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Market coupling and the organization of counter-trading: separating energy and transmission again?

Giorgia OGGIONI [1] and Yves SMEERS[2]

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Abstract

The horizontal integration of the energy market and the organization of transmission services remain two open issues in the restructured European electricity sector. The coupling of the French, Belgian and Dutch electricity markets (the trilateral market) in November 2006 was a real success that the inclusion of Germany to the trilateral market should soon prolong. But the extension of market coupling whether in Central Western Europe or in other European regions encounters several difficulties and the future remains far from clear. The highly meshed grid of continental Europe complicates things and it is now sometimes recognized that the penetration of wind will further exacerbate these difficulties. The nodal system could go a long way towards solving these problems, but its implementation is not yet foreseen in the EU.
This paper analyzes versions of market coupling that differ by the organization of counter- trading. While underplayed in current discussions, counter-trading will become a key element of market coupling as its geographic coverage expands and wind penetration develops. We consider a stylized six node example found in the literature and simulate market coupling for different assumptions of zonal decomposition and coordination of TSOs. We show that these assumptions matter: market coupling can be quite vulnerable to the particular situation on hand; counter-trading can work well or completely fail depending on the case and it is not clear beforehand what will prevail. Our analysis relies on standard economic notions such as social welfare, Nash and Generalized Nash equilibrium. But the use of these notions is probably novel. We also simplify matters by assuming away strategic behaviour. The nodal organization is the reference first best scenario: different zonal decompositions and degrees of coordinations are then studied with respect to this first best solution.

Keywords: market coupling, counter-trading, European electricity market, Generalized Nash equilibrium.

JEL Classification: D52, D58, Q40

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1 Introduction

Congestion management is still an open issue in the restructured European electricity sector. Grid congestion occurs when available infrastructure capacities, in parts of the transmission system, constraint the energy market. Grid congestion evolves with short and long term changes (increase and decrease) of generation and consumption and with the development of the grid infrastructure. One can deal with congestion by resorting to an organization that clears the energy market while remaining within infrastructure capacities: parts of the grid are congested, but in a way that does not endanger its security. This is the case of the so-called nodal pricing or flow-based system. The nodal system clears the energy market while simultaneously taking into account the real possibilities of the grid: it fully integrates energy and transmission and is thus the perfect implementation of “implicit auction” (EU parlance refers to implicit auction, but the EU does not implement nodal pricing). The theory of the model goes back to Hogan [20] and its first implementation to 1998 in the PJM market. The model has since worked successfully in several regions of the US and in New Zealand (see the country survey of Sioshansi and Pfaffenberger [29]). Due to the perfect integration of energy and transmission, Power Exchanges (PXs) and Transmission System Operators (TSOs) form a unique entity (an ISO in U.S. parlance) that clears both the energy and transmission markets in order to maximize social welfare while satisfying energy balance and transmission constraints. In this market organization, congestion costs are directly passed into the electricity prices and contribute to differentiate them over nodes.

Other market designs separate energy and transmission markets and consequently the action of the of PXs and TSOs on the respective market. This is what is currently happening in Europe where PXs organize the energy market on the basis of the “Market Coupling” rules; while TSOs operate counter-trading in the transmission market. Market coupling organizes congestion management on a zonal basis. This means that energy market is split in several zones controlled by different PXs. These zones are linked by (possibly capacitated) inter-connectors that provide a simplified representation of the grid. Transmission System Operators announce the capacity of these inter-connectors capacities to PXs before the clearing of the energy market. The energy market clears on the basis of these capacities and the resulting injections and withdrawals are communicated back to TSOs. If the capacities announced by system operators before the clearing are small enough compared to the real possibilities of the grid, the energy market clears without creating congestion. If not, overflows occur and TSOs will then intervene and restore grid feasibility buying incremental or decremental power flows from consumers and generators in order to restore the feasibility of the grid. These operations are denoted as counter-trading or re-dispatching. Counter-trading costs that arise from the TSOs’ activities are charged to users of the grid. These represent the link between the energy and transmission markets.

Market coupling deals with spatial arbitrage between different markets, which is thus also the subject of this paper (a work on inter-temporal arbitrage in a wind intensive market is underway). The basic idea of the mechanism is depicted in Figure 1.

