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The political economy of derived pension rights

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Abstract

Derived pension rights exist in most Social Security systems but with variable generosity. They are mainly targeted towards non-working wives and widows and are viewed as a means to alleviate poverty among older women living alone. The purpose of this paper is to explain how they can emerge from a political economy process when the Social Security is a combination of Bismarckian and Beveridgian pillars. It also shows that derived rights tend to encourage stay-at-home wives thus revealing an unpleasant trade-o§ between female labor participation and poverty alleviation.

Keywords: social security, derived pension rights, majority voting, individualisation of pension rights.

JEL Classification: D72, D78, H55

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1 Introduction

Most OECD pension systems offer protection for non-working widows and divorcees through the so-called derived pension rights. Derived pension rights represent an important reality in two ways. They first represent a non negligible part of Social Security spending and second they contribute to poverty alleviation among elderly women living alone.[1] The concept of derived pension rights concern Social Security benefits, which accrue to an individual but which originate from and depend on their relationship with another person, usually of parenthood, marriage or cohabitation. They most often concern women, widows, divorcees or even non-working wives.

To illustrate the type of derived pension rights we have in mind, let us take the case of Belgian Social Security. The three main derived pension rights are (i) the survivors' benefits for working or non-working widows (80% of the deceased's pension plus own pension but the combined amount cannot exceed more than 110% of own pension entitlement), (ii) the divorcees' pension benefits (37.5% of a former spouse's average earnings over the duration of marriage, less their own pension rights accrued during the same period), (iii) the spousal benefits which are a form of dependents' allowance and are usually provided as a supplement to the main pension benefit (15% of the insured's average lifetime earnings; indeed, the replacement ratio is 60% for singles while it is 75% for couples).[2] Note that besides these benefits from derived rights, the elderly is also entitled to a basic or targeted pension which is granted to all older persons who did not earn enough pension entitlements.

In spite of their importance, derived pension rights are not a well-researched area of pension system analysis. There are only a few studies and all of them are empirical. The focus is mostly on gender inequality and on spouses as dependents or survivors, while the rights of dependent children are largely excluded. Male dependents are also excluded from most analyses, reflecting their minimal share in total beneficiaries of derived pension rights. Most studies examine the economic situation of elderly women living alone.[3]

A few articles also compare derived pension rights across countries and study how the eco-

[1] In France, in 2006, derived pension rights were estimated to represent 14% of total Social Security expenditure (see les Comptes de la Protection Nationale, DREES, 2007). In Belgium, figures are even bigger: for the same year, spousal benefits plus the survival benefits amounted to 32% of the private sector pensions expenditures.

[2] These figures are taken from Choi (2006).

[3] In the case of France, see for example, Bonnet et Houriez (2009a, b).

nomic situation of women is modified after the death of a spouse. For example, Burkhauser et al. (2005) show that, for the four countries under study (the US, Germany, Canada and Great Britain), the change in the economic well-being of women following the death of their husband is comparable, even though these countries have a different mix of public and private pension schemes; differences in outcomes are mostly related to the age at death of the spouse, the age of the survivor and whether there are surviving children. Thompson and Carasso (2002) also analyze pension benefits of 16 countries from the perspective of women, including survivors' benefits and pension rights in case of divorce. Unlike Burkhauser et al. (2005), these authors find that survivors' benefits mainly depend on the type of main pension schemes for workers, in particular whether they are flat rate or earnings-related. As for divorce, countries address this issue in various ways: no benefit provision, pension splitting and creation of special benefits. Hurd and Wise (1997) show that a restructuring of pension benefits in the US could have an important effect on poverty of elderly women living alone. Poverty rates of widows could be strongly reduced by an increase in survivor benefits which would be funded by a reduction in the benefits of couples, even though this reduction could result in a small increase in the poverty rate of couples. According to Choi (2006), in the OECD countries under study, non-working widows and working widows receive an average pension level of 36 and 50% respectively, compared to an average level for couples of nearly 60% of average earnings.

Most importantly for the purpose of our paper, the existence of such derived rights makes the Social Security redistributes resources between individuals with different marital status. As it was shown by Galasso (2002), the magnitude of such redistribution is also surprisingly large. For instance, one-earner couples get the highest internal return from the Social Security, followed by two-earner couples with 70/30 earnings split; returns are equal for two-earner couples with a 50/50 earnings split and single women, while single men are the most disadvantaged. The difference in returns observed between singles and married couples, either one-earner or two-earner, can be explained by the so-called "derived pension rights". Several countries, like France, provide the surviving spouse (more often the woman) with a survivor benefit, while some other countries provide one-earner couples with a higher replacement rate than the one applied to single men; some countries, like Belgium or Japan, provide both types of derived benefits.[4] The marital status and the generosity of the system towards the non-working spouse is then likely

[4]For example, in Belgium, the supplementary pension is evaluated to 1/4 of the working spouse pension. As shown in Gruber and Wise (1999), derived pension rights may take very different forms depending on the country.

to play an important role in the support for a pension system. This is the gist of our paper.

Unlike these empirical papers, ours is theoretical. Our objective is twofold. First, using a political economy model in which individuals vote over the level of derived rights, we want to identify the factors that are likely to influence the emergence of such rights but also what are the consequences on the size and the generosity of the general pension system (either Beveridgian or Bismarckian). Our intuition is that it should be related to the cost of housework, to the potential wage of the non-working spouse but also to the characteristics of the welfare state and to the political process we consider. Our second objective is to show that such a system, whose primary goal is to give financial protection to non-working women and to the poorest one-breadwinner couples, favours the existence of such couples and of stay-at-home women who it intends to protect, as compared to a pure market economy situation. There is thus a trade-off between poverty alleviation and women's labour participation.

To do so, we model a society composed of men and women who are part of either one- or two-breadwinner couples.[5] For simplicity, we assume away single individuals. Besides these differences in the participation to the labour market, couples have also different productivity. We assume that there exists a pension system which is a combination of Bismarckian and Beveridgian systems. This implies that such a system is partly contributive (individuals get a pension benefit which is related to their previous contributions) and partly redistributive (the pension benefit includes a flat part). In addition, we assume that one-breadwinner couples benefit from derived rights and that such rights account for both survivor benefits and/or dependence allowances in order to reduce poverty in these couples.[6] We first characterize laissez-faire, first- and second-best solutions as benchmarks but our objective is positive rather than normative. Hence, in order to understand how the political process can favour the emergence of derived rights which would supplement the existing Social Security system, we study the majority equilibrium one observes in such a society. We assume that individuals vote on the level of derived rights while the mix Beveridge-Bismarck is set at the constitutional level. At this level, the criterion chosen is the Rawlsian maximin objective. Other specifications could have been chosen for sure. Instead of the two stages adopted here, first a normative one and then a positive one, we could have used sequential voting or even simultaneous one. These two options are analytically more difficult and

[5] For simplicity, we consider only heterosexual couples.

[6] There is no possibility of divorce and thus there is no divorcee derived pension benefit included in the derived pension scheme.

would not necessarily bring more interesting results. As to the Rawlsian criterion it is standard at the constitutional level; it is in the spirit of Rawls' view of the veil of ignorance. Here too other options would have been possible at the cost of additional complexity.

Anticipating on the following we show that if the decisive voter is a one-breadwinner couple a contributive pension system tends to be desirable as it implies less tax distortion than a Beveridgian system. We also show that whether the decisive voter is a one breadwinner depends on the opportunity cost of a second earner but also on the Bismarck-Beveridge mix.

This paper contributes to the literature on the political economy of pensions.[7] In his seminal contribution, Browning (1975) focused on age differences and showed that, if the old favour generous pensions and the young prefer private savings, the decisive voter is the median age one. More recent models include wage differences alongside age differences. In such a framework, Casamatta et al. (2000) show that the pension system is chosen by a majority made of rich and poor workers who collude against a coalition of retirees and middle-class workers: this is the so-called *ends against the middle* outcome. We adopt a different approach in the sense that we claim that the marital situation and the labour force participation decisions inside couples also affect the support for pension systems when they include spousal benefits. In this respect, the most closely related paper is the one by Leroux et al. (2010). In this paper, individuals differ according to two characteristics, gender and marriage. Also it is assumed that the partition of the population between singles and couples and between one- and two-breadwinners is given and that the pension system is purely Beveridgian. Further the size of derived rights is exogenous. It is shown that the majority voting outcome depends on the relative number of one-breadwinner couples and on the size of these derived rights. In contrast in the present paper, we assume that the generosity of derived rights and wives' labor participation are endogenous and as such depend on policy instruments which agents vote on.

The rest of the paper is organized as follows. In the next section, we present our model. In Section 3, we derive individuals' decisions and the laissez faire. In section 4, we solve the first- and second-best optima. Section 5 solves the majority voting equilibrium assuming first a discrete distribution of types and then a continuous distribution. Last section concludes.

[7] For good surveys, see Galasso and Profeta (2002) and de Walque (2005).

2 The model

We assume that individuals differ in gender. There is a mass one of men as well as of women. Individuals live for two periods. All men and *some* (we become clearer on this point later on) women work in the first period of their life and retire in the second one. The first period is of length 1 while the length of the second period differs for men and women: $\pi_m = 1$ for men and $\pi_f = \beta$ for women, with $\beta \geq 1$. In addition to the difference in longevity, men and women with the same productivity w, have different wages: $\omega_m = w$ for men and $\omega_f = \alpha w$ for women with $\alpha \leq 1$. For the time being, we do not specify any specific productivity distribution w, only that it is distributed over an interval $[\underline{w}, \bar{w}]$. In Section 4, we assume successively a discrete and a continuous distribution of productivity.

