2010/43

Utilitarianism and unequal longevities: A remedy?

Marie-Louise Leroux and Gregory Ponthiere

CORE

DISCUSSION PAPER

Center for Operations Research
and Econometrics

Voie du Roman Pays, 34
B-1348 Louvain-la-Neuve
Belgium
http://www.uclouvain.be/core

CORE DISCUSSION PAPER
2010/43

Utilitarianism and unequal longevities: A remedy?

Marie-Louise LEROUX [1] and Gregory PONTHIERE[2]

July 2010

Abstract

This paper re-examines a counterintuitive corollary of utilitarianism under unequal longevities: the tendency to redistribute resources from short-lived towards long-lived agents, against any intuition of compensation. It is shown that this corollary prevails not only under time-additive lifetime welfare, but, also, in general, under non-additive lifetime welfare, so that this counterintuitive redistributive corollary is a robust argument against utilitarianism. This paper studies a remedy to that counterintuitive corollary. This consists in imputing, when solving the social planner's problem, the consumption equivalent of a long life to the consumption of long-lived agents. We identify the conditions under which such a modified utilitarian optimum involves a compensation of short-lived agents with respect to the laissez-faire. That remedy is also applied to an economy with risky longevity, where short-lived agents are penalized not only by the limited opportunities to spread resources over time (due to a shorter life), but, also, by lost savings (due to unanticipated death).

Keywords: utilitarianism, differential longevity, compensation, redistribution, consumption equivalent.

JEL Classification: D63, I12, I18, J18

---

[1] Université catholique de Louvain, CORE, B-1348 Louvain-la-Neuve, Belgium. E-mail: marie-louise.leroux@uclouvain.be

[2] Ecole Normale Supérieure, Paris, and Paris School of Economics, F-75014 Paris, France.
E-mail: Gregory.ponthiere@ens.fr

The authors would like to thank David de la Croix, Louis Eeckhoudt, Marc Fleurbaey, Thibault Gajdos, François Maniquet, André Masson, Patrick Moyes, Pierre Pestieau, Erik Schokkaert, Luigi Siciliani, Salvador Valdes-Prieto and two anonymous referees for their helpful comments on this paper.

This paper presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the authors.

1 Introduction

Although widely used by taxation theorists, utilitarianism exhibits nonetheless a quite counterintuitive corollary in the particular context of unequal longevities. Actually, under standard assumptions such as the expected utility hypothesis and additive lifetime welfare, utilitarianism recommends nothing less than the redistribution of resources from *short-lived* agents to *long-lived* agents.

That corollary, which contradicts any intuition of compensation, can be explained as follows. Take the simple case of deterministic longevities varying across agents. Under time-additive lifetime welfare, a utilitarian social planner can hardly distinguish between, on the one hand, one life of x periods, and, on the other hand, x lives of one period.[1] Hence, provided Gossen's First Law (1854) - i.e. the law of declining marginal utility of consumption per period - holds, it is always optimal, for a utilitarian planner, to give the same consumption per period to all agents, whatever their length of life is. As a consequence, long-lived agents do not only live longer: they benefit also, at the social optimum, from more resources. Hence, provided living long is a good thing (or, at least, not a bad thing *per se*), short-lived people are penalized *twice*: once by Nature and once by Bentham.[2]

This double penalization is quite counterintuitive, especially when longevity differentials are exogenous. Clearly, in that case, one would like short-lived agents to be compensated for their short life, as they cannot be regarded as responsible for this. Note that the intuition for compensation may also be strong even when longevity differentials are partly endogenous. For instance, shorter lives due to a strong taste for sin goods, or a large disutility from physical activity may be also regarded as caused by factors that are exogenous to the agent, and, as such, which would support some compensation.

Classical utilitarianism can hardly do justice to such intuitions. All this does not really come as a surprise: as shown by Mirrlees (1982), utilitarianism can, at best, serve as an ethical standard in the special case of a society of *identical* individuals, because, in that case, the totality of all individuals can be regarded as a single individual. However, once some heterogeneity is introduced in the fundamentals (e.g. preferences, handicap, etc.), utilitari-

[1] For simplicity, we abstract here from pure time preferences. Natural discounting through survival probabilities is discussed in the second part of the paper.

[2] As this is well-known, the classical utilitarian doctrine was first stated by Jeremy Bentham (1789). Note, however, that the principle of the largest happiness for the largest number can also be found in the earlier writings of Cessare Beccaria (1764) on the most desirable laws and institutions for justice.

anism can only be used as a useful approximation, and may lead to counterintuitive results.[3] Given that a variation in the length of life can hardly be regarded as non-fundamental, it is not surprising that utilitarianism yields here some counterintuitive consequences.

But even if the difficulties faced by classical utilitarianism under longevity differentials could be expected, this leaves us nonetheless with a quite uncomfortable position. The origin of this discomfort lies in the universality of longevity differentials. Actually, as shown by demographers, longevity differentials within a given cohort have always been large, and remain significant today. For instance, according to the United Nations Development Program (2008), the life expectancy of women is, in the U.S., about 5.2 years larger than the one for men in 2007 (80.4 years against 75.2). There exist also large disparities in survival conditions according to the education, the income, the ethnicity, and the employment status.[4] Hence, if the mere existence of longevity differentials suffices to reject the use of classical utilitarianism, there remains little room for using that ethical doctrine.

Should we then abandon utilitarianism when considering policy discussions in which agents have unequal lengths of life, that is, in almost all policy issues? Whereas one may be tempted to answer affirmatively, it should be stressed that various solutions can be brought, in order to keep the utilitarian framework, but *without* the undesirable redistribution from short-lived to long-lived agents.

A first solution consists in relaxing the assumption of additive lifetime welfare, and in representing lifetime utility by a concave transform of the sum of temporal utilities. That solution, proposed by Bommier (2006) and Bommier *et al* (2009, 2010), introduces a distinction between one life of x periods and x lives of one period, so that a utilitarian planner is less likely to redistribute from the short-lived to the long-lived. Another solution, explored in Leroux and Ponthiere (2009), consists in relaxing the expected utility hypothesis, which is another way to avoid the double penalization. However, those approaches, which rely on complex representations of individual preferences, do not lead to analytically tractable solutions, neither at the laissez-faire, nor at the first-best. Moreover, it is far from obvious that those solutions suffice to save utilitarianism from counterintuitive redistribution.

The goal of this paper is twofold. First, we examine the conditions under which utilitarianism exhibits the undesirable tendency of redistributing resources from short-lived towards

[3] For instance, Arrow (1971) and Sen (1973) showed that, given that a handicaped person is likely to have a lower marginal utility of consumption than other persons, utilitarianism would give him fewer resources than to much better off persons, which is quite paradoxical.

[4] See Rogot *et al* (1992).

long-lived agents. Then, having shown the generality of those conditions, as well as the roots of the problem, we shall propose our own "remedy", and apply it to utilitarianism.

It follows from those two goals that the present study will remain entirely in the utilitarian tradition. Note that our exclusive focus on utilitarianism does not reveal any adherence to its foundations, whose ethical plausibility has been largely questioned.[5] On the contrary, our emphasis on utilitarianism is due to the large popularity of that ethical doctrine in the field of optimal taxation. That large popularity, taken jointly with the counterintuitive results implied by that doctrine under unequal longevities, provides the major motivation for this work.[6] Having stressed this, let us now briefly present our study in more details.

To study the conditions under which utilitarianism redistributes from short-lived towards long-lived agents, we shall first concentrate on a two-period model with unequal deterministic longevities. In that model, all agents know the date of their death, but there can be a large welfare loss due to a shorter life. Provided the (exogenous) endowment of agents is sufficiently large, there exists a welfare loss resulting from the limited opportunity of short-lived agents to spread their endowment over time (because of Gossen's First Law). Moreover, in order to isolate the exact role played by the form of lifetime welfare, we shall assume that lifetime welfare can take either a standard time-additive form, or can be a concave transform of the sum of temporal utilities. Comparing the laissez-faire equilibrium with the utilitarian optimum will then allow us to show that assuming non-additive lifetime welfare does not, in general, suffice to avoid the counterintuitive redistribution from short-lived towards long-lived agents. The reason why this is so lies in the fact that the utilitarian planner, in his problem, does not take into account a fundamental source of injustice among agents, namely that a given amount of consumption does not have the same capacity to produce welfare across agents with unequal longevities.[7] Actually, provided one forgets that source of injustice, the concavity of temporal welfare tends to favour an equalization of consumption per period, and the non-additive nature of lifetime welfare can only play against

[5] By utilitarianism, we mean an ethical doctrine based on welfarism, sum-ranking and consequentialism. See Sen and Williams (1982) on the critique of those three pillars.

[6] By remaining in the utilitarian tradition, the present study differs from ethical frameworks relying on a broader informational basis (e.g. primary goods, functionings), on an non-aggregative objective (e.g. maximin), and paying attention to the relation between means and ends (e.g. responsibility-based approaches). The study of those alternative frameworks under unequal longevities is left for future research.

[7] In some sense, Gossen's First Law makes short-lived agents like handicapped persons, who can only reach, in comparison with long-lived agents, a lower lifetime welfare level from a *given* amount of resources. The introduction of non-additive lifetime welfare tends to mitigate the welfare gap between short-lived and long-lived agents, but can hardly make that gap disappear completely.

that tendency, but not eradicate it.[8] Therefore that undesirable redistribution constitutes a quite robust argument against utilitarianism in the context of unequal longevities.

Given that this counterintuitive feature of utilitarianism comes from its neglect of a cause of injustice (the unequal capacities of consumptions to generate welfare across short-lived and long-lived agents), a natural "remedy" to that problem consists in the addition, to the social planner's problem, of *compensation contraints*. Those constraints account for the fact that consuming a given amount of resources when being short-lived or when being long-lived are, in general, not equivalent in welfare terms. In short, those constraints define homogenized consumptions for short-lived and long-lived agents, by counting the *consumption equivalent of a long life* as part of the consumption of long-lived agents.[9] The underlying idea is that this consumption equivalent is a measure of the advantage of the long-lived, and should be counted as such. The remedy, which can be called "compensation-constrained utilitarianism", consists then of solving a modified social planning problem where all consumptions - either of short-lived or of long-lived agents -, are homogenized by the above procedure. Homogenizing consumptions has crucial consequences on redistribution. By introducing the consumption equivalent of a long life in the social planner's problem, the undesirable redistribution from short-lived to long-lived agents is contradicted, and may even be turned into a redistribution from long-lived to short-lived agents.

This paper aims at examining under which conditions that remedy allows the compensation of short-lived agents or, at least, reduces the - counterintuitive - transfers from short-lived to long-lived agents. For that purpose, we will first contrast the compensation-constrained utilitarian optimum with the laissez-faire and the standard utilitarian optimum in the basic two-period model with deterministic longevities. Then, for the sake of generality, we will consider a more general framework, where longevity is risky, by introducing a probability of survival from the first to the second period of life. The introduction of risky longevity allows us to identify two distinct welfare costs from a shorter life. On the one hand, short-lived agents suffer, as under deterministic longevities, from reduced opportunities to spread their endowment over time. On the other hand, the presence of risk brings an additional welfare loss, since some savings are lost as a result of unanticipated death. That second welfare loss exacerbates the disadvantage of short-lived agents, and invites a larger

[8] However, the *extent* of counterintuitive redistribution depends on the form of lifetime welfare (see below).

[9] That remedy is close to what Broome (2004) proposes in his attempt to account for the value of longevity in a utilitarian framework, but in a goods metrics (and not utility metrics).

compensation. Here again, utilitarianism can hardly provide such a compensation, making a remedy even more needed. Note, however, that the presence of risk, by preventing the identification of long-lived agents *ex ante*, complicates also the task of the social planner, who must deal with *ex ante* unobservable heterogeneity.

Finally, it is also important, at this stage, to show how this paper relates to the existing literature. Actually, this paper makes a bridge between two literatures, which, as far as we know, have remained largely unconnected despite a common focus on longevity and welfare.

The first literature, to which we refered above, focused on the optimal tax-transfer policy under unequal longevities. Bommier (2006) and Bommier *et al* (2009, 2010) studied, in the context of unequal longevities, the optimal redistribution in a utilitarian framework. However, those papers did not identify the conditions under which long-lived agents are better off than short-lived agents at the laissez-faire. This constitutes the first task of the present paper. Moreover, those previous works highlight that concavifying the sum of temporal utilities may reduce the tendency of utilitarianism to redistribute towards the long-lived, but without identifying the conditions under which compensation takes place. The present study will pay a particular attention to that issue, which serves as a starting point for proposing a remedy against utilitarianism's undesirable redistributive corollaries.

The second branch of the literature to which this paper is related consists of the applied welfare analysis on the valuation of longevity differentials across time and space. Since Usher's (1973) pioneer calculation of the consumption-equivalent of a longer life on the basis of empirical estimates of the value of a statistical life (VSL), various empirical studies have studied the monetization of longevity gains (see Nordhaus, 2003; Becker *et al*, 2005). However, while those studies were highly informative about how to measure the progress of societies, their major output - i.e. the empirical estimates of consumption-equivalents of a long life - remained largely unexploited. The present study shows how those estimates can be used to homogenize consumptions across agents having different longevities, and, then, how the so-modified utilitarian problem can lead to a more intuitive allocation of resources.

This paper is organized as follows. Section 2 presents the standard utilitarian redistribution problem under unequal longevities, in the cases of additive and non-additive lifetime welfare. Section 3 introduces the remedy, and contrasts the modified first-best with the standard utilitarian optimum. Section 4 examines how the remedy can be applied under risky longevity. Numerical illustrations are given in Section 5. Section 6 concludes.

2 The basic model: deterministic longevity

Let us first consider the standard utilitarian problem of redistribution under differential longevities. For that purpose, we shall consider here two types of agents, $i = 1, 2$, with different longevities. Type-1 agents live one period (of length normalized to 1), while type-2 agents live two periods. For simplicity, we shall, without loss of generality, assume that there exists one member within each of those groups.

As usually assumed in the literature, the utility of death is fixed to zero. However, the lifetime welfare is here assumed to have a more general form than the standard time-additive form. For the sake of generality, we suppose that the lifetime utility of agents is given by[10]

$$\begin{aligned} U^1 &= G\left(u\left(c^1\right)\right) \\ U^2 &= G\left(u\left(c^2\right) + u\left(d^2\right)\right) \end{aligned}$$

where c^i and d^i are first- and second-period consumptions for agent $i = 1, 2$, $u(\cdot)$ is the standard temporal utility of consumption (with $u'(\cdot) > 0$ and $u''(\cdot) < 0$), and $G(\cdot)$ is an increasing and concave transform of individual temporal utilities (i.e. $G'(\cdot) > 0$ and $G''(\cdot) \leq 0$).[11] The standard time-additive lifetime welfare coincides with the case where $G(\cdot)'' = 0$. As shown by Bommier (2006), the linearity of $G(\cdot)$ implies that agents are net risk neutral with respect to the life duration, in the sense that agents with no impatience are strictly indifferent between loteries with the same expected length of life (under a constant consumption per period). On the opposite, by relying on a concave function $G(\cdot)$, we allow here for the possibility that individuals are net risk averse with respect to the length of life.

Regarding the specification of temporal utility functions $u(\cdot)$, we shall also assume, in the rest of the paper, that the utility function takes the form $u(c) = v(c) + \beta$, where $v(0) = 0$, and $\lim_{c\to\infty} v(c) = \bar{v}$, with $\beta < \bar{v} < \infty$.[12]

[10] For simplicity, we abstract here from pure time preferences. See Section 4 for a more complete model with (natural) time discounting.

[11] Temporal utility functions $u(\cdot)$ and concave transforms $G(\cdot)$ are assumed to be the same for all agents.

[12] As we shall see, this decomposition into a concave function and an intercept greatly helps when considering whether short-lived agents are, *ceteris paribus*, penalized by a short life or not.

2.1 The laissez-faire

At the laissez-faire, and assuming that each agent has one half of the total endowment W of resources, the optimal consumptions are

$$\begin{aligned} c^1 &= \frac{W}{2} \\ c^2 &= d^2 = \frac{W}{4} \end{aligned}$$

Contrary to common beliefs, nothing guarantees, in general, that the long-lived agent is, at the laissez-faire, better off than the short-lived agent. However, under mild conditions on individual temporal utility functions $u(.)$ and the available resources W, long-lived people are necessarily better off than short-lived persons, and, thus, advantaged by Nature, despite the equality of resources available for each of them.

Proposition 1 *If $u(0) \geq 0$ (i.e. $\beta \geq 0$), type-2 agents are, at the laissez-faire, better off than type-1 agents, whatever the total amount of resources W is. If $u(0) < 0$ (i.e. $\beta < 0$), type-2 agents are, at the laissez-faire, better off than type-1 agents if and only if $W > W^S$, where W^S is such that $2v\left(\frac{W^S}{4}\right) + \beta = v\left(\frac{W^S}{2}\right)$.*

Proof. See the Appendix. ■

Hence, it follows from Proposition 1 that, under either $u(0) \geq 0$ or $W > W^S$, short-lived agents enjoy, at the laissez-faire, a lower welfare level than the one of long-lived agents. Thus, under those conditions, short-lived agents are said to be disadvantaged by Nature, as they enjoy, for an *equal* amount of resources, a lower lifetime utility level than long-lived agents. Note that the condition for a natural disadvantage of short-lived agents is invariant to the transform $G(\cdot)$, and, as such, is robust to the functional form for lifetime welfare.

