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An Endogenous Growth Model with Productive Public Spending and Uncertain Lifetime Consumers

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Abstract

I develop an endogenous growth model in which sustained long-run growth is due to investment in public capital, the government provides lump-sum transfers, public consumption and investment subsidies, and consumers have uncertain lifetimes. A flexible framework capable of analysing the growth effects of fiscal policy in both infinite and finite horizons cases is provided. The Barro rule (Barro, 1990) for the optimal provision of public investment is extended to the finite horizons case. In contrast with Mourmouras and Lee (1999), the growth maximizing income tax rate is lower in the latter scenario and decreasing in the probability of death parameter. The growth hampering effect of unproductive public spending is depicted in the finite horizons as well as in the infinite horizons case. However, increases in either public consumption or lump-sum transfers reduce long-run economic growth less in the former than in latter case. Furthermore, the growth maximizing level of public investment is increasing in other fiscal policy tools regardless the assumption of uncertain lifetime. Finally, an optimal rule for investment subsidies provision is analytically derived.

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J.E.L. Classification: E62, H54, O41

1 Introduction

This paper focuses on an endogenous growth model in which sustained long-run growth is due to investment in public capital, the government provides lump-sum transfers, public consumption and investment subsidies, and consumers have uncertain lifetimes. The model is an extension of the framework provided by Greiner (1999), departing from it by modelling consumers' lifetime according to the perpetual youth overlapping generation model (Blanchard, 1985). The aim is to analyse the growth effects of varying fiscal policy parameters in infinite as well as finite horizons scenarios, reducing some recent theoretical contributions on this branch of the literature to special cases of a more general framework.

Barro (1990) predicts the existence of an optimal level of public investment financed by a flat rate income tax[1]. Greiner (1999) provides an extension by dividing productive government spending between investment in public capital and subsidies to private investment and including

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[1]On the other hand, with lump-sum taxes, the long-run rate of growth is monotonically increasing in government provision of infrastructure.

in his theoretical framework lump-sum transfers to households and public consumption[2]. His main findings are such that the growth maximizing income tax rate τ_{max} is monotonically increasing in the levels of public consumption, lump-sum transfers to households and subsidies to private investment.

On the other hand, Mourmouras and Lee (1999) analyse the effects of productive government expenditure on growth — abstracting from any other type of government expenditure — combining Blanchard-type consumers with endogenous growth. However, their model is not solved analytically and the finite and infinite horizons scenarios are compared using a numerical simulation. Long-run growth is always lower under the assumption of finite lives compared to the infinite horizons case, but the assumption of uncertain lifetime does not affect the Barro rule. Indeed, the Barro curve is obtained for both the finite and the infinite horizons cases[3], with the optimal level of government investment on infrastructure equating the share of public services in the aggregate production function in both cases.

In this paper, I develop a Ramsey-type model with endogenous growth due to government spending in public capital. As Mourmouras and Lee (1999), I refer to Blanchard (1985) to model consumers' lifetime horizons. However, the optimal lifetime consumption plan is determined as in a standard representative agent model, the only difference being a rate of time preference augmented by a probability of death λ. Such a device makes it possible to build a general framework collapsing to the infinite horizons scenario by simply setting to zero the parameter λ. Furthermore, this allows to analytically derive the role of finite lives in affecting the optimal provision of government spending whereas, in Mourmouras and Lee (1999), λ affects the long-run rate of growth but τ_{max} is independent of λ.

For a null λ, the model departs from Barro (1990) solely for the presence of fiscal policy parameters, others than government expenditure on infrastructure. Namely, lump-sum transfers to households φ_1, public consumption φ_2 and investment subsidies θ_S.

For a positive λ, the model is populated by uncertain lifetime consumers *à la* Blanchard, departing from Mourmouras and Lee (1999) for the fact that I explicitly take into account the effect of a positive λ not only on the long-run rate of growth, but also on the optimal income tax rate. Thus, the Barro rule is extended to the case of finite-lived consumers and it turns out to be dependent on the horizon index. I will refer to such an extension as a *modified Barro rule*: for $\lambda = 0$, the optimal provision of public investment equates the share of public capital in the aggregate production function. However, if consumers live finite lives, such an optimal level will be lowered by the consumption externality due to λ.

The assumption of uncertain lifetime consumers also affects the relationships relating other fiscal policy parameters to long-run economic growth and, as a consequence, their respective impacts on the optimal public investment provision rule.

As for the growth effects of other fiscal policy tools, the long-run rate of growth of the economy γ is lowered by either higher lump-sum transfers to households or public consumption, regardless the value of λ. However, increases in either φ_1 or φ_2 of the same amounts reduce γ less for $\lambda > 0$ than for $\lambda = 0$.

[2]This author, in contrast with Barro, also analyses the different effects of both variations in income and consumption taxation under the alternative assumptions that labour is supplied either inelastically or elastically within an economy populated by an infinitely-lived representative agent.

[3]In contrast with Barro, instead, they depict the existence of the Barro curve in the finite horizons case even in the case of government expenditure financed by lump-sum taxes.

On the other hand, the growth effect of rising investment subsidies is ambiguous for any value of λ. The difference under the assumption of finite lives is that for a positive λ, the growth maximizing level of θ_S is negatively related to λ: as the consumer life horizon increases, the optimal value of θ_S is reached before.

As for the relationships linking τ_{max} to other categories of public expenditure, for $0 \leq \lambda \leq 1$, τ_{max} is increasing in φ_1, φ_2 and θ_S.

The remainder of the paper is organized as follows. The theoretical model will be built in section 2, introducing the behavioral assumptions imposed upon households, firms and the government. The two differential equations describing the overall behavior of the economy will be derived, depicting the role played by the uncertain lifetime hypothesis in decelerating long-term economic growth on the BGP. Section 3 is devoted to the analysis of fiscal policy in the model. The growth effects of fiscal policy tools are derived analytically, with particular attention paid to the definition of the *modified Barro rule* and its relationships with other fiscal policy tools. The model is solved numerically in section 4, with the aim of assessing the existence of the *Barro curve* in the finite horizons case and providing more insight on the expected effects of fiscal policy on growth, as analytically derived in the previous section. Some conclusions will be drawn in section 5. The proofs of all propositions are reported in the Appendix.

2 The Model

The economy is composed of consumers who maximize their lifetime utilities, profit-maximizing competitive firms and the government. Consumers supply labour inelastically and — for a positive λ — can have their savings insured by an insurance company. The aggregate production function shows diminishing returns to scale in private and public capital separately and constant returns to scale in the two forms of capital taken together. The government runs a balanced budget constraint, financing investment in infrastructure through a flat income tax rate, and providing public consumption, investment subsidies to firms and lump-sum transfers to households.

2.1 Firms

The production side of the economy is described by a Cobb-Douglas production function in private capital K and productive public services G. Following Barro (1990), the government purchases a share of private output and uses these purchases to provide free public services to the private sector. These services are assumed to be non rival and non excludable. Since the use of G by a firm does not prevent other users from benefiting from them, it is the total amount of publicly provided services that matters for the firms and enters the production function. This assumption is useful to model a broad concept of public capital, which can be thought as the infrastructure network of a country. Under the assumption that all the services belonging to G are publicly provided with no user fees, G represents an unpaid input of production and, indeed, it plays the role of a positive externality in enhancing the marginal product of private capital[4]. Given these assumptions, the aggregate production function is:

$$Y = K^{1-\alpha}G^{\alpha} = K\left(\frac{G}{K}\right)^{\alpha}; \quad 0 < \alpha < 1 \tag{2.1}$$

[4]This concept of external economy due to G dates back to Meade (1952).

This production function[5] shows diminishing returns to scale in G and K separately, and constant returns to scale in K and G taken together:

$$Y_K > 0; \quad Y_{KK} < 0 \tag{2.2}$$

An increase in G leads to an increase in the marginal product of private capital, which implies $Y_{KG} > 0$. Thus, productive public services cannot easily be provided by the private sector.

Assuming competitive markets, the first order condition for the firms' profit maximization problem requires the real interest rate to equalize the physical marginal product of private capital. This condition is expressed by:

$$r = (1-\alpha)\left(\frac{G}{K}\right)^{\alpha} \tag{2.3}$$

From the definition of the production function (2.1) and the first order condition (2.3), the following condition is derived:

$$rK = (1-\alpha)\,Y < Y; \quad 0 < \alpha < 1 \tag{2.4}$$

Therefore, this model allows the output of the economy (Y) to be larger than the payments to the owners of private capital (rK). This circumstance is due to the additional income induced by public spending through the positive effect on the marginal product of private capital.

The following conditions are introduced and they will be used later in order to obtain the dynamic expression describing the evolution in time of private capital:

$$w + (r+\lambda)\,K + \pi = K^{1-\alpha}G^{\alpha} \tag{2.5}$$

$$T_p = \varphi_1 T = \varphi_1 \tau K^{1-\alpha}G^{\alpha} \tag{2.6}$$

where w is labour income, π are profits, λ is the probability of death faced by households and the term $(r+\lambda)\,K$ is introduced in analogy with the Blanchard model[6].

Condition (2.5) simply states that total income must equate total output of the economy, while (2.6) comes from the definition of government lump-sum transfers to households T_p, as clarified below in section 2.3.

2.2 Consumers

Households are assumed to have uncertain lifetimes according to the model by Blanchard (1985). Hence, we shall assume that they face a constant instantaneous probability of death λ throughout their life. Their expected remaining life is $1/\lambda$ and it is constant throughout their life. Agents are of different ages and have different levels of wealth, but they all have the same propensity to consume. This approach allows for flexibility: the expected life $1/\lambda$ is interpreted as an horizon index that can be chosen anywhere between 1 and infinity to study the effects of the horizons of agents on the behavior of the economy. The limiting case of infinite horizons will occur by letting λ go to zero since this implies that $1/\lambda$ tends to infinity. It is assumed that there is no inter-generational

[5] Where the labour input L is normalized to 1.

[6] Under the assumptions of perfectly competitive markets, constant returns to scale and inelastically labour supply, the terms w and π vanish.

bequest motive which, together with the assumption of uncertain lifetime, implies — as we will see later — a role for an insurance market.

Each consumer does not consider any choice regarding the allocation of her time endowment between labour and leisure. In other words, labour is inelastically supplied and the consumer supplies a constant amount of labour. The expected lifetime utility of the individual i born at time s is given by:

$$U^i = \int_t^\infty \ln C^i e^{-(\rho+\lambda)t} dt \tag{2.7}$$

The instantaneous utility function is assumed to have a logarithmic form. The rate of time preferences ρ is increased by the probability of death λ. The higher the probability of death, the more heavily consumers discount the future[7] and given the assumption that λ is constant throughout consumers' life, it is possible to assume a constant propensity to consume as well. This way of modelling the case of finite horizons can be regarded as an application of Blanchard (1985) to a standard Ramsey-type model with endogenous growth.

The Blanchard model has the merit of allowing for aggregation in OLGs models. On the other hand, it suffers from the backward of abstracting from the life-cycle aspect of the individual consumption behavior. This limitation can be overcome by combining the Blanchard-type consumer with the standard representative agent model. Indeed, by doing so, no aggregation procedure is needed and the relationship between finite horizons and the evolution in time of consumption can be analytically determined by referring the analysis to the representative agent.

The inter-temporal budget constraint faced by the consumer must take into consideration the role played in the economy by the government. For this reason, the consumer budget constraint proposed by Greiner (1999) is modified in order to adapt it to our framework:

$$\dot{K} = \{[w + (r+\lambda) K + \pi](1-\tau) + T_p - C\} \left(\frac{1}{1-\theta_S}\right) \tag{2.8}$$

The rationale behind this budget constraint is that the insurance covers only asset wealth: the consumer receives (pays) λK for every period of her life from (to) the insurance company and the amount K is paid to (cancelled by) the insurance company when the consumer dies. By using the conditions (2.5) e (2.6) and solving the budget constraint for $\dot{K}/K$ we obtain:

$$\frac{\dot{K}}{K} = K^{-\alpha} G^{\alpha} \frac{1-\tau(1-\varphi_1)}{1-\theta_S} + \frac{\lambda(1-\tau)}{1-\theta_S} - \frac{C}{K(1-\theta_S)} \tag{2.9}$$

This expression generalizes the dynamic equation of private capital in Greiner (1999), differing from the latter for the term that includes the parameter λ. We could then apply this expression to both cases of infinitely lived and uncertain lifetime consumers by simply imposing this parameter either equal to zero or to some positive value.

The existence of a unique solution to the households' optimization problem is subject to the condition that K and G are bounded by the increasing function $e^{\gamma t}$, where $0 < \gamma < (\rho + \lambda)$. Provided that such a condition holds, the Pontryagin's maximum principle can be used to derive an optimal solution to (2.7) subject to (2.8), to which is associated the following Hamiltonian:

$$H = \ln C e^{-(\lambda+\rho)t} + \psi \frac{1}{1-\theta_S} \{[w + (r+\lambda) K + \pi](1-\tau) + T_p - C\} \tag{2.10}$$

[7] Cass and Yaari (1967) provide a theoretical proof of the fact that the effect of the probability to death is to raise the individual rate of time preference.

