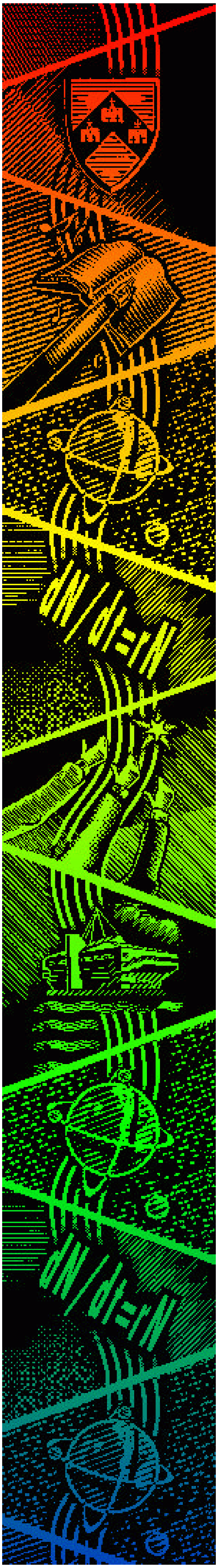

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Responsibility, Compensation and Income Distributions

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Abstract

According to the opportunity egalitarian ethics, inequalities due to factors beyond the individual responsibility are inequitable and to be compensated by society; whereas inequalities due to personal responsibility are equitable and not to be compensated. In this paper we try to explore how this conception can be translated into a concrete public policy, when the individual responsibility level is unobservable. To cope with this informational constraint, we adopt Roemer's (1993) statistical solution. We first derive normative criteria for unambiguous social rankings of income distributions. Then, we characterize an opportunity egalitarian income tax and we formulate criteria for choosing among alternative tax systems.

Keywords: Opportunity, responsibility, egalitarian, income inequality, redistribution.

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1 Introduction

Recent contributions, in the philosophical debate, have proposed opportunity, instead of outcomes, as the appropriate "currency of egalitarian justice"(Cohen, 1989). The appealing feature of the equality of opportunity view is that it offers a version of egalitarianism which does not deny individual responsibility. For this reason, it is argued, equality of opportunity is the prevailing conception of social justice in western liberal democracies. One can easily state the central intuition behind these theories[1], in terms of an ethical division of labour between society and its individual members (Arneson, 1998): It is not society 's business to make people happy or to make them achieve any other valuable outcome they wish to. But it is society 's business to secure to its members an equal distribution of opportunities to achieve whatever outcome they care about. The fact that equal opportunities will not lead to equal achievements is not a failure of this conception; it is, on the contrary, a consequence of the society's respect for the individuals' different "ambition plans". Once the means or opportunities to reach a valuable outcome have been equally split, which particular opportunity, from those open to her, the individual chooses, is outside the scope of justice.

Now, the choices that are likely to confront an opportunity egalitarian policy maker will involve the evaluation of different public policies on the basis of the opportunity redistribution they introduce, which in turn requires comparing situations where individuals have different opportunities. Hence, if we want to give operational content to the opportunity egalitarianism, we have to address the following issues: (i) how to measure the degree of inequality present in a distribution of opportunities? (ii) what redistribution mechanisms can be designed to increase the degree of 'opportunity equality'?

A natural answer, for the redistribution problem, would consist in equalizing the individual opportunity sets and, once this equalization is achieved, in letting the individuals choose from their opportunity set their preferred option[2]. This solution corresponds to performing a direct exercise of measurement of inequality in a distribution of opportunity sets[3].

This approach is surely correct in principle; however, given the high levels of measurability and comparability of opportunities required, it seems unlikely to be

[1]We can refer, in particular, to Rawls (1971), Sen (1980, 1985, 1992), Dworkin (1981a, b),Cohen (1989), Arneson (1989), Barry (1991) , Roemer (1993). For a reconstruction of the philosophical debate on opportunity egalitarianism, see Roemer (1996).

[2]This is basically the program proposed by Dworkin (1981a, b).

[3]In this direction are, for instance, the contributions by Alergi and Nieto (1998), Herrero (1997), Kranich (1996, 1997), Ok and Kranich (1998)). For a recent survey of the relevant literature, with attention also to the related issue of ranking opportunity sets, see Peragine (1998a). A different, "indirect" approach is adopted by Van de gaer *et al. (1998) in a paper exploring the link between the measurement of intergenerational mobility and the notion of equality of opportunity.*

useful for operational purposes. In addition to the measurability limitations, consider that the elements entering a person's opportunity set will be, in general, social circumstances as well as individual native talents. Hence, it is likely that some of these opportunities are fixed, personal and cannot be redistributed. Thus a phisical equalization of opportunities could not be carried out.

These implementability constraints motivate an indirect approach to the opportunity egalitarian project, where the focus is not on the distribution (and redistribution) of opportunities "per se"; rather, it is on the consequences of a given distribution of opportunities on some form of individual advantage. A consistent reformulation of the opportunity egalitarian conception could be the following: Inequalities in a given distribution of outcomes, which are due to differences in factors beyond the individual control (opportunities), are to be considered inequitable and are to be compensated by society; whereas inequalities due to factors within the personal responsibility are equitable and are not to be compensated.

The first part of the principle is sometimes called the "Principle of Compensation" (**PC**)(Fleurbaey,1995); the latter part, the "Principle of Natural Reward" (Fleurbaey, 1995), or the "Principle of Responsibility" (**PR**)(Barry, 1991).

Hence, the Compensation Principle, egalitarian in spirit, says that the institutions of a society should operate in such a way as to counteract the effects of factors beyond the individual control. Notice that the Compensation Principle depends for its application on our having, as a benchmark for the compensation of people with low endowments of opportunities, some idea of the welfare (or advantage) level appropriate to those similarly placed, except for having suffered the low opportunity level.