Given that it is only under a very low level of resources that short-lived are advantaged by Nature, we shall, throughout this paper, pay a larger attention to the case where short-lived are disadvantaged, and leave the other, less plausible case, aside.

2.2 The utilitarian optimum

Let us now examine how a social planner would distribute a given amount W of resources. The problem of the social planner can be written as:

$$\begin{aligned} &\max_{c^1,c^2,d^2} G\left(u\left(c^1\right)\right) + G\left(u\left(c^2\right) + u\left(d^2\right)\right) \\ &\text{s.to } c^1 + c^2 + d^2 \leq W \end{aligned}$$

From the FOCs of that simple optimization problem, we have

$$\begin{aligned} G'\left(u\left(c^1\right)\right)u'\left(c^1\right) &= G'\left(u\left(c^2\right)+u\left(d^2\right)\right)u'\left(c^2\right) \\ &= G'\left(u\left(c^2\right)+u\left(d^2\right)\right)u'\left(d^2\right) \end{aligned}$$

Given $G''(.) \leq 0$, per period consumptions are equal for the type-2 agent, i.e. $c^2 = d^2$.

Regarding how the consumptions of type-1 and type-2 agents differ, the solution depends on the precise form of the transform $G(\cdot)$. If $G(.)$ was linear, so that lifetime welfare takes a standard time-additive form, the optimal allocation would be such that $c^1 = c^2 = d^2 = W/3$. Thus, total consumption for type-1 agent would be $W/3$, while it would be $2W/3$ for a type-2 agent. As a consequence, utilitarianism would then redistribute from short-lived towards long-lived agents. Under $G(.)$ strictly concave, we may have $c^1 < c^2$ or $c^1 > c^2$, depending on the level of the intercept β. As shown in Proposition 2, we have $c^1 > c^2$ under β sufficiently large, but we may also have $c^1 < c^2$ when β is low.[13]

Proposition 2 *The utilitarian optimum is such that*

- *If $G''(\cdot) = 0$, we have: $c^1 = c^2 = d^2 = W/3$.*
- *If $G''(\cdot) < 0$, we have:*
  - *under $\beta > -v\left(\frac{W}{3}\right)$, $c^1 > c^2 = d^2$.*
  - *under $\beta = -v\left(\frac{W}{3}\right)$, $c^1 = c^2 = d^2 = W/3$.*
  - *under $\beta < -v\left(\frac{W}{3}\right)$, $c^1 < c^2 = d^2$.*

Proof. See the Appendix. ■

Under standard time-additive lifetime welfare (i.e. $G''(\cdot) = 0$), all life periods are treated equally by the social planner, so that consumption is smoothed across people and periods. Hence, in comparison with the laissez-faire, utilitarianism redistributes resources from type-1 agents (i.e. short-lived agents) to type-2 agents (i.e. long-lived agents), contrary to any intuition of compensation.[14] Clearly, given that the agent of type 1 lives a shorter life, and is, under the mild conditions of Proposition 1, disadvantaged by Nature, one may be tempted to give him some compensation, that is, some additional consumption. But utilitarianism, under time-additive lifetime welfare, yields the opposite result: the long-lived agent will

[13] Note that, in Bommier *et al.* (2010), it is assumed that the intercept of $u(\cdot)$ is strictly positive.

[14] Note that this redistribution violates, under the conditions of Proposition 1, what Sen (1973) called the Weak Equity Axiom: when an agent has a lower utility level than another for all levels of income, the optimal allocation must not give less income to him than to the other.

benefit from the same consumption per period as the short-lived, and, thus, will also benefit from a higher total consumption (aggregated on the whole lifetime).

However, once lifetime welfare is a concave transform of the sum of temporal utilities, the utilitarian allocation involves a higher consumption per period for short-lived agents provided the intercept of the temporal utility function β exceeds the threshold $-v\left(\frac{W}{3}\right)$. This amounts to saying that, in that case, the utilitarian social planner will, under a strictly concave transform $G(\cdot)$, give more to the short-lived per period than to the long-lived. Hence the introduction of a concave transform $G(\cdot)$ affects the optimum allocation of resources for a utilitarian social planner: this leads, in general (i.e. under $\beta > -v\left(\frac{W}{3}\right)$), to a higher consumption (per period) to the short-lived in comparison to the utilitarian solution under standard time-additive lifetime welfare.

Note that this result does not imply that utilitarianism necessarily compensates the short-lived agent in comparison with the laissez-faire, even under $G''(\cdot) < 0$. Remind that, at the laissez-faire, the short-lived consumption equals $W/2$, so that a compensation for a short life requires the utilitarian optimum to involve $c^1 > W/2$. In the light of Proposition 2, this is not always the case. Under $G''(\cdot) = 0$, there is a double penalization of short-lived, as $c^1 = W/3 < W/2$. Moreover, under $G''(\cdot) < 0$ and $\beta > -v\left(\frac{W}{3}\right)$, we have $c^1 > W/3$, which does not guarantee compensation (i.e. $c^1 > W/2$). Actually, it is only under strict conditions that utilitarianism compensates short-lived agents with respect to the laissez-faire.

Proposition 3 *Assume $W > W^S$. Under utilitarianism, short-lived agents receive a compensation in comparison with the laissez-faire, i.e. $c^1 > W/2$, if and only if:*

$$\frac{G'\left(v\left(\frac{W}{2}\right)+\beta\right)}{G'\left(2v\left(\frac{W}{4}\right)+2\beta\right)} > \frac{v'\left(\frac{W}{4}\right)}{v'\left(\frac{W}{2}\right)}$$

Note that, under $W \leq W^S$, short-lived agents never receive a compensation: $c^1 < W/2$.

Proof. See the Appendix. ■

To interpret Proposition 3, let us consider some polar cases. If lifetime welfare takes a standard time-additive form, the LHS of the condition of Proposition 3 equals 1, whereas the RHS is larger than 1, so that there cannot be any compensation of short-lived agents. It is even worse, as $c^1 < W/2$ involves a double penalization of short-lived agents: once by Nature (shorter life) and once by Bentham (lower total consumption). Thus the concavity of the transform $G(\cdot)$ is required for the existence of a compensation of short-lived agents under standard utilitarianism. But although necessary, that condition is not sufficient. The reason why this is not so has to do with the non-linearity of temporal welfare, which supports the opposite of compensation. Indeed, if $v(\cdot)$ was linear, the RHS of the condition in Proposition 3 would be equal to 1, to that, provided W lies strictly above the threshold level W^S, the LHS of the condition would exceed 1, and some compensation of the short-lived would take place. However, temporal utility is likely to be concave, leading a RHS larger than 1, and this explains why non-additive lifetime welfare does not suffice, on its own, to bring a compensation of short-lived agents with respect to the laissez-faire.

Actually, it is quite likely that $G(\cdot)$ is "not concave enough" in comparison to $v(\cdot)$, so that utilitarianism does not compensate short-lived agents in comparison with the laissez-faire, and reinforces welfare inequalities induced by Nature.[15] The transform $G(\cdot)$ is probably closer to a linear function than the function $v(\cdot)$, on the grounds that temporal welfare "accumulates" quite well over time periods (i.e. with a low degree of declining marginal "returns" from temporal welfare), so that the LHS of the condition must be close to 1, unlike the RHS, where the difference between $v'(\frac{W}{4})$ and $v'(\frac{W}{2})$ is likely to be large, implying that the RHS exceeds 1. Thus utilitarianism does not, in general, compensate short-lived agents, and does even penalize them, by transfering resources towards long-lived agents. In the light of this, a "double penalization" is, despite $G(\cdot)'' < 0$, most likely, the short-lived being penalized once by Nature (shorter life) and once by Bentham (less total consumption).[16]

The above discussion has obvious consequences in welfare terms. If $G(\cdot)'' = 0$, the inequality in lifetime welfare between long-lived agents and short-lived agents is, under $W > W^S$, larger at the utilitarian first-best than under the laissez-faire, as a result of the double penalization of the short-lived.[17] If, on the contrary, $G(\cdot)'' < 0$, the outcome depends

[15] Note, however, that such a claim should be treated with cautiousness, as we would ideally need here a formal measure of concavity, which can only be local, making our claims relative to some values of W.

[16] Another way to show that the condition of Proposition 3 is likely to be violated is to rewrite it as

$$\frac{G'(y)}{G'((1+k)y)} > \frac{v'(x)}{v'(2x)}$$

with $0 < k < 1$, $x \equiv W/4$ and $y \equiv v(W/2) + \beta$. Note first that $0 < k < 1$ can be justified as follows. The sum of the temporal welfares induced by two periods of life with consumption $W/4$ is, under $W > W^S$, necessarily larger than the welfare induced by one period of life with consumption $W/2$, implying $0 < k$, but also necessarily less than twice the welfare induced by one period of life with consumption $W/2$, implying $k < 1$. Back to the condition, one can see that, given $k < 1$, the *relative* change (from the numerator to the denominator) considered in the arguments of the two functions $v(\cdot)$ and $G(\cdot)$ is strictly larger on the RHS than on the LHS (since x doubles, whereas y is multiplied by less than 2). Hence we can expect that the effect of those relative changes on the marginal derivative is larger on the RHS than on the LHS, implying that the RHS exceeds the LHS, contrary to the condition of Proposition 3.

[17] Indeed, at the laissez-faire, the inequality in lifetime welfare between the long-lived and the short-lived

on $G(\cdot)$, $u(\cdot)$, W and β. If these are such that the condition of Proposition 3 is satisfied, then utilitarianism reduces lifetime welfare inequalities between long-lived and short-lived agents. Otherwise utilitarianism exacerbates inequalities between long-lived and short-lived in comparison with the laissez-faire. That undesirable corollary is most plausible, since the degree of concavity of temporal welfare is likely to be larger than the one of lifetime welfare.

From this section, it is clear that the undesirable tendency of utilitarianism to redistribute from short-lived towards long-lived agents remains despite the non-additivity of utilities across time-periods. Even though the concave transform $G(\cdot)$ supports a redistribution towards the short-lived in comparison with the double-additive case, this does *not* suffice, on its own, to bring the compensation of short-lived agents in comparison with the laissez-faire. The reason why this does not suffice has to do with the non-linearity of temporal welfare, which supports an equalization of consumption per period per person despite longevity inequalities. The introduction of non-additive lifetime welfare mitigates that tendency, but does not suffice to compensate short-lived agents in comparison to the laissez-faire.

How should then one proceed to obtain a compensation of short-lived agents? Undoubtedly, there exist several ways to implement the idea of compensation.[18] One may, for instance, consider that the short life is something for which agents are not responsible at all, and, which, as such, requires a complete compensation of disadvantaged agents. As a consequence, the compensation in consumption should be sufficiently large to equalize, if possible, the lifetime welfare of all agents, either short-lived or long-lived.[19] Whereas such an approach is intuitively appealing, this forces us to depart from the utilitarian doctrine, since such an egalitarian approach leads to the neglect of aggregative concerns.

Given that the present paper concentrates on utilitarianism, we shall propose, in the next section, an alternative way to implement the compensation *within* the utilitarian framework,

agents is, under $W > W^S$:

$$G\left(2v\left(\frac{W}{4}\right)+2\beta\right)-G\left(v\left(\frac{W}{2}\right)+\beta\right)>0$$

At the utilitarian optimum, inequality in lifetime welfare is

$$G\left(2v\left(c^2\right)+2\beta\right)-G\left(v\left(c^1\right)+\beta\right)>0$$

If $G(\cdot)$ is linear, $c^1=c^2=d^2=W/3$, so that the first term at the optimum is larger than the first term at the laissez-faire, while the second term is smaller. Hence the lifetime welfare inequality under utilitarianism is then strictly larger than under the laissez-faire.

[18] For a survey on compensation in recent normative economics, see Fleurbaey and Maniquet (2006).

[19] Note that such an approach does not require the homogeneity of preferences, since the compensation could be carried out on an equalitarian-equivalent basis, that is, on the basis of how agents locate with respect to an allocation of reference (see Fleurbaey, 2005).

and, which, as such, does not abandon all aggregative concerns on the grounds of compensation and equality.[20] The approach that we shall develop in the next section keeps the utilitarian (aggregative) goal, but adds a concern for compensation. One can thus refer to it as some form of "compensation-constrained utilitarianism".

3 Compensation-constrained utilitarianism

Our starting point for rethinking the utilitarian allocation of resources in the context of unequal longevities consists of an observation made in the previous section. In Proposition 3, we show that, despite the introduction of non-additive lifetime welfare, an undesirable redistribution from short-lived towards long-lived agents is most likely to be recommended by utilitarianism, on the grounds of the concavity of temporal welfare. The reason why such a result holds lies in the fact that standard utilitarianism does not take into account that short-lived agents are, in general, disadvantaged in comparison with the long-lived (since a given amount of resources generates a lower welfare for the short-lived than for the long-lived, as shown in Proposition 1).[21] The utilitarian social planner does not regard that piece of information as relevant. This explains why, because of Gossen's Law, there is a strong tendency towards transfering to the long-lived.[22]

It is that precise postulate that has to be relaxed to provide a compensation of short-lived agents. The present section proposes one way to carry out such a distinction. For that purpose, we shall first define what we call the consumption equivalent of a long life (subsection 3.1), and, then, redefine the social planner's problem by counting this consumption equivalent as a part of the consumption of long-lived agents (subsection 3.2).

3.1 The consumption equivalent of a long life

In order to distinguish consumptions depending on the number of periods lived, one approach consists in introducing, within the social planner's problem, a fictive consumption equivalent of a long life, and to solve the so-constructed modified utilitarian problem. Such a consumption equivalent captures the idea that living long is a kind of consumption, which, as such, has to be taken into account as a consumption in the planner's problem.

[20] On the compensation of short-lived agents in an equivalent-egalitarian basis, see Fleurbaey *et al* (2010).
[21] It is only under linear temporal welfare that the "handicap" of short-lived agents disappears.
[22] That tendency is only partly contradicted by the non-linearity of lifetime welfare (see Proposition 3).

For that purpose, let us first denote by α the fictive consumption equivalent to a long life. The consumption equivalent to a long life α corresponds to the value, expressed in the (unique) consumption good, of enjoying a long life, that is, in the present context, a life of two periods. Alternatively, α can also be interpreted as reflecting the consumption equivalent of the continuity of life across periods. In the rest of this paper, we shall assume that such a consumption equivalent exists, in the sense that, for any longevity differential, it is possible to find a compensation in terms of the consumption good.[23]

The consumption equivalent of a long life α makes the agent indifferent between, on the one hand, a short life with that additional consumption, and, on the other hand, a long life:

$$G\left(u\left(c^{1*}+\alpha\right)\right)=G\left(u\left(c^{1**}\right)+u\left(d^{1**}\right)\right) \tag{1}$$

where c^{1*}, c^{1**} and d^{1**} are the consumptions under the laissez-faire, when the agent faces one period of life [i.e. problem (*)] or two periods of life [i.e. problem (**)].[24] From Section 2, we have $c^{1*}=W/2$ and $c^{1**}=d^{1**}=W/4$ (as a type-1 agent with two periods of life would behave as a type-2 agent).

The level of α depends on the utility functions of agents. However, given that the concave transform $G(\cdot)$ appears on both sides of the above expression, we can simplify it to

$$u\left(c^{1*}+\alpha\right)=u\left(c^{1**}\right)+u\left(d^{1**}\right)$$

Hence, the consumption equivalent of a long life α is invariant to the precise form of the transform $G(\cdot)$: whatever $G(\cdot)$ is concave or not, this does not affect the definition of α. This invariance of α to the precise form of $G(\cdot)$ implies that the remedy we shall propose below is distinct from applying a concave transform to the sum of temporal utilities.

The consumption equivalent of a long life α depends only on the shape of the temporal utility function. If, for instance, $u(\cdot)$ was linear and if $u(0)=0$, we would have $u(c^{1*}+\alpha)=W/2+\alpha$, and $u\left(c^{1**}\right)+u\left(d^{1**}\right)=W/2$, so that α would be equal to zero: to make an agent indifferent between a short life and a long life, no compensation is required, as it suffices to transfer second-period consumption to the first period. Alternatively, if $u(.)$ is affine with an intercept β, we have $u(c^{1*}+\alpha)=W/2+\alpha+\beta$, and $u\left(c^{1**}\right)+u\left(d^{1**}\right)=W/2+2\beta$, which yields $\alpha=\beta$. However, if $u(.)$ is concave, and if $u(0)$ is not too low or if resources W are sufficiently large, we have $u(W/2)<2u(W/4)$, so that $\alpha>0$.

[23] That assumption is far from weak, especially if the longevity differentials considered are large. In that case, a consumption equivalent may not exist.

[24] Note that this would also be true for type-2 agents.

Given that temporal utility is concave in consumption, and that resources W can be regarded as sufficiently large, the intuition tends to assign to α a positive sign: this captures the idea that it is better to have, *ceteris paribus*, a long life rather than a short life, that is, it is better, *for a given amount of resources*, to live long. Actually, the conditions guaranteeing a positive α are, by construction, the same as the ones that lead to a double penalization by Nature and by Bentham, i.e. the conditions of Proposition 1.[25]

At this stage, it is also crucial to notice that, as we have *identical* temporal utility functions for all agents, the consumption equivalent of a long life α does not only have the capacity to equalize the lifetime utility under a short life and a long life, but it can also *compensate* a short-lived agent by giving him as much utility as a long-lived agent.