The necessary optimality conditions are obtained by setting the partial derivative of H with respect to the control variable C equal to zero, the partial derivative of H with respect to the state variable K equal to minus the time derivative of the co-state variable ψ and the partial derivative of H with respect to ψ equal to minus the time derivative of K:

$$\frac{\partial H}{\partial C} = 0 \Rightarrow \frac{1}{C} e^{-(\lambda+\rho)t} = \frac{\psi}{1-\theta_S} \tag{2.11}$$

$$\frac{\partial H}{\partial K} = -\dot{\psi} \Rightarrow \psi \frac{(r+\lambda)(1-\tau)}{(1-\theta_S)} = -\dot{\psi} \tag{2.12}$$

$$\frac{\partial H}{\partial \psi} = \dot{K} \Rightarrow \frac{1}{1-\theta_S} \{[w + (r+\lambda) K + \pi](1-\tau) + T_p - C\} = \dot{K} \tag{2.13}$$

The first order conditions (2.11)-(2.13) are also sufficient if the following transversality condition is satisfied:

$$\lim_{t\to\infty} e^{-(\rho+\lambda)t} \psi (K - K^*) \geq 0 \tag{2.14}$$

Substituting (2.11) into (2.12), we obtain:

$$(1-\theta_S) \frac{1}{C} e^{-(\lambda+\rho)t} \frac{(r+\lambda)(1-\tau)}{(1-\theta_S)} = -\dot{\psi} \tag{2.15}$$

where:

$$-\dot{\psi} = \left\{ (1-\theta_S) \left[-\frac{1}{C^2} \dot{C} e^{-(\lambda+\rho)t} - \frac{1}{C} (\lambda+\rho) e^{-(\lambda+\rho)t} \right] \right\} \tag{2.16}$$

Substituting (2.16) into (2.15), the Euler equation is derived:

$$\frac{\dot{C}}{C} = \frac{r(1-\tau) - \lambda(\tau+\theta_S) - \rho(1-\theta_S)}{(1-\theta_S)} \tag{2.17}$$

Imposing $\tau = \theta_S = \lambda = 0$, it is possible to obtain $\dot{C} = (r-\rho) C$, which gives the Euler equation for the case of a CRRA instantaneous utility function with elasticity of substitution equal to 1, when no government issues are considered and consumers live infinite lives.

We can now substitute the profit maximizing condition in the Euler equation in order to derive the equation of motion for consumption:

$$\frac{\dot{C}}{C} = \frac{(1-\tau)}{1-\theta_S} (1-\alpha) \left(\frac{G}{K} \right)^{\alpha} - \frac{\lambda(\tau+\theta_S)}{1-\theta_S} - \rho \tag{2.18}$$

where (2.18) states that consumption is decreasing over time in the subjective rate of discount as well as in the probability of death parameter. A higher value of the discount factor ρ will reduce consumption growth and this effect will be even stronger in the presence of a positive probability of death. The role played by λ in decelerating consumption growth over time can also be seen from the expression for the dynamics of private capital (2.9) where such a parameter enters with a positive sign.

2.3 Government

The government collects taxes T from total income produced in the economy and uses taxes to finance public consumption C_p, lump-sum transfers to households T_p, investment in public capital $\dot{G}$ and investment subsidies to firms $\theta_S \dot{K}$. No public debt issues are considered in the model and the government budget constraint is the same as in Greiner (1999).

Recalling the definition of the production function in (2.1) and assuming that the government uses shares φ_1 and φ_2 of tax revenue for lump-sum transfers to households and public consumption respectively (with φ_1 and φ_2 defined between 0 and 1; $\varphi_1 + \varphi_2 < 1$), the budget constraint is written as follows:

$$\tau K^{1-\alpha} G^{\alpha} = \dot{G} + (\varphi_1 + \varphi_2)\, \tau K^{1-\alpha} G^{\alpha} + \theta_S \dot{K} \tag{2.19}$$

Substituting $\dot{K}$ from the consumer budget constraint, we derive the dynamic equation of public capital:

$$\begin{aligned} \frac{\dot{G}}{G} &= K^{1-\alpha} G^{\alpha-1} \left\{ \tau (1 - \varphi_1 - \varphi_2) - \frac{\theta_S}{1-\theta_S} [1 - \tau (1 - \varphi_1)] \right\} \\ &- \frac{\theta_S}{1-\theta_S} \left[\frac{\lambda K (1-\tau)}{G} - \frac{C}{G} \right] \end{aligned} \tag{2.20}$$

This equation differs from the one in Greiner (1999) for the term $\theta_S/(1-\theta_S) \cdot \lambda K(1-\tau)/G$, which is equal to zero for a null λ. Thus, as for the dynamic equations of consumption and private capital, we have an expression capable of treating the infinite horizon scenario as a limiting case.

2.4 The economy

The economy is described by the following system of differential equations:

$$\frac{\dot{C}}{C} = \frac{(1-\tau)}{1-\theta_S} (1-\alpha) \left(\frac{G}{K} \right)^{\alpha} - \frac{\lambda (\tau + \theta_S)}{1-\theta_S} - \rho \tag{2.21}$$

$$\frac{\dot{K}}{K} = K^{-\alpha} G^{\alpha} \frac{1 - \tau (1 - \varphi_1)}{1-\theta_S} + \frac{\lambda (1-\tau)}{1-\theta_S} - \frac{C}{K (1-\theta_S)} \tag{2.22}$$

$$\begin{aligned} \frac{\dot{G}}{G} &= K^{1-\alpha} G^{\alpha-1} \left\{ \tau (1 - \varphi_1 - \varphi_2) - \frac{\theta_S}{1-\theta_S} [1 - \tau (1 - \varphi_1)] \right\} \\ &- \frac{\theta_S}{1-\theta_S} \left[\frac{\lambda K (1-\tau)}{G} - \frac{C}{G} \right] \end{aligned} \tag{2.23}$$

The system (2.21)-(2.23) does not have a rest point and the usual procedure to determine the steady state and to analyse its properties fails to provide a unique solution for it. Therefore, the variables C and G are normalized by K by defining the ratios $x = G/K$ and $c = C/K$, which respectively express public capital and private consumption in terms of private capital. The definition of the two new variables x and c implies:

$$\frac{\dot{x}}{x} = \frac{\dot{G}}{G} - \frac{\dot{K}}{K}; \quad \frac{\dot{c}}{c} = \frac{\dot{C}}{C} - \frac{\dot{K}}{K} \tag{2.24}$$

By doing so, the system (2.21)-(2.23) is reduced to:

$$\begin{aligned}\dot{x} &= x^{\alpha}\left\{\tau\left(1-\varphi_1-\varphi_2\right)-\frac{\theta_S}{1-\theta_S}\left[1-\tau\left(1-\varphi_1\right)\right]\right\}-x^{\alpha+1}\frac{1-\tau\left(1-\varphi_1\right)}{1-\theta_S}\\ &+ \left(x+\theta_S\right)\left[\frac{c}{1-\theta_S}-\frac{\lambda\left(1-\tau\right)}{1-\theta_S}\right]\end{aligned} \tag{2.25}$$

$$\begin{aligned}\dot{c} &= c\left[x^{\alpha}\frac{\left(1-\tau\right)}{1-\theta_S}\left(1-\alpha\right)-\frac{\lambda\left(1+\theta_S\right)}{1-\theta_S}-\rho\right]-cx^{\alpha}\frac{1-\tau\left(1-\varphi_1\right)}{1-\theta_S}\\ &+ \frac{c^2}{1-\theta_S}\end{aligned} \tag{2.26}$$

Proposition 1 *There exists a BGP with endogenous growth for the economy described by (2.25)-(2.26) and such a BGP is unique.*

The system (2.25)-(2.26) will have a steady state solution which will correspond to the balanced growth path (BGP) of the original system (2.21)-(2.23). In such a steady state the variables in the model will grow at the same rate and the long-run growth rate of the economy will be given by:

$$\gamma=\frac{\dot{C}}{C}=\frac{\left(1-\tau\right)}{1-\theta_S}\left(1-\alpha\right)\left(\frac{G}{K}\right)^{\alpha}-\frac{\lambda\left(\tau+\theta_S\right)}{1-\theta_S}-\rho \tag{2.27}$$

Hence, the long-run rate of growth is decreasing both in the rate of time preferences ρ and in the probability of death parameter λ. Thus, we are able to capture the decelerating effect on economic growth caused by λ. From (2.27) it is clear that:

$$\left.\frac{\dot{C}}{C}\right|_{(\lambda=0)}>\left.\frac{\dot{C}}{C}\right|_{(\lambda>0)} \tag{2.28}$$

Consumers with infinite lives are willing to postpone consumption in the future and to increase current saving. This behavior leads to a higher long-run growth rate. An increase in λ, *ceteris paribus*, is always associated with a lower long-run rate of growth of the economy (and with higher steady state values for both x and c).

The system (2.25)-(2.26) is similar to the one in Greiner (1999), who in turn refers to the model by Futagami *et al.* (1993). The system that we have produced departs from Greiner (1999) due to the inclusion of the probability of death and then it can be easily reduced to that form in order to make our results comparable with his conclusions. Moreover, our system collapses to that used by Futagami *et al.* (1993) when $\theta_S=\varphi_1=\varphi_2=0$ and $\lambda=0$.

The economy in Futagami *et al.* (1993) is characterized by saddle-path stability but it is assumed that tax revenues are used for public investment only. On the other hand, Greiner (1999) proves that the model is both locally and globally determinate, arguing that: "With inelastic labour supply there exists at most one BGP with endogenous growth and the Jacobian matrix of the system has one positive and one negative real root, *i. e.*, the rest point of the system is the saddle path". This implies that there exists a unique value for the initial level of consumption, which can be chosen freely by the household, such that the economy converges to the stable BGP in the long-run.

Proposition 2 *The Jacobian matrix of the system (2.25)-(2.26) has one positive and one negative real root, which implies that the unique BGP is stable.*

Departing from the literature cited above for the assumption of uncertain lifetime consumers, the model needs to be analysed in detail about its mathematical properties. In the light of this, sections B and C in the Appendix (pages 33-36) report the proofs of propositions 1 and 2.

3 Fiscal Policy

Given the theoretical framework provided above, it is possible to analyse the growth effects of changes in fiscal parameters, depicting the role played by the uncertain lifetime hypothesis. The following subsections 3.1-3.3 deal with the relationships between fiscal policy tools and long-term economic growth. In particular, the growth hampering effect of a rise in either public consumption or lump-sum transfers is described in 3.1. In subsection 3.2, it is shown the growth maximizing income tax rate and how its value is influenced by the presence of public consumption and transfers to households. Finally, subsection 3.3 focuses on the ambiguous effect of investment subsidies on growth and their relationship with the growth maximizing income tax rate. All the analytical results shown below are derived in detail in the Appendix (see section D, pages 36-39).

3.1 Public consumption and lump-sum transfers

The share of government expenditure devoted to public consumption has been modelled with the parameter φ_2. Starting with the infinite horizons case, an increase in public consumption implies that more resources will be devoted to unproductive purposes as opposed to public investment and private investment subsidies. As a direct consequence, productive public expenditure will decrease, which in turn will negatively affect growth. The expected effect of an increase in the parameter φ_2 is hence a decline in the balanced growth rate γ. Such a decelerating effect on long-term economic growth will be reflected by a smaller steady state value of $x = G/K$, as shown in equation (2.27).

Given a positive probability of death, let us consider the consequences of increasing public consumption. The key feature to be taken into account is that, compared to the infinite horizons scenario and other things being equal — as shown in equation (2.27) — the economy will always grow at a lower rate in the long-run. An increase in φ_2, is still expected to impact negatively long-term economic growth, but will such an impact be more or less effective than in the infinite horizons scenario?

Proposition 3 *The long-run rate of growth of the economy γ is decreasing in public consumption φ_2. Increases in φ_2 of the same amount reduce γ less for $\lambda > 0$ than for $\lambda = 0$:*

$$\frac{\partial \gamma}{\partial \varphi_2} < 0 \tag{3.1}$$

$$\left.\frac{\partial \gamma}{\partial \varphi_2}\right|_{(\lambda>0)} < \left.\frac{\partial \gamma}{\partial \varphi_2}\right|_{(\lambda=0)} \tag{3.2}$$

According to proposition 3, the decrease in the long-run rate of growth caused by an increase in φ_2 will be lower when $\lambda > 0$. As formally proved in the Appendix — see subsection D.1 — this is due to a smaller negative impact of the increase in φ_2 on the steady state value of x.

Increasing transfers to households (a higher value for the parameter φ_1) will lead to two opposite effects. On one side, a smaller share of total government expenditure will be devoted to productive uses, implying a reduction in the balanced growth rate. On the other hand, an income effect will take place, making consumers richer than before. However, provided that transfers are lump-sum, these will not affect decisions concerning the allocation of private resources between consumption and savings. Thus, the only expected effect will be the first one and a lower long-run growth rate is predictable. This is also true in the presence of a positive λ, but once again the question of interest is whether or not an increase in households transfers will affect economic growth in the same way.

Proposition 4 *The long-run rate of growth of the economy γ is decreasing in lump-sum transfers φ_1. Increases in φ_1 of the same amount reduce γ less for $\lambda > 0$ than for $\lambda = 0$:*

$$\frac{\partial \gamma}{\partial \varphi_1} < 0 \tag{3.3}$$

$$\left.\frac{\partial \gamma}{\partial \varphi_1}\right|_{(\lambda>0)} < \left.\frac{\partial \gamma}{\partial \varphi_1}\right|_{(\lambda=0)} \tag{3.4}$$

As for the case of higher public consumption, when the government decides to increase transfers to households, under the hypothesis of uncertain lifetime consumers, the decline in the long-run rate of growth of the economy will be lower compared to the infinite horizons scenario.