On the other hand, the Responsibility principle says that social arrangements should be such that people finish up with the outcomes of their voluntary acts. Differences in individuals achievements which can be unambiguosly attributed to differences in the responsibility exercised, are not compensable at the bar of justice, for if they were, the individual responsibility would be denied.

It is evident that the two principles above remain two empty boxes, until one defines precisely what are the factors beyond, as opposed to within, the individual responsibility. A conservative or "rightist" interpretation would recognize the principle of compensation, and yet try to reduce the space of individual achievements to be attributed to social circumstances or opportunities. On the other hand, a "leftist" interpretation would recognise the responsibility principle, but considering it as not having any application in a world of universal causality: individual preferences could themselves be considered as determined by social circumstances; in other words, the space of individual responsibility would be much reduced (Barry, 1991).

However, the scope of this paper is not to define the proper domain of individual responsibility; rather, we study tools which are potentially useful to such a discussion. More precisely, given any concept of responsibility a society decides to adopt, we first address the question of how to rank different "outcome distributions" on the basis of opportunity inequality. Then we try to characterize a public policy coherent with the opportunity egalitarian conception[4].

One way of addressing the previous questions can be illustrated by the following (informal) argument. Consider a given population and a distribution of a particular form of advantage (income, utility, etc.). The advantage, for each individual, is function of two classes of variables: factors beyond the individual's control (or opportunities) and factors for which the individual is responsible. Now consider, in such population, the group of people who have exercised the same degree of responsibility (let us call it a $Tranche$). Since the individual advantage is determined only by opportunities and responsibility, then any outcome inequality observed in a tranche can only reflect differentials in opportunities: therefore, according to the Compensation principle, any outcome inequality within a tranche is inequitable and it has to be compensated[5].

Hence, one way to measure the opportunity inequality is that of focusing on the outcome inequality within the group of people who have exercised the same degree of responsibility; and to do this for any degree of responsibility (for any tranche). The problem would be, in principle at least, simple, were we able to observe, to measure and to compare the responsibility level of individuals. Such observability assumption, however, would be a heroic one, to say the least. Therefore, to make our proposal interesting from an operational point of view, we consider the case of non-observability of the responsibility level and, to cope with this informational constraint, we adopt Roemer (1993)'s "percentile approach". To explain in details the strategy we propose, it is time to introduce a more formal model.

1.1 A formal model

We start our formal analysis by selecting the income as the relevant advantage. Consider a population represented by a finite set N of individuals. Each individual's income $x, x \in X$, is causally determined by two kinds of factors: factors beyond the individual control, represented by a person's opportunity set $O, O \in \Omega$, and factors for

[4]There is a growing body of literature which studies redistribution mechanisms inspired by the opportunity egalitarian ethics. See, in particular, the contributions by Bossert (1995), Bossert and Fleurbaey (1996), Bossert et al. (1996), Fleurbaey (1994, 1995a,b), Fleurbaey and Maniquet (1996), Iturbe-Ormaetxe (1996), Iturbe-Ormaetxe and Nieto (1995), Roemer (1993, 1996, 1998). For a recent survey, see Fleurbaey (1998).

[5]In the limiting case, the Compensation principle requires "equal outcomes for people who have exercised equal responsibility". Actually, this requirement, together with another property inspired by the Responsibility principle (requiring an equal transfer for people equally endowed in opportunities), plays a crucial role in Bossert (1995)'s and Fleurbaey (1994)'s characterizations of an opportunity redistributive mechanism. Moreover, Fleurbaey (1994) and Bossert (1995) show that, unless the advantage function is additively separable in responsible and non responsible characteristics, there exist not redistribution mechanism satisfying the two properties above.

which the individual is fully responsible, represented by a scalar variable $w, w \in W$. Hence we have:

$$x = g(O, w).$$

We do not know the form of the function g ;we know, however, that it is the same for all individuals[6].

Income is supposed to be continuously distributed, with cumulative distribution function $F(x)$ and $X = [0, z]$.

A person's opportunity set O is observable, and we denote by

$$\Omega = \left\{O^1, O^2, ..., O^i, ..., O^n\right\}$$

the finite set of all possible opportunity sets. We next assume that there is a general political agreement on the following complete ordering $\succ$ over all possible opportunity sets[7]:

$$O^n \succ ... \succ O^i \succ ... \succ O^1$$

so that, in general, we have: $O^{i+1} \succ O^i$. It seems moreover reasonable to assume that, for any fixed level of responsibility $w, w \in W$, the higher the level of opportunities O^i a person is endowed with, the higher her income level x is:

Assumption 1

$$\forall i, j \in (1, ..., n), \forall w \in W, \; O^i \succ O^j \rightarrow g(O^i, w) > g(O^j, w).$$

The responsibility variable w is unobservable and individuals have the same degree of access to the same set of responsible choices W. We next introduce the following assumption on the individual income function g:

Assumption 2

$$\forall w \in W, \; g \; is \; continuous \; and \; monotonic \; in \; w.$$

Following Bossert (1995) and Bossert *et al.* (1996), in our model the distribution of both O and w is not altered by the public policy enforced; however, income is perfectly transferable. This is quite reasonable for the opportunities, especially if one thinks to the native talents as an important element of the individual opportunity set. As for the responsibility variable, the assumption could be justified by thinking that it would be hardly acceptable to held a person fully responsible for w, were this variable depending on the public policy. We now introduce two notions:

[6]Notice, in particular, that we are not assuming the income function to be separable in the opportunity and responsibility variable; a property which plays a crucial role in some of the models proposed so far in the literature: see Bossert (1995) and Fleurbaey (1994) for the characterization of redistributive mechanisms, and Peragine (1998b) for the measurement of opportunity inequality.

[7]Actually, the problem of deriving a ranking rule of opportunity sets has been addressed in a recent and growing literature in the field of social choice, starting with the seminal articles by Jones and Sugden (1982), Suppes (1987) Pattanaik and Xu (1990).

