Estimating the Black Economy through Monetary Approach: A Case Study of Pakistan

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Abstract

In the recent years, the 'black economy' has held enormous appeal for policy makers. Presence of black economy creates critical misrepresentation of macroeconomic variables in official estimates that lead to the false determination and delusional impact of economic policies. Similarly, black economy represents the unrecorded potential of the economy vis-à-vis resource generation and mobilization. The Economy of Pakistan underwent several minor tax reforms since 1960's. However, the tax and tariff reform of 1990's, committed under international pressure, was the first comprehensive exercise and therefore it becomes highly desirable to gauge its impact on the black economy and tax evasion practices. This paper, with some modifications, uses the standard monetary approach to obtain the latest estimates of the size of black economy and its macroeconomic implications thereof.

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1. Introduction

An ample share of economic activities takes place outside the official or recorded economy all over the world, in general, and in developing economies, in particular. There are at least three major concerns associated with the existence of underground or black economy. First, the possibility of biased evaluation of economic and social conditions of economic agents, and thus sub-optimal policies, if one disregards the hidden economy. Second, loss of precious tax revenue escaped from collection channels thereby increasing the cost of providing public services. And third, indication of shaken trust between government and economic agents[1]. Furthermore, the evaded or lost income shows the potential immobilized resources that should have been a part of national income. If the underground economy is large and significant, there is a clear evidence of market distortions, poor governance and/or disproportionate administrative regulations.

Albeit there is a consensus on the presence of underground economy, the phenomenon has been discussed and defined in the literature under many different names such as unofficial, informal, unregistered, unobserved, shadow, subterranean, parallel, hidden, invisible and irregular. Nevertheless, the purpose behind all these definitions is to link the underground economy to official national income so as to compare and add these figures to GNP. Conceptually, there are four classifications of underground economy made according to the particular institutional rules they violate. These are, illegal Economy, unreported Economy, unrecorded Economy and informal Economy[2]. Moreover, as Schneider and Frey (2001) points out, the notion of underground or black economy should not only be identified with illegality. Most of the activities are perfectly legal but the taxes are evaded due to different reasons and loopholes.

The present effort addresses the issue of the size of unreported economy in Pakistan and therefore we are explicitly interested in estimating the resources that are lost due to tax evasion and avoidance. In this regard, we use, with some modifications, the methodology adopted by Ahmed and Ahmed (1995). The revisit to this issue is of critical importance especially after taking into account the tax reforms exercise started in early 90s under the influence of IMF and stringently beefed up in late 90s.

The paper is organized as follows: Section 2 presents the review of selected literature, section 3 illustrates the methodology, section 4 elucidates the results and finally section 5 concludes the discussion with some policy implications.

2. Review of Selected Literature

Frey et al (1984) pointed out four approaches for measuring the size of black economy. These are tax evasion approach, national accounts approach, employment approach and monetary approach. Out of these methods, monetary approach is by far the most widely used methodology in estimating the black economy due to its simplicity. The pioneering efforts in this area are of Gutmann (1977), Feige (1979) and Tanzi (1983). Gutmann (1977) estimated the size of United States' black economy and came up with the figure of $200 billion. He first calculated the ratio of currency to demand deposits for the period 1937-41 (the benchmark period) and then for 1976. He then calculated the difference of ratios and thus the extra currency between these two periods. Multiplied by the ratio of GNP to legal money, this extra currency gave the size of black economy. Criticism on Gutmann's approach is the use of demand deposits in the numerator. This approach assumes that the increase in to ratio would attract people to hold more currency by withdrawing from demand deposits. However, they may instead convert their money from demand to time deposits. Feige (1979) presented his model based on quantity theory of money by formulating