3.2 Public investment

Rising the income tax rate τ yields two effects operating in opposite directions. In the infinite horizons case, given an increase in τ, the first effect to be taken into consideration is the higher taxation on returns on capital, which implies a disincentive to save and, as a consequence, a reduction in private investment with the effect of lowering long-term economic growth. However, an opposite effect will take place: for a given level of income, a higher income tax rate implies higher tax revenues which in turn leads to higher investment in public capital and accelerates economic growth. Thus, the net effect of an increase in τ might be either positive or negative, depending on whether the second effect offsets the first one. The well-known *Barro rule* states that the optimal provision of public investment implies that a unit increase in government spending implies a unit increase in output. With a Cobb-Douglas production function, this means that the optimal government spending is equal to its share in the production function.

Greiner (1999) concludes that higher levels of unproductive public spending and/or generous investment subsidies, will force the government to increase productive investment in order to compensate for the negative effect on the long-run rate of growth.

The model presented in the previous section assumes the same fiscal policy tools proposed by Greiner (1999), generalizing his framework to include the case of finite horizons. Hence, the question of how the optimal level of τ might be affected by the uncertain lifetime hypothesis arises. Mourmouras and Lee (1999) find τ_{max} to be independent of the probability of death parameter. In other words, the optimal level of public investment provision does not depend on the consumption externality due to the uncertain lifetime hypothesis. They find the *Barro rule* to be satisfied both in finite and infinite lives scenarios, pointing out that this rule is only determined by the production side of the economy, which they model as in Barro (1990). On the other hand, we expect that the new growth maximizing level of τ will be affected by the different effect on the disincentive to save caused by the higher taxation on returns on capital. Indeed, once the probability of death is introduced in the model, the growth-maximizing income tax rate becomes:

$$\tau_{max}|_{(\lambda>0)} = \frac{\alpha x^{\alpha}(\alpha-1)(x+\theta_S)}{x^{\alpha+1}(\alpha-1)-\lambda x+\lambda\alpha(x+\theta_S)} \tag{3.5}$$

Proposition 5 *There exists a growth maximizing income tax rate τ_{max} both in the infinite and the finite horizons scenarios, the first one being higher than the latter one:*

$$\tau_{max}|_{(\lambda>0)} < \tau_{max}|_{(\lambda=0)} \tag{3.6}$$

Hence, the growth maximizing level of the income tax will be reached before if consumers have uncertain lifetime than in the infinite horizons case. In contrast with Mourmouras and Lee (1999), optimal public investment turns out to be dependent on the horizon index. Such a relationship is depicted in (3.5), but it can be easily seen by directly compare our set up with the model provided by Barro (1990). This can be done by simply deleting θ_S in (3.5). By doing so, a *modified Barro rule* is obtained:

$$\tau_{max}|_{(\lambda>0,\theta_S=0)} = \frac{x\alpha}{x+\lambda} \tag{3.7}$$

For $\lambda = 0$, (3.7) is equivalent to the *Barro rule*: the optimal provision of public investment is given by the share of public capital in the aggregate production function. However, if consumers live finite lives, such an optimal level will be lowered by the consumption externality due to λ.

Under the hypothesis of uncertain lifetime consumers, the relationships linking such an optimal rule for public investment to other fiscal policy tools will be also affected. As for unproductive public expenditure our results are summarized in proposition 6.

Proposition 6 *For $0 \leq \lambda \leq 1$, the growth maximizing income tax rate τ_{max} is increasing in both public consumption φ_2 and lump-sum transfers to households φ_1.*

$$\left.\frac{\partial \tau_{max}}{\partial \varphi_i}\right|_{(0\leq\lambda\leq1)} > 0, \quad i = 1,2 \tag{3.8}$$

Hence, regardless the value of λ, in the presence of higher unproductive public expenditures, the need for a higher provision of public investment turns out to be always necessary[8].

3.3 Investment subsidies

The last fiscal policy tool to be considered are investment subsidies, represented by the parameter θ_S. As for the two categories of unproductive public spending, let us consider the growth effect of varying θ_S and the relationship linking this parameter to τ_{max}.

It is evident from the F.O.C. (2.11) that a change in θ_S affects consumers' marginal utility: higher investment subsidies lead to a reduction in the marginal utility for each given level of

[8]For the proofs of propositions 5 and 6, see subsection D.2 in the Appendix.

consumption. Moreover, from the consumer budget constraint it turns out that an increase in the parameter θ_S causes private investment to be cheaper. These two effects combined together will shift resources from consumption to investment in the private sector by increasing the opportunity cost of consumption. Thus, one will expect that the rate of growth of the economy will increase after the government's decision to provide the private sector with higher investment subsidies.

On the public side of the economy, however, devoting more resources to investment subsidies implies a depletion of resources from investment in public capital and thus a lower long-run rate of growth. As a consequence, the net effect resulting from the combination of the two effects in the private and in the public sectors is ambiguous. However, Greiner (1999) claims that there exists a growth-maximizing value for investment subsidies and that if it is in the interior $(0, 1)$ it will be determined by the elasticity of x with respect to θ_S on the balanced growth rate. The analysis provided by Greiner (1999) refers to the infinitely lived representative consumer case; what if we impose a positive probability of death? After the decision of the government to increase θ_S, the two opposite effects described above will occur again. On the public side of the economy, the decline in the share of public spending devoted to productive use will be the same as in the infinite lives scenario. On the other hand, we expect that the growth enhancing effect due to the decline in the marginal utility for each level of consumption will be larger than in the former case because of the presence of the probability of death. Hence, the growth maximizing level of θ_S will have a smaller value: it should be reached earlier than in the infinite horizons case.

Proposition 7 *There exists a growth maximizing value for investment subsidies, and such a value is decreasing in the probability of death parameter λ:*

$$\frac{\partial \gamma}{\partial \theta_S}|_{(\lambda=0)} > (\leq) 0 \quad \text{when} \quad \frac{\partial x}{\partial \theta_S}\frac{\theta_S}{x} > (\leq) - \frac{\theta_S}{\alpha(1-\theta_S)} \tag{3.9}$$

$$\left.\frac{\partial \gamma}{\partial \theta_S}\right|_{(\lambda>0)} > (\leq) 0 \quad \text{when} \quad \frac{\partial x}{\partial \theta_S}\frac{\theta_S}{x} > (\leq) - \frac{\theta_S}{\alpha(1-\theta_S)} + \frac{\lambda(1+\tau)}{(1-\theta_S)^2} \tag{3.10}$$

As for the impact of higher investment subsidies on the growth maximizing income tax rate, in analogy with public consumption and lump-sum transfers to households, our result is summarized in proposition 8[9].

Proposition 8 *For $0 \leq \lambda \leq 1$, the growth maximizing income tax rate τ_{max} is increasing in investment subsidies.*

$$\left.\frac{\partial \tau_{max}}{\partial \theta_S}\right|_{(0\leq\lambda\leq1)} > 0 \tag{3.11}$$

4 Simulation

The model has been solved numerically with two main objectives. The first aim is the comparison of the steady state solution of the system (2.25)-(2.26) across alternative life horizons scenarios. This aim is pursued in subsection 4.1 by evaluating the steady state values of x and c and the

[9]For the proofs of proposition 7 and 8, see subsection D.3 in the Appendix.

corresponding long-run rate of growth γ, for alternative values of the probability of death parameter λ.

The second objective is related to the analysis of fiscal policy under the assumption of uncertain lifetime consumers and its comparison with the infinite lives scenario. Subsections 4.2-4.4 report the results of a number of simulations of the model aimed at evaluating the growth impacts of varying fiscal policy parameters under alternative values of λ.

The numerical values of all the parameters involved in this analysis have been chosen to make the results as much as possible comparable with the ones obtained in previous studies. The initial values used for φ_1 and φ_2 are 0.35 and 0.40 respectively. The value of the public capital share α usually used in the literature is around 0.30. Greiner (1999) uses 0.30 and so we choose to adopt this value. On the other hand, Barro (1990) uses 0.25 in his simulation, which implies a higher private capital share. With regard to the rate of time preferences ρ, the range of the values commonly used is between 0.01 and 0.04, which implies that the consumer is assumed to use an annual discount rate varying between 1% and 4%. Following Greiner (1999) and recalling that the model is concerned with the behavior of the economy in the long-run, we assume that one time period includes a spell of five years and we set the annual discount rate at 0.04, which will imply imposing $\rho = 0.2$. Finally, the income tax rate and the investment subsidies parameters are initially set at 0.15 and 0.10 respectively.

The complete set of results is reported in Tables 1-12 (see Appendix A, pages 22-32). Alternative horizons scenarios are compared letting λ assuming the values 0, 0.03 and 0.06.

4.1 The impact of the horizon index

The system used in the numerical simulation for solving the model is given by:

$$\begin{aligned}\frac{\dot{x}}{x} &= x^{\alpha-1}\left\{\tau\left(1-\varphi_1-\varphi_2\right)-\frac{\theta_S}{1-\theta_S}\left[1-\tau\left(1-\varphi_1\right)\right]\right\} \\ &- x^{\alpha}\frac{1-\tau\left(1-\varphi_1\right)}{1-\theta_S}+\left(1+\frac{\theta_S}{x}\right)\left[\frac{c}{1-\theta_S}-\frac{\lambda\left(1-\tau\right)}{1-\theta_S}\right] \end{aligned} \tag{4.1}$$

$$\frac{\dot{c}}{c}=\left[x^{\alpha}\frac{\left(1-\tau\right)}{1-\theta_S}\left(1-\alpha\right)-\frac{\lambda\left(1+\theta_S\right)}{1-\theta_S}-\rho\right]-x^{\alpha}\frac{1-\tau\left(1-\varphi_1\right)}{1-\theta_S}+\frac{c}{1-\theta_S} \tag{4.2}$$

which is equivalent to (2.25)-(2.26). By imposing $\dot{c}/c = \dot{x}/x = 0$, and solving for x and c, we find the steady state solutions of the two variables. When the analysis is carried out in the finite horizons case, one would expect the steady state solutions to change. This is due to the fact that the steady state values for x and c are affected by the probability of death: the higher the probability of death, the higher they will be. Since for a higher probability of death, households have a disincentive to postpone consumption in the future, the steady state level of consumption to private capital ratio is increasing in λ. The steady state solution for x provides the value of the ratio G/K at which consumption is constant over time. As the probability of death increases, x will increase, since a larger amount of government spending is required in order to promote economic growth, thus compensating for the negative effect on growth caused by the higher level of current consumption.

Figures 8-10 in the Appendix show the curves $\dot{c}/c = 0$ and $\dot{x}/x = 0$ for $\lambda = 0$, $\lambda = 0.03$ and $\lambda = 0.06$ respectively. In each of these three cases, the interception of the two curves gives steady state values for x and c. Both x and c steady state values increase with the probability of death.

The solution of the model for $\lambda = 0$ is ($x = 0.0711$, $c = 0.3191$). When the probability of death is set at 0.03 the new steady state solution is given by $x = 0.0758$ and $c = 0.3549$; when $\lambda = 0.06$, we find $x = 0.0808$ and $c = 0.3906$.

The balanced growth rate is $\gamma = 0.01983$ in the infinite horizons case, $\gamma = 0.01934$ for $\lambda = 0.03$, and $\gamma = 0.01883$ when $\lambda = 0.06$. Thus, the balanced growth rate is lower under the uncertain lifetime assumption and is decreasing in the probability of death parameter. The role played by λ is to increase the propensity to consume: the higher the probability of death, the higher the willingness of consuming today, and this circumstance negatively affects the long-run rate of growth. In the general case $0 \leq \lambda \leq 1$, γ is linked to λ by the linear relationship shown in Figure 1.

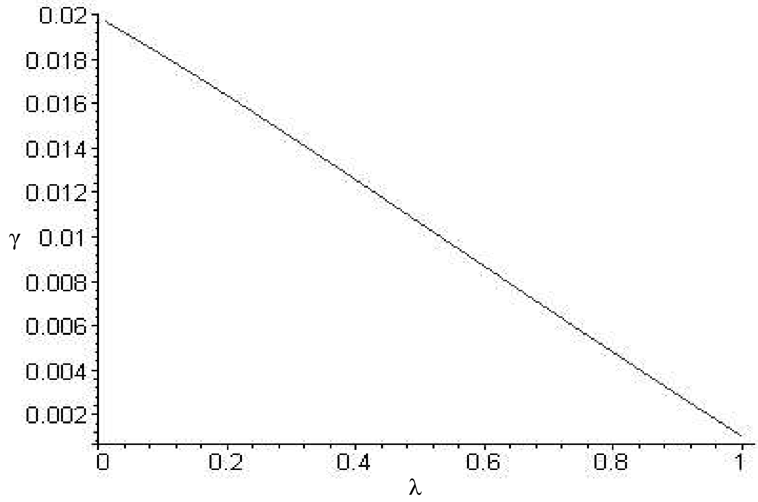

Figure 1: The Relationship between γ and λ

4.2 The impact of public consumption and lump-sum transfers

Let us consider the impact of increasing public consumption. After increasing φ_2 from 0.40 to 0.45, the balanced growth rate is lowered as expected in the three scenarios considered ($\lambda = 0$, $\lambda = 0.03$ and $\lambda = 0.06$, see Table 1, Appendix A, page 22). A higher share devoted to public consumption causes a decline in the share of government expenditure devoted to productive purposes and, as a consequence, economic growth is negatively affected. The peculiar property of the finite horizons case is the following: given the same increase in φ_2, the negative effect on growth is smaller than in the infinite horizons case. The percentage decline in γ is denoted with γ_1 in Table 1. Starting from the initial steady state, after an increase in public consumption the growth rate decreases less in the finite horizon case than in the infinite one.