To see this, note first that, given that agents have the same utility functions, the RHS of expression (1) is also equal to $G\left(u\left(c^{2**}\right)+u\left(d^{2**}\right)\right)$, because all agents would solve the consumption program similarly if put in the same situation under identical utility functions. It follows from this that the consumption equivalent α does not only bring the equality of utility between the short life and the long life for a given individual, but it also *equalizes the lifetime utilities* of agents having different lengths of life:

$$G\left(u\left(c^{1*}+\alpha\right)\right)=G\left(u\left(c^{2**}\right)+u\left(d^{2**}\right)\right) \tag{2}$$

as $c^{1**}=c^{2**}$ and $d^{1**}=d^{2**}$ for agents solving the same problem (**). The advantage of expression (2) over expression (1) is that (2) relies on empirically observable choices: type-2 agents' consumptions at the laissez-faire.

Note that, given that α is invariant to the transform $G(\cdot)$, its construction can be made on the mere basis of information on the temporal utility function $u(\cdot)$. The construction of the consumption equivalent of a long life is illustrated on Figure 1. To find the value of α, we compute the sum of temporal utilities of a type-2 agent under the laissez-faire, equal to $u(c^2)+u(d^2)$, and look for the level of c^1 that yields the same temporal welfare $u(c^1)$. On the left graph, that level is equal to $W/2+\alpha$. Hence, the consumption equivalent of a long life, α, corresponds to the thick horizontal segment on the left graph of Figure 1.

The consumption equivalent of a long life α consists, by construction, in a measure of the damage suffered by the short-lived agent, or, alternatively, a measure of the advantage of the long-lived agent. As such, the consumption equivalent constitutes an adequate tool

[25] To see this, note first that, under $u(0)\geq 0$, or $u(0)<0$ and $W>W^S$, we have, at the laissez-faire, $u\left(\frac{W}{2}\right)<2u\left(\frac{W}{4}\right)$, so that only a positive α could make the LHS equal to the RHS: $u\left(\frac{W}{2}+\alpha\right)=2u\left(\frac{W}{4}\right)$.

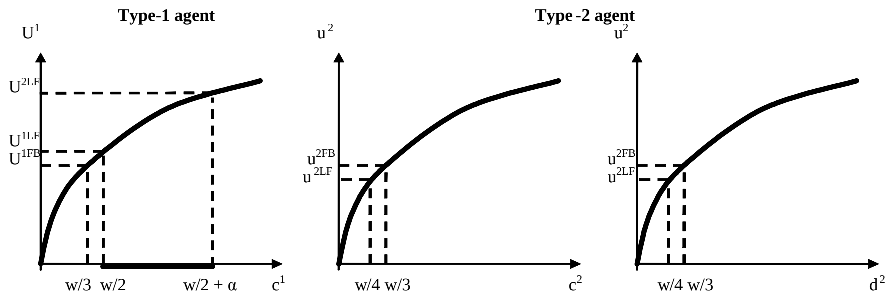

Figure 1: The consumption-equivalent

for transforming the consumptions of short-lived and long-lived agents into homogeneous, i.e. comparable, variables, which can then serve as a basis for the social planner's problem. Whereas the standard utilitarian planning problem used to treat all consumptions as equivalent, one can now consider an alternative social planning problem, based on homogeneous consumptions. Actually, by including the consumption equivalent of a long life α, as a fictive consumption of long-lived agents, the social planner can take into account the fact that long-lived agents are, in general, advantaged by Nature (in the sense that their resource endowment has a different impact in welfare terms), and allocate resources accordingly.[26]

3.2 The modified planner's problem

The modified utilitarian problem differs from the standard one (see Section 2.2) in a single aspect: the planner, instead of defining the consumption flows c^1, c^2 and d^2 while not taking into account longevity differentials among agents, will now distinguish between consumptions that are enjoyed by short-lived and by long-lived agents. Thus the social planner still solves a social welfare maximization problem here, but on the basis of homogenized consumptions $\tilde{c}^1$, $\tilde{c}^2$ and $\tilde{d}^2$, i.e. consumptions taking longevity differentials into account.

In order to take longevity differentials into account, a natural way to proceed is to incorporate the consumption equivalent of a long life α as part of the long-lived agents' consumption, that is, to include it in $\tilde{c}^2$ and $\tilde{d}^2$. Indeed, if α measures the advantage of the long-lived, it makes sense to count this as a consumption enjoyed by the long-lived. This

[26] Note that if $\alpha < 0$, then the social planner would rather take into account the fact that the long-lived are disadvantaged by Nature.

amounts to incorporate α within consumptions $\tilde{c}^2$ and $\tilde{d}^2$.

Note that there exist several ways to introduce α in the consumption of the long-lived agent. *A priori*, the social planner might count the whole consumption equivalent of a long life as a part of the second-period consumption (i.e. $\tilde{c}^2 = c^2$ and $\tilde{d}^2 = d^2 + \alpha$), or, alternatively, as a part of the first-period consumption (i.e. $\tilde{c}^2 = c^2 + \alpha$ and $\tilde{d}^2 = d^2$). However, if α captures the lifetime - rather than instantaneous - value of the continuation of life as a whole, it makes more sense to spread that consumption equivalent of a long life *equally* on all periods lived by a long-lived agent.

Hence the homogenized consumptions $\tilde{c}^1$, $\tilde{c}^2$ and $\tilde{d}^2$ can be defined as:

$$\begin{aligned} \tilde{c}^1 &= c^1 \\ \tilde{c}^2 &= c^2 + \frac{\alpha}{2} \\ \tilde{d}^2 &= d^2 + \frac{\alpha}{2} \end{aligned}$$

By allocating the resources among agents on the basis of those homogenized consumptions, the social planner will distinguish between the consumptions enjoyed by a long-lived agent and the consumption enjoyed by a short-lived agent.

Thus, the social planner's problem becomes:

$$\begin{gathered} \max_{c^1,c^2,d^2} G\left(u\left(\tilde{c}^1\right)\right) + G\left(u\left(\tilde{c}^2\right) + u(\tilde{d}^2)\right) \\ \text{s.to } \tilde{c}^1 = c^1 \\ \text{s.to } \tilde{c}^2 = c^2 + \tfrac{\alpha}{2} \\ \text{s.to. } \tilde{d}^2 = d^2 + \tfrac{\alpha}{2} \\ \text{s.to } c^1 + c^2 + d^2 \leq W \end{gathered}$$

where α is obtained from equation (2).[27] The introduction of homogenized consumptions $\tilde{c}^1$, $\tilde{c}^2$ and $\tilde{d}^2$ can be regarded here as the addition of *compensation constraints*, aimed at making the planner internalize longevity differentials as an ethically relevant piece of information for the allocation problem at stake.[28] Note, however, that the resource constraint remains the same as before, since the consumption-equivalent of a long life α is something fictive, which does not coincide with actual resources, whose total amount remains equal to W.

[27] Note that α cannot be seen as a satisfaction parameter for having a long life, as α is in good metrics, not utility metrics. This point is worth being stressed, as an additive satisfaction parameter expressed in utility terms would not affect at all the problem of the social planner.

[28] The standard utilitarian problem coincides with a special case, where $\tilde{c}^2 = c^2$ and $\tilde{d}^2 = d^2$.

The FOCs of that modified problem are

$$\begin{aligned}
G'\left(u\left(c^1\right)\right)u'\left(c^1\right) &= \lambda \\
G'\left(u\left(c^2+\frac{\alpha}{2}\right)+u\left(d^2+\frac{\alpha}{2}\right)\right)u'\left(c^2+\frac{\alpha}{2}\right) &= \lambda \\
G'\left(u\left(c^2+\frac{\alpha}{2}\right)+u\left(d^2+\frac{\alpha}{2}\right)\right)u'\left(d^2+\frac{\alpha}{2}\right) &= \lambda
\end{aligned}$$

where λ is the Lagrange multiplier associated with the resource constraint. The compensation-constrained utilitarian optimum is presented in Proposition 4.

Proposition 4 *The compensation-constrained utilitarian optimum is such that:*

- *If $G''(\cdot) = 0$, we have $c^1 = W/3 + \alpha/3$ and $c^2 = d^2 = W/3 - \alpha/6$.*
- *If $G''(\cdot) < 0$, we have:*
  - *under $\beta > -v\left(\frac{W}{3}+\frac{\alpha}{3}\right)$, $c^1 > W/3 + \alpha/3$ and $c^2 = d^2 < W/3 - \alpha/6$.*
  - *under $\beta = -v\left(\frac{W}{3}+\frac{\alpha}{3}\right)$, $c^1 = W/3 + \alpha/3$ and $c^2 = d^2 = W/3 - \alpha/6$.*
  - *under $\beta < -v\left(\frac{W}{3}+\frac{\alpha}{3}\right)$, $c^1 < W/3 + \alpha/3$ and $c^2 = d^2 > W/3 - \alpha/6$.*

Proof. See the Appendix. ■

Let us contrast those results with the standard utilitarian allocation. For that purpose, take first the baseline case of additive lifetime welfare. In that case, we had, under standard utilitarianism, $c^1 = c^2 = d^2 = W/3$. Thus, under $\alpha > 0$, the social planner now gives more per-period consumption to short-lived agents than to long-lived agents.[29] Indeed, in addition to the first-best allocation $W/3$, short-lived agents now also receive a third of the consumption equivalent of a long life. On the contrary, the long-lived agents now undergo a reduction of their consumption in the first and second periods, equal to $\alpha/6$. Hence the monetization of longevity gains in the long-lived's consumption bundle affects the optimum, by implying a larger consumption for the short-lived (under $\alpha > 0$).[30]

Turning now to the case where $G''(\cdot) \neq 0$, we obtain, as in Proposition 2, that whether the social planner allocates more resources to the short-lived or not in comparison to the case where $G''(\cdot) = 0$ depends on the intercept of the temporal utility function β. Thus, as

[29] Alternatively, under $\alpha < 0$, we would have $c^1 < c^2 = d^2$.

[30] Note also that, provided one imposes a particular functional form on $u(\cdot)$, it is possible to use the analytical solutions for c^1 and c^2 from Proposition 4 in such a way as to derive an explicit solution for α. For instance, if $v(c) = c^{\frac{1}{k}}$ and $\beta = 0$, we obtain, by substituting for c^1 and c^2 in the expression $u(c^1+\alpha) = u(c^2) + u(d^2)$, that $\alpha = W\frac{2^k-1}{4+2^{k-1}}$. Under that formulation, α is increasing in k, from $W/5$ when $k = 1$ to $2W$ when $k = +\infty$.

above, the introduction of non-additive lifetime welfare may or may not reinforce the compensation given to the short-lived. But what is more important is that the compensation-constrained utilitarian optimum yields a larger consumption to short-lived agents in comparison with the standard utilitarian case. For instance, under $\beta > -v\left(\frac{W}{3}\right)$, we had $c^1 > W/3$ under standard utilitarianism, whereas we now have, under the less stringent condition $\beta > -v\left(\frac{W}{3} + \frac{\alpha}{3}\right)$, that $c^1 > W/3 + \alpha/3$, which involves more consumption to the short-lived (under $\alpha > 0$).

Hence, monetizing longevity gains and counting these in the consumption bundles of long-lived agents does significantly affect the optimum allocation of resources, in favour of the short-lived. But is this necessarily the case that short-lived agents will be compensated for their shorter life, that is, do they now receive a higher consumption in comparison to the laissez-faire? Proposition 5 provides the answer to that question.

Proposition 5 *Assume $W > W^S$. Under the compensation-constrained utilitarian optimum, short-lived agents receive a compensation in comparison with the laissez-faire, i.e. $c^1 > W/2$, if and only if:*

$$\frac{G'\left(v\left(\frac{W}{2}\right) + \beta\right)}{G'\left(2v\left(\frac{W}{4} + \frac{\alpha}{2}\right) + 2\beta\right)} > \frac{v'\left(\frac{W}{4} + \frac{\alpha}{2}\right)}{v'\left(\frac{W}{2}\right)}$$

Note that, under $W \leq W^S$, short-lived agents never receive a compensation: $c^1 \leq W/2$.

Proof. See the Appendix. ■

The necessary and sufficient condition for compensation of short-lived agents under our modified utilitarian framework (with respect to the laissez-faire) can be compared with the corresponding condition under standard utilitarianism, i.e. the one in Proposition 3. It is easy to see that, provided α is positive, the denominator of the LHS is reduced in comparison to the LHS of the condition in Proposition 3, whereas the numerator of the RHS of the condition is reduced. Hence counting the consumption equivalent of a longer life as a part of the consumption of long-lived agents raises, *ceteris paribus*, the likelihood of compensation of the short-lived in comparison with the standard utilitarian approach.

Note that, as under standard utilitarianism, the compensation of short-lived agents depends here also on the curvature of the transform $G(\cdot)$. However, even if lifetime welfare was a mere sum of temporal utilities (i.e. $G''(\cdot) = 0$), so that the LHS of the above condition equals 1, there may still be some compensation of the short-lived here, simply because the

RHS of the condition may be lower than 1, in the case where $\alpha > W/2$. Such a compensation would have been impossible under standard utilitarianism, where the whole compensation is driven by the non-linearity of lifetime welfare. Here the introduction of the consumption equivalent of a longer life as a part of the consumption of long-lived agents pushes towards compensation independently from the shape of $G(\cdot)$.

In sum, Proposition 3 showed that the concavity of $G(\cdot)$ was necessary but not sufficient for the occurence of a compensation of short-lived agents under standard utilitarianism. It was also most likely that compensation would not take place, except for strongly concave $G(\cdot)$. What we show here is that once we count the consumption equivalent of a longer life as consumption of the long-lived, the concavity of $G(\cdot)$ is no longer necessary for a compensation of the short-lived in comparison to the laissez-faire. On the contrary, there can be some compensation here despite a purely linear lifetime welfare. As such, it appears that the homogenization of consumptions by means of the consumption equivalent of a longer life does not only complement the role of the transform $G(\cdot)$ as a determinant of the compensation or non-compensation, but may also play as a substitute allowing compensation even when lifetime welfare takes its standard time-additive form.

The above discussion can be easily translated in terms of lifetime welfare inequalities. Compensation-constrained utilitarianism, by raising, under $W > W^S$, the consumption of the short-lived, and by reducing the one of the long-lived, tends also to reduce welfare inequalities between long-lived and short-lived agents in comparison with the utilitarian optimum. Moreover, provided the condition of Proposition 5 is satisfied, compensation-constrained utilitarianism will also bring a compensation of short-lived agents with respect to the laissez-faire when $W > W^S$. That compensation may be sizeable, depending on the fundamentals of the economy, i.e. $u(\cdot)$, $G(\cdot)$ and W, which determine α.

Note, however, that, although compensation-constrained utilitarianism can, under some conditions, bring a sizeable compensation of short-lived agents with respect to the laissez-faire, it does not necessarily bring the exact equalization of lifetime utility across agents with different lengths of life. Nothing guarantees the equalization of lifetime utilities under compensation-constrained utilitarianism. Actually that solution is distinct from a Maximin solution equalizing agents' utilities. The present approach remains utilitarian, even though it is a utilitarianism that is constrained by a broader definition of consumption bundles taking interpersonal longevity differentials into account.

In sum, this section shows how the introduction of the consumption equivalent of a long life in a modified utilitarian framework can contribute to avoid an undesirable corollary of utilitarianism, where short-lived agents were, under general conditions, penalized twice: once for a shorter life, and once for enjoying less consumption. The homogenization of consumptions through counting the consumption-equivalent of a long life as long-lived agents's consumption contradicts such a double penalization of short-lived agents, and may also support a compensation of these. In particular, this modified approach allows the compensation of the short-lived even when lifetime welfare takes an additive form, that is, even when utilitarianism recommends necessarily a double penalization of the short-lived.

4 The general model: risky longevity

A major limitation of the preceding analysis was to focus on a purely deterministic world, where short-lived and long-lived agents can be identified *ex ante* by the social planner. As a consequence, the planner could use the consumption equivalent of a long life α to provide some compensation, knowing exactly all agents' longevity.

In reality, it is quite difficult to proceed in that way, as the length of life is inherently risky. Some agents have, because of some characteristics, a higher propensity to die, but this does not guarantee that each of those agents will necessarily enjoy a shorter life. In other words, the perspective of a higher *expected* length of life (or life expectancy) *ex ante* does not necessarily imply the enjoyment of a longer life *ex post*.[31]

In order to take into account the difference between the expected length of life and the actual length of life, we now assume that agents of type $i = 1, 2$ all live a first period of life with certainty, but reach the second period with a probability π^i.[32] Agents are thus not equal, and differ with respect to an exogenous characteristic influencing their longevity prospects, so that their life expectancies $1 + \pi^i$ are unequal. As above, we assume that type-1 agents suffer from some disadvantage with respect to type-2 agents:

$$\pi^1 < \pi^2$$

In comparison with the previous sections (where $\pi^1 = 0$ and $\pi^2 = 1$), we now have four

[31] For instance, although women exhibit a higher life expectancy than men, some women have a shorter life than some men, so that the groups with distinct survival prospects do not correspond to the groups with distinct actual longevities.

[32] For simplicity, we assume here that this probability is exogenous. On optimal tax policy under endogenous survival probabilities through health spending, see Leroux *et al* (2010).

types of agents: besides long-lived type-2 agents and short-lived type-1 agents, we have also long-lived type-1 agents and short-lived type-2 agents. The former enjoy, despite a low life expectancy, a long life, while the latter have a short life, despite a high life expectancy.