$$PT = MV + M'V' \quad (1)$$

Where M and M' are currency notes and demand deposits respectively, V and V' are Velocities (or average turnover) of money respectively of currency notes and demand deposits, P is composite price index of existing and newly produced goods and T is volume of transactions. In this approach, the estimated PT is divided by the observed income to GNP ratio to get the size of black economy where observed income is the product of price index of newly produced goods and real income of the economy. He formulized that the derived nominal GNP and official GNP are the same in the absence of black economy. Tanzi (1980, 1983) formulated his methodology based on Cagen's (1958) work. Cagen explained that the long run behavior of currency-money supply-ratio depends upon expected real per capita income, volume of retail trade, volume of travel per capita, degree of urbanization and tax rate on transactions. According to him, the higher the tax rates, the more the transactions made by currency payments to avoid reporting to tax collector. Tanzi (1980) re-hypothesized the same link between tax rate and currency-money supply ratio to obtain the alternative estimates of US black economy. He assumed that currency is used to carry out transactions in the black economy and high taxes are the forces behind the size of black economy. Tanzi (1980, 1983) postulated the currency in circulation to money supply ratio $(C/M2)$ as a function of top bracket statutory tax rate, weighted average rate on interest income, ratio of personal income tax to personal income net of transfers $(Ti's)$, share of wages and salaries in national income (ws/ni), interest rate (r) and per capita income (Y_p). The expected signs for both Y_p and r are negative. This is because economic development in the country, measured by per capita income, is assumed to lead to the replacement of currency by cheques, thus causing a fall in $C/M2$. Higher interest rates encourage people to invest currency holdings in time and other forms of deposits and thereby reducing the volume of currency in circulation. On the other hand, higher taxes motivate people in indulge in tax evading activities that are facilitated by the use of currency. Moreover, as the wages are paid in currency, especially of daily workers, an increase in wages will require more currency. Tanzi

(1982) also estimated the extant of evaded taxes by multiplying the underground economy by tax to GNP ratio. Tanzi's methodology is considered to be the most sophisticated one in estimating black economy through monetary approach

In the context of Pakistan, all attempts followed the Tanzi's approach with some modifications. Shabsigh (1995) used the ratio of currency in circulation to demand deposits (M2 minus currency in circulation) as the dependant variable, while the independent variables were real per capita income, real interest rate, per capita banking services, taxes on imports, exports and domestic services. He concluded that the size of black economy was 21 percent of GDP in 1975 declined slightly to 20.4 percent of GDP in 1990. Ahmed and Ahmed (1995) came up with the result that this size was declined from 52 percent in 1960 to 35 percent in 1990. Iqbal et al (1998) used currency in circulation to M2 ratio as dependant variable while domestic tax to GDP ratio, international taxes to GDP ratio, real interest rate, real per capita income growth and banking services are used as explanatory variables. They also used a dummy for the period 1988 to 1996 to capture the effect of structural adjustment program. They concluded that the underground economy has increased from 20 percent of GDP in 1973 to 51 percent in 1996. Aslam (1998) introduced ratio of currency in circulation and foreign currency accounts to M2 as dependant variable. Independent variables include: total tax revenue to GDP ratio, interest rate on time deposits and a dummy for the period 1991-1998 in order to capture the effect of introduction of the foreign currency accounts in 1991. He reveals that the underground economy has been increased from 29 percent of GDP in 1960 to 44 percent in 1990. Kemal (2003) used the same dependent variable as Aslam (1998) and the explanatory variables were tax to GDP ratio, banking services, GDP growth rate and a dummy for the period 1990-2002. He came up with the results that underground economy has increased from 20 percent of GDP in 1974 to 54 percent in 1998 and then declined again to 37 percent in 2002. He also used a lagged dependant variable to capture the inertia in variables. Yasmin and Rauf (2004), using the similar methodology, found out that the growth rate of underground economy remained 12.7% and of tax evasion 10.9% between 2001-2002 as against 5.9% growth of GDP. They also ran two other equations with GDP

as dependant variable in both. Explanatory variables in first equation were tax evasion and lag of GDP and in second equation size of underground economy and lag of GDP. They confirmed significant negative effect of both tax evasion and underground economy on GDP.

As evident, a comparison of these studies in the context of Pakistan reveals contradicting results vis-à-vis the size of black economy. Econometrically, the bases of these alternative results may include (i) the choice of variables, (ii) choice of the estimation period and (iii) choice of the functional form and underlying assumptions. Short run fluctuations in the selected macroeconomic variables may also distort the inference. Moreover, despite the use of different dummies, no study tried to capture the impact of taxation reforms of 1990s on the underground economy and this is one of the focuses of our study.

