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Macroeconomic Sources of FOREX Risk

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Abstract

This paper considers the problem of measuring macroeconomic sources of financial risk.

1. It aims to provide a general theory of asset pricing suitable for taking account of macroeconomic sources of risk. Stochastic discount factor theory is used to provide the theoretical framework. This is capable of embracing most of the approaches in the literature, including general equilibrium theory. Market structure needs to be added to this.

2. It is shown that many of the models used in the empirical literature of asset pricing have a fundamental flaw: they admit unlimited arbitrage opportunities. High profile suites of computer programs just produced and sold world-wide suffer the same problem, and hence should not be used.

3. Modelling the exchange rate is key to much of monetary policy (eg the Bank of England's Monetary Policy Committee), and to testing FOREX market efficiency. The forward premium puzzle lies at the heart of the difficulty of doing this. The theoretical results of this paper are used to re-examine the distribution of exchange rate movements and to try to resolve this puzzle.

Stochastic discount factor theory is used to derive expressions for the risk premia for domestic and foreign investors. It is shown that these are likely to be different. A combined theory of market risk when both types of investor are trading is then obtained. The cases of complete and incomplete markets are considered. It is shown how macroeconomic sources of risk can be introduced by modelling the stochastic discount factor using observable macroeconomic variables. Three SDF models are compared: a benchmark

model which provides a reformulation of traditional tests of FOREX efficiency; inter-temporal consumption-based CAPM; and the monetary model of the exchange rate, a familiar macroeconomic model of FOREX which can be interpreted as arising from traditional hedging concerns.

The joint distribution of the excess return to foreign exchange and the macro factors is specified in a way that satisfies the no-arbitrage assumption. It is assumed that the joint distribution has multivariate GARCH and it is shown that to eliminate arbitrage opportunities it is necessary for the conditional distribution of the excess return to exhibit GARCH-in-mean. The omission of the conditional covariance between the excess return and the sources of risk is the reason why nearly all financial statistical packages are not suitable for use in financial econometrics. The presence of this term implies that the analysis must be conducted in a multi-variate and not a uni-variate framework.

The theory admits the possibility that domestic and foreign investors may have different attitudes to risk. This is incorporated into the model by introducing a switching formulation of the conditional covariance structure. Extreme changes in exchange rates suggest that the usual assumption of log-normality may fail to capture the excess kurtosis of excess returns. The model is therefore also estimated assuming a log t-distribution. It is notoriously difficult to achieve convergence in multi-variate GARCH models, and GARCH-in-mean effects increase the difficulty. This is a major limitation in the practicality of the whole approach. It is shown that assuming constant correlation greatly simplifies the estimation without sacrificing any essential elements.

Tests are conducted to enable a comparison of different SDF models, different market structures, different attitudes to risk, and differences between the SDF model and the Fama approach. The empirical work is based on monthly data for the sterling-dollar exchange rate 1975-1997. Our main new finding is that the evidence is more consistent with the FOREX risk premium arising from traditional partial equilibrium models of currency risk that form the basis of hedging than with consumption-CAPM, a general equilibrium theory. In particular, US and UK output appear to be important sources of FOREX risk.

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1 Introduction

This paper considers how to measure macroeconomic sources of risk using stochastic discount factor theory to provide the asset pricing framework. This is implemented by modelling the joint distribution of the excess returns and the observable sources of macroeconomic risk using a multi-variate GARCH-in-mean framework. This approach is used to re-examine the FOREX risk premium puzzle using monthly data for 1975-1997 on the sterling-dollar exchange rate.

The aim is to obtain a direct estimate of the contribution of macroeconomic, and other variables, to an asset's risk premium. This approach is in contrast to the latent variable method familiar in the analysis of the term structure in which the factors are backed out of the estimation and then sometimes given a macroeconomic interpretation *ex post*. The advantage of being able to directly estimate macroeconomic sources of risk is that investors can then better assess whether to increase or decrease their exposure to macroeconomic events.

To achieve this a number of problems have to be addressed. First, a suitable theory of asset pricing is required. This should be capable of embracing both general equilibrium models of asset pricing and other theories. Stochastic discount factor (SDF) theory achieves this, and it can be used for observable as well as latent factors. Three SDF theories are compared: a benchmark model which provides a reformulation of traditional tests of FOREX efficiency; consumption-based CAPM, a standard general equilibrium model; and the monetary model of the exchange rate, a familiar macroeconomic model of FOREX. Traditional CAPM, which can also be given an SDF interpretation, forms the basis of standard hedging procedures where the aim is avoid pure currency risk. The monetary model identifies the sources of this risk. Another factor affecting the choice of model is the market structure. This depends on whether investors are dollar or sterling based, or whether both types of investors are participating; whether domestic and foreign investors have different attitudes to risk; and whether markets are complete or incomplete.

Second, the econometric model needs to be specified so that it does not admit arbitrage opportunities. Failure to do this is a common fault in empirical finance. Two of the most high-profile and recent financial statistical packages to be marketed do not contain a model that satisfies this requirement. According to SDF theory, a time-varying risk premium is due to time-varying covariation between excess returns (relative to the risk-free asset) and the (macroeconomic) factors. This means that to avoid arbitrage opportunities it is necessary to model the joint distribution of excess returns and the macroeconomic factors and for the conditional mean of the distribution of the excess returns to depend on the conditional covariances of the joint distribution. Thus a multi-variate, not a univariate, econometric model is required. A widely used way of modelling time-varying variances and covariances is through ARCH and GARCH processes. To eliminate arbitrage opportunities, it would then be necessary to use multi-variate GARCH-in-mean.

The third problem is that multi-variate GARCH-in-mean models are difficult to estimate. Possibly due to the large number of parameters, to sensitivity to starting values, or to extreme values it is hard to achieve convergence, and even when successful, estimation can be a lengthy task. To make this methodology a practical proposition it is necessary to seek models that do not sacrifice any of the essential features described above, yet are computationally viable. The constant conditional correlation model of Bollerslev (1992) seems to have this property and is used in this paper.

A limitation of this approach is the lack of availability of high frequency macroeconomic data. Very few macroeconomic time series are available on a monthly or weekly basis; most are available either quarterly or annually. This means that unless interpolation methods are used, there is a limit to the risk premia that it is possible to estimate in this way.

Instead of setting out the theory described above in general terms, we develop it in relation to a specific problem: the measurement of the sterling-dollar FOREX risk premium between 1975-1997. To give the application additional focus, we examine whether our measure of the FOREX risk premium is capable of resolving the FOREX risk premium puzzle. By including a measure of the risk premium in models of the excess return to foreign exchange, we attempt to eliminate the bias in the coefficient of the forward premium found in traditional tests of FOREX market efficiency.

The paper is set out as follows. In Section 2 we discuss the forward premium puzzle. In Section 3 we derive the FOREX risk premium using stochastic discount factors. We also consider the effect of the market structure on the risk premium. In Section 4 we show how to specify the arbitrage-free joint conditional distribution of the excess return to foreign exchange and the macroeconomic variables. The econometric model is a multi-variate GARCH-in-mean model with a constant conditional correlation matrix. In

Section 5 we describe the models that we estimate. These differ according to the SDF theory chosen and the market structure. To allow for the possibility that domestic and foreign investors have different attitudes to risk we introduce a switching formulation of the conditional covariance structure. Extreme changes in exchange rates suggest that the usual assumption of log-normality may fail to capture the excess kurtosis of excess returns. The model is therefore also estimated assuming a log t-distribution. Estimates based on the various different SDF models, different market structures and different attitudes to risk are reported in Section 5. The FOREX risk-premium puzzle is re-examined in Section 6. Our conclusions are presented in Section 7.

2 The forward premium puzzle

There is a conflict between theory and evidence on the behaviour the foreign exchange (FOREX) market that remains unresolved. The stylized facts of the FOREX market are not consistent with standard theories: they suggest that it is not efficient, and they imply an arbitrage opportunity that is highly implausible. It is still not clear whether these findings are due to omitting to take account of (or inadequately modelling) a time-varying FOREX risk premium, or to other causes such as a peso effect or non-rational expectations. In her comprehensive survey of these tests Lewis (1995, p1949), concluded that

"no risk premium model with believable measures of risk aversion has yet been able to generate the variability in predictable excess returns that are observed in the data."(p1949),

A similar conclusion was reached by Mark and Wu (1998) who found that the inter-temporal asset pricing model is unable to predict FOREX risk premia with the correct sign, and that survey expectations data gives only fragmentary support to the noise-trader model. Lewis argued that future research will need to integrate the various explanations for the rejection. Engel (1996) in his survey identified four general directions in which the literature might go forward. One of these was to extend the analysis of the risk premium.

We begin by explaining briefly what the forward premium puzzle is. This also serves to establish notation. Consider two countries (domestic and foreign) each of which issues a one-period bond that is risk-free in terms of its own currency. Let $R(t+1)$ denote the excess return to domestic investors from investing at time t in the foreign bond. Thus

$$R(t+1) = i^*(t) + \Delta s(t+1) - i(t) \tag{1}$$

where $i(t)$ and $i^*(t)$ are the domestic and foreign one-period nominal interest rates, respectively and $s(t)$ is the logarithm of the domestic price of foreign exchange. If investors are risk neutral and rational then the expectation of $R(t+1)$ conditional on information at time t is

$$E_t[R(t+1)] = 0 \tag{2}$$

the uncovered interest parity condition. But if investors are risk averse then

$$E_t[R(t+1)] = \phi(t) \tag{3}$$

where, for the moment, $\phi(t)$ will be given the interpretation of a risk premium. If only domestic investors are exposed to exchange risk (i.e. foreign investors hold only their own bond) then $\phi(t) \geq 0$. The sign is reversed if only foreign investors are exposed to exchange risk. More generally, $\phi(t)$ can be positive or negative depending on the relative magnitudes of these portfolio composition effects.

If the logarithm of the forward rate is denoted by

$$f(t) = s(t) + i(t) - i^*(t) \tag{4}$$

then the excess return can be written

$$\begin{aligned} R(t+1) &= s(t+1) - f(t) \\ &= \Delta s(t+1) - [f(t) - s(t)] \end{aligned} \tag{5}$$

where $f(t) - s(t)$ is the forward premium. If the rational expectations innovation is defined as

$$\varepsilon(t+1) = R(t+1) - E_t[R(t+1)] \tag{6}$$

then equation (3) can be expressed as

$$R(t+1) = \alpha + \beta[f(t) - s(t)] + e(t+1) \tag{7}$$

where

$$e(t+1) = \phi(t) + \varepsilon(t+1) \tag{8}$$

Equation (7) - or more commonly a variant in which $R(t+1)$ is replaced by $\Delta s(t+1)$ - is the basis of most tests of the efficiency of the foreign exchange market. Efficiency implies that $\alpha = \beta = 0$ and rationality implies that $E_t[\varepsilon(t+1)] = 0$. If, in addition, investors are risk neutral then $\phi(t) = 0$, and so $E_t[e(t+1)] = 0$. When all these assumptions hold the OLS estimators of α and β will be consistent.[1]

Using the convention that the currency is measured as US dollars per unit of domestic currency (i.e. US bonds are the domestic asset), the forward premium puzzle is that in practice the OLS estimate of β is negative. For the US dollar-sterling data set used in this paper we obtain the following estimates[2]

$$\begin{aligned} R(t+1) \quad &= \quad \underset{(2.24)}{-5.756} \; - \; \underset{(4.29)}{2.706} \; [f(t) - s(t)] \\ R^2 \quad &= \quad 0.063, \; LM^{AR} = 44.1, \; LM^{ARCH} = 18.6 \end{aligned}$$

This estimate of β of -2.7, is typical of those reported in the surveys of Lewis(1995) and Engel (1996). It will also be noted that the disturbances show significant serial correlation and heteroskedasticity.

