2010/81

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December 2010

Abstract

Educational tracking is a very controversial issue in education. The tracking debate is about the virtues of uniformity and vertical differentiation in the curriculum and teaching. The pro-tracking group claims that curriculum and teaching better aimed at children's varied interest and skills will foster learning efficacy. The anti-tracking group claims that tracking systems are inefficient and unfair because they hinder learning and distribute learning inequitably. In this paper we provide a detailed within-country analysis of a specific educational system with a long history of early educational tracking between schools, namely the Flemish secondary school system in Belgium. This is interesting place to look because it provides a remarkable mix of excellence and inequality. Indeed the Flemish school system is repeatedly one of the best performer in the international harmonized PISA tests in math, science and reading; whereas it produces some of the most unequal distributions of learning between schools and students. Combining evidence from the PISA 2006 data set at the student and school level with recent statistical methods, we show first the dramatic impact of tracking on social segregation; and then, the impact of social segregation on equality of educational opportunity (adequately measured). It is shown that tracking, via social segregation, has a major effect on inequality of opportunity. Children of different economic classes will have different access to knowledge.

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1 Introduction

'*What is tracking?*'

Tracking is the grouping of students into classes by ability/prior achievements and organizing curriculum by its level of difficulty. Track assignments are based on successful completion of prerequisite courses, prior achievements, and teacher recommendations (The tracking decision is no longer based on IQ test as in the past in the US). But still tracking remains a very controversial issue. Pro-tracking groups claim that students at different levels of ability require different types of instruction. '*Expecting all children the same age to learn from the same materials is like expecting all children the same age to wear the same size clothing.*'(Madeline Hunter). Opponents of tracking have argued that high achieving students serve as role models for less able, struggling students. In truth, tracking is a very divisive and complex issue involving some fundamental equity /efficiency trade offs as well described in Tom Loveless (1999) '*The Tracking Wars*'. Following Loveless (1999), there is much confusion in the debate simply because tracking is a practice that means different things to different people. It is indeed useful to distinguish tracking from ability grouping. Both systems denote the practice of grouping students of similar ability or prior achievement together for instruction. In this report, we adhere to the conventional definitions employed by researchers, using "ability grouping" to refer to the grouping of students by ability within classes with uniform curriculum for all, which is primarily an elementary school practice, and "tracking" to refer to the grouping of students by ability between classes with differentiated curriculum, a strategy common in middle and high schools. The tracks cover distinctly different curricula across subjects, and lead to different destinations upon graduation. Three tracks are common: (1) a high track, with college-preparatory or honors courses that prepare students for admission to top colleges and universities; (2) a middle track that served as a catch-all for the group of students in the middle, and (3) a low track, consisting of vocational courses and a smattering of low-level academic offerings, serving mainly low functioning and indifferent students. After graduation, low track students frequently drop out, go to work, or get unemployed.

'*Why it is unpopular?*'

One of the main reasons that tracking has become unpopular has less to do with the outcomes the practice generates than with the types of students who tend to be assigned to the different tracks. A major concern is that tracking is used to segregate students on the basis of family background and race, as well as ability. In fact, the primary charges against tracking are (i) that it doesn't accomplish anything and (ii) that it unfairly creates unequal opportunities for academic achievement. This critique has fueled in the 90's with the very influential book by Anne Wheelock (1992), '*Crossing the Tracks. How Untracking Can Save America's Schools*'.

'What is the evidence?'

Generally speaking, research fails to produce conclusive evidence. Loveless (1999) reports a large set of empirical research comparing tracking and non-tracking systems based on the US National Education Longitudinal Study started in 1988. When students are ability grouped into separate classes and given an identical curriculum, there is no appreciable effect on achievement. However when the curriculum is adjusted to better match ability level, it appears that student achievement is boosted, especially for high ability students receiving an accelerated curriculum, but that low ability students may suffer from assignment to lower tracks.[1] To sum up, heterogeneous classes appear to benefit low ability students, but may depress the achievement of average and high achieving students. The central difficulty with such non-experimental evidence is the identification problem: it is almost impossible to identify the causal effect of school tracking on pupils' achievement because of selection bias. We could observe a positive peer group effect simply because those forming the same group share similar unobserved characteristics (selection effect), and not because there is some positive social interaction (incentive effect). This is called the reflection problem by reference to the co-movement between a person and his image in a mirror.[2] Does the group reflect your own image, or do you match the image of the group? With random selection the effective impact of a treatment can be identified. It is the reason why randomized experiments are so valuable to evaluate policy impact. So far there is few experimental evidence on tracking and its impact on education outcomes. The main exceptions are Duflo et al. (2009) for primary education in Kenya where there is random assignment of pupils (grade 1) and teachers among 60 tracking and 61 non-tracking schools and Guyon et al. (2010) for secondary education in Northern Ireland where there is discontinuity in the share of pupils in elite-schools across cohorts and across regions. In the random assignment of Duflo et al. (2009), in tracking schools, students were ranked by prior achievements and the top and bottom halves were assigned to different classes. They found that students in tracking schools scored 0.14 standard deviation higher on average, regardless of their initial score. They argued that greater homogeneity in the classroom allowed teachers to tailor their teaching to what the student can learn, and this differentiation benefits everyone including those assigned to the bottom classes. Obviously the difficulty is to extend this potential (experimental) benefit for the lower-achieving pupils of a teaching more appropriate to them in a more general (non-experimental) context where teaching differentiation can promote segregation and create unequal opportunities. In contrast to the Duflo et al. (2009) experiment, Guyon et al. (2010) find evidence against the

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[1]See also Rees et al. (1996)

[2](Manski, 1993)

efficiency argument from tracking. Using cohort data from Northern Ireland as a natural experiment, they show via a discontinuity regression approach that there is a significant positive net-effect on educational outcomes of the 'de-tracking' refrom in 1989. In addition, the authors find clear evidence for a positive association between the area-level variation of the share of pupils in elite schools and the proportion of successful students at age 16 and 18.

'*Social segregation*'

Tracking, to be sure, links a student's present and past track level. As illustrated in Figure 1, if past academic achievement is related to parental background, then tracking will link present track to family background. As a result, students may be placed in low tracks because of the socio-economic status of their family.[3] If we believe that teaching follows a hierarchical sequence, exposing students to increasingly difficult skills and complex knowledge, early tracking can lock in students with low socio-economic background in low tracks and induce progressive segregation. The consequence is unequal access to knowledge. This is getting worse, as evidence seems to suggest, if low tracks attracts less experienced teachers and hinders the motivation and aspiration of students with lower expectations; and if parents intervention into tracking decision is more common with highly educated parents pushing for high track placements. This is where unequal opportunities comes into the debate.
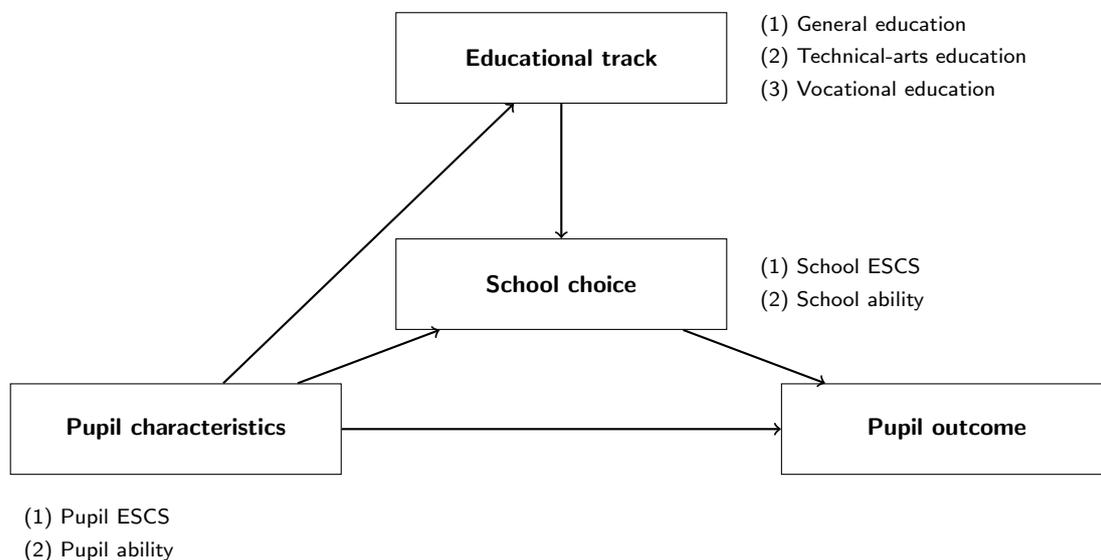

Figure 1: School tracking and inequality of opportunity

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[3]PIRLS 2006 displays clearly strong correlation between early reading literacy (grade 4) and parental education in all countries (chapter 3, exhibit 3.5, pp120-121).

