviate substantially from the full information benchmark. However, the results indicate too much correlation of consumption changes with lagged income but not smooth enough consumption. Nevertheless, heterogeneity in income and incomplete information seem to account for a large portion of the deviations from the full information case.

An informal examination of the potential implications of mismeasurement of the stochastic part of micro income reveal that the predictions will be even higher with a better measure of the true income process. Relaxing some other restrictive assumptions of the model makes the results look more favorable. Accounting for the presence of durables, allowing for finite lifetimes and retirement savings of consumers and advance information about income reduces the regression coefficient for lagged income and yields smoother consumption. All these influences combined seem to give a reasonable account of what we observe in aggregate data.

Rational expectations models with incomplete aggregate information have mostly used the assumption that aggregate information arrives with a one period lag. In the present context, the no information model seems to yield somewhat better results than the lagged information model but does not clearly dominate it. Some combination of the two models will probably improve the predictions and certainly seems more reasonable as a description of reality. Aggregate information may enter into individual decision making not because people actively pursue the collection of such information but purely by interacting with a lot of other individuals. Formalizing models in which aggregate information arrives more slowly should be an area that deserves more attention.

The feature that drives the results in this paper is that the model yields an autocorrelated process for aggregate consumption changes. Galí (1991) has shown that excess smoothness of consumption can be characterized in the frequency domain with less restrictive assumptions than in Deaton (1987) or Campbell and Deaton (1989). Essentially, his results stem from the autocorrelation in consumption changes and are therefore consistent with the predictions from the no information model.

A number of other models have been suggested that lead to autocorrelated consumption. A simple model of habit formation (Deaton, 1987) or slow adjustment of consumers to income shocks (Attfield, Demery, and Duck, 1989) also leads to an AR(1) for consumption changes. Unlike for my model, the micro parameters are generally not estimable in these cases so the models cannot be subjected to the same stringent test. Furthermore, these models imply that consumption should have the same autocorrelation structure in micro and in aggregate data. This seems to be at odds with the empirical findings.

Although in this paper I have focussed on implications of the no information model for aggregate data the model is roughly consistent with previous findings on micro data for consumption. It predicts correctly that the orthogonality conditions should not be rejected in panel data. The approach taken by Altonji and Siow (1987), Zeldes (1989) and Runkle (1991) is consistent with the model presented here. These studies find little evidence against the permanent income model with food consumption data from the PSID. While this is a very limited consumption concept, results from the Consumer Expenditure Survey (Lusardi, 1991) seem to confirm these findings with broader measures of nondurables. But incomplete information may even play a role if the permanent income model fails in micro data, as long as the individual consumption processes are less correlated with lagged income than in the aggregate.

Zeldes (1989) finds some evidence for such correlations for low wealth consumers in the PSID, interpreting them as liquidity constraints. It seems quite reasonably a priori that part of the population is liquidity constraint. It would be interesting to know how liquidity constraints and possibly other features interact with the incomplete information assumption. Deaton (1991) attempts to do so. One of his models combines liquidity constraints and precautionary savings with the incomplete information structure of the model of this paper. In numerical simulations Deaton finds a regression coefficient of consumption growth on lagged income growth of 0.42 and a smoothness

ratio just below one. His results are for logs of the variables and are therefore not directly comparable to mine. Nevertheless, it seems that incomplete information may be the major factor driving these results.

Since many of the specifications in this paper are very restrictive future research should incorporate incomplete information into more sophisticated models. Consumption research is certainly not the only area where considerations of aggregation bias may turn out to be important. The work of Froot and Perold (1991) offers a very convincing account of autocorrelations in aggregate stock returns based on the same idea. Hopefully, investigations of the robustness of the representative agent model in other areas of macroeconomics will become an important topic in future research.