Although theory predicts that the dollar will depreciate if the forward premium is positive, the implication of a negative value of β is that it will appreciate. Thus, instead of the interest differential $i - i^*$ compensating for an expected future exchange rate depreciation, this evidence implies that it is accompanied by an exchange rate appreciation. Or, put differently, the greater the interest differential of the foreign over the domestic bond $i^* - i$, the larger will be the excess return to doing so. In general, therefore, the appropriate investment strategy would be to hold the bond with the higher interest rate; the subsequent exchange change will usually reinforce

[1] A more familiar way of writing this model is in terms of $\Delta s(t+1)$ instead of $R(t+1)$ when equation (7) becomes

$$\Delta s(t+1) = \alpha + (\beta + 1)[f(t) - s(t)] + e(t+1)$$

[2] The data are described more fully below. They refer to the monthly US dollar- sterling exchange rate for the period 1975-1997.

this advantage. In practice, this would be bound to lead to destabilizing FOREX speculation; investing in the bond with the higher domestic currency return is therefore a one-way bet. The implausibility of the presence of such an arbitrage opportunity suggests that there must be another explanation.

One explanation is that the estimate of β is biased downwards due to the presence of the risk premium in the error term of the regression. This is also consistent with the finding of significant serial correlation and heteroskedasticity in the residuals. Assuming that the FOREX market is efficient and investors are rational, Fama (1984) has shown that the bias in β can be expressed as

$$\begin{aligned} bias &= cov[f(t) - s(t), \phi(t)]/var[f(t) - s(t)] \\ &= \rho \left[\frac{var[\phi(t)]}{var[f(t) - s(t)]} \right]^{\frac{1}{2}} \end{aligned} \tag{9}$$

where ρ is the correlation between $f(t) - s(t)$ and $\phi(t)$, (i.e. between the forward and risk premia). Negative bias implies, therefore, that $\rho < 0$. This can be interpreted as meaning that for US investors, the greater the expected depreciation of domestic currency, the lower is the required risk premium for holding foreign assets.

In effect, therefore, the horizontal line coincident with the x-axis that is predicted by theory is shifting up or down due to changes in the risk premium: and the greater the expected depreciation, the smaller the shift. Figure 1 is a scatter diagram of the excess return $R(t+1)$ against $f(t) - s(t)$ for the US dollar-sterling exchange rate for the data used in this study. The regression line of $R(t+1)$ on $f(t) - s(t)$ slopes negatively. This gives a rough idea of the order of magnitude of the shifts in the horizontal line that a time-varying risk premium would need to induce to account for the forward premium puzzle.

The puzzle is deepened by the fact that general equilibrium models of the risk premium typically do not produce a risk premium that is capable of generating the sorts of bias observed in practice. Since the estimate of β is typically negative and $\rho < 1$ the variance of the risk premium would need to be considerably greater than the variance of the forward premium. Since the maximum value of $\rho^2 = 1$, equation (9) implies that

$$\begin{aligned} var[\phi(t)] &= bias^2\, var[f(t) - s(t)]/\rho^2 \\ &\geq bias^2\, var[f(t) - s(t)] \end{aligned} \tag{10}$$

This gives a lower bound to the variance of $\phi(t)$, the equivalent of the Hansen-Jagannathan (1991) bound. For a typical bias of the order of -2, the variance of the risk premium would need to be a factor four greater than the variance of the forward premium.[3]

3 Stochastic Discount Factors

Instead of using standard SDF models in which the factors are unobservable latent variables, we pursue a different approach with wider potential in asset pricing. We assume that the SDF can be proxied by observable macroeconomic variables that are jointly distributed with the excess return on foreign exchange. The tests surveyed by Engel (1996) and Lewis (1995) and that carried out by Mark and Wu (1998) are based on a special case of the SDF model with observable factors, the inter-temporal consumption-based capital asset pricing model. Our more general framework enables us to examine a broader range of macroeconomic variables in a theoretically consistent way. Instead of using the familiar Cox-Ingersoll-Ross (CIR) model to describe the factors, we employ a vector GARCH-in mean model.

The attractions of the SDF model are: it is consistent with most theories of asset pricing, including general equilibrium models of asset pricing as special cases; it does not depend on an explicit specification of risk aversion; and it has the flexibility to generate the required degree of variability in the discount factor.[4] The best known of the SDF models is the Duffie-Kan (1996) class of affine models. This involves the use of unobservable factors which are extracted from the asset returns. SDF models of asset pricing are commonly used for the term structure. Papers that have used unobservable affine factor models for the term structure include Duffie and Kan (1996), Dai and Singleton (2000), Backus, Foresi and Telmer (1998, 2001) and Remolona, Wickens and Gong (1998).

The main problem in using SDF theory is how to model the discount factor. In a critique of SDF methodology Kan and Zhou (1999) argue that it ignores a fully specified model for asset returns and therefore the estimate of the risk premium is unreliable when asset returns follow a linear model. They

[3]For our estimate of equation (7) a lower bound to the proportion of the variation of $R(t+1)$ due to $\phi(t)$ is 6.5%(=$V[\phi]/V[R]$)

[4]Adetailed analysis of the use of SDF theory in asset pricing is provided by Cochrane (2001).

claim that traditional methodologies typically incorporate a fully specified model and perform substantially better. They also argue that specification tests have low power. These conclusions have been challenged by Cochrane (2000) who claims that they are due to giving traditional methodologies a false information advantage. He finds that the SDF and traditional methodologies behave almost identically in traditional linear i.i.d environments. A traditional theory like inter-temporal CAPM is also an SDF model. In this case there can be no difference between the two. Nonetheless, the choice of discount factor is arbitrary in the SDF model; it is not clear whether it is better to base the choice on traditional theories, other economic theories or purely statistical criteria. What is clear, and is shown later, is that a failure to correctly model the discount factor is likely to affect the results.

There are a number of new conceptual problems that arise in using the SDF model to price currency compared with pricing bonds. The aim is to price currency risk and hence derive the foreign exchange risk premium. Pure currency risk arises when the underlying assets are risk free in terms of their domestic currencies. When investors are risk-neutral, the arbitrage condition is given by uncovered interest parity. In this case there is no risk premium and the domestic and foreign investor is treated symmetrically. When investors are risk-averse, the currency risk premium for domestic investors may be different from that of the foreign investor. The relative size of domestic and foreign investors may also matter. In other words, there may be portfolio effects, and these could reflect differences in attitudes to risk between investors. In the case of complete markets these complications do not arise.

The use of the SDF model to price currency is new and there have been very few studies to date. Backus, Foresi and Telmer (2001) use the Duffie-Kan approach, not the observable SDF model. Hollifield and Yaron (1999) use a higher order expansion of the no–arbitrage condition with two observable factors (money and inflation) generated by a CIR model. They estimate the model using GMM on the moment condition. Hollifield and Yaron draw some interesting conclusions: the model must have significant real risk, and the monetary shocks should result in small inflation risk but lead to volatility in the real pricing kernel.

Prior to the use of factor models, most studies of the FOREX market were based on inter-temporal CAPM, a general equilibrium model. For example, Mark (1983) based a test of efficiency on the Euler condition and used GMM estimation. Implausibly large estimates of the coefficient of relative risk aversion (CRRA) were obtained, and the restrictions of the theory were rejected. But Kaminsky and Peruga (1990) adopted an approach to testing the general equilibrium model that is similar to the SDF model, and they employed a vector GARCH specification of the error structure. Their findings were similar to those of Mark in that they obtained an implausibly large estimate the CRRA, but they could not reject the theoretical restrictions. These two studies were based on monthly data. In commenting on Kaminsky and Peruga's findings, Baillie and Bollerslev (1991) argued that one reason for the weak results may be the lack of sufficient conditional heteroskedasticity in exchange rate data. Baillie and Bollerslev therefore used weekly data, and they allowed for moving average dynamics of the conditional mean of the excess return. They also used a univariate GARCH model for each variable from which they derived an estimate of the risk premium. Their findings were, however, similar to those of Kaminsky and Peruga in that all of the ARCH-in-mean effects were insignificant.

3.1 Theory

We consider how to obtain an expression for the foreign exchange risk premium using a version of the stochastic discount model based on observable, but stochastic, macroeconomic factors. These factors are jointly distributed with the excess return on foreign exchange, the only asset we consider. First we outline SDF theory as relevant for FOREX, we then take account of the measurement of the factors.

The SDF model can be expressed as

$$1 = E_t[M(t+1)(1 + i^*(t) + \Delta s(t+1))] \tag{11}$$

where $M(t)$ is the discount factor, or pricing kernel (see Singleton(1990)). In other words, $M(t+1)$ is the discount factor required to make the present value of the total income $(1 + i^*(t))$ from holding a foreign bond and converted to domestic currency equal to one unit of domestic curency. The only source of uncertainty here is the one-period ahead spot exchange rate as both $i(t)$ and $i^*(t)$ are known at time t. Taking logarithms of equation (11) and assuming log-normality gives

$$E_t[m(t+1)+i^*(t)+\Delta s(t+1)]+\frac{1}{2}V_t[m(t+1)+i^*(t)+\Delta s(t+1)] = 0 \tag{12}$$

where $m = \ln(M)$, and it is assumed that $\ln(1 + x) \approx x$ for small x. Hence, $-m$ is the discount rate. Replacing $R(t + 1)$ in equations (11) and (12) by the risk free rate $i(t)$ gives

$$E_t[m(t+1) + i(t)] + \frac{1}{2}V_t[m(t+1)] = 0 \tag{13}$$

Subtracting equation (13) from equation (12) gives

$$E_t[R(t+1)] + \frac{1}{2}V_t[R(t+1)] = -Cov_t[m(t+1), R(t+1)] \tag{14}$$

The last term on the left hand-side of equation (14) is the Jensen effect due to taking the expectations of a non-linear function of Normally distributed variables - i.e. the logarithm. The term of the right hand-side is the FOREX risk premium for the US investor. Comparing equation (14) with equation (3) implies that $\phi(t)$ is not in fact just the risk premium but is

$$\phi(t) = -\frac{1}{2}V_t[R(t+1)] - Cov_t[m(t+1), R(t+1)] \tag{15}$$

This implies that $\phi(t)$ will have a higher variance than the FOREX risk premium which will be of some assistance in helping to generate the additional variability required in $\phi(t)$.[5]

Because equation (14) involves conditional expectations, and given equation (5), it can be expressed in other ways, for example, as[6]

$$E_t[R(t+1)] + \frac{1}{2}V_t[\Delta s(t+1)] = -Cov_t[m(t+1), \Delta s(t+1)] \tag{16}$$

This shows explicitly that uncertainty about the future spot exchange rate is a necessary element in the risk premium. The larger the predicted covariance between the rate of appreciation of domestic currency and the discount rate, the smaller the risk premium of domestic investors holding foreign denominated assets. Although these domestic investors only suffer a loss when domestic currency appreciates, the larger the discount rate, the less this loss is.