Table 2 in the Appendix refers to a fiscal policy experiment similar to the one described above, the only difference being the increase in φ_1 instead of φ_2. Thus, the objective of the analysis is to describe the impact of increasing lump-sum transfers to households on long-term economic growth, other things being equal. Letting φ_1 varying from 0.35 to 0.40, the long-run rate of growth is lowered both when $\lambda = 0$ and for a positive probability of death, the latter case being characterized by a smaller reduction in γ (see γ_1 in Table 2).

Hence, increases in lump-transfers and public consumption are both less effective in lowering the long-run rate of growth of the economy under the assumption of uncertain lifetime than in the infinite horizon scenario. We note that the only distinguishing feature is that when the government switches resources from public consumption to lump-sum transfers, the steady state value of c becomes higher.

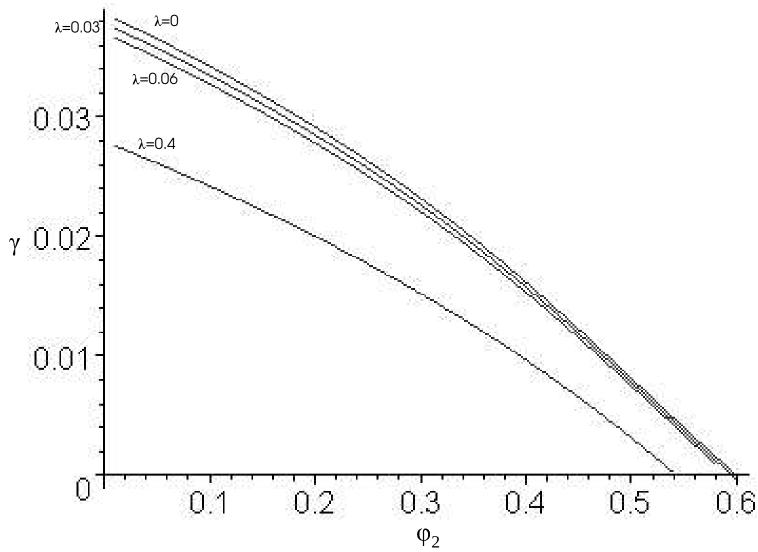

Figure 2: The relationship between γ and φ_2

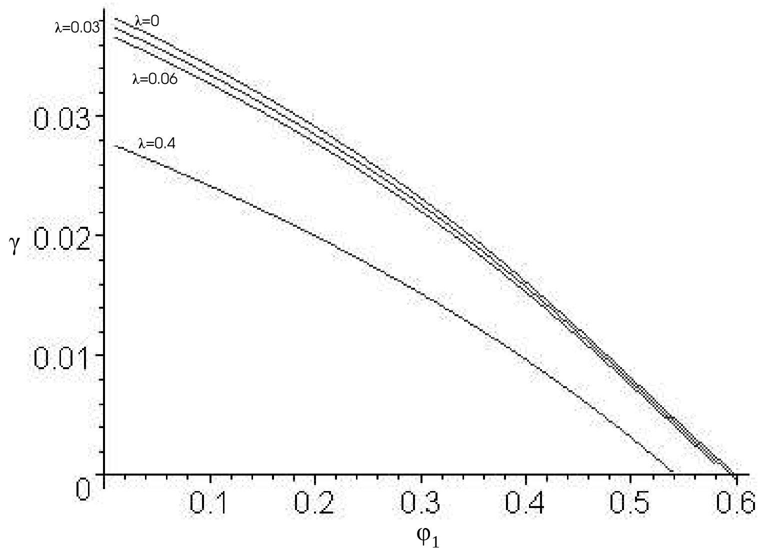

Figure 3: The relationship between γ and φ_1

Figures 2 and 3 illustrate the relationship between γ and φ_i $(i = 1, 2)$, in a more general case. Indeed, the two pictures are obtained by letting vary φ_2 (Figure 2) and φ_1 (Figure 3) between 0 and 0.60, holding fixed the share devoted to the other category of unproductive government expenditure. In both cases, the three top curves (starting with the highest one) refer to values for λ of 0, 0.03, and 0.06 respectively. The lowest and flattest curve represents the limiting case of $\lambda = 0.4$[10]. As pointed out in propositions 3 and 4, an increase in either φ_1 or φ_2 leads to smaller and smaller reductions in the balanced growth rate, as the probability of death parameter is set to a higher value.

4.3 The impact of public investment

In order to simulate an increase in public investment in infrastructure services, we let vary the income tax rate parameter τ between 0.15 and 0.50. The main interest of this experiment is to analyse the differences that emerge by assuming different values for λ. Namely, we want to test the existence of the Barro curve. This result is non trivial for two main reasons. First, the model includes additional categories of expenditures with respect to Mourmouras and Lee (1999). Second and more importantly, the consumption externality due to the finite lives assumption is likely to affect the determination of τ_{max}. Holding fixed the values of all other parameters at their respective starting levels, the model is solved for values of τ varying from 0.15 to 0.50. The results of this simulation are shown in Tables 3-5 in the Appendix (pages 23-25).

Starting from the infinitely lived consumers case (Table 3), it can be seen that γ increases for higher values of τ up to a point, after which it starts falling. The balanced growth rate reaches its maximum value for $\tau = 0.383$. Thus, this is the optimal value of the income tax rate. A similar behavior can be depicted under the uncertain lifetime hypothesis in Tables 4 and 5. For increasing values of the income tax rate, the balanced growth rate increases up to a point and then it goes down: the relationship between γ and τ takes the form of a hump-shaped curve in the finite horizons case.

In Figure 4, the values of γ are plotted against the values taken by τ when $\lambda = 0$, $\lambda = 0.03$ and $\lambda = 0.06$. The highest curve refers to the infinite horizon case, the lowest one to a probability of death equal to 0.06. As argued above, the finite horizon case is always characterized by a lower balanced growth rate and this implies a Barro curve closer to the x-axis. For each given value of τ, γ is decreasing in λ for the role played by the probability of death in reducing economic growth. Hence, the higher λ, the lower the Barro curve. This result is in contrast with Mourmouras and Lee (1999), who find the optimal role for public investment provision to be independent of λ, due to the fact that the Barro rule only arises from the production side of the economy. The present framework, instead, captures the consumption externality effect of λ, explicitly accounting for its impact on optimal fiscal policy.

Indeed, as shown in Tables 4 and 5, when the probability of death is fixed at 0.03 the growth maximizing income tax rate is lower with respect to the case of $\lambda = 0$ and it is even lower for $\lambda = 0.06$. In the first case, the maximum value of γ (0.02830) is achieved when $\tau_{max} = 0.359$, while the growth maximizing income tax rate becomes 0.335 when λ is set at 0.06 (with $\gamma = 0.02614$).

This is coherent with the result summarized in proposition 5. The negative relationship between λ and τ_{max} is shown in Figure 5 for the more general case $0 \leq \lambda \leq 1$.

[10]Both maximum values of φ_i and λ, have been chosen to satisfy the condition $0 < \gamma < (\rho + \lambda)$.

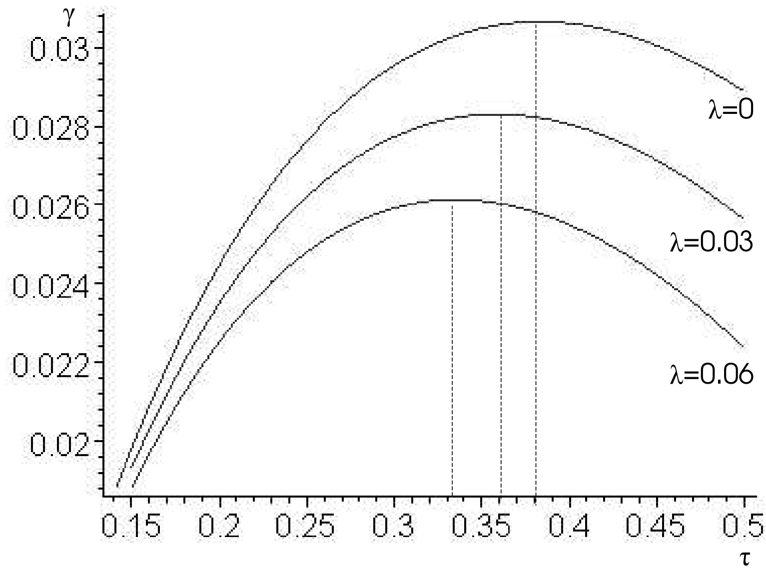

Figure 4: Barro Curve, Finite and Infinite Horizons

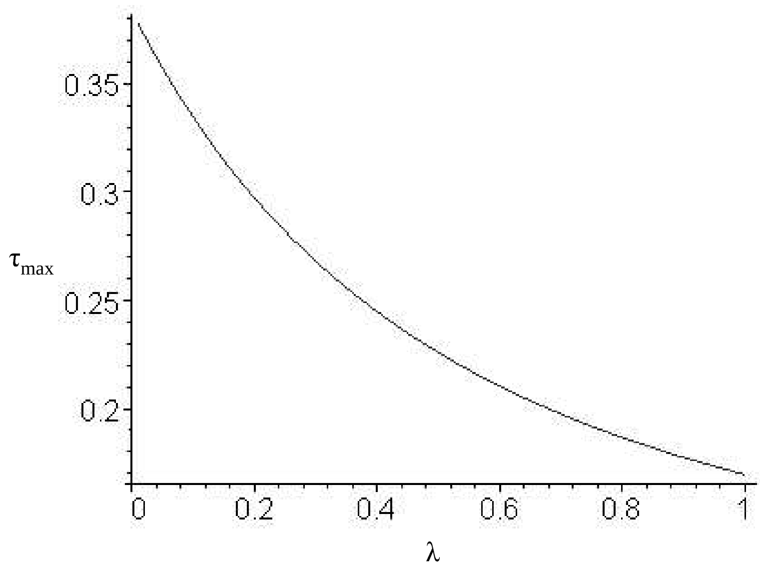

Figure 5: The relationship between τ_{max} and λ

4.3.1 The impact of public investment with higher public consumption and higher lump-sum transfers

As argued in proposition 6, when the government provides the economy with higher shares of either transfers to households or public consumption, it will also need to increase public investment in order to offset the negative effect on economic growth caused by the higher unproductive use of its resources. In order to consider this fact, we solve the model assuming that the government devotes a share $\varphi_2 = 0.45$ to public consumption, comparing the outcome with the original scenario in which φ_2 was set at 0.40 (Tables 3-5). The expected outcome will be a higher optimal income tax rate and the results of this experiment are shown in Tables 6-8 (pages 26-28) in the Appendix. In the infinite horizons case τ_{max} is 0.394 (Table 6), which confirms our expectation. For values of the probability of death equal to 0.03 and 0.06, the same behavior is observed: τ_{max} increases to 0.368 (Table 7) and 0.342 (Table 8) respectively.

As shown in section 4.2, an increase of the same amount either in φ_2 or in φ_1 will have an identical impact on the balanced growth rate. As a consequence, after an increase of either φ_2 or φ_1, τ_{max} will be affected in the same way. For this reason, the results of an increase in φ_1 are not shown in the Appendix.

Figure 6 shows the increasing relationship between unproductive uses of government resources and the optimal income tax rate.

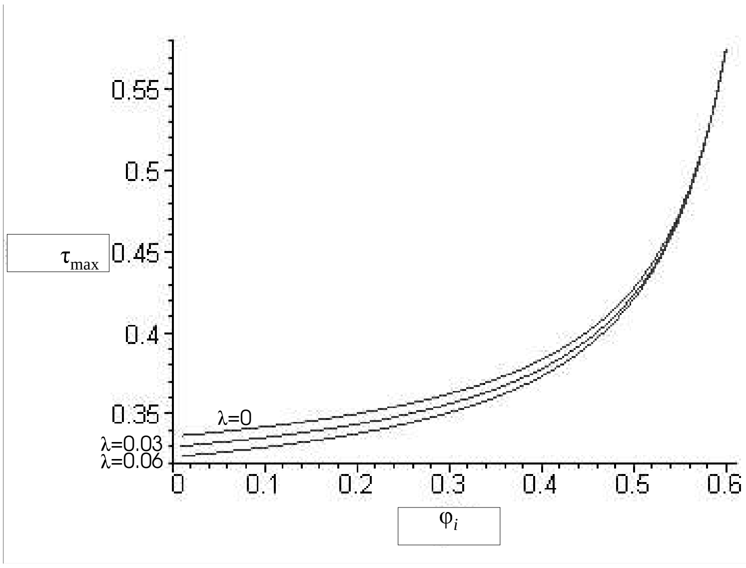

Figure 6: The relationship between τ_{max} and φ_i

4.3.2 The impact of public investment with higher investment subsidies

According to Proposition 8, the growth maximizing income tax rate is increasing not only in unproductive government expenditure, but also in the investment subsidies parameter θ_S. Tables 9-11 report the results of an increase in θ_S from its starting value of 0.10 to 0.12, when λ is set to 0, 0.03 and 0.06 respectively. As stated before, increasing investment subsidies leads to the need for a higher optimal provision of investment in infrastructure. Such an impact on τ_{max} is evident by comparing the results in Tables 9-11 with the ones reported in Tables 3-5: in the infinite horizons scenario, τ_{max} increases from 0.383 to 0.399 (Table 9); for $\lambda = 0.03$, τ_{max} increases from 0.367 to 0.374 (Table 10); when $\lambda = 0.06$, τ_{max} increases from 0.335 to 0.35 (Table 11). In the more general case of θ_S ranging from 0 to 0.15, such an increasing relationship between τ_{max} and θ_S is shown in Figure 7.