3. Data and Estimation Methodology

The model subject to estimation is based on the methodology of Ahmed and Ahmed (1995) with a slight modification[3]. We formulate

$$Lncm2 = \beta_0 + \beta_1 Ln(1+tgr) + \beta_2 Lnr + \beta_4 d_{97} + e \qquad (2)$$

where *cm2* is the Ratio of currency in circulation to money supply m2, *tgr* is overall tax to GDP ratio, *r* is weighted average rate of return on deposits and d_{97} is the tax reform dummy, taking the value of 1 from 1997 onwards. We hypothesize a positive link between tax to GDP ratio and currency to M2. An increase in the tax rate stimulates people to evade them through using currency transactions instead of cheques. On the other hand, increase in the rate of return would induce people to invest in deposits and thereby reducing the currency-money supply ratio.

Besides the currency ratio equation, another dependant variable is used in a separate regression with same explanatory variables. This dependant variable includes bearer bonds along with the currency in circulations. Bearer bonds were introduced in mid 80's to enhance the savings and investment in the economy. These bonds were particularly attractive for black money since they can be obtained in unlimited quantities and without any cumbersome procedure. Since they are easily convertible into cash at any time, they can serve as currency themselves. Therefore, the second regression is

$$Lncbm2 = \beta_0 + \beta_1 Ln(1 + tgr) + \beta_2 Lnr + \beta_4 d_{97} + e \qquad (3)$$

Where *cbm2* is (currency + bearer bonds' value)/(M2 + bearer bonds' value). Moreover, as mentioned above, most of the macroeconomic variables are subject to short run fluctuations and thus the resulting variance may distort the results. To cope with this problem, we use the HP filter (Hodrick and Prescott, 1997) to remove cyclical variations from independent variables. More specifically, let y_t be a series composed of two components: a cyclical component (c_t) and a trend component (τ_t). HP filter isolates c_t from τ_t by minimizing the variance of y_t. To do this, HP filter uses a penalty parameter λ to control the smoothness of the series τ_t. The larger the value of λ, the smoother the series and τ_t becomes a perfect linear trend as $\lambda \to \infty$. There are various critiques on the use of HP filter for smoothing a series and researchers point out some of the undesirable properties associated with it (Ahumada and Garegnani, 1999 and Ravn and Uhlig, 1997). Ravn and Uhlig (1997, p.1), nonetheless, suggest that '*none of these shortcomings and undesirable properties are particularly compelling: HP filter has withstood the test of the time and the fire of discussion remarkably well'*. Likewise, Ahumada and Gargnani (1999, p.18) conclude that the criticized drawbacks of HP filter '*do not appear to have had great effects on its wide use in empirical research'*. Figures 1(a), 1(b) and 1(c) in the appendix-A compare the actual and filtered series used in regression. Different values of smoothing penalty λ are chosen for different variables

depending upon the empirical practices[4]. Furthermore, we use the Moving Average (MA) technique to deal with the problem of autocorrelation in both equations.

Rest of the analysis follows the typical path. From equation 2, predicted value of ratio *cm2* is computed for each year first with tax variable ($cm2_t$) and then without tax variable ($cm2_{wt}$). The difference between tax and without tax ratio gives us an indication regarding the level of currency holdings stimulated by taxes. This difference is multiplied by M2 to obtain the level of illegal money. Mathematically:

$$\text{Illegal Money (IM)} = (cm2_t - cm2_{wt}) \,.\, M2 \tag{4}$$

Size of the Black economy can be obtained by multiplying illegal money by velocity of money. Velocity of money equals the ratio of GNP to legal money. Moreover, total money in the economy can either be legal or illegal. Therefore, legal money is computed by taking the difference between total money supply and illegal money. Mathematically:

$$Legal\ Money\ (LM) = M2 - IM \tag{5}$$

$$Velocity\ of\ Money\ (v) = \frac{GNP}{LM} \tag{6}$$

$$Black\ Economy\ (BE) = IM \,.\, v \tag{7}$$

Finally, level of tax evasion is obtained by multiplying the size of black economy to the ratio of tax-to-GNP.

$$Tax\ Evasion = BE.\left(\frac{Taxes}{GNP}\right) \tag{8}$$

The same process is applied for equation 3. Furthermore, we assume that the velocity of money is the same for both illegal and legal money. Rationally, when the black money is used in regular markets for transactions, it should behave in the same manner as white money in order to appear regular and trustful. The data for our analysis covers the period 1960 to 2003 and obtained from various issues of Pakistan Economic Survey and State Bank of Pakistan's Annual Reports.