The existence of $2 \times 2 = 4$ types of agents raises the question of the dimension along which compensation is to be made: do we want to compensate people *ex ante*, that is, to compensate them for a low life expectancy? Or do we want to compensate people *ex post*, that is, to compensate them for having a shorter life? The form taken by the compensation - and, thus, the remedy to be implemented - depends on whether one adopts an *ex ante* or an *ex post* approach. In the rest of this section, we shall consider compensation *ex post*, on the grounds that what really matters, at the end of the day, is the actual standards of living enjoyed by agents, rather than what could have been enjoyed by them.[33]

4.1 The laissez-faire

Under the laissez-faire, each agent of type i, who is assumed to be an expected utility maximizer, chooses consumptions in such a way as to maximize

$$\pi^i G\left(u(c^i) + u(d^i)\right) + \left(1 - \pi^i\right) G\left(u(c^i)\right)$$

subject to the budget constraints

$$\begin{aligned} c^i &\leqslant \frac{W}{2} - s^i \\ d^i &\leqslant s^i R^i \end{aligned}$$

where s^i denotes savings and R^i is the return on savings. It is assumed, for simplicity, that the pure interest rate is zero, and that a perfect, class-specific, annuity market exists, which yields an actuarially fair return, so that the return on savings R^i is $1/\pi^i$.

The first order conditions yield:

$$\frac{\left[\pi^i G'\left(u(c^i) + u(d^i)\right) + \left(1 - \pi^i\right) G'\left(u(c^i)\right)\right]}{G'\left(u(c^i) + u(d^i)\right)} u'(c^i) = u'(d^i)$$

Under a linear $G(\cdot)$, we obtain that $c^i = d^i$, so that substituting into the individual's lifetime budget constraint, laissez-faire levels of consumptions are

$$\begin{aligned} c^1 &= d^1 = \frac{W}{2\left(1 + \pi^1\right)} \\ c^2 &= d^2 = \frac{W}{2\left(1 + \pi^2\right)} \end{aligned}$$

[33] Note, however, that it may also be worth comparing such an *ex post* compensation approach with an *ex ante* compensation approach. This is left for future research.

Thus, in the laissez-faire, agents with a shorter life expectancy consume more than agents with a high life expectancy, simply because they face a lower chance to ever be able to consume in the second period.[34]

On the contrary, under $G''(\cdot) < 0$, the fraction on the LHS of the FOC is larger than one, which leads to $c^i > d^i$, and to the following ranking:

$$\begin{aligned} d^1 &< \frac{W}{2\left(1+\pi^1\right)} < c^1 \\ d^2 &< \frac{W}{2\left(1+\pi^2\right)} < c^2 \end{aligned}$$

By consuming more in the first period than in the second period, agents partially insure themselves against the risk of incuring a low level of utility both because they had a shorter life and because they consumed less.

The conditions under which short-lived agents of type i are disadvantaged in comparison with long-lived agents of the same type are summarized in Proposition 6.

Proposition 6 *Consider a set of agents with a survival probability* $0 < \pi^i < 1$. *If* $u(0) \geq 0$ *(i.e.* $\beta \geq 0$*), long-lived agents are, at the laissez-faire, better off than short-lived agents, whatever the total amount of resources* W *is. If* $u(0) < 0$ *(i.e.* $\beta < 0$*), long-lived agents are, at the laissez-faire, better off than short-lived agents if and only if* $W > W^{S\prime}$*, where* $W^{S\prime}$ *is such that* $v\left(\frac{W^{S\prime}}{2(1+\pi^i)}\right) + \beta = 0$.

Proof. See the Appendix. ■

Proposition 6 states that when the intercept of the temporal utility function is non-negative, short-lived agents are always penalized at the laissez-faire in comparison to long-lived agents. However, under a negative β, things become more complex, and it is only if the resources available for the group W exceed some threshold $W^{S\prime}$ that short-lived persons are worse-off than long-lived agents with the same life expectancies. Proposition 7 compares the threshold $W^{S\prime}$ with its counterpart in absence of risk, i.e. W^S.

Proposition 7 *Suppose that* W^S *defines the critical resource level above which short-lived agents are, at the laissez-faire, worse off than long-lived agents in a riskless world. Suppose that* $W^{S\prime}$ *defines the critical resource level above which short-lived agents are, at the laissez-faire, worse off than long-lived agents in a world where the survival probability is* π^i *(*$0 < \pi^i < 1$*). We have:* $W^S > W^{S\prime}$.

[34] This is due to the assumption of type-specific perfect annuity markets yielding actuarially fair returns to agents of types 1 and 2.

Proof. See the Appendix. ■

The necessary condition for the disadvantage of being short-lived under risky longevity is *weaker* than the corresponding condition under deterministic longevity (Proposition 1), as the threshold of resources $W^{S\prime}$ is inferior to W^S. The intuition behind this is that, under deterministic longevities, short-lived people can avoid some part of the damage from being short-lived by consuming their all endowment in the first period (because they perfectly anticipate that they will die in the end of the first period). However, in a risky world, such a behaviour is not optimal (as it is then rational to save resources for the case where they survive to the second period), and the mere fact of savings resources for the old days leads to a higher damage from having a short life. Hence, short-lived agents are more penalized in a risky world, since beyond the cost of non-spreading consumption on several periods, there is an additional cost due to the resources that are lost due to unanticipated death.

4.2 The utilitarian optimum

The social planner aims at maximizing average lifetime welfare, subject to the budget constraint of the economy:

$$\begin{aligned}
&\max_{c^1,d^1,c^2,d^2} \sum_{i=1,2} \pi^i G\left(u(c^i)+u(d^i)\right) + \left(1-\pi^i\right) G\left(u(c^i)\right) \\
&\text{s.to } \sum_{i=1,2} \left(c^i + \pi^i d^i\right) \leq W
\end{aligned}$$

The first order conditions are then

$$\begin{aligned}
\left[\pi^i G'\left(u(c^i)+u(d^i)\right) + \left(1-\pi^i\right) G'\left(u(c^i)\right)\right] u'(c^i) &= \lambda \\
u'(d^i) G'\left(u(c^i)+u(d^i)\right) &= \lambda
\end{aligned}$$

where λ is the Lagrange multiplier associated with the resource constraint of the economy. Proposition 8 describes the first-best optimum depending on the form of the function $G(\cdot)$.

Proposition 8 *The utilitarian optimum is such that:*

- *If* $G''(\cdot) = 0$*, we have:* $c^1 = d^1 = c^2 = d^2 = \frac{W}{2+\pi^1+\pi^2}$

- *If* $G''(\cdot) < 0$*, we have:*

  - *under* $\beta > -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, $d^2 < d^1 < \frac{W}{2+\pi^1+\pi^2} < c^1 < c^2$ *or* $d^1 < d^2 < \frac{W}{2+\pi^1+\pi^2} < c^2 < c^1$.

  - *under* $\beta = -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, $c^2 = c^1 = d^1 = d^2 = \frac{W}{2+\pi^1+\pi^2}$.

- *under* $\beta < -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, $c^2 < c^1 < \frac{W}{2+\pi^1+\pi^2} < d^1 < d^2$ *or* $c^1 < c^2 < \frac{W}{2+\pi^1+\pi^2} < d^2 < d^1$.

Proof. See the Appendix. ■

Proposition 8 confirms the crucial influence of the form of lifetime welfare on the utilitarian optimum. When lifetime welfare takes a standard time-additive form, i.e. $G(\cdot)$ is linear, utilitarianism equalizes consumptions across all agents and all periods. Such an equalization has an important redistributive impact. At the laissez-faire, type-1 agents enjoyed $\frac{W}{2(1+\pi^1)}$ at all periods of life, whereas type-2 enjoyed $\frac{W}{2(1+\pi^2)}$ at all periods. Given that $\pi^1 < \pi^2$, utilitarianism, by yielding a consumption equal to $\frac{W}{2+\pi^1+\pi^2}$ for everyone, contributes to reduce the consumption of type-1 agents and to raise the consumption of type-2 agents. Utilitarianism tends thus to redistribute from agents with a low life expectancy to agents with a high life expectancy, which is ethically questionable.

Under a concave tranform $G(\cdot)$, the utilitarian optimum is less easy to analyze, as its form depends on the level of the intercept of the temporal utility function β. Depending on its level, utilitarianism will recommend consumption profiles that are increasing or decreasing with the age.[35] Regarding the comparison of type-1 and type-2 agents, we know that utilitarianism will not give an advantage to some type of agents at all periods, but, rather, will give more consumption to some agents at one period and to the other agents at the other period. However, we cannot say which group will enjoy the highest consumption at a particular period, since this depends on the precise form of the temporal utility function $u(\cdot)$, of the concave transform $G(\cdot)$, as well as on the levels of π^1 and π^2. Hence, it is also difficult to see how utilitarianism redistributes resources in comparison with the laissez-faire. Proposition 9 summarizes our results regarding how utilitarianism affects inequalities between short-lived and long-lived agents within each group.

Proposition 9 *Utilitarianism affects lifetime welfare inequalities as follows:*

- *Under* $G''(\cdot) = 0$ *and for any* β, *utilitarianism reduces inequalities between the long-lived and the short-lived for type-*1 *agents, and does the opposite for type-*2 *agents.*

- *Under* $G''(\cdot) < 0$, *the impact of utilitarianism on inequalities between the long-lived and the short-lived within each group is the following:*

[35] More precisely, if β is sufficiently large, consumption profiles should be declining (to compensate the disutility from a short life), while the opposite holds of β is very low. In that latter case, consumption profiles should be increasing (to compensate the disutility from a long life).

- *under* $\beta > -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, *utilitarianism has ambiguous effects.*

- *under* $\beta = -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, *utilitarianism reduces inequalities between the long-lived and the short-lived for type-*1 *agents, and does the opposite for type-*2 *agents.*

- *under* $\beta < -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, *utilitarianism has ambiguous effects.*

Proof. See the Appendix. ■

Therefore, whether utilitarianism tends to reinforce or reduce inequalities between short-lived and long-lived within a group i of agents with a life expectancy $1+\pi^i$ depends on the precise form of lifetime welfare (i.e. additive or not). If $G(\cdot)$ is linear, utilitarianism reduces inequalities between short-lived and long-lived within one group, but raises these in the other. The same result prevails under $G(\cdot)$ non-linear when $\beta = -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$. However, in the other cases, the utilitarian optimum depends on the temporal utility function $u(\cdot)$, the concave transform $G(\cdot)$ and the other fundamentals of our economy (W, π^1 and π^2), so that no precise conclusion can be drawn whithout imposing further assumptions.

Although the above analytical results are incomplete, and, as such, invite some numerical explorations, these suffice, nonetheless, to have serious doubts on the capacity of utilitarianism to compensate short-lived agents. Actually, in the previous section, which focused on the special case where $\pi^1 = 0$ and $\pi^2 = 1$, there was already a serious questioning of the ability of utilitarianism to compensate short-lived. Indeed, Section 3 showed that it was only provided the transformation $G(\cdot)$ was strongly concave that some compensation of the short-lived would take place, but not otherwise. Given that result, it is no surprise that, in the general case where $0 \leq \pi^1 \leq \pi^2 \leq 1$, the capacity of utilitarianism to compensate short-lived agents is ambiguous. That limited capacity of utilitarianism to compensate short-lived agents invites, as in the determinitic case, a remedy.

4.3 Compensation-constrained utilitarianism

The social planning problem described in Section 4.2 suffers from the same weakness as the one in the basic model with deterministic longevities. It does not take into account a major source of injustice across agents, namely that the capacity of consumption to generate lifetime welfare varies depending on the longevity of consumers (because of Gossen's First Law).[36] Such an unequal capacity of temporal consumptions to produce welfare is not

[36] Note, once again, that this handicap of short-lived agents is only mitigated, but not eliminated, by the introduction of a concave transform $G(\cdot)$ (see Proposition 3).

regarded as a problem by the social planner in the standard utilitarian problem, and this is what causes the counterintuitive redistribution from the short-lived to the long-lived. Therefore, as in Section 3, a more reasonable approach to the allocation of resources requires first to homogenize consumptions, and, then, to solve the associated planning problem under the introduced compensation constraints.

In order to define homogeneous consumptions, i.e. consumptions $\tilde{c}^1$, $\tilde{c}^2$ and $\tilde{d}^2$ that take longevity differentials into account, we shall, here again, count the consumption-equivalent of a longer life as a part of the long-lived consumption bundle. The consumption equivalent of a longer life can be defined so as to equalize the utility of a short-lived and of a long-lived agent. The consumption equivalent, denoted by α^i, is such that

$$G\left(u\left(c^{i*}+\alpha^i\right)\right) = G\left(u\left(c^{i*}\right)+u\left(d^{i*}\right)\right) \tag{3}$$

where c^{i*} and d^{i*} are the consumptions chosen by a type-i agent under the laissez-faire (i.e. before knowing whether he survives to period 2 or not). A major difference with respect to Section 3 is that consumption equivalents are here type-specific, as agents make, at the laissez-faire, distinct consumption choices (because they face different survival prospects). Note, however, that, although type-specific, consumption equivalents keep a fundamental invariance property: as under certain lifetimes, $G(\cdot)$ does not play any role in the equation defining the consumption equivalent:

$$\begin{aligned} u\left(c^{1*}+\alpha^1\right) &= u\left(c^{1*}\right)+u\left(d^{1*}\right) \\ u\left(c^{2*}+\alpha^2\right) &= u\left(c^{2*}\right)+u\left(d^{2*}\right) \end{aligned}$$

Regarding the sign of the consumption equivalent, it is no surprise that it depends on the conditions guaranteeing that short-lived agents have a lower actual lifetime utility than long-lived agents, that is, on the conditions of Proposition 5.[37] As far as the difference between the levels of α^1 and α^2 is concerned, we do not know whether $\alpha^1 \lesseqgtr \alpha^2$.

Let us now use the consumption-equivalents in such a way as to modify the utilitarian planning problem. At this stage, it is crucial to notice a fundamental difference between the deterministic longevity case studied in Section 3 and the risky longevity case studied here. In Section 3, under certain lifetime, it was possible to identify *ex ante* who would be long-lived

[37] To see this, note that, at the laissez-faire, we have, under the assumptions that $\beta > 0$ or $\beta < 0$ but $W > W^s$, $u\left(c^i\right) < u\left(c^i\right)+u\left(d^i\right)$. Hence only a positive α^i can restaure the equality, so that $u\left(c^i+\alpha^i\right) = u\left(c^i\right)+u\left(d^i\right)$.

and who would be short-lived, and such a possibility allowed us to count the consumption-equivalent of a long-lived as a part of his consumption during both periods of his life without any possibility of penalizing short-lived agents. In the present framework, however, there is some risk about the length of life. As a consequence, it is no longer possible to know *ex ante* who will live long or not. The presence of risk has thus an important impact on the precise manner under which the consumption-equivalent of a long life should be counted as a part of the consumption of the long-lived. Clearly, since no one knows *ex ante* the longevity of individuals, one can no longer divide the consumption equivalent α^i equally on the two periods, as we used to do in the model without risk.[38] Given that it is only in the second period that long-lived agents can be identified, the entire consumption-equivalent of a long life α^i should be counted as a part of second-period consumption. This leads to the following homogenized consumptions $\tilde{c}^1$, $\tilde{c}^2$ and $\tilde{d}^2$:

$$\begin{aligned}
\tilde{c}^1 &= c^1 \\
\tilde{d}^1 &= d^1 + \alpha^1 \\
\tilde{c}^2 &= c^2 \\
\tilde{d}^2 &= d^2 + \alpha^2
\end{aligned}$$

Let us now rederive the compensation-constrained utilitarian optimum under that alternative implementation of the remedy. Assigning the entire consumption equivalent of a long life as a part of second-period consumption, the problem of a social planner becomes:

$$\begin{array}{l}
\max_{c^1,d^1,c^2,d^2} \sum \left(1-\pi^i\right) G\left(u\left(\tilde{c}^i\right)\right) + \pi^i G\left(u\left(\tilde{c}^i\right) + u(\tilde{d}^i)\right) \\
\text{s.to } \tilde{c}^i = c^i \\
\text{s.to } \tilde{d}^i = d^i + \alpha^i \\
\text{s.to } \sum_{i=1,2} \left(c^i + \pi^i d^i\right) \leq W
\end{array}$$

where α^i is estimated from expression (3), at the laissez-faire. Here again, the homogenization of consumptions through the introduction of a consumption equivalent in the long-lived's consumption can be regarded as the addition of compensation constraints to

[38] Indeed, counting part of the consumption-equivalent of a long life as first-period consumption would, in the present context, penalize even more the short-lived in comparison with standard utilitarianism, by reducing his lifetime welfare even more.

the planner's problem.[39] First-order conditions can be rearranged as

$$\begin{aligned}
\left[\left(1-\pi^1\right) G'\left(u\left(c^1\right)\right)+\pi^1 G'\left(u\left(c^1\right)+u(d^1+\alpha^1)\right)\right] u'(c^1) &= \lambda \\
\left[\left(1-\pi^2\right) G'\left(u\left(c^2\right)\right)+\pi^2 G'\left(u\left(c^2\right)+u(d^2+\alpha^2)\right)\right] u'(c^2) &= \lambda \\
G'\left(u\left(c^1\right)+u(d^1+\alpha^1)\right) u'(d^1+\alpha^1) &= \lambda \\
G'\left(u\left(c^2\right)+u(d^2+\alpha^2)\right) u'(d^2+\alpha^2) &= \lambda
\end{aligned}$$

where λ is the Lagrange multiplier associated with the resource constraint of the economy. Here again, we expect that the optimum must depend on the particular specification for lifetime welfare. Proposition 10 summarizes our results.