'What is our contribution?'

A first contribution of this paper is to estimate the effect of systematic school tracking on social segregation in schooling. In Duflo et al. (2009), educational tracking is studied without attention to social segregation because of the random assignment of students and staff to different treatment groups. In non experimental contexts, many more effects come in. Educational tracking can lead to racial and class bias in track assignment with curriculum differentiation, difference in teacher experience and quality, difference in resources, and difference in students expectations and motivations.

To investigate empirically the link between school tracking and social segregation, we study systematic '*between-school*' tracking as is implemented extensively in among others Belgium, Hungary, Switzerland, Austria, Luxembourg, the Netherlands and Germany. Pupils are sorted in general (high), technical-arts (middle) and vocational (low –track) study programmes based on prior achievements, with almost no probability to have lessons together with pupils from different tracks and almost no possibility to share the same teachers. Each track has its own curriculum and end goals. Most schools organize only one track so that pupils in different tracks are in different schools. There is almost no upward track mobility.

This is clearly different from '*within-school*' tracking as discussed in Duflo et al. (2009) and is the case in among others the US[4]. We focus on the Flemish community of Belgium (Flanders), which has a long tradition of educational tracking at the age of 12 (grade 7). This is interesting place to look at because it combines high average achievements (repeatedly in the top of international PISA Tests in Math, Science and Reading) with extensive achievements inequality between schools and between students (bottom in the international PISA test variances). Belgium is also a country that displays very extensive social segregation in education (see Jenkins et al. (2008)). Using Hutchens decomposition method, we show that most of social segregation in the Flemish community takes place between tracks. It is also shown that the private/public school dimension has little impact on social segregation.

Secondly, by linking school tracking to social segregation, we can indirectly measure the effect of school tracking on educational opportunities in a cross-sectional microlevel study. We adopt recent empirical methods with strong theoretical underpinnings to study how school tracking affects equality of opportunity in schooling via social segregation. We estimate the existence of inequality of opportunity in schooling by comparing conditional (on socio-economic status) distributions of test scores and by estimating the 'Gini opportunity' index as in Lefranc et al. (2008). Then, the determinants of inequality of opportunity are investigated in a multilevel regression approach that is closely related to Bourguignon et al. (2007). This multilevel

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[4]Within-school tracking is discussed in among others Epple et al. (2002), Figlio and Page (2002).

regression analysis relates the test score of students to the social composition of the school and the socio-economic status of students. To accommodate the hierarchical clustering of pupils in schools, we include random school effects as well a a specific random individual effect. We complement this conditional mean regression analysis, with a broader conditional quantile regression analysis to see how not only the mean of the distribution, but also the full distribution is affected (possibly differently) by the family background and the school composition. Explicit considerations of these effects via quantile regression can provide a more nuanced view of the stochastic relationship between socio-economic variables and test scores and therefore a more informative empirical analysis. Results show that school tracking has via social segregation a profound effect on inequality of opportunity in schooling. The quantile regression analysis reveals that the social composition of school is the main influence on the conditional quartiles of test scores. This means that students with the same family background achieve significantly different test score threshold depending with whom they go to school.

2   Related literature

Recent empirical research has shown the importance of educational achievements for (1) individual earnings, (2) the distribution of income, and (3) economic growth (Barro, 2001; Bishop, 1992; Nickell, 2004; Hanushek and Wößmann, 2008). As a result, the issue of equal earnings opportunities is closely related to equal educational opportunities (Brunello and Checchi, 2007). Understanding the drivers of equal opportunities in education is thus a major issue. Since the Coleman (1966) report, the impact of family background and peer effects on the quality of education has been investigated in a wide range of literature. Using data from respectively the US and Brazil, Betts and Roemer (2005) and Waltenberg and Vandenberghe (2007) show that not the redistribution of public budgets, but institutional features are the key in increasing equality of opportunity in schooling.(see also survey of Betts (2010)). As shown in OECD (2006), school composition is important in explaining educational achievement. In this paper, we study the causes and effects of social segregation in a within-country approach.

The study of school tracking is closely related to the study of peer effects. In equilibrium analyses of among others Epple and Romano (1998), de Bartolome (1990), Benabou (1996), Nechyba (2000) and Epple et al. (2002), peer group effects are incorporated to study the impact of school vouchers, private-public school sorting and community structure. In empirical studies however, no consensus is reached on the relation between peer effects and educational

outcomes (Brunello and Checchi, 2007). This, amongst others, because it is difficult to separate peer effects from other confounding effects. To overcome this problem, Hanushek et al. (2003) control for family background, school settings, student and school-by-grade fixed effects in a longitudinal panel study on a set of pupils in Texas. Hanushek et al. (2003) find evidence for a linear relationship between peer group quality and educational outcomes. Consequently, at an aggregate level, altering peer-group quality by educational tracking is expected to have no impact on the average education quality, but strong impact on inequality of educational outcomes. However, exploiting the same student variation in test scores across subjects, Lavy et al. (2009) find in a within-pupil regression approach on pupils in secondary education in England evidence that only the top 5% and bottom 5% students matter to explain individual variations across subjects. In addition, evidence is found that only academic achievement matters, not family background and that peer effects are heterogeneous in gender and student's ability.

In a recent randomized experiment of Duflo et al. (2009) in 121 primary schools in Kenya, new evidence is found that within-school tracking - separating pupils with different abilities in different classes within the same school - is an effective instrument to improve the performance of pupils with respectively low and high prior perceived cognitive ability. The random nature of the experiment is assured by the random attribution of teachers and schools in the two installed systems (tracking versus non-tracking) and by assigning pupils in two tracks by initial achievement (bottom halve of class selected into lower track, upper half to higher track). As both teachers and schools are randomly selected, there are no institutional effects by assumption. As the effect of school tracking is found to be positive for both pupils in the low and high tracks, there is no equity-efficiency trade-off. However, Guyon et al. (2010) show that the results of Duflo et al. (2009) do hot hold in a between-school tracking setting in secondary education in developed countries. In a natural experiment, Guyon et al. (2010) investigate the impact of the 1989 'de-tracking' reform in secondary education in Northern Ireland with an increase of the relative size of pupils in elite track schools from around 31% to 35%. Information is used from 22 self-constructed areas in Northern Ireland for cohorts between 1974 and 1982. Discontinuity in educational outcomes is found across cohorts, showing a positive net-effect of de-tracking. In addition, a positive association between the area-level change in size of the reform and the area-level educational performance change shows the robustness of this positive net-effect of de-tracking.

In contrast to the Duflo et al. (2009) experiment, a vast literature of cross-country and country case studies show the detrimental effects of school tracking for equality of opportunity in education. In Schutz et al. (2008), data from TIMMS and TIMMS-REPEAT is used to estimate and explain the effect of family background on student performance, using cross-

country variations. Evidence is found that late tracking and a long pre-school cycle are beneficial for equality of opportunity (EOp) in schooling. Ammermüller (2005) and Hanushek and Wößmann (2006) use PISA and PIRLS data to estimate the effect of institutions on educational opportunities by applying a difference-in-difference approach. Both studies find evidence for a negative effect of educational tracking. Ammermüller (2005) finds in addition a negative effect of school systems with a large private school sector. However, using a difference-in-difference approach with data from PISA, TIMMS and PIRLS, Waldinger (2007) shows that the evidence from these cross-country difference-in-difference studies on educational tracking is vulnerable to specification changes.

