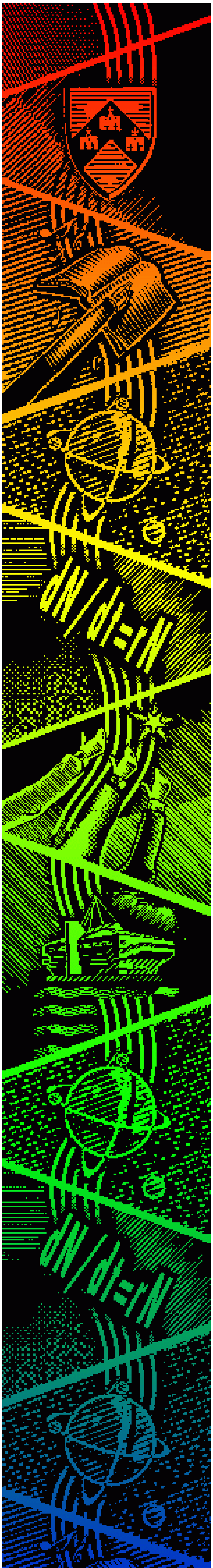

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Corporate Bond Valuation with Both Expected and Unexpected Default

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CORPORATE BOND VALUATION WITH BOTH EXPECTED AND UNEXPECTED DEFAULT

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Abstract

This paper presents three variants of a tractable structural model in which default may take place both expectedly and unexpectedly. The model has the merit of predicting realistically high short term credit spreads. Closed form solutions are provided for corporate bonds (and default swaps) when interest rates are constant or stochastic and when the bond recovery value is exogenous or endogenous to the model. The analysis suggests that, in order for the observed short term yield spreads on high grade corporate bonds to be compensation for credit risk, bond holders must believe that a dramatic sudden plunge in the firm's assets value is possible, even if extremely unlikely.

Key words: corporate bond valuation, structural model, unexpected default, short term credit spreads, endogenous bond recovery value, plunge

of assets value.

JEL classification: G13;G33.

1 Introduction

This work presents closed form solutions for pricing corporate bonds and credit default swaps using a structural model approach. Following Cathcart and El-Jahel (2003), the specificity of the model is that default can occur in two different ways, i.e. either when the value of the firm's assets first drops to the level of the default barrier, or when an unexpected sudden event causes the firm to default. Within this framework closed form solutions are presented whereby the recovery value of the bond upon default is either exogenous or endogenous to the model. Under exogenous bond recovery value, the default free term structure can be flat and constant or it can be stochastic.

When the default free term structure is flat and constant, the exogenous bond recovery value is assumed to be a fraction of its face value. When the default free short rate is stochastic, the exogenous recovery value of the bond is assumed to be a fraction of its default free value, which is a typical assumption in the presence of a stochastic default free term structure (see e.g. Longstaff and Schwartz, 1995). This is a special case of the model by Cathcart and El-Jahel (2003), for which a new closed form solution is presented. Under a flat and constant term structure, the endogenous recovery value of the bond is a function of the value of the debtor's assets at the time of default. All these model variants can provide a solution to a well know shortcoming of the "traditional" structural models of credit risk, in that they are capable to predict realistically high short term credit spreads.

The analysis with endogenous debt recovery value suggests that, if the observed short term yield spreads for high grade borrowers are compensation for credit risk, bond holders must believe there is a tiny probability (e.g. less than 1% per annum) of a dramatic sudden drop in assets value, a drop which can plunge assets value down to approximately the level of the default barrier.

1.1 Literature

In the last decade structural models and reduced form models of credit risk have been "competing" to price default risky corporate bonds. Structural models have the advantage of making effective use of balance sheet and equity price information in a theoretically coherent way, but have been unable to generate realistically high short term credit spreads. Reduced form models have been able to generate realistically high short term credit spreads, but do not make full use of firm specific information or are based on rating information which is usually not timely or exhaustive.

The under-prediction of short term credit spreads by structural models is at least partly due to the fact that empirically observed yield spreads on corporate bonds incorporate a liquidity premium over and above a credit risk premium (see e.g. Perraudin 2003). Moreover, such yield spreads may also partly be due to the often less favorable tax treatment of corporate bond coupons with respect to government bond coupons. Nether-the-less short term credit spreads predicted by structural models still look disturbingly lower than observed corporate yield spreads.

Recent literature on credit risk valuation, aware of the respective advantages and disadvantages of structural models and reduced form models, has repeatedly tried to reconcile the structural and the reduced form approaches in order to achieve some sort of advantageous synthesis of two. Notable such examples are Duffie and Lando (2001), who show how credit spreads are affected by the often incomplete accounting information available to bond holders, and Zhou (2001), who assumes that the value of the firm's assets follows a jump diffusion process and values bonds through simulation. These two models are capable of predicting higher short term credit spreads than implied by usual structural models at the expense of model tractability. Alternatively, the best practice, e.g. Pan (2001), has assumed a stochastic default barrier capable to predict non-negligible short term credit spreads on corporate bonds. Pan's model as well as Duffie and Lando's have the drawback of predicting excessively high short term spreads when default is likely and the bond is close to maturity.

This paper is more close in spirit to Cathcart and El-Jahel's (2003) in that both expected and unexpected default can take place. But, unlike in Cathcart and El-Jahel, full closed form solutions are derived when the default hazard rate is constant, when the default free interest rate is constant or stochastic and when the recovery value of the defaulted bond is endogenous.

Hereafter, section 2 proposes new closed form solutions for bond and default swap values under constant and stochastic interest rates given that the recovery value of the defaulted bond is exogenous. Section 3 proposes a closed form solution for bond value under constant interest rates whereby the recovery value of the defaulted bond depends on the firm's assets value at the time of unexpected default.

2 The bond valuation model with exogenous debt recovery value

The bond valuation model assumptions are standard. The model assumes universal risk neutrality. The firm's assets value risk neutral process follows a geometric Brownian motion

$$dV = (r - b) \cdot V \cdot dt + s \cdot V \cdot dz \tag{1}$$

where r is the default free short interest rate assumed constant over time, where b is the assets pay-out ratio, where s is the assets volatility parameter, where dz is the differential of a Wiener process; b and s are are constant. The firm has issued a bond security with value of $D(V,t)$, with face value of P, with maturity of T and yearly coupon flow (assumed to be continuously paid) of C. Default can occur in an expected way the first time V drops to the level V_b. Thereupon the recovery value of $D(V,t)$, denoted by R, is an exogenous fraction α of the bond face value P, i.e. $R = \alpha P$ with $0 \leq \alpha \leq 1$.

Default can occur also in an unexpected way in any infinitesimal time period

dt. In any period dt there is a probability λdt that default may be unexpectedly precipitated causing $D(V,t)$ to fall to the constant recovery value $R_\lambda = \alpha_1 P$, where again $0 \leq \alpha_1 \leq 1$. Thus R need not equal R_λ. Note that in this section the bond recovery value upon default is an exogenous fraction of the par value of the bond. Different recovery assumptions will be made later in the paper.