4. Estimation Results

Table 1 shows the results from regression equations 2 and 3. All variables are highly significant at 1 percent level. Moreover, goodness of fit and F-ratio are also quite high. In both regressions, tax-to-GDP ratio (*tgr*) possesses the positive sign confirming the hypothesis of increasing currency-money supply ratio with increasing tax rates. Sign of weighted average rate of return (*r*) is negative, which also confirms our hypothesis that the higher the rates of returns on deposits, the higher the savings and the lower the currency-M2 ratio. The coefficients of these two variables, especially *tgr*, are quite higher than Ahmed and Ahmed (1995). The reason is the use of filtered *tgr* and *r*, which removed the cyclical fluctuations, thus increasing the coefficients by making the trend components in both the series stronger.

Table 1
Regression results

	Equation 2			**Equation 3**		
	Currency Ratio (*cm2*)			Currency + Bond Ratio (*cbm2*)		
	Coefficient	Standard Error	*p*-value	Coefficient	Standard Error	*p*-value
Constant	-1.100	0.170	0.0000	-1.229	0.162	0.0000
tgr	8.805	2.496	0.0011	10.311	2.346	0.0001
r	-0.597	0.075	0.0000	-0.584	0.072	0.0000
d_{97}	-0.344	0.057	0.0000	-0.233	0.059	0.0003
N	44			44		
R^2	0.88			0.85		
dw-statistic	1.77			1.70		
F-statistic	77.90			57.44		

Dummy variable for tax reforms (d_{97}) is also highly significant bearing the negative sign. Tax reform exercise reached at its peak in 1997 when quite a few important steps were taken. These steps include, inter alia, the substantial decrease in the personal income tax rate to 20 per cent, reduction in corporate tax rate, withdrawal of turnover tax and some of the withholding taxes, strengthening the tax administration and improvements in documentation. Negative sign of dummy in both the equations implies that these exercises were significant in reducing the cm_2 and cbm_2 ratios, and thus contracting the size of black economy. After the estimation, we obtained the size of black economy explained by equations 4 through 8. The results are shown in Table 2 and 3. Black economy in Pakistan turns out to be highest in early 60s when the corporate and personal income tax rates were high. Corporate income tax rate was 30 percent plus 30 percent super tax. This (aggregate corporate income and super tax) rate was dropped to 40 percent in the later part of 80s. Likewise, the maximum personal income tax rate was 75 percent during 1960-64 causing the black economy to remain well above 30 percent of GDP during the same period. Black economy kept declining during 1965-75 when this rate was brought down in between 60 and 70 percent (Qureshi, 1989, pp.23). Furthermore, this rate was 56 percent during 1980 –1986 brought down to 39 percent in 1988 and further to 28 percent in 1993 – the effect of which is subsequently reflected by the squeezing black economy in the periods under review. This is also interesting to note that Kemal (2003) reports an increasing trend of black economy between 1995 and 1998– contrary to our results. Possible explanation could be the tax reform effect, which was absent in Kemal (2003). Furthermore, results of Kemal (2003) are based on a special specification where lagged dependent variable is used as explanatory variable with high positive coefficient.