Appendix A

It is straightforward to extend the examples in sections 3 and 4 to more general processes for income; all the intuition is the same. First return to the version of the model with no information. Let the first differences in individual income be stationary. This is a fairly general framework since it allows for stationarity in the levels as well, if the first differenced process has an MA unit root. Income consists of an aggregate and an individual component given by their respective Wold representations:

$$\Delta y_{it} = \phi(L)\varepsilon_t + \theta(L)u_{it} \quad (A1)$$

$$\text{where } \phi(z) = \sum_{i=0}^{\infty} \phi_i z^i$$

$$\theta(z) = \sum_{i=0}^{\infty} \theta_i z^i$$

Average per capita income is then given by

$$\Delta y_t = \phi(L)\varepsilon_t \quad (A2)$$

Given stationarity, the process for individual income changes has a Wold representation

$$\Delta y_{it} = A(L)\eta_{it} \quad (A3)$$

Individual consumption will follow

$$\Delta c_{it} = \frac{r}{1+r} \frac{A\left(\frac{1}{1+r}\right)}{1 - \frac{1}{1+r}} \eta_{it} = A\left(\frac{1}{1+r}\right) \eta_{it} \quad (A4)$$

Define $\bar{\eta}_t$ as the mean of η_{it} . Equating (A1) and (A3) and summing over individuals yields

$$A(L)\bar{\eta}_t = \phi(L)\varepsilon_t \quad (A5)$$

The lag polynomial $A(L)$ has a unit root if both $\phi(L)$ and $\theta(L)$ have a unit root, i.e. if income is stationary in levels. If $A(L)$ has no unit root (i.e. at least one of the two components is integrated) we can invert it to obtain

$$\Delta c_t = A\left(\frac{1}{1+r}\right)\bar{\eta}_t = A\left(\frac{1}{1+r}\right)A^{-1}(L)\phi(L)\varepsilon_t \quad (A6)$$

If individual income is stationary in levels, i.e. both $A(L)$ and $\phi(L)$ have a unit root these will cancel in (A5) and we can rewrite the equation as

$$A^*(L)\bar{\eta}_t = \phi^*(L)\varepsilon_t \quad (A7)$$

$$\text{where } A^*(L)(1-L) = A(L)$$

$$\phi^*(L)(1-L) = \phi(L)$$

Using (A7) we obtain the aggregate consumption response

$$\Delta c_t = A\left(\frac{1}{1+r}\right)\bar{\eta}_t = A\left(\frac{1}{1+r}\right)A^{*-1}(L)\phi^*(L)\varepsilon_t \quad (A8)$$

If both the aggregate and the individual component are described by an MA(2) then $A(L) = 1 + a_1L + a_2L^2$. The roots of this polynomial are defined by $\mu^2 + a_1\mu + a_2 = 0$. Writing consumption changes in its series representation (see e.g. Priestley (1981), p. 125)

$$\Delta c_t = \frac{A\left(\frac{1}{1+r}\right)}{\mu_1 - \mu_2} \sum_{i=0}^{\infty} (\mu_1^{i+1} - \mu_2^{i+1})(\varepsilon_{t-i} + \phi_1\varepsilon_{t-1-i} + \phi_2\varepsilon_{t-2-i}) \quad (A9)$$

This can be used to derive the regression coefficient of consumption changes on lagged income changes

$$\beta = \frac{A\left(\frac{1}{1+r}\right)}{(\mu_1 - \mu_2)(1 + \phi_1^2 + \phi_2^2)} \quad (A10)$$

$$\times \{(\mu_1 - \mu_2 + \mu_1^3 - \mu_2^3)(\phi_1 + \phi_1\phi_2) + (\mu_1^2 - \mu_2^2)(1 + \phi_1^2 + \phi_2^2) + (\mu_1^4 - \mu_2^4)\phi_2\}$$