The event that triggers unexpected default is a completely unexpected event that is independent of the market value of the firm's assets V, such as may be the discovery of substantial misgivings in the firm's accounts. Employing standard valuation arguments, we know that the value $D(V,t)$ of the corporate bond satisfies the following equation

$$\frac{dD(V,t)}{dt} + \frac{d^2 D(V,t)}{dV^2} s^2 V^2 + \frac{dD(V,t)}{dV}(r-b)V - (r+\lambda) D(V,t) + C + \lambda R_\lambda = 0 \tag{2}$$

$$D(V,T) = P \tag{3}$$

$$D(V_b,t) = R \tag{4}$$

$$D(V \to \infty, t) \to \frac{\lambda R_\lambda + C}{r+\lambda}\left(1 - e^{-(r+\lambda)(T-t)}\right) + e^{-(r+\lambda)(T-t)} P \tag{5}$$

Equation 2 reflects the facts that a continuous coupon flow is received by bond holders at the yearly rate C, that the bond is subject to unexpected default with constant hazard rate λ, that the bond holders recover R upon expected default and R_λ upon unexpected default. Condition 3 is the terminal condition if the debtor is solvent. Condition 4 sets the recovery value of the bond when the default barrier is first reached. Condition 5 reflects the fact that, as the firm's assets become very valuable, default can take place only in an unexpected way. The right hand side of condition 5 is the value of the bond when only unexpected default can occur with intensity λ.

In deriving equation 2 universal risk neutrality is assumed, but such assumption does not seem restrictive. In fact expected default is driven by the dynamics of assets value V, which can be replicated at least in principle so to make the market complete. And we know from Harrison and Kreps (1979) that market completeness is tantamount to the risk neutrality assumption. Moreover λ is both the real and the risk-neutral intensity of unexpected default. In fact the risk of unexpected default commands no premium, since such risk is assumed to have no systematic component, unexpected default being a complete surprise. The solution to equation 2 and to the respective conditions is

$$D(V,t) = D(V) - O_D(V,t) + O_P(V,t) \tag{6}$$

$$D(V) = \frac{\lambda R_\lambda + C}{r+\lambda} + \left(-\frac{\lambda R_\lambda + C}{r+\lambda} + R\right)\left(\frac{V}{V_b}\right)^q \tag{7}$$

$$q = \frac{-\left(r - b - \frac{1}{2}s^2\right) - \sqrt{\left(r - b - \frac{1}{2}s^2\right)^2 + 2(r+\lambda)s^2}}{s^2} \tag{8}$$

$$O_D(V,t)=\frac{\lambda R_\lambda+C}{r+\lambda}e^{-(r+\lambda)(T-t)}\cdot\Omega\left(d\left(\frac{V}{V_b},1\right),d\left(\frac{V_b}{V},1\right),0,1\right)$$
$$-\frac{-\frac{\lambda R_\lambda+C}{r+\lambda}+R}{(V_b)^q}e^{(-(r+\lambda)+n(q))(T-t)}\cdot\Omega\left(d\left(\frac{V}{V_b},q\right),d\left(\frac{V_b}{V},q\right),q,q\right) \tag{9}$$

$$O_P(V,t)=P\cdot e^{-(r+\lambda)(T-t)}\cdot\Omega\left(d\left(\frac{V}{V_b},1\right),d\left(\frac{V_b}{V},1\right),0,1\right) \tag{10}$$

$$n(w)=(r-b)w+\frac{1}{2}w(w-1)s^2 \tag{11}$$

$$d(z,w)=\frac{w\ln(z)+\left(n(w)+\frac{1}{2}s^2w^2\right)(T-t)}{ws\sqrt{T-t}} \tag{12}$$

$$\Omega(k_1,k_2,k_3,w)=V^{k_3}N(k_1)-\left(\frac{V}{V_B}\right)^{-2\frac{n(w)}{w\cdot s^2}}(V_B)^{k_3}N(k_2) \tag{13}$$

These formulae allow to derive a closed form solution also for the value of a credit default swap $S(V,t)$, which pays $P-R_\lambda$ in case of unexpected default and $P-R$ in case of expected default before time T, and which requires the protection buyer (bondholder) to continuously pay a constant premium at the yearly rate C_s. Then the value of one such credit default swap for the protection buyer is

$$S(V,t)=S(V)-O_S(V,t) \tag{14}$$

$$S(V) = \frac{\lambda(P - R_\lambda) - C_s}{r + \lambda} + \left(-\frac{\lambda(P - R_\lambda) - C_s}{r + \lambda} + R\right)\left(\frac{V}{V_b}\right)^q \tag{15}$$

$$q = \frac{-\left(r - b - \frac{1}{2}s^2\right) - \sqrt{\left(r - b - \frac{1}{2}s^2\right)^2 + 2(r + \lambda)s^2}}{s^2} \tag{16}$$

$$\begin{aligned} O_S(V,t) = {} & \frac{\lambda(P - R_\lambda) - C_s}{r + \lambda} e^{-(r+\lambda)(T-t)} \cdot \Omega\left(d\left(\frac{V}{V_b}, 1\right), d\left(\frac{V_b}{V}, 1\right), 0, 1\right) \\ & - \frac{\frac{\lambda(P-R_\lambda)-C_s}{r+\lambda} + R}{(V_b)^q} e^{(-(r+\lambda)+n(q))(T-t)} \cdot \Omega\left(d\left(\frac{V}{V_b}, q\right), d\left(\frac{V_b}{V}, q\right), q, q\right) \end{aligned} \tag{17}$$

Figure 1 shows the model predicted term structure of credit spreads. The figure shows that the advantage of allowing both expected and unexpected default to occur is that short term credit spreads are realistically high. The figure assumes a base case with $V = 100$, $V_b = 30$, $b = 5\%$, $\sigma = 20\%$, $r = 4\%$, $T = 10$, $F = 30$, $C = 5\% \cdot F$, $\lambda = 0.25\%$, $R_\lambda = 0.5$ and $R = 0.5$. Notice that if $\lambda = 0.25\%$ the unexpected drop in assets value takes place every $\frac{1}{\lambda} = 400$ years on average.

The figure shows that as λ increases so that unexpected default becomes more likely, credit spreads on even high grade bonds widen. Such are the bonds for which structural models usually predict too low short term credit spreads.

So far a flat and constant term structure of default free interest rates has been assumed. Now this assumption is relaxed.

2.1 When default free interest rates are stochastic

A closed form solutions for corporate bonds valuation is now presented when the term structure of interest rates is stochastic and, as before, default can be either expected or unexpected. In keeping with other structural models that assume a stochastic default free term structure, this section assumes that debt holders recover a fraction of the default free value of the cash flows promised by the bond ("recovery of Treasury" assumption, see e.g. Longstaff and Schwartz (1995)).