In the present study, the impact of tax reforms is dominating and evident from both tables 2 and 3. Black economy as percentage of GDP was declined by nine percentage points in case of both currency ratio and currency bearer bond equation during 1996 and 1997. The corresponding decline in tax evasion as percentage of GDP was 39 percent and 32 percent respectively for both methods. Figure 1 and 2 respectively plot *tgr* and *r* against black economy to GDP ratio estimated using equation 2[5]. Black

economy remained relatively high during early 90's at around 26% of GDP. During that period, tax-to-GDP ratio was almost stagnant at 13% and rate of return on deposits was falling - a disincentive to withdraw from black economic activities. Nevertheless, during 1996-97, tax-to-GDP ratio dropped to 12.7% after touching its peak of 14% coupled with the increase in rate of return from 6.4% to 6.8%. Both these factors, especially tax reform effect, played significant role in slashing the black economy. This is also imperative to note that the declination of tax to GDP further in 1999-00, which does not appear to have great impact on the size of black economy, was actually the result of re-basing of country's GDP. On the other hand, the sharp decline of rate of return on deposits from 1998 onwards acted as a hurdle in reducing the size of black economy.

Figure 1: Black Economy-GDP & Tax-GDP ratios

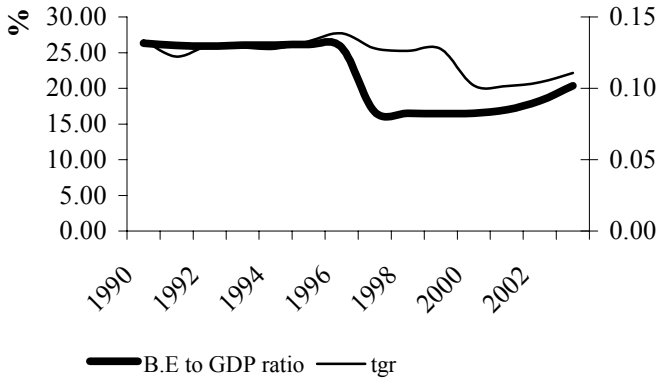
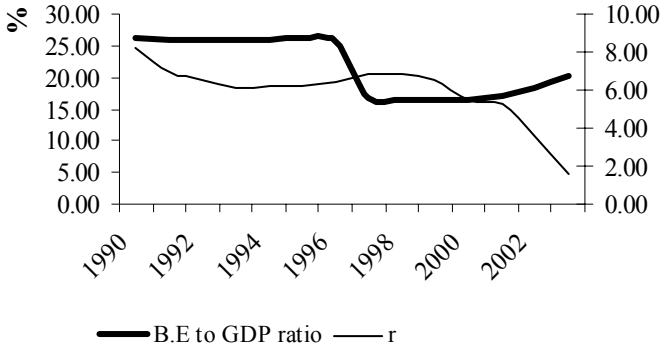


Figure 2: Black Economy-GDP ratio & Rate of Return



The inclusion of bearer bond with currency in circulation, as table-2 shows, does not alter the direction of outcomes. It, however, is imperative to increases the magnitude of black economy as percentage of GDP. This suggests that bearer bonds are quite reliable medium of exchange in the underground economy alongside currency. Roughly, it can be seen that the inclusion of bearer bond increases, on average, the black economy as percentage of GDP by 5 percentage points each year. Bearer bonds were introduced during mid 80's to promote savings in Pakistan. They became a handy medium of exchange later on due

to their limited physical quantity required for any transaction as against currency and their hassle-free acquisition. This is quite interesting to note that the annual compound growth rate of currency in circulation and bearer bonds during last two decades remained almost the same; i.e. 12 percent. Moreover, note also that the size of black economy has slightly increased from 2000 onwards. This is, perhaps, due to the reduction of rate of return on deposits, which was declined by more than 30 per cent between 2000 and 2003 revealing the loose stance of monetary policy. On the other hand, the effective coverage of indirect as well as direct taxes was increased during the same period. This brought some of the untaxed sectors into the tax net, causing the tax to GDP ratio to increase slightly by 0.26 percent during the same period.