The structure of the society is such that it is only composed of couples, in which the husband always works and the wife does not necessarily do so.[8] We also assume positive assortative mating: a man with productivity (and thus wage) equal to w always marries a woman who has the same productivity, w (and thus, a wage equal to αw).[9]

Let us now define the intertemporal utility of couples, which is likely to depend on whether one or both members of the couple work. In both cases, however, we assume that it is quasi-linear (linear in the first-period consumption). This is undoubtedly a strong assumption as it assumes away income effects but it is needed to obtain clear results. If the couple comprises two breadwinners, their joint intertemporal utility is represented by

$$U^{c2}(c, l_f, l_m, d) = 2c - v(l_f) - v(l_m) + (\pi_f + \pi_m) u(d) - k \tag{1}$$

where c and d denote the first- and second-period consumptions, respectively. Second-period utility function $u(.)$ is such that $u'(.) > 0$ and $u''(.) < 0$. The labour supply is denoted l_i and for simplicity, we assume that the disutility of labour $v(l_i)$ is quadratic and equal to $l_i^2/2$ for both genders $i = f, m$. Moreover, in a couple where both members work, the couple also incurs a constant utility cost, k which can be regarded as the value of housework (the reservation wage).[10] Note also that our specification implies a unitary decision making within the couple.

[8] This assumption is made for simplicity. Adding also single individuals would not change our conclusions. In a subsequent work, we plan to make marriage endogeneous in the same way as wife's labor participation is made here endogeneous.

[9] The papers of Mare (1991), Pencavel (1998) and Qian (1998) find strong evidence of positive assortative mating with respect to education. Education can be regarded as a good proxy for income.

[10] We could have assumed that k depends on the couple's productivity, w. We leave it for future work.

This is at odds with a number of alternative household models, ranging from bargaining to non-cooperative models.[11]

In the case where the couple comprises only one breadwinner, the lifetime utility is simply

$$U^{c1}\left(c, l_m, d\right) = 2c - v\left(l_m\right) + \left(\pi_f + \pi_m\right) u\left(d\right) \tag{2}$$

The utility function of a one-breadwinner couple differs from that of a two-breadwinner in that the wife does not incur disutility of work and the couple does not support the forgone value of housework k. Note that, if one member of the couple does not work, it is always the woman as for the same productivity, she obtains a lower wage than her husband.

Let's now turn to the definition of the budget constraint. This will depend on whether couples comprise one or two earners. Let first consider the intertemporal budget constraint of a two-breadwinner couple. In this case, both members of the couple work, contribute to the pension system, consume and save in the first period. In the second period, they retire and receive a full pension benefit. Their budget constraint has thus the following form,

$$\left(\omega_m l_m + \omega_f l_f\right)\left(1 - \tau - \theta\right) + \pi_m p_m + \pi_f p_f \geq 2c + \left(\pi_f + \pi_m\right) d \tag{3}$$

where, on the left-hand side, total resources comprise both members' net income and their pension benefits, $p_i \forall i = m, f$ which they receive over a length π_i. As we shall explain in more details below, the tax rates τ and θ serve to finance, respectively, the pension benefits, $p_i \;\; \forall i = m, f$ and the derived rights, g. In our setting, even though a two-breadwinner couple does not receive derived rights (i.e. $g = 0$ in the above constraint), he contributes to it. On the right-hand side, total spending are made of first- and second-period consumptions for both members of the couple.

Let now turn to the budget constraint of a one-breadwinner couple. It is slightly different from the two-breadwinner case and such that,

$$\omega_m l_m \left(1 - \tau - \theta\right) + \pi_f g + \pi_m p_m \geq 2c + \left(\pi_f + \pi_m\right) d \tag{4}$$

Only the man in the couple pays taxes (for an amount $\omega_m l_m \left(\tau + \theta\right)$) and thus receives a pension benefit, p_m over a length π_m. Nevertheless, the non-working wife, even though she did not contribute to the pension system, receives a lump sum g over the length of her second period of life, π_f. This lump sum represents the "derived rights": a married woman will receive it,

[11] This would complicate the model and we believe that this would not change our main results.

thanks to her marital situation. This supplementary pension benefit is designed as a way to avoid poverty in these couples in which only one member of the couple was working and as a way to avoid poverty of widows (since they receive it over a period $\pi_f > \pi_m$).

Let now define in details the pension system. If agents work, they contribute to a general pension system for an amount $\tau\omega_i l_i$. In exchange, they receive a full pension benefit over the length of the second period of life, which depends on their gender and is such that,

$$\pi_m p_m = \gamma\tau\omega_m l_m + \pi_m b \quad (5)$$

$$\pi_f p_f = \gamma\tau\omega_f l_f + \pi_f b \quad (6)$$

Here, the benefits are partially contributive and the parameter $\gamma > 0$ gives the degree of contributiveness. The parameter b is a lump-sum pension benefit which does not depend on individuals' previous contributions. Hence, if $\gamma = 0$, the system is "purely Beveridgian" and individuals receive the same lump sum independently of their previous contributions. The only difference is that women receive more as they live longer ($\beta > 1$). On the contrary, if $b = 0$ or alternatively $\gamma = 1$, the system is "purely Bismarckian".[12] In the case where $b = 0$, $\pi_m p_m > \pi_f p_f$ as men gross earnings, $\omega_m l_m$ are higher than women earnings, $\omega_f l_f$. As a consequence p_m is definitively higher that the corresponding p_f. It is important to observe that our pure Bismarckian system provides benefits that are longevity dependent, which is not the case of actual Bismarckian systems but can be found in some notional accounts pensions.

In addition to the general pension system, non-working spouses receive a supplementary benefit which is financed through additional taxation of *all* individuals in the society but redistributed only to these women. Hence, all workers pay $\theta\omega_i l_i$ while working which is redistributed to all non-working spouses, through a lump sum g over the length of their second period of life, π_f. While the flat benefit b is distributed to all retired workers, the derived benefit g concerns only non-working retirees.

[12] To see this, we combine (5) and (6) with the revenue constraint made explicit below and we obtain:

$$(1-\gamma)\,\tau E\left[\omega_m l_m + \omega_f l_f\right] = \left(\pi_m + \pi_f\right) b$$

so that $\gamma = 1$ leads to $b = 0$.

3 Individuals decisions

3.1 Labour decisions

In this section, we characterize the individuals' labour decisions taking the parameters of the pension system (τ, θ, b, g) as given. If the couple comprises two earners, it solves the following problem

$$\max_{c,\, d,\, l_f,\, l_m} U^{c2}(c, d, l_f, l_m) = 2c - l_f^2/2 - l_m^2/2 + (\pi_m + \pi_f)\, u(d) - k \qquad (A)$$
$$\text{s.t. } (\omega_m l_m + \omega_f l_f)(1 - \tau - \theta) + \pi_m p_m + \pi_f p_f \geq 2c + (\pi_f + \pi_m)\, d$$

Replacing for the expressions of the pension benefits (5) and (6), problem A is equivalent to solving

$$\max_{c,\, d,\, l_m,\, l_f} (\omega_m l_m + \omega_f l_f)(1 - \tau - \theta) + \gamma\tau\omega_m l_m + \pi_m b + \gamma\tau\omega_f l_f + \pi_f b$$
$$-l_f^2/2 - l_m^2/2 + (\pi_f + \pi_m)\left[u(d) - d\right] - k$$

From the first order conditions, we obtain

$$\begin{aligned} u'(d^*) &= 1 \\ l_m^* &= w\left(1 - (1-\gamma)\tau - \theta\right) \\ l_f^* &= \alpha w\left(1 - (1-\gamma)\tau - \theta\right) \end{aligned}$$

Hence the labour supply of individuals is distorted here for two reasons. First, individuals face a net tax rate $(1-\gamma)\tau$ to finance the pension system. We call it "net" so as to emphasize the fact that, in the second period, individuals get back a fraction of their earlier contributions, as the system is partly contributive. Second, individuals face an additional tax rate θ which aims at financing the derived rights. Hence, if $\theta = 0$ and if the system were purely contributive, that is $\gamma = 1$, the agent would not face any labour distortion. On the contrary, if $\gamma = 0$, the distortion is maximum. For further use, we define the effective tax rate, t_e as

$$t_e = (1-\gamma)\tau + \theta \qquad (7)$$

To the extent that we take b as given and that the Bismarckian pension is identical to private saving, what really matters is $(1-\gamma)\tau$ and not the values of γ and τ.

From this, the indirect utility function of a two-breadwinner couple with productivity w is written as

$$V^{c2}(w, t_e, b) = \frac{\left(1+\alpha^2\right) w^2 (1-t_e)^2}{2} + \nabla + (1+\beta)\, b - k \tag{8}$$

where $\nabla = (\pi_f + \pi_m)(u(d^*) - d^*)$ is constant given the quasi-linear utility assumption.