In this section we also assume that the market in incomplete, so that the stochastic dynamics of the firm's assets value cannot be replicated by trading in market securities. If follows that now the risk neutral process for V is

$$dV = V \cdot (m - \lambda_V \cdot s) \cdot dt + V \cdot s \cdot dz \tag{18}$$

where λ_V is a constant that denotes the market price of V risk. Such risk neutral process is assumed also in Cathcart and El-Jahel (2003). Moreover, the risk neutral process for r is now assumed to follow a generic process

$$dr = u\left(r,t\right) + \sigma\left(r,t\right) dz_r$$

where $u\left(r,t\right)$ and $\sigma\left(r,t\right)$ are continuously differetiable functions of the short default free interest rate r and of time t, and where dz_r is the differential of a Wiener process. V and r are instantaneously uncorrelated, i.e. $E\left(dV \cdot dr\right) = 0$ or $E\left(dz \cdot dz_r\right) = 0$, as is assumed also in Cathcart and El-Jahel (2003). Then, given the "recovery of Treasury" assumption, the pricing equation for the value of a zero coupon bond $D(V,r,t)$ with face value of 1 now satisfies the following equation

$$\frac{dD(V,r,t)}{dt}+\frac{d^2D(V,r,t)}{dV^2}s^2V^2+\frac{d^2D(V,r,t)}{dV^2}r\sigma^2+\frac{dD(V,r,t)}{dV}(m-\lambda_V\cdot s)\cdot V+ \tag{19}$$

$$+\frac{dD(V,r,t)}{dr}\alpha(\mu-r)-(r+\lambda)D(V,r,t)+\lambda(1-a)Z(r,t)=0$$

$$D(V\rightarrow\infty,r,t)\rightarrow Z'(r,t) \tag{20}$$

$$D(V_b,r,t)=(1-a)\cdot Z'(r,t) \tag{21}$$

$$D(V,r,T)=1 \tag{22}$$

where $Z'(r,t)$ is the price of a defaultable zero coupon bond when default can be just unexpected. In other words $Z'(r,t)=e^{-\lambda(T-t)}\cdot Z(r,t)$, where $Z(r,t)$ is the price of a default free zero coupon bond, which we can leave unspecified for our purposes. The solution to equation 19 and to the associated boundary conditions is

$$D(V,r,t)=e^{-\lambda(T-t)}\cdot Z(r,t)\cdot H(V,t)$$

$$H(V,t)=H(V)-O(H(V),t)+O(1,t) \tag{23}$$

$$H(V)=\frac{\lambda(1-a)}{r+\lambda}-\left(\frac{\lambda(1-a)}{r+\lambda}\right)\left(\frac{V}{V_b}\right)^q \tag{24}$$

$$q=\frac{-\left(m-\lambda_V\cdot s-\frac{1}{2}s^2\right)-\sqrt{\left(m-\lambda_V\cdot s-\frac{1}{2}s^2\right)^2+2\lambda s^2}}{s^2} \tag{25}$$

$$\begin{aligned} O\left(H\left(V\right),t\right) &= \frac{\lambda\left(1-a\right)}{r+\lambda}\cdot\Omega\left(d\left(\frac{V}{V_b},1\right),d\left(\frac{V_b}{V},1\right),0,1\right) \\ &+ \frac{-\frac{\lambda(1-a)}{r+\lambda}}{\left(V_b\right)^q}e^{n(q)(T-t)}\cdot\Omega\left(d\left(\frac{V}{V_b},q\right),d\left(\frac{V_b}{V},q\right),q,q\right) \end{aligned} \tag{26}$$

$$O_1\left(V,t\right)=\Omega\left(d\left(\frac{V}{V_b},1\right),d\left(\frac{V_b}{V},1\right),0,1\right) \tag{27}$$

It follows that the credit spread on a zero coupon bond is

$$cs=\lambda-\frac{\ln H\left(V,t\right)}{\left(T-t\right)} \tag{28}$$

These formulae give the price of a corporate zero coupon bond, but are relevant also to the valuation of a coupon bond, since a coupon bond is equivalent to a portfolio of zero coupon bonds and can be valued accordingly. Note that in such case coupons are assumed to be discretely paid rather than continuously paid. This model is a special case of Cathcart and El-Jahel (2003), whereby the default intensity λ is a constant rather than a function of the default free short rate r. But, unlike in Cathcart and El-Jahel, here we have a closed form solution for bond valuation. Cathcart and El-Jahel (2003) have already shown the credit spreads predicted by this type of model.

So far the recovery value of the bond has been exogenous to the model. Next such recovery value is made endogenous to the model.

3 The bond valuation model with endogenous recovery

So far the recovery value of the defaulted bond has been assumed to be exogenous, whereas now it is assumed to be endogenous to the model and to depend on the firm's assets value at the time of default, both when default is expected and unexpected. The focus is on the bond recovery value after unexpected default and the analysis is confined to a flat term structure environment. Unexpected default may again take place with constant intensity λ, it occurs at a random time denoted by t_λ and is assumed to be associated with a sudden drop in the value of the firm's assets from V to $V(1-j)$, with $0 \leq j \leq 1$. Such drop at the time of unexpected default corresponds to the sudden stock price fall one often observes at default and may be due to a reassessment of firm value on part of investors. Asymmetric information between the firm and investors and incomplete accounting information may cause this phenomenon as investors suddenly re-adjust their estimates of the firm's value after the default event.

After unexpected default, the firm enters bankruptcy only if the drop in assets value renders the firm irreversibly insolvent. If the firm enters bankruptcy, it incurs bankruptcy costs that are a fraction a of $V(1-j)$ and debt value at t_λ would be

$$R_\lambda(V) = \min\left[V(1-j)(1-a), P\right]. \tag{29}$$

Since default and "cross default" debt provisions would accelerate debt maturity, the solvency condition at t_λ is

$$V(1-j)(1-a) \geq P. \tag{30}$$

This condition implies that the firm cannot issue new equity to be able to pay P. The firm can only liquidate the assets at a proportional cost of a or issue new debt. But the firm cannot issue new debt with a fair value of least P if the assets collateral value after default $V(1-j)(1-a)$ is less than P. Thus, only if $V \geq V_\lambda = \frac{P}{(1-j)(1-a)}$ creditors can recover the full face value P because the firm can refinance by issuing new debt and only if $V < V_\lambda$ would the firm be irreversibly insolvent after unexpected default, enter bankruptcy and incur bankruptcy costs.

Instead, debt value after expected default is

$$D(V_b) = \min\left(V_b(1-a), P\right) \tag{31}$$

and debt payoff at maturity T is

$$D(V_T, T) = \min\left[V_T(1-a), P\right].$$

Again, if $V_T \leq V_P = \frac{P}{(1-a)}$ the firm could not keep solvent by refinancing its debt through liquidation of its assets or issuance of new debt. Again the firm cannot issue new equity. Typically V_P would be higher than V_b.

Employing standard valuation arguments (see e.g. Wilmott 1998 at chapter 26) and assuming that the value of the firm's assets follows the process of equation 1 we can now re-write the debt pricing equation as

$$\frac{dD(V,t)}{dt} + \frac{d^2D(V)}{dV^2}s^2V^2 + \frac{dD(V)}{dV}(r-b+\lambda j)V - (r+\lambda)D(V) + C + \lambda R_\lambda(V) = 0 \tag{32}$$

$$D(V_T,T) = V_T(1-a)\cdot 1_{V_T \le V_P} + P \cdot 1_{V_T > V_P} \tag{33}$$

$$D(V_b,t) = \min(V_b(1-a), P) \tag{34}$$

$$D(V\to\infty,t) \to \frac{C+\lambda P}{r+\lambda}\left(1-e^{-(r+\lambda)(T-t)}\right) + e^{-(r+\lambda)(T-t)}P \tag{35}$$