[5] If logarithms are not taken, and the excess return is defined as $1 + R(t+1) = \frac{(1+i^*(t))S(t+1)}{(1+i(t))S(t)}$ then the arbitrage relation would be $E_t[R(t+1)] = -Cov_t[M(t+1), R(t+1)]$ which does not involve the Jensen effect.

[6] $\Delta s(t+1)$ could be replaced in equation (16) by $s(t+1)$.

It has been implicitly assumed that the risk is being borne by domestic investors through their holding of foreign bonds. This would imply that the discount factor is that appropriate for domestic investors. In practice, of course, foreign investors are exposed to the same FOREX risk in reverse.[7] Measuring returns and the discount factor in foreign currency would give

$$E_t[R^*(t+1)] + \frac{1}{2}V_t[R^*(t+1)] = -Cov_t[m^*(t+1), R^*(t+1)] \tag{17}$$

where $-m^*$ is the foreign investor's discount rate and is measured in foreign currency, and $R^* = -R$. This implies that for the UK investor the expected excess return is determind by

$$E_t[-R(t+1)] + \frac{1}{2}V_t[R(t+1)] = Cov_t[m^*(t+1), R(t+1)] \tag{18}$$

where $Cov_t[m^*(t+1), R(t+1)]$ is the risk premium.

Subtracting equation (18) from (14) gives

$$E_t[R(t+1)] = -Cov_t[\frac{1}{2}(m(t+1) + m^*(t+1)), R(t+1)] \tag{19}$$

Thus the Jensen effect disappears. The combined risk premium is the difference between the individual investor risk premia and is due to covariation between the average of the discount factors of the domestic and foreign investors and the excess return defined for the domestic investor (or, equivalently, $\Delta s(t+1)$). Adding equations (19) and (14) gives

$$\begin{aligned} V_t[R(t+1)] &= Cov_t[m^*(t+1), \Delta s(t+1)] \\ &\quad -Cov_t[m(t+1), \Delta s(t+1)] \end{aligned} \tag{20}$$

This implies that

$$\Delta s(t+1) = m^*(t+1) - m(t+1) + \eta(t+1) \tag{21}$$

[7] It is also possible for domestic investors to hold short positions. In this case the source of risk would be the same as that of foreign investors, though the discount factor would still be that of the domestic investor. We ignore this complication in our discussion. It is probable that a relatively small proportion of FOREX transactions are of this type.

with $Cov_t[\Delta s(t+1), \eta(t+1)] = 0$. Equation (20) reveals that there is a linear relation between $V_t[\Delta s(t+1)]$, $Cov_t[m^*(t+1), \Delta s(t+1)]$ and $Cov_t[m(t+1), \Delta s(t+1)]$ and only two terms are required, as in equation (19). There is an important proviso to this result. If, as is likely, there is measurement error in the proxy for the discount factor, then it will not hold in practice in the data. This is not a weakness of SDF theory *per se*, but an indication of the likely effects of modelling the discount factor incorrectly.

In the case of complete markets the two discount factors are identical when measured in the same currency, see Backus, Foresi and Telmer (2001). Hence

$$m^*(t+1) = m(t+1) + \Delta s(t+1) \tag{22}$$

This would imply that equations (16) and (18) are then identical.

3.2 SDF model with observable factors

In general the stochastic discount factor is not observable. We consider two cases where it can be expressed in terms of observable variables: an inter-temporal general equilibrium model, consumption-based CAPM , and a partial equilibrium model, traditional CAPM which is based on mean-variance analysis.

3.2.1 Consumption-based CAPM (C-CAPM)

In this case, as is well known, the SDF gives a the general equilibrium pricing kernel equal to the inter-temporal marginal rate of substitution. The precise form of the SDF depends on the choice of value function. For example, for the power utility function $U[C(t)] = [C(t)^{1-\sigma} - 1]/(1-\sigma)$ with coefficient of relative risk aversion σ, the SDF is

$$\begin{aligned} M(t+1) &= \delta \left[\frac{U'[C(t+1)]}{U'[C(t)]}\right] \frac{P(t)}{P(t+1)} \\ &= \delta \left[\frac{C(t+1)}{C(t)}\right]^{-\sigma} \frac{P(t)}{P(t+1)} \end{aligned} \tag{23}$$

where $C(t)$ is nominal consumption, and includes both domestic and foreign goods and services, $P(t)$ is the price level and δ is the rate of discount of utility. Taking logarithms gives

$$m(t+1) = \ln\delta - \sigma\Delta c(t+1) - \Delta p(t+1) \tag{24}$$

Thus, the risk premium for domestic investors in foreign bonds is smaller, the larger the predicted covariation between the appreciation of domestic currency and the rate of growth of consumption and the inflation rate.

3.2.2 CAPM

In traditional CAPM the value function is defined in terms of the mean and variance of financial wealth rather than consumption. For the two period problem this gives

$$M(t+1) = \sigma_t \frac{W_{t+1}}{W_t} = \sigma_t(1 + R_{t+1}^W) \tag{25}$$

where W_t is nominal financial wealth and R_{t+1}^W is the nominal return on wealth. The discount factors can be obtained from the variables that explain this portfolio return. As explained later, we assume that the portfolio consists of hedged and unhedged currency and so the element that is unknown in R_{t+1}^W is the future spot exchange rate. We then use the monetary model of the exchange rate to explain this, and hence to provide the macroeconomic factors.

3.2.3 The observable factors

These two models suggest a simple way to generalize the choice of discount rate. It can be assumed that $m(t)$ is a linear function of observable macroeconomic variables $z(t)$, namely

$$m(t+1) = \beta' z(t+1) + \xi(t+1) \tag{26}$$

$z(t+1)$ may include a constant. C-CAPM gives equation (26) which is a special case of (24) in which there are two macro factors, $\Delta c(t+1)$ and $\Delta p(t+1)$. The term $\xi(t)$ is included to represent the possibility that the macro factors do not capture $m(t)$ perfectly. It is assumed that $\xi(t)$ is orthogonal to $z(t)$. An alternative interpretation is that $z(t)$, a single variable, measures $m(t)$ with error. In this case $\xi(t)$ is correlated with $z(t)$ but not $m(t)$. The problem of how to choose $z(t)$ remains. Particular general equilibrium models will suggest a set of variables. The conjecture is that these variables can be approximated by equation (26) and hence, as a practical matter, the measurement of risk can be confined to including variables in this way.

Equation (14) can now be written

$$\begin{aligned} E_t[R(t+1)] + \frac{1}{2}V_t[R(t+1)] &= -\beta' Cov_t[z(t+1), R(t+1)] \\ &\quad - Cov_t[\xi(t+1), R(t+1)] \end{aligned} \tag{27}$$

The aim is to proxy the risk premium by the first term on the right hand-side of equation (27) and compute the conditional covariance from the joint conditional distribution of $\{R(t+1),\ z(t+1)\}$ together with the conditional variance of $R(t+1)$. β will need to be estimated, and the last term in equation (27) is ignored.

In general, the presence of $\xi(t)$ would mean that the last term in equation (27) is not zero; nor will the two conditional covariances be uncorrelated. Thus omitting this term will introduce a bias and reduce the power of tests of the model. This is related to Kan and Zhou's (1999) point. Only using the correct discount factor will avoid this bias. If, however, $\xi(t)$ is due to pure measurement error, then the last term may be expected to be zero. There will then be no bias, only estimation inefficiency.

4 Econometric model

The excess return is a function of three variables, the exchange rate and the two interest rates. After transforming these variables to stationarity we obtain the excess return, the forward premium and the change in one of the interest rates. The joint conditional distribution of three variables is the starting point for the econometric model. To these variables we can add others to help proxy the SDF.

We assume that the conditional covariance structure of the FOREX market can be closely approximated by ARCH. Thus we model the joint conditional distribution of asset returns by a vector autoregressive GARCH-in-mean process. This allows the conditional mean of the distribution to be affected by lagged levels and by the conditional covariance matrix, and it models the conditional covariance matrix by a multivariate GARCH process.[8]

This approach permits us to model the excess return and the macroeconomic factors jointly, and it can capture all of the features in the theory above. For the model to be consistent with the absence of arbitrage it is necessary to impose restrictions. These restrictions also provide a test of market efficiency. This will be broadly the correct way to specify models of asset prices when using ARCH. More generally, it will be necessary to use multivariate non-linear models with stochastic volatility, of which the CIR and Vasicek models are well-known univariate examples. The reason for including lagged variables (the VAR part of the model) is that we are able to obtain a better representation of the error terms. Also, by including the lagged forward premium in the equation for the excess return, we can directly observe whether, by including a measure of the risk premium, the forward premium becomes insignificant as SDF theory predicts.

We define the following vector of stationary variables $x(t+1) = \{R(t+1), z(t+1)'\}'$ and assume that it is generated by the vector autogressive GARCH-in-mean

$$\mathbf{x}(t+1) = \boldsymbol{\alpha} + \boldsymbol{\Gamma}\mathbf{x}(t) + \boldsymbol{\Phi}\mathbf{g}(t) + \varepsilon(\mathbf{t+1}) \tag{28}$$

where the distribution of $\varepsilon(t+1)$ conditional on information available at time t, $\Psi(t)$, is

$$\boldsymbol{\varepsilon}(\mathbf{t+1}) \mid \Psi(t) \sim N[0, \mathbf{H}(t+1)] \tag{29}$$

$$\mathbf{g}(t) = vech\{\mathbf{H}(\mathbf{t+1})\} \tag{30}$$

$$\mathbf{h}_{ij}(t+1) = \rho_{ij}[\mathbf{h}_{ii}(t+1) \times \mathbf{h}_{jj}(t+1)]^{\frac{1}{2}} \tag{31}$$

$$\mathbf{h}_{ii}(t+1) = v_i + a_i\mathbf{h}_{ii}(t) + b_i\boldsymbol{\varepsilon}_i(t)^2 \tag{32}$$

where for the variables $i, j = 1, ..., n$ each $\mathbf{h}_{ii}(t+1)$ has a GARCH(1,1) structure and ρ_{ij} is the (constant) correlation between $\boldsymbol{\varepsilon}_i(t+1)$ and $\boldsymbol{\varepsilon}_j(t+1)$. This model is the constant correlation multivariate GARCH-in-mean.