As to the one-breadwinner couple, he solves the following problem:

$$\begin{aligned} &\max_{c,\ d,\ l_m} 2c - l_m^2/2 + (\pi_f + \pi_m)\, u(d) \\ &\text{s.t. } \omega_m l_m (1-\tau-\theta) + \pi_f g + \pi_m p_m \geq 2c + (\pi_f + \pi_m)\, d \end{aligned} \tag{B}$$

In this case, only the working spouse contributes to the pension system and receives a pension when retired but in supplement the non-working spouse receives a lump-sum subsidy g during a retirement of length π_f. Again, from the FOCs,

$$\begin{aligned} u'(d^*) &= 1 \\ l_m^* &= w(1-(1-\gamma)\tau-\theta) = w(1-t_e) \end{aligned}$$

and we get the indirect utility function of a one-breadwinner couple, with productivity w:

$$V^{c1}(w, t_e, b, g) = \frac{w^2 (1-t_e)^2}{2} + b + \beta g + \nabla \tag{9}$$

3.2 Labour participation within the couple

We now turn to the choice of labor participation within the couple. The decision of whether one or both members of the couple should be working is likely to depend on the value of housework, k, on the disutility of labour supply and on the level of the wage αw which the couple has to forgo if the wife is not working. It also depends on the features of the pension system, that is on the ratio of contributions to payments and on the existence of derived rights, so that the return obtained from the pension system is likely to be different whether one or both spouses work.

To do so, we determine the productivity threshold $\hat{w}$ that separates one-breadwinner and two-breadwinner couples. This threshold is likely to be modified by the introduction of the pension system and of derived pension rights. It is such that, at this level, the couples are indifferent between both working and incurring the cost k, or having a non-working spouse. Hence, the threshold level $\hat{w}$ is obtained by solving the equality $V^{c1}(\hat{w}, t_e, b, g) = V^{c2}(\hat{w}, t_e, b)$. Replacing for the expressions of the indirect utility functions, one obtains

$$\frac{\alpha^2 \hat{w}^2 (1-t_e)^2}{2} + \beta(b-g) - k = 0 \tag{10}$$

When the solution is interior, $\hat{w} \equiv w(t_e, b, g)$ is a function of the pension parameters and it is implicitly defined by the above equation. Hence, if the productivity of a couple is such that $w \geq \hat{w}$, both spouses work while if $w \leq \hat{w}$, only the husband works. However, it may be the case that the solution is never interior and that for any $w \in [\underline{w}, \bar{w}]$ (even for the couple with the smallest productivity, $\underline{w}$) $V^{c1}(w, t_e, b, g) < V^{c2}(w, t_e, b)$ so that every couples in this society comprise two breadwinners and $\hat{w} \rightarrow \underline{w}$. On the other hand, it may also be the case, that for any $w \in [\underline{w}, \bar{w}]$ (even for the couple with the highest productivity, $\bar{w}$), $V^{c1}(w, t_e, b, g) > V^{c2}(w, t_e, b)$ so that every couples are one-breadwinner couples and $\hat{w} \rightarrow \bar{w}$; this latter case may arise for instance, if the cost of house work, k is high.

3.3 The government budget constraint

A feasible pension scheme must satisfy the following government budget constraint:

$$
\begin{aligned}
(\tau + \theta)\left(\int_{\underline{w}}^{\bar{w}} \omega_m l_m f(w)\, dw + \int_{\hat{w}}^{\bar{w}} \omega_f l_f f(w)\, dw\right) \quad \geq \quad & \int_{\underline{w}}^{\bar{w}} (\gamma \tau \omega_m l_m + b) f(w)\, dw \\
& + \int_{\hat{w}}^{\bar{w}} (\gamma \tau \omega_f l_f + \beta b) f(w)\, dw + \beta \int_{\underline{w}}^{\hat{w}} g f(w)\, dw
\end{aligned}
$$

On the left-hand side, a mass one of men contribute to the pension system, while only women with productivity between $[\hat{w}, \bar{w}]$ work and thus contribute. This implies that on the right-hand side, every man receives the full pension benefit while only women with productivity between $[\hat{w}, \bar{w}]$ receive it. Non-working women, i.e. those with productivity $[\underline{w}, \hat{w}]$ receive derived rights, g. Recalling that the effective tax rate, t_e is defined by (7), $l_m = w(1 - t_e)$ and $l_f = \alpha w(1 - t_e)$, the above inequality can be rewritten as

$$
(1 - t_e)\, t_e \left(\int_{\underline{w}}^{\bar{w}} w^2 f(w)\, dw + \alpha^2 \int_{\hat{w}}^{\bar{w}} w^2 f(w)\, dw\right) \geq b + \beta \left(b \int_{\hat{w}}^{\bar{w}} f(w)\, dw + g \int_{\underline{w}}^{\hat{w}} f(w)\, dw\right) \tag{11}
$$

In equilibrium, the above equation holds with equality. Contrary to many models dealing with pension design, the tax base (i.e. the expression inside parenthesis on the left-hand side) is not fixed as the labour participation decision and thus the threshold $\hat{w}$, depend on the features of the pension system. Note also that this threshold also determines how much derived rights are going to be distributed in the second period to non-working women. These are crucial points of our model.

3.4 The Laissez Faire

In order to understand how the introduction of a pension system and of derived rights modifies the equilibrium, let us first assume there is no government intervention and thus, no pension system. In this case, individuals simply choose how much labour to supply and whether one or both members of the couple should be working.

Substituting for $\tau = \theta = p_m = p_f = 0$, into (8) and (9), we obtain couples' indirect utility functions,

$$\begin{aligned} V^{1c}(w) &= \frac{w^2}{2} + (\pi_f + \pi_m)\left[u(d^*) - d^*\right] = \frac{w^2}{2} + \nabla \\ V^{2c}(w) &= \frac{\left(1+\alpha^2\right)w^2}{2} + \nabla - k \end{aligned}$$

where optimal labour supplies are not distorted and equal to individuals' wage,

$$l_m^* = w \text{ and } l_f^* = \alpha w. \tag{12}$$

From this, we turn to the choice of labor participation within the couple. This decision of whether one or both members of the couple should be working now depends only on the value of housework, k, on the disutility of labour supply and on the level of the wage αw which the couple has to forgo if the wife is not working. Both wife and husband decide to work if their joint utility is such that $V^{2c}(w) \geq V^{1c}(w)$ that is if

$$w \geq \hat{w} = \frac{(2k)^{1/2}}{\alpha} \tag{13}$$

where w is the same for both members of a two-breadwinner couple as we assumed assortative mating. On the contrary, if the productivity of a couple is such that $w < \hat{w}$, only the husband works. There is indifference between being one- or two-breadwinner couples if the utility cost of working for the wife, k, is just equal to the utility gain of such a move, namely $(\alpha w)^2/2$. This threshold $\hat{w}$ is increasing in k and decreasing in the gender wage gap, α. Indeed, if the reservation wage k is very high, the productivity level w must be very high for the two members of a couple to be working. Similarly, if α is very low, the potential gain for a woman working is very low so that her productivity must be very high for her to accept working.

4 Optimal solution: first- and second-best

Even though our approach is mainly positive, it is worth looking at the optimal solution. To do so, we assume that the social welfare function takes a general utilitarian form. We will consider successively the first-best solution and then the second-best solution, in which we use the fiscal instruments available to the social planner, $(t_e, b, g, \hat{w})$. We compare them with the market equilibrium solution and this will give interesting benchmark for interpreting the voting equilibrium.

We first study the first best assuming that the social welfare function is the sum of a concave transformation $\Psi(\cdot)$ of individuals' utility:[13]

$$\max \int_{\underline{w}}^{\hat{w}} \Psi\left(U^{c1}(c, l_m, d)\right) f(w)\, dw + \int_{\hat{w}}^{\bar{w}} \Psi\left(U^{c2}(c, l_f, l_m, d)\right) f(w)\, dw.$$

with $\Psi' > 0, \Psi'' < 0$. From the first-order conditions, we obtain both (12) and (13) so that the laissez-faire conditions on the segmentation between one-breadwinner and two-breadwinner couples and on labour supplies are efficient. Moreover, we obtain that

$$U^{c1}(c, l_m, d) = U^{c2}(c, l_f, l_m, d)\, \forall w$$

which is a direct consequence of the quasi-linearity assumption. In this case, we obtain unitary marginal utilities of income for all individuals. Hence, the optimum implies equal utilities for all couples, independently of their productivity and of whether they comprise one or two breadwinners.

This optimum can be decentralized with individualized lump-sum taxes and transfers, from high-productivity toward low-productivity couples and from two-breadwinner toward one-breadwinner couples.

This solution however cannot be achieved with our instruments so that we now study the second-best framework, in which we assume that fiscal instruments are the same as the ones we consider in the voting process.

Under the constrained optimum, the optimal levels of $(t_e, b, g, \hat{w})$ are a solution to the following problem:

$$\max_{t_e, b, g, \hat{w}} \int_{\underline{w}}^{\hat{w}} \Psi\left(\frac{w^2(1-t_e)^2}{2} + b + \beta g + \nabla\right) f(w)\, dw \qquad (C)$$
$$+ \int_{\hat{w}}^{\bar{w}} \Psi\left(\frac{(1+\alpha^2)\, w^2 (1-t_e)^2}{2} + \nabla + (1+\beta)\, b - k\right) f(w)\, dw$$
$$\text{s. to} \int_{\underline{w}}^{\hat{w}} \left(t_e(1-t_e)\, w^2 - b - \beta g\right) f(w)\, dw + \int_{\hat{w}}^{\bar{w}} \left(t_e(1-t_e)\, \alpha^2 w^2 - (1+\beta)\, b\right) f(w)\, dw \geq 0$$

[13] Without this transformation, there is no need for redistribution, with quasi-linear utility functions.