This is the same model as in section one, but the recovery value after unexpected default is now endogenous. The solution to 32 is:

$$D(V,t) = D(V) - O_{D_h}(V,t) - O_{D_l}(V,t) + O_P(V,t) \tag{36}$$

$$D(V) = D_h(V)\cdot 1_{V>V_P} + D_l(V)\cdot 1_{V\le V_P} \tag{37}$$

where $1_{x>y}$ is the indicator function such that, $1_{x>y} = 1$ if $x > y$ and $1_{x>y} = 0$ if $x \le y$;

$$D_h\left(V\right)=\frac{\lambda P+C}{r+\lambda}+\left(-\frac{\lambda P+C}{r+\lambda}+D_l\left(V_\lambda,t\right)\right)\left(\frac{V}{V_\lambda}\right)^{q_2} \tag{38}$$

$$D_l\left(V\right)=\frac{\lambda\left(1-a\right)\left(1-j\right)V+C}{b+\lambda}+k_1V^{q_1}+k_2V^{q_2} \tag{39}$$

$$q_1 = \frac{-\left(r-b+\lambda j-s^2\right)+\sqrt{\left(r-b+\lambda j-s^2\right)^2+4s^2\left(r+\lambda\right)}}{2s^2} \tag{40}$$

$$q_2 = \frac{-\left(r-b+\lambda j-s^2\right)-\sqrt{\left(r-b+\lambda j-s^2\right)^2+4s^2\left(r+\lambda\right)}}{2s^2} \tag{41}$$

$$\begin{aligned}O_{D_h}\left(V,t\right)&=\frac{\lambda+C}{r+\lambda}e^{-(r+\lambda)(T-t)}\cdot\Omega\left(d_-\left(\frac{V}{V_b}\right),d_-\left(\frac{V_b^2}{V\cdot V_\lambda}\right),0,1\right) \qquad (42)\\ &+\frac{\left(-\frac{\lambda+C}{r+\lambda}+D_l\left(V_\lambda,t\right)\right)}{\left(V_\lambda\right)^{q_2}}\cdot e^{(-(r+\lambda)+n(q_2))(T-t)}\cdot\Omega\left(d\left(\frac{V}{V_b},q_2\right),d\left(\frac{V_b^2}{V\cdot V_\lambda},q_2\right),q_2,q_2\right)\end{aligned}$$

$$O_{D_l}(V,t) = \frac{C}{b+\lambda} e^{-(r+\lambda)(T-t)} \cdot \tag{43}$$

$$\left[\Omega\left(d_-\left(\frac{V}{V_b}\right), d_-\left(\frac{V_b^2}{V \cdot V_P}\right), 0, 1\right) - \Omega\left(d_-\left(\frac{V}{V_b}\right), d_-\left(\frac{V_b^2}{V \cdot V_\lambda}\right), 0, 1\right)\right] \tag{44}$$

$$+\frac{\lambda(1-a)(1-j)}{b+\lambda} e^{-\lambda(T-t)} \cdot$$

$$\left[\Omega\left(d\left(\frac{V}{V_b}, 1\right), d\left(\frac{V_b^2}{V \cdot V_P}, 1\right), 1, 1\right) - \Omega\left(d\left(\frac{V}{V_b}, 1\right), d\left(\frac{V_b^2}{V \cdot V_\lambda}, 1\right), 1, 1\right)\right] \tag{45}$$

$$+k_1 \cdot e^{(-(r+\lambda)+n(q_1))(T-t)} \cdot$$

$$\left[\Omega\left(d\left(\frac{V}{V_b}, q_1\right), d\left(\frac{V_b^2}{V \cdot V_P}, q_1\right), q_1, q_1\right) - \Omega\left(d\left(\frac{V}{V_b}, q_1\right), d\left(\frac{V_b^2}{V \cdot V_\lambda}, q_1\right), q_1, q_1\right)\right] \tag{46}$$

$$+k_2 \cdot e^{(-(r+\lambda)+n(q_2))(T-t)} \cdot$$

$$\left[\Omega\left(d\left(\frac{V}{V_b}, q_2\right), d\left(\frac{V_b^2}{V \cdot V_P}, q_2\right), q_2, q_2\right) - \Omega\left(d\left(\frac{V}{V_b}, q_2\right), d\left(\frac{V_b^2}{V \cdot V_\lambda}, q_2\right), q_2, q_2\right)\right] \tag{47}$$

$$O_P(V,t) = P \cdot e^{-(r+\lambda)(T-t)} \cdot \Omega\left(d\left(\frac{V}{V_b}, 1\right), d\left(\frac{V_b^2}{V \cdot V_P}, 1\right), 0, 1\right) \tag{48}$$

$$+e^{-\lambda(T-t)} \cdot (1-a) \cdot \tag{49}$$

$$\left[\Omega\left(d\left(\frac{V}{V_b}, 1\right), d\left(\frac{V_b}{V}, 1\right), 1, 1\right) - \Omega\left(d\left(\frac{V}{V_b}, 1\right), d\left(\frac{V_b^2}{V \cdot V_P}, 1\right), 1, 1\right)\right] \tag{50}$$

where

$$k_1 = \frac{-k_2 V_b^{q_2} + (1-a)V_b - \frac{\lambda(1-a)(1-j)}{b+\lambda} V_b}{V_b^{q_1}} \tag{51}$$

$$k_2 = \frac{\left(-\frac{\lambda P}{r+\lambda} + D_l(V_\lambda, t)\right) q_2 \frac{1}{V_\lambda} - \frac{\lambda(1-a)(1-j)}{b+\lambda} - \frac{-k_2 V_b^{q_2} + (1-a)V_b - \frac{\lambda(1-a)(1-j)}{b+\lambda} V_b}{V_b^{q_1}} q_1 V^{q_1 - 1}}{q_2 V^{q_2 - 1}} \tag{52}$$

and where $d_{-}(x) = \frac{\ln(x)+\left(r-b+\lambda j+\frac{1}{2}s^2\right)(T-t)}{s\sqrt{T-t}}$ and where $V_\lambda = \frac{P}{(1-j)(1-a)}$.

We could reinterpret this model also as follows. At t_λ assets value falls because accounting mis-givings are detected and the true assets value becomes known to the market. This event gives debt holders the right to accelerates debt maturity, which does precipitate the firm's insolvency if the firm cannot refinance by issuing new debt in order to pay P.

3.1 Comparative statics

Comparative statics using the model in this section are summarised in Table 1. The base case scenario assumes $V = 100$, $V_b = 30$, $b = 5\%$, $\sigma = 20\%$, $r = 4\%$, $T = 10$, $F = 30$, $C = 5\% \cdot F$, $a = 20\%$, $\lambda = 0.25\%$, $j = 0.7$.

In the base case credit spreads are very low but they short term spreads and long term spreads are similar in magnitude. The third and fourth columns show how the credit spreads increase as V lowers to 90 or to 80, ceteris paribus. Notice how short term spreads do not vanish even for short maturities. The fifth and sixth columns show how the magnitude of the unexpected default intensity λ drives the level of short term credit spreads. When $\lambda = 0$, short term spreads vanish because unexpected default is ruled out. The two rightmost columns show that a dramatic downwards jump in assets value is required to prevent short term spreads form vanishing. When $j = 0.5$ short term spreads are much lower than long term spreads and in fact nearly disappear despite the possibility of unexpected default. This highlights that it is only an unexpected dramatic fall in assets value, even if very unlikely, that can boost short term spreads, i.e.

justify higher short term credit spreads than the spreads predicted by "classic" structural models. In fact, creditors can recover the full face value of debt if assets are valuable enough *after unexpected default* (i.e. if $V_\lambda > \frac{P}{(1-j)(1-a)}$), so that short term spreads for high credits are tend to zero despite possible unexpected default.