[8] For an extensive review of multi-variate ARCH models and alternative parameterizations see Bollerslev, Chou and Kroner (1992), Bollerslev, Engle and Nelson (1994) and Pagan (1996). And for a discussion of the specification of multi-variate ARCH models in financial models see Flavin and Wickens (1998).

The computational problems in estimating multivariate GARCH models are well documented, see for example Bollerslev (2001). The main difficulty is to achieve convergence of the associated likelihood function as multivariate GARCH models are sensitive both to extreme data points and starting values of the iteration. These problems are greatly exacerbated when there are also ARCH-in-mean effects, as here, and when exchange rate data are involved. We sought an econometric methodology that would meet our theoretical requirements, that would have wider applicability, and would prove computationally feasible. The constant correlation model of heteroskedasticity we chose is less general than desirable, for example, the multivariate GARCH model of the BEKK model described and generalized in Engle and Kroner (1995), but is supported by the results of Ding and Engle (1994) who find that it gives fairly good performance in comparison with the more general BEKK model. Even in implementing the correlation model we have found it necessary to employ a further simplification as described below.

The specification of the excess return equation in the model requires appropriate restrictions to avoid arbitrage possibilities. Assuming that $z(t)$ accurately approximates the stochastic discount factor, so that the last term of equation (27) can be ignored, the restrictions are given by equation (27). This determines the row of $\mathbf{\Phi}$ associated with $R(t+1)$. As, in general, β is unrestricted, the coefficients corresponding to the conditional covariances of the other variables with $R(t+1)$ will be unrestricted. Furthermore, since ρ_{ij} is constant, the conditional covariances in the mean of the excess return equation can be replaced by the product of the conditional standard deviation of the excess return with the conditional standard deviation of the other variables. The coefficients of the conditional covariances in the excess return equation are then $\phi_j = \rho_{1j}\beta_j$ for each variable j, where variable 1 is the excess return. As the ρ_{ij} are assumed constant, they can be consistently estimated by the corresponding unconditional correlation matrix of the variables. The choice of dependent variable and the explanatory variables that appear in the excess return equation depend on the model being estimated, as explained in the next section.

Instead of using a full information systems estimator, we proceeded in a sequence of steps. First we used a VAR estimator for the whole system. This gives a consistent estimator of the long-run variance for each variable and is used as the starting value for univariate GARCH(1,1) estimation for each variable. The GARCH(1,1) results were used as estimates of the conditional standard deviations of the macroeconomic variables. Since we do not need the conditional covariances themselves in the conditional mean of the excess return equation (only the product of the conditional standard deviations is required), the final step is to estimate the excess return equation on its own using quasi maximum likelihood estimation, conditional on these estimates of the conditional standard deviations of the macroeconomic variables. The resulting estimates will not be fully asymptotically efficient, but they will be consistent. The loss of efficiency requires further investigation, but we conjecture that this loss will not be large, and will be a small price for the considerable gain in the tractability of the procedure. Even using this approach, we found it necessary to add further restrictions to ensure that the conditional variance remained non-negative. The GARCH coefficients were constrained to be positive and their sum was constrained not to exceed unity, i.e. $a_1, b_1 > 0$, and $a_1 + b_1 < 1$.

5 Empirical models

The models estimated differ in the assumptions made about the structure of the FOREX market, and the hypotheses used to generate the stochastic discount factors. The differences in the market structures reflect whether the investor is assumed to be dollar or sterling based, whether both types of investor are trading FOREX, and whether markets are complete. Also, we estimate a more general model representing an alternative hypothesis not constrained to satisfy the no-arbitrage restriction.

5.1 Market structure

The four models of the excess return that reflect different assumptions about the market structure are

US investor

$$R(t+1) + \frac{1}{2}V_t[R(t+1)] = \phi^{us\prime}C_t^{us}(t+1) + \varepsilon_1(t+1)$$

UK investor

$$R(t+1) - \frac{1}{2}V_t[R(t+1)] = \phi^{uk\prime}C_t^{uk}(t+1) + \varepsilon_1(t+1)$$

US and UK investors

$$R(t+1) = \phi^{us\prime} C_t^{us}(t+1) + \phi^{uk\prime} C_t^{uk}(t+1) + \boldsymbol{\varepsilon}_1(t+1)$$

General alternative model

$$\begin{aligned} R(t+1) &= \gamma_1 R(t) + \gamma_2[f(t) - s(t)] + V_t[R(t+1)] \\ &\quad + \phi^{us\prime} C_t^{us}(t+1) + \phi^{uk\prime} C_t^{uk}(t+1) + \boldsymbol{\varepsilon}_1(t+1) \end{aligned}$$

where $C_t(t+1)$ is a vector with j^{th} element $[\mathbf{h}_{11}(t+1) \times \mathbf{h}_{jj}(t+1)]^{\frac{1}{2}}$. Here we are using $\boldsymbol{\varepsilon}_1(\mathbf{t+1})$ to refer to the error term in the excess return equation no matter which model is chosen. In practice the error terms in each model would be different.

The variables in $C_t(t+1)$ depend on the choice of SDF model. In the case of complete markets, the first three models are identical. The fourth model is designed to relax the arbitrage restrictions and to allow comparison with equation (7), the original FOREX model.

5.2 Alternative SDF models

We estimate four different models of the stochastic discount factors: (i) a benchmark model designed to represent the joint distribution of the three variables entering into the FOREX excess return, (ii) C-CAPM, (iii) traditional CAPM based on the monetary model of the exchange rate and (iv) a model that combines all three.

Benchmark model

The benchmark model is derived from traditional tests of FOREX market efficiency based on uncovered interest parity. It shows how the variables that appear in these traditional tests - i.e. equation (7) - should be modelled when there is no other source of risk. Since there are three independent sources of randomness (the exchange rate and the two risk-free rates), a three-variable GARCH-in-mean model is estimated. The three variables are $R(t)$, $f(t)-s(t)$ and $\Delta i^{us}(t)$. For this model there are two conditional covariances in the mean of the excess return equation. They are the conditional covariances of $R(t+1)$ with $f(t+1)-s(t+1)$ and $\Delta i^{us}(t+1)$, and they are the only covariance terms to appear in each of the market structure models. The difference between the US and the UK investor models is just due to the dependent variable.

C-CAPM

For the US investor model, it follows from equation (24), that there are three variables: $R(t+1)$, the rate of growth of consumption $\Delta c^{us}(t+1)$ and the inflation rate $\Delta p^{us}(t+1)$. The risk premium is based on the conditional covariances of the excess return with the other two variables. The excess return equation can therefore be written

$$\begin{aligned} R(t+1)+\frac{1}{2}V_t[R(t+1)] &= \sigma^{us}Cov_t[c^{us}(t+1), R(t+1)] \\ &\quad +Cov_t[\Delta p^{us}(t+1), R(t+1)]+\boldsymbol{\varepsilon}_1(t+1) \end{aligned} \tag{33}$$

The equation explaining $R(t+1)$ for the UK investor is

$$\begin{aligned} R(t+1)-\frac{1}{2}V_t[R(t+1)] &= -\sigma^{uk}Cov_t[c^{uk}(t+1), R(t+1)] \\ &\quad -Cov_t[\Delta p^{uk}(t+1), R(t+1)]+\boldsymbol{\varepsilon}_1(t+1) \end{aligned} \tag{34}$$

All four conditional covariances appear in the joint investor model and in the model on the alternative hypothesis.

Traditional CAPM and the monetary model

In the monetary model, the exchange rate is determined by future expected relative money supplies and output levels, see for example Frenkel (1976) and Obstfeld and Rogoff (1998). This would suggest that exchange risk might be due to forecast covariation between today's exchange rate and tomorrow's domestic and foreign money supplies and output.

The monetary model of the exchange rate, which is based on UIP, is best known as a macroeconomic and not a finance explanation. But it is easy to give it additional finance credentials, and even to justify it as an example of SDF theory, by interpreting it as an example of traditional CAPM. The relevance of CAPM to FOREX is the widespread use of mean-variance analysis in hedging FOREX risk. The uncertainty about the pay-off to the possibly partly-hedged portfolio then arises from the future return on the portfolio.

In the case of FOREX this is just pure currency risk and anything correlated with this - such as tomorrow's domestic and foreign money supplies and output - could be used to help reduce it.

We use the index of industrial production as our monthly output measure, and narrow money as our measure of money. For the US investor model, we use only US industrial production and the US money supply, and for the UK investor model, only UK industrial production and the UK money supply. The combined investor model includes all four variables

General model

This model takes no position on which (if any) of the three previous models is the correct source of stochastic discount factors. Instead, all of the variables in these models are used. A possible theoretical justification for the combined model is that the monetary model assumes purchasing power parity, i.e. perfect price flexibility. If prices were sticky, as seems probable, exchange rates would be slower to adjust and could also be affected by the rate of inflation.

5.3 Switching conditional variance structure

If investors have different attitudes to risk then there is a strong case incorporating this more fully in the econometric specification of the combined model. A dollar-based investor holding sterling assets faces losses from exchange risk when the dollar unexpectedly appreciates, i.e. when $\Delta s(t+1) - E_t \Delta s(t+1) < 0$; the interest differential is supposed to compensate for any expected appreciation. Similarly, sterling-based investors face losses from exchange risk when $\Delta s(t+1) - E_t \Delta s(t+1) > 0$. This suggests that in computing the conditional covariance matrix in the combined model, a different model of the conditional variance of the excess return - i.e. of $\Delta s(t+1)$ - should be used depending on whether $\Delta s(t+1) - E_t \Delta s(t+1) = \boldsymbol{\varepsilon}_1(t) \lesseqgtr 0$. Hence, for the case where there are both types of investor we investigate whether the conditional variance of the excess return is better modelled by

$$\mathbf{h}_{11}(t+1) = v_1 + a_1 \mathbf{h}_{11}(t) + b_1 \boldsymbol{\varepsilon}_1(t)^2 + \delta b_1^* \boldsymbol{\varepsilon}_1(t)^2 \tag{35}$$

than by equation (32), where $\delta = 0$ if $\Delta s(t+1) \leqslant 0$ and $= 1$ if $\Delta s(t+1) > 0$. In order to obtain estimates of the conditional variance for the general model we found it necessary to add the further restriction that $a_1 + b_1 + \delta b_1^* < 1$.