Consider two markets North (N) and South (S) with supply and demand bids in each of them. Assume that there are two generators in N. Denote them as “gen1” and “gen2”. Gen1 disposes of a 100 MW plant whose marginal cost is 5 €/MWh, while gen2 can at maximum run 400 MW at a marginal cost of 20 €/MWh. They have to satisfy a demand of 200 MWh. In

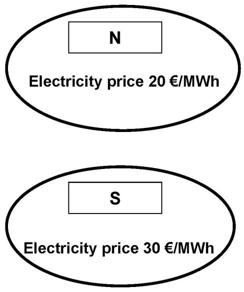

Figure 1: North (N) and Southern (S) zones

market S, there are other two generators "gen3" and "gen4". Both generators have an available capacity of 300 MW, but gen3 has a marginal cost of 20 €/MWh, while gen4 operates at a marginal cost of 30 €/MWh. The electricity demand in S amounts to 600 MWh. Note that generation in N is cheaper than in S.

Consider first the equilibrium in each market taken in isolation as depicted in Figure 1. The equilibrium price in market S is higher than the equilibrium price in market N as shown in Figure 1. This creates an arbitrage opportunity between the two markets: electricity should move from market N to S. Suppose now that the two markets are linked by a Transfer Capacity (TC) as depicted in Figure 2. If this transfer capacity is large enough the usual arbitrage reasoning will imply a flow between the N and S markets that equalizes the prices in both markets: this is shown on Figure 2a. If this transfer capacity is limited, the arbitrage will be limited to what the TC allows (in our case 250 MWh, as illustrated in Figure 2b) and the electricity prices will be zonal.

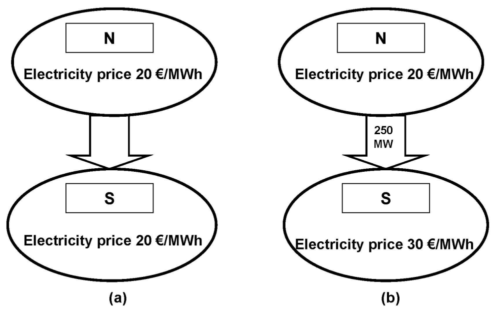

Figure 2: North (N) and Southern (S) zones with TC

The following section describes the successes and the failures of the application of market coupling and counter-trading in the European energy system.

2 Market coupling in the European energy system

The coupling of the Belgian, Dutch and French electricity markets (Market Coupling or MC, or the "trilateral market" when limited to these three countries) went live on 21 November 2006. The three countries are part of "CWE", the Central-Western Region of the European Regional Initiative or "ERI" (see European Energy Regulators or ERGEG web site) that also comprises Germany and Luxembourg. The coupling of these three markets was probably the first achievement of the ERI and became the success story of the restructuring of the European electricity sector. Market coupling quickly led to the convergence of the prices on the power exchanges of the trilateral market (Den Ouden [5], Dijkgraaf and Janssen [7]), which in turn showed the efficiency of "implicit auctions" (a partial integration of energy and transmission markets) compared to so-called "explicit auctions" that fully separate energy and transmission. Implicit auctions eventually became the preferred option for cross border trade in Regulation No 714/2009 (European Commission [13]) which is part of the third legislative package that will become law in March 2011. Other Market Coupling projects are now foreseen: the linkage of Germany to the existing trilateral market in CWE should be achieved on 7 September 2010[1]; this should pave the way to a new phase of the coupling of both Eastern and Western Denmark to Northern Germany (the coupling of DK East and West with Germany went to different phases since its first launching in 2008). A new MC project is also foreseen between Poland, the Czech Republic and Austria (see EXAA [15]); last a more ambitious project aims at coupling the Scandinavian, Spanish, Italian markets with CWE[2].

But the success story has its limits. The coupling of the Danish and North German markets initiated in October 2008 had to be interrupted after one week of operations (DERA [6]) and was only reactivated in November 2009[3]. A project to improve the current trilateral system in CWE through the introduction of a better representation of the transmission system[4] failed to meet its deadline target in 2009, without clear indication of what went wrong and how one is now trying to fix the problem. The completion of the current MC in CWE by the inclusion of Germany, now foreseen for 7 September 2010, has also been delayed a few times, again without much explanation besides that it is "a challenging project". This will in turn require relaunching the Danish-German coupling. Somewhat hidden behind these headlines, domestic congestion management remains as opaque as ever: Duthaler et al. [8] use AC load flow simulations on the UCTE area to show the important role of domestic congestion in the management of the European grid. But companies hardly confirm. RTE's and Elia's annual reports (RTE [28], Elia [9])) barely mention congestion. The Dutch operator seems to be much more concerned (TenneT web site includes a whole discussion of congestion management), but these concerns are not necessarily shared: Groeneveld speaking in the name of E.ON Benelux (see E.ON Benelux [10]) explains that Dutch worries about congestion in the Netherlands, as studied in D-Cision and The Brattle Group' report [3], are largely unfounded ("congestions should be incidental in

[1] For more details see TenneT [32] and EPEX Spot [11].