1. A *Type,* denoting the subset of the total population N having the same opportunity set: for $O^i \in \Omega$, we call "type i" the set of individuals whose opportunity set is O^i. Since O is observable, we can easily construct a finite partition of the population into types. Within each type there will be a distribution of income $F^i(x)$, with density distribution $f^i(x)$, and population N_i.

2. A *Tranche,* denoting the subset of the total population N having exercised the same level of responsibility: for $w \in W$, a tranche is the set of individuals whose responsibility level is equal to w. We are however considering the realistic case of non-observability of the responsibility variable; to define a tranche in a workable way, we need therefore to deduce the degree of responsibility exercised from some observable behaviour.

Consider that, once we have included in the specification of the type all relevant factors beyond a person's control, the differences in the income level among people in the same type are, by definition, within their own responsibility. Since the income function is supposed to be increasing in the responsibility variable, we can say that, in a given type, the higher is the income level, the higher the responsibility exercised. However, how to compare the responsibility level of people belonging to different types? We need some measure or some proxy of the individual responsibility which allow us to perform inter-types comparisons.

The problem can be stated as follow: there is an unobservable variable, responsibility (w), which is distributed in some way among the members of a society; say according to a distribution

$$R(w) = \Pr(W : W \leq w).$$

The individual income is generated, in our model, trough the function $g\ (x = g(O, w))$, which is supposed to be continuous and monotone in w. For any $O^i \in \Omega$, F_i is the income distribution in type i, and $F = \Sigma_{i=1}^{n} q_i^F F_i$, were q_i^F is the population share in type i of distribution F, is the income distribution for the whole society. So that we can write $F(x) = \Pr(X : X \leq x)$ and

$$\begin{aligned} F_i(x) &= \Pr\left(g\left(O^i, W\right) : g\left(O^i, W\right) \leq x\right) \\ &= \Pr\left(W : W \leq h\left(O^i, x\right)\right) \\ &= R\left(h\left(O^i, x\right)\right) \end{aligned}$$

where h is defined by:

$$h\left(O^i, x\right) = w \Leftrightarrow g\left(O^i, w\right) = x.$$

Thus, for example, $F_i(\tilde{x})$ is the fraction of type i population with income less or equal to $\tilde{x}$. By the monotonicity assumption, $\tilde{x}$ corresponds to some unique value of responsibility, say $\tilde{w}_i$, in the type population, and $F_i(\tilde{x}) = R(\tilde{w}_i)$. That is, the person who exercised responsibility less than or equal to $\tilde{w}$ are precisely the persons who achieved the income level less than or equal to $\tilde{x}$. Thus, starting from a responsibility distribution $R(w)$, for any given opportunity level $O^i \in \Omega$, we can generate the income distribution for type i:

$$R \longrightarrow F^i = R \circ h_i$$

Then, for any given partition of the population into types, i.e. for any set of $\left\{q_i^F\right\}_{i\in(1,\ldots,n)}$, we can generate the overall income distribution F :

$$F^i \longrightarrow F.$$

Let us denote by Ψ the set of income distributions generated by the relevant responsibility distributions. Now, let $x^F(p,i)$ be the income level at the p^{th} percentile of the income distribution in type i and let $w(p)$ be the responsibility level at p : $R(w(p)) = p$. Hence we have:

$$F_i\left(x^F(p,i)\right) = p$$

and

$$x^F(p,i) = g\left(O^i, w(p)\right)$$

i.e. the fraction of type i persons who have exercised effort $w \leq w(p)$ is exactly p.

From this construction it follows (a slight modification of) Roemer (1993)' s "statistical" solution to the problem of comparing individuals' responsibility levels: "people in different types have exercised a comparable degree of responsibility if they are at the same percentile of their own type income distributions". That is, the position of the individual, as expressed by the percentile, in her own type income distribution, is our proxy for the unobservable degree of responsibility exercised: a "tranche" is therefore constituted by all individuals at the same centile of their own type income distributions. What is the intuitive interpretation for this choice?

The idea is that since, by construction, everyone in a given type is endowed with the same set of opportunities, where (i.e. at which centile) in her own type distribution a person locates is due to her own autonomous choice. She could have placed herself, with a given application of effort, at any centile. Thus, the degree of effort or responsibility exercised uniquely determines the location of an individual in her type income distribution[8]; on the other hand, the income level (in absolute terms) corresponding to that location, is not under the individual control: it is a characteristic of the type; that is, it is a function of the opportunity set.

[8]Acually, this is the case because in the present model, as in all the models so far proposed in the literature on equality of opportunity, we are implicitly assuming that the individual (responsible) choices can be directly mapped to a consequence in terms of individual advantage, without considering the possible interaction with the choices made by other individuals.

Hence, considering types $(1, 2, ..., i, ...n)$, we can now define the tranche p in population N as the subset of individuals whose incomes are at the p^{th} percentile of their respective type distributions F_1, F_2, ..., F_i, ..., F_n. More precisely, consider, within each type i, a continuum of values[9] of $p, p \in [0, 1]$; for any fixed dp we define dx^i by $dp = f^i\left(x^F(p,i)\right)dx^i$, so that dp represents the proportion of type i population in $[x^F(p,i), x^F(p,i) + dx^i]$. We can now define the set of incomes between p and $p + dp$ in type i as:

$$\chi_i^F(p, dp) = \{x \mid x \in [x^F(p,i), x^F(p,i) + dx^i]\}. \tag{1}$$

Hence the subset of population, identified by type, who have exercised responsibility p in the whole population N is represented by the following $Tranche$ $T_F(p, dp)$:

$$T_F(p, dp) = \bigotimes_{i=1}^{n} \chi_i^F(p, dp). \tag{2}$$

As a matter of notation, consider that, if N_i denotes the population in type i, $n^F(p, dp, i) = N_i dp = N_i f_i(x(p,i))dx^i$ is the number of income units between $x(p,i)$ and $x(p,i) + dx^i$. Letting $1^F(p, dp, i)$ be the unit vector of length $n^F(p, dp, i)$, we can therefore describe the tranche at p, as defined in (2), by the following $Tranche$ p distribution $T_F(p, dp)$:

$$T_F(p, dp) = \left\{1^F(p, dp, 1)x^F(p, 1), ..., 1^F(p, dp, i)x^F(p, i), ..., 1^F(p, dp, n)x^F(p, n)\right\}. \tag{3}$$