Proposition 10 *The compensation-constrained utilitarian optimum is such that:*

- *If $G''(.) = 0$, we have:*

$$\begin{aligned}
c^1 &= c^2 = \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} \\
d^1 &= \frac{W+\pi^2\alpha^2-\alpha^1\left(2+\pi^2\right)}{2+\pi^1+\pi^2} \\
d^2 &= \frac{W+\pi^1\alpha^1-\alpha^2\left(2+\pi^1\right)}{2+\pi^1+\pi^2}
\end{aligned}$$

- *If $G''(\cdot) < 0$, we have, under $\alpha^i > 0$:*
  - *under* $\beta > -v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$, $c^1 > \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} > d^1; c^2 > \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} > d^2$
  - *under* $\beta = -v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$, $c^1 = c^2 = \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}; d^1 = \frac{W+\pi^2\alpha^2-\alpha^1\left(2+\pi^2\right)}{2+\pi^1+\pi^2}; d^2 = \frac{W+\pi^1\alpha^1-\alpha^2\left(2+\pi^1\right)}{2+\pi^1+\pi^2}$.
  - *under* $\beta < -v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$, $c^1 < \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} < d^1; c^2 < \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} < d^2$.

Proof. See the Appendix. ■

In order to interpret the modified utilitarian optimum in that general framework, we shall here first concentrate on the benchmark case where lifetime welfare takes a standard time-additive form (i.e. $G''(\cdot) = 0$), and leave the interpretation of the case where $G(\cdot)$ is concave for the end of this section.

In the standard case where $G(\cdot)'' = 0$, first-period consumptions are equalised, $c^1 = c^2$ and this consumption level is also, under either $\alpha^1 > 0$ and/or $\alpha^2 > 0$, higher than under

[39] The standard utilitarian problem coincides with the special case where $\tilde{c}^i = c^i$ and $\tilde{d}^i = d^i$ for all agents of type $i = 1, 2$.

standard utilitarianism, so that short-lived agents are, in comparison with standard utilitarianism, better off. However, second-period consumptions d^1 and d^2, which are now different for type-1 and type-2 agents, are, in general, smaller than under standard utilitarianism. Actually, this is the case provided $\pi^2\left(\alpha^2-\alpha^1\right)-2\alpha^1<0$ and $\pi^1\left(\alpha^1-\alpha^2\right)-2\alpha^2<0$, which are mild conditions (as the difference between α^1 and α^2 cannot be extremely large, given that the difference between π^1 and π^2 must belong to $[0,1]$).

Still taking the simplest case of $G''(.)=0$, let us now compare *ex post* welfare inequalities by proceeding as before and by comparing lifetime utilities with the utilitarian case:

		compensation-constrained utilitarianism		utilitarianism
Type 1	Long-lived	$u\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)+u\left(\frac{W+\pi^2\alpha^2-\alpha^1\left(2+\pi^2\right)}{2+\pi^1+\pi^2}\right)$	$\lesseqgtr$	$2u\left(\frac{W}{2+\pi^1+\pi^2}\right)$
	Short-lived	$u\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$	$>$	$u\left(\frac{W}{2+\pi^1+\pi^2}\right)$
Type 2	Long-lived	$u\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)+u\left(\frac{W+\pi^1\alpha^1-\alpha^2\left(2+\pi^1\right)}{2+\pi^1+\pi^2}\right)$	$\lesseqgtr$	$2u\left(\frac{W}{2+\pi^1+\pi^2}\right)$
	Short-lived	$u\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$	$>$	$u\left(\frac{W}{2+\pi^1+\pi^2}\right)$

Table 1: Some welfare comparisons

Thus compensation-constrained utilitarianism increases the welfare of all short-lived agents with respect to utilitarianism, while this may not be the case for long-lived agents in comparison with the situation under utilitarianism, depending on the values of α^1 and α^2.

Regarding the inequalities between long-lived and short-lived agents inside a given group (either type-1 or type-2), we find that inequalities inside group 1 are reduced as compared to utilitarianism if $\pi^2\left(\alpha^2-\alpha^1\right)-2\alpha^1<0$ and inequalities inside group 2 are also reduced if $\pi^1\left(\alpha^1-\alpha^2\right)-2\alpha^2<0$. This is due to the simple fact that compensation-constrained utilitarianism raises first-period consumptions and reduces, under mild conditions, second-period consumptions with respect to utilitarianism, so that this reduces inequalities of lifetime welfare between the short-lived and the long-lived of a given type i. This reduction of lifetime welfare inequalities within each group occurs under general conditions, that is, if one excludes extreme cases where the differential between the two consumption equivalents α^1 and α^2 is extremely large (i.e. cases where survival probabilities differ strongly).[40]

[40] Under extreme differentials between α^1 and α^2, lifetime welfare inequalities within one group may be increased under our remedy (but not in the other group). However, such a large gap is implausible, especially if the groups under study do not exhibit large differences in group-specific life expectancies.

Proposition 11 *In comparison with utilitarianism and under $G''(\cdot) = 0$ and $\alpha^i > 0$, compensation-constrained utilitarianism:*

- increases the welfare of short-lived agents, but may or may not increase the welfare of long-lived agents;

- reduces lifetime welfare inequalities within each group $i = 1, 2$ between long-lived agents and short-lived agents, provided $\pi^2 \left(\alpha^2 - \alpha^1\right) - 2\alpha^1 < 0$ and $\pi^1 \left(\alpha^1 - \alpha^2\right) - 2\alpha^2 < 0$.

Proof. The proof follows from the above table. ■

The intuition behind these two latter conditions is the following. If, for instance, the consumption equivalent for one group is extremely large, let us say, if $\alpha^1 >> \alpha^2$, then, under compensation-constrained utilitarianism, there would be a reduction of lifetime welfare inequalities between short-lived and long-lived type-1 agents, but, because of redistributions across types favouring type-2 agents, a rise of lifetime welfare inequalities between the short-lived and the long-lived type-2 agents. But such a specific case is hardly plausible, so that, in general, compensation-constrained utilitarianism reduces inequalities between long-lived and short-lived in each group.

Besides the impact on inequalities, the change in the way in which α^i is spread on the lifecycle has also important consequences for other aspects of the social optimum. Remember that, because of the impossibility to identify *ex ante* who would be long-lived or short-lived due to risk about the length of life, compensation-constrained utilitarianism could not, unlike in the basic deterministic model, spread the consumption-equivalent of a long life on the whole lifecycle of agents. Hence it had to be counted entirely as second-period consumption. A major corollary of this is that the second-period consumption of agents d^i is now lower than their first-period consumption c^i. This results from the planner's will to compensate short-lived agents. The non-equalization of marginal utilities of consumption over time can be regarded as an efficiency loss: from an efficiency perspective, consumption should be smoothed, for each individual, across all his life periods. However, the compensation requires to give a higher first-period consumption to all agents than to survivors in the second period, which implies that consumption cannot be smoothed across periods for long-lived agents. Therefore a tension arises between compensation and efficiency concerns.

Let us now add a few observations on the general case where $G''(\cdot) < 0$. In that case, the compensation-constrained utilitarian optimum depends on several determinants, including the intercept of the temporal utility function β. This result does not surprise us, as that intercept was playing a similar role in the benchmark model without risk. Here again, compensation-constrained utilitarianism recommends decreasing or increasing consumption profiles with the age, depending on the level of β. There is, however, a significant difference with respect to the benchmark deterministic model. In the riskless model, the consumption-equivalent of a long life was equal for all agents, and this uniqueness allowed us to characterize almost completely the modified utilitarian solution. On the contrary, in the model with risky longevity, the consumption-equivalent of a long life is type-specific.[41] This is the reason why it is hard to say analytically more on the modified utilitarian solution under non-additive lifetime welfare. Nonetheless, given that lifetime welfare may not be additive over time, it makes sense to complement the above analytical discussion by some numerical simulations. This is the task of the next section.

5 A numerical illustration

Let us now illustrate, by means of numerical simulations, how the laissez-faire, the utilitarian optimum and the compensation-constrained utilitarian optimum differ in a simple economy with risky longevity of the kind studied in Section 4. For that purpose, we need first to impose some functional forms for agents' utility functions $G(\cdot)$ and $u(\cdot)$, and to calibrate the type-specific survival probabilities π^1 and π^2 as well as the total endowment W.

Regarding the temporal utility function $u(\cdot)$, we assume, for simplicity, that it has a simple CES form with an intercept:

$$u(c^i) = \frac{c^{i1-\sigma}}{1-\sigma} + \beta$$

where σ is the elasticity of intertemporal substitution, while β is an intercept. Note the crucial role played by those two preference parameters for the computation of the consumption equivalent of a long life. If, for instance, utility is linear in consumption, we have $\sigma = \beta = 0$, so that the consumption equivalent is zero. However, for other values of preference parameters, α^i is likely to vary significantly.[42] Note that, under $\sigma < 1$, if β is too large (i.e. the utility from mere survival is large), there cannot be any consumption equivalent of a long life, as no consumption can compensate for the fact of facing a shorter life.

[41] Indeed the consumption-equivalent of a long life reflects the actual consumption choices of agents, which depend on agents's beliefs about survival. Hence the differences in life expectancies $1+\pi^1$ and $1+\pi^2$, by leading to a differential in the savings decision at the laissez-faire, tend also to affect the levels of the consumption-equivalent, and, thus, the optimal consumption profiles under compensation-constrained utilitarianism.

[42] For an empirical study on that topic, see Ponthiere (2008).

Regarding the calibration of σ and β, we shall proceed as follows. In all cases, we will assume that $\sigma = 0.5$ but we will make β vary and take three distinct values: 0, -2 and 2.[43] In the case where the lifetime utility function is linear in the temporal utility of consumption, we also make a more realistic calibration of the intercept, on the basis of empirical estimates of the value of a statistical life, and show, in the Appendix, that a plausible value for the intercept of the temporal utility function is $\beta = 4.472$.

As far as the functional form for lifetime welfare is concerned, we shall here rely on a simple form for the concave transform $G(\cdot)$:

$$G\left(u(c^i) + u(d^i)\right) = \frac{\left(u(c^i) + u(d^i)\right)^{\kappa}}{\kappa}$$

When κ equals 1, we are back to the standard case where lifetime welfare is the sum of temporal utilities. On the contrary, when κ is less than 1, lifetime welfare takes a non-additive form close to the one discussed in Bommier (2006).

To illustrate the model of Section 4, we need also to identify two groups with different survival prospects. As an example, we shall here suppose that type-1 agents are males and type-2 agents are females. In the U.S., life expectancy at birth for males is about 75 years, equal here to $1 + \pi^1 = 1.25$.[44] For women, life expectancy is 80.5 years, equal here to $1 + \pi^2 = 1.40$. Thus, one has $\pi^1 = 0.25$ and $\pi^2 = 0.40$. Finally, we make the assumption of a total endowment equal to $W = 20$.

Let us now compare the laissez-faire, the utilitarian optimum and the compensation-constrained utilitarian optimum. For that purpose, Table 2 concentrates on the benchmark case where lifetime welfare is the mere sum of temporal utilities (i.e. $\kappa = 1$), and provides estimates of the consumptions and the welfare levels for all agents, who can be either of type 1 or type 2, and can be either short-lived (i.e. superscript SL) or long-lived (i.e. superscript LL). For the sake of completeness, Table 1 provides those various estimates for different values of the intercept parameter β.

Let us first consider, as a benchmark example, the case where $\beta = 0$. At the laissez-faire, consumption is smoothed across periods, but is larger for type-1 agents, who face a lower chance of surviving till the second period. Moreover, short-lived agents are worse-off than long-lived agents who belong to the same group, illustrating the first part of Proposition 6.

[43] Note that, while empirical studies of σ yield an estimate of 0.83 (see Blundell *et al*, 1994), such estimates cannot be used here, as these rely on a model where the period is a year, unlike in the present model.

[44] Indeed, if a period is of length 40 years, and starting at the age of 25, we obtain that a life expectancy of 75 years involves 65 years (i.e. 25 years + the first period) + 10 years, equal to 0.25 period.

At the utilitarian optimum, consumption is now equalized at all periods for all agents, so that it reduces inequalities between short-lived and long-lived agents of the type-1 group, whereas it increases them for type-2 agents, in conformity with Proposition 9.[45]

$\kappa = 1$		c^1	c^2	d^1	d^2	U^{1SL}	U^{1LL}	U^{2SL}	U^{2LL}
$\beta = -2$	Laissez-faire	8.00	7.14	8.00	7.14	3.66	7.31	3.35	6.69
	Utilitarianism	7.55	7.55	7.55	7.55	3.49	6.99	3.49	6.99
	CC Utilitarianism	10.00	10.00	0.00	0.00	4.32	2.32	4.32	2.32
	α^1= 13.69, α^2= 11.74								
$\beta = 0$	Laissez-faire	8.00	7.14	8.00	7.14	5.66	11.31	5.35	10.69
	Utilitarianism	7.55	7.55	7.55	7.55	5.49	10.99	5.49	10.99
	CC Utilitarianism	10.00	10.00	0.00	0.00	6.32	6.32	6.32	6.32
	α^1= 24.00, α^2= 21.43								
$\beta = 2$	Laissez-faire	8.00	7.14	8.00	7.14	7.66	15.31	7.35	14.69
	Utilitarianism	7.55	7.55	7.55	7.55	7.49	14.99	7.49	14.99
	CC Utilitarianism	10.00	10.00	0.00	0.00	8.32	10.32	8.32	10.32
	α^1= 36.31, α^2= 33.12								
$\beta = 4.472$	Laissez-faire	8.00	7.14	8.00	7.14	10.13	20.26	9.82	19.63
	Utilitarianism	7.55	7.55	7.55	7.55	9.97	19.93	9.97	19.93
	CC Utilitarianism	10.00	10.00	0.00	0.00	10.80	15.27	10.80	15.27
	α^1= 54.30, α^2= 50.33								

Table 2: Outcomes under time-additive lifetime welfare

Turning now to the compensation-constrained utilitarian optimum, we can notice that this leads to a corner solution: the imputation of the consumption-equivalent of a long life (estimated at the laissez-faire) to the long-lived second-period consumption implies that the total endowment W should be divided entirely between the first-period consumptions of types 1 and 2, whereas nothing should be left for second-period consumption.[46] As a consequence, it appears that compensation-constrained utilitarianism implies a rise in the welfare of short-lived agents, as well as a fall in the welfare of long-lived agents, in comparison with the laissez-faire and the standard utilitarian optimum. Thus, under $\kappa = 1$ and $\beta = 0$, the condition stated in Proposition 11 is satisfied, in the sense that the compensation-constrained utilitarian optimum involves, in comparison with standard utilitarianism, a reduction of inequalities between the long-lived and the short-lived. But more importantly,

[45] Moreover, (short-lived and long-lived) type-2 agents end up with a higher utility than under the laissez-faire, whereas type-1 agents are always worse-off, independently from living one or two periods.

[46] Note also that the consumption equivalents of a longer life (estimated at the laissez-faire) are such that $\alpha^1 > \alpha^2$, which was unclear from our theoretical part.

the modified utilitarian optimum brings also a compensation to the short-lived even in comparison with the laissez-faire, since compensation-constrained utilitarianism raises the welfare of the short-lived and reduces the welfare of the long-lived in comparison to the laissez-faire. Given that second-period consumptions are set to 0 and β equals also 0, compensation-constrained utilitarianism yields here a perfect equalization of lifetime welfare across all agents, either short-lived or long-lived, whatever their survival prospects were. Thus, in this special case, compensation-constrained utilitarianism treats all agents equally, independently from their initial survival chance and from their actual longevity.

Let us now examine how sensitive those results are to the intercept β. First note that the value of the intercept does not play any role on consumption levels at the laissez-faire and at the standard utilitarian optimum and compensation-constrained utilitarian optimum.[47] Only welfare levels and consumption equivalents α^i differ, since these are increasing in the level of β. Thus, as under $\beta = 0$, compensation-constrained utilitarianism, by concentrating all resources on first-period consumption, brings a compensation to short-lived agents in comparison with the laissez-faire. The unique difference is that, when comparing *ex post* lifetime welfare levels, long-lived agents remain better off than short-lived agents under $\beta > 0$, since, in that case, it is impossible to make the short-lived as well off as the long-lived on the mere basis of a higher first-period consumption. Due to risky longevity, one cannot identify *ex ante* who will be short-lived or long-lived, and this explains the incapacity to provide a full equality of lifetime welfare.

In sum, Table 2, by assuming a linear lifetime utility, illustrates the capacity of compensation-constrained utilitarianism to operate some compensation of the short-lived in comparison with the laissez-faire and the standard utilitarian optimum. The remedy proposed here compensates all short-lived agents (men and women) equally, by sharing equally all resources in the first-period and by leaving nothing for the second period.

Table 3 assumes $\kappa = 1/2 < 1$, to examine how assumptions on lifetime welfare affect our results.[48] In the laissez-faire, first-period consumptions are always higher than second-period ones for any value of β. This is in conformity with the analytical findings of Section 4. Moreover, we find also that per-period consumptions are higher for type-1 agents than

[47] Note that first order conditions are independant from β, so that the laissez-faire and the utilitarian levels of consumption are identical for any value of β.