We can conclude that when the bond recovery value is endogenous in that it is a function of the firm's assets value, unexpected default can explain realistically high short term credit spreads for low grade debtors, but less so for high grade debtors. If the observed short term yield spreads on high grade corporate bonds are compensation for credit risk, investors must gauge that unexpected default is possible, even if it is unlikely, and that it is associated with an exceptional drop in assets value.

3.2 Random jump in assets value and random bankruptcy costs

The model of this section can be easily adapted if the size of the assets value jump j is random and distributed according to a discrete probability distribution. Suppose j is distributed such that $j = i$ with probability $p(i)$, i is such that $0 \leq i \leq n \leq 1$ and $\sum_{i=0}^{n} p(i) = 1$. Then, denoting with $D(V, t, j^*)$ the bond value when j is distributed as just described, from the concavity of $R_\lambda(V)$ with respect to j it follows that

$$D(V, t, j^*) = \sum_{i=0}^{n} p(i) \cdot D(V, t, i) \leq D(V, t, E(i)) \qquad (53)$$

where $D(V,t,i)$ is the bond value as per equation 36 when j is certain and equal to i and where $D(V,t,E(i))$ is the bond value when j is equal to the expected value $E(i)$ with certainty. Note that $E(i) = \sum_{i=0}^{n} i \cdot p(i)$. In a similar way it is possible to adapt the proposed model when the bankruptcy cost parameter a is random and the distribution for a is a discrete one.

Table 2 shows model predicted credit spreads as j is uniformly distributed such that $i = \frac{0}{100}, \frac{1}{100}, \frac{2}{100}, ..$up to $\frac{100}{100}$ and $p(i) = \frac{1}{101}$ for any i. The uniform distribution corresponds to maximum uncertainty about the magnitude of the drop in assets value. Table 2 assumes the same parameters as in Table 1 and shows how, when j is uniformly distributed, high short term credit spreads are possible and they are driven by the magnitude of the default intensity λ. It is the extreme uncertainty about j that causes credit spreads in Table 2 to be higher than in Table 1. The fifth column of Table 2 also shows that higher assets volatility can significantly increase credit spreads for bond maturities beyond one year or so.

Overall, the uncertainty about the magnitude of the unexpected jump in assets value can contribute to increase the model predicted credit spreads up to the empirically observed levels of yield spreads between corporate and Treasury bonds.

4 Conclusion

This paper has presented three variants of a structural model of credit risk in closed form, whereby both expected and unexpected defaults may take place. Closed form solutions have been provided for corporate bonds and credit default swaps, both when interest rates are constant and stochastic, and both when the recovery value of the bond after default is exogenous or endogenous to the model.

With exogenous bond recovery value, the model has the merit of predicting non negligible credit spreads even for short maturities and even for the best credits. This is true both under constant and under stochastic interest rates and the reason is that unexpected default increases credit spreads of all maturities. But this is often not true of short term credit spreads with endogenous bond recovery value.

When the bond recovery value is endogenous, unexpected default may be associated with a sudden significant drop in the firm's assets value, as is suggested by the sharp drop in stock price one often observes at the time of default. Asymmetric information between the firm and investors and less than transparent accounting information may explain this phenomenon, as the drop in assets value at default may correspond to a reassessment of firm value on part of investors. Such drop can render *a good credit economically insolvent* and cause the firm's assets to be insufficient to fully satisfy the claim of bondholders after default. But *such drop must be exceptionally large to render a good credit insolvent.* Overall, when recovery value is endogenous, unexpected default together with a simultaneous sudden and potentially large drop in the firm's assets value can explain why short term credit spreads should be higher than "classic" structural models of credit risk do predict. Moreover, extreme uncertainty about the magnitude of the potential plunge in assets value contributes to boost model predicted short term credit spreads.

High short term credit spreads seem plausible for low grade debtors, but less so for high grade debtors. If the observed short term yield spreads for high grade borrowers are compensation for credit risk, investors must believe there is a tiny probability, e.g. around 0.25% per annum, of a dramatic sudden plunge in assets value down to approximately the default barrier level.

A When interest rates are constant

The solution to equation 2 and to the respective boundary conditions is

$$D\left(V,t\right)=D\left(V\right)-O_{D}\left(V,t\right)+O_{P}\left(V,t\right) \tag{54}$$

where

$$\frac{dO_D\left(V,t\right)}{dt}+\frac{d^2O_D\left(V,t\right)}{dV^2}s^2V^2+\frac{dO_D\left(V,t\right)}{dV}\left(r-b\right)V-\left(r+\lambda\right)O_D\left(V,t\right)=0 \tag{55}$$

$$O_D\left(V,T\right)=D\left(V\right) \tag{56}$$

$$O_D\left(V_b,t\right)=0 \tag{57}$$

$$O_D\left(V\to\infty,t\right)\to e^{-(r+\lambda)(T-t)}D\left(V\right) \tag{58}$$

$$\frac{d^2D\left(V\right)}{dV^2}s^2V^2+\frac{dD\left(V\right)}{dV}\left(r-b\right)V-\left(r+\lambda\right)D\left(V\right)+\lambda R_\lambda+C=0 \tag{59}$$

$$O\left(V_b,t\right)=R \tag{60}$$

$$D\left(V\to\infty,t\right)\to\frac{\lambda R_\lambda+C}{r+\lambda} \tag{61}$$

$$\frac{dO_P\left(V,t\right)}{dt}+\frac{d^2O_P\left(V,t\right)}{dV^2}s^2V^2+\frac{dO_P\left(V,t\right)}{dV}\left(r-b\right)V-\left(r+\lambda\right)O_P\left(V,t\right)=0 \tag{62}$$

$$O_P\left(V,T\right)=P \tag{63}$$

$$O_P\left(V_b,t\right)=0 \tag{64}$$

$$O_P\left(V\to\infty,t\right)\to e^{-(r+\lambda)(T-t)}P \tag{65}$$

The solutions to the above partial differential equations and to the respective boundary conditions are the formulae reported in the text. In particular the solution for $O_D(V,t)$ is found as follows. First $D(V)$ is found to be

$$D(V) = \frac{\lambda R_\lambda + C}{r+\lambda} + \left(-\frac{\lambda R_\lambda + C}{r+\lambda} + R\right)\left(\frac{V}{V_b}\right)^q$$

with

$$q = \frac{-\left(r - b - \frac{1}{2}s^2\right) - \sqrt{\left(r - b - \frac{1}{2}s^2\right)^2 + 2(r+\lambda)s^2}}{s^2}$$