Moreover, notice that from Assumption 1 it follows that the tranche distributions $T_F(p, dp)$ are ordered at any p :

$$\forall F, \forall i \in (1, ..., n), \forall p \in [0, 1], \; x^F(p, i) \leq x^F(p, i + 1). \tag{4}$$

In the framework we have introduced, the focus of concern for an analysis of equality of opportunity for income is clearly the Tranche distribution: we can compare any two income distributions F, $G \in \Psi$, on the basis of opportunity inequality, by comparing the inequality present in every tranche distribution $\mathrm{T}_G(p, dp)$ and $T_F(p, dp)$; on the other hand, an opportunity egalitarian public policy will be intended to decrease the degree of inequality present at every tranche.

It is also useful to define the types-mean distribution F_μ obtained from a distribution $F \in \Psi$. Let us denote by μ_i^F the mean income of type i of distribution F, defined as:

$$\mu_i^F = \int_0^1 x^F(p, i)\, dp.$$

[9] In application, of course, the data come in discrete form; income distribution data are for instance available in discrete quantile form, not in continuous density function form. Nevertheless, in this paper we formulate the problem in the continuous form, for simplicity of presentation.

Recalling that N_i^F denotes the population in type i, and denoting by 1_i^F the unit vector of length N_i^F, we can define the types-mean distribution F_μ , obtained from distribution F, as follow:

$$F_\mu = \left\{\mu_1^F 1_1^F, ..., \mu_i^F 1_i^F, ..., \mu_n^F 1_n^F\right\}. \tag{5}$$

Notice that Assumption 1 implies that the type mean distribution F_μ, obtained from any F is ordered, such that:

$$\mu_1^F < ... < \mu_i^F < ... < \mu_n^F.$$

The analysis in the rest of the paper is organized as follows. In section 2 we derive dominance criteria for unambiguous social rankings of income distributions. In section 3 we characterize an income tax which is consistent with both the Principle of Compensation and the Principle of Responsibility. Section 4 provides some concluding remarks.

2 Ranking income distributions in terms of equality of opportunity

2.1 Opportunity Lorenz Partial Ordering

Consider any two income distributions F , $G \in \Psi$. Given the reasoning developed above, a natural criterion to rank distributions F and G in terms of opportunity inequality can be formulated by focusing on the income inequality present within each tranche. To perform such a comparison, we introduce two partial orderings which, loosely speaking, say that one income distribution F "Opportunity Lorenz Dominateś"($\succcurlyeq_{OL}$) ("Opportunity Generalized Lorenz Dominates", $\succcurlyeq_{OGL}$) another distribution G if, and only if, the former Lorenz dominates (Generalized Lorenz Dominates) the latter at the limit within each tranche. Hence we have the following:

Definition 1 *For all $F, G \in \Psi$,*

$$F \succcurlyeq_{OL} G \Longleftrightarrow$$

$$\sum_{i=1}^{k} \frac{q_i^F x^F(p,i)}{\sum_{j=1}^{n} q_j^F x^F(p,j)} \geq \sum_{i=1}^{k} \frac{q_i^G x^G(p,i)}{\sum_{j=1}^{n} q_j^G x^G(p,j)}, \forall k \in (1,...,n), \forall p \in [0,1] \tag{6}$$

and

$$F \succcurlyeq_{OGL} G \Longleftrightarrow$$

$$\sum_{i=1}^{k} q_i^F x^F(p,i) \geq \sum_{i=1}^{k} q_i^G x^G(p,i), \forall k \in (1,...,n).,\forall p \in [0,1]. \tag{7}$$

A weaker condition[10] for comparing the opportunity inequality in any two distributions would be that of requiring Lorenz dominance not within each tranche distribution, but just for the mean distributions. Hence we formally define the two partial orderings $\succcurlyeq_{OL(\mu)}$ and $\succcurlyeq_{OGL(\mu)}$:

Definition 2 *For all $F, G \in \Psi$,*

$$F \succcurlyeq_{OL(\mu)} G \iff$$

$$\sum_{i=1}^{k} \frac{q_i^F \mu_i^F}{\sum\limits_{j=1}^{n} q_j^F \mu_j^F} \geq \sum_{i=1}^{k} \frac{q_i^G \mu_i^G}{\sum\limits_{j=1}^{n} q_j^G \mu_j^G}, \forall k \in (1, ..., n) \tag{8}$$

and

$$F \succcurlyeq_{OGL(\mu)} G \iff$$

$$\sum_{i=1}^{k} q_i^F \mu_i^F \geq \sum_{i=1}^{k} q_i^G \mu_i^G, \forall k \in (1, ..., n). \tag{9}$$

2.2 Opportunity Egalitarian SEF

We now try to capture our intuition about social justice, as expressed by the two principles of Responsibility and Compensation, into the formulation of a Social Evaluation Function (SEF); then we'll try to obtain suitable conditions for social dominance. To this end we propose a generalization of the Yaari Dual SEF (Yaari, 1988) to the case of income distributions which can be decomposed into homogeneous sub-groups. According to the standard Yaari Dual SEF, social preferences over income distributions are represented by a weighted average of ordered incomes, where each income is weighted according to its position in the ranking. Hence, using a standard Dual SEF, the welfare of type i would be expressed as:

$$W_F^i = \int_0^1 U(p)\, x^F(p, i)\, dp$$

[10] *Actually, the ordering $\succcurlyeq_{OL}$ is properly defined as: $F \succcurlyeq_{OL} G$ if and only if*

$$\sum_{i=1}^{k} \frac{n^F(p, dp, i)\, x^F(p, i)}{\sum_{j=1}^{n} n^F(p, dp, j)\, x^F(p, j)} \geq \sum_{i=1}^{k} \frac{n^G(p, dp, i)\, x^G(p, i)}{\sum_{j=1}^{n} n^G(p, dp, j)\, x^G(p, j)}, \forall k \in (1, ..., n), \forall p \in [0, 1].$$