Rearranging first-order conditions with respect to b and g, we obtain

$$\int_{\hat{w}}^{\bar{w}} \left(\Psi'\left(V^{c2}\right) - \lambda\right) f(w)\, dw = 0 \tag{14}$$

$$\int_{\underline{w}}^{\hat{w}} \left(\Psi'\left(V^{c1}\right) - \lambda\right) f(w)\, dw = 0 \tag{15}$$

From these formula, we see that g and b are there to equalize marginal utilities of one- and two-breadwinner couples. In appendix, we also show that the condition for t_e is such that

$$\frac{t_e}{1-t_e} = \frac{-\left[cov\left[\Psi'\left(V^{ci}\right), w^2\right] + \alpha^2\, cov\left[\Psi'\left(V^{c2}\right), w^2\right]\Big|_{\hat{w}}^{\bar{w}}\right]}{\lambda\left[Ew^2 + \alpha^2\, Ew^2\big|_{\hat{w}}^{\bar{w}}\right]} \tag{16}$$

where λ is the Lagrange multiplier associated to the resource constraint. Note that the first covariance in the numerator is defined over the all interval; it is the covariance between marginal utility of income and men's gross earnings (independently of whether they belong to a one- or a two-breadwinner couple). The second covariance is defined only over the interval $[\hat{w}, \bar{w}]$ and is related to working women. The same distinction is made in the denominator for the average square productivity. If there was no working women the second terms in the numerator and the denominator would be zero and we would have the usual formula for linear income taxation.

The denominator of (16) is the standard efficiency term as taxation creates distortions on the labour supply. This efficiency term depends on the cost of public funds, λ and on the derivative of labour income with respect to the tax. With a quadratic labour disutility and a quasi linear utility function, the derivative of the labour income with respect to t_e is $-2(1-t_e)\,w^2$ for men and $-2(1-t_e)\,\alpha^2 w^2$ for women. On the contrary, the numerator is the standard equity term and is positive since $cov\left[\Psi'\left(V^{ci}\right), w^2\right]$ and $cov\left[\Psi'\left(V^{c2}\right), w^2\right]$ are negative, as the level of earnings and the marginal utility are negatively correlated. If $\Psi(.)$ was linear, there would be no redistributive objective and this term would cancel out.

5 The majority voting equilibrium

We now turn to the study of the voting equilibrium. In order to understand better the case in which the productivity distribution is continuous, we will first start by solving a (more simple) model in which the distribution of productivity is discrete. Only in the second part of this section, we assume a continuous distribution of productivity.

In any case, we will assume the following timing. At time $t = 0$, the level of the flat pension benefit b is fixed at the constitutional level so as to satisfy a Rawlsian objective. This also implies that implicitly, the effective tax rate t_e will always be different from zero (except if both $b = g = 0$). At time $t = 1$, couples choose simultaneously their labour participation (whether to be one- or two-breadwinner couples) and they vote over the level of derived rights g. As usual in this type of problem, we proceed backward: couples first vote on the level of g for a given level of b and decide to be one- or two-breadwinner and then, we determine the level b which maximizes the social welfare function. This gives us the equilibrium outcome (t_e^*, g^*, b^*).[14]

5.1 Discrete distribution of productivity

5.1.1 Analytical solution

Let first assume 3 categories of couples, with productivity, $w_1 < w_2 < w_3$. We assume that they are in proportions, p_1, p_2 and p_3 such that $\Sigma_{i=1}^3 p_i = 1$ and that $w_2 \leq E(w)$ (distributions are either symmetric or right-skewed). For the moment, we assign no specific values to p_i and w_i and solve the general case. Only in the numerical example below, we assume different distributions. We also set that the couple with productivity w_1 is always a one-breadwinner couple while the couple with productivity w_3 is always a two-breadwinner couple. As we have only three types, the median-type-decisive voter is always the individual with productivity w_2 so that the voting outcome corresponds to his preferred policy platform, which of course, depends on whether he is a one- or a two-breadwinner couple.

In order to solve the political equilibrium, we proceed in the following way. We first assume that the median agent is a one-breadwinner couple and further that he is a two-breadwinner. In each case, we derive his utility level under his preferred policy (since this is the equilibrium outcome) for a given level of b; the solution corresponds to the case in which he obtains the highest utility. Using a numerical example, we find the optimal level for the flat benefit, which maximizes the Rawlsian objective.

Let first assume that type-2 agent is a one-breadwinner couple. In this case, the budget constraint of the government is such that[15]

$$t_e (1 - t_e) \left[Ew^2 + p_3 \alpha^2 w_3^2\right] \geq (1 + p_3 \beta) b + \beta (p_1 + p_2) g$$

[14] The way we proceed may not be the unique one. For instance, we could have as well assumed that the level of derived rights, g are decided at the constitutional level while the flat benefit b is decided by majority voting. We could also have assumed sequential voting. This is left for future work.

[15] This budget constraint is equivalent to (11) except that, because we consider a discrete distribution of productivity, the threshold $\hat{w}$ disappears. This is implicit in assuming that w_2 is either one- or two-breadwinner couple. This simplifies a lot our computations and this allows us to obtain clear analytical results.

where on the left-hand side of this equation, we have total contributions paid: all men contribute to the system, for an amount $t_e(1-t_e)Ew^2$ while only women from the highest productivity group, contribute for an amount $t_e(1-t_e)\alpha^2 w_3^2$. On the right-hand side, we have total benefits distributed: all men are working so that they receive b and only women with a high productivity work so that they receive a pension benefit b for a length β; non-working women, those who belong to couples with productivity w_1 and w_2, receive derived rights, g over a length β. Rearranging the above condition, we obtain the equation for the level of the derived pension benefit, as a function of t_e and b:

$$g(t_e,b)=\frac{t_e(1-t_e)\left[Ew^2+p_3\alpha^2 w_3^2\right]-(1+p_3\beta)b}{\beta(p_1+p_2)}$$

Using this equation, we find the preferred tax rate of the median voter (i.e. a one-breadwinner couple, with productivity w_2). His indirect utility function is

$$V^{c1}(w_2,t_e,b,g)=\frac{w_2^2(1-t_e)^2}{2}+b+\beta g(t_e,b)+\nabla$$

Note that we assume that the median agent vote on the level of t_e (and thus on the level of derived rights) without considering that the choice of a specific pension policy might change the partition of the society between one-breadwinner and two-breadwinner couples. In other words, he does not see the impact that the pension policy may have on type-1 and type-3 agents' labour participation decision (who may now prefer to be two- breadwinner couples or the reverse). He takes this partition as given. Hence, his preferred tax rate is obtained from solving

$$\max_{t_e} V^{c1}(w_2,t_e,b,g)=\frac{w_2^2(1-t_e)^2}{2}+b+\frac{t_e(1-t_e)\left[Ew^2+p_3\alpha^2 w_3^2\right]-(1+p_3\beta)b}{(p_1+p_2)}+\nabla$$

which yields[16]

$$t_e^{c1}=\frac{\left[Ew^2+p_3\alpha^2 w_3^2\right]-w_2^2(p_1+p_2)}{2\left[Ew^2+p_3\alpha^2 w_3^2\right]-w_2^2(p_1+p_2)}$$

In case the median voter is a one-breadwinner couple, he will always vote for a tax rate greater than the minimum one required to finance the flat benefit and thus he always prefer $g^*>0$. Hence, for a given b, the voting equilibrium is characterized by

$$(t_e^*,g^*)=\left(\frac{\left[Ew^2+p_3\alpha^2 w_3^2\right]-w_2^2(p_1+p_2)}{2\left[Ew^2+p_3\alpha^2 w_3^2\right]-w_2^2(p_1+p_2)},\frac{t_e^*(1-t_e^*)\left[Ew^2+p_3\alpha^2 w_3^2\right]-(1+p_3\beta)b}{\beta(p_1+p_2)}\right) \tag{17}$$

[16] A corner solution is never possible. Using the Jensen inequality, it always the case that $w_2\leq E(w)<\sqrt{Ew^2}<\sqrt{\frac{Ew^2+p_3\alpha^2w_3^2}{p_1+p_2}}$, which makes impossible to have $\partial V^{c1}(w_2,t_e,b,g)/\partial t_e<0$.

Note that the equilibrium tax rate is independent of the level of b. The explanation is straightforward: since the flat benefit is fixed, an increase in the effective tax rate directly increases the level of derived rights; individuals then choose the best trade-off between taxation which create labour distortions and the level of derived rights they can obtain in return. We also find that the level of derived rights, g^* is always decreasing in b: for a given tax rate, an increase in the flat rate benefit given to all working agents has to be compensated by a decrease in the level of the benefit toward non-working spouses, in order to keep the budget balanced.

Let now turn to the constitutional stage, in which b is fixed. If the poorest individual has a zero productivity, he is a one-breadwinner couple and his income is simply $(b+\beta g^*)$. Replacing for the value of g^*, it is straightforward to show that this income is always decreasing in b. Hence, at the constitutional level, it is optimal to set the flat rate pension benefit to zero: $b^* = 0$.[17]

Let now assume that the couple with median productivity w_2 is a two-breadwinner couple. In this case, his preferred tax rate is likely to be different as well as the government budget constraint. First, as he is a two-breadwinner couple, he would contribute to the derived rights system, without receiving, so that he always prefers a zero level of derived rights. Since he is the decisive voter, the voting equilibrium is characterized by $g^* = 0$ and the equilibrium tax rate is only determined by the government budget constraint.