5. Conclusion and Policy Implications

In this paper, we attempted to estimate the size of unreported part of the economy that is caused by tax evasions. This becomes of special importance once the impact of taxation reforms is incorporated. Overall, black economy has a declining trend as percentage of GDP. This is due to the tax reforms involving rationalization of tax rates. Inclusion of bearer bonds as a medium of exchange significantly increases the size of the black economy. After their inception in mid 80's, the volume of bearer bonds is growing almost at the same pace as currency in circulation. Despite the fact that black economy as a percentage of GDP has decreased, the annual compound growth rate of black economy during the sample period remained more than 11 percent. At disaggregated level, this growth remained 2 percent during 60s, 17 percent during 70's, 15 percent during 80s and 13 percent during 90s and afterwards (see table 3). Similarly, tax evasion grew at the rate of 12 percent. This growth remained 5 percent during 60s, 19 percent during 70s, 16 percent during 80s and 11 percent during 90s and afterwards. These results are approximately same for both equations. This is also worth mentioning that due to the strong underlying assumptions of Tanzi (1980, 1983), the estimates of black economy cannot be taken as precise measures. They can, nevertheless, be effectively used to deduce the broad trend and directions. In the light of above

discussion, therefore, several suggestions pertaining to the policy actions can be made. Although Increase in the direct tax revenue is vital for a developing country because of its redistributive effects, the medium of this increase, nonetheless, cannot solely be the increase in tax rates since this give rise to tax evasion. Instead, broadening the tax base would be an ultimate solution. To supplement these efforts, official administration regarding the detection and hammering of tax evasion should be improved. Tax reform process should be consolidated and integrated with other macroeconomic reforms. System loopholes and prevailing corruption among the tax collection authorities cannot be neglected when dealing with the issue of evasion. These inefficiencies must be dealt accordingly in order to curb the tax deadweight loses and to reduce the cost of being part of the reported economy.

Table-2
Estimates of Black Economy via Equation-2

Year	Illegal Money (Million Rs.)	Legal Money (Million Rs.)	Velocity of Money	Black Economy (Million Rs.)	Black Economy as % of GDP	Tax Evasion (Million Rs.)	Tax Evasion as % of GDP
1960	1,994	3,862	4.6	9,206	51.6	722	4.0
1961	1,820	4,058	4.8	8,784	44.8	705	3.6
1962	1,745	4,360	4.7	8,207	40.0	687	3.3
1963	1,863	5,124	4.3	8,000	36.3	611	2.8
1964	1,990	5,950	4.2	8,276	33.4	628	2.5
1965	2,043	6,578	4.3	8,752	31.0	726	2.6
1966	3,152	10,852	2.9	9,061	29.0	740	2.4
1967	3,355	12,279	2.9	9,698	27.3	894	2.5
1968	3,436	13,349	2.9	9,945	25.7	864	2.2
1969	3,599	14,849	2.8	10,015	24.2	1,019	2.5
1970	3,787	16,653	2.9	10,860	22.7	1,077	2.3
1971	3,845	18,068	2.8	10,792	21.2	960	1.9
1972	3,864	19,339	2.8	10,943	20.0	869	1.6
1973	4,313	22,755	3.0	12,879	18.8	1,072	1.6
1974	4,738	25,941	3.4	16,205	18.3	1,451	1.6
1975	5,035	28,039	4.0	20,173	18.1	1,705	1.5
1976	6,382	35,269	3.8	24,132	18.4	1,801	1.4
1977	8,163	43,610	3.6	29,057	19.3	2,174	1.4
1978	10,508	53,151	3.5	37,280	21.0	3,962	2.2
1979	13,658	64,954	3.2	44,083	22.5	4,936	2.5
1980	16,837	75,587	3.3	56,315	24.0	6,844	2.9
1981	19,790	84,831	3.5	70,193	25.2	8,517	3.1
1982	22,621	93,889	3.7	83,649	25.8	9,621	3.0
1983	28,812	117,213	3.4	99,254	27.2	11,424	3.1
1984	32,486	130,781	3.5	114,113	27.2	12,580	3.0
1985	36,733	147,172	3.5	127,409	27.0	14,457	3.1
1986	42,223	168,888	3.3	138,974	27.0	17,227	3.3
1987	48,057	191,966	3.2	152,420	26.6	19,764	3.5
1988	54,021	215,493	3.3	176,604	26.1	22,300	3.3
1989	58,338	232,119	3.4	200,262	26.0	26,519	3.4
1990	68,775	272,477	3.3	224,810	26.3	28,775	3.4
1991	81,192	319,452	3.3	264,487	26.0	31,595	3.1
1992	103,197	402,372	3.0	312,317	25.9	40,222	3.3
1993	122,315	473,075	2.8	347,238	26.0	44,336	3.3
1994	145,027	558,371	2.8	406,505	26.0	51,844	3.3
1995	169,971	654,761	2.9	488,024	26.2	64,394	3.5
1996	192,455	746,225	2.8	544,964	25.7	75,802	3.6
1997	151,794	901,439	2.7	405,646	16.7	52,316	2.2
1998	172,373	1,033,947	2.6	442,339	16.5	56,356	2.1
1999	182,318	1,098,228	2.7	483,563	16.5	62,267	2.1
2000	200,718	1,199,914	3.1	626,533	16.5	64,703	1.7
2001	223,955	1,302,089	3.2	706,590	17.0	72,668	1.7
2002	270,627	1,490,743	3.0	803,375	18.3	83,381	1.9
2003	342,507	1,736,262	2.9	981,029	20.3	105,340	2.2