[48] Here again, the allocations under study are provided under $\beta = -2$, 0 and 2. We exclude the case where $\beta = 4.472$ as this value, based on the value of a statistical life, was computed under the assumption that $G(\cdot)$ was linear. Taking this change into account would complicate our computations of the new intercept, without giving more insights.

for type-2 agents. This last point was ambiguous in the theoretical section. Under utilitarianism, consumption profiles are now always decreasing with age, in conformity with Proposition 8. We find also that type-1 agents now get higher first-period consumption but lower second-period consumption than type-2 agents. Again, our analytical part could not establish this clearly. We still find that inequalities between short-lived and long-lived agents of type-1 are reduced, while it is the reverse for type-2 agents, yet the differences are smaller than in the linear case. This illustrates that departing from the standard time-additive lifetime welfare postulates reduces the welfare inequalities between short-lived and long-lived under utilitarianism. However, those inequalities remain large, so that it can hardly be said that utilitarianism jointly with non-additive lifetime welfare suffices to bring a compensation to short-lived agents. On the contrary, the compensation-constrained utilitarian optimum, by giving maximum consumption in the first period and zero consumption in the second period, brings a compensation of short-lived persons. Thus, even though the introduction of a concave transform $G(\cdot)$ can partly mitigate the natural tendency of utilitarianism to redistribute resources from short-lived towards long-lived, this numerical example suggests that a compensation of short-lived agents cannot be carried out within standard utilitarianism, but requires the introduction of compensation constraints.[49]

$\kappa = 0.5$		c^1	c^2	d^1	d^2	U^{1SL}	U^{1LL}	U^{2SL}	U^{2LL}
$\beta = -2$	Laissez-faire	8.59	7.81	5.66	5.48	3.93	5.14	3.79	5.01
	Utilitarianism	8.38	7.99	5.53	5.61	3.89	5.10	3.82	5.06
	CC Utilitarianism	10.00	10.00	0.00	0.00	4.16	3.05	4.16	3.05
	$\alpha^1 = 9.98,\ \alpha^2 = 9.30$								
$\beta = 0$	Laissez-faire	8.63	7.86	5.48	5.36	4.85	6.50	4.74	6.40
	Utilitarianism	8.48	7.99	5.38	5.45	4.83	6.47	4.76	6.43
	CC Utilitarianism	10.00	10.00	0.00	0.00	5.03	5.03	5.03	5.03
	$\alpha^1 = 19.23,\ \alpha^2 = 18.34$								
$\beta = 2$	Laissez-faire	8.65	7.88	5.38	5.30	5.62	7.62	5.52	7.54
	Utilitarianism	8.54	7.99	5.31	5.37	5.60	7.60	5.53	7.56
	CC Utilitarianism	10.00	10.00	0.00	0.00	5.77	6.43	5.77	6.43
	$\alpha^1 = 30.56,\ \alpha^2 = 29.44$								

Table 3: Outcomes under non-time-additive lifetime welfare

[49] We have also run these simulations for other values of κ, such as $\kappa = 0.75$. Our results are robust to a variation of κ. Under compensation-constrained utilitarianism, it is always optimal to distribute equally all resources in the first period and to leave nothing for the second one. Thus, individuals with the same actual length of life obtain the same utility. Only the utility gaps between short- and long-lived agents within the same initial group increase when κ increase.

In sum, this section illustrates that, whereas classical utilitarianism can hardly compensate short-lived agents - even under non-additive lifetime welfare - compensation-constrained utilitarianism can operate such a compensation, by raising first-period consumption while reducing second-period consumption. Those results are robust to the precise specification of preferences. Nonetheless, the size of welfare inequalities between short-lived and long-lived agents depends on the specification of preferences. Some welfare inequalities may still prevail despite the remedy we propose, simply because of the risky nature of longevity, which prevents a differentiated treatment of agents in the first period.

6 Concluding remarks

This paper starts from a paradoxical result of classical utilitarianism: a tendency, under standard assumptions, to redistribute resources from short-lived to long-lived agents, implying, under mild conditions, a double penalization of short-lived agents: one penalty by Nature, one by Bentham. We proposed a re-examination of the treatment of unequal longevities under utilitarianism, on the basis of a simple two-period model with deterministic or risky longevities. The major contributions of the paper are twofold.

Firstly, we identified formal conditions under which short-lived agents are worse off than long-lived agents at the laissez-faire. The crucial role of the intercept of the temporal utility function and of the society's endowment was highlighted. It was also shown that the introduction of risk about the length of life weakens the conditions under which short-lived agents are disadvantaged with respect to long-lived agents, because of lost savings due to unanticipated death. We also identified conditions under which utilitarianism can compensate short-lived agents with respect to the laissez-faire. Those conditions are never satisfied under standard time-additive lifetime welfare, and may only be satisfied under a high degree of concavity of the transform applied to the sum of temporal utilities. The reason why departing from time-additive lifetime welfare does not, in general, suffice to bring a compensation of the short-lived with respect to the laissez-faire has to do with the fact that the standard utilitarian problem does not take into account the fundamental source of injustice between short-lived and long-lived agents, namely that consumption has a higher capacity to generate lifetime welfare for long-lived agents than for short-lived agents (because of Gossen's First Law). Note that taking a concave transform of the sum of temporal utilities only mitigates - but does not eliminate - that problem.

Secondly, and on the basis of that first observation, we proposed a remedy to that absence of compensation. What we proposed is to solve a utilitarian social planning problem that concerns homogenized consumptions, i.e. consumptions that are made comparable despite unequal longevities. In what can be called a compensation-constrained utilitarianism, the consumption-equivalent of a long life is counted as a part of the consumption of the long-lived agents, in such a way as to take into account the natural advantage of long-lived agents. The imputation of the consumption equivalent of a long life (estimated at the laissez-faire) to the consumption of long-lived agents at all periods was shown to yield a compensation to the short-lived agents with respect to utilitarianism, and was also shown to reinforce the likelihood of a compensation of the short-lived with respect to the laissez-faire. Hence, whereas utilitarianism tends to maintain - and sometimes to exacerbate - welfare inequalities caused by Nature (depending on the form of lifetime welfare), compensation-constrained utilitarianism implies, on the contrary, a much more intuitive treatment of agents disadvantaged by Nature. We also showed numerically that the large welfare inequalities that may subsist under standard utilitarianism despite non-additive lifetime welfare are necessarily reduced under compensation-constrained utilitarianism.

Thanks to those two contributions, the present study casts new light on a variety of problems of redistribution involving longevity inequalities. Longevity differentials are present in many policy debates, concerning pensions, long-term care, etc. Hence, in all those issues, if one acts as a standard utilitarian policy maker, there is hardly any compensation of short-lived agents, and we may even have large transfers towards long-lived agents, in opposition with basic ethical intuition. More importantly, such a counterintuitive redistribution is most likely to hold despite non-additive lifetime welfare, so that one can harldy rely on such a solution. This is the reason why the present study proposed an alternative road, which, as we showed, yields some compensation to the short-lived under plausible conditions.

Finally, three extensions of this paper should be mentionned. First, while the social planner takes here the consumption equivalent of a long life as a constant, which is estimated on the basis of laissez-faire choices, one may argue that the social planner should solve his modified problem while taking the consumption equivalent as a *variable*, which depends on his *own* allocation of resources.[50] Second, whereas this paper concentrates on first-best

[50] That alternative approach, which captures the idea that the consumption equivalent of a long life depends on what life is, would be worth being pursued, but is not trivial, as the endogeneity of α requires

optimum (without and with risk about the length of life), one may also want to explore the second-best problem, under *non observability* of type-specific life expectancies, and, thus, of the consumption equivalents. Third, although this paper concentrates on a disease (and a cure) for utilitarianism under unequal longevities, it would be worth considering whether other ethical frameworks suffer from the same kind of problem, and, more generally, to consider the issue of compensation of unequal longevities *outside utilitarianism*.[51]

Hence much work remains to be done. In any case, taking longevity differentials into account properly is not, for a government, optional. In the light of the central position of longevity as a determinant of human lifetime welfare, we believe that it is a necessity.

7 Appendix

7.1 Proof of Proposition 1

Take the case where $u(0) \geq 0$ (i.e. $\beta \geq 0$). In that case, we want to show that

$$G\left(u\left(\frac{W}{2}\right)\right) < G\left(u\left(\frac{W}{4}\right) + u\left(\frac{W}{4}\right)\right)$$

is always true. Given $G'(.) > 0$, that inequality is equivalent to

$$u\left(\frac{W}{2}\right) < u\left(\frac{W}{4}\right) + u\left(\frac{W}{4}\right)$$

That inequality is, under $u(0) \geq 0$ (i.e. $\beta \geq 0$) and $u''(\cdot) < 0$, always satisfied. Thus the first part of Proposition 1 follows from the concavity of $u(.)$.

However, under $u(0) < 0$ (i.e. $\beta < 0$), that inequality is satisfied only if

$$2v\left(\frac{W}{4}\right) - v\left(\frac{W}{2}\right) + \beta > 0$$

The LHS of that expression is negative at $W = 0$, but tends to $\bar{v} + \beta > 0$ when W tends to infinity. Hence, by continuity, there must exist a resource level W^S at which that expression equals zero. For higher resource levels, the above inequality is strictly satisfied, so that short-lived agents are, at the laissez-faire, worse off than long-lived agents, despite the equality of endowment $W/2$. Note that this condition is invariant to the transform $G(\cdot)$, so that whether short-lived agents are worse off or better off than long-lived agents has nothing to do with the precise shape of the lifetime welfare function, i.e. time-additive or not.

an additional constraint to be imposed, in order to avoid a multiplicity of optima.

[51] A first step in that direction is provided by Fleurbaey *et al* (2010).

7.2 Proof of Proposition 2

The case of $G(\cdot)$ linear is trivial. Indeed, if $G'(\cdot)$ equals a constant k, we have, from the FOCs, $ku'\left(c^1\right) = ku'\left(c^2\right) = ku'(d^2)$, from which it follows that $c^1 = c^2 = d^2$.

Let us thus focus on the case where $G''(\cdot) < 0$. From the FOCs it is obvious that we have $c^2 = d^2$ in all cases.

Assume first that $\beta > -v\left(\frac{W}{3}\right)$. Let us now prove by *reductio ad absurdum* that $c^1 > W/3$. For that purpose, assume instead that $c^1 \leq W/3$ and $c^2 = d^2 \geq W/3$. Under that assumption, we have $v\left(c^2\right) + v\left(d^2\right) + 2\beta > v\left(c^1\right) + \beta$. Hence $G'\left(v\left(c^1\right) + \beta\right) > G'\left(v\left(c^2\right) + v\left(d^2\right) + 2\beta\right)$. Moreover, if $c^1 \leq W/3$ and $c^2 = d^2 \geq W/3$, we also have $v'\left(c^1\right) \geq v'(c^2)$. Hence it follows that $G'\left(v\left(c^1\right) + \beta\right) v'\left(c^1\right) > G'\left(v\left(c^2\right) + v\left(d^2\right) + 2\beta\right) v'\left(c^2\right)$, in contradiction with the FOC $G'\left(v\left(c^1\right) + \beta\right) v'\left(c^1\right) = G'\left(v\left(c^2\right) + v\left(d^2\right) + 2\beta\right) v'\left(c^2\right)$. Hence a contradiction is reached. It must be the case that $c^1 > W/3$ and $c^2 = d^2 < W/3$, so that $c^1 > c^2 = d^2$.

Take now the case where $\beta = -v\left(\frac{W}{3}\right)$. In that case, it is easy to see that, under $c^1 = W/3$ and $c^2 = d^2 = W/3$, we have $v\left(c^2\right) + v\left(d^2\right) + 2\beta = 0 = v\left(c^1\right) + \beta$, so that $G'\left(v\left(\frac{W}{3}\right) + \beta\right) = G'\left(v\left(\frac{W}{3}\right) + v\left(\frac{W}{3}\right) + 2\beta\right)$, so that the FOC of the social planner's problem is satisfied. Hence $c^1 = c^2 = d^2 = W/3$ is the solution.

Finally, take the case where $\beta < -v\left(\frac{W}{3}\right)$. Here again, we show that $c^1 < c^2 = d^2$ by *reduction ad absurdum*. For that purpose, assume that $c^1 \geq W/3$ and $c^2 = d^2 \leq W/3$. From this, we have $v\left(c^2\right) + v\left(d^2\right) + \beta < v\left(c^1\right)$, so that $v\left(c^2\right) + v\left(d^2\right) + 2\beta < v\left(c^1\right) + \beta$. Hence $G'\left(v\left(c^1\right) + \beta\right) < G'\left(v\left(c^2\right) + v\left(d^2\right) + 2\beta\right)$. Moreover, if $c^1 \geq W/3$ and $c^2 = d^2 \leq W/3$, we also have $v'\left(c^1\right) \leq v'(c^2)$. Hence it follows that $G'\left(v\left(c^1\right) + \beta\right) v'\left(c^1\right) < G'\left(v\left(c^2\right) + v\left(d^2\right) + 2\beta\right) v'\left(c^2\right)$, in contradiction with the FOC $G'\left(v\left(c^1\right) + \beta\right) v'\left(c^1\right) = G'\left(v\left(c^2\right) + v\left(d^2\right) + 2\beta\right) v'\left(c^2\right)$. Hence a contradiction is reached. It must be the case that $c^1 < W/3$ and $c^2 = d^2 > W/3$.

7.3 Proof of Proposition 3

Assume $W > W^S$, so that long-lived agents are necessarily better off than short-lived agents at the laissez-faire. Short-lived agents receive a compensation when the utilitarian optimum involves $c^1 > W/2$. This is the case when, from the social planner's perspective, we have:

$$G'\left(u\left(\frac{W}{2}\right)\right) u'\left(\frac{W}{2}\right) > G'\left(u\left(\frac{W}{4}\right) + u\left(\frac{W}{4}\right)\right) u'\left(\frac{W}{4}\right)$$

that is, the marginal utility gain from raising the consumption of the short-lived beyond his laissez-faire consumption $W/2$ (i.e. the LHS) exceeds the marginal utility loss from reducing the consumption of the long-lived.

One can rewrite that expression as

$$\frac{G'\left(v\left(\frac{W}{2}\right)+\beta\right)}{G'\left(2v\left(\frac{W}{4}\right)+2\beta\right)}>\frac{v'\left(\frac{W}{4}\right)}{v'\left(\frac{W}{2}\right)}$$

Note that W^S is such that $2v\left(\frac{W^S}{4}\right)+2\beta=v\left(\frac{W^S}{2}\right)+\beta$. From this, it is easy to see that, if $W\leq W^S$, we have, by definition of W^S, $2v\left(\frac{W}{4}\right)+2\beta\leq v\left(\frac{W}{2}\right)+\beta$. Hence the LHS of the above condition is lower than 1. Given that the RHS exceeds 1 by the concavity of $v(.)$, it is certain that the condition is not satisfied, so that $c^1\leq W/2$.

However, if $W>W^S$, there may be compensation of the short-lived or not, depending on the curvatures of $v(\cdot)$, $G(\cdot)$ and the levels of W and β.

7.4 Proof of Proposition 4

The proof is close to the one of Proposition 2.

Take first the case where $G(\cdot)$ is linear. In that case, we have, under the conditions of Proposition 1,

$$c^1>c^2=d^2$$

When using both the FOCs and the budget constraint, it is easy to see that we have

$$\begin{aligned} c^1 &= W/3+\alpha/3 \\ c^2 &= d^2 \;=\; W/3-\alpha/6 \end{aligned}$$

When $G''(\cdot)<0$, we still have $c^2=d^2$, but the level of c^1 depends on the level of β.

Take the case where $\beta>-v\left(\frac{W}{3}+\frac{\alpha}{3}\right)$. Let us show that $c^1>W/3+\alpha/3$ and $c^2=d^2<W/3-\alpha/6$ by *reduction ad absurdum*. For that purpose, assume first that $c^1\leq W/3+\alpha/3$ and $c^2=d^2\geq W/3-\alpha/6$. In that case, we have $u'\left(c^1\right)\geq u'(c^2+\frac{\alpha}{2})$ and, provided $\beta>-v\left(W/3+\alpha/3\right)$, we have $v\left(c^1\right)+\beta<v\left(c^2+\frac{\alpha}{2}\right)+v\left(c^2+\frac{\alpha}{2}\right)+2\beta$. Hence $G'\left(v\left(c^1\right)+\beta\right)>G'\left(2v\left(c^2+\frac{\alpha}{2}\right)+2\beta\right)$. Therefore $G'\left(v\left(c^1\right)+\beta\right)v'\left(c^1\right)>G'\left(2v\left(c^2+\frac{\alpha}{2}\right)+2\beta\right)v'\left(c^2+\frac{\alpha}{2}\right)$, which is in contradiction with the FOC of the planner's problem. It must be the case that $c^1>W/3+\alpha/3$ and $c^2=d^2<W/3-\alpha/6$.

Take now the case where $\beta=-v\left(\frac{W}{3}+\frac{\alpha}{3}\right)$. In that case, it is easy to see that, under $c^1=W/3+\alpha/3$ and $c^2=d^2=W/3-\alpha/6$, we have $v\left(c^2+\frac{\alpha}{2}\right)+v\left(c^2+\frac{\alpha}{2}\right)+2\beta=0=$

$v\left(c^1\right)+\beta$, so that $G'\left(v\left(\frac{W}{3}\right)+\beta\right) = G'\left(v\left(\frac{W}{3}\right)+v\left(\frac{W}{3}\right)+2\beta\right)$, so that the FOC of the social planner's problem is satisfied. Hence $c^1 = W/3+\alpha/3$ and $c^2 = d^2 = W/3-\alpha/6$ is the solution.