5.4 Excess kurtosis

It is well known that exchange rates exhibit excess kurtosis relative to the Normal distribution. One way of trying to take account of this is to use the t-distribution instead of the Normal, though this is not without problems too[9]. After regressing the excess return on a VAR the estimated kurtosis exceeds 3, the value for the Normal. The implied degrees of freedom for a t-distribution varies with the variables in the VAR. Without industrial production we estimate that there are 9 degrees of freedom, but with industrial production there are 10 degrees of freedom.

6 Estimates

The data used in this paper are monthly from 1975.1 - 1997.12 of the US dollar - sterling exchange rate. One month euro-dollar and euro-sterling interest rates are used. The price indices are the CPI for the US and the RPI for the UK. As consumption data are not available monthly, we use deflated retail sales data. As noted, the output series are the indices of industrial production. The money supply is the money base for the US and M0 for the UK. The correlations of the excess return with each of the variables is reported in Table 1[10]. These are required in order to sign the coefficients of the conditional covariances in the excess return equation.

Due to the large number of variables used and models estimated, we report only the results for the excess return equation. There are significant ARCH effects for all of the variables, and most variables show that the own conditional variance is significant in its own conditional mean. We consider each stochastic discount factor model in turn. There are four sets of two tables, one for the Normal distribution and one for the t-distribution. The first column gives the results for the dollar-based investor holding sterling assets, the second column for the sterling-based investor, the third column for where there are both types of investor. In columns one and two the

[9]There is a technical problem with using the t-distribution. If the logs of the excess return and the discount rate have a multi-variate t-distribution, then the log of the term on the RHS of equation (11) will no longer exist because the moment-generating function of the t-distribution doesn't exist.

[10]An alternative estimate are the correlations of the residuals of the individual equations obtained from the GARCH estimation of each equation. The results are very similar.

coefficient of the conditional variance of the excess return is imposed, not estimated. As the data are expressed in terms of annualised returns or rates of change the coefficient on the own conditional variance is restricted to be $\frac{1}{2400} = 0.0004166$, and not 0.5 as above. Due to the presence of the variance and covariances in the excess return model, the estimating equation is not linear and so is not free from the unit of measurement. In column three this coefficient is set to zero. The fourth column is a general unrestricted version of the other models that drops the restrictions on the own conditional variance and conditional covariances, and includes the lagged excess return and forward premium. The switching effect is only appropriate for the two investor model and the general model. Columns three and four have no switching effect. Columns five and six repeat three and four, but also allow for switching.

6.1 Benchmark model

The benchmark model aims to specify the joint distribution of the variables used in traditional tests of FOREX market efficiency in way that satisfies the no-arbitrage condition of SDF theory. Estimates of the excess return equation for the benchmark model are reported in Tables 2a and 2b. There is little qualitative difference between the two tables, and the following comments apply to both.

None of the conditional covariances with the excess return is significant in any of the models or with either distribution. One measure of being able to successfully measure the FOREX risk premium is that it eliminates the forward premium puzzle. In other words, the biases in the estimate of the forward premium should be removed so that the estimate is insignificant from zero. Having failed to provide a significant model of the FOREX premium, it is not surprising that the forward premium retains its significance. We also find that the lagged excess return is significant. Thus, the model used in traditional tests of the FOREX market when reformulated so that the conditional heteroskedasticity in the joint distribution of the variables is taken into account, is unable to provide a significant measure of the FOREX risk premium.[11]

[11] There is an alternative explanation for the significance of the lagged excess return. Suppose that the FOREX risk premium is persistent, displaying first-order autocorrelation, then a first order autoregressive transformation of the excess return would take account of this. This argument would also suggest that the risk premium is imperfectly measured

6.2 C-CAPM

C-CAPM, a general equilibrium model of asset pricing, specifies that the excess return on FOREX should be modelled jointly with consumption growth and inflation. In the absence of monthly consumption data, we use retail sales data for the US and UK which are available monthly. We use the CPI (for the US) and the RPI (for the UK) for consumer price inflation. Estimates of the excess return equation are reported in Tables 3a and 3b.

The results for the Normal and t-distributions are broadly similar. The conditional covariance terms in the two single investor models (columns 1 and 2) are now significant at the 10% level for the Normal distribution. For the US investor model using the t-distribution the conditional covariances are less significant. In the two investor model (column 3) only the conditional covariance with US consumption is significant. In the general model (column 4) none of the conditional covariances is significant for the Normal distribution, but the covariance with UK consumption is significant. Tests of the restriction on the coefficient of the own conditional variance do not reject the restriction. The outcome of this test is the same for all of the models. This is mainly because the theoretical value is so small and the unrestricted coefficient is not estimated precisely enough.

These results can be related more closely to C-CAPM theory. Equation (33) is the excess return model for the US investor predicted by the theory. The estimates in Table 3 are for the coefficients of the product of the conditional standard deviations (i.e. of $\rho_{1,j}^{us}\beta_j^{us}$), not for the conditional covariances required by the theory (i.e. of β_j^{us}). But by using the unconditional correlations with the excess return reported in Table 1, it is possible to recover estimates of the β_j^{us}. From equation (33) the coefficient of the covariance of the excess return with consumption growth is σ^{us}, the coefficient of relative risk aversion, and that with inflation is unity. A further adjustment is needed to allow for the fact that as the data are measured in annual terms, the coefficient is actually an estimate of $1200\rho_{1,j}^{us}\beta_j^{us}$. Based on the results for the Normal distribution, the implied estimates of these coefficients for the US investor are -289 and 43800, respectively.

In obtaining the corresponding results for the UK investor it should be recalled that the equation (34) is used to explain $R(t+1)$ whereas the excess return for the UK investor is $-R(t+1)$. The resulting estimate of σ^{uk} is -283 and of the coefficient of the covariance with inflation is 10320. For the

by the SDF model.

combined model only the coefficient of the conditional covariance with US consumption is significant and this is -410.

All of the estimates of the coefficient of relative risk aversion therefore have the wrong sign and are very large. The same is true when the t-distribution is used. The signs for the covariance with inflation are correct in the single investor and combined models, but are significant only in the single investor models. The size of the coefficients is however far too large. Moreover, in the general model none of the conditional covariances is significant and, as would be expected in view of this, the lagged excess return and forward premium retain their significance. In other words, the forward premium puzzle is not resolved.

The broad conclusion that emerges from these results about C-CAPM is similar to those of Mark and Wu (1998), Lewis (1995) and Engel (1996). As specified above with power utility, the estimates are not consistent with the theoretical predictions and the theory does not seem able to provide a satisfactory model of the FOREX risk premium.

6.3 Monetary model

The monetary model of the exchange rate seeks to explain exchange rate changes with relative rates of growth of the domestic and foreign money supplies, and output. As noted above, this model is relevant to a theory of FOREX risk based on pure currency risk and could arise from standard hedging considerations, thereby making it a version of CAPM. It may also be noted that in the absence of monthly consumption data, narrow money could serve as proxy for nominal consumption expenditures. We specify the joint distribution of the excess return to FOREX and the rates of nominal money growth, the money base (for the US) and M0 (for the UK). The estimates are reported in Tables 4a and 4b.

The theoretical predictions are that the coefficient on conditional covariances should be positive for US money and negative for US output, and these signs should be reversed for the UK variables. Thus, taking account of the signs of the unconditional correlations with the excess return in Table 1, the estimates for the UK investor have the correct sign and are significant. The single US investor model performs best assuming a t-distribution, when the estimates are bordering on significance and also have the correct sign. For the two investor model all the signs are correct, but the covariances with UK money are not significant. In the general model the output covariances are the most significant. These results therefore show considerable support for the monetary model and hence for the traditional models of currency risk. The continued significance of the lagged excess return and forward premium in the general model indicates, however, that the forward premium puzzle is not resolved even if output, and in some cases money, seem to be significant sources of FOREX risk.

6.4 Combined model

This model has no explicit theoretical justification. The aim is simply to model the joint distribution of all of the variables previously considered to provide a general alternative model for the purposes of comparison with previous results. The two single investor models are restricted to include only domestic sources of risk and, in addition, the variables in the benchmark model. As before, the combined model excludes the conditional variance of the excess return. The results are reported in Tables 5a and 5b.

In the two single investor models and the two investor model the conditional covariances with consumption and output are the only significant variables, but only the output terms have the theoretically correct sign. This suggests that the factors may come from a mixture of the C-CAPM and CAPM models. In the general model the lagged excess return and the forward premium retain their significance, showing once more that the forward premium puzzle remains.

6.5 Switching model

The aim with the switching model is to allow investors to have different attitudes to risk. This would only affect the two investor and the general models. The columns 5 and 6 of Tables 2-5 are a re-estimate of columns 3 and 4 and include an additional term in the expression for the conditional variance of the excess return to allow for a shift in the impact of last period's error. The switching term is significant in some of the models. It is most important for the two investor case within the monetary and combined models, and slightly more significant for the Normal than the t-distribution estimates.

Including the switching term has a major impact on the estimates of the coefficients of the conditional covariances in the monetary model. In the two investor model, all of these coefficents are significant and have the correct sign. In the general model, the estimates are similar to those without switching effects. Thus, these results indicate that US and UK investors may have different attitudes to risk. They also lend strong support for the monetary model, but still without eliminating the forward premium puzzle.

6.6 Graphical comparison of risk premia

We can compare the different estimates of the FOREX risk premium graphically. The four panels of Figure 2 plot the risk premia (plus the Jensen effects) for the four different SDF models. And each panel displays four estimates, reflecting the four different market structures. The risk premia can be positive or negative. A positive (negative) risk premium implies that the US (UK) investor bares the risk.

For the benchmark model there is little difference between the four estimates of the risk premia. Apart from 1980, a period when US interest rates were unusually volatile due to monetary policy taking the form of money-base targeting, and when the dollar depreciated considerably against sterling, it is mainly positive. Since about 1986 it has also been quite small. In May 1980, a common outlier for all of the graphs, there was a sharp fall in the covariance between the excess return and the forward premium.

The risk premia for C-CAPM demonstrate further the problems with this model. The risk premia for the US investor are positive most of the time. Apart from 1975 and 1980 the risk premia for the other three models are mainly positive too. This seems to be due primarily to a positive sign on the covariance between the excess return and domestic consumption in both US and UK investor models, which is contrary to the predictions of C-CAPM.[12]

The risk premia for the monetary model are larger and more variable than those for the other models. The UK and combined investor models, in particular, show positive risk premia for 1979-80 (not negative as before) and for 1984-88 and negative risk premia for 1981-3. In general, UK output is more volatile than US output and this volatility difference is most pronounced in the two periods of large positive risk premia. In general US money growth has a much higher volatility than UK money growth and this difference is greatest in 1981-2 when the risk premia were negative. Broadly,

[12] The outlier in the UK investor results in mid-1979 appears to be due to changes in consumption that are related to tax rate changes that were announced before the election of that year.

the implications of this are that US investors of UK bonds require a higher risk premium when the UK is going through periods of output fluctuations due particularly to a downturn in GDP. In contrast, UK investors require a higher risk premium when US monetary policy is volatile. These findings are all quite plausible and provide further support for the monetary model.