Table-3
Estimates of Black Economy via Equation-3

Year	Illegal Money (Million Rs.)	Legal Money (Million Rs.)	Velocity of Money	Black Economy (Million Rs.)	Black Economy as % of GDP	Tax Evasion (Million Rs.)	Tax Evasion as % of GDP
1960	2,202	3,654	4.9	10,743	60.2	843	4.7
1961	1,996	3,883	5.0	10,066	51.3	808	4.1
1962	1,899	4,206	4.9	9,258	45.1	775	3.8
1963	2,014	4,972	4.4	8,912	40.4	681	3.1
1964	2,157	5,783	4.3	9,231	37.2	700	2.8
1965	2,247	6,374	4.4	9,935	35.2	824	2.9
1966	3,545	10,460	3.0	10,572	33.8	863	2.8
1967	3,871	11,763	3.0	11,681	32.9	1,077	3.0
1968	4,045	12,741	3.0	12,264	31.7	1,066	2.8
1969	4,278	14,170	2.9	12,476	30.2	1,270	3.1
1970	4,456	15,984	3.0	13,313	27.9	1,320	2.8
1971	4,397	17,516	2.9	12,730	25.1	1,133	2.2
1972	4,263	18,940	2.9	12,327	22.5	979	1.8
1973	4,612	22,456	3.0	13,957	20.4	1,162	1.7
1974	4,954	25,725	3.4	17,085	19.3	1,530	1.7
1975	5,201	27,873	4.0	20,958	18.8	1,771	1.6
1976	6,641	35,010	3.8	25,294	19.3	1,888	1.4
1977	8,800	42,973	3.6	31,789	21.1	2,378	1.6
1978	11,967	51,692	3.6	43,651	24.6	4,639	2.6
1979	16,304	62,308	3.4	54,854	28.0	6,142	3.1
1980	20,658	71,766	3.5	72,774	31.0	8,844	3.8
1981	24,386	80,235	3.8	91,451	32.9	11,096	4.0
1982	27,530	88,980	3.9	107,421	33.1	12,355	3.8
1983	34,463	111,562	3.6	124,733	34.2	14,357	3.9
1984	38,284	124,983	3.7	140,721	33.5	15,513	3.7
1985	43,058	140,847	3.6	156,056	33.1	17,707	3.8
1986	49,645	161,466	3.4	170,918	33.2	21,187	4.1
1987	56,702	183,321	3.3	188,322	32.9	24,419	4.3
1988	63,679	205,835	3.4	217,946	32.3	27,521	4.1
1989	68,464	221,993	3.6	245,742	32.0	32,542	4.2
1990	80,064	261,188	3.4	273,025	32.0	34,947	4.1
1991	93,873	306,771	3.4	318,436	31.3	38,040	3.7
1992	119,473	386,096	3.2	376,814	31.3	48,529	4.0
1993	142,737	452,653	3.0	423,494	31.8	54,073	4.1
1994	171,428	531,970	2.9	504,352	32.3	64,323	4.1
1995	203,738	620,994	3.0	616,788	33.1	81,384	4.4
1996	232,542	706,138	3.0	695,855	32.8	96,791	4.6
1997	204,086	849,147	2.8	578,976	23.8	74,670	3.1
1998	228,704	977,616	2.7	620,712	23.2	79,082	3.0
1999	236,883	1,043,663	2.8	661,133	22.5	85,132	2.9
2000	254,500	1,146,132	3.3	831,687	21.9	85,889	2.3
2001	280,262	1,245,782	3.3	924,212	22.2	95,049	2.3
2002	339,024	1,422,346	3.1	1,054,809	24.0	109,477	2.5
2003	433,071	1,645,698	3.0	1,308,692	27.1	140,524	2.9