This constraint is now modified; the structure of the society is different as couples with productivity w_2 now comprise two earners and their desired level of derived rights is zero. The budget constraint takes the following form:

$$t_e\left(1-t_e\right)\left[Ew^2+\alpha^2\left(p_2 w_2^2+p_3 w_3^2\right)\right] \geq \left(1+\left(p_2+p_3\right)\beta\right) b$$

where on the left-hand side, total contributions are made by a mass one of working-men and women with productivity w_2 and w_3. On the right-hand side, pension benefits are provided to these agents who contributed in the previous period and no derived rights are given to non-working spouses ($g^* = 0$). Hence, the tax rate, $t_e^{c2}(b)$ that satisfies the government budget constraint, for a given b when the median couple is a two-breadwinner couple is implicitly determined by

$$b = t_e^{c2}(b)\left(1-t_e^{c2}(b)\right)\frac{Ew^2+\alpha^2\left(p_2 w_2^2+p_3 w_3^2\right)}{1+\left(p_2+p_3\right)\beta}. \tag{18}$$

[17] We would have obtained the same result if we had assumed that the productivity of the poorest individual was non null, as t_e^* is independent of b.

We now find what should be the labour participation decision of the median voter (whether it is a one- or a two-breadwinner couple). This, of course, will induce a different political equilibrium outcome.[18]

For a given b, if the median voter is a one-breadwinner, the political outcome is characterized by (17) while if he is a two-breadwinner, the solution is given by (18) and $g^* = 0$. Let define the net utility obtained by the median voter from being a one-breadwinner and choosing his preferred policy platform as $\Delta = V^{c1}(w_2, t_e^*, b, g^*) - V^{c2}\left(w_2, t_e^{c2}(b), b\right)$, which can be rewritten as

$$\Delta = \frac{w_2^2 (1 - t_e^*)^2}{2} + \beta g^* - \left(\frac{\left(1 + \alpha^2\right) w_2^2 \left(1 - t_e^{c2}(b)\right)^2}{2} + \beta b - k \right) \tag{19}$$

where we used (8) and (9). If this difference is positive, the median voter chooses to be one breadwinner which implies that at the voting equilibrium, derived rights are always positive and the voting equilibrium is characterized by (17). On the contrary, if this difference is negative, the median voter is a two-breadwinner couple so that he always prefers a zero level of derived rights and the equilibrium is $(t_e^*, g^*) = \left(t_e^{c2}(b), 0\right)$.

Let finally make some comparative static analysis, and study in particular the factors that influence the labour participation decision of the median voter. To do so, we consider the differences in utility obtained by the median type from being one-breadwinner or two-breadwinner using expression (19). First, note that independently from the case considered, the higher is the cost of housework, the higher is the net utility, so that the more likely the median type is a one-breadwinner couple. Indeed, unless the wage of the second earner (here the woman with wage αw) is very high, a couple may prefer to be one-breadwinner and avoid a high cost k (and eventually get some derived rights).

Let now see how these differences vary with the level of the flat benefit. The effect is ambiguous. On the one hand, the tax rate is increasing in b, since we should be on the increasing part of the Laffer curve: $t_e^{c2}(b) > 0$ (recall that t_e^* and g^* are independent of b), which increases labour distortions. On the other hand, an increase in b makes more likely the median voter to be a two-breadwinner (as he would get more resources from being a two-breadwinner couple). In the following, we find the overall effect of an increase in b on the labour participation decision.

[18] The single-crossing condition defined by Gans and Smart (1996) is effectively satisfied in our framework, even though we have two subgroups in the popuplation. The marginal rate of substitution between t_e and g is monotonically decreasing in w for one-breadwinner couples and two-breadwinner couples always prefer zero derived rights. This guarantees that a political equilibrium exists under pure majority rule and that the Condorcet winner is the preferred tax rate of the median productivity individual.

In the next part, we assume specific productivity distributions and find what should be the level of b decided at the constitutional stage.

5.1.2 Numerical illustration

First, we assume that $\alpha = 0.8$ and $\beta = 1.2$. In France, it is estimated that women life expectancy at 60 is 20% higher than that of men and that the pay gap is around 20%.[19] In the following, we study the political outcome under three types of distributions:

1. A centered distribution, with $w_1 = 0, w_2 = 1, w_3 = 2$ with proportions $p_1 = p_2 = p_3 = 1/3$. In this case, $E(w) = w_m = 1$ and $Ew^2 = 5/3$.

2. A right-skewed distribution with $w_1 = 0, w_2 = 3/4, w_3 = 2$ and proportions $p_1 = 2/9, p_2 = 4/9, p_3 = 3/9$. In this case, $w_m = 3/4 < E(w) = 1$ but $E(w)$ is held constant with respect to the previous example. We also have that $Ew^2 = 19/12$.

3. A centered distribution, with equalizing transfers which are mean preserving.[20] We assume that $w_1 = 1/2, w_2 = 1, w_3 = 3/2$ with proportions $p_1 = p_2 = p_3 = 1/3$ so that $E(w) = w_m = 1$. In this case, $Ew^2 = 7/6$.

Before going into the details of the simulations, we first have to define an interval for k, which satisfies our initial assumptions that type-1 agents are one-breadwinner and type-2 agents are two-breadwinner couples. For distributions 1 and 2, one needs to have, in the laissez-faire, that

$$\begin{aligned} V^{c1}(w_1) &\geq V^{c2}(w_1) \Leftrightarrow k \geq 0 \\ V^{c2}(w_3) &\geq V^{c1}(w_3) \Leftrightarrow k \leq 1.28 \end{aligned}$$

By the same procedure, we find that in distribution 3, $k \in [0.08, 0.72]$. Also, as a benchmark case, we find that, in the laissez-faire, the median voter would be indifferent between being one- or two-breadwinner if $k = 0.32, 0.18$ and 0.32 for the three distributions respectively. Hence for any smaller (resp. higher) level of k, the median voter is a two-breadwinner (resp. one-breadwinner) couple, in the laissez-faire. Note that these conditions are equivalent to the continuous case condition (13).

[19] Figures are taken from the French National Institute of Statistics, INSEE (See www.insee.fr).
[20] On this type of distribution, see Atkinson (1983).

b	0	0.1	0.125	0.173	0.2	0.3	0.40
$k=0$							
Δ	0.269	0.051	0		-0.155	-0.337	$\rightarrow$-0.425
t_e^*	0.42	0.42	0.42		-	-	-
$t_e^{c2}(b)$	-	-	0.089		0.16	0.27	$\rightarrow$0.5
g^*	0.77	0.59	0.55		0	0	0
$\beta g^* + b$ / $(1+\beta)\,b$	0.924 / 0	0.808 / 0.22	0.785 / 0.275		0.2 / 0.44	0.3 / 0.66	$\rightarrow$0.4 / 0.88
$k=0.1$							
Δ	0.369	0.151		0	-0.055	-0.237	$\rightarrow$-0.325
t_e^*	0.42	0.42		0.42	-	-	-
$t_e^{c2}(b)$	-			0.13	0.16	0.27	$\rightarrow$0.5
g^*	0.77	0.59		0.47	0	0	0
$\beta g^* + b$ / $(1+\beta)\,b$	0.924 / 0	0.808 / 0.22		0.737 / 0.38	0.2 / 0.44	0.3 / 0.66	$\rightarrow$0.4 / 0.88

Table 1: Political equilibrium outcome under distribution 1.

Results for distribution 1 are provided in Table 1.[21]

Since, in the above example, we set $k < 0.32$ in the laissez-faire the median agent would be a two breadwinner. The introduction of a pension system and of derived rights (whose level are determined by voting) suffices to change the labour participation decision of the median voter. Indeed, we find that, for $b \in [0, 0.125]$ when $k = 0$ (or $b \in [0, 0.173]$ when $k = 0.1$), $\Delta \geq 0$ so that the median voter now prefers to be one-breadwinner and to benefit from derived rights. Only for $b > 0.125$ (or 0.173), the median voter prefers to comprise two breadwinners. In this case, the net gain from being a two-breadwinner couple (i.e. an additional full pension benefit net of taxes on the second earner and of housework cost) exceeds the net benefit to be one breadwinner (i.e. the derived rights). We also insert a row, $\beta g^* + b$, which corresponds to the income earned by the poorest agent.[22] The level of b chosen by the Rawlsian planner will be such that $\beta g^* + b$ is maximum. From this table, it is clear that at the constitutional level, b will be set to zero and equilibrium values are taken from the first column, $(t^*, b, g^*) = (0.42, 0, 0.77)$, which are independent of the level of k.

We also checked that this result is robust by introducing a last row with the values of $(1+\beta)\,b$, which represents the amount that a low-productivity individual would obtain from the pension system if he belonged to a two-breadwinner couple. Indeed for $b < 0.125$ (resp. 0.173), the poorest individual has always interest in being a one breadwinner as $(1+\beta)\,b - k < b + \beta g$. On

[21] Note that the level of b is constrained by the fact that we remain on the good side of the Laffer curve.

[22] The poorest agent is a one-breadwinner couple, with $w_1 = 0$ so that his utility is simply $V^{c1}(0, t_e^*, b, g^*) = b + \beta g^* + \nabla$ where ∇ is a constant, set to 0 for simplicity.

Distribution 2	b	0	0.1	0.2	0.21	0.25	0.3	0.34
$k=0$	Δ	0.528	0.270	0.021	0		-0.203	$\rightarrow$-0.245
	t_e^*	0.46	0.46	0.46	0.46		-	-
	$t_e^{c2}(b)$	-	-	-	0.19		0.336	$\rightarrow 0.5$
	g^*	0.76	0.58	0.41	0.39		0	0
	βg^*+b	0.907	0.797	0.687	0.678		0.3	0.34
$k=0.1$	Δ	0.628	0.370	0.121		0	-0.102	$\rightarrow -0.146$
	t_e^*	0.46	0.46	0.46		0.46	-	-
	$t_e^{c2}(b)$	-	-	-		0.25	0.336	$\rightarrow 0.5$
	g^*	0.76	0.58	0.41		0.32	0	
	βg^*+b	0.907	0.797	0.687		0.634	0.3	0.34

Table 2: Political equilibrium outcome under distribution 2.

the contrary, for $b>0.125$ (resp. 0.173), a type-1 couple would obtain a higher utility if both members were working (and thus receiving a double pension) than if only the man were working (and receiving one full pension plus derived rights), as $(1+\beta)\,b-k>b+\beta g$. Yet, the utility from being a two breadwinner in that case is still always lower than the one of being one breadwinner at the political equilibrium $(t^*,b,g^*)=(0.42,0,0.77)$. Hence the political equilibrium outcome is robust.