The general model is similar to the monetary model except that in 1979-80 the risk premium becomes negative. Three periods of higher than usual risk premia occur: 1979, 1985 and 1993. All are periods of higher than usual UK output volatility. The last is after the UK left the ERM and suffered a mild output downturn.

In Figure 3 the risk premia for the two investor version of four SDF models are shown together with the excess return, R_{t+1}. The additional information relates to the relation between the excess return and the risk premia. It is clear that noise dominates the behaviour of the excess return. Periods of high exchange rate volatility are associated with the largest risk premia. Also periods of predominantly positive excess returns tend to be associated with positive risk premia (in 1979-80 and 1985) while periods of negative excess returns tend to be associated with negative risk premia (1981-1984). After 1986, the risk premia seem to be smaller, even when the volatility of the excess returns is high.

7 Conclusions

In this paper we have considered the problem of measuring macroeconomic sources of financial risk. We have used the stochastic discount factor model to provide a general theory of asset pricing and have described in detail how this can be implemented empirically using the multivariate GARCH-in-mean model. We have argued that ARCH-in mean effects must be included in order that the empirical model may satisfy the no-arbitrage condition, and that as the risk premium is a conditional covariance, the model must also be multivariate.

We have then used this methodology to measure the FOREX risk premium and to examine whether incorporating this will resolve the forward premium puzzle. We have shown that it is important to take account of the fact that market participants may be dollar or sterling based and have obtained a no-arbitrage model suitable for the case when both types of investor are present. When markets are complete the model reduces to that for a single type of investor.

To implement SDF theory it is necessary to select the variables determining the discount factor. We use inter-temporal CAPM, a formal general equilibrium model, and other less formal models to generate possible variables. One of these models, our benchmark model, involves using the joint distribution of the variables in traditional tests of FOREX market efficiency based on UIP. Another model is based on the monetary model of the exchange rate, a leading macroeconomic model of the exchange rate.

The empirical results provide no support for inter-temporal consumption-based CAPM, which confirms the findings of other studies. One of the most interesting results of the paper is the support provided for the monetary model. We find that positive risk premia emerge particularly in 1979-80 and again around 1985. These seem to be due mainly to US investors requiring a risk premium to compensate for higher UK output volatility. Negative risk premia around 1981-82 seem to be due to volatile US monetary policy.

The market structure that performs best is the two investor monetary model which includes switching effects to allow for different attitudes to risk among US and UK investors. The zero restriction on the own conditional variance is satisfied in all of the estimates of the two investor model. There is little to choose between the Normal and t-distribution estimates, but the former are to be preferred on the grounds that they are slightly better, and there is a logical difficulty with using the t-distribution in the SDF framework.

The main problem that remains is the continued presence of the forward premium puzzle. Even our preferred model does not eliminate this.

Modelling risk using observable factors within the SDF framework is, in our view, a considerable advance on existing work. Discovering the potential usefulness of the monetary model to capture the FOREX premium is a further advance in our knowledge. It has the interesting implication that the FOREX risk premium may be more associated with pure currency risk than general equilibrium considerations.

A number of problems still remain. Once general equilibrium models fail it is not clear how to choose the variables from which to measure the discount factor. The SDF model itself provides no guidance. It has been suggested that mis-measuring the discount factor, for example, by using the wrong variables, may greatly impair the usefulness of the SDF model. The use of observable sources of macroeconomic risk makes severe data demands. Ideally high frequency macro data is needed, but most macroeconomic data are not widely available and then mainly at monthly intervals. This curtails the amount of heteroskedasticity in the explanatory variables. In order to provide an adequate representation of the theory, the VGARCHM model must be highly parameterized. This, together with the lack of heteroskedasticity in the data makes the numerical convergence difficult and the optimization a lengthy procedure. Further advances in the use of this general approach will depend in large part in finding satisfactory solutions to these problems.

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Figure 1. Plot of R(t+1) against f(t)-s(t)

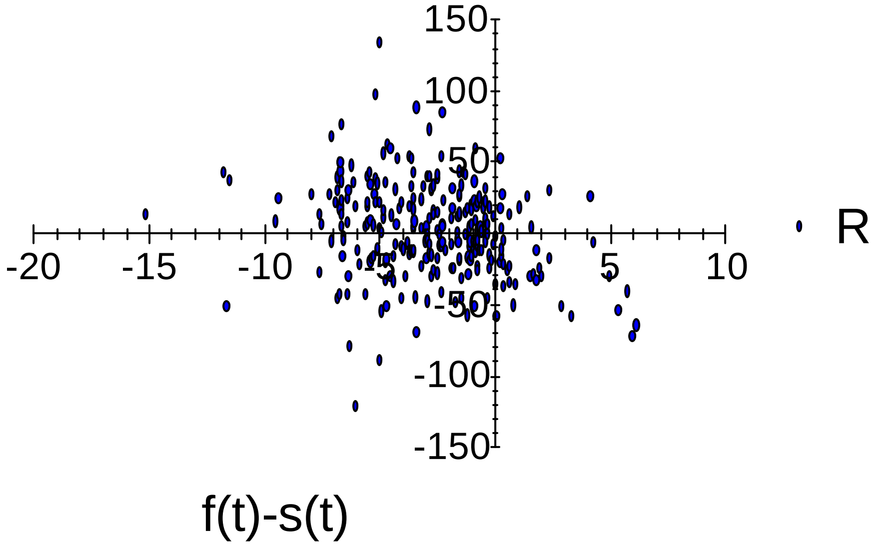

Figure 1:

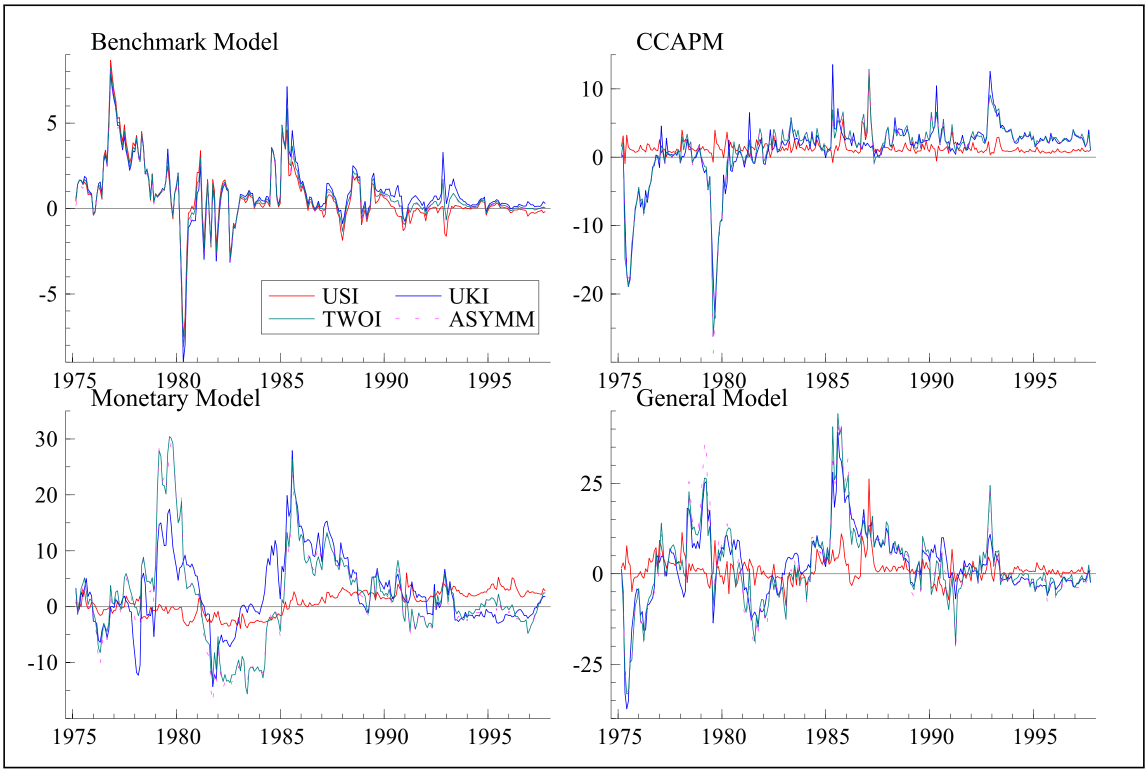

Figure 2: **Risk Premia (incl. Jensen effect) : All SDF Models and Market Structures.** USI: US investor, UKI: UK investor, TWOI: US & UK two investor, ASYMM: Two investor switching model.

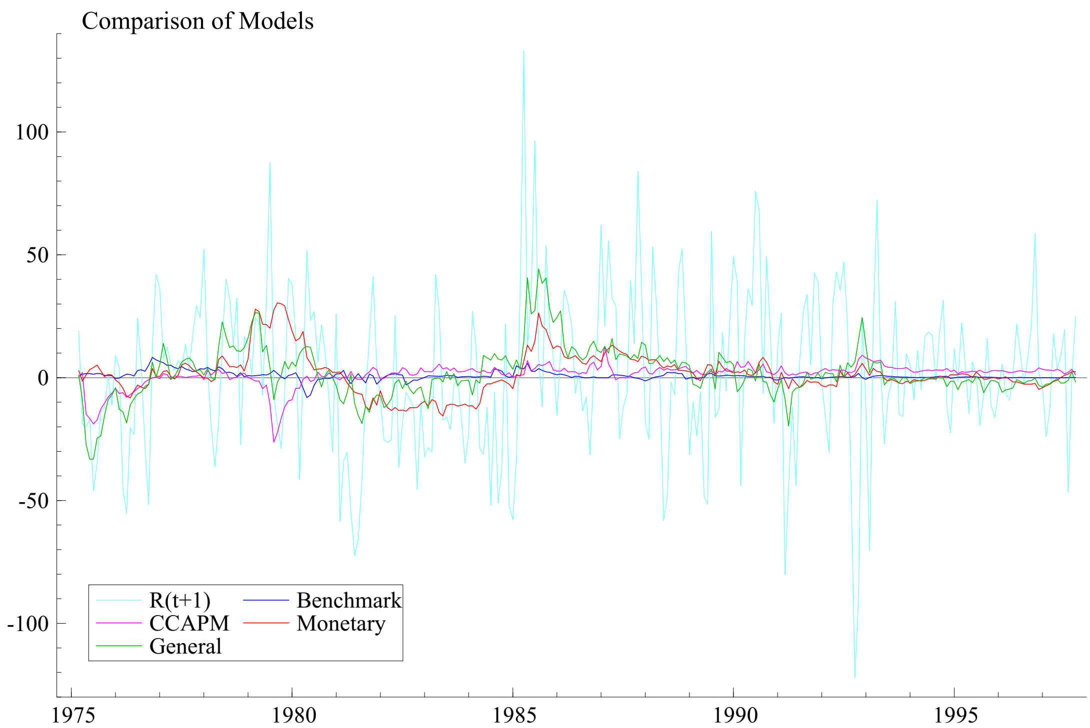

Figure 3: **Excess Return and the Risk Premia : the Two Investor Model**

Table 1
Unconditional correlation with the excess return (ρ_{1j})