Using cohort data from 4 datasets, Brunello and Checchi (2007) do find significant effects of educational tracking for educational attainment and early wages. Brunello and Checchi (2007) thus find an effect of educational tracking on equality of opportunity in income via inequality of opportunity in schooling.

In the majority of country case studies - referenced in Brunello and Checchi (2007) - educational reforms that reduce educational tracking are found to be associated with less impact of family background on educational attainment. A well-known example is the case of Finland - where in 1972-1977 a two-track system was progressively replaced with a comprehensive school system till the age of 16. Pekkarinen et al. (2009) find in a difference-in-difference approach an increase of intergenerational income mobility as consequence of the reform.

The role of social segregation in explaining the effect of educational tracking on EOp in schooling is clarified by Checchi and Flabbi (2007). They show in a theoretical model of school-track choice that ability tracking is harmful for EOp in schooling if tracking is based on family background. If not - and there is full information on ability - ability tracking is not harmful for EOp in schooling. Put differently, ability tracking can result in inequality of opportunity in schooling when the family background determines the track choices, given the level of ability (Contini et al., 2008).

Using the PISA 2000 and PISA 2003 dataset, Jenkins et al. (2008) find an association between social segregation and school choice for a sample of OECD countries. Using pupil and school-level data from the PISA 2006 dataset, Alegre and Ferrer (2010) find effect of social composition of schools on educational performance. By estimating social segregation at country-level, Alegre and Ferrer (2010) find association between social segregation and selection of pupils by schools and into ability tracks.

The remainder of the paper is structured as follows. In section 3, we give an overview of our filtered sample with Flemish data from the PISA 2006 dataset. In section 4, we discuss

the methodology to estimate and explain inequality of opportunity, and to estimate and decompose social segregation. In section 5, we present and discuss the results. In section 6 we provide some concluding remarks and discussions.

3 Data

The PISA 2006 dataset is used. In Belgium, education is organized by the Flemish community, the French-speaking community and the German-speaking community. We focus on the Flemish community. The Programme for International Student Assessment (PISA) is *'an internationally standardized assessment that assesses how far students near the end of compulsory education have acquired some of the knowledge and skills that are essential for full participation in society. In all cycles, the domains of reading, mathematical and scientific literacy are covered not merely in terms of mastery of the school curriculum, but in terms of important knowledge and skills needed in adult life.'*(OECD, 2006)

There are 3 cycles: PISA 2000, PISA 2003 and PISA 2006. In 2006, the survey was implemented in 57 countries. The PISA dataset is characterized by richness on variables related to educational achievement, family background and school level institutional settings. Although the main focus of PISA 2006 is on science, each participating pupil is asked to complete a standardized test on math, science and reading and fill out a survey with questions related to their family background, views on issues related to science, the environment, careers, learning time and teaching and learning approaches of science. Each principal of the participating schools is asked to complete a survey with questions on the characteristics of the school.

Tests are typically constructed to have assessed between 4500 and 10000 students of age 15 in each country. To sample the target population of 15-year old pupils that are at least in grade 7, PISA 2006 has implemented a two-stage stratified sample design. In a first-stage, for each strata[5], schools are sampled proportional to size from a list of schools in the region (PPS sampling). The target was 150 schools in each region. In a second-stage, 35 pupils are randomly drawn with equal probability from a list of 15-year old pupils in the school.[6] Final student weights are constructed to correct for varying selection probabilities of the students.[7] To incorporate sampling variation, a Balanced Repeated Replication (BRR) procedure with 80 replication estimates - described in OECD (2005)- can be used to construct standard errors

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[5]A group of schools, formed to improve the precision of sample based estimates.

[6]If the school size is lower than 35, all pupils are included in the sample.

[7]This occurs because of certain subgroups that are over- or under-sampled, the information of school size at the time is not completely correct, school non-response, student non-response and the inclusion of trimming weight to ensure stable estimates.(OECD, 2009)

(OECD, 2009). Alternatively, to do statistical inference, bootstrap resampling approaches can be used[8]. Both approaches incorporate the final student weights.

PISA 2006 makes use of a plausible value approach to estimate the pupil performance in respectively mathematics, science and reading literacy. These plausible values are random values from the posterior distribution and may not be aggregated at pupil level (OECD, 2005). Therefore, in what follows, we use the first plausible value to estimate educational outcomes in math, science and reading at pupil level.

Pupils that are in special education or part-time education are deleted from the sample. By this, the sample is reduced to 4125 observations in the Flemish community of Belgium. Sub-schools are defined to investigate the importance of school tracking in general, technical-arts and vocational education. A sub-school is defined as a unit that provides either general, technical-arts, or vocational education. When a school provides both general and technical or arts education (which is relatively rare), then the school is treated as two separate (sub-)schools.[9] The sample consists of 269 Flemish (sub-)schools.

Table 1 shows descriptive statistics of key variables to study the relation between school tracking and educational opportunities. Standardized test scores for math, science and reading are high in the Flemish community (PISA average is 500). In addition, the high standard deviation of educational outcomes in the Flemish community shows that there is high inequality in individual test scores. To relate inequality in outcomes to family background, we consider 2 circumstance variables: socio-economic status and migration status. First, family socio-economic status is estimated by PISA as a composite index of the Economic and Socio-Cultural Status (ESCS) of a pupil, derived from (1) the highest occupational status of each student's parents,(2) their highest educational level, and (3) a summary measure of household possessions (OECD, 2009). Second, for migration status, three proxies are used. First-generation immigrants and second-generation immigrants are respectively defined as pupils that are not born in Belgium and pupils that are born in Belgium, but are children of immigrants. Pupils that are first- or second-generation immigrant and do not speak the school language at home are grouped in a latter category of non-native pupils. Table 1 shows that there are less than 8 percent immigrant pupils in the sample. As mentioned above, only 35 pupils are sampled in each school. Consequently, a detailed look at segregation between

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[8]In this approach, for a sample of n observations, each bootstrap sample is a random sample of n observations selected with replacement from the original sample of observations. Consequently, in a bootstrap sample that is drawn with replacement, some of the n original sample observations are more than once in the bootstrap sample. Other original sample observations are not in an individual bootstrap sample. By replicating the bootstrap samples many times, we approximate the true population.

[9]This is necessary to construct 'between school track' and 'within school track' social segregation estimates

migrants (small minority) and non-migrants (large majority) would lead to inaccurate estimates. Therefore, we do not study ethnic segregation in this paper. The focus is thus on social segregation and equality of opportunity.

In Belgium, secondary education starts in general at age 12 and ends at age 18. In the Flemish community, pupils in ordinary education are tracked in the first year of secondary education in general education, technical education, arts education and vocational education based on prior achievements. In our filtered sample, around 50 percent of pupils are in general education (high track), 28 percent are in technical-arts education (middle track) and 20 percent in vocational education (low track).[10]

If a pupil has not reached the basic skills, determined by the 'end goals' in a school year, grade repetition and re-orientation to lower tracks are used. In our filtered sample, 77 percent of pupils are 'on time'.

In comparison with the French speaking community, the Flemish community is characterized by early tracking (age 12 versus age 15) and low use of grade repetition (23 percent vs 47 percent).

Private-granted schools are only a negligible proportion of the school population. There are mainly public schools (under control of community, provinces, cities or municipalities) and private operating, public-granted schools (e.g. Catholic schools, non-confessional schools). In our filtered sample, 74 percent of pupils are in private-operating schools.

< Table 1 about here >

To provide descriptive information on the disproportionate representation of pupils with low family background in low tracks, we rank pupils in the sample by their ESCS level and the top and bottom halves are assigned to the low and high social position groups.

Descriptive statistics in Table 2 indicate that in both communities, pupils with low social position have significantly less probability to attend general education (about two times less likely to attend the high track) and much higher probability to lag behind than pupils with a high social position (about twice more likely). Only 27 percent of pupils with a low social position are in general education without lagging behind. For pupils with a high social position, this is 58 percent. In addition, descriptive statistics indicate greater representation of pupils with low social position in public schools. In sum, pupils with a low social position

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[10]We merge the technical and arts tracks together because the two tracks do not dominate each other in curriculum difficulty and test scores and because there is only a small proportion of pupils that are in the arts track.

have relatively higher probability to lag behind, to be in a lower track or to go in a public school.