Finally, take the case where $\beta < -v\left(\frac{W}{3}+\frac{\alpha}{3}\right)$. Here again, we show that $c^1 < W/3+\alpha/3$ and $c^2 = d^2 > W/3-\alpha/6$ by *reduction ad absurdum*. For that purpose, assume that $c^1 \geq W/3+\alpha/3$ and $c^2 = d^2 \leq W/3-\alpha/6$. From this, we have $v\left(c^2+\frac{\alpha}{2}\right)+v\left(d^2+\frac{\alpha}{2}\right)+2\beta < v\left(c^1\right)+\beta$. Hence $G'\left(v\left(c^1\right)+\beta\right) < G'\left(v\left(c^2+\frac{\alpha}{2}\right)+v\left(d^2+\frac{\alpha}{2}\right)+2\beta\right)$. Moreover, if $c^1 \geq W/3+\alpha/3$ and $c^2 = d^2 \leq W/3-\alpha/6$, we also have $v'\left(c^1\right) \leq v'(c^2+\frac{\alpha}{2})$. Hence it follows that $G'\left(v\left(c^1\right)+\beta\right)v'\left(c^1\right) < G'\left(v\left(c^2+\frac{\alpha}{2}\right)+v\left(d^2+\frac{\alpha}{2}\right)+2\beta\right)v'\left(c^2+\frac{\alpha}{2}\right)$, in contradiction with the FOC $G'\left(v\left(c^1\right)+\beta\right)v'\left(c^1\right) = G'\left(v\left(c^2+\frac{\alpha}{2}\right)+v\left(d^2+\frac{\alpha}{2}\right)+2\beta\right)v'\left(c^2+\frac{\alpha}{2}\right)$. It must be the case that $c^1 < W/3+\alpha/3$ and $c^2 = d^2 > W/3-\alpha/6$.

7.5 Proof of Proposition 5

Proposition 5 can be proved as follows. Suppose $W > W^S$, so that long-lived agents are better than short-lived agents at the laissez-faire. Short-lived agents can be said to be compensated with respect to the laissez-faire when c^1 exceeds $W/2$ under the modified utilitarian solution. This happens if and only if

$$G'\left(u\left(\frac{W}{2}\right)\right)u'\left(\frac{W}{2}\right) > G'\left(u\left(\frac{W}{4}+\frac{\alpha}{2}\right)+u\left(\frac{W}{4}+\frac{\alpha}{2}\right)\right)u'\left(\frac{W}{4}+\frac{\alpha}{2}\right)$$

that is, the marginal utility gain from raising c^1 above $W/2$ is larger than the marginal utility loss from reducing c^2 and d^2 below $W/4$. That expression can be rewritten as the condition of Proposition 5.

Under $W \leq W^S$, long-lived agents are not better off than short-lived agents at the laissez-faire. Hence, as a consequence of the definition of α, i.e. $u\left(c^{1*}+\alpha\right) = u\left(c^{2**}\right)+u\left(d^{2**}\right)$, we have $\alpha \leq 0$. Moreover, we have, by definition of W^S, that, under $W \leq W^S$, $v\left(\frac{W}{2}\right)+\beta \geq 2v\left(\frac{W}{4}\right)+2\beta$. To have $c^1 > W/2$ under the remedy, we would need

$$G'\left(u\left(\frac{W}{2}\right)\right)u'\left(\frac{W}{2}\right) > G'\left(u\left(\frac{W}{4}+\frac{\alpha}{2}\right)+u\left(\frac{W}{4}+\frac{\alpha}{2}\right)\right)u'\left(\frac{W}{4}+\frac{\alpha}{2}\right)$$

Given that $\alpha < 0$, it is clear that, under $W \leq W^S$, $u\left(\frac{W}{4}+\frac{\alpha}{2}\right)+u\left(\frac{W}{4}+\frac{\alpha}{2}\right) < u\left(\frac{W}{2}\right)$, leading $G'\left(u\left(\frac{W}{4}+\frac{\alpha}{2}\right)+u\left(\frac{W}{4}+\frac{\alpha}{2}\right)\right) > G'\left(u\left(\frac{W}{2}\right)\right)$. It is also obvious that $u'\left(\frac{W}{4}+\frac{\alpha}{2}\right) > u'\left(\frac{W}{2}\right)$. Hence the above inequality is necessarily violated, implying $c^1 < W/2$.

7.6 Proof of Proposition 6

At the laissez-faire, the short-lived agents with survival probability π^i are disadvantaged with respect to the long-lived if and only if

$$G\left(u\left(c_i\right)\right) < G\left(u\left(c_i\right) + u\left(d_i\right)\right)$$

that is, if and only if

$$v\left(d_i\right) + \beta > 0$$

This is always true under $u(0) \geq 0$ (i.e. $\beta \geq 0$), whatever we have $G''\left(.\right) = 0$ or $G''\left(.\right) < 0$. However, under $u(0) < 0$, it is not necessarily the case that long-lived are always better off than short lived. Indeed, under $\beta < 0$, the LHS is negative at $W = 0$. However, as W tends to infinity, the LHS tends to $\bar{v} + \beta > 0$. Hence, by continuity, there must exist a critical level of total endowment $W^{S\prime}$ for each agent with survival probability π^isuch that a strict equality holds. This level is such that

$$v\left(\frac{W^{S\prime}}{2\left(1+\pi^i\right)}\right) + \beta = 0$$

7.7 Proof of Proposition 7

Note that, in the absence of risk about the length of life, the threshold was defined as

$$2v\left(\frac{W^S}{4}\right) - v\left(\frac{W^S}{2}\right) + \beta = 0$$

Combining the definitions of thresholds, we have

$$v\left(\frac{W^{S\prime}}{2\left(1+\pi^i\right)}\right) = 2v\left(\frac{W^S}{4}\right) - v\left(\frac{W^S}{2}\right)$$

from which it is trivial to see that $W^S > W^{S\prime}$. Let us proof this by contradiction. Suppose first that we have $W^{S\prime} = W^S = W$. Hence we would have

$$v\left(\frac{W}{2\left(1+\pi^i\right)}\right) + v\left(\frac{W}{2}\right) = v\left(\frac{W}{4}\right) + v\left(\frac{W}{4}\right)$$

which is necessarily false, as the LHS always exceeds the RHS under $0 < \pi^i < 1$. Assume now that $W^S < W^{S\prime}$. Hence we should have

$$v\left(\frac{W^{S\prime}}{2\left(1+\pi^i\right)}\right) + v\left(\frac{W^S}{2}\right) = v\left(\frac{W^S}{4}\right) + v\left(\frac{W^S}{4}\right)$$

which is also false, because the LHS always exceeds the RHS under $0 < \pi^i < 1$. Therefore it must be true that $W^S > W^{S\prime}$.

7.8 Proof of Proposition 8

The case of G linear is trivial. In this case, $G'(\cdot)$ is equal to a constant and from the first order condition, $c^1 = d^1 = c^2 = d^2$. Replacing into the ressource constraint, we obtain the consumption levels, $W/\left(2 + \pi^1 + \pi^2\right)$.

Let now turn to the case where $G''(\cdot) < 0$.

Assume first that $\beta > -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$. Let us first show that $c^i > \frac{W}{2+\pi^1+\pi^2} > d^i$. Assume instead that $d^i > \frac{W}{2+\pi^1+\pi^2} > c^i$. From the first FOC we have

$\left[\pi^i G'\left(u(c^i) + u(d^i)\right) + \left(1 - \pi^i\right) G'\left(u(c^i)\right)\right] u'(c^i) = \lambda$. From the second FOC we have $u'(d^i) G'\left(u(c^i) + u(d^i)\right) = \lambda$. However, it is easy to see that there must be a contradition here. Given $\beta > -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, we have $u(c^i) + u(d^i) > u(c^i)$. Hence $G'\left(u(c^i) + u(d^i)\right) < G'\left(u(c^i)\right)$. Hence $\pi^i G'\left(u(c^i) + u(d^i)\right) + \left(1 - \pi^i\right) G'\left(u(c^i)\right) > G'\left(u(c^i) + u(d^i)\right)$. But if $d^i > c^i$, we have also $u'(c^i) > u'(d^i)$. Hence it must be the case that

$\left[\pi^i G'\left(u(c^i) + u(d^i)\right) + \left(1 - \pi^i\right) G'\left(u(c^i)\right)\right] u'(c^i) > u'(d^i) G'\left(u(c^i) + u(d^i)\right)$. Therefore, given that the LHS and the RHS of that expression are equal to λ, a contradiction is reached, confirming that $c^i > \frac{W}{2+\pi^1+\pi^2} > d^i$. Therefore we have $c^1 > d^1$ and $c^2 > d^2$. Hence 6 rankings are possible:

$$\begin{aligned}
d^2 &< c^2 < d^1 < c^1; d^1 < c^1 < d^2 < c^2 \\
d^2 &< d^1 < c^2 < c^1; d^1 < d^2 < c^1 < c^2 \\
d^1 &< d^2 < c^2 < c^1; d^2 < d^1 < c^1 < c^2
\end{aligned}$$

Let us eliminate some of these by contradiction.

Suppose $d^2 < c^2 < d^1 < c^1$. From the FOCs, we have

$u'(d^1) G'\left(u(c^1) + u(d^1)\right) = u'(d^2) G'\left(u(c^2) + u(d^2)\right)$. But given that $u(c^2) + u(d^2) < u(c^1) + u(d^1)$, it must be true that $G'\left(u(c^1) + u(d^1)\right) < G'\left(u(c^2) + u(d^2)\right)$. But as $u'(d^1) < u'(d^2)$, we have $u'(d^1) G'\left(u(c^1) + u(d^1)\right) < u'(d^2) G'\left(u(c^2) + u(d^2)\right)$, which contradicts the equality $u'(d^1) G'\left(u(c^1) + u(d^1)\right) = u'(d^2) G'\left(u(c^2) + u(d^2)\right)$. Hence that ranking is not possible.

Similar arguments can be used to show by contradiction that the rankings $d^1 < c^1 < d^2 < c^2$, $d^2 < d^1 < c^2 < c^1$, and $d^1 < d^2 < c^1 < c^2$ are not possible. We are left with two rankings: $d^1 < d^2 < c^2 < c^1$ or $d^2 < d^1 < c^1 < c^2$.

Assume that $\beta = -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$. Hence, if one imposes $c^2 = c^1 = d^1 = d^2 = \frac{W}{2+\pi^1+\pi^2}$, one obtains, by substituting in the FOCs, that

$$\begin{aligned}
\left[\pi^1 G'(0) + \left(1-\pi^1\right) G'(0)\right] u'\left(\frac{W}{2+\pi^1+\pi^2}\right) &= \lambda \\
\left[\pi^2 G'(0) + \left(1-\pi^2\right) G'(0))\right] u'\left(\frac{W}{2+\pi^1+\pi^2}\right) &= \lambda \\
u'\left(\frac{W}{2+\pi^1+\pi^2}\right) G'(0) &= \lambda
\end{aligned}$$

from which it appears clearly that $c^2 = c^1 = d^1 = d^2 = \frac{W}{2+\pi^1+\pi^2}$ is the unique solution to the planner's problem.

Assume now that $\beta < -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$. Let us first show that $c^i < \frac{W}{2+\pi^1+\pi^2} < d^i$. Assume instead that $d^i < \frac{W}{2+\pi^1+\pi^2} < c^i$. From the first FOC we have

$\left[\pi^i G'\left(u(c^i)+u(d^i)\right) + \left(1-\pi^i\right) G'\left(u(c^i)\right)\right] u'(c^i) = \lambda$. From the second FOC we have $u'(d^i) G'\left(u(c^i)+u(d^i)\right) = \lambda$. However, it is easy to see that there must be a contradition here. Given $\beta < -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, we have $u(c^i)+u(d^i) < u(c^i)$. Hence $G'\left(u(c^i)+u(d^i)\right) > G'\left(u(c^i)\right)$. Hence

$\pi^i G'\left(u(c^i)+u(d^i)\right) + \left(1-\pi^i\right) G'\left(u(c^i)\right) < G'\left(u(c^i)+u(d^i)\right)$. But if $d^i < c^i$, we have also $u'(c^i) < u'(d^i)$. Hence it must be the case that

$\left[\pi^i G'\left(u(c^i)+u(d^i)\right) + \left(1-\pi^i\right) G'\left(u(c^i)\right)\right] u'(c^i) < u'(d^i) G'\left(u(c^i)+u(d^i)\right)$. Therefore, given that the LHS and the RHS of that expression are equal to λ, a contradiction is reached, confirming that $c^i < \frac{W}{2+\pi^1+\pi^2} < d^i$. Therefore we have $c^1 < d^1$ and $c^2 < d^2$. Hence 6 rankings are possible:

$$\begin{aligned}
c^2 &< d^2 < c^1 < d^1; c^1 < d^1 < c^2 < d^2 \\
c^2 &< c^1 < d^2 < d^1; c^1 < c^2 < d^1 < d^2 \\
c^1 &< c^2 < d^2 < d^1; c^2 < c^1 < d^1 < d^2
\end{aligned}$$

By using the same kind of proof as above, we can rule out the first four rankings by contradiction, and thus we keep $c^1 < c^2 < d^2 < d^1$ and $c^2 < c^1 < d^1 < d^2$.

7.9 Proof of Proposition 9

Assume first that $G(\cdot)$ is linear. Laissez-faire inequalities between a long-lived and a short-lived with the same survival probability π^i are equal to $v\left(W/2\left(1+\pi^i\right)\right) + \beta$, while, in the first-best utilitarian optimum, these inequalities are independant of the survival probability and equal to $v\left(W/\left(2+\pi^1+\pi^2\right)\right) + \beta$. For agents with survival probability π^1, it is easy to show that inequalities are higher in the laissez-faire than in the first-best, while for agents with survival probability π^2, it is the opposite.

Under $G(\cdot)$ non linear, the utilitarian optimum depends on various elements (see Proposition 7).

If $\beta > -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, the difference in lifetime welfare between long-lived and short-lived type-1 agents is, at the utilitarian optimum, smaller than $v\left(\frac{W}{2+\pi^1+\pi^2}\right)+\beta$, as $d^{1FB} < \frac{W}{2+\pi^1+\pi^2}$. At the laissez-faire, that inequality was smaller than $v\left(\frac{W}{2(1+\pi^1)}\right)+\beta$, as $d^{1LF} < \frac{W}{2(1+\pi^1)}$. Given that $\pi^1 < \pi^2$, we have $\frac{W}{2+\pi^1+\pi^2} < \frac{W}{2(1+\pi^1)}$. But this does not allow us to conclude on whether the inequality is reduced or increased by utilitarianism. For type-2 agents, the difference in lifetime welfare between long-lived and short-lived agents is, at the utilitarian optimum, smaller than $v\left(\frac{W}{2+\pi^1+\pi^2}\right)+\beta$, as $d^{2FB} < \frac{W}{2+\pi^1+\pi^2}$. At the laissez-faire, that inequality was smaller than $v\left(\frac{W}{2(1+\pi^2)}\right)+\beta$, as $d^{2LF} < \frac{W}{2(1+\pi^2)}$. Given that $\pi^1 < \pi^2$, we have $\frac{W}{2+\pi^1+\pi^2} > \frac{W}{2(1+\pi^2)}$. Here again, we cannot conclude on whether the inequality is raised or reduced by utilitarianism.

It is only in the special case where $\beta = -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$ that we can say for sure that utilitarianism reduces inequalities between short-lived and long-lived within type-1 agents, and does the opposite for type-2 agents.

If $\beta < -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, the difference in lifetime welfare between long-lived and short-lived type-1 agents is, at the utilitarian optimum, smaller than $v\left(\frac{W}{2+\pi^1+\pi^2}\right)+\beta$, as $d^{1FB} > \frac{W}{2+\pi^1+\pi^2}$. At the laissez-faire, that inequality was smaller than $v\left(\frac{W}{2(1+\pi^1)}\right)+\beta$, as $d^{1LF} > \frac{W}{2(1+\pi^1)}$. Given that $\pi^1 < \pi^2$, we have $\frac{W}{2+\pi^1+\pi^2} < \frac{W}{2(1+\pi^1)}$, but here again one cannot draw conclusions on how utilitarianism affects inequalities within type-1 agents. The same is true for type-2 agents.

7.10 Proof of Proposition 10

Take the case where $G''(\cdot) = 0$. From the FOCs, we have:

$$\begin{aligned} u'(c^1) &= u'(c^2) \\ u'(c^1) &= u'(d^1+\alpha^1) \\ u'(c^2) &= u'(d^2+\alpha^2) \\ u'(d^1+\alpha^1) &= u'(d^2+\alpha^2) \end{aligned}$$

from which it is trivial to see that, under $\alpha^i > 0$, $c^1 = d^1 + \alpha^1$, $c^2 = d^2 + \alpha^2$, and $c^1 = c^2$. Substituting for all this in the economy's resource constraint yields the solutions in the Proposition.

Take now the case where $G''(\cdot) < 0$, and focus on the case where $\alpha^i > 0$.