We now consider a more realistic distribution, with $w_m<E(w)$ in Table 2.[23]

This does not modify substantially our results, except that the switch from one- to two-breadwinner couples now arises at a higher level of b. Hence under a more realistic productivity distribution, the political equilibrium is still characterized by positive levels of derived rights and a zero flat rate benefit: $(t^*,b,g^*)=(0.458,0,0.756)$.[24]

We finally study the equilibrium in the case of the last productivity distribution. Our results are reported in Table 3.

In this last case, the Rawlsian social planner will consider $V^{c1}(w_1,t^*,b,g^*)$ where t^* takes either the value t_e^* or $t_e^{c2}(b)$ rather than βg^*+b, as under distribution 3, the poorest individual has a productivity different from zero, so that the tax rate will also affect his utility, through labour distortions. Note also that, when $\Delta=0$, both political equilibrium are possible so that the first value in * gives the utility level $V^{c1}(w_1,t^*,b,g^*)$ in case the political outcome is $t^*=t_e^*$ and $g^*\neq 0$ while the second one gives the utility level in case the outcome is $t^*=t_e^{c2}(b)$ and $g^*=0$. From this table, we find that the political equilibrium should be such

[23] Note that the level of b is constrained by the fact that we remain on the good side of the Laffer curve.
[24] We checked that these results are robust.

b	0	0.032	0.05	0.095	0.1	0.2	0.26
$k=0.1$							
Δ	0.054	0	-0.029		-0.107	-0.231	$\rightarrow$-0.1874
t_e^*	0.373	0.373	-		-	-	-
$t_e^{c2}(b)$	-	0.032	0.051		0.108	0.2624	$\rightarrow 0.5$
g^*	0.481	0.425	0		0	0	0
$V^{c1}(w_1, t^*, b, g^*)$	0.63	*0.59 / 0.15	0.16		0.20	0.27	$\rightarrow$0.29
$k=0.2$							
Δ	0.154		0.070	0	-0.007	-0.131	$\rightarrow$-0.0874
t_e^*	0.373		0.373	0.373	-	-	-
$t_e^{c2}(b)$	-		-	0.102	0.108	0.262	$\rightarrow 0.5$
g^*	0.481		0.393	0.315	0	0	0
$V^{c1}(w_1, t^*, b, g^*)$	0.63		0.57	*0.52 / 0.20	0.20	0.27	$\rightarrow$0.29

Table 3: Political equilibrium outcome under distribution 3

that $(t_e^*, b, g^*) = (0.373, 0, 0.481)$ for any value of k, as with $b = 0$, the utility of the poorest individual is maximized.

In unreported simulations, we checked that this equilibrium is robust. For $b \leq 0.032$ (resp. ≤ 0.095), type-1 agents are always better-off being one- breadwinner than two-breadwinner, i.e. $V^{c2}(w_1, t_e^*(b), b) \leq V^{c1}(w_1, t_e^*, b, g^*)$. Moreover, we have checked that, for any value of k, for $b > 0.032$ (or $b > 0.095$), type-1 couples would obtain higher utility from being a two-breadwinner couple than being one-breadwinner, that is $V^{c2}\left(w_1, t_e^{c2}(b), b\right) > V^{c1}\left(w_1, t_e^{c2}(b), b, 0\right)$; however, in such a case, the utility of being two-breadwinner is always lower than at the political equilibrium $(t^*, b, g^*) = (0.373, 0, 0.481)$.

Finally, it should be mentioned that, for any distribution, the equilibrium outcome is invariant to the level of k. The only difference is that the higher is k, the higher is Δ and the more likely it is that the median voter is one breadwinner. This confirms our analytical results. These three tables also clearly show that, taking into account both direct and indirect (through the level of taxation) impacts of b on the differences in utility Δ, it is always decreasing in the flat benefit. Hence, the higher is b, the smaller is g and the more likely the median voter is a two-breadwinner couple.

5.2 Continuous productivity distribution

Let now turn to an alternative model, in which the distribution of productivity is continuous. The crucial difference with the previous model is that, we now have to make use of $\hat{w}$ as defined by equation (10) so as to find whether the median type, w_m is a one- or two-breadwinner couple, i.e. whether $w_m \lesseqgtr \hat{w}$ as it is clear from Section 3.2.[25] However, as equation (10) shows, the threshold productivity is endogenous and depends itself on the pension system instruments.

Hence, in order to simplify the model and to be able to get some analytical conclusions, we will make the assumption that w is uniformly distributed over $[0,1]$.[26] This simplifies a lot our model as $\hat{w}$ defined by (10) will also give the number of couples which comprise only one breadwinner. This assumption is standard in models of mobility and of occupational choice.

Under a uniform productivity distribution, the government budget constraint (11) has the following form:

$$t_e\left(1-t_e\right)\frac{1+\alpha^2\left(1-\hat{w}^3\right)}{3} \geq b\left(1+\beta\left(1-\hat{w}\right)\right)+\beta g\hat{w} \tag{20}$$

The timing of the model is the same as before, so that we proceed in the same way. First we determine the preferred tax rate of the median voter couple and his labour participation decision, for a given level of b. Given our assumption of a uniform distribution of productivity, the median voter will always be the individual with productivity, $w_m = 1/2$. Once knowing his preferred policy platform, which also gives the political outcome, we find the level of b which maximizes the utility of the poorest couple, that with $w = 0$.

5.2.1 Median voter preferred tax rate and labour participation decision

First, we determine the median voter's preference for the tax rate or equivalently his preference for a given level of derived rights, which depends on his labour participation decision.

We also assume that agents do not see the impact that the choice of a specific tax rate and of derived rights has on the partition between one- and two-breadwinner couples. In other words, they take $\hat{w}$ as given and they do not consider a general equilibrium model in which $\hat{w}$ effectively depends on pension parameters (as it is clear from equation 13).

Let first assume that the median voter is a two-breadwinner couple. His problem consists in maximizing his indirect utility (8) in which $w = 1/2$ subject to the government budget constraint (20). As before, this couple votes for $g = 0$. Hence, the effective tax rate is implicitly defined

[25] In the discrete type distribution, such a threshold did not appear as we were making assumptions on k so as to ensure that w_1- couple was a one breadwinner couple and w_3- couple was a two-breadwinner couple. Only for the median type couple, his labour decision was unclear, so that we were assuming successively that he was one- or two-breadwinner and solving the model accordingly.

[26] A more realistic assumption would be to consider a right-skewed distribution or a distribution defined on a different interval $[x,1]$ with $x > 0$. As we showed in the discrete-productivity distribution, this does not change substantially our results.

by (20) in which $g = 0$:

$$t_e(b)(1 - t_e(b)) \frac{1 + \alpha^2 (1 - \hat{w}^3)}{3} = b(1 + \beta(1 - \hat{w})) \tag{21}$$

Note that in such a case, we need to have $\hat{w} < 1/2$ as by assumption, the median voter is a two-breadwinner couple.

Let us now assume instead that the median voter is a one-breadwinner couple. In this case, his problem amounts to maximize his utility (9) in which $w = 1/2$ subject to the government budget constraint (20). His preferred policy platform will be[27]

$$(t_e, g) = \left(\frac{\frac{1+\alpha^2(1-\hat{w}^3)}{3\hat{w}} - \frac{1}{4}}{\frac{2}{3}\frac{1+\alpha^2(1-\hat{w}^3)}{\hat{w}} - \frac{1}{4}}, \frac{t_e(1-t_e)\frac{1+\alpha^2(1-\hat{w}^3)}{3} - b(1+\beta(1-\hat{w}))}{\beta\hat{w}} \right)$$

where g^* is obtained from solving (20) and one needs to have $\hat{w} > 1/2$. Note that, as it was already the case in the discrete distribution example, the above solution for t_e is independent of the level of b.

We now derive the labour participation decision of the median voter. We need here to resort to simulations in order to find what will be the labour participation decision of the median voter. To do so, let us distinguish two cases:

- **Case 1:** The median voter is a one-breadwinner couple with productivity such that $w_m = 1/2 < \hat{w}$.

In this case the political outcome is, for a given b,

$$(t_e^*, g^*) = \left(\frac{\frac{1+\alpha^2(1-\hat{w}^3)}{3\hat{w}} - \frac{1}{4}}{\frac{2}{3}\frac{1+\alpha^2(1-\hat{w}^3)}{\hat{w}} - \frac{1}{4}}, \frac{t_e^*(1-t_e^*)\frac{1+\alpha^2(1-\hat{w}^3)}{3} - b(1+\beta(1-\hat{w}))}{\beta\hat{w}} \right)$$

where $\hat{w}$ is the solution to (10) replacing for (t_e^*, g^*).