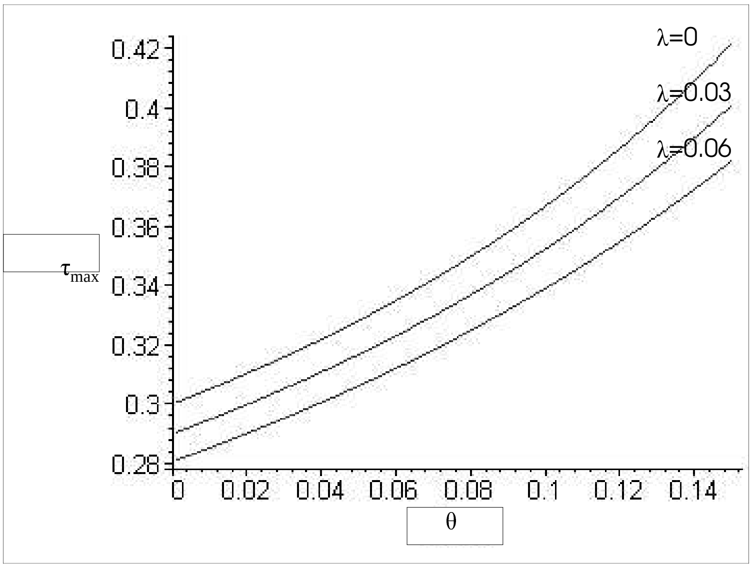

Figure 7: The relationship between τ_{max} and θ_S

4.4 The impact of investment subsidies

The last experiment assesses the existence of a growth maximizing value for investment subsidies as stated in proposition 7. For this simulation, the income tax rate is set at 0.425[11] and the results are reported in Table 12 in the Appendix. The optimal value of θ_S in the infinite horizon case is found to be 0.113. For this value of θ_S, γ reaches it maximum value (0.03041). A slightly lower θ_{max} (0.012) is obtained by re-running the experiment for a value of λ equal to 0.03. This provides a confirmation of the intuitive explanation provided to justify a smaller value for θ_{max} in the finite horizons case. By setting a probability of death at 0.06, θ_{max} becomes 0.104.

5 Conclusions

This paper was aimed at studying the growth effects of fiscal policy in a Barro-type endogenous growth model with finite horizons. The model provides a flexible framework capable of studying the growth effects of fiscal policy both in infinite and finite horizons scenarios and reducing to limiting cases some recent Barro-type models. The optimal lifetime consumption plan has been determined within a standard representative agent model, the only difference being a rate of time preference augmented by a positive probability of death parameter. The government was assumed to run a balanced budget constraint, equating total expenditures to total revenues collected by levying a flat-rate income tax. I have distinguished between productive and unproductive categories of government expenditures. Productive public spending includes investment in public capital and private investment subsidies. On the other hand, public consumption and lump-sum transfers to households were assumed to be unproductive.

Comparing the two alternative scenarios of finite and infinite horizons, I have obtained results

[11]The same value used by Greiner (1999) has been chosen for comparability purposes.

on (i) the growth effects of each category of government expenditures on long-run economic growth, and (ii) the relationships relating the Barro rule to the other categories of government expenditure.

Regarding the first set of conclusions, both categories of unproductive government spending are shown to have a decelerating effect on long-run growth. This result is in line with the existing literature and verified regardless the assumption of uncertain lifetime. However, a rise of either lump-sum transfers to households or public consumption reduces the long-run rate of growth less in the finite than in the infinite horizon scenario. On the other hand, the growth effects of the two categories of productive expenditures are ambiguous, and for both I have derived a growth maximizing value. As for public investment, the Barro rule still holds in the infinite horizon scenario but, in contrast with the existing literature, is negatively linked to the probability of death parameter. This implies that the growth maximizing level of public investment is lower under the assumption of uncertain lifetime. Similarly, the growth maximizing level of private investment subsidies is reached earlier in the finite than in the infinite horizons scenario.

Relative to the second set of conclusions, the effects of public consumption, lump-sum transfers to households and investment subsidies on the optimal provision of public investment are similar. Indeed, it is shown that the growth maximizing level of public investment tends to increase in the presence of higher levels of other categories of expenditures. This result takes place regardless the assumption on uncertain lifetime.

The model is based on some restrictive hypothesis and, as a consequence, can be extended along a number of directions. For instance, the assumption of no labour-leisure choice could be relaxed in favor of the assumption of endogenous labour supply, providing an extension on the consumption side. A further extension to the model could interest the government side, where it is assumed the absence of public debt and it is solely considered the case of proportional income taxation. Hence, a further extension of the model could cover the analysis of alternative mixes of different categories of expenditures and different structures of taxation — distortionary and non-distortionary — and/or alternative sources of financing — deficit or taxation. Finally, it would be worthwhile to use this theoretical framework in order to account for welfare considerations.

Appendix

A Fiscal Policy Experiment

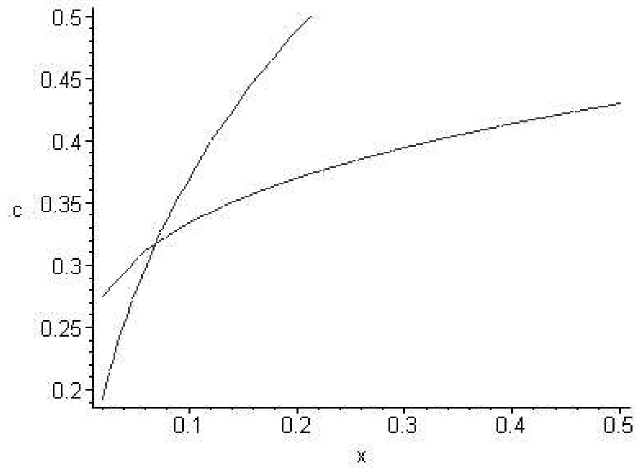

Figure 8: Steady State solution ($\lambda = 0$)

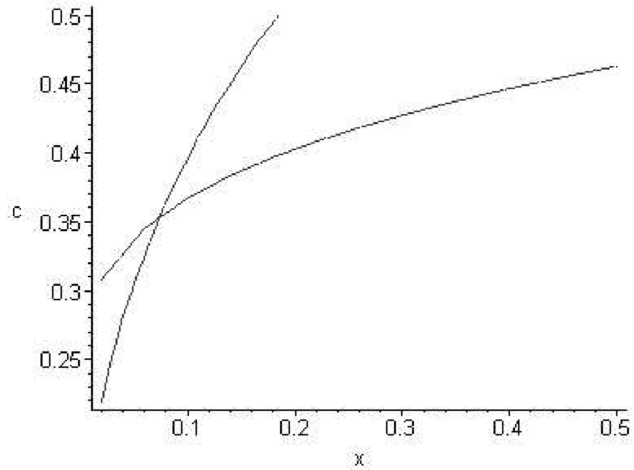

Figure 9: Steady State solution ($\lambda = 0.03$)

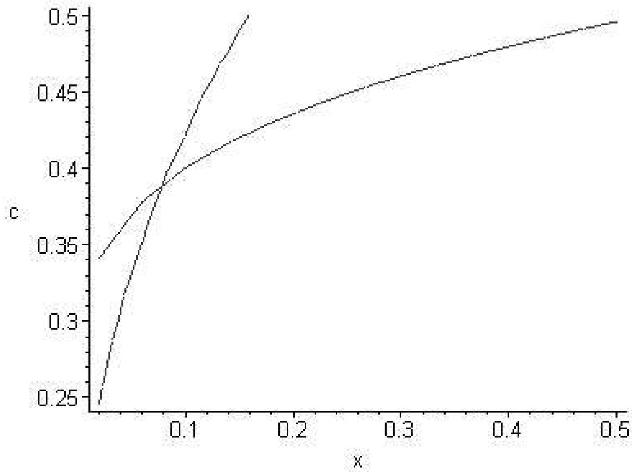

Figure 10: Steady State solution ($\lambda = 0.06$)

	$\lambda = 0$		$\lambda = 0.03$		$\lambda = 0.06$	
	$\varphi_2 = 0.40$	$\varphi_2 = 0.45$	$\varphi_2 = 0.40$	$\varphi_2 = 0.45$	$\varphi_2 = 0.40$	$\varphi_2 = 0.45$
x	0.0711339	0.057632	0.075898	0.062004	0.080825	0.066544
c	0.319145	0.310630	0.358338	0.346528	0.394106	0.382389
γ	0.019831	0.016170	0.019340	0.015749	0.018835	0.015312
γ_1	-0.00366		-0.00359		-0.00352	
$\varphi_1 = 0.35, \alpha = 0.3,\ \rho = 0.2, \tau = 0.15, \theta = 0.1$						

Table 1: An increase in public consumption

	$\lambda = 0$		$\lambda = 0.03$		$\lambda = 0.06$	
	$\varphi_1 = 0.35$	$\varphi_1 = 0.40$	$\varphi_1 = 0.35$	$\varphi_1 = 0.40$	$\varphi_1 = 0.35$	$\varphi_1 = 0.40$
x	0.071134	0.057632	0.075898	0.062004	0.080825	0.066544
c	0.319145	0.313816	0.35488	0.349785	0.390580	0.385715
γ	0.019831	0.016170	0.019340	0.015749	0.018835	0.015312
γ_1	-0.00366		-0.00359		-0.00352	
$\varphi_2 = 0.40, \alpha = 0.3,\ \rho = 0.2, \tau = 0.15, \theta = 0.1$						

Table 2: An increase in lump-sum transfers

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.0711	0.0784	0.0861	0.0942	0.1027	0.1116	0.1209	0.1306	0.1408
c	0.3191	0.3235	0.3279	0.3321	0.3364	0.3406	0.3445	0.3489	0.353
γ	0.01983	0.02089	0.02188	0.02281	0.0236	0.0244	0.0252	0.0258	0.025
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1513	0.1623	0.1737	0.1856	0.198	0.218	0.2243	0.2382	0.2528
c	0.3571	0.3611	0.3651	0.3692	0.3732	0.3772	0.3812	0.3852	0.3892
γ	0.027	0.02761	0.02809	0.02852	0.0289	0.02924	0.02954	0.02979	0.03002
τ	0.33	0.34	0.35	0.36	0.37	0.383	0.39	0.40	0.41
x	0.2675	0.2837	0.3002	0.3174	0.3354	0.3599	0.3738	0.3943	0.4159
c	0.3932	0.3972	0.4013	0.4054	0.4095	0.4136	0.4178	0.422	0.4263
γ	0.0302	0.03035	0.03047	0.03056	0.03063	0.03064	0.03063	0.0361	0.03054
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.4385	0.4621	0.487	0.5131	0.5407	0.5697	0.6002	0.6325	0.6667
c	0.4307	0.435	0.4395	0.444	0.4486	0.4532	0.458	0.4628	0.4678
γ	0.03045	0.03033	0.0302	0.03003	0.02985	0.02963	0.0294	0.02915	0.02887
$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3,\ \rho = 0.2, \theta = 0.1$									

Table 3: An increase in public investment ($\lambda = 0$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.07590	0.08347	0.09145	0.09982	0.1086	0.1178	0.1273	0.1373	0.1477
c	0.3549	0.3592	0.3635	0.3678	0.372	0.372	0.3803	0.3844	0.3885
γ	0.01934	0.02030	0.02120	0.02203	0.02279	0.02350	0.02415	0.02475	0.02529
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1586	0.1699	0.1816	0.1939	0.2066	0.2198	0.2337	0.2480	0.2630
c	0.3925	0.3967	0.4006	0.4046	0.4086	0.4126	0.4166	0.4207	0.4247
γ	0.02577	0.02621	0.02660	0.02695	0.02725	0.02751	0.02773	0.02791	0.02806
τ	0.33	0.34	0.35	0.359	0.37	0.38	0.39	0.40	0.41
x	0.2786	0.2949	0.3119	0.3278	0.3482	0.3676	0.3878	0.4091	0.4313
c	0.4287	0.4323	0.4368	0.4409	0.4451	0.4492	0.4535	0.4577	0.462
γ	0.02817	0.02824	0.02829	0.02830	0.02828	0.02823	0.02815	0.02805	0.02791
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.4546	0.4791	0.5049	0.5319	0.5604	0.5905	0.6222	0.6557	0.6911
c	0.4664	0.4708	0.4753	0.4799	0.4845	0.4892	0.494	0.499	0.5039
γ	0.02775	0.02757	0.02736	0.02713	0.02687	0.02659	0.02629	0.02596	0.02562
	$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3,\ \rho = 0.2,\ \theta = 0.1$								