[2] See EPEX Spot [12].

[3] See ICIS Heren. 2010. News. CWE/Nordic market coupling seeks "stop-gap" measures. Available at http://www.icis.com/heren/articles/2010/03/08/9340801/cwenordic-market-coupling-seeks-stop-gap-measures.html

[4] See the "Memorandum of Understanding of the Pentalateral Energy Forum on Market Coupling and Security of Supply in the Central Western Europe". Available at:
http://www.benelux.be/pdf/pdf_nl/dos/dos14_PentalateralMoUMarketCouplingAndSecurityOfSupply.pdf

nature and limited in size"). In contrast, EWIS [14] analyzes wind development in CWE and explains that congestion will make the situation unsustainable without additional investments in the grid. All this sounds like "déjà vu" and reminds us of now outdated discussions in California (well before the meltdown) and more recently in Texas (before the decision to go nodal). In both cases local congestion and counter-trading turned out to be much more important than initially foreseen.

Difficulties with Market Coupling are hardly surprising: while the linkage of the Belgian, Dutch and French markets is a true milestone in the lengthy progress towards an internal European electricity market, the overall approach suffers from several design flaws that cannot, but progressively materialize as one moves ahead. These difficulties should not endanger the more ambitious coupling of the Scandinavian, Spanish and Italian markets with CWE provided the integration of CWE is successful: indeed, the radial coupling of CWE with Scandinavia and the Iberian and Italian peninsulas should be relatively easy to handle compared to the integration of the meshed CWE. But a zonal Market Coupling in a highly meshed system such as the one of CWE is effectively a "challenging project" and the integration of wind power at the level stated in EU objectives in that meshed system may indeed make the current market design "unsustainable".

System integration is at the core of the internal electricity market. It is based on both spatial and inter-temporal arbitrage of electricity and its financial derivatives. The analysis conducted in the Introduction on a two zone market can be applied to the Central Western European electricity market. The implementation of market coupling generalizes that framework to the current trilateral market including France (F), Belgium (B) and the Netherlands (NL) as shown on Figure 3a, and after the coupling with Germany (G) to a four country system market (referred to as the pentalateral market as Luxembourg has part of this system integrated both with Germany and Belgium[5]) as depicted in Figure 3b.

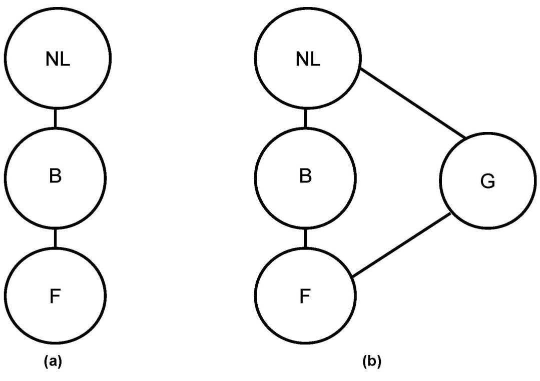

Figure 3: Stylized representation of the Central Western European power market

The notions of arbitrage and transfer capacities introduced in Figure 2 extend as such to the more general cases of Figure 3 as well as to the future extension to the Nordic market and the

[5]See again "Memorandum of Understanding of the Pentalateral Energy Forum on Market Coupling and Security of Supply in the Central Western Europe". Available at:
http://www.benelux.be/pdf/pdf_nl/dos/dos14_PentalateralMoUMarketCouplingAndSecurityOfSupply.pdf

two Southern peninsulas. Leaving aside issues related to the representation of the characteristics of the generators such as bloc bids (see Appendix A for a short discussion of the comparison of the treatment of machine characteristics in US and EU systems), the question raised by market coupling boils down to the determination of the transfer capacities linking the zonal systems and to whether electricity transmission can really be represented by transfer capacities. The relevance and importance of these questions is well acknowledged. Hogan in [21] quotes the US federal electricity regulator stating that transfer capacities are artificial constructs without economic or physical reality that were inherited from the regulatory period when trade was not a key matter. Less assertively, European Transmission System Operators cautiously advice that they do not guarantee the validity of the transmission capacities that they publish. Last but not least, one can observe that it is now several years that TSOs postpone the publication of their method for computing transmission capacities. At least we know some of the principles that they use (see Rious et al. [26] for a detailed explanation) and will invoke them later in the discussion.