$f - s$	Δi^{us}	Δc^{us}	Δc^{uk}	Δp^{us}	Δp^{uk}	Δm^{us}	Δm^{uk}	Δy^{us}	Δy^{uk}
-0.209	-0.076	-0.079	-0.072	0.002	0.055	0.095	-0.002	-0.179	-0.092

Table 2a

Estimates of the benchmark model of the FOREX excess return

The excess return equation in the multivariate GARCH in mean model

Normal distribution

Dependent Variable	**US**	**UK**	**US & UK**	**General**	**US & UK**	**General**
			Non-switching model		**switching model**	
Constant				1.636 (0.31)		1.684 (0.31)
$V_t(R_{t+1})$	-0.0004	0.0004		-0.004 (0.64)		-0.005 (0.63)
$C_t(R_{t+1}, f_{t+1} - s_{t+1})$	0.063 (0.66)	0.047 (0.49)	0.057 (0.62)	-0.084 (0.78)	0.050 (0.56)	-0.087 (0.79)
$C_t(R_{t+1}, \Delta i_{t+1}^{us})$	-0.073 (0.55)	-0.078 (0.58)	-0.078 (0.58)	0.048 (0.38)	-0.065 (0.49)	0.048 (0.38)
R_t				0.352 (5.68)		0.345 (5.33)
$f_t - s_t$				-1.683 (2.51)		-1.790 (2.36)
Δi_t^{us}				-1.899 (1.03)		-1.914 (1.01)
Conditional variance						
Constant	1251.6 (3.30)	1246.3 (3.35)	1257.1 (2.89)	914.9 (5.25)	1177.7 (3.57)	912.8 (5.38)
$\sigma_t^2(R_{t+1})$	0.617 (4.96)	0.622 (5.00)	0.620 (5.10)	0.662 (5.02)	0.659 (6.02)	0.679 (4.85)
ε_t^2	0.266 (2.73)	0.261 (2.71)	0.265 (2.56)	0.164 (2.34)	0.290 (3.57)	0.173 (2.32)
$\delta\varepsilon_t^2$					-0.123 (1.31)	-0.040 (0.37)

Notes: 1. Dependent variable is R_{t+1}

2. $C_t(x_{t+1}, y_{t+1}) = \sqrt{h_{xx}(t+1).h_{yy}(t+1)}$, where $h_{xx}(t+1)$ is the conditional variance of x_{t+1}

3. Numbers in parentheses are t-statistics

Table 2b
Estimates of the benchmark model of the FOREX excess return
The excess return equation in the multivariate GARCH in mean model
t-distribution

Dependent Variable	US	UK	US & UK	General	US & UK	General
			Non-switching model		switching model	
Constant				4.551 (0.68)		4.751 (0.68)
$V_t(R_{t+1})$	-0.0004	0.0004		-0.009 (1.03)		-0.010 (1.01)
$C_t(R_{t+1},\ f_{t+1}-s_{t+1})$	0.088 (0.97)	0.069 (0.76)	0.081 (0.88)	-0.115 (1.08)	0.063 (0.69)	-0.117 (1.08)
$C_t(R_{t+1}, \Delta i^{us}_{t+1})$	-0.103 (0.82)	-0.106 (0.85)	-0.109 (0.86)	0.074 (0.62)	-0.079 (0.63)	0.074 (0.61)
R_t				0.334 (5.62)		0.327 (5.27)
$f_t - s_t$				-2.271 (3.37)		-2.357 (3.17)
Δi^{us}_t				-1.843 (1.04)		-1.863 (1.03)
Conditional variance						
Constant	1402.4 (2.01)	1383.0 (2.06)	1395.4 (2.02)	867.4 (5.49)	1271.0 (2.66)	869.6 (5.51)
$\sigma^2_t(R_{t+1})$	0.638 (4.49)	0.648 (4.60)	0.643 (4.55)	0.624 (3.93)	0.652 (4.17)	0.645 (3.95)
ε^2_t	0.263 (2.24)	0.254 (2.21)	0.259 (2.23)	0.149 (2.00)	0.291 (2.61)	0.162 (1.99)
$\delta\varepsilon^2_t$					-0.100 (0.97)	-0.042 (0.42)

Notes: 1. Dependent variable is R_{t+1}

2. $C_t(x_{t+1}, y_{t+1}) = \sqrt{h_{xx}(t+1).h_{yy}(t+1)}$, where $h_{xx}(t+1)$ is the conditional variance of x_{t+1}

3. Numbers in parentheses are t-statistics

4. Degrees of freedom = 9

Table 3a

Estimates of the C-CAPM for the FOREX excess return

The excess return equation in the multivariate GARCH in mean model

Normal distribution

Dependent Variable	**US**	**UK**	**US & UK** (Non-switching model)	**General** (Non-switching model)	**US & UK** (switching model)	**General** (switching model)
Constant				-0.282 (0.031)		1.379 (0.15)
$V_t(R_{t+1})$	-0.0004	0.0004		-0.003 (0.36)		-0.005 (0.50)
$C_t(R_{t+1}, \Delta c_{t+1}^{us})$	0.024 (2.09)		0.027 (2.20)	0.021 (1.39)	0.028 (1.91)	0.015 (0.87)
$C_t(R_{t+1}, \Delta c_{t+1}^{uk})$		-0.016 (1.92)	-0.012 (1.23)	-0.013 (1.45)	-0.037 (0.56)	-0.013 (1.49)
$C_t(R_{t+1}, \Delta p_{t+1}^{us})$	-0.104 (1.68)		-0.030 (0.22)	-0.061 (0.35)	0.020 (0.13)	-0.011 (0.06)
$C_t(R_{t+1}, \Delta p_{t+1}^{uk})$		0.039 (2.02)	-0.013 (0.21)	0.005 (0.066)	-0.162 (1.57)	-0.010 (0.15)
R_t				0.351 (5.14)		0.330 (5.13)
$f_t - s_t$				-1.722 (2.70)		-2.057 (2.31)
Conditional variance						
Constant	1290.3 (2.96)	1252.2 (3.19)	1283.1 (2.87)	921.3 (5.08)	1167.5 (3.67)	953.1 (4.49)
$\sigma_t^2(R_{t+1})$	0.627 (5.45)	0.607 (5.28)	0.624 (5.56)	0.676 (5.76)	0.674 (6.73)	0.737 (6.11)
ε_t^2	0.269 (2.92)	0.277 (3.08)	0.273 (3.05)	0.175 (2.87)	0.298 (4.16)	0.211 (2.72)
$\delta\varepsilon_t^2$					-0.162 (1.57)	-0.136 (1.40)

Notes: 1. Dependent variable is R_{t+1}

2. $C_t(x_{t+1}, y_{t+1}) = \sqrt{h_{xx}(t+1).h_{yy}(t+1)}$, where $h_{xx}(t+1)$ is the conditional variance of x_{t+1}

3. Numbers in parentheses are t-statistics

Table 3b

Estimates of the C-CAPM for the FOREX excess return

The excess return equation in the multivariate GARCH in mean model

t-distribution

Dependent Variable	US	UK	US & UK	General	US & UK	General
			Non-switching model		switching model	
Constant				0.624 (0.07)		0.905 (0.12)
$V_t(R_{t+1})$	-0.0004	0.0004		-0.015 (1.48)		-0.014 (1.25)
$C_t(R_{t+1}, \Delta c_{t+1}^{us})$	0.019 (1.85)		0.016 (1.50)	0.013 (1.06)	0.019 (1.22)	0.007 (0.45)
$C_t(R_{t+1}, \Delta c_{t+1}^{uk})$		-0.017 (2.17)	-0.017 (1.97)	-0.020 (2.19)	-0.017 (2.35)	-0.022 (2.69)
$C_t(R_{t+1}, \Delta p_{t+1}^{us})$	-0.073 (1.34)		-0.087 (1.20)	0.034 (0.28)	-0.086 (1.00)	0.001 (0.01)
$C_t(R_{t+1}, \Delta p_{t+1}^{uk})$		0.043 (2.33)	0.047 (1.83)	0.061 (1.98)	0.044 (1.50)	0.054 (1.63)
R_{t+1}				0.332 (5.00)		0.309 (5.10)
$f_t - s_t$				-2.432 (3.95)		-2.664 (2.68)
Conditional variance						
Constant	1424.4 (1.94)	1366.0 (2.03)	1426.3 (1.76)	850.7 (5.56)	1298.0 (2.58)	969.3 (3.28)
$\sigma_t^2(R_{t+1})$	0.642 (4.56)	0.632 (4.63)	0.636 (4.79)	0.649 (3.98)	0.666 (6.83)	0.752 (6.78)
ε_t^2	0.262 (2.26)	0.268 (2.36)	0.273 (2.37)	0.145 (2.03)	0.300 (3.74)	0.218 (2.47)
$\delta\varepsilon_t^2$					-0.124 (1.15)	-0.168 (1.74)

Notes: 1. Dependent variable is R_{t+1}

2. $C_t(x_{t+1}, y_{t+1}) = \sqrt{h_{xx}(t+1).h_{yy}(t+1)}$, where $h_{xx}(t+1)$ is the conditional variance of x_{t+1}

3. Numbers in parentheses are t-statistics

4. Degrees of freedom = 9

Table 4a

Estimates of the monetary model of the FOREX excess return

The excess return equation in the multivariate GARCH in mean model

Normal distribution

Dependent Variable	**US**	**UK**	**US & UK**	**General**	**US & UK**	**General**
			Non-switching model		**switching model**	
Constant				21.898 (2.04)		23.150 (1.86)
$V_t(R_{t+1})$	-.0004	0.0004		0.011 (1.02)		0.009 (0.66)
$C_t(R_{t+1}, \Delta m_{t+1}^{us})$	0.131 (1.25)		0.121 (1.48)	-0.343 (1.58)	0.141 (1.82)	-0.331 (1.25)
$C_t(R_{t+1}, \Delta m_{t+1}^{uk})$		-0.089 (2.27)	-0.034 (0.84)	-0.029 (0.73)	-0.080 (2.15)	-0.044 (1.03)
$C_t(R_{t+1}, \Delta y_{t+1}^{us})$	-0.041 (1.16)		-0.109 (2.84)	-0.070 (1.77)	-0.094 (2.71)	-0.071 (1.61)
$C_t(R_{t+1}, \Delta y_{t+1}^{uk})$		0.042 (2.55)	0.054 (3.41)	0.034 (2.11)	0.064 (4.69)	0.039 (2.20)
R_t				0.321 (5.13)		0.306 (4.68)
$f_t - s_t$				-1.333 (2.17)		-1.635 (1.97)
Conditional variance						
Constant	1185.0 (3.97)	1181.2 (3.46)	1099.6 (3.76)	946.3 (3.93)	952.6 (6.31)	990.3 (4.10)
$\sigma_t^2(R_{t+1})$	0.621 (4.75)	0.628 (5.29)	0.634 (5.53)	0.698 (6.43)	0.621 (8.08)	0.713 (6.99)
ε_t^2	0.244 (2.78)	0.250 (2.68)	0.227 (2.54)	0.179 (2.58)	0.369 (4.81)	0.236 (3.07)
$\delta\varepsilon_t^2$					-0.332 (3.49)	-0.141 (1.66)