<Table 2 about here >

4 Methodology

4.1 Defining inequality of opportunity

The main focus of the literature on equality of opportunity is on separating sources of inequality of outcomes that are morally acceptable and morally unacceptable.[11] In the seminal book of Roemer (1998) *"equality of opportunity"*, it is shown that not all inequality of outcomes is ethically immoral. Only inequality that is outside the realm of individual choice - referred as *circumstances* - should be eliminated by public intervention. On the other hand, inequality in outcomes that is a consequence of factors where individuals are judged to be responsible for - referred as *effort* - are morally acceptable and should not be eliminated or compensated for.[12] There is a priori no reason to suspect that equality of outcomes and equality of opportunity goes hand in hand (Lefranc et al., 2008). Typical examples of circumstances are family background and individual attributes such as race, gender and place of birth. Examples of effort are own education, annual working hours and migration (Ramos and Van de Gaer, 2009).

EOp is achieved when no particular vector of circumstances is preferred to another vector of circumstances by all individuals (Lefranc et al., 2008). As shown in Van de gaer (1993), Roemer (1998) and Lefranc et al. (2008), EOp amounts in comparing distributions of outcomes, conditional on circumstances.

Consider a situation where individuals are allowed to choose their circumstances s - referred as *type* - before they know their level of effort. EOp prevails between circumstances s and s' if s is not preferred to s' by all individuals, and vice versa. In other words, agents cannot order the opportunity sets of s and s'. The opportunity set of an individual can be represented by the conditional distribution function $F(x|s)$. Under the (weak) assumption that preferences satisfy the criteria of first order stochastic dominance (FSD) and second-order stochastic dominance (SSD), stochastic dominance tests can be used to rank conditional distribution functions. A formal definition of first order stochastic dominance (FSD) and second order stochastic dominance (SSD) is given in respectively (1) and (2).

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[11]Seminal works are among others Arneson (1989), Barry (1991), Cohen (1989), Dworkin (1981a), Dworkin (1981b), Roemer (1993), Roemer (1998) and Sen (1980).

[12]See also Fleurbaey (2008) for detailed account of equality of opportunity.

Suppose inequality of opportunity where circumstance s is preferred to circumstance s' by all individuals. Inequality of opportunity defined as first order stochastic dominance between s and s' means that the distribution of outcome x conditional on s is for all x below the distribution of x conditional on circumstance s'.

However, it can easily be shown that this is a very weak definition of EOp. Indeed, suppose a situation where the outcome distribution of type s always dominates the outcome distribution of type s', except at the top (possibly when they exert maximal effort). Under the definition of inequality of opportunity as first order stochastic dominance, EOp is not rejected in this setting. But it is unfair because type s' must exert maximal effort to get a chance to outperform type s.

Second order stochastic dominance provides extra restrictions. Under the definition of inequality of opportunity as second order stochastic dominance - under the assumption of a Von Neumann-Morgenstern utility function- EOp prevails when the expected value derived from distribution $F(y|s)$ is not greater than the one derived from $F(y|s')$.

$$s \succeq_{FSD} s' \text{ iff } F(x|s) \leq F(x|s'), \forall x \in \Re_+. \tag{1}$$

$$s \succeq_{SSD} s' \text{ iff } \int_0^x F(y|s)dy \leq \int_0^x F(y|s')dy, \forall x \in \Re_+. \tag{2}$$

4.2 Measuring inequality of opportunity

Econometric stochastic dominance techniques can be used to test FSD and SSD of conditional distributions. A large advantage of this approach - against among other looking at the marginal effect of circumstances on the conditional mean of educational outcomes such as in Schutz et al. (2008)- is that empirical tests have a strong theoretical underpinning[13]. As robustness test, we also estimate inequality of opportunity with a Gini-type index. We use a Gini Opportunity index as proposed by Lefranc et al. (2008). This index is based on the equivalence between SSD and Lorenz dominance as shown by Shorrocks (1983). The Gini opportunity (GO) index is defined in (3)

$$GO(x) = \frac{1}{\mu} \sum_{i=1}^{k} \sum_{j>i} p_i p_j (\mu_j(1 - G_j) - \mu_i(1 - G_i)), \tag{3}$$

with k types, μ the mean of the population, μ_k the mean of group k, p_k the population weight of group k and G the Gini coefficient. The GO index computes the sum of all pairwise differences of the opportunity sets of all types, where the opportunity sets are defined as twice

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[13]References of papers that use this approach to study equality of opportunity are among others Checchi and Peragine (2010), Lefranc et al. (2008), Peragine and Serlenga (2008) and Pistolesi (2009).

the area under Generalized Lorenz curve, $\mu_s(1 - G_s)$ for type s (Ramos and Van de Gaer, 2009). The GO index is in the interval $[0, 1]$. A value of 0 indicates full EOp. Bootstrapping can be used to do statistical inference.

4.3 Explaining inequality of opportunity

4.3.1 Conditional mean regression approach

To estimate the effects of school factors and family background on student achievement, a multilevel regression analysis is carried out where covariates are distributed at two levels: the students and schools. In an educational setting, unobserved school effects are expected from school-level disparities in e.g. the academic culture of school staff. As students are clustered in different schools, the assumption of independent noise is violated. It is thus necessary to include random school effects into the empirical analysis to obtain unbiased estimates (Raudenbush and Bryk, 2002).

The main purpose here is to identify two distinct channels for the impact of family background on individual educational attainments: a direct effect through the parental socio-economic status, and an indirect effect through school choice. In addition we will assess how much of the educational variation across schools can be explained by the school's average social position, after controlling for the rest of the individual family background and school level variables. As a result we can examine how and to what extent the social composition of schools (and so the social segregation between schools) perpetuates indirectly to student achievement inequalities across schools.

To separate the direct and the indirect school choice effect of parental circumstances on individual achievement, we follow a regression approach closely related to Bourguignon et al. (2007). We consider that the educational outcome of an individual (O) is defined as a function of circumstance variables (C), effort variables (E) and unobserved determinants or random noise.[14] Individual ability is unobserved variable and we do not possess information on ability scores or earlier academic achievements. To allow for hierarchically clustered noise, we define θ_j as the random effect of school j and ϵ_i as the pupil-level errors. We further relax the i.i.d. assumption of ϵ_i by allowing for clustering within strata and by the introduction of probability weights to correct for unequal selection probabilities as proposed in Pfeffermann et al. (1998).

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[14]Random noise is a combination of the effect of unobserved variables, measurement error and luck. We follow Lefranc et al. (2009) in defining luck as *"random determinants that are seen as a fair source of inequality provided that they are even-handed"*. Hence, we do not categorize random noise under effort or circumstances.

$$O_{ij} = f(C_i, E(C_i)) + \theta_j + \epsilon_i \text{ , with i=1,...,n and j=1,...,m.} \tag{4}$$

Circumstances are supposed to have a 'direct' effect on outcomes and an 'indirect' effect via 'effort' (Bourguignon et al., 2007). The school choice (or track choice) is the only observable 'effort' variable we will consider (since other effort variables are not observables). The school choice is related to circumstances variables. Inequality of opportunity measures the overall effect of circumstances variables on educational attainments both via the direct effect of circumstances on educational achievements and the indirect effect through the effect of circumstances on the effort vairables (school choice). The central feature of this paper is to include this indirect effect of circumstances on school choice via social segregation. For this, we include the school's average social position (S). Consequently, if we define n pupils in m schools, we obtain a model as defined in (5).

$$O_{ij} = f(C_i, S_j) + \theta_j + \epsilon_i \text{ , with i=1,...,n, j=1,..,m.} \tag{5}$$

The non-observable effort variables are captured by the unobserved determinants both at school levels θ_j and individual level ϵ_i. We specify a linear econometric model with varying intercepts as

$$O_{ij} = \alpha_j + \beta C_i + \epsilon_i$$
$$\alpha_j = \alpha + bS_j + \theta_j \tag{6}$$

Where C_i and S_j are covariates at respectively the student and school level, ϵ_i and θ_j are independent errors at each level, β is a vector of coefficients for the circumstances C and b is the coefficient for the school's average social position S. Substituting the group level equation into the individual level equation gives the reduced form which can be estimated by maximum likelihood estimation as:

$$O_{ij} = \alpha + \beta C_i + bS_j + \theta_j + \epsilon_i \text{ , with i=1,...,n, j=1,..,m.} \tag{7}$$

It is worth noting that estimates can be biased because of standard omitted variable problem due to the non-observable ability variable, and possible correlation between S and θ. If there correlation between ability and the circumstance variables C, then the residual terms are not orthogonal to the regressors. Therefore, in (7), we make the implicit assumption that (1) the social position of a pupil is unrelated to his true " ability" (no genetic transmission of cognitive ability) and (2) that the school's average social position is independent of the random school effects. To the extent, that we treat ability as non-observable circumstance variables for which pupils are not responsible, the overall effect of the observable circumstances variables is

underestimated. Our estimates provide a lower-bound on the overall impact of circumstances on educational outcomes.