Let us first show that, under $\beta > -v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$, we have $c^i > \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} > d^i$. Let us show it by contradiction. According to the FOC, we have:

$\left[\left(1-\pi^i\right)G'\left(u\left(c^i\right)\right) + \pi^i G'\left(u\left(c^i\right) + u(d^i+\alpha^i)\right)\right]u'(c^i) = G'\left(u\left(c^i\right) + u(d^i+\alpha^i)\right)u'(d^i + \alpha^i)$. Suppose now $c^i < \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} < d^i$. Then, under $\alpha^i > 0$, $u'(c^i) > u'(d^i+\alpha^i)$. Moreover, under $\beta > -v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$, we have $G'\left(u\left(c^i\right)\right) > G'\left(u\left(c^i\right) + u(d^i+\alpha^i)\right)$. Therefore it must be true that $\left[\left(1-\pi^i\right)G'\left(u\left(c^i\right)\right) + \pi^i G'\left(u\left(c^i\right) + u(d^i+\alpha^i)\right)\right]u'(c^i) > G'\left(u\left(c^i\right) + u(d^i+\alpha^i)\right)u'(d^i+\alpha^i)$, in contradiction with the FOCs. Hence it must be the case that $c^i > \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} > d^i$. Therefore we have $c^1 > d^1$ and $c^2 > d^2$. Hence 6 rankings are possible:

$$\begin{aligned}
d^2 &< c^2 < d^1 < c^1; d^1 < c^1 < d^2 < c^2 \\
d^2 &< d^1 < c^2 < c^1; d^1 < d^2 < c^1 < c^2 \\
d^1 &< d^2 < c^2 < c^1; d^2 < d^1 < c^1 < c^2
\end{aligned}$$

Note, however, that we have $d^i < c^j$.[52] Hence we can eliminate the rankings $d^2 < c^2 < d^1 < c^1$ and $d^1 < c^1 < d^2 < c^2$. However, it is not possible to eliminate the others, since the two types of agents have distinct consumption-equivalents α^1 and α^2. We are left with four rankings: $d^2 < d^1 < c^2 < c^1$, $d^1 < d^2 < c^1 < c^2$, $d^1 < d^2 < c^2 < c^1$ or $d^2 < d^1 < c^1 < c^2$.

Under $\beta = -v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$, it is easy to see that if one substitutes for $c^1 = c^2 = \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}; d^1 = \frac{W+\pi^2\alpha^2-\alpha^1\left(2+\pi^2\right)}{2+\pi^1+\pi^2}; d^2 = \frac{W+\pi^1\alpha^1-\alpha^2\left(2+\pi^1\right)}{2+\pi^1+\pi^2}$ in the FOCs, we get:

$$\begin{aligned}
\left[\left(1-\pi^1\right)G'(0) + \pi^1 G'(0)\right]u'(c^1) &= G'(0)\,u'(d^1+\alpha^1) \\
\left[\left(1-\pi^2\right)G'(u(0)) + \pi^2 G'(0)\right]u'(c^2) &= G'(0))\,u'(d^2+\alpha^2)
\end{aligned}$$

From which it is obvious that $c^1 = c^2 = \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}; d^1 = \frac{W+\pi^2\alpha^2-\alpha^1\left(2+\pi^2\right)}{2+\pi^1+\pi^2}; d^2 = \frac{W+\pi^1\alpha^1-\alpha^2\left(2+\pi^1\right)}{2+\pi^1+\pi^2}$ is the solution of the planning problem, as this satisfies the FOCs.

Assume now that $\beta < -v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$. Let us first show that $c^i < \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} < d^i$. Assume instead that $d^i < \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} < c^i$. From the first FOC we have

[52] Indeed $c^i > \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} > d^i$ is true for all i, so that it is also true that $c^i > d^j$.

$\left[\pi^i G'\left(u(c^i)+u(d^i+\alpha^i)\right)+\left(1-\pi^i\right)G'\left(u(c^i)\right)\right]\ u'(c^i)=\lambda$. From the second FOC we have $u'(d^i+\alpha^i)G'\left(u(c^i)+u(d^i+\alpha^i)\right)=\lambda$. However, it is easy to see that there must be a contradition here. Given $\beta<-v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$, we have $u(c^i)+u(d^i+\alpha^i)<u(c^i)$. Hence $G'\left(u(c^i)+u(d^i+\alpha^i)\right)>G'\left(u(c^i)\right)$. Hence

$\pi^i G'\left(u(c^i)+u(d^i+\alpha^i)\right)+\left(1-\pi^i\right)G'\left(u(c^i)\right)<G'\left(u(c^i)+u(d^i+\alpha^i)\right)$. But if $d^i<\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}<c^i$, we have also $u'(c^i)<u'(d^i+\alpha^i)$. Hence it must be the case that $\left[\pi^i G'\left(u(c^i)+u(d^i+\alpha^i)\right)+\left(1-\pi^i\right)G'\left(u(c^i)\right)\right]u'(c^i)<u'(d^i+\alpha^i)G'\left(u(c^i)+u(d^i+\alpha^i)\right)$. Therefore, given that the LHS and the RHS of that expression are equal to λ, a contradiction is reached, confirming that $c^i<\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}<d^i$. Therefore we have $c^1<d^1$ and $c^2<d^2$. Hence 6 rankings are possible:

$$\begin{aligned}
c^2 &< d^2<c^1<d^1; c^1<d^1<c^2<d^2\\
c^2 &< c^1<d^2<d^1; c^1<c^2<d^1<d^2\\
c^1 &< c^2<d^2<d^1; c^2<c^1<d^1<d^2
\end{aligned}$$

By using the same kind of proof as above, we can rule out the first two rankings by contradiction, and thus we keep $c^2<c^1<d^2<d^1$, $c^1<c^2<d^1<d^2$, $c^1<c^2<d^2<d^1$ and $c^2<c^1<d^1<d^2$.

7.11 Calibration of β under $G''(\cdot)=0$

Regarding the calibration of the intercept β under $G''(\cdot)=0$, note first that the value of statistical life is, in the present model, equal to

$$VL\left(c^i\right)\equiv\frac{u\left(c^i\right)-u'\left(c^i\right)c^i}{u'\left(c^i\right)}$$

Indeed, the expected lifetime utility of a type-i agent is $U^i=\left(1+\pi^i\right)u\left(c^i\right)$, where $c^i=w/\left(1+\pi^i\right)$ when consumption is smoothed across periods and where w is the initial wealth. The value of life can be interpreted as the amount of wealth the agent is willing to give up in order to increase his survival probability, for a given level of expected utility, i.e.

$$\begin{aligned}
VL\left(c^i\right) &\equiv \left.\frac{dw}{d\pi^i}\right|_{\bar{U}}=-\left.\frac{\partial U^i/\partial\pi^i}{\partial U^i/\partial w}\right|_{\bar{U}}\\
&= \frac{u\left(c^i\right)-u'\left(c^i\right)c^i}{u'\left(c^i\right)}
\end{aligned}$$

Substituting for $u(c^i)$ in $VL(c^i)$ yields:

$$VL\left(d^i\right)\equiv\beta\left(c^i\right)^\sigma+c^i\left(\frac{1}{1-\sigma}-1\right)$$

Assuming that the initial endowment equals 10 and that consumption is smoothed across periods, the VSL is

$$VL\left(5\right)=\beta\left(5\right)^{0.5}+5\left(\frac{1}{1-0.5}-1\right)=\beta\left(5\right)^{0.5}+5$$

Given that the VSL amounts to about 120 times income per head per year (see Miller, 2001), which amounts to $\frac{120}{40}$ income per period of 40 years, we have, given the two-period structure, $VL\left(5\right)=\frac{120}{40}\left(5\right)=15$, from which it follows that $\beta=4.472$.

8 References

Arrow, K.J. (1971). 'A utilitarian approach to the concept of equality in public expenditures', Quarterly Journal of Economics, 85, pp. 409-415.

Beccaria, C. (1764). Des délits et des peines. French version introduced by P. Audegean (2009), ENS Edition, Lyon.

Becker, G.S., Philipson, T. & Soares, R. (2005). 'The quantity and the quality of life and the evolution of world inequality', American Economic Review, 95(1), pp. 277-291.

Bentham, J. (1789). Introduction to the Principles of Morals and Legislation. In J.S. Mill: Utilitarianism, edited by M. Warnock (1962), Fontana Press, London.

Blundell, R., Browning, M. & Meghir, C. (1994). 'Consumer demand and the life-cycle allocation of household expenditures', Review of Economic Studies, 61(1), pp. 57-80.

Bommier, A. (2006). 'Uncertain lifetime and intertemporal choice: risk aversion as a rationale for time discounting', International Economic Review, 47(4), pp. 1223-1246.

Bommier, A., M-L. Leroux and J-M. Lozachmeur (2009). 'On the public economics of annuities with differential mortality', CORE Discussion Paper 2009-30.

Bommier, A., M-L. Leroux and J-M. Lozachmeur (2010). 'Social Security and differential mortality'. Canadian Journal of Economics, forthcoming.

Broome, J. (2004). Weighing Lives. Oxford University Press, New-York.

Fleurbaey, M. (2005). 'Health, wealth and fairness', Journal of Public Economic Theory, 7, pp. 253-284.

Fleurbaey, M., Leroux, M-L. and Ponthiere, G. (2010): 'Unequal longevities and compensation', mimeo.

Gossen, H.E. (1854). The Laws of Human Relations and the Rules of Human Action Derived Therefrom (english version, MIT Press, 1983).

Leroux, M-L., Pestieau, P. & Ponthiere, G. (2010). 'Optimal linear taxation under endogenous longevity', Journal of Population Economics, forthcoming.

Leroux, M-L., & Ponthiere, G. (2009). 'Optimal tax policy and expected longevity: a mean and variance utility approach', International Tax and Public Finance, 16(4), pp. 514-537.

Miller, T. (2000). 'Variations between countries in values of statistical life'. Journal of Transport Economics and Policy, 34(2), pp. 169-188.

Mirrlees, J. (1982). 'The economic uses of utilitarianism', in A. Sen and B. Williams (eds.): Utilitarianism and Beyond, Cambridge University Press, Cambridge.

Nordhaus, W. (2003). 'The health of nations: the contribution of improved health to living standards', in K. Murphy and R. Topel (eds.): Measuring the Gains from Medical Research: An Economic Approach, The University of Chicago Press.

Ponthiere, G. (2008). 'A study of the sensibility of longevity-adjusted income measures', Oxford Development Studies, 36(3), pp. 339-361.

Rogot, E., Sorlie, P. & Johnson, N. (1992). 'Life expectancy by employment status, income, and education in the National Longitudinal Mortality Study', Public Health Reports, 107(4), pp. 457-461.

United Nations Development Program (UNDP) (2008): The Human Development Report, Oxford University Press, New-York.

Sen, A., 1973. On Economic Inequality, Oxford, Clarendon Press.

Sen, A. & Williams, B. (1982): Utilitarianism and Beyond, Cambridge University Press, Cambridge.

Usher, D. (1973). 'An imputation to the measure of economic growth for changes in life expectancy', in M. Moss (ed): The Measurement of Economic and Social Performance, NBER, New-York.

Recent titles

CORE Discussion Papers

2009/86. Ilke VAN BEVEREN and Hylke VANDENBUSSCHE. Product and process innovation and the decision to export: firm-level evidence for Belgium.

2010/1. Giorgia OGGIONI and Yves SMEERS. Degree of coordination in market-coupling and counter-trading.

2010/2. Yu. NESTEROV. Efficiency of coordinate descent methods on huge-scale optimization problems.

2010/3. Geert DHAENE an Koen JOCHMANS. Split-panel jackknife estimation of fixed-effect models.

2010/4. Parkash CHANDER. Cores of games with positive externalities.

2010/5. Gauthier DE MAERE D'AERTRYCKE and Yves SMEERS. Liquidity risks on power exchanges.

2010/6. Marc FLEURBAEY, Stéphane LUCHINI, Christophe MULLER and Erik SCHOKKAERT. Equivalent income and the economic evaluation of health care.

2010/7. Elena IÑARRA, Conchi LARREA and Elena MOLIS. The stability of the roommate problem revisited.

2010/8. Philippe CHEVALIER, Isabelle THOMAS and David GERAETS, Els GOETGHEBEUR, Olivier JANSSENS, Dominique PEETERS and Frank PLASTRIA. Locating fire-stations: an integrated approach for Belgium.

2010/9. Jean-Charles LANGE and Pierre SEMAL. Design of a network of reusable logistic containers.

2010/10. Hiroshi UNO. Nested potentials and robust equilibria.

2010/11. Elena MOLIS and Róbert F. VESZTEG. Experimental results on the roommate problem.

2010/12. Koen DECANCQ. Copula-based orderings of multivariate dependence.

2010/13. Tom TRUYTS. Signaling and indirect taxation.

2010/14. Asel ISAKOVA. Currency substitution in the economies of Central Asia: How much does it cost?

2010/15. Emanuele FORLANI. Irish firms' productivity and imported inputs.

2010/16. Thierry BRECHET, Carmen CAMACHO and Vladimir M. VELIOV. Model predictive control, the economy, and the issue of global warming.

2010/17. Thierry BRECHET, Tsvetomir TSACHEV and Vladimir M. VELIOV. Markets for emission permits with free endowment: a vintage capital analysis.

2010/18. Pierre M. PICARD and Patrice PIERETTI. Bank secrecy, illicit money and offshore financial centers.

2010/19. Tanguy ISAAC. When frictions favour information revelation.

2010/20. Jeroen V.K. ROMBOUTS and Lars STENTOFT. Multivariate option pricing with time varying volatility and correlations.

2010/21. Yassine LEFOUILI and Catherine ROUX. Leniency programs for multimarket firms: The effect of Amnesty Plus on cartel formation.

2010/22. P. Jean-Jacques HERINGS, Ana MAULEON and Vincent VANNETELBOSCH. Coalition formation among farsighted agents.

2010/23. Pierre PESTIEAU and Grégory PONTHIERE. Long term care insurance puzzle.

2010/24. Elena DEL REY and Miguel Angel LOPEZ-GARCIA. On welfare criteria and optimality in an endogenous growth model.

2010/25. Sébastien LAURENT, Jeroen V.K. ROMBOUTS and Francesco VIOLANTE. On the forecasting accuracy of multivariate GARCH models.

2010/26. Pierre DEHEZ. Cooperative provision of indivisible public goods.

2010/27. Olivier DURAND-LASSERVE, Axel PIERRU and Yves SMEERS. Uncertain long-run emissions targets, CO_2 price and global energy transition: a general equilibrium approach.

2010/28. Andreas EHRENMANN and Yves SMEERS. Stochastic equilibrium models for generation capacity expansion.

2010/29. Olivier DEVOLDER, François GLINEUR and Yu. NESTEROV. Solving infinite-dimensional optimization problems by polynomial approximation.

2010/30. Helmuth CREMER and Pierre PESTIEAU. The economics of wealth transfer tax.

Recent titles

CORE Discussion Papers - continued

2010/31. Thierry BRECHET and Sylvette LY. Technological greening, eco-efficiency, and no-regret strategy.
2010/32. Axel GAUTIER and Dimitri PAOLINI. Universal service financing in competitive postal markets: one size does not fit all.
2010/33. Daria ONORI. Competition and growth: reinterpreting their relationship.
2010/34. Olivier DEVOLDER, François GLINEUR and Yu. NESTEROV. Double smoothing technique for infinite-dimensional optimization problems with applications to optimal control.
2010/35. Jean-Jacques DETHIER, Pierre PESTIEAU and Rabia ALI. The impact of a minimum pension on old age poverty and its budgetary cost. Evidence from Latin America.
2010/36. Stéphane ZUBER. Justifying social discounting: the rank-discounting utilitarian approach.
2010/37. Marc FLEURBAEY, Thibault GAJDOS and Stéphane ZUBER. Social rationality, separability, and equity under uncertainty.
2010/38. Helmuth CREMER and Pierre PESTIEAU. Myopia, redistribution and pensions.
2010/39. Giacomo SBRANA and Andrea SILVESTRINI. Aggregation of exponential smoothing processes with an application to portfolio risk evaluation.
2010/40. Jean-François CARPANTIER. Commodities inventory effect.
2010/41. Pierre PESTIEAU and Maria RACIONERO. Tagging with leisure needs.
2010/42. Knud J. MUNK. The optimal commodity tax system as a compromise between two objectives.
2010/43. Marie-Louise LEROUX and Gregory PONTHIERE. Utilitarianism and unequal longevities: A remedy?

Books

J. GABSZEWICZ (ed.) (2006), La différenciation des produits. Paris, La découverte.

L. BAUWENS, W. POHLMEIER and D. VEREDAS (eds.) (2008), *High frequency financial econometrics: recent developments*. Heidelberg, Physica-Verlag.

P. VAN HENTENRYCKE and L. WOLSEY (eds.) (2007), *Integration of AI and OR techniques in constraint programming for combinatorial optimization problems*. Berlin, Springer.

P-P. COMBES, Th. MAYER and J-F. THISSE (eds.) (2008), *Economic geography: the integration of regions and nations*. Princeton, Princeton University Press.

J. HINDRIKS (ed.) (2008), *Au-delà de Copernic: de la confusion au consensus* ? Brussels, Academic and Scientific Publishers.

J-M. HURIOT and J-F. THISSE (eds) (2009), *Economics of cities*. Cambridge, Cambridge University Press.

P. BELLEFLAMME and M. PEITZ (eds) (2010), *Industrial organization: markets and strategies*. Cambridge University Press.

M. JUNGER, Th. LIEBLING, D. NADDEF, G. NEMHAUSER, W. PULLEYBLANK, G. REINELT, G. RINALDI and L. WOLSEY (eds) (2010), *50 years of integer programming, 1958-2008: from the early years to the state-of-the-art*. Berlin Springer.

CORE Lecture Series

C. GOURIÉROUX and A. MONFORT (1995), Simulation Based Econometric Methods.

A. RUBINSTEIN (1996), Lectures on Modeling Bounded Rationality.

J. RENEGAR (1999), A Mathematical View of Interior-Point Methods in Convex Optimization.

B.D. BERNHEIM and M.D. WHINSTON (1999), Anticompetitive Exclusion and Foreclosure Through Vertical Agreements.

D. BIENSTOCK (2001), Potential function methods for approximately solving linear programming problems: theory and practice.

R. AMIR (2002), Supermodularity and complementarity in economics.

R. WEISMANTEL (2006), Lectures on mixed nonlinear programming.