Appendix A: Comparison of Filtered vs. Actual Series

Figure 1(a): *tgr* series in Equation 2

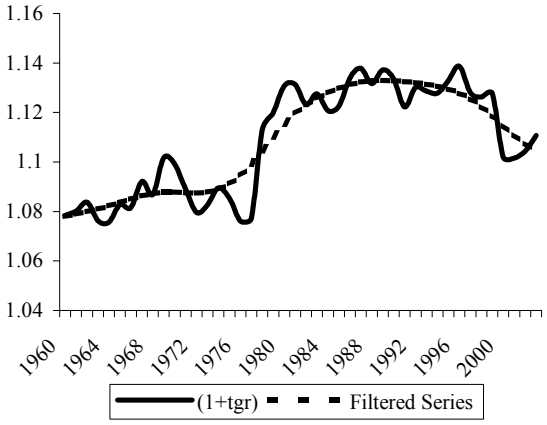


Figure 1(b): *tgr* series in Equation 3

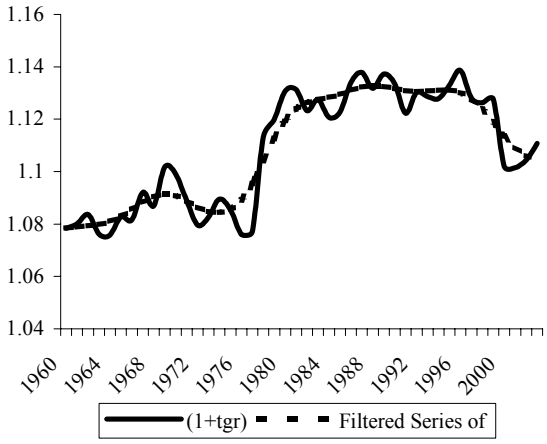
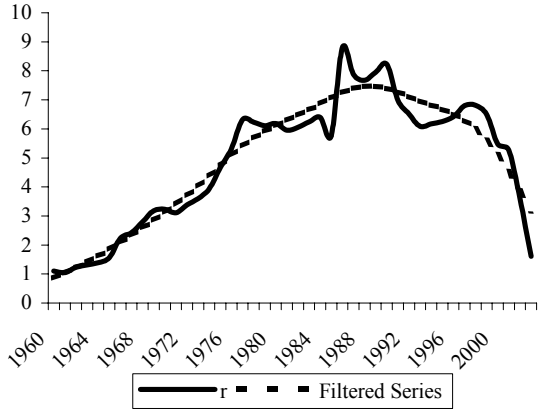


Figure 1(c): *r* series in Equation 2 and 3



Notes

[1] See Schneider and Frey (2001) for further discussion.

[2] See, for instance, Fiege (1990) for an account of this taxonomy.

[3] Ahmed and Ahmed (1995) uses a dummy for 1960-1971 period based on the separation of Bangladesh (former East Pakistan) in 1971, which is replaced by the tax reform dummy in the present exercise.

[4] λ for *tgr* takes the value 50 and 10 respectively in equation 2 and 3 while λ is 50 for *r* in both the equations. Empirically, Hodrick and Prescott (1997) suggest $\lambda \leq 100$ and Maravall and del Rio (2001) propose $6 \leq \lambda \leq 14$ in case of annual data

[5] Plotting these two variables with black economy to GDP ratio estimated using equation 3 brings the similar conclusion.

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