In short and barring questions of bloc bids and inter-temporal arbitrage that we do not consider here, the problem of spatial arbitrage in Market Coupling concentrates on the transmission system, whether represented by transfer capacities or otherwise, and its saturation. This is commonly referred to as congestion and occurs when one element of this grid is saturated because of the transactions in the energy market. This paper therefore deals with congestion management in market coupling. It is organized as follows. Section 3 presents a test problem that consists of two configurations of a two-zone system such as commonly found in the papers of the Transmission System Operators (see Entso papers on ENTSO-E website[6]). Section 4 provides further discussion on market coupling and describes different possible organizations of counter-trading to relieve congestion when it occurs. Numerical results of the nodal pricing model and the market coupling and counter-trading problems applied to these two different market configurations are respectively discussed in Sections 5, 6 and 7. Finally, Section 8 summarizes the finding of the analysis.

3 The test problem

We analyze the economic problems raised by Market Coupling on the basis of a six node network taken from Chao and Peck [2]. According to Stoft [30], we conduct the whole analysis on an hourly basis or in MWh. Capacities are thus expressed in MWh and not in MW. The system is depicted on Figure 4. Its functioning is explained in the following subsections.

3.1 The energy market

The network connects six nodes where generators and consumers are located. Power is injected in nodes 1, 2 and 4 while withdrawals are in nodes 3, 5 and 6. Power production and consumption economics are summarized by linear marginal costs and inverse demand functions. These are listed in Table 1.

[6]http://www.entsoe.eu/index.php?id=80

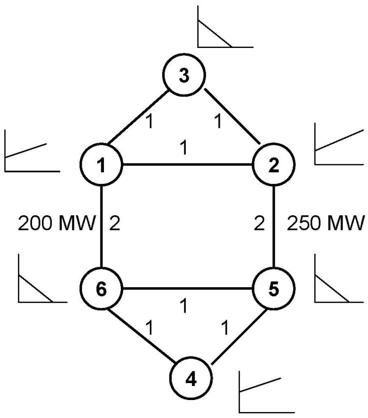

Figure 4: Six node market (Chao and Peck (1998))

Node	Function Type	Function
1	Marginal Cost	10+0.05q
2	Marginal Cost	15+0.05q
3	Inverse Demand	37.5-0.05q
4	Marginal Cost	42.5+0.025q
5	Inverse Demand	75-0.1q
6	Inverse Demand	80-0.1q

Table 1: Inverse demand and marginal cost functions of the 6 node market

3.2 The grid

The network contains eight lines and it is assumed to be without loss. All lines have impedances equal to 1 except lines (1-6) and (2-5) that have impedances equal to 2. Similarly all lines have unlimited capacities except lines (1-6) and (2-5) that respectively have capacities equal to 200 and 250 MWh. The network model is a standard representation of load flow equations through a Power Transmission Distribution Factor (PTDF) matrix. Defining a hub node where the market clears (here node 6), the PTDF of a certain node for a certain line is equal to the portion of the flow injected at that node and withdrawn at the hub, which passes through that line of the network. The PTDFs of the hub (node 6) are all zero. We are only interested in the PTDFs involving lines (1-6) and (2-5) since these are the only ones with limited capacities. These PTDFs are given in Table 2.

Power (1 MWh) Injected at Node	Power flow on link 1 → 6 (MWh)	Power flow on link 2 → 5 (MWh)
1	0.625	0.375
2	0.5	0.5
3	0.5625	0.4375
4	0.0625	-0.0625
5	0.125	-0.125
6 (hub)	0	0

Table 2: PTDF of the 6 node market

4 Market coupling and different possible counter-trading organizations

4.1 Market coupling

Consider the market in Figure 4 and split it into two zones, North and South, that are linked by an inter-connector that may have a limited transfer capacity. We consider two decompositions of the network into zones. The first is depicted in Figure 5 and we denote it as the (3-3) two zone problem. In the (3-3) configuration, North includes nodes 1, 2 and 3, while nodes 4, 5 and 6 are in the South.