The variance of consumption changes can be found from (A6) by solving the Yule-Walker equations. Now turn to the model with lagged information. Rewrite (A1) as

$$\Delta y_{it} = \varepsilon_t + u_{it} + \bar{\phi}(L)\varepsilon_{t-1} + \bar{\theta}(L)u_{it-1} \quad (A11)$$

$$\text{where } \bar{\phi}(z) = \sum_{i=1}^{\infty} \phi_i z^i$$

$$\bar{\theta}(z) = \sum_{i=1}^{\infty} \theta_i z^i$$

Define v_{it} again as the contemporaneous innovation. Since all the previous values of the aggregate shocks can be observed and all the previous values of the individual shocks can be inferred we can again think of information consisting of the new information v_{it} and the correction for the error made before. Equation (16) in the text still defines the error made last period in attributing parts of the innovation to the aggregate and the individual processes. Analogously to equation (18) we obtain for the change in individual consumption

$$\Delta c_{it} = \quad (A12)$$

$$\left\{ \phi \left(\frac{1}{1+r} \right) \omega + \theta \left(\frac{1}{1+r} \right) (1-\omega) \right\} v_{it} + (1+r) \left\{ \phi \left(\frac{1}{1+r} \right) - \theta \left(\frac{1}{1+r} \right) \right\} \xi_{it-1}$$

Aggregating yields²⁰

$$\Delta c_t = \quad (A13)$$

$$\left\{ \phi \left(\frac{1}{1+r} \right) \omega + \theta \left(\frac{1}{1+r} \right) (1-\omega) \right\} \varepsilon_t + (1+r) \left\{ \phi \left(\frac{1}{1+r} \right) - \theta \left(\frac{1}{1+r} \right) \right\} (1-\omega) \varepsilon_{t-1}$$

The regression coefficient of consumption changes on lagged income changes is given by

²⁰ Equations (A10) and (A11) correspond to equations (11) and (12) in Goodfriend (1991).

$$\beta = \frac{(1+r)\left(\phi\left(\frac{1}{1+r}\right) - \theta\left(\frac{1}{1+r}\right)\right)(1-\omega)}{\sum_{i=0}^{\infty} \phi_i^2} \quad (A14)$$

To see whether the no information model is consistent with the predictions of Campbell and Mankiw's (1989, 1990) λ - model, form the conditional expectations of income and consumption changes. Using equations (A2) and (A6) and the notation from above

$$E_{i-1}(\Delta y_i) = \bar{\phi}(L)\varepsilon_{i-1} \quad (A15)$$

$$E_{i-1}(\Delta c_i) = \bar{A}(L)\Delta c_{i-1} + A\left(\frac{1}{1+r}\right)\bar{\phi}(L)\varepsilon_{i-1}$$

Proportionality of the conditional expectations requires that $\bar{A}(L) = 0$. Unless the aggregate and individual income processes are suitably restricted there is no reason for this condition to hold.

Appendix B

Construction of the SIPP Sample. The 1984 Survey of Income and Program Participation was conducted in nine interview waves. Households were interviewed on a rolling basis, starting October 1983 for the first rotation group and ending July 1986 with the last rotation group. For waves 2, rotation group 2 was not interviewed, for wave 8 there is no interview for rotation group 3. In each interview, questions were asked about income for each of the previous four months. Thus monthly income data are available for all rotation groups from September 83 to March 86. Since I intend to construct quarterly observations I started with the October 83 variables.

I started by matching household heads from the nine interview waves. This resulted in 12,874 matches. I then restrict the matched sample as described in the text by selecting continuous heads for the period of analysis, that did not change marital status or their level of schooling in any month. Per capita family income is constructed by subtracting property income (F*-PROP) from total family income (F*TOTINC) and deflating by the monthly CPI for urban consumers (1982-1984 base) and by family size. Finally, I corrected reported age of the head so that age increments by one every four quarters. The final sample contains quarterly variables from the last quarter in 1983 to the first quarter in 1986. The sample only includes heads that were older than 16 years and younger than 70 years throughout the sample. The final sample has 8,170 observations.

Construction of the Aggregate Series. I created the consumption and income series from the National Income and Product Accounts largely following Blinder and Deaton (1985). The labor income series consists of labor and transfer income (the Citibase Series GW + GPOL + GPT) less social insurance contributions (GPSIN). To subtract the portion of taxes on labor income I created the ratio of wages, salaries and other labor income to income including interest, dividends and rents. Personal tax payments (GPTX) were multiplied by this ratio and the result subtracted from income. Proprietors' income (GPROP) was multiplied by the same ratio before adding it to the income series. Unlike Blinder and Deaton I did not add nontax payments to state and local governments to income and consumption because Citibase only reports this series starting from 1958. Income was adjusted in the second quarter of 1975 by subtracting the tax rebate and social security bonus. The numbers for this adjustment were taken from Blinder (1981), table 2.