Put $D(V) = \frac{\lambda R + C}{r+\lambda} + AV^q$, with $A = \left(-\frac{\lambda R + C}{r+\lambda} + R\right) \cdot (V_b)^{-q}$. By applying Ito's lemma it can be shown that

$$d(D(V)) = d(AV^q) = n(q) \cdot AV^q \cdot dt + AV^q \cdot qs \cdot dz$$

where $n(w) = (r-b)w + \frac{1}{2}w(w-1)s^2$. Then it is known that the value at time t of a claim $Q(D(V), t, V_b, T)$ that pays $D(V)$ at time T if $V_T \geq V_{bT}$ and that pays nothing otherwise is:

$$Q(D(V), t, V_b, T) = Q(AV^q, t, V_{bT}, T) + Q\left(\frac{\lambda R_\lambda + C}{r+\lambda}, t, V_{bT}, T\right)$$

$$Q(AV^q, t, V_b, T) = A \cdot e^{(-r+n(q))(T-t)} \cdot V^q N\left(d\left(\frac{V}{V_{bT}}, q\right)\right)$$

$$Q\left(\frac{\lambda R_{\lambda}+C}{r+\lambda},t,V_b,T\right)=e^{-r(T-t)}\frac{\lambda R_{\lambda}+C}{r+\lambda}N\left(d\left(\frac{V}{V_{bT}},1\right)-s\sqrt{T-t}\right)$$

where $N(x)$ is the cumulative of the standard normal density with x as the upper limit of integration and $d(z,w)=\frac{w\ln(z)+\left(n(w)+\frac{1}{2}s^2w^2\right)(T-t)}{ws\sqrt{T-t}}$. Similarly, using results for valuing down-and-out barrier options (see e.g. Wilmott (1998) at page 192), the value of a claim that pays $D(V)$ at time T if $V_T\geq V_{bT}$ and if $V_t\geq V_b$ for any time $t<T$ and that pays nothing otherwise is:

$$Q(D(V),t,V_b,V_{bT},T)=Q(AV^q,t,V_b,V_{bT},T)+Q\left(\frac{\lambda R_{\lambda}+C}{r+\lambda},t,V_b,V_{bT},T\right)$$

$$\begin{aligned}&Q(AV^q,t,V_b,V_{bT},T)=Ae^{(-r+n(q))(T-t)}\\&\cdot\left(V^qN\left(d\left(\frac{V}{V_{bT}},q\right)\right)-\left(\frac{V}{V_b}\right)^{-2\frac{n(q)}{q\cdot s^2}}(V_b)^q\cdot N\left(d\left(\frac{(V_b)^2}{V\cdot V_{bT}},q\right)\right)\right)\end{aligned}$$

$$\begin{aligned}&Q\left(\frac{\lambda R_{\lambda}+C}{r+\lambda},t,V_b,V_{bT},T\right)=\frac{\lambda R_{\lambda}+C}{r+\lambda}e^{-r(T-t)}\\&\cdot\left(N\left(d\left(\frac{V}{V_{bT}},1\right)-s\cdot\sqrt{T-t}\right)-\left(\frac{V}{V_b}\right)^{1-2\frac{r-b}{s^2}}N\left(d\left(\frac{(V_b)^2}{V\cdot V_{bT}},1\right)-s\cdot\sqrt{T-t}\right)\right).\end{aligned}$$

Substituting a rearranging gives the formulae for $O_D(V,t)$ in the text given that $V_b = V_{bT}$.

B When interest rates are stochastic

The derivation of proposition 2 is as follows. Assume $Z'(r,t)$ is the price of a defaultable zero coupon bond whereby default can be just unexpected default.

Thus $Z'(r,t)$ satisfies

$$\frac{dZ'(r,t)}{dt} + \frac{d^2 Z'(r,t)}{dr^2} r\sigma^2 + \frac{dZ'(r,t)}{dr} \alpha(\mu - r) - (r+\lambda) Z'(r,t) = 0 \tag{66}$$

$$Z'(r,T) = 1 \tag{67}$$

$$Z'(r \to \infty, t) \to 0 \tag{68}$$

$$Z'(r \to 0, t) = finite. \tag{69}$$

$Z'(r,T) = e^{-\lambda(T-t)} \cdot Z(r,t)$, where $Z(r,t)$ is the value of a default free zero coupon bond as per Cox-Ingersoll-Ross (1985).

Then from equation 19 we can write $D(V,r,t) = H(V,t) \cdot Z'(r,t)$, where

$$\frac{dH(V,t)}{dt}+\frac{d^2H(V,t)}{dV^2}s^2V^2+\frac{dH(V,t)}{dV}(m-\lambda_x\cdot s)V+\lambda(1-a)=0 \quad (70)$$

$$H(V\to\infty,r,t)\to 1 \quad (71)$$

$$H(V_b,t)=(1-a) \quad (72)$$

$$H(V,T)=1 \quad (73)$$

The solution to this partial differential equations for $H(V,t)$ is derived as follows. First recognise that

$$H(V,t)=H(V)-O(H(V),t)+O_1(V,t) \quad (74)$$

where $H(V)$ is such that

$$\frac{d^2H(V,t)}{dV^2}s^2V^2+\frac{dH(V,t)}{dV}(m-\lambda_x\cdot s)V+\lambda(1-a)=0 \quad (75)$$

$$H(V\to\infty,r,t)\to 1 \quad (76)$$

$$H(V_b,t)=(1-a) \quad (77)$$

where $O(H(V),t)$ is the value of a claim that pays $H(V)$ at time T if $V_T\geq V_{bT}$ and if $V_t\geq V_b$ for any time $t<T$ and that pays nothing otherwise, and where $O_1(V,t)$ is such that

$$\frac{dO_1\left(V,t\right)}{dt}+\frac{d^2O_1\left(V,t\right)}{dV^2}s^2V^2+\frac{dO_1\left(V,t\right)}{dV}\left(m-\lambda_x\cdot s\right)V=0 \tag{78}$$

$$O_1\left(V\rightarrow\infty,r,t\right)\rightarrow 1 \tag{79}$$

$$O_1\left(V_b,t\right)=0 \tag{80}$$

$$O_1\left(V,T\right)=1 \tag{81}$$

$O_1\left(V,t\right)$ is the value of a digital barrier option, whose closed form solution is well-known and is reported in the text.

Then the solution for $H\left(V\right)$ is

$$H\left(V\right)=\frac{\lambda\left(1-a\right)}{r+\lambda}-\left(\frac{\lambda\left(1-a\right)}{r+\lambda}\right)\left(\frac{V}{V_b}\right)^{q_h} \tag{82}$$

$$q_h=\frac{-\left(m-\lambda_V\cdot s-\frac{1}{2}s^2\right)-\sqrt{\left(m-\lambda_V\cdot s-\frac{1}{2}s^2\right)^2+2\lambda s^2}}{s^2}. \tag{83}$$

Then, from Appendix 1 the value of a claim that pays $H\left(V\right)$ at time T if $V_T\geq V_{bT}$ and if $V_t\geq V_b$ for any time $t<T$ and that pays nothing otherwise is:

$$Q\left(H\left(V\right),t,V_b,V_{bT},T\right)=Q\left(\frac{\lambda\left(1-a\right)}{r+\lambda},t,V_b,V_{bT},T\right)-Q\left(A'V^q,t,V_b,V_{bT},T\right)$$

with $A'=-\frac{\lambda(1-a)}{r+\lambda}\frac{1}{(V_b)^{q_h}}$, with

$$Q\left(A'V^q, t, V_b, V_{bT}, T\right) = A' \cdot e^{(-r+n_h(q_h))(T-t)}$$
$$\cdot \left(V^{q_h} N\left(d\left(\frac{V}{V_{bT}}, q_h\right)\right) - \left(\frac{V}{V_b}\right)^{-2\frac{n_h(q_h)}{q_h \cdot s^2}} (V_b)^{q_h} \cdot N\left(d\left(\frac{(V_b)^2}{V \cdot V_{bT}}, q_h\right)\right)\right)$$

with $n_h(q_h) = (m - \lambda_V \cdot s) \cdot q_h + \frac{1}{2} q_h (q_h - 1) \cdot s^2$, with

$$Q\left(\frac{\lambda(1-a)}{r+\lambda}, t, V_b, V_{bT}, T\right) = \frac{\lambda(1-a)}{r+\lambda} \cdot e^{-r(T-t)}$$
$$\cdot \left(N\left(d\left(\frac{V}{V_{bT}}, 1\right) - s \cdot \sqrt{T-t}\right) - \left(\frac{V}{V_b}\right)^{1-2\frac{r-b}{s^2}} N\left(d\left(\frac{(V_b)^2}{V \cdot V_{bT}}, 1\right) - s \cdot \sqrt{T-t}\right)\right)$$

and with $V_b = V_{bT}$. Substitutions and re-arrangements give the formula for $O(H(V), t)$ in the text.