However, since $n^F(p, dp, i) = N_i^F dp$, we can cancel dp and, dividing by N^F, we obtain $q_i^F = \frac{N_i^F}{N^F}$. Now, letting $dp \longrightarrow 0$, we finally obtain: $F \succcurlyeq_{OL} G$ if and only if

$$\sum_{i=1}^{k} \frac{q_i^F x^F(p, i)}{\sum_{j=1}^{n} q_j^F x^F(p, j)} \geq \sum_{i=1}^{k} \frac{q_i^G x^G(p, i)}{\sum_{j=1}^{n} q_j^G x^G(p, j)}, \forall k \in (1, ..., n), \forall p \in [0, 1].$$

where $x^F(p,i)$ is defined as before, $U(p)$ is a function expressing the weight attached by society to any income at centile p of type i distribution, and $U(p)\,x^F(p,i)$ therefore represents the contribution to the total social evaluation of a person at centile p of type i. Different value judgments are expressed in this framework by selecting different classes of "social weight" functions.

A generalization to the heterogeneous population case is obtained by expressing the social welfare as an aggregate of the welfare of each type, represented by a standard Dual SEF, weighted by the relevant population share. We obtain the following Social Evaluation Function:

$$W_F = \sum_{i=1}^{n} q_i^F \int_0^1 U_i(p)\, x^F(p,i)\, dp \tag{10}$$

where $U_i(p), i \in (1, ..., n)$, are the possibly different social weightings given to different types. Now we express the intuition behind the Compensation and Responsibility principles by restricting the class of the weight functions $U_i(p)$.

The first property we impose, expression of the "Principle of Responsibility", requires our SEF to express income inequality neutrality within each type. Since, by construction, all people in the same type are endowed with the same opportunities, the within type income inequality can only be due to different degrees of responsibility exercised. Therefore, according to the Responsibility principle, these inequalities are equitable and not to be compensated: our SEF should consequently express neutrality with respect to such inequality. Following Yaari (1988), this requirement is translated by assigning an equal weight to each income in the same type, whatever is the centile they are located:

Property 1 (PR) $\forall p \in [0,1], \forall i \in (1, ..., n)$, $U_i(p) = \beta_i \geq 0$.

The second property is expression of the "Principle of Compensation". Any income inequality within the group of people having exercised the same degree of responsibility is inequitable; therefore, the SEF should express income inequality aversion within such group. This requirement is translated by imposing that, comparing two individuals in different types but at the same centile of their own type distribution, the SEF give higher weight, the lower is the type (the opportunity set):

Property 2 (PC) $\forall p \in [0,1], \forall i \in (1, ..., n-1), U_i(p) > U_{i+1}(p)$.

If Property 1 holds, then Property 2 can simply be expressed as: $\forall i \in (1, ..., n-1), \beta_i > \beta_{i+1}$.

Hence, the weights are constant within types, and decreasing with respect to opportunities within tranches. Let us denote by W^{OE} the set of Opportunity Egalitarian SEFs constructed as in (10) and such that the $U_i(p), \forall i \in (1, ..., n)$, satisfy properties 1 and 2. We then introduce the following:

Definition 3 *For all* $F, G \in \Psi$,

$$F \succcurlyeq_{WOE} G \Longleftrightarrow W_F \geq W_G, \forall W \in W^{OE}.$$

2.3 Dominance results

We are now ready to obtain unambiguous social rankings of income distributions, based on the opportunity egalitarian principles. The following result proves that if a given distribution $F \in \Psi$ Opportunity Generalized Lorenz dominates another income distribution $G \in \Psi$, then F is ranked above G according to any Opportunity Egalitarian SEF.

Theorem 1 *For all $F, G \in \Psi$,*

$$F \succcurlyeq_{OGL} G \Longrightarrow F \succcurlyeq_{WOE} G.$$

Proof. : See the Appendix.

Hence, unambiguous social rankings are achieved if one distribution Generalized Lorenz dominates another at any tranche. This theorem gives normative significance to the statistic-descriptive concept of Opportunity (Generalized) Lorenz Dominance; the normative judgment being based on the opportunity egalitarian ethics.

Notice that Generalized Lorenz dominance at each tranche is a sufficient condition for social dominance; not a necessary one. Indeed, it is easy to show that a much weaker condition is both necessary and sufficient for social dominance, according to all the opportunity egalitarian SEFs:

Theorem 2 *For all $F, G \in \Psi$,*

$$F \succcurlyeq_{WOE} G \Longleftrightarrow F \succcurlyeq_{GL(\mu)} G.$$

Proof. : See the Appendix.

That is, Generalized Lorenz dominance of the mean distribution is a necessary and sufficient condition for welfare dominance.