Table 4: An increase in public investment ($\lambda = 0.03$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.08082	0.08865	0.09689	0.1055	0.1146	0.1240	0.1339	0.1442	0.1549
c	0.3906	0.3949	0.3991	0.4034	0.4076	0.4117	0.4158	0.4199	0.4240
γ	0.01883	0.01970	0.02050	0.02124	0.02191	0.02253	0.02309	0.02360	0.02405
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1660	0.1776	0.1898	0.2023	0.2155	0.2291	0.2433	0.2581	0.2736
c	0.4281	0.4321	0.4361	0.4401	0.4441	0.4481	0.4521	0.4562	0.4602
γ	0.02445	0.02481	0.02512	0.02538	0.02560	0.02578	0.02592	0.02603	0.02610
τ	0.33	0.335	0.35	0.36	0.37	0.38	0.39	0.40	0.41
x	0.2896	0.2979	0.3239	0.3422	0.3613	0.3813	0.4023	0.4242	0.4472
c	0.4642	0.4683	0.4724	0.4765	0.4807	0.4849	0.4891	0.4934	0.4978
γ	0.02613	0.02614	0.02610	0.02604	0.02595	0.02582	0.02567	0.02550	0.02529
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.4731	0.4967	0.5233	0.5513	0.5808	0.6120	0.6449	0.6796	0.7164
c	0.5021	0.5066	0.5111	0.5157	0.5204	0.5252	0.53	0.5345	0.54
γ	0.02506	0.0248	0.02453	0.02423	0.02390	0.02355	0.02318	0.02279	0.02237
$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3,\ \rho = 0.2,\ \theta = 0.1$									

Table 5: An increase in public investment ($\lambda = 0.06$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.05763	0.06340	0.06953	0.07602	0.08267	0.09007	0.09763	0.1057	0.1139
c	0.3138	0.3181	0.3225	0.3268	0.3311	0.3354	0.3397	0.3440	0.3483
γ	0.01617	0.01712	0.01802	0.01888	0.01969	0.02044	0.02115	0.02181	0.02242
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1225	0.1317	0.1411	0.1510	0.1613	0.1721	0.1833	0.1951	0.2073
c	0.3526	0.3569	0.3612	0.3654	0.3697	0.3740	0.3784	0.3827	0.3871
γ	0.02298	0.02350	0.02397	0.02440	0.02479	0.02514	0.02546	0.02573	0.02598
τ	0.33	0.34	0.35	0.36	0.37	0.38	0.394	0.40	0.41
x	0.2201	0.2335	0.2474	0.2620	0.2773	0.2933	0.3169	0.3275	0.3459
c	0.3915	0.3959	0.4003	0.4048	0.4093	0.4139	0.4186	0.4232	0.4280
γ	0.02618	0.02636	0.02650	0.02661	0.02670	0.02675	0.02678	0.02677	0.02675
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.3652	0.3855	0.4068	0.4293	0.4530	0.4780	0.5043	0.5322	0.5617
c	0.4328	0.4380	0.4426	0.4470	0.4528	0.4581	0.4634	0.4688	0.4744
γ	0.02669	0.02661	0.02651	0.02639	0.02624	0.02606	0.02587	0.02566	0.02542
$\varphi_1 = 0.35, \varphi_2 = 0.45, \alpha = 0.3,\ \rho = 0.2,\ \theta = 0.1$									

Table 6: An increase in public investment with higher public consumption ($\lambda = 0$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.06200	0.06802	0.07440	0.08114	0.08824	0.09570	0.1035	0.1117	0.1203
c	0.3498	0.3541	0.3584	0.3227	0.3670	0.3713	0.3756	0.3798	0.3841
γ	0.01575	0.01661	0.01742	0.01818	0.01889	0.01955	0.02017	0.02073	0.02126
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1293	0.1387	0.1484	0.1587	0.1693	0.1805	0.1921	0.2042	0.2168
c	0.3884	0.3927	0.3969	0.4012	0.4055	0.4098	0.4142	0.4185	0.4229
γ	0.02173	0.02216	0.02255	0.02290	0.02321	0.02348	0.02371	0.02391	0.02407
τ	0.33	0.34	0.35	0.36	0.368	0.38	0.39	0.40	0.41
x	0.2301	0.2439	0.2583	0.2734	0.2892	0.3057	0.3230	0.3412	0.3602
c	0.4273	0.4317	0.4362	0.4407	0.4453	0.4499	0.4545	0.4592	0.4640
γ	0.02420	0.02430	0.02436	0.02440	0.02441	0.02439	0.02434	0.02426	0.02416
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.3803	0.4013	0.4234	0.4468	0.4713	0.4973	0.5247	0.5537	0.5845
c	0.4689	0.4738	0.4788	0.4839	0.4891	0.4944	0.4998	0.5053	0.5109
γ	0.02404	0.02389	0.02371	0.02352	0.02330	0.02306	0.02280	0.02251	0.02220
$\varphi_1 = 0.35, \varphi_2 = 0.45, \alpha = 0.3,\ \rho = 0.2,\ \theta = 0.1$									

Table 7: An increase in public investment with higher public consumption ($\lambda = 0.03$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.06654	0.07281	0.07944	0.08643	0.09379	0.1015	0.1096	0.1181	0.1270
c	0.3857	0.3899	0.3943	0.3985	0.4028	0.4071	0.4114	0.4156	0.4199
γ	0.01531	0.01607	0.01679	0.01746	0.01808	0.01865	0.01918	0.01966	0.02009
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1362	0.1459	0.1560	0.1666	0.1776	0.1891	0.2011	0.2136	0.2267
c	0.4242	0.4284	0.4327	0.4370	0.4413	0.4456	0.4499	0.4543	0.4587
γ	0.02048	0.02083	0.02113	0.02140	0.02162	0.02181	0.02196	0.02208	0.02217
τ	0.33	0.342	0.35	0.36	0.37	0.38	0.39	0.40	0.41
x	0.2403	0.2575	0.2695	0.2851	0.3015	0.3186	0.3365	0.3553	0.3751
c	0.4631	0.4676	0.4472	0.4766	0.4812	0.4858	0.4905	0.4953	0.5001
γ	0.02222	0.02224	0.02223	0.02219	0.02212	0.02203	0.02191	0.02176	0.02159
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.3958	0.4177	0.4406	0.4649	0.4904	0.5174	0.5459	0.5761	0.6081
c	0.5050	0.5099	0.5150	0.5202	0.5254	0.5307	0.5362	0.5418	0.5475
γ	0.02139	0.02117	0.02092	0.02066	0.02037	0.02005	0.01972	0.01937	0.01899
$\varphi_1 = 0.35, \varphi_2 = 0.45, \alpha = 0.3,\ \rho = 0.2,\ \theta = 0.1$									

Table 8: An increase in public investment with higher public consumption ($\lambda = 0.06$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.5924	0.06566	0.07251	0.07978	0.08748	0.09561	0.1042	0.1131	0.1225
c	0.3077	0.3121	0.3164	0.3207	0.3250	0.3293	0.3335	0.3377	0.3419
γ	0.01792	0.01903	0.02009	0.02110	0.02204	0.02293	0.02376	0.02454	0.02526
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1324	0.1427	0.1534	0.1647	0.1764	0.1886	0.2014	0.2147	0.2286
c	0.3461	0.3502	0.3544	0.3585	0.3625	0.3667	0.3707	0.3748	0.3789
γ	0.02592	0.02653	0.02709	0.02760	0.02807	0.02848	0.02886	0.02919	0.02948
τ	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.399	0.41
x	0.2430	0.2582	0.2739	0.2904	0.3077	0.3257	0.3445	0.3622	0.3849
c	0.3830	0.3872	0.3913	0.3954	0.3996	0.4038	0.4081	0.4119	0.4167
γ	0.02973	0.02994	0.03012	0.03026	0.03037	0.03045	0.03049	0.03050	0.03049
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.4066	0.4294	0.4533	0.4784	0.5049	0.5327	0.5621	0.5932	0.6260
c	0.4210	0.4255	0.4299	0.4345	0.4391	0.4438	0.4485	0.4534	0.4584
γ	0.03044	0.03037	0.03026	0.03012	0.02998	0.02980	0.02960	0.02937	0.02912
$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3,\ \rho = 0.2,\ \theta = 0.12$									

Table 9: An increase in public investment with higher investment subsidies ($\lambda = 0$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.06477	0.07152	0.07869	0.08630	0.09434	0.1028	0.1117	0.1210	0.1308
c	0.3449	0.3492	0.3535	0.3578	0.3620	0.3663	0.3705	0.3746	0.3788
γ	0.01765	0.01866	0.019612	0.02051	0.02135	0.02214	0.02286	0.02354	0.02416
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1410	0.1517	0.1628	0.1745	0.1866	0.1993	0.2125	0.2263	0.2407
c	0.3829	0.3871	0.3912	0.3953	0.3994	0.4035	0.4075	0.4116	0.4157
γ	0.02472	0.02524	0.02571	0.02613	0.02650	0.02683	0.02712	0.02736	0.02757
τ	0.33	0.34	0.35	0.36	0.374	0.38	0.39	0.40	0.41
x	0.2557	0.2714	0.2878	0.3049	0.3302	0.3415	0.3611	0.38162	0.4032
c	0.4116	0..4240	0.4281	0..4323	0..4382	0.4407	0.4450	0.4493	0.4536
γ	0.02773	0.02787	0.02796	0.02803	0.02806	0.02805	0.02802	0.02795	0.02786
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.4257	0.4494	0.4744	0.5006	0.5282	0.5573	0.5880	0.6204	0.6547
c	0.4580	0.4625	0.4670	0.4716	0.4763	0.4810	0.4859	0.4908	0.4958
γ	0.02774	0.02759	0.02741	0.02721	0.02698	0.02673	0.02645	0.02615	0.02583
$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3, \ \rho = 0.2, \ \theta = 0.12$									

Table 10: An increase in public investment with higher investment subsidies ($\lambda = 0.03$)

τ	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23
x	0.0705	0.0776	0.08514	0.0931	0.1014	0.1103	0.1195	0.1292	0.1393
c	0.3820	0.3863	0.39053	0.3948	0.3989	0.4032	0.4074	0.4115	0.4157
γ	0.0173	0.01823	0.01911	0.01990	0.02064	0.02132	0.02195	0.02253	0.02305
τ	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32
x	0.1499	0.1610	0.1726	0.1847	0.1973	0.2104	0.2241	0.2384	0.2533
c	0.4198	0.4239	0.4279	0.4321	0.4361	0.4402	0.4443	0.4484	0.4525
γ	0.02352	0.02394	0.02432	0.02465	0.02493	0.02517	0.02538	0.02554	0.02566
τ	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.40	0.41
x	0.2689	0.2852	0.3022	0.3199	0.3386	0.3580	0.3784	0.3997	0.4221
c	0.4566	0.4608	0.4649	0.4691	0.4733	0.4776	0.4819	0.4862	0.4906
γ	0.02574	0.02579	0.02581	0.02579	0.02574	0.02566	0.02555	0.02541	0.02524
τ	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50
x	0.4456	0.4703	0.4963	0.5236	0.5524	0.5828	0.6149	0.6488	0.6847
c	0.4951	0.4996	0.5042	0.5088	0.5135	0.5183	0.5232	0.5282	0.5333
γ	0.02504	0.02482	0.02457	0.02429	0.02399	0.02366	0.0233	0.02294	0.02255
$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3,\ \rho = 0.2,\ \theta = 0.12$									

Table 11: An increase in public investment with higher investment subsidies ($\lambda = 0.06$)

	θ	x	c	γ
$(\lambda = 0)$	0.07	0.4995	0.4469	0.03029
	0.08	0.483	0.442	0.03034
	0.09	0.4665	0.4375	0.03037
	0.10	0.4502	0.4328	0.03040
	0.113	0.4339	0.4281	0.03041
	0.12	0.4179	0.4233	0.03040
	0.13	0.4019	0.4184	0.03039
	0.14	0.3861	0.4135	0.03036
	0.15	0.3705	0.4085	0.03031
$(\lambda = 0.03)$	0.07	0.5115	0.4808	0.02759
	0.08	0.4965	0.4768	0.02762
	0.09	0.4815	0.4727	0.02765
	0.10	0.4667	0.4686	0.02766
	0.112	0.4520	0.4644	0.02767
	0.12	0.4374	0.4603	0.02766
	0.13	0.4230	0.4561	0.02765
	0.14	0.4086	0.4518	0.02762
	0.15	0.3944	0.4475	0.02752
$(\lambda = 0.06)$	0.07	0.5236	0.5148	0.024901
	0.08	0.5103	0.5113	0.024920
	0.09	0.4970	0.5079	0.024933
	0.104	0.4838	0.5044	0.024940
	0.11	0.4708	0.5009	0.024939
	0.12	0.4578	0.4973	0.024932
	0.13	0.4450	0.4938	0.024918
	0.14	0.4322	0.4902	0.024896
	0.15	0.4196	0.4866	0.024867
$\varphi_1 = 0.35, \varphi_2 = 0.40, \alpha = 0.3, \tau = 0.425$				

Table 12: Growth maximizing level of investment subsidies

B Uniqueness

Proof of proposition 1 *There exists a BGP with endogenous growth for the economy described by (2.25)-(2.26) and such a BGP is unique.*

In order to prove proposition 1, we first set $\dot{c} = 0$,

$$cx^{\alpha}\frac{(1-\tau)(1-\alpha)}{(1-\theta_S)} - c\frac{\lambda(1+\theta_S)}{(1-\theta_S)} - c\rho - cx^{\alpha}\frac{[1-\tau(1-\varphi_1)]}{(1-\theta_S)} + \frac{c^2}{(1-\theta_S)} = 0 \tag{B.1}$$

Solving (B.1) for c and substituting the result in (2.25) yields

$$\begin{aligned} F(x,\cdot) &= x^{\alpha}\left\{\tau(1-\varphi_1-\varphi_2) - \frac{\theta_S(1-\tau)(1-\alpha)}{(1-\theta_S)}\right\} \\ &- x^{\alpha+1}\frac{(1-\tau)(1-\alpha)}{(1-\theta_S)} + (x+\theta_S)\left[\rho + \frac{\lambda(\tau+\theta_S)}{(1-\theta_S)}\right] \end{aligned} \tag{B.2}$$