Alternatively, one could think of using instrumental variable approach. However, this strategy is not promising in our educational setting, because it is unlikely to find a variable that is correlated to circumstance variables and has no direct effect on educational outcome (Bourguignon et al., 2007). Alternatively, following Bourguignon et al. (2007), one could explore the magnitude of the potential biases by a monte-carlo approach where a wide range of correlations between the residual terms and covariates are explored. However, extension of the proposed approach from OLS to a two-stage maximum likelihood estimation procedure is not pursued in this paper. We postpone to the concluding section the discussion of the implication of the omitted ability variable on our analysis and results.

4.3.2 Conditional quantile regression approach

To obtain a more complete picture of the conditional distribution of pupil performance, Koenker and Bassett (1978) introduced the estimation of conditional quantiles rather than the conditional mean.[15] Following this method, the conditional αth quantile ($\alpha \in (0,1)$) is defined as the test score threshold such that α percent of the pupils of the reference group perform worse and $1 - \alpha$ percent perform better. It is given by the inverse of the conditional CDF:

$$q_\alpha(x) = \inf\{y : F(y|x) \geq \alpha\} = F^{-1}(\alpha|x). \tag{8}$$

For example, if y is the pupil test score and x her socio-economic status, 25 % of the pupils in the same reference group x performs worse than the score threshold $q_{0.25}(x = 0)$. Recently, Li and Racine (2008) proposed a nonparametric kernel approach to estimate conditional CDF functions in a multivariate setting with both continuous and discrete variables.[16] By using kernel weights, no a priori parametric assumptions need to be imposed on the quantile regression. The main features of the Li and Racine (2008) approach in comparison to other kernel quantile regression approaches are that 1) it admits smoothing of both discrete and continuous covariates, 2) irrelevant variables are 'smoothed out' with high probability via the data-driven bandwidth selection, 3) also the dependent variable can be smoothed to improve estimation and 4) optimal bandwidths are selected in an automatic data-driven approach.[17] A

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[15]See Koenker (2005) for an overview of parametric quantile regression approaches

[16]This approach is implemented in the programing software R as package 'np' of Hayfield and Racine (2008). See Li and Racine (2007) for an excellent overview of nonparametric econometrics.

[17]As recommended in Li and Racine (2008), we use least-squares cross-validation for bandwidth selection. A second-order Gaussian kernel is used for nonparametric weighting.

disadvantage of this nonparametric quantile regression approach is that it is - to our knowledge - not possible to include random school effects.[18] The smooth estimate of $F(y|x)$ that allows the inclusion of mixed discrete ($X^d \in \Re^r$) and continuous covariates ($X^c \in \Re^q$) can be defined as:

$$\hat{F}(y|x) = \frac{n^{-1} \sum_{i=1}^{n} G(\frac{y-Y_i}{h_0}) K_\gamma(X_i, x)}{\hat{\mu}(x)} \tag{9}$$

Where $\hat{\mu}(x) = n^{-1} \sum_{i=1}^{n} W_h(X_i, x)$ is the kernel estimate of $\mu(x)$. h_0 is the smoothing parameter associated with y. With generalized product kernel $K_\gamma(X_i, x) = W_h(X_i^c, x^c) L_\lambda(X_i^d, x^d)$. Product kernel of X^c is $W_h(X_i^c, x^c) = \prod_{i=1}^{q} h_s^{-1} w((X_{is}^c - X_s^c)/h_s)$, with $w(\cdot)$ a univariate kernel function and h a smoothing parameter associated with X^c. Product kernel of X^d is $L_\lambda(X_i^d, x^d) = \prod_{s=1}^{r} l(X_{is}^d, x_s^d, \lambda_s)$, with univariate kernel function $l(X_{is}^d, x_s^d, \lambda_s) = \mathbf{1}(X_{is}^d = x_s^d) + \lambda_s \mathbf{1}(X_{is}^d \neq x_s^d)$ and smoothing parameter $\lambda \in (0, 1)$. $G(\cdot)$ is a CDF, e.g. the standard normal CDF. To obtain the conditional quantile estimate $\hat{q}_\alpha(x)$, we minimize the following objective function:[19]

$$\hat{q}_\alpha(x) = \arg \min_q |\alpha - \hat{F}(q|x)|. \tag{10}$$

4.4 Social segregation

Social segregation - that is the uneven distribution of social groups across schools - can be represented by a segregation curve or as a numerical measure.[20] The main drawback of segregation curves is that only an incomplete or partial ordering is provided. If segregation curves intersect, the segregation curve approach is silent about which distribution is more segregated (Hutchens, 2004). Unlike segregation curves, cardinal measures produce complete rankings. It is shown in Hutchens (2004) that a square root index is the only measure of segregation that satisfies seven properties needed for a complete and additive decomposable ordering.[21] We use the Hutchens (2004) square root index to study social segregation in

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[18]Inclusion of fixed school effects is not possible in this setting because it would induce an identification problem between the school level covariate S and the school level fixed effects (analogously to the multicollinearity problem in parametric models).

[19]An alternative is to use a check function approach as described in Koenker (2005) and Li and Racine (2007).

[20]A segregation curve plots the cumulative fraction of type 1 people (vertical axis) and type 2 people (horizontal axis), both fractions being ranked from low to high values of x_{1j}/x_{2j}, with x_{1j} (and x_{2j}) respectively the fraction of type 1 (2) in school j (Hutchens, 2004).

[21](1) *Scale invariance*: if N_1 and/or N_2 are multiplied by a positive scalar and the share of both types in the S schools remains the same then segregation does not change, with N_1 (N_2) the number of observations of type 1 (type 2) in the region in question. (2) *Symmetry in groups*: measure is unaffected if all the people in group i trade places with those in group j. (3) *Movement between groups*: for example, if a pupil with a low social position in a 'rich' school moves to a 'poor' school, segregation increases. (4) *Insensitivity to proportional divisions*. (5) *Additive decomposability*: the segregation measure can be decomposed in as sum

education - this is the segregation between socio-economic groups in schooling.

The Hutchens (2004) square root index is defined as the sum, over all schools, of each school's gap from proportional representation. As formulated in (11), for each school, this gap is the difference between the geometric mean of the proportional representation of children from different reference groups and the geometric mean of the actual group proportions.(Jenkins et al., 2008)

$$H = \sum_{i=1}^{S} \left[\sqrt{\frac{p_i}{P} * \frac{r_i^{\text{no seg}}}{R}} - \sqrt{\frac{p_i}{P} * \frac{r_i}{R}} \right],$$ (11)

with p_i the number of children with a low social position in school $i = 1, ..., S$. Low social position can be defined as the first, second or third quartile of ESCS distribution. r_i is the proportion of children with a high social position in school $i = 1, ..., S$. P and R are the proportions of children in the population with respectively a low and a high social position. If the low social position is defined as the first quartile, then $P = 0.25$ and $R = 0.75$. If there is no segregation, there is proportional representation of each group in each school so that $\frac{p_i}{P} = \frac{r_i}{R}$ in every school. Thus, (11) can be written as (12).

$$H = \sum_{i=1}^{S} \left[\frac{p_i}{P} - \sqrt{\frac{p_i}{P} * \frac{r_i}{R}} \right]$$ (12)

By the additive decomposability property, (12) can be written as (13). Social segregation can be decomposed as the sum of between-group segregation ($H_{between}$) and within-group segregation (H_{within}).

$$H = H_{within} + H_{between} \text{ ,where } H_{within} = \sum_{g=1}^{G} w_g H_g, \text{ with } w_g = (P_g/P)^{0.5}(R_g/R)^{0.5}, \quad (13)$$

with $g = 1, ..., G$ groups, w_g the weight of group g, P_g and R_g the number of pupils in group g with respectively a low and high social position.