The real consumption series is constructed by adding the constant dollar expenditures on nondurables and services and subtracting expenditures on clothing and shoes because these have rather durable characteristics (GCN82 + GCS82 - GCNC82). The consumption deflator obtained by dividing the nominal consumption series by the real series is used to deflate income. Both income and consumption are divided by the total population (GPOP).

Finally, to make the scale of the consumption series comparable to the income series it is multiplied by the ratio of total expenditures (GC82) to expenditures on nondurables and services. Quarterly NIPA series are reported at annual rates. I divided all series by four to obtain quarterly amounts.

Appendix C

This appendix I discuss how to calculate the utility loss the household suffers by ignoring aggregate information in consumption decisions. The basic setup is taken from the appendix in Cochrane (1989, pp. 334-335). The second part gives the matrix representations of the full information model and the no information model used in the utility calculations.

Utility for the quadratic model can be written as

$$U(X_t) = E_t \sum_{j=0}^{\infty} \beta^j X_{t+j}' R X_{t+j} \quad (C1)$$

where $\beta = 1/(1+r)$ and X_t represents the state vector of the system which evolves according to

$$X_t = A X_{t-1} + \Gamma \xi_t \quad (C2)$$

$$E_t(\xi_{t+1}) = 0$$

$$E_t(\xi_t \xi_t') = \Sigma$$

Equation (C1) can be solved as

$$U(X_t) = X_t' P X_t + \frac{1+r}{r} \text{Trace}(P \Gamma \Sigma \Gamma') \quad (C3)$$

where

$$P = R + \beta A' P A \quad (C4)$$

P will be a symmetric matrix; therefore (C4) cannot be solved directly. Cochrane shows, however, that

$$M \text{vec}(P) = (I - \beta M (A' \otimes A') N)^{-1} M \text{vec}(R) \quad (C5)$$

where M is a transformation matrix that deletes the redundant rows of a stacked symmetric matrix, i.e.

$$\text{vech}(P) = M \text{vec}(P)$$

and N does the opposite operation.²¹

Cochrane uses (C3) and (C5) to solve analytically for $U(X_t)$. Instead, once the model is expressed in the form (C1) and (C2), these equations can easily be used in Gauss to calculate utility numerically. I took this latter route.

The full information model. Instead of comparing the no information model to Goodfriend's model with lagged information I chose to use a model with full contemporaneous information on aggregate variables as the benchmark. This model will yield higher utility than Goodfriend's. The utility comparisons I present will therefore be upper bounds for the choice relevant to the consumer.

Since all the variables refer to a single household and the distinction between aggregate and individual variables is not important here I suppress i subscripts for notational convenience. Income in the full information model is given by (22) in the text.

$$\Delta y_t = (1 + \phi_1 L + \phi_2 L^2) \varepsilon_t + (1 - \alpha_1 L - \alpha_2 L^2) \mu_t \quad (C6)$$

²¹ See Henderson and Searle (1979) for details.

Optimal consumption is given by

$$c_t = \frac{r}{1+r} \left[A_t + \sum_{i=0}^{\infty} \frac{E_t y_{t+i}}{(1+r)^i} \right] \quad (C7)$$

$$= \frac{r}{1+r} A_t + y_t + \left[\frac{\phi_1}{1+r} + \frac{\phi_2}{(1+r)^2} \right] \varepsilon_t + \frac{\phi_2}{1+r} \varepsilon_{t-1} - \left[\frac{\alpha_1}{1+r} + \frac{\alpha_2}{(1+r)^2} \right] u_t - \frac{\alpha_2}{1+r} u_{t-1}$$

and assets follow

$$A_t = (1+r)[A_{t-1} + y_t - c_t] \quad (C8)$$

$$= A_{t-1} - \left[\phi_1 + \frac{\phi_2}{1+r} \right] \varepsilon_{t-1} - \phi_2 \varepsilon_{t-2} + \left[\alpha_1 + \frac{\alpha_2}{1+r} \right] u_{t-1} + \alpha_2 u_{t-2}$$