C When debt recovery value is endogenous

The solution to equation 32 is found as follows. First recognise that

$$D(V,t) = D(V) - O_{D_h}(V,t) - O_{D_l}(V,t) + O_P(V,t) \tag{84}$$

where

$$D(V) = D_h(V) \cdot 1_{V > V_\lambda} + D_l(V) \cdot 1_{V \le V_\lambda} \tag{85}$$

and $D_h(V)$ and $D_l(V)$ are such that

$$\frac{d^2 D_h(V)}{dV^2} s^2 V^2 + \frac{dD_h(V)}{dV}(r - b + \lambda j) V - (r+\lambda) D_h(V) + \lambda P + C = 0 \tag{86}$$

$$\frac{d^2 D_l(V)}{dV^2} s^2 V^2 + \frac{dD_l(V)}{dV}(r - b + \lambda j) V - (r+\lambda) D_l(V) + \lambda(1-a)(1-j) V + C = 0 \tag{87}$$

subject to

$$\lim_{V\to\infty} D_h(V) \to \frac{\lambda P}{r+\lambda} \tag{88}$$

$$D_h(V_\lambda) = D_l(V_\lambda, t) \tag{89}$$

$$\left[\frac{dD_h(V)}{dV}\right]_{V=V_\lambda} = \left[\frac{dD_l(V)}{dV}\right]_{V=V_\lambda} \tag{90}$$

$$D_l(V_b) = (1-a) V_b \tag{91}$$

with $V_\lambda = \frac{P}{(1-j)(1-a)}$. The solutions to the above ODE's are:

$$D_h(V) = \frac{\lambda P + C}{r+\lambda} + \left(-\frac{\lambda P + C}{r+\lambda} + D_l(V_\lambda, t)\right)\left(\frac{V}{V_\lambda}\right)^{q_2} \tag{92}$$

$$D_l(V) = \frac{\lambda(1-a)(1-j)V + C}{b+\lambda} + k_1 V^{q_1} + k_2 V^{q_2} \tag{93}$$

with

$$
\begin{aligned}
q_{1,2} &= \frac{-\left(r-b+\lambda j-s^2\right) \pm \sqrt{\left(r-b+\lambda j-s^2\right)^2+4s^2\left(r+\lambda\right)}}{2s^2} \\
k_1 &= \frac{-k_2 V_b^{q_2} + (1-a)\,V_b - \frac{\lambda(1-a)(1-j)}{b+\lambda} V_b}{V_b^{q_1}} \\
k_2 &= \frac{\left(-\frac{\lambda P}{r+\lambda} + D_l\left(V_\lambda, t\right)\right) q_2 \frac{1}{V_\lambda} - \frac{\lambda(1-a)(1-j)}{b+\lambda} - \frac{-k_2 V_b^{q_2} + (1-a)V_b - \frac{\lambda(1-a)(1-j)}{b+\lambda} V_b}{V_b^{q_1}} q_1 V^{q_1 - 1}}{q_2 V^{q_2-1}}.
\end{aligned}
$$

Then, the value of a claim that pays $\min\left(P, V\left(1-a\right)\right)$ at time T if $V_t \geq V_b$ for any time $t < T$ and that pays nothing otherwise is:

$$
O_P\left(V,t\right) = P \cdot e^{-(r+\lambda)(T-t)} \cdot \Omega\left(d\left(\frac{V}{V_b},1\right), d\left(\frac{V_b^2}{V \cdot V_P},1\right),0,1\right) \tag{94}
$$

$$
+e^{-\lambda(T-t)} \cdot (1-a) \cdot \tag{95}
$$

$$
\left[\Omega\left(d\left(\frac{V}{V_b},1\right), d\left(\frac{V_b}{V},1\right),1,1\right) - \Omega\left(d\left(\frac{V}{V_b},1\right), d\left(\frac{V_b^2}{V \cdot V_P},1\right),1,1\right)\right] \tag{96}
$$

Then, the value of a claim that pays $D_h\left(V\right)$ at time T if $V_T \geq V_P$ and if $V_t \geq V_b$ for any time $t < T$ and that pays nothing otherwise is:

$$
O_{D_h}\left(V,t\right) = \frac{\lambda + C}{r+\lambda} e^{-(r+\lambda)(T-t)} \cdot \Omega\left(d_-\left(\frac{V}{V_b}\right), d_-\left(\frac{V_b^2}{V \cdot V_\lambda}\right),0,1\right) \tag{97}
$$

$$
+ \frac{\left(-\frac{\lambda+C}{r+\lambda} + D_l\left(V_\lambda,t\right)\right)}{\left(V_\lambda\right)^{q_2}} \cdot e^{\left(-(r+\lambda)+n(q_2)\right)(T-t)} \cdot \Omega\left(d\left(\frac{V}{V_b},q_2\right), d\left(\frac{V_b^2}{V \cdot V_\lambda},q_2\right),q_2,q_2\right)
$$

Then, the value of a claim that pays $D_l\left(V\right)$ at time T if $V_\lambda \geq V_T \geq V_P$ and if $V_t \geq V_b$ for any time $t < T$ and that pays nothing otherwise is:

$$O_{D_l}(V,t) = \frac{C}{b+\lambda} e^{-(r+\lambda)(T-t)} \cdot \tag{98}$$

$$\left[\Omega\left(d_-\left(\frac{V}{V_b}\right), d_-\left(\frac{V_b^2}{V \cdot V_P}\right), 0, 1\right) - \Omega\left(d_-\left(\frac{V}{V_b}\right), d_-\left(\frac{V_b^2}{V \cdot V_\lambda}\right), 0, 1\right)\right] \tag{99}$$

$$+\frac{\lambda(1-a)(1-j)}{b+\lambda} e^{-\lambda(T-t)} \cdot$$

$$\left[\Omega\left(d\left(\frac{V}{V_b}, 1\right), d\left(\frac{V_b^2}{V \cdot V_P}, 1\right), 1, 1\right) - \Omega\left(d\left(\frac{V}{V_b}, 1\right), d\left(\frac{V_b^2}{V \cdot V_\lambda}, 1\right), 1, 1\right)\right] \tag{100}$$

$$+k_1 \cdot e^{(-(r+\lambda)+n(q_1))(T-t)} \cdot$$

$$\left[\Omega\left(d\left(\frac{V}{V_b}, q_1\right), d\left(\frac{V_b^2}{V \cdot V_P}, q_1\right), q_1, q_1\right) - \Omega\left(d\left(\frac{V}{V_b}, q_1\right), d\left(\frac{V_b^2}{V \cdot V_\lambda}, q_1\right), q_1, q_1\right)\right] \tag{101}$$

$$+k_2 \cdot e^{(-(r+\lambda)+n(q_2))(T-t)} \cdot$$

$$\left[\Omega\left(d\left(\frac{V}{V_b}, q_2\right), d\left(\frac{V_b^2}{V \cdot V_P}, q_2\right), q_2, q_2\right) - \Omega\left(d\left(\frac{V}{V_b}, q_2\right), d\left(\frac{V_b^2}{V \cdot V_\lambda}, q_2\right), q_2, q_2\right)\right] \tag{102}$$