As in Jenkins et al. (2008), as robustness check, the results are compared to estimates with the Duncan and Duncan (1955) dissimilarity index, also called the 'displacement index'. In this setting, the Duncan and Duncan (1955) dissimilarity index measures the fraction of pupils with low social position that would need to be displaced to 'rich' schools, without replacing them by other children, in order that every school has the same proportions of children with low and high social background.(Duncan and Duncan, 1955)

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of between-group segregation and within-group segregation.(6) *Symmetry in types*: the segregation measure is unaffected if pupils with a low social position are named group 1 or group 2. (7) *Index in interval* $[0, 1]$, with 0 no segregation and 1 full segregation.(Hutchens, 2004)

$$D = \frac{1}{2} \sum_{i=1}^{S} \left| \frac{p_i}{P} - \frac{r_i}{R} \right| \tag{14}$$

5 Results

5.1 The Extent of Inequality of Opportunity

In this section, we use a conditional distribution approach to investigate the existence of inequality of opportunity in schooling. We study inequality of opportunity in schooling, caused by socio-economic status (ESCS). The impact of migration status is handled in a multivariate regression approach in the following subsection. This because ESCS and migration status are highly related.

Visual inspection of Figure 2 shows that the distribution of pupils in higher ESCS quartile dominates by FSD the distribution of pupils in lower ESCS quartiles. This indicates that there exists inequality of opportunity in schooling associated to family background.

Table 3 shows the results from the Gini Opportunity (GO) estimates. In this univariate analysis, inequality of opportunity between pupils in 4 quartiles of ESCS is studied. For this, the dependent variable is the first principal component (FPC) of pupil performance on the PISA 2006 standardized tests for math, science and reading. Higher values indicate higher inequality of opportunity in schooling.

In the Flemish community, a Gini Opportunity of 0.016 is found, which is significantly different from zero. Consequently, we find evidence for a significant inequality of opportunity in schooling, caused by ESCS.

The robustness of this result is shown by estimation of EOp between 2 equal quantiles of pupils and 6 equal quantiles of pupils.[22] In addition, to test the sensitivity of the results for inclusion of migration status, we estimate Gini Opportunity for a sample of native pupils.

<Figure 2 about here >
<Table 3 about here >

5.2 The Impact of Social Segregation on Inequality of Opportunity

5.2.1 The Conditional mean regression

OECD (2006) shows that in Belgium 40.7 percent of the between-school variance in science

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[22]We thus split the groups at respectively the region ESCS median and at ESCS quantiles (1/6 , 1/3, 1/2, 2/3, 5/6).

performance and 1 percent of the within-school variation are explained by respectively the ESCS of schools and pupils. In the Flemish community, this is 38.6 and 1.9 percent. The OECD average is 20.5 and 3.8 percent. In sum, OECD (2006) shows that there are strong indications that the school ESCS has a large impact on inequality of opportunity in Belgium. This is in line with Jenkins et al. (2008), where Belgium is ranked as a high social segregation country. By implementing the Hutchens (2004) square root index on the PISA 2000 and 2003 datasets, Jenkins et al. (2008) show that only Hungary has higher social segregation than Belgium (H=0.142) in a sample of 30 OECD countries.

However, the effect of school ESCS is underestimated in PISA 2006. In the Flemish community, different tracks can coexists. Because the peer effects and institutional effects are mainly influenced by the situation within a given track, we expect a larger effect if we study the effect of sub-schools ESCS. For this - as mentioned in section 3 - we treat separately different tracks in the same school as different sub-schools.

We extend the PISA 2006 results in two ways. First, we allow for random (sub-)school effects by estimating a two-level regression model as discussed in section 4. As in OECD (2006), we choose an additive linear functional form (HLM model). With level 1 the pupil and level 2 the sub-school. Second, instead of investigating pupil performance in science, we proxy educational outcomes by the standardized first principal component of math, reading and science.

Results show firstly, that only 2.87 percent of within-school variation in performance is explained by ESCS and the proxies for migration status. If only ESCS of the pupils is considered, this is only 0.43 percent. The significant coefficients and explanatory power of the proxies of migration status indicate that migration status matters. In the Flemish community, non-native pupils perform significantly worse than other native pupils.

Secondly, and more related to the issue of social segregation, the results in Table 4 show that school ESCS can explain around 60 percent of the between-school variation. Pupils in a school with many pupils with an unfavorable family background perform on average lower. The school's average ESCS should not be interpreted as a pure peer effect, but rather as the effect of social segregation, which includes both peer effects and institutional effects. The large explanatory power of school ESCS indicates that social segregation with large variation in school ESCS is a powerful predictor of variation in educational achievements between schools. We find a large coefficient for school ESCS- respectively 1.70, which implies a 0.75 standard deviation disparity in educational outcomes between the first and third quartile school ESCS. Differently put, the large coefficient of school ESCS indicates large indirect effects of circumstances via the (self-)selection of pupils with a low (high) social position in schools where the social position of pupils is on average low (high). Obviously, it is not possible to say

whether they perform better because of the positive influence in the classroom (endogenous effect) or because they share similar unobserved favorable characteristics (correlated effect) (see Manski (1993)).[23]

<Table 4 about here >

5.2.2 The Conditional quantile regression

The nonparametric quantile regression approach brings finer results. In a preliminary regression, we constructed a model with 1 dependent variable (FPC of math, science and reading), 3 continuous covariates (school ESCS, pupil ESCS and social diversification index) and 3 discrete variables (dummy for first-generation immigrant, second generation immigrant and immigrant that does not speak official Belgian language at home). The dummies for first-generation and second generation immigrants were 'smoothed out' and are thus estimated to be irrelevant. In addition, no (interaction) effects was found for within-school diversification. Therefore, we re-estimated the nonparametric model with only 2 continuous covariates (ESCS and school ESCS) and 1 discrete covariate (Language at home). Figure 3 shows that we only find a small positive effect of pupil ESCS on the conditional first quartile output($q_{0.25}$), median output($q_{0.50}$) and third quartile output ($q_{0.75}$). For the school ESCS, profound effects on pupil performance are found. Figure 4 shows that the three respective conditional quantiles are a clear positive function of school ESCS. The finding of a strong effect of school ESCS is thus robust for altering the estimation methodology. Figure 6 illustrates further that the effect of ESCS is almost completely captured by the indirect effect of social segregation (school ESCS). Only where school ESCS is low and individual ESCS is high - and the data is thus very sparse- we find a strong positive direct effect of pupil ESCS. For the language spoken at home, we find small, but negative effects (see Figure 5). Thus the finding that immigration status has an effect besides the effect of ESCS is thus robust for altering the estimation methodology.

<Figure 3 about here >
<Figure 4 about here >
<Figure 5 about here >
<Figure 6 about here >

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[23]Gender and interaction between individual ESCS and school ESCS were excluded because no significant effect was found. Results on the effect of social segregation are robust for altering the definition of school composition to first-quartile, median and third quartile school ESCS and for the introduction of social diversification in the model.