2.4 Roemer's SEF

We now aim to illustrate the connection between the SEF we are adopting and Roemer's. Roemer (1996) proposes a SEF represented by a weighted average of the minimum outcome of each tranche, the weight being the population shares of the relevant tranche. Now consider our Social Evaluation Function $W \in W^{OE}$:

$$W_F = \sum_{i=1}^{n} q_i^F \int_0^1 U_i(p)\, x^F(p,i)\, dp$$

Clearly, our SEF shares the kind of aggregation "across tranches" proposed by Roemer; in fact, W_F is obtained by summing the welfare level (as measured by the social planner) within each tranche, and the welfare of each tranche is then weighted by the relevant population share (through $q_i^F dp$,which represents the proportion of people at centile p of type i). As for the evaluation of each tranche distribution, recall that we have expressed the within-tranche income inequality aversion by imposing Property 2 (PC) on the class of weighting functions U_i. Following Roemer, we now impose a condition which actually strengthens Property 2: it requires that, within each tranche, only the welfare of the worst off type (in opportunity terms) matters. That is, in evaluating an income-opportunity-effort distribution, the social planner focuses on each tranche; and within each tranche he gives a positive weight only to the group of people endowed with the lowest level of opportunities. We call this property "Extreme within-tranche inequality aversion":

Property 3 (EWTIA) $U_i(p) > 0$, *i=1*, $\forall p \in [0,1]$ & $U_i(p) = 0, \forall p \in [0,1], \forall i \in (2, ..., n)$.

If we restrict the class of U_i by using Property 3 instead of Property 2, our SEF now becomes :

$$W_F^R = \int_0^1 U_1(p) x^F(p,1)\, q_1^F dp \tag{11}$$

which is clearly no other than Roemer' s proposed form of SEF[11]. This social evaluation function is utilitarian (or additive) among tranches and Rawlsian (in the sense of using the maximin criterion) within each tranche. Notice that W_F^R reduces to the utilitarian criterion if all opportunity sets are equal (i.e. when opportunity is not recognised as a relevant and distinctive individual characteristics for normative judgments); it reduces to maximin when all responsibilities are equal. Hence, denoting by W_F^R the set of Social Evaluation Functions constructed as in 10 and such that the $U_i(p), \forall i \in (1, ..., n)$, satisfy properties 1 and 3, the condition for social welfare dominance now becomes:

$$\forall F, G \in \Psi, \;\; \left(W_F^R - W_G^R\right) > 0 \Longleftrightarrow q_1^F \mu_1^F > q_1^G \mu_1^G. \tag{12}$$

According to condition (12), the focus of welfare comparisons becomes simply the total income of the types with the lowest endowment of opportunities; it is a maximin criterion applied to the type-mean distribution, where each type-mean is weighted by the relevant type population share.

Notice that the dominance condition derived in Theorem 2 (using our SEF) requires condition (12) and much more: as we have seen, it requires Generalized Lorenz dominance of one mean distribution over another.

[11]Notice however that our environment, and therefore our SEF, differs in an important way from that of Roemer: we are not considering the responsiveness of individual behaviour to the tax system. In this respect, our model is similar to the one developed by Bossert (1995), and to the general literature on income redistribution and tax progressivity.

3 An Opportunity Egalitarian Income Tax

In this section we consider the problem of defining a redistributive public policy inspired by the opportunity egalitarian ethics. Given the case of non-transferability of the individual opportunities and full transferability of income, we do not seek a mechanism to redistribute opportunities; rather, we try to characterize an income policy which compensates individuals for income inequalities due to differences in endowments of opportunities, without interfering with the inequalities due to autonomous choices.

Considering the previous analysis of opportunity inequality, it seems reasonable to use the opportunity Lorenz criteria constructed earlier in comparing the pre-tax and the post-tax distributions; therefore, we can say that an opportunity redistributive public policy is a tax policy such that, after its application to a given income distribution F, the post-tax distribution F_T , compared to the pre-tax distribution F by means of the Lorenz criterion, exhibits: exhibits:

1. a lower degree of inequality within any tranche $p, p \in [0,1]$;
2. the same degree of inequality within any type $i, i \in (1, ..., n)$.

In more formal terms, we first define the standard Lorenz partial ordering applied to any two type i distributions F_i and G_i, $i \in (1, ..., n)$, belonging respectively to distribution $F, G \in \Psi$:

$$\forall F, G \in \Psi,\ \forall i \in (1, ..., n), \forall p \in [0,1],\ \ F_i \succcurlyeq_L G_i \Longleftrightarrow$$

$$\frac{\int_0^p F_i^{-1}(t)\,dt}{\int_0^1 F_i^{-1}(t)\,dt} \geq \frac{\int_0^p G_i^{-1}(t)\,dt}{\int_0^1 G_i^{-1}(t)\,dt}, \forall p \in [0,1]$$

with $\approx_L$ and $\succ_L$ being respectively the symmetric and asymmetric components of $\succcurlyeq_L$. Now, letting F_T be the post-tax distribution obtained by applying the tax T to a distribution $F \in \Psi$, and recalling the definition of Opportunity Lorenz Partial Ordering ($\succcurlyeq_{OL}$), we can introduce the following

Definition 5 *A tax policy T is an Opportunity Redistributive Tax Policy if and only if*

$$\forall F \in \Psi, \forall i \in (1, ..., n),\ F^i \approx_L F_T^i\ \ \&\ \ F_T \succcurlyeq_{OL} F.$$

We now formulate some axioms inspired by the two principles, Responsibility and Compensation, which are driving all our analysis.

According to the **Principle of Responsibility** (**PR**), differences in individuals achievements which can be unambiguously attributed to differences in the responsibility exercised, are not compensable at the bar of justice, for if they were, the individual responsibility would be denied. Now, by construction, the group of people in the same type are endowed with the same set of opportunities; therefore, differences in their income levels can be unambiguously attributed to differences in the responsibility exercised. The principle of responsibility requires that such differences be not compensated by the tax system; in other words, no redistribution is ethically grounded within types. We capture this ethical principle by requiring within-type proportionality of the tax system.

Letting $T_{p,i}(x)$ be the tax paid by an individual with income x, in tranche p of type i, we can formulate the following:

Axiom 1 (PR)*:*

$$\forall i \in (1, 2, ..., n), \forall p, q \in [0, 1], \forall x, y \in X$$
$$\frac{T_{p,i}(x)}{x} = \frac{T_{q,i}(y)}{y}$$

where $\frac{T_{p,i}(x)}{x}$ is the average tax rate for individuals with income x, in type i, tranche p. Thus, an income tax $T_{p,i}(x)$ reflecting Axiom 1 (**PR**) should be proportional within each type; this ensures that within types there will be no inequality reduction.