A solution to $F(x,\cdot) = 0$ gives a BGP for the economy. For $x = 0$ we have $F(0,\cdot) > 0$

$$F(0,\cdot) = \theta_S\left[\frac{\lambda(\tau+\theta_S)}{(1-\theta_S)} + \rho\right] > 0 \tag{B.3}$$

We now calculate the sign of $\partial F(x,\cdot)/\partial x$:

$$\begin{aligned} \frac{\partial F(x,\cdot)}{\partial x} &= \alpha x^{\alpha-1}\left\{\tau(1-\varphi_1-\varphi_2) - \frac{\theta_S(1-\tau)(1-\alpha)}{(1-\theta_S)}\right\} \\ &- (\alpha+1)x^{\alpha}\frac{(1-\tau)(1-\alpha)}{(1-\theta_S)} + \frac{\lambda(\tau+\theta_S)}{(1-\theta)} + \rho \end{aligned} \tag{B.4}$$

where

$$\gamma = \frac{\dot{C}}{C} = x^{\alpha}\frac{(1-\alpha)(1-\tau)}{(1-\theta_S)} - \frac{\lambda(\tau+\theta_S)}{(1-\theta_S)} - \rho$$

Thus, by substituting this result into (B.4):

$$\begin{aligned} \frac{\partial F(x,\cdot)}{\partial x} &= \alpha x^{\alpha-1}\left\{\tau(1-\varphi_1-\varphi_2) - \frac{\theta_S(1-\tau)(1-\alpha)}{(1-\theta_S)}\right\} \\ &- \alpha x^{\alpha}\frac{(1-\tau)(1-\alpha)}{(1-\theta_S)} - \gamma \end{aligned} \tag{B.5}$$

From

$$\gamma = \frac{\dot{G}}{G} = x^{\alpha-1}\left\{\tau(1-\varphi_1-\varphi_2) - \frac{\theta_S[1-\tau(1-\varphi_1)]}{(1-\theta_S)}\right\} - \frac{\lambda\theta_S(1-\tau)}{x(1-\theta_S)} + \frac{c\theta_S}{x(1-\theta_S)}$$

It follows that

$$\alpha x^{\alpha-1}\tau(1-\varphi_1-\varphi_2) = \alpha\left\{\gamma + x^{\alpha-1}\frac{\theta[1-\tau(1-\varphi_1)]}{(1-\theta_S)} + \frac{\lambda\theta_S(1-\tau)}{x(1-\theta_S)} - \frac{c\theta_S}{x(1-\theta_S)}\right\}$$

Thus, by substituting this expression in (B.5), we obtain

$$\frac{\partial F(x,\cdot)}{\partial x} = (\alpha-1)\gamma + \alpha x^{\alpha-1}\frac{\theta_S[1-\tau(1-\varphi_1)]}{(1-\theta_S)} - \alpha x^{\alpha}\frac{(1-\tau)(1-\alpha)}{(1-\theta_S)} + \frac{\alpha\lambda\theta_S(1-\tau)}{x(1-\theta_S)} - \frac{\alpha c\theta_S}{x(1-\theta_S)} - \alpha x^{\alpha-1}\frac{\theta_S(1-\tau)(1-\alpha)}{(1-\theta_S)} \tag{B.6}$$

From (B.1), it follows that

$$\frac{\alpha c\theta_S}{x(1-\theta_S)} = \frac{\alpha\theta_S\lambda(1+\theta_S)}{x(1-\theta_S)} + \frac{\alpha\rho\theta_S}{x} - \alpha x^{\alpha-1}\frac{\theta_S}{(1-\theta_S)} \{(1-\tau)(1-\alpha) - [1-\tau(1-\varphi_1)]\} \tag{B.7}$$

Inserting this result in (B.6) yields

$$\frac{\partial F(x,\cdot)}{\partial x} = (\alpha-1)\gamma - \alpha x^{\alpha}\frac{(1-\tau)(1-\alpha)}{(1-\theta_S)} + \frac{\alpha\theta_S\lambda(1-\tau)}{x(1-\theta_S)} - \frac{\alpha\theta_S\lambda(1+\theta_S)}{x(1-\theta_S)} - \frac{\alpha\rho\theta_S}{x} \tag{B.8}$$

Using again the definition of $\gamma = (\dot{C}/C)$ leads to

$$\frac{\partial F(x,\cdot)}{\partial x} = -\gamma - \left[\alpha\rho + \frac{\alpha\lambda(\tau+\theta_S)}{(1-\theta_S)}\right]\left(1+\frac{\theta_S}{x}\right) \tag{B.9}$$

From (B.9) it follows that $\partial F(x,\cdot)/\partial x < 0$ always holds on the BGP and, as a consequence, $F(x,\cdot)$ cannot cross the horizontal axis from below. Since $F(0,\cdot) > 0$ and $F(x,\cdot)$ is a continuous function the BGP is unique.

C Stability

Proof of proposition 2 *The Jacobian matrix of the system (2.25)-(2.26) has one positive and one negative real root, which implies that the unique BGP is saddle path.*

In order to prove proposition 2 we need to evaluate the partial derivatives of (2.25)-(2.26) at the steady state.

$$\frac{\partial \dot{x}}{\partial x} = \alpha x^{\alpha-1}\left\{\tau(1-\varphi_1-\varphi_2) - \frac{\theta_S[1-\tau(1-\varphi_1)]}{(1-\theta_S)}\right\} - (\alpha+1)x^{\alpha}\frac{[1-\tau(1-\varphi_1)]}{(1-\theta_S)} + \frac{c}{(1-\theta_S)} - \frac{\lambda(1-\tau)}{(1-\theta_S)} \tag{C.1}$$

Setting $\dot{x} = 0$ implies that

$$\frac{c}{(1-\theta_S)} = -x^{\alpha-1}\left\{\tau(1-\varphi_1-\varphi_2) - \frac{\theta_S[1-\tau(1-\varphi_1)]}{(1-\theta_S)}\right\} + x^{\alpha}\frac{[1-\tau(1-\varphi_1)]}{(1-\theta_S)} - \frac{c\theta_S}{x(1-\theta_S)} + \frac{\lambda(1-\tau)(1+\theta_S)}{(1-\theta_S)} \tag{C.2}$$

By substituting this result in (C.1), we obtain

$$\begin{aligned}\frac{\partial \dot{x}}{\partial x} &= (\alpha-1)\,x^{\alpha-1}\left\{\tau\,(1-\varphi_1-\varphi_2)-\frac{\theta_S\,[1-\tau\,(1-\varphi_1)]}{(1-\theta_S)}\right\} \\ &- \alpha x^{\alpha}\frac{[1-\tau\,(1-\varphi_1)]}{(1-\theta_S)}-\frac{c\theta_S}{x\,(1-\theta_S)}+\frac{\lambda\theta_S\,(1-\tau)}{x\,(1-\theta_S)}\end{aligned} \tag{C.3}$$

$$\frac{\partial \dot{x}}{\partial c}=\frac{x+\theta_S}{1-\theta_S} \tag{C.4}$$

$$\frac{\partial \dot{c}}{\partial x}=c\alpha x^{\alpha-1}\left\{\frac{(1-\tau)\,(1-\alpha)}{(1-\theta_S)}-\frac{[1-(1-\varphi_1)]}{(1-\theta_S)}\right\} \tag{C.5}$$

$$\frac{\partial \dot{c}}{\partial c}=x^{\alpha}\frac{(1-\tau)\,(1-\alpha)}{(1-\theta_S)}-\frac{\lambda\,(1+\theta_S)}{(1-\theta_S)}-\rho-x^{\alpha}\frac{[1-(1-\varphi_1)]}{(1-\theta_S)}+\frac{2c}{(1-\theta_S)} \tag{C.6}$$

Setting $\dot{c}=0$ implies that

$$\frac{c}{(1-\theta_S)}=\frac{\lambda\,(1+\theta_S)}{(1-\theta_S)}+\rho-x^{\alpha}\left\{\frac{(1-\tau)\,(1-\alpha)}{(1-\theta_S)}-\frac{[1-(1-\varphi_1)]}{(1-\theta_S)}\right\}$$

By substituting this result in (C.6), we obtain

$$\frac{\partial \dot{c}}{\partial c}=\frac{c}{(1-\theta_S)} \tag{C.7}$$

Thus, the Jacobian matrix of the system (2.25)-(2.26) is given by

$$J=\begin{bmatrix}(\alpha-1)\,x^{\alpha-1}\phi_1-\alpha x^{\alpha}\phi_2-\frac{c\theta_S}{x(1-\theta_S)}+\frac{\lambda\theta_S(1-\tau)}{x(1-\theta_S)} & \frac{x+\theta_S}{1-\theta_S} \\ c\alpha x^{\alpha-1}\phi_3 & \frac{c}{(1-\theta_S)}\end{bmatrix} \tag{C.8}$$

where

$$\phi_1=\left\{\tau\,(1-\varphi_1-\varphi_2)-\frac{\theta_S\,[1-\tau\,(1-\varphi_1)]}{(1-\theta_S)}\right\} \tag{C.9}$$

$$\phi_2=\frac{[1-\tau\,(1-\varphi_1)]}{(1-\theta_S)} \tag{C.10}$$

$$\phi_3=\left\{\frac{(1-\tau)\,(1-\alpha)}{(1-\theta_S)}-\frac{[1-(1-\varphi_1)]}{(1-\theta_S)}\right\} \tag{C.11}$$

The determinant of the Jacobian matrix is:

$$\begin{aligned}Det\ J &= \frac{cx}{(1-\theta_S)}\left\{(\alpha-1)\,x^{\alpha-2}\tau\,(1-\varphi_1-\varphi_2)-\frac{c\theta_S}{x^2\,(1-\theta_S)}\right\} \\ &+ x^{\alpha-1}c\frac{\theta_S\,[1-\tau\,(1-\varphi_1)]}{(1-\theta_S)^2}-(x+\theta_S)\,c\alpha x^{\alpha-1}\frac{(1-\tau)\,(1-\alpha)}{(1-\theta_S)^2} \\ &+ \frac{c\lambda\theta_S\,(1-\tau)}{x\,(1-\theta_S)}\end{aligned} \tag{C.12}$$

From the definition of $\gamma = (\dot{G}/G)$ it follows that

$$\begin{aligned} cx^{\alpha-1}\frac{\theta_S\left[1-\tau\left(1-\varphi_1\right)\right]}{\left(1-\theta_S\right)^2} &= -\frac{c\gamma}{\left(1-\theta_S\right)}+cx^{\alpha-1}\frac{\tau\left(1-\varphi_1-\varphi_2\right)}{\left(1-\theta_S\right)} \\ &- \frac{c\lambda\theta_S\left(1-\tau\right)}{x\left(1-\theta_S\right)^2}+\frac{c^2\theta_S}{x\left(1-\theta_S\right)^2} \end{aligned} \tag{C.13}$$

Substituting this result into (C.12)

$$\begin{aligned} Det\ J &= \frac{c\alpha}{\left(1-\theta_S\right)}x^{\alpha-1}\left\{\tau\left(1-\varphi_1-\varphi_2\right)-\frac{\theta_S\left(1-\tau\right)\left(1-\alpha\right)}{\left(1-\theta_S\right)^2}\right\} \\ &- \frac{c\gamma}{\left(1-\theta_S\right)}-c\alpha x^{\alpha}\frac{\left(1-\tau\right)\left(1-\alpha\right)}{\left(1-\theta_S\right)^2} \end{aligned} \tag{C.14}$$

From (B.4), it follows that

$$\begin{aligned} &\frac{c\alpha}{\left(1-\theta_S\right)}x^{\alpha-1}\left\{\tau\left(1-\varphi_1-\varphi_2\right)-\frac{\theta_S\left(1-\tau\right)\left(1-\alpha\right)}{\left(1-\theta_S\right)^2}\right\}= \\ &c\alpha x^{\alpha}\frac{\left(1-\tau\right)\left(1-\alpha\right)}{\left(1-\theta_S\right)^2}-\frac{c\alpha}{x\left(1-\theta_S\right)}\left(x+\theta_S\right)\left[\rho+\frac{\left(\tau+\theta_S\right)}{\left(1-\theta_S\right)}\right] \end{aligned} \tag{C.15}$$

Substituting this result in (C.14)

$$Det\ J = -c\left\{\frac{\gamma}{\left(1-\theta_S\right)}+\frac{\alpha\left(x+\theta_S\right)}{x\left(1-\theta_S\right)}\left[\rho+\lambda\frac{\left(\tau+\theta_S\right)}{\left(1-\theta_S\right)}\right]\right\} \tag{C.16}$$

Since $Det\ J < 0$, proposition 2 is proved.