5.3 The Impact of tracking on social segregation

Pupils have a low social position if they have ESCS below the median and they are with a high social position otherwise. In the descriptive statistics, we have shown that pupils with a low social position are unevenly distributed (1) amongst public and private-operating publicly funded schools and (2) amongst school tracks (general, technical-arts and vocational). In this section, we decompose social segregation into school types and into school tracks. For this, we decompose the Hutchens (2004) square root index as in (14) and (16). This with w the weight of the given subgroup, H_{group} the within-group Hutchens (2004) square root index of social segregation and $H_{between}$ the between-group social segregation.

$$H = w_{general}H_{general} + w_{technical-arts}H_{technical-arts} + w_{vocational}H_{vocational} + H_{between} \quad (15)$$

$$H = w_{public}H_{public} + w_{private}H_{private} + H_{between} \quad (16)$$

Table 5 shows that social segregation prevails. A Hutchens (2004) square root index (H) is found of 0.14 and a Dissimilarity index (D) of 0.39. This latter index is easy to interpret: 39 percent of pupils with low social position need to be displaced from 'poor' to 'rich' schools-without replacing them by other children - in order that every school has the same share of children with low and high social background.

School tracking is found to be a main driver of social segregation. Indeed, between-track segregation explains 50.89 percent percent of social segregation in schools. Decomposition of the Hutchens (2004) square root index shows that only 6.56 percent of social segregation can be explained by school type.

In sum, we find evidence by decomposition that social segregation is for a large part driven by school tracking.

As robustness test, we define differently pupils into low social positions as those in the first quartile ESCS. Results are given in Table 6. We find similar results with some variation in the size of the tracking impact on social segregation. The main conclusion that school tracking is far more important than school type in explaining social segregation is robust. However, social segregation between school types is now significantly different from zero.

<Table 5 about here >
<Table 6 about here >

6 Concluding remarks and discussions

In this paper we have investigated the importance of school tracking for inequality of opportunity in education. For this, we used Flemish pupil and school level data from the PISA 2006 dataset. Firstly, we have shown the existence of inequality of opportunity in schooling by stochastic dominance testing on conditional distributions. We found strong evidence for inequality of opportunity. Secondly, using a bootstrapped version of the Gini Opportunity index, evidence was found of the extent of inequality of opportunity in schooling in the Flemish community. Thirdly, we showed in a two-level regression (with school specific effect) that social segregation is a main driver of inequality of opportunity in schooling in Flanders. Over 60 percent of the variation between schools in educational outcomes can be explained by the variation in the school social composition. Using a conditional quantile regression approach, we showed that conditional quantiles are mostly influenced by the school socio-economic composition, with almost no influence of individual socio-economic status. This result suggests that the individual school opportunity set depends much more on the school memberships than on the individual family background. To link tracking to social segregation, we decomposed the Hutchens square root index of social segregation between tracks and within tracks. We find strong evidence for a crucial role of school tracking in explaining social segregation. Only 6.56 percent of social segregation can be explained by school type while the between-track segregation explains 50.89 percent of social segregation in schools. In sum, results show that school tracking is - via social segregation - detrimental for equality of opportunity in schooling in Flanders.

Although this paper shows the high association between social segregation, inequality of opportunity and school tracking, we cannot really provide a causal relation. The fundamental reason is that we cannot control for unobserved ability levels. For instance, it could be that there is no social bias in the track assignment after controlling for the cognitive ability of the students. Moreover the regression estimates can overestimate the impact of family background on (average) test scores if (unobserved) cognitive ability is positively correlated to parental educations. In fact it is now well documented that there is parental transmission of cognitive ability. See Holmlund et al. (2008), Plug and Vijverberg (2003) and various articles in Nature magazine. Our response to this genetic transmission issue is threefold. Firstly, it is fair to say, that there is a risk that this genetic transmission of ability will legitimate social inequality in education achievements as natural. Secondly, we have been less preoccupied with the ability transmission in this work because we have concentrated mostly on explaining average differences between groups and not between individuals. We have shown that the social composition of the school is a very powerful predictor of individual test scores. To

keep this result in perspective, it should be emphasized that average differences in cognitive ability between groups are small compared with the range of individual difference between groups. Thirdly, the purpose of our analysis was to contribute to the debate on school tracking by pointing to its possible societal implications (i.e. social segregation) and ethical issues (unequal access to knowledge). In this respect, we very much adhere to Judith Harris's conclusion in her book 'The Nurture Assumption' (1998) '*We may not hold their tomorrows in our hands but we surely hold their todays, and we have the power to make their todays very miserable*'.

7 Tables and Figures

Variable	Mean	S.E.
Output		
PISA 2006 Performance in math, filtered sample (FS)	555.940	(3.054)
PISA 2006 Performance in reading, FS	537.757	(5.644)
PISA 2006 Performance in science, FS	541.023	(2.611))
PISA 2006 Standard deviation of performance in math, FS	89.190	(1.464)
PISA 2006 Standard deviation of performance in reading, FS	92.401	(1.793)
PISA 2006 Standard deviation of performance in science, FS	84.007	(1.372)
Circumstances		
Economic and Socio-Cultural Status (ESCS)	0.272	(0.025)
Proportion of first-generation immigrants	0.030	(0.006)
Proportion of second-generation immigrants	0.026	(0.005)
Imm. that speak non-off. Belgian language at home	0.022	(0.004)
Educational system		
School type (public=1, private-operating=0)	0.263	(0.019)
General education	0.479	(0.016)
Technical-arts education	0.325	(0.014)
Vocational education	0.196	(0.012)
Grade 10	0.771	(0.008)
Age of school tracking	12	
Number of observations	4125	

Table 1: A SAS procedure for a Balanced Repeated Replication procedure with 80 replication estimates, described in OECD (2005), is used to construct the mean and standard error. Standard errors between brackets

Group	Pupils with low social position	Pupils with high social position
General education without lagging behind	0.272	0.575
	(0.251 , 0.293)	(0.552 , 0.594)
General education	0.313	0.643
	(0.293 , 0.334)	(0.621 , 0.664)
Technical or arts education	0.392	0.273
	(0.356 , 0.397)	(0.256 , 0.294)
Vocational education	0.310	0.085
	(0.289 , 0.330)	(0.071 , 0.096)
Lagging behind	0.283	0.160
	(0.262 , 0.304)	(0.143 , 0.177)
Public school	0.321	0.207
	(0.298 , 0.341)	(0.190 , 0.225)

Table 2: Bootstrap approach with 999 replications and 95% basic confidence intervals between brackets, package 'boot' in R
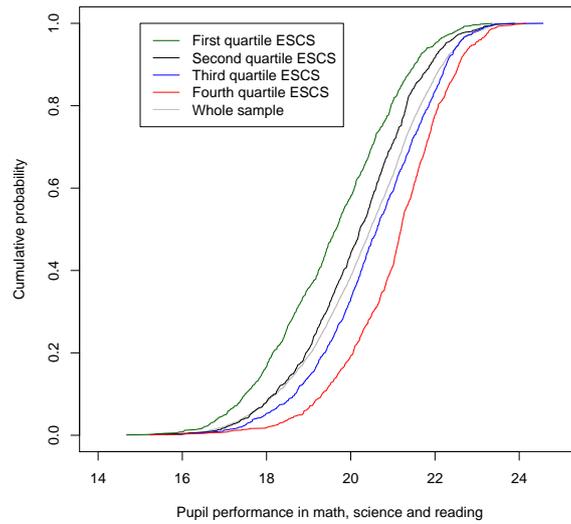

Figure 2: Conditional distribution of pupil achievement

Variable	Estimates
Gini opportunity (GO)$\times$ 100 - 4 groups	1.647
	(1.522 , 1.832)
Gini opportunity (GO)$\times$ 100 - 2 groups	1.270
	(1.152 , 1.416)
Gini opportunity (GO)$\times$ 100 - 6 groups	1.693
	(1.545 , 1.843)
Gini opportunity (GO)$\times$ 100 - 4 groups - native pupils	1.560
	(1.413 , 1.709)

Table 3: Bootstrapping with replacement, 999 replications, package 'boot' in R, 95 % basic confidence intervals between brackets

Variable	Model I	Model II
ESCS of pupil	0.121***	0.102***
	(0.026)	(0.024)
Sub-school average ESCS	1.752***	1.701***
	(0.117)	(0.121)
First-generation immigrant		-0.429***
		(0.126)
Second-generation immigrant		-0.443***
		(0.132)
Immigrant that does not speak official		
Belgian language at home		-0.568***
		(0.133)
Constant	19.675***	19.732***
	(0.059)	(0.057)
Log likelihood	-5966.299	-5919.216
Between-sub-school variation explained	59.532%	59.100%
Within-sub-school variation explained	0.428 %	2.869%
Number of level 1 units	4125	4125
Number of level 2 units	269	269