The Axiom **PR** could be also derived as an application of the classical Horizontal Equity principle to the current context. The Horizontal Equity Principle requires the "equal treatment of equals**"**. Giving operative meaning to this command requires to define in a precise way the concepts of "equals" and of "equal treatment". In the current context, it seems natural to evaluate the "equal position" in terms of opportunity: are "equals", in a normative sense, individuals with equal opportunities. Hence the HE rule in the present context refers to the treatment of people belonging the same type. If we interpret the "equal treatment " command as equal average tax rate we are lead to the formulation of the **PR** Axiom.

On the other hand, according to the **Principle of Compensation (PC)**, differences of achievements which can be attributed to differences in opportunities are considered inequitable: they should be compensated at the bar of justice. Therefore, inequalities in achievements, among persons who have exercised the same degree of responsibility, should be reduced (compensated), by a public policy. In our model, this requires that, within each tranche, the tax be inequality reducing. Using the same notation as before, and requiring that there be no reranking after the introduction of the tax, we can express formally this requirement and formulate the following:

Axiom 2 (PC)*:*

$$\forall F \in \Psi, \forall i \in (1, 2, ..., n), \forall p \in [0, 1], \forall x, y \in X,$$

$$\begin{aligned} \frac{T_{p,i}(x)}{x} &\leq \frac{T_{p,i+1}(y)}{y} \quad \& \\ x^F(p,i) - T_{p,i}(x) &\leq x^F(p,i+1) - T_{p,i+1}(x). \end{aligned}$$

The PC axiom could be derived as an application of the classical Vertical Equity principle to the current context. The Vertical Equity Principle requires an "appropriately unequal treatment of unequals". In the current context, people are "unequals", normatively speaking, when they are endowed with different sets of opportunities; hence the Vertical equity principle here refers to the appropriate differentiation of treatment of people with different opportunities; that is, of people belonging to different types. If we interpret, as before, the "unequal treatment " as unequal average tax rate, and require also, as part of the Vertical Equity principle, that there be no reranking after the introduction of the taxes, we end up with the formulation of the Axiom **PC**.

In the following result the Responsibility Axiom (**PR**) and the Compensation Axiom (**PC**) are used to characterize the Opportunity Redistributive Tax Policy.

Theorem 3 *The tax policy T satisfies Axioms* **PR** *and* **PC** *if and only if T is an Opportunity Redistributive Tax Policy.*

Proof. : See the Appendix.

We can now analyse the Opportunity Redistributive Tax Policy from a normative point of view; to do this, we employ the already defined $\succcurlyeq_{WOE}$ ordering. Letting T be a tax satisfying Axioms **PC** and **PR**, and denoting by F_P the post tax distribution obtained after the application of a fully proportional tax P raising the same revenue as T, we obtain the following

Theorem 4 *For any distribution* $F \in \Psi$, *and the proportional tax P raising the same revenue as T, if T satisfies* **PC** *and* **PR,** *then* $F_T \succcurlyeq_{WOE} F_P$.

Proof. : See the Appendix.

A consequence of these results is that we now have a criterion to discriminate among alternative tax systems on the basis of a clear normative judgement. In fact, *given two tax systems* T^1 *and* T^2, *raising the same total revenue, we say that* T^1 *is preferred to* T^2 *according to the Opportunity Egalitarian Ethics, if and only if the post-tax distribution* F_{T^1} *Opportunity Lorenz dominates the post-tax distribution* F_{T^2}*:* $F_{T^1} \succcurlyeq_{OL} F_{T^2}$.

4 Concluding remarks

The philosophy of equality of opportunity is this: society should indemnify people against poor outcomes that are the conseqence of causes beyond their control, but not against outcomes that are the consequence of causes within their control, and therefore for which they are personally responsible. In this paper we have tried to explore how the opportunity egalitarian ethics can be translated into a concrete public policy, when the individual responsibility level is unobservable. To cope with this informational constraint, we have adopted Roemer's (1993) statistical solution.

In section 2, by capturing (some of) the opportunity egalitarian principles into the formulation of a social evaluation function, dominance criteria for unambiguous social rankings of income distributions have been proposed. Then (section 3), appealing properties have been formulated and used to characterize an income tax that compensates individuals for income inequalities due to differences in opportunities, without interfering with the inequalities due to autonomous choices.

It is possible to indicate (at least) three possible extensions of this work. First, by investigating more complete orderings which are consistent with the (Opportunity) Lorenz ranking. The idea is that of using an additively decomposable inequality index, then interpreting the within-tranche inequality as opportunity inequality, and the between tranches inequality as inequality due to individual responsibility. Second, by taking into account the effects of the public policy enforced on the individual behaviour. Finally, by considering the possible interaction between the individual's autonomous choice of responsibility with the choices made by others. By stating that the degree of effort exercised uniquely determines the location (centile) of an individual in her type income distribution, we have implicitly assumed that the individual (responsible) choices can be directly mapped to a consequence in terms of individual advantage, without considering the possible interaction with the choices made by other individuals. Once we have recognised the crucial difference between the action a person chooses, among those open to her, and the consequence of this action, which is the result of simultaneous actions chosen by all the individuals in the considered society, it is not clear at all where, in such a context, to draw the bounds of the individual responsibility. An explicit consideration of this distinction seems to be a promising direction for future exercises of modelling the equality of opportunity problem.

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5 Appendix

Proof. of Theorem 1

The result is basically a consequence of the following Lemma, which we state and prove:

Lemma 1 $\sum_{k=1}^{n} v_k w_k \geq 0$ *for all sets of numbers* $\{v_k\}$ *such that* $v_k \geq v_{k+1} \geq 0$, $\forall k \in (1, 2, ..., n)$ *, if and only if* $\sum_{i=1}^{k} w_i \geq 0$ *,* $\forall k \in (1, 2, ..., n)$ *.*

Proof. of Lemma 1.