Let now turn to the constitutional stage, in which b is fixed. The poorest individual has a zero productivity so that he is a one-breadwinner couple and his income is simply $(b + \beta g^*)$. Replacing for the value of g^*, it is straightforward to show that this income is always decreasing in b, *if* the threshold $\hat{w}$ was assumed to be exogenous so that, in this case, it would be optimal to set the flat pension benefit to zero: $b^* = 0$.[28] Note however that in our simulations, we take

[27] Note that a corner solution is never possible. Using the Jensen inequality, it always the case that $E(w) = 1/2 < \sqrt{Ew^2} < \sqrt{\frac{1+\alpha^2(1-\hat{w}^3)}{3\hat{w}}}$, with $Ew^2 = 1/3$ which makes impossible to have $\partial V^{c1}(1/2, t_e, b, g)/\partial t_e < 0$.

[28] We would have obtained the same result if we had assumed that the productivity of the poorest individual was non null, as t_e^* is independent of b.

into account that $\hat{w}$ varies with b, so that typically, we will find that at the optimum, the tax system is not entirely Bismarckian. To see that let us differentiate the income of the poorest couple with respect to b. We obtain:

$$\frac{\partial(b+\beta g)}{\partial b}=1-\frac{((1+\beta(1-\hat{w}))}{\hat{w}}+\frac{d\hat{w}}{db}\left[\frac{\beta b}{\hat{w}}-\frac{g}{\hat{w}}-\hat{w}t_e^*(1-t_e^*)\,\alpha^2\right].$$

The first two terms of the RHS are negative implying a pure Bismarckian system as in the previous section. However the third term which cannot be signed without further restrictions could lead to a positive value of b.

In this case 1, the utility of the median one-breadwinner couple at the political equilibrium outcome is

$$V^{c1}\left(1/2,t_e^*,b,g^*\right)=\frac{(1/4)\left(1-t_e^*\right)^2}{2}+b+\beta g^*+\nabla$$

where t_e^*, g^* and $\hat{w}$ are obtained from solving a system of three equations-three unknowns.

- **Case 2:** The median voter is a two-breadwinner couple with productivity such that $\hat{w} < w_m = 1/2$.

In this case the political outcome is, for a given b, $(t_e^*, g^*) = (t_e(b), 0)$ with $t_e(b)$, the solution to (21) but where now $1/2 > \hat{w}$ and the threshold $\hat{w}$ is again defined by (10) where $g^* = 0$. In this case 2, the utility of the median two-breadwinner couple is

$$V^{c2}\left(1/2,t_e(b),b\right)=\frac{\left(1+\alpha^2\right)(1/4)\left(1-t_e(b)\right)^2}{2}+(1+\beta)\,b+\nabla-k$$

In order to find what is the labour participation decision of the median voter, we check, in the simulations, whether these cases could be possible (under our assumptions on $\hat{w}$) and we compare the utility levels he obtains under his preferred policy in each case. The political outcome corresponds to the policy platform that gives him the highest utility level.[29]

Before going into the details of the simulations, let us mention that as in the discrete type case, we keep $\alpha = 0.8$ and $\beta = 1.2$ and $\nabla = 0$. Under these assumptions, we find that, in the laissez faire, $\hat{w} = 1/2$ if the cost of housework k is equal to 0.08. In other words, for this value, the population would be equally segmented between one-breadwinner and two-breadwinner couples

[29] Since utility functions are the identical under a discrete and a continuum productivity distribution, we still obtain that the single-crossing condition defined by Gans and Smart (1996) is satisfied in our framework. Hence, a political equilibrium exists under pure majority rule and the Condorcet winner is the preferred tax rate of the median productivity individual.

if there was no governmental intervention. In the following, this value $k = 0.08$ is going to be used as a benchmark.

The following tables report the voting equilibrium outcomes assuming successively different values of k. In these tables, we report the preferred policy platform of the median voter (t_e^*, g^*) which depends on his labour participation decision (whether $1/2 \lesseqgtr \hat{w}$), for a given level of the flat benefit.[30] Given that the median individual is the decisive one, this is also the equilibrium outcome. The last row in each table reports the utility of the poorest individual, $V^{c1}(0, t_e^*, b, g^*)$. Since the Rawlsian objective consists in maximizing the welfare of this agent, the social planner will choose the level of b that gives the highest utility level to this individual.[31]

Let us first study the case in which $k = 0.08$ (Table 4). For this value at the laissez-faire, the society would be equally divided between one- and two-breadwinner couples. We find that the introduction of a pension system with derived rights has ambiguous effects, due to the modification of the partition of the society. For instance, for a low level of b, we find that there is now a majority of one-breadwinner couples, who would vote for a positive level of derived rights. In such a case, the higher is b, the higher is the tax rate but the lower will be the level of derived rights. Hence, when b increases, the number of two-breadwinner couples increases so that we find that, for intermediate levels of b, there will now be a majority of two-breadwinner couples who would prefer zero derived rights. Again, we find that increasing b increases the tax rate and thus labour distortions so that, for a high level of b, a majority of couples will again prefer to have a non-working spouse.[32] Finally, we obtain that the level of the flat benefit that maximizes the utility of the poorest individual is $b = 0.06$ so that in equilibrium, the policy platform chosen is $(t_e^*, b^*, g^*) = (0.393, 0.06, 0.039)$. Hence contrary to the discrete type distribution, it should not be always the case that the flat benefit is null, in order to maximize the utility of the worst-off.

It is also the case that compared to the laissez-faire situation, which is a first-best outcome, the political equilibrium outcome encourages more women to stay at home, since we find that

[30] In unreported simulations, we computed the solutions under the two cases for different values of b and k. It happened that for each value of b, only one solution was always possible (for instance, the other did not fall into the intervals).

[31] As in the discrete case, it may happen that the poorest-one-breadwinner couple, would have interest in being two-breadwinner after the introduction of pension system as for high values of b, one may have that $V^{c2}(0, t_e^*, b) = (1+\beta)b - k > V^{c1}(0, t_e^*, b, g^*) = b + \beta g^*$. However, it is always less than what this couple obtains at the Rawlsian solution, being a one-breadwinner couple.

[32] Our interval for b is limited by the pic of the Laffer curve.

b	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07
t_e^*	0.263	0.283	0.303	0.324	0.155	0.214	0.393	0.430
g^*	0.063	0.060	0.057	0.053	0	0	0.039	0.026
$\hat{w}$	0.94	0.92	0.89	0.86	0.37	0.32	0.68	0.51
Median Voter	1bw	1bw	1bw	1bw	2bw	2bw	1bw	1bw
$V^{c1}\left(1/2, t_e^*, b, g^*\right)$	0.143	0.146	0.149	0.151	-	-	0.152	0.142
$V^{c2}\left(1/2, t_e\left(b\right), b\right)$	-	-	-	-	0.154	0.157	-	-
$V^{c1}\left(0, t_e^*, b, g^*\right)$	0.0756	0.082	0.088	0.094	0.04	0.05	0.107	0.101

Table 4: Political equilibrium outcome for k=0.08.

b	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.075
t_e^*	0.246	0.266	0.286	0.307	0.328	0.351	0.375	0.406	0.48
g^*	0.057	0.054	0.051	0.047	0.0436	0.039	0.033	0.024	0
$\hat{w}$	0.96	0.94	0.917	0.89	0.85	0.80	0.74	0.63	0.34
Median Voter	1bw	1bw	1bw	1bw	1bw	1bw	1bw	1bw	2bw
$V^{c1}\left(1/2, t_e^*, b, g^*\right)$	0.140	0.142	0.145	0.147	0.149	0.149	0.148	0.143	
$V^{c2}\left(1/2, t_e\left(b\right), b\right)$	-	-	-	-	-	-	-	-	0.120
$V^{c1}\left(0, t_e^*, b, g^*\right)$	0.068	0.068	0.812	0.086	0.092	0.097	0.100	0.099	0.075

Table 5: Political equilibrium outcome for k=0.10.

$\hat{w} = 0.68$.

In the following two tables, we study cases in which $k > 0.08$, so that, under the laissez-faire, there is a majority of one-breadwinner couples.[33] We find that, for any value of b (except if it is very high and close to the pic of the Laffer curve), there should be a majority of one-breadwinner couples. Hence the introduction of a pension system with derived rights increases the number of one-breadwinner couples with respect to the laissez-faire. Yet, as soon as the level of the pension benefit increases, the tax rate increases and derived rights decrease, so that the advantage obtained from the pension system of being a one-breadwinner decreases. This is why, for a high level of b and a high level of the tax rate, we should observe a majority of two-breadwinner couples in the society, who would prefer zero derived rights. Again we find that the utility of the poorest individual is maximized at $b = 0.06$ so that the political equilibrium is characterized by $(t_e^*, b^*, g^*) = (0.375, 0.06, 0.033)$.

Assuming that $k = 0.12$, we find that the political outcome should now be such that $(t_e^*, b^*, g^*) = (0.358, 0.06, 0.028)$.

From these three tables, it is clear that the higher is the cost of housework, the smaller are the

[33] In the laissez-faire, $\hat{w} = 0.56$ for $k = 0.1$ and $\hat{w} = 0.61$ for $k = 0.12$.

b	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
t_e^*	0.228	0.248	0.269	0.289	0.311	0.334	0.358	0.385	0.445
g^*	0.052	0.049	0.046	0.042	0.038	0.034	0.028	0.02	0
$\hat{w}$	0.978	0.960	0.938	0.912	0.88	0.84	0.79	0.70	0.49
Median Voter	1bw	1bw	1bw	1bw	1bw	1bw	1bw	1bw	2bw
$V^{c1}\left(1/2, t_e^*, b, g^*\right)$	0.137	0.139	0.142	0.144	0.145	0.146	0.145	0.141	
$V^{c2}\left(1/2, t_e\left(b\right), b\right)$	-	-	-	-	-	-	-	-	0.119
$V^{c1}\left(0, t_e^*, b, g^*\right)$	0.062	0.069	0.075	0.080	0.086	0.091	0.094	0.094	0.08

Table 6: Political equilibrium outcome for k=0.12.

equilibrium tax rate and the level of derived rights, for an equal level of pension benefit. Finally, we also find, from these tables, that the introduction of a pension system always increases the number of one-breadwinner couples with respect to the efficient laissez-faire situation. Logically, this number also increases with the cost of housework.