Define the state vector as

$$X_t = [1 \ A_t \ y_t \ \varepsilon_t \ \varepsilon_{t-1} \ u_t \ u_{t-1}]' \quad (C9)$$

Using (C7) and (C9) we can write

$$c_t - \bar{c} = \begin{bmatrix} -\bar{c} & \frac{r}{1+r} & 1 & \frac{\phi_1}{1+r} + \frac{\phi_2}{(1+r)^2} & \frac{\phi_2}{1+r} & -\left[\frac{\alpha_1}{1+r} + \frac{\alpha_2}{(1+r)^2} \right] & -\frac{\alpha_2}{1+r} \end{bmatrix} X_t \\ \equiv F' X_t \quad (C10)$$

Then R in (C1) is given by

$$R = -\frac{1}{2} F F' \quad (C11)$$

The transition equation for the system in (C2) becomes

$$\begin{bmatrix} 1 \\ A_t \\ y_t \\ \varepsilon_t \\ \varepsilon_{t-1} \\ u_t \\ u_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\left(\phi_1 + \frac{\phi_2}{1+r} \right) & -\phi_2 & \alpha_1 + \frac{\alpha_2}{1+r} & \alpha_2 \\ 0 & 0 & 1 & \phi_1 & \phi_2 & -\alpha_1 & -\alpha_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ A_{t-1} \\ y_{t-1} \\ \varepsilon_{t-1} \\ \varepsilon_{t-2} \\ u_{t-1} \\ u_{t-2} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ u_t \end{bmatrix} \\ \Sigma = \begin{bmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_u^2 \end{bmatrix} \quad (C12)$$

The no information model. The income process to the household in the no information model looks like

$$\Delta y_{it} = (1 - \theta_1 L - \theta_2 L^2) \eta_t \quad (C13)$$

Consumption is given by

$$c_t = \frac{r}{1+r} A_t + y_t - \left[\frac{\theta_1}{1+r} + \frac{\theta_2}{(1+r)^2} \right] \eta_t - \frac{\theta_2}{1+r} \eta_{t-1} \quad (C14)$$

and assets follow

$$A_t = A_{t-1} + \left[\theta_1 + \frac{\theta_2}{1+r} \right] \eta_{t-1} + \theta_2 \eta_{t-2} \quad (C15)$$

Define the state vector as

$$X_t = [1 \ A_t \ y_t \ \eta_t \ \eta_{t-1}]' \quad (C16)$$

Using (C14) and (C16)

$$c_t - \bar{c} = \left[-\bar{c} \quad \frac{r}{1+r} \quad 1 - \left[\frac{\theta_1}{1+r} + \frac{\theta_2}{(1+r)^2} \right] - \frac{\theta_2}{1+r} \right] X_t \equiv F' X_t \quad (C17)$$

The transition equation becomes

$$\begin{bmatrix} 1 \\ A_t \\ y_t \\ \eta_t \\ \eta_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \theta_1 + \frac{\theta_2}{1+r} & \theta_2 \\ 0 & 0 & 1 & -\theta_1 & -\theta_2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ A_{t-1} \\ y_{t-1} \\ \eta_{t-1} \\ \eta_{t-2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \eta_t \quad (C18)$$

Once both models have been solved for the level of utility attained the utility difference is converted to quarterly rates by multiplying by $r/(1+r)$. To convert the utility loss to dollar terms divide the utility loss by the expected value of marginal instantaneous utility

$$\text{\$ loss/quarter} = \frac{r}{1+r} \frac{\Delta U}{Eu'(c_t)} = \frac{r}{1+r} \frac{\Delta U}{(\bar{c} - \bar{y})} = \frac{r}{1+r} \frac{\gamma \Delta U}{\bar{y}} \quad (C19)$$

where γ is the coefficient of relative risk aversion. The calculations in the paper are for a coefficient of relative risk aversion of two and a mean income level of \$2,646.

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