Applying Abel's decomposition:

$$\sum_{k=1}^{n} v_k w_k = \sum_{k=1}^{n} (v_k - v_{k+1}) \sum_{i=1}^{k} w_i.$$

It is obvious that, if $\sum_{i=1}^{k} w_i \geq 0$, $\forall k \in (1, 2, ..., n)$, then $\sum_{k=1}^{n} v_k w_k \geq 0$. As for the necessity part, suppose that $\sum_{k=1}^{n} v_k w_k \geq 0$ for all sets of numbers $\{v_k\}$ such that $v_k \geq v_{k+1} \geq 0$, but $\exists\ j \in (1, 2, ..., n)$: $\sum_{i=1}^{j} w_i < 0$. Consider what happens when $(v_k - v_{k+1}) \searrow 0$, $\forall k \neq j$:

$$\sum_{k=1}^{n} v_k w_k \rightarrow (v_j - v_{j+1}) \sum_{i=1}^{j} w_i < 0$$

which is the desired contradiction. ■

Now we want to find a sufficient condition for $\Delta W \geq 0, \forall W \in W^{OE}$, where:

$$\Delta W = (W_F - W_G) = \int_0^1 \sum_{i=1}^{n} \beta_i \left[q_i^F x^F (p, i)\, dp - q_i^G x^G (p, i) \right] dp \geq 0.$$

Considering that, by property 1 and 2, $\beta_i \geq \beta_{i+1} \geq 0, \forall i \in (1, ..., n)$, from Lemma 1 we obtain that:

$$\sum_{i=1}^{n} \beta_i \left[q_i^F x^F (p, i) - q_i^G x^G (p, i) \right] \geq 0, \forall p \in [0, 1]$$

$$\Leftrightarrow \sum_{i=1}^{k} \left[q_i^F x^F (p, i) - q_i^G x^G (p, i) \right] \geq 0, \forall k \in (1, ..., n), \forall p \in [0, 1]. \quad (13)$$

Condition (13) is no other than "Opportunity Generalized Lorenz dominance". Clearly this condition implies welfare dominance. ■

Proof. of Theorem 2

The condition for welfare dominance is that $\Delta W \geq 0, \forall W \in W^{OE}$, which holds if and only if

$$\sum_{i=1}^{n} \beta_i \left[q_i^F \int_0^1 x^F(p,i)\, dp - q_i^G \int_0^1 x^G(p,i)\, dp \right] \geq 0$$

for all β_i satisfying Properties 1 and 2. Let $S_i^F(p) = \left[q_i^F \int_0^1 x^F(p,i)\, dp - q_i^G \int_0^1 x^G(p,i)\, dp \right]$, so that $\Delta W = \sum_{i=1}^{n} \beta_i S_i^F(p)$. From Lemma 1 we obtain that:

$$\begin{aligned}
\Delta W &\geq 0 \Longleftrightarrow \sum_{i=1}^{k} S_i^F(p) \geq 0, \forall k \in (1,...,n) \\
&\Longleftrightarrow \sum_{i=1}^{k} \left[q_i^F \int x^F(p,i)\, dp - q_i^G \int x^G(p,i)\, dp \right] \geq 0, \forall k \in (1,...,n) \\
&\Longleftrightarrow \sum_{i=1}^{k} q_i^F \mu_i^F \geq \sum_{i=1}^{k} q_i^G \mu_i^G, \forall k \in (1,...,n)
\end{aligned}$$

which is Generalized Lorenz dominance of the type-means distribution F_μ over G_μ. ■

Proof. of Theorem 3

First, let us focus on the within tranche (between types) redistributive effect. Consider any tranche $T_F(p, dp)\,, p \in [0,1]\,$, and let $T_{F-T}(p, dp)$ be the post-tax distribution at tranche p, obtained after the application of the tax T. We have to prove that

$$\begin{aligned}
T_{F-T}(p,dp) &\succcurlyeq {}_{OL}T_F(p,dp) \Leftrightarrow \\
\sum_{i=1}^{k} \frac{q_i^F \left(x^F(p,i) - T_{p,i}(x) \right)}{\sum_{i=1}^{n} q_i^F \left(x^F(p,i) - T_{p,i}(x) \right)} &\geq \sum_{i=1}^{k} \frac{q_i^F x^F(p,i)}{\sum_{i=1}^{n} q_i^F x^F(p,i)}, \forall k \in (1,...,n)
\end{aligned}$$

if and only if T satisfies PC and PR. Considering the progressivity of T within tranche p (imposed by axiom PC) and the absence of reranking, the result is ensured by the Jakobsson-Fellman theorem. This will be true at any $p, p \in [0,1] : F_T \succcurlyeq_{OL} F$.

As for the second claim of Theorem 3, again the Jakobsson-Fellman theorem ensures that the within type proportionality required by axiom PR is a necessary and sufficient condition for $F^i \approx_L F_T^i, \forall i \in (1,...,n)\,$. ■

Proof. of Theorem 4

Just a consequence of theorems 1 and 3. By Theorem 3,

$$T \quad satisfies \;\; PC \quad and \quad PR \Leftrightarrow F_T \succcurlyeq_{OL} F.$$

By definition: $F_P \approx_{OL} F$, and

$$F_T \succsim_{OL} F \quad \& \quad F_P \approx_{OL} F \Rightarrow F_T \succsim_{OL} F_P.$$

Therefore: $T \quad satisfies \;\; PC \quad and \quad PR \Rightarrow F_T \succsim_{OL} F_P$.

Now, considering that, by Theorem 1, $\forall F, G \in \Psi, F \succsim_{OL} G \Longrightarrow F \succsim_{WOE} G$, we finally obtain that

$$T \quad satisfies \;\; PC \quad and \quad PR \Rightarrow F_T \succsim_{WOE} F_P.$$

■