6 Conclusion

Derived pension rights are pension entitlements which are not earned as the result of a worker's own contributions and earnings history but based on a family relationship, typically the relationship between a husband and wife. Benefits resulting from derived rights encompass spousal benefits as a supplement to a worker's pension, benefits for divorced spouses, and survivor benefits for widows. They contribute to alleviate poverty in old age for women living alone. Despite their financial weight in total pension expenditure, derived pension rights have not yet been studied from a theoretical viewpoint.

This paper tries to fill this gap by showing how to account for the existence of derived pension rights from both a normative and a positive viewpoint. We show that in a society with individuals differing in their productivity and the type of couple they live in, a pension system with positive derived rights is likely to emerge. In order to increase the redistribution toward the poorest couples, it is desirable to increase the level of these derived rights and to have a Social Security system as contributive as possible. This positive level of derived rights leads to having more stay-at-home wives, so that we show an unavoidable conflict between two objectives: poverty alleviation and individualization of pension rights that seeks to foster female labor participation.

Still, our paper makes some assumptions in order to make the problem tractable. Some of them may not be crucial, like, for instance, assuming that the cost of housework is independent of agents' productivity. Some others may certainly be more important. For instance, the quasi-linearity of the utility function is made here for simplicity but it has some non negligible consequences as one- or two-breadwinner couples have the same second-period consumption, independently of their productivity. Furthermore, we assume that inside couples, there is an equal bargaining power, while this may not be the case in reality. Specifically, it may be reasonable to assume that, in one-breadwinner couples, the working spouse may have higher bargaining power. Finally, we assume that derived rights are chosen by voting while the level of the flat pension benefit is chosen at the constitutional level. Alternatively we could have assumed sequential voting or that derived rights are fixed by the constitution while individuals vote on the level of the flat benefit. This certainly influences our results.

Finally, our paper could be extended in several ways. A direct extension would be to make the decision of marriage endogenous so that it would certainly depend on pension parameters. In this model, we would assume a society composed of singles, one- and two-breadwinner couples. This is on our research agenda.

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A Second-best solution

The Lagrangian of problem (C) has the following expression

$$\begin{aligned}
\pounds &= \int_{\underline{w}}^{\hat{w}} \Psi\left(\frac{w^2\left(1-t_e\right)^2}{2}+b+\beta g+\nabla\right) f\left(w\right) dw \\
&+\int_{\hat{w}}^{\bar{w}} \Psi\left(\frac{\left(1+\alpha^2\right) w^2\left(1-t_e\right)^2}{2}+\nabla+\left(1+\beta\right) b-k\right) f\left(w\right) dw \\
&+\lambda\left[\int_{\underline{w}}^{\hat{w}}\left(t_e\left(1-t_e\right) w^2-b-\beta g\right) f\left(w\right) dw+\int_{\hat{w}}^{\bar{w}}\left(t_e\left(1-t_e\right)\left(1+\alpha^2\right) w^2-\left(1+\beta\right) b\right) f\left(w\right) dw\right]
\end{aligned}$$

with λ the Lagrange multiplier associated to the ressource constraint. First-order conditions are

$$\begin{aligned}
\frac{\partial \pounds}{\partial t_e} &= -\left(1-t_e\right)\left[\int_{\underline{\mathrm{w}}}^{\hat{w}} w^2\Psi'\left(V^{c1}\right)f\left(w\right)dw + \left(1+\alpha^2\right)\int_{\hat{w}}^{\bar{w}} w^2\Psi'\left(V^{c2}\right)f\left(w\right)dw\right] \\
+ & \quad \lambda\left(1-2t_e\right)\left[\int_{\underline{\mathrm{w}}}^{\hat{w}} w^2 f\left(w\right)dw + \left(1+\alpha^2\right)\int_{\hat{w}}^{\bar{w}} w^2 f\left(w\right)dw\right] = 0 \qquad (22) \\
\frac{\partial \pounds}{\partial b} &= \int_{\underline{\mathrm{w}}}^{\hat{w}} \Psi'\left(V^{c1}\right)f\left(w\right)dw + \left(1+\beta\right)\int_{\hat{w}}^{\bar{w}} \Psi'\left(V^{c2}\right)f\left(w\right)dw \\
- & \quad \lambda\left[\int_{\underline{\mathrm{w}}}^{\hat{w}} f\left(w\right)dw + \left(1+\beta\right)\int_{\hat{w}}^{\bar{w}} f\left(w\right)dw\right] = 0 \qquad (23) \\
\frac{\partial \pounds}{\partial g} &= \beta\int_{\underline{\mathrm{w}}}^{\hat{w}} \Psi'\left(V^{c1}\right)f\left(w\right)dw - \beta\lambda\int_{\underline{\mathrm{w}}}^{\hat{w}} f\left(w\right)dw = 0 \qquad (24) \\
\frac{\partial \pounds}{\partial \hat{w}} &= \Psi\left(\frac{\hat{w}^2\left(1-t_e\right)^2}{2} + b + \beta g + \nabla\right)f\left(\hat{w}\right) - \Psi\left(\frac{\left(1+\alpha^2\right)\hat{w}^2\left(1-t_e\right)^2}{2} + \nabla + \left(1+\beta\right)b - k\right)f\left(\hat{w}\right) \\
+ & \quad \lambda\left[t_e\left(1-t_e\right)\left(1-\alpha^2\right)\hat{w}^2 - \beta\left(g-b\right)\right]f\left(\hat{w}\right) = 0 \qquad (25)
\end{aligned}$$

where for simplicity, we drop the arguments in the expressions of indirect utility functions and set $V^{c1} \equiv V^{c1}\left(w, t_e, b, g\right)$ and $V^{c2} \equiv V^{c2}\left(w, t_e, b\right)$. Combining (24) with (23), we obtain

$$\begin{aligned}
\int_{\hat{w}}^{\bar{w}} \left(\Psi'\left(V^{c2}\right) - \lambda\right)f\left(w\right)dw &= 0 \\
\int_{\underline{\mathrm{w}}}^{\hat{w}} \left(\Psi'\left(V^{c1}\right) - \lambda\right)f\left(w\right)dw &= 0
\end{aligned}$$

Rearranging (22), and substituting for the above equalities, we obtain

$$\begin{aligned}
\frac{\partial \pounds}{\partial t_e} &= -\left(1-t_e\right)\left[\int_{\underline{\mathrm{w}}}^{\hat{w}} w^2\Psi'\left(V^{1c}\right)f\left(w\right)dw - \int_{\underline{\mathrm{w}}}^{\hat{w}} w^2 f\left(w\right)dw \int_{\underline{\mathrm{w}}}^{\hat{w}} \Psi'\left(V^{1c}\right)f\left(w\right)dw\right. \\
& \left. + \int_{\hat{w}}^{\bar{w}} \left(1+\alpha^2\right)w^2\Psi'\left(V^{2c}\right)f\left(w\right)dw - \int_{\hat{w}}^{\bar{w}} \left(1+\alpha^2\right)w^2 f\left(w\right)dw \int_{\hat{w}}^{\bar{w}} \Psi'\left(V^{2c}\right)f\left(w\right)dw\right] \\
& -\left(1-t_e\right)\lambda\left[\int_{\underline{\mathrm{w}}}^{\bar{w}} w^2 f\left(w\right)dw + \alpha^2\int_{\hat{w}}^{\bar{w}} w^2 f\left(w\right)dw\right] \\
& + \lambda\left(1-2t_e\right)\left[\int_{\underline{\mathrm{w}}}^{\bar{w}} w^2 f\left(w\right)dw + \alpha^2\int_{\hat{w}}^{\bar{w}} w^2 f\left(w\right)dw\right] = 0
\end{aligned}$$

The above function can be rewritten as

$$\begin{aligned}
\frac{\partial \pounds}{\partial t_e} &= -(1-t_e)\left[\int_{\underline{w}}^{\hat{w}} w^2\Psi'\left(V^{1c}\right)f(w)\,dw + \int_{\hat{w}}^{\bar{w}} w^2\Psi'\left(V^{2c}\right)f(w)\,dw\right. \\
&\quad -\int_{\underline{w}}^{\hat{w}} w^2 f(w)\,dw \int_{\underline{w}}^{\hat{w}} \Psi'\left(V^{1c}\right)f(w)\,dw - \int_{\hat{w}}^{\bar{w}} w^2 f(w)\,dw \int_{\hat{w}}^{\bar{w}} \Psi'\left(V^{2c}\right)f(w)\,dw\Big] \\
&\quad +\int_{\hat{w}}^{\bar{w}} \alpha^2 w^2\Psi'\left(V^{2c}\right)f(w)\,dw - \int_{\hat{w}}^{\bar{w}} \alpha^2 w^2 f(w)\,dw \int_{\hat{w}}^{\bar{w}} \Psi'\left(V^{2c}\right)f(w)\,dw\Big] \\
&\quad -\lambda t_e\left[\int_{\underline{w}}^{\bar{w}} w^2 f(w)\,dw + \alpha^2\int_{\hat{w}}^{\bar{w}} w^2 f(w)\,dw\right] = 0
\end{aligned}$$

which yields expression (16).

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