Significance levels : $*: 5\%$ $**: 1\%$ $***: 0.1\%$

Table 4: Dependent variable: first principal component of first plausible value of test scores reading, math and science. Standard errors between brackets. The model, estimated in STATA with GLLAMM, is a two-level model with level 1 the pupils and level 2 the sub-schools. We allow for clustering within strata. Final student weights are introduced as probability weights as proposed in Pfeffermann et al. (1998). To obtain between- and within-sub-school variation explained, we compare the model with only a constant with the model with explanatory variables, as suggested in OECD (2009).
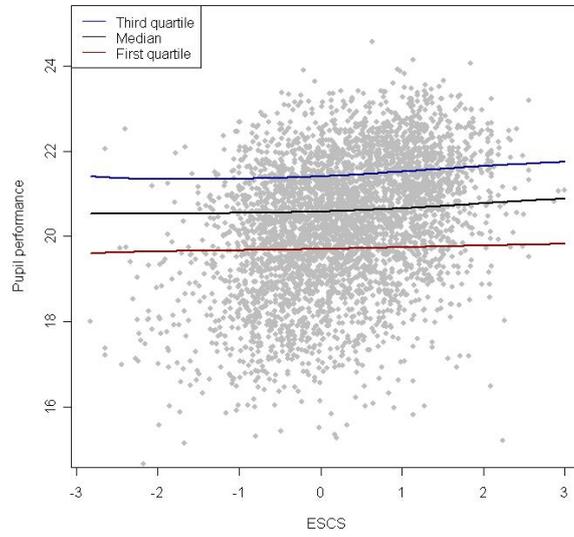
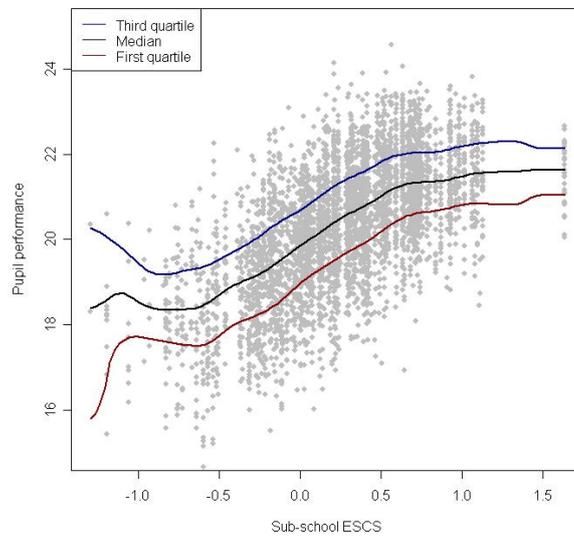**Figure 3:** Condtional quantile estimates: effect ESCS

Figure 4: Conditional quantile estimates: effect school ESCS
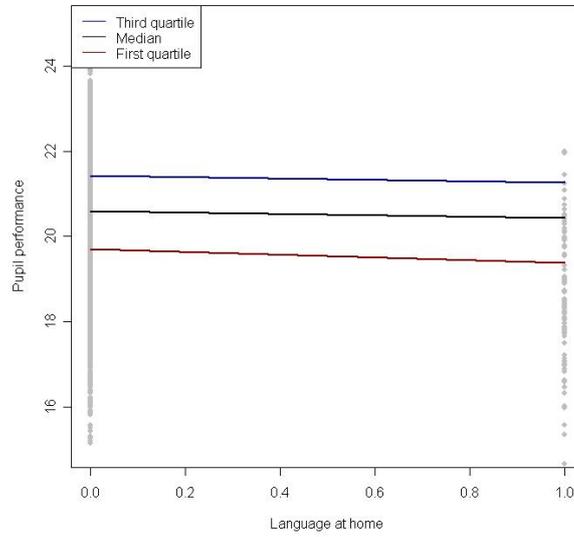
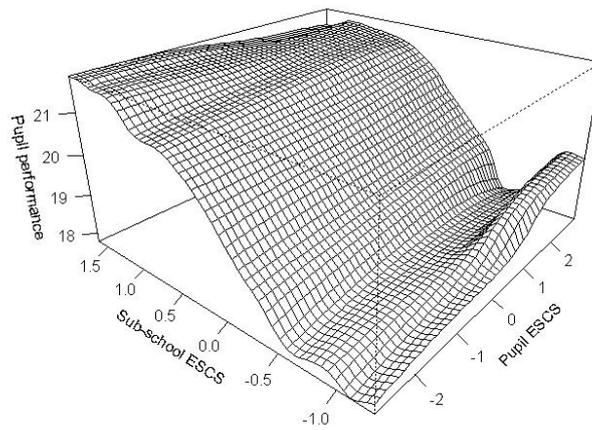**Figure 5:** Conditional quantile estimates: effect language at home

Figure 6: Conditional quantile surface: effect ESCS and school ESCS on median output

Segregation index	Estimates
Square root index (H)	0.135
	(0.115 , 0.158)
Dissimilarity index (D)	0.389
	(0.358 , 0.425)
In general education (H_{general})	0.060
	(0.044 , 0.077)
In technical-arts education ($H_{\text{technical-arts}}$)	0.051
	(0.038 , 0.064)
In vocational education ($H_{\text{vocational}}$)	0.144
	(0.098 , 0.192)
Within track segregation (H_{within})	0.066
	(0.055 , 0.080)
Between track segregation (H_{between})	0.069
	(0.051 , 0.085)
Within track segregation (H_{within} as % of H)	49.1 %
	(42.2 , 56.5)
Between-track segregation(H_{between} as % of H)	50.887 %
	(43.5 , 57.8)
In public schools (H_{public})	0.147
	(0.106 , 0.192)
In private-operating schools (H_{private})	0.121
	(0.097 , 0.146)
Within school type segregation (H_{within})	0.127
	(0.107 , 0.148)
Between school type segregation (H_{between})	0.009
	(-0.001 , 0.014)
Within school type segregation (H_{within} as % of H)	93.4 %
	(89.4 , 100.5)
Between school type segregation(H_{between} as % of H)	6.6 %
	(-0.5 , 10.6)
Sample of sub-schools	269
Sample of pupils	4125

Table 5: Bootstrapping with replacement, 999 replications, package 'boot' in R. 95% basic confidence intervals between brackets

Segregation index	Estimates
Square root index (H)	0.168
	(0.139 , 0.200)
Dissimilarity index (D)	0.413
	(0.377 , 0.453)
In general education (H_{general})	0.159
	(0.111 , 0.210)
In technical-arts education ($H_{\text{technical-arts}}$)	0.075
	(0.053 , 0.095)
In vocational education ($H_{\text{vocational}}$)	0.105
	(0.065 , 0.143)
Within track segregation (H_{within})	0.109
	(0.090 , 0.132)
Between track segregation (H_{between})	0.059
	(0.039 , 0.075)
Within track segregation (H_{within} as % of H)	65.0 %
	(57.8 , 73.8)
Between-track segregation(H_{between} as % of H)	35.0 %
	(26.2 , 42.2)
In public schools (H_{public})	0.145
	(0.086 , 0.195)
In private-operating schools (H_{private})	0.160
	(0.128 , 0.197)
Within school type segregation (H_{within})	0.154
	(0.124 , 0.181)
Between school type segregation (H_{between})	0.014
	(0.003 , 0.024)
Within school type segregation (H_{within} as % of H)	91.4 %
	(86.0 , 97.9)
Between school type segregation(H_{between} as % of H)	8.6 %
	(2.1 , 14.0)
Sample of sub-schools	269
Sample of pupils	4125

Table 6: Pupils with low social position are defined as pupils with an ESCS score below the first quartile of ESCS in the region. Bootstrapping with replacement, 999 replications, package 'boot' in R. 95% basic confidence intervals between brackets

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