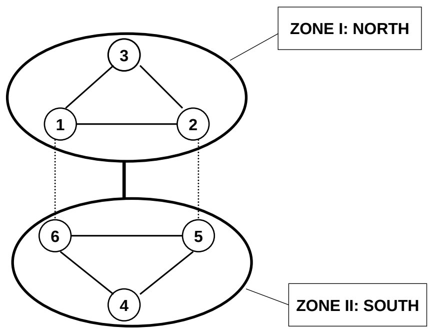

Figure 5: (3-3) two zone configuration

Figure 6 depicts an alternative zonal decomposition. We refer to it as the (4-2) configuration since the Northern zone includes four nodes (1, 2, 3 and 6), while the Southern zone only two (4 and 5).

In both configurations, the two zones are linked by an inter-connector with a limited transfer capacity that we set according to a method presented in Section 4.2. In the (3-3) market organization, there is no transmission constraint inside the zones; in the (4-2) line (1-6) with a capacity of 200 MWh is now internal to the Northern zone. Recall that the inter-connector stands for the simplified representation of the grid with which TSOs provide PXs.

Each zone is controlled by one TSO and one PX. We assume, in line with the current

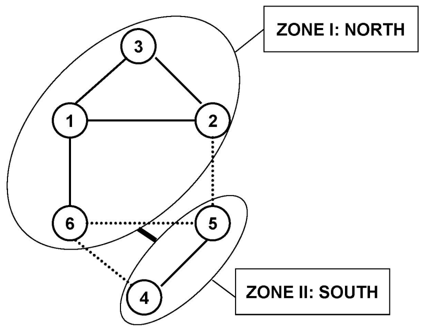

Figure 6: (4-2) two zone configuration

implementation of Market Coupling that the PXs of the two zones clear the energy market in a coordinated way. In these simple models where one does not consider complications such as bloc bids, this amounts to solving a welfare maximization problem over the six nodes subject to the sole constraints on imports/exports between the two zones, that is excluding constraints on domestic lines in the zones (see Appendix C). As already explained in the Introduction, PXs clear the market taking into account this simplified representation of the network. The resulting flows may or may not be compatible with the physical characteristic of the network and then TSOs operate counter-trading in order to eliminate overflows and restore feasibility.

4.2 Computing transfer capacities: the principles

Transfer capacities are a key element of the organization of the European electricity system. The notion is controversial: the US Federal regulator recognizes that it lacks both physical and economic senses (Hogan [21]). European systems operators are not as affirmative, but they carefully announce "Indicative Values for Net Transfer Capacities" that they further qualify as "non-binding". The EU Directorate General for Competition seems to be the only one to firmly believes in transfer capacities that it uses in its different proceedings.

System operators have so far refrained from publishing the details of their method for calculating ATC, but the overall principle is known: it is the worst case analysis of the possible flows on the lines. We follow Rious et al. [26] description of the approach, taking advantage of the simplistic two-zone nature of our problem and further simplifying the analysis by dropping all considerations of N-1 security criterion. These authors first explain that "*The TSO knows only the difference between total generation and total load on its control area*" and "*he must anticipate the nodal sharing of total generation and load to forecast the base case power flows*". In other words, the TSO must first estimate the power flow that would exist before any cross border trade. TSOs can only do this on the basis of their own experience with their grid. Rious et al. [26] therefore cannot offer any suggestion on how to get the "base case power flow". We accordingly set the tentative base case power flow to zero; in other words, we assume that domestic transactions do not imply any significant flows on the interconnections. The computation of the ATC then boils down to the following very simple considerations. Selecting a pair of nodes, each located in a zone, one considers an import/export between the two zones originating in one of these nodes and ending in the other. The transfer capacity is then the maximal import/export that can be accommodated by the interconnections whatever the originating and ending nodes.

Consider first the case of the (3-3) configuration. It is easy to see that the worst possible loading of the inter-connection is obtained by exporting from node 1 and importing in node 6. The inter-connection results from the combination of lines (1-6) and (2-5). Since the flow is asymmetric, it puts a maximal load on line (1-6) that has the smallest capacity, that is 200 MWh. A unit export from N to S will then imply a flow on line (1-6) computed as $PTDF_{1,(1-6)} - PTDF_{6,