Notes: 1. Dependent variable is R_{t+1}

2. $C_t(x_{t+1}, y_{t+1}) = \sqrt{h_{xx}(t+1).h_{yy}(t+1)}$, where $h_{xx}(t+1)$ is the conditional variance of x_{t+1}

3. Numbers in parentheses are t-statistics

Table 4b

Estimates of the monetary model of the FOREX excess return

The excess return equation in the multivariate GARCH in mean model

t-distribution

Dependent Variable	**US**	**UK**	**US & UK**	**General**	**US & UK**	**General**
			Non-switching model		**switching model**	
Constant				13.139 (1.29)		13.803 (1.32)
$V_t(R_{t+1})$	-0.0004	0.0004		-0.001 (0.058)		-0.001 (0.06)
$C_t(R_{t+1}, \Delta m^{us}_{t+1})$	0.153 (1.68)		0.140 (1.85)	-0.093 (0.47)	0.168 (1.93)	-0.102 (0.50)
$C_t(R_{t+1}, \Delta m^{uk}_{t+1})$		-0.080 (2.23)	-0.020 (0.50)	-0.013 (0.34)	-0.053 (1.43)	-0.019 (0.44)
$C_t(R_{t+1}, \Delta y^{us}_{t+1})$	-0.049 (1.59)		-0.115 (3.20)	-0.080 (2.20)	-0.110 (3.07)	-0.083 (2.08)
$C_t(R_{t+1}, \Delta y^{uk}_{t+1})$		0.038 (2.49)	0.048 (3.29)	0.027 (1.75)	0.056 (3.94)	0.029 (1.76)
R_{t+1}				0.306 (5.04)		0.292 (4.64)
$f_t - s_t$				-1.928 (2.80)		-2.110 (2.49)
Conditional variance						
Constant	1216.8 (2.83)	1286.0 (2.29)	1153.5 (3.01)	866.0 (4.69)	1013.6 (5.30)	911.8 (4.12)
$\sigma^2_t(R_{t+1})$	0.644 (4.20)	0.642 (4.54)	0.647 (5.49)	0.687 (4.87)	0.631 (5.42)	0.734 (6.29)
ε^2_t	0.231 (2.15)	0.251 (2.22)	0.230 (2.41)	0.147 (2.08)	0.359 (3.08)	0.191 (2.49)
$\delta\varepsilon^2_t$					-0.321 (2.83)	-0.110 (1.28)

Notes: 1. Dependent variable is R_{t+1}

2. $C_t(x_{t+1}, y_{t+1}) = \sqrt{h_{xx}(t+1).h_{yy}(t+1)}$, where $h_{xx}(t+1)$ is the conditional variance of x_{t+1}

3. Numbers in parentheses are t-statistics

4. Degrees of freedom = 10

Table 5a

Estimates of the combined model of the FOREX excess return

The excess return equation in the multivariate GARCH in mean model

Normal distribution

Dependent Variable	**US**	**UK**	**US & UK**	**General**	**US & UK**	**General**
			Non-switching model		**switching model**	
constant				8.806 (0.48)		10.159 (0.59)
$V_t(R_{t+1})$	-0.0004	0.0004		-0.006 (0.31)		-0.005 (0.25)
$C_t(R_{t+1}, f_{t+1} - s_{t+1})$	0.133 (0.94)	-0.121 (0.92)	-0.182 (1.15)	-0.300 (1.47)	-0.143 (0.91)	-0.277 (1.41)
$C_t(R_{t+1}, \Delta i_{t+1}^{us})$	-0.114 (0.76)	-0.039 (0.34)	-0.011 (0.08)	0.200 (1.51)	-0.058 (0.45)	0.194 (1.47)
$C_t(R_{t+1}, \Delta c_{t+1}^{us})$	0.033 (2.43)		0.023 (1.64)	0.016 (0.97)	0.019 (1.35)	0.017 (1.07)
$C_t(R_{t+1}, \Delta c_{t+1}^{uk})$		-0.022 (2.63)	-0.030 (4.44)	-0.028 (2.90)	-0.025 (2.98)	-0.028 (3.24)
$C_t(R_{t+1}, \Delta p_{t+1}^{us})$	0.179 (0.91)		-0.043 (0.22)	-0.312 (1.51)	0.031 (0.16)	-0.310 (1.57)
$C_t(R_{t+1}, \Delta p_{t+1}^{uk})$		-0.037 (0.84)	0.222 (1.47)	0.129 (0.61)	-0.018 (0.12)	0.131 (0.64)
$C_t(R_{t+1}, \Delta m_{t+1}^{us})$	-0.087 (0.53)		-0.124 (0.70)	-0.022 (0.072)	0.090 (0.51)	-0.057 (0.20)
$C_t(R_{t+1}, \Delta m_{t+1}^{uk})$		-0.027 (0.52)	-0.018 (0.35)	0.010 (0.186)	-0.029 (0.58)	0.009 (0.16)
$C_t(R_{t+1}, \Delta y_{t+1}^{us})$	-0.093 (1.89)		-0.208 (3.22)	-0.105 (1.42)	-0.131 (2.18)	-0.107 (1.52)
$C_t(R_{t+1}, \Delta y_{t+1}^{uk})$		0.063 (3.23)	0.070 (4.33)	0.059 (3.26)	0.095 (6.34)	0.059 (3.34)
R_t				0.300 (4.80)		0.310 (4.85)
$f_t - s_t$				-1.775 (2.61)		-1.657 (2.55)
Conditional variance						
Constant	1236.3 (3.34)	1163.0 (3.22)	1166.2 (2.82)	867.8 (4.89)	904.1 (5.70)	726.1 (3.02)
$\sigma_t^2(R_{t+1})$	0.600 (4.45)	0.612 (5.09)	0.538 (4.13)	0.634 (4.46)	0.658 (9.38)	0.628 (4.60)
ε_t^2	0.274 (2.86)	0.270 (2.79)	0.324 (2.70)	0.190 (2.30)	0.332 (4.73)	0.173 (2.10)
$\delta\varepsilon_t^2$					-0.293 (2.62)	0.078 (0.57)

Notes: 1. Dependent variable is R_{t+1}

2. $C_t(x_{t+1}, y_{t+1}) = \sqrt{h_{xx}(t+1).h_{yy}(t+1)}$, where $h_{xx}(t+1)$ is the conditional variance of x_{t+1}

3. Numbers in parentheses are t-statistics

Table 5b

Estimates of the combined model of the FOREX excess return

The excess return equation in the multivariate GARCH in mean model

t-distribution

Dependent Variable	**US**	**UK**	**US & UK**	**General**	**US & UK**	**General**
			Non-switching model		**switching model**	
Constant				0.644 (0.037)		0.556 (0.04)
$V_t(R_{t+1})$	-0.0004	0.0004		-0.019 (1.09)		-0.019 (1.17)
$C_t(R_{t+1}, f_{t+1} - s_{t+1})$	0.152 (1.18)	-0.077 (0.58)	-0.159 (1.02)	-0.382 (2.00)	-0.071 (0.46)	-0.391 (2.11)
$C_t(R_{t+1}, \Delta i_{t+1}^{us})$	-0.129 (0.99)	-0.088 (0.77)	0.009 (0.07)	0.234 (1.76)	-0.049 (0.38)	0.237 (1.82)
$C_t(R_{t+1}, \Delta c_{t+1}^{us})$	0.028 (2.43)		0.023 (1.55)	0.016 (0.98)	0.023 (1.62)	0.016 (1.01)
$C_t(R_{t+1}, \Delta c_{t+1}^{uk})$		-0.025 (2.98)	-0.033 (4.03)	-0.035 (3.10)	-0.027 (2.98)	-0.036 (3.29)
$C_t(R_{t+1}, \Delta p_{t+1}^{us})$	0.272 (1.70)		0.026 (0.14)	-0.162 (0.85)	0.074 (0.41)	-0.165 (0.87)
$C_t(R_{t+1}, \Delta p_{t+1}^{uk})$		-0.009 (0.20)	0.132 (1.26)	0.091 (0.63)	0.040 (0.38)	0.096 (0.69)
$C_t(R_{t+1}, \Delta m_{t+1}^{us})$	-0.031 (0.24)		-0.051 (0.38)	0.135 (0.64)	0.038 (0.30)	0.138 (0.72)
$C_t(R_{t+1}, \Delta m_{t+1}^{uk})$		-0.036 (0.71)	-0.002 (0.03)	0.049 (0.89)	-0.027 (0.53)	0.049 (0.91)
$C_t(R_{t+1}, \Delta y_{t+1}^{us})$	-0.136 (3.00)		-0.177 (3.47)	-0.105 (2.04)	-0.150 (3.07)	-0.107 (2.10)
$C_t(R_{t+1}, \Delta y_{t+1}^{uk})$		0.056 (3.36)	0.064 (4.10)	0.053 (2.92)	0.071 (4.57)	0.054 (2.91)
R_t				0.273 (4.63)		0.277 (4.23)
$f_t - s_t$				-2.511 (3.69)		-2.483 (3.53)
Conditional variance						
Constant	1184.3 (2.84)	1166.2 (2.48)	1087.5 (2.95)	764.9 (5.92)	983.8 (4.37)	733.9 (3.78)
$\sigma_t^2(R_{t+1})$	0.629 (3.99)	0.626 (4.58)	0.580 (4.56)	0.644 (4.38)	0.640 (8.50)	0.643 (4.40)
ε_t^2	0.241 (2.18)	0.261 (2.34)	0.281 (2.49)	0.138 (2.01)	0.350 (4.64)	0.131 (1.66)
$\delta\varepsilon_t^2$					-0.259 (2.23)	0.017 (0.18)

Notes: 1. Dependent variable is R_{t+1}

2. $C_t(x_{t+1}, y_{t+1}) = \sqrt{h_{xx}(t+1).h_{yy}(t+1)}$, where $h_{xx}(t+1)$ is the conditional variance of x_{t+1}

3. Numbers in parentheses are t-statistics

4. Degrees of freedom = 10