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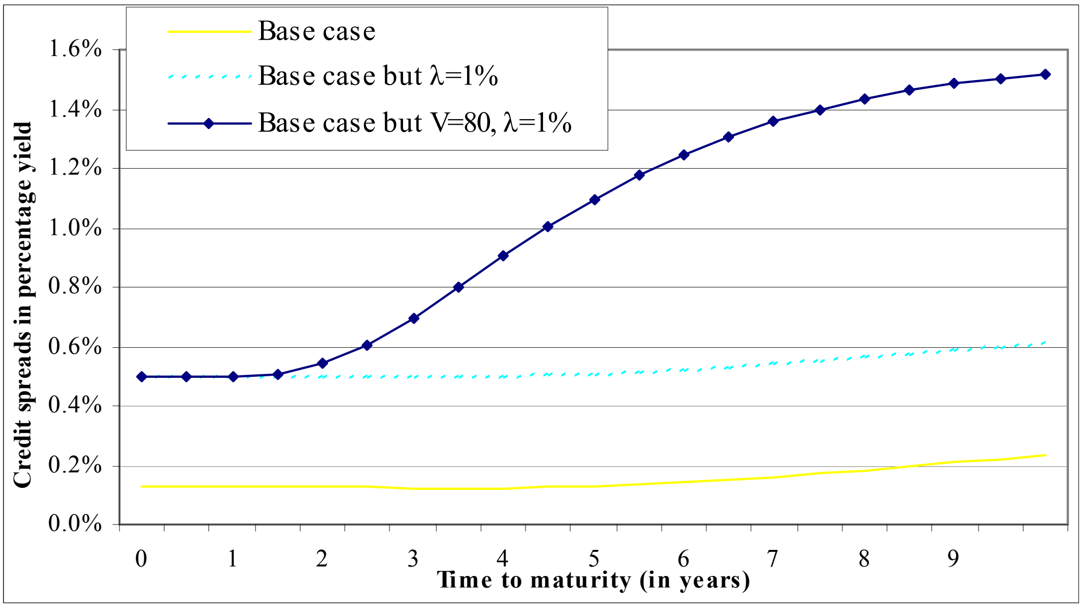
Base case
Base case but λ=1%
Base case but V=80, λ=1%
1.6%
1.4%
1.2%
1.0%
0.8%
0.6%
0.4%
0.2%
0.0%
Credit spreads in percentage yield
0
1
2
3
4
5
6
7
8
9
Time to maturity (in years)

Figure 1:

Maturity in years: (T-t)	Base case	Base case but V=90	Base case but V=80	Base case but V=80, λ=0	Base case but V=80, λ=1%	Base case but V=80, λ=1% and j=0.6	Base case but V=80, λ=1% and j=0.5
0.1	0.06%	0.07%	0.09%	0.00%	0.37%	0.16%	0.00%
0.5	0.06%	0.07%	0.09%	0.00%	0.37%	0.16%	0.02%
1.0	0.06%	0.07%	0.09%	0.00%	0.37%	0.17%	0.04%
1.5	0.06%	0.08%	0.10%	0.01%	0.38%	0.18%	0.06%
2.0	0.06%	0.08%	0.12%	0.03%	0.40%	0.20%	0.09%
2.5	0.06%	0.09%	0.15%	0.06%	0.43%	0.23%	0.12%
3.0	0.06%	0.11%	0.19%	0.10%	0.46%	0.27%	0.16%
3.5	0.06%	0.13%	0.23%	0.14%	0.49%	0.30%	0.21%
4.0	0.06%	0.14%	0.27%	0.18%	0.53%	0.34%	0.24%
4.5	0.06%	0.16%	0.30%	0.21%	0.55%	0.37%	0.28%
5.0	0.06%	0.18%	0.33%	0.25%	0.58%	0.40%	0.31%
5.5	0.06%	0.20%	0.36%	0.27%	0.60%	0.42%	0.34%
6.0	0.06%	0.21%	0.38%	0.30%	0.62%	0.44%	0.36%
6.5	0.07%	0.22%	0.40%	0.32%	0.64%	0.46%	0.38%
7.0	0.07%	0.23%	0.42%	0.34%	0.65%	0.47%	0.40%
7.5	0.08%	0.24%	0.44%	0.36%	0.66%	0.49%	0.42%
8.0	0.08%	0.25%	0.45%	0.37%	0.67%	0.50%	0.43%
8.5	0.08%	0.26%	0.46%	0.38%	0.68%	0.51%	0.44%
9.0	0.09%	0.27%	0.47%	0.39%	0.68%	0.51%	0.45%
9.5	0.09%	0.28%	0.48%	0.40%	0.68%	0.52%	0.46%
10.0	0.10%	0.28%	0.48%	0.41%	0.69%	0.52%	0.46%

Table 1

Maturity in years: (T-t)	Base case	Base case but V=90	Base case but V=80	Base case but V=80, s=30%	Base case but V=80, λ=0.5%	Base case but V=80, λ=1%
0.1	0.25%	0.25%	0.25%	0.25%	0.50%	1.00%
0.5	0.25%	0.25%	0.25%	0.25%	0.50%	1.00%
1.0	0.25%	0.25%	0.25%	0.34%	0.50%	1.00%
1.5	0.25%	0.25%	0.26%	0.50%	0.51%	1.01%
2.0	0.25%	0.26%	0.28%	0.67%	0.53%	1.02%
2.5	0.25%	0.27%	0.31%	0.80%	0.56%	1.05%
3.0	0.25%	0.28%	0.34%	0.89%	0.59%	1.09%
3.5	0.25%	0.30%	0.38%	0.95%	0.63%	1.12%
4.0	0.25%	0.32%	0.42%	0.99%	0.66%	1.15%
4.5	0.25%	0.34%	0.45%	1.00%	0.70%	1.18%
5.0	0.25%	0.35%	0.49%	1.01%	0.73%	1.21%
5.5	0.25%	0.37%	0.51%	1.01%	0.75%	1.23%
6.0	0.25%	0.38%	0.54%	1.01%	0.77%	1.25%
6.5	0.26%	0.39%	0.56%	1.00%	0.79%	1.27%
7.0	0.26%	0.41%	0.57%	0.99%	0.81%	1.28%
7.5	0.26%	0.42%	0.59%	0.98%	0.82%	1.29%
8.0	0.27%	0.42%	0.60%	0.97%	0.83%	1.30%
8.5	0.27%	0.43%	0.61%	0.96%	0.84%	1.30%
9.0	0.28%	0.44%	0.62%	0.95%	0.85%	1.31%
9.5	0.28%	0.44%	0.63%	0.94%	0.85%	1.31%
10.0	0.28%	0.45%	0.63%	0.93%	0.86%	1.31%

Table 2