D Fiscal Policy

D.1 Public consumption and lump-sum transfers

Proof of propositions 3 and 4 *The long-run rate of growth of the economy γ is decreasing in public consumption φ_2 and lump-sum transfers φ_1. Increases in φ_i ($i = 1, 2$) of the same amount reduce γ less for $\lambda > 0$ than for $\lambda = 0$.*

The impact of public consumption and lump-sum transfers to households is derived by differentiating the long-run rate of growth γ with respect to φ_i, $i = 1, 2$

$$\frac{\partial\gamma}{\partial\varphi_i}=\frac{\partial\gamma}{\partial x}\frac{\partial x}{\partial\varphi_i}=\alpha\frac{\left(1-\tau\right)}{1-\theta_S}\left(1-\alpha\right)x^{\alpha-1}\frac{\partial x}{\partial\varphi_i},\quad i=1,2 \tag{D.1}$$

where

$$\frac{\partial x}{\partial\varphi_i}=-\frac{\partial F\left(x,\cdot\right)/\partial\varphi_i}{\partial F\left(x,\cdot\right)/\partial x}=\frac{\tau x^{\alpha}}{\partial F\left(x,\cdot\right)/\partial x}<0,\quad i=1,2 \tag{D.2}$$

From the proof of proposition 1, we know that $\partial F\left(x,\cdot\right)/\partial x < 0$. Hence, $\partial x/\partial\varphi_i < 0$ and

$$\frac{\partial\gamma}{\partial\varphi_i}<0,\quad i=1,2 \tag{D.3}$$

Moreover — from (B.9) — $|\partial F(x,\cdot)/\partial x|_{(\lambda>0)} > |\partial F(x,\cdot)/\partial x|_{(\lambda=0)}$. Thus $|\partial x/\partial \varphi_i|_{(\lambda>0)} > |\partial x/\partial \varphi_i|_{(\lambda=0)}$ and

$$\left.\frac{\partial \gamma}{\partial \varphi_i}\right|_{(\lambda>0)} < \left.\frac{\partial \gamma}{\partial \varphi_i}\right|_{(\lambda=0)}, \quad i = 1, 2 \tag{D.4}$$

Hence, propositions 3 and 4 are proved.

D.2 The growth maximizing income tax rate

Proof of proposition 5 *There exists a growth maximizing income tax rate τ_{max} both in the infinite and the finite horizons scenarios, the first one being higher than the latter one.*

In order to calculate the growth maximizing level of income taxation in the finite horizons case, the derivative of (2.27) with respect to τ is evaluated as follows

$$\frac{\partial \gamma}{\partial \tau} = x^{\alpha}\frac{(1-\alpha)}{(1-\theta_S)}\left[-1+\frac{\alpha(1-\tau)}{\tau}\frac{\partial x}{\partial \tau}\frac{\tau}{x}\right] - \frac{\lambda}{(1-\theta_S)} \tag{D.5}$$

where

$$\left.\frac{\partial x}{\partial \tau}\right|_{F(x,\cdot)=0} = -\frac{\partial F(x,\cdot)/\partial \tau}{\partial F(x,\cdot)/\partial x} = \frac{-x^{\alpha}\left[(1-\varphi_1-\varphi_2)+\frac{(\theta_S+x)(1-\alpha)}{(1-\theta_S)}\right]-\frac{\lambda(\theta_S+x)}{(1-\theta_S)}}{-\gamma-\left[\alpha\rho+\frac{\alpha\lambda(\theta_S+\tau)}{(1-\theta_S)}\right]\left(1+\frac{\theta_S}{x}\right)} \tag{D.6}$$

Solving $F(x,\cdot)$ for ρ yields

$$\rho = x^{\alpha}\left[\frac{(1-\alpha)(1-\tau)}{(1-\theta_S)} - \frac{\tau(1-\varphi_1-\varphi_2)}{(\theta_S+x)}\right] - \frac{\lambda(\theta_S+\tau)}{(1-\theta_S)} \tag{D.7}$$

Let us now substitute (D.7) into (D.6) and the resulting expression for $\partial x/\partial \tau$ back into (D.5). By doing so, it is possible to solve $\partial \gamma/\partial \tau = 0$ for τ and to obtain the growth maximizing level of income taxation in the presence of a positive λ:

$$\tau_{max}|_{(\lambda>0)} = \frac{\alpha x^{\alpha}(\alpha-1)(x+\theta_S)}{x^{\alpha+1}(\alpha-1)-\lambda x+\lambda\alpha(x+\theta_S)} \tag{D.8}$$

which for $\lambda = 0$ simplifies to

$$\tau_{max}|_{(\lambda=0)} = \alpha\left(1+\frac{\theta_S}{x}\right) \tag{D.9}$$

where[12]

$$\alpha\left(1+\frac{\theta_S}{x}\right) > \frac{\alpha x^{\alpha}(\alpha-1)(x+\theta_S)}{x^{\alpha+1}(\alpha-1)-\lambda x+\lambda\alpha(x+\theta_S)} \tag{D.10}$$

Hence

$$\tau_{max}|_{(\lambda>0)} < \tau_{max}|_{(\lambda=0)} \tag{D.11}$$

And proposition 5 is proved.

Proof of proposition 6 *For $0 \le \lambda \le 1$, the growth maximizing income tax rate τ_{max} is increasing in both public consumption φ_2 and lump-sum transfers to households φ_1.*

For $\lambda = 0$, τ_{max} is given by (D.9).

[12] This inequality has been evaluated using Maple 7.0 for the following values of the parameters: $x > 0, 0 \le \alpha \le 1, 0 \le \theta_S \le 1$ and $0 \le \lambda \le 1$.

Let us define the implicit function Γ:

$$\Gamma\left(x\left(\tau,\theta_S,\varphi_i\right),\tau,\theta_S,\varphi_i\right) \equiv \tau_{max} - \alpha\left(1+\frac{\theta_S}{x}\right) \equiv \tau_{max} - \bar{\Gamma} = 0 \tag{D.12}$$

Totally differentiating (D.9) with respect to φ_i, $i = 1,2$ and applying the implicit function theorem to (D.12):

$$\begin{aligned}\left.\frac{\partial \tau_{max}}{\partial \varphi_i}\right|_{(\lambda=0)} &= -\frac{\partial\Gamma/\partial\varphi_i}{\partial\Gamma/\partial\tau} = -\frac{\frac{\partial\bar{\Gamma}}{\partial x}\frac{\partial x}{\partial\varphi_i}}{1-\frac{\partial\bar{\Gamma}}{\partial x}\frac{\partial x}{\partial\tau}} = -\frac{\frac{\alpha\theta_S}{x^2}\frac{\partial x}{\partial\varphi_i}}{1+\frac{\alpha\theta_S}{x^2}\frac{\partial x}{\partial\tau}} = \\ &= -\frac{\frac{\alpha\theta_S}{x^2}\frac{\partial x}{\partial\varphi_i}}{\frac{\alpha\theta_S}{x^2}\left(\frac{x^2}{\alpha\theta_S}+\frac{\partial x}{\partial\tau}\right)} = -\frac{\frac{\partial x}{\partial\varphi_i}}{-\left(\frac{\partial\bar{\Gamma}}{\partial x}\right)^{-1}+\frac{\partial x}{\partial\tau}}, \quad i=1,2\end{aligned} \tag{D.13}$$

where from (D.2) $\partial x/\partial\varphi_i < 0$ and from (D.6), $\partial x/\partial\tau > 0$. Hence, we obtain:

$$\left.\frac{\partial \tau_{max}}{\partial \varphi_i}\right|_{(\lambda=0)} = -\frac{\partial x/\partial\varphi_i}{x^2/\alpha\theta_S + \partial x/\partial\tau} > 0, \quad i=1,2 \tag{D.14}$$

For $0 < \lambda \leq 1$, τ_{max} is defined by (D.8).

Let us define the implicit function Γ_2:

$$\begin{aligned}\Gamma_2\left(x\left(\tau,\theta_S,\varphi_i\right),\tau,\theta_S,\varphi_i\right) &\equiv \tau_{max} - \frac{\alpha x^\alpha\left(\alpha-1\right)\left(x+\theta_S\right)}{x^{\alpha+1}\left(\alpha-1\right)-\lambda x+\lambda\alpha\left(x+\theta_S\right)} \equiv \\ &\equiv \tau_{max} - \bar{\Gamma}_2 \equiv \\ &\equiv \tau_{max} - \frac{A}{B} = 0\end{aligned} \tag{D.15}$$

Totally differentiating (D.8) with respect to φ_i, $i = 1,2$ and applying the implicit function theorem to (D.15):

$$\left.\frac{\partial \tau_{max}}{\partial \varphi_i}\right|_{(0<\lambda\leq 1)} = -\frac{\partial\Gamma_2/\partial\varphi_i}{\partial\Gamma_2/\partial\tau} = -\frac{\frac{\partial\bar{\Gamma}_2}{\partial x}\frac{\partial x}{\partial\varphi_i}}{1-\frac{\partial\bar{\Gamma}_2}{\partial x}\frac{\partial x}{\partial\tau}} = -\frac{\frac{\partial x}{\partial\varphi_i}}{-\left(\frac{\partial\bar{\Gamma}_2}{\partial x}\right)^{-1}+\frac{\partial x}{\partial\tau}}, \quad i=1,2$$

where $\partial x/\partial\varphi_i < 0$, $\partial x/\partial\tau > 0$, and — for $x > 0, 0 \leq \alpha \leq 1, 0 \leq \theta_S \leq 1$ and $0 \leq \lambda \leq 1$ — the sign of the term $\left(\frac{\partial\bar{\Gamma}_2}{\partial x}\right)^{-1}$ is negative[13]:

$$\begin{aligned}\left(\frac{\partial\bar{\Gamma}_2}{\partial x}\right)^{-1} &= \left\{\left[\alpha\left(\alpha-1\right)\left(\alpha+1\right)x^\alpha+\alpha^2\theta_S\left(\alpha-1\right)x^{\alpha-1}\right]B^{-1}\right\} \\ &- \left\{B^{-2}\left[\left(\alpha-1\right)\left(\alpha+1\right)x^\alpha-\lambda\left(\alpha+1\right)\right]A\right\} < 0\end{aligned} \tag{D.16}$$

Therefore, for all the values of the parameters coherent with our theoretical model:

$$\left.\frac{\partial \tau_{max}}{\partial \varphi_i}\right|_{(0<\lambda\leq 1)} = -\frac{\partial x/\partial\varphi_i}{-\left(\frac{\partial\bar{\Gamma}_2}{\partial x}\right)^{-1}+\partial x/\partial\tau} > 0, \quad i=1,2 \tag{D.17}$$

[13] This inequality has been evaluated using Maple 7.0 for the following values of the parameters: $x > 0, 0 \leq \alpha \leq 1, 0 \leq \theta_S \leq 1$ and $0 \leq \lambda \leq 1$.

D.3 Investment subsidies

Proof of proposition 7 *There exists a growth maximizing value for investment subsidies, and such a value is decreasing in the probability of death parameter* λ.

Differentiating γ with respect to θ_S leads to

$$\frac{\partial \gamma}{\partial \theta_S} = \frac{(1-\alpha)(1-\tau)}{(1-\theta_S)^2} x^{\alpha} \left[1 + \frac{\alpha(1-\theta_S)}{\theta_S} \frac{\partial x}{\partial \theta_S} \frac{\theta_S}{x} \right] - \frac{\lambda(1+\tau)}{(1-\theta_S)^2} \tag{D.18}$$

Since from (D.22) $\partial x / \partial \theta_S < 0$, for $\lambda = 0$

$$\left. \frac{\partial \gamma}{\partial \theta_S} \right|_{(\lambda=0)} > (\leq) 0 \quad \text{if} \quad \frac{\partial x}{\partial \theta_S} \frac{\theta_S}{x} > (\leq) - \frac{\theta_S}{\alpha(1-\theta_S)} \tag{D.19}$$

However, for a positive λ we obtain

$$\left. \frac{\partial \gamma}{\partial \theta_S} \right|_{(\lambda>0)} > (\leq) 0 \quad \text{if} \quad \frac{\partial x}{\partial \theta_S} \frac{\theta_S}{x} > (\leq) - \frac{\theta_S}{\alpha(1-\theta_S)} + \frac{\lambda(1+\tau)}{(1-\theta_S)^2} \tag{D.20}$$

Proof of proposition 8 *For* $0 \leq \lambda \leq 1$, *the growth maximizing income tax rate* τ_{max} *is increasing in investment subsidies.*

Differentiating (D.8) with respect to θ_S

$$\frac{\partial \tau_{max}}{\partial \theta_S} = - \frac{\frac{\partial \bar{\Gamma}_2}{\partial x} \frac{\partial x}{\partial \theta_S}}{1 - \frac{\partial \bar{\Gamma}_2}{\partial x} \frac{\partial x}{\partial \tau}} = - \frac{\frac{\partial x}{\partial \theta_S}}{-\left(\frac{\partial \bar{\Gamma}_2}{\partial x} \right)^{-1} + \frac{\partial x}{\partial \tau}} > 0 \tag{D.21}$$

where $\left(\frac{\partial \bar{\Gamma}_2}{\partial x} \right)^{-1} < 0$ is defined in (D.16) and $\partial x / \partial \theta_S$ is derived by totally differentiating $F(x, \cdot) = 0$ and using $\gamma = \dot{C}/C$:

$$\frac{\partial x}{\partial \theta_S} = - \frac{\partial F / \partial \theta_S}{\partial F / \partial x} = - \frac{-\gamma - x^{\alpha} \frac{(1-\tau)(1-\alpha)}{(1-\theta_S)^2} (x + \theta_S) + \frac{\lambda \tau (1+\theta_S)}{(1-\theta_S)^2}}{-\gamma - \left[\alpha \rho + \frac{\alpha \lambda (\tau + \theta_S)}{(1-\theta_S)} \right] \left(1 + \frac{\theta_S}{x} \right)} < 0 \tag{D.22}$$

Hence, for $0 \leq \lambda \leq 1$, $\partial \tau_{max} / \partial \theta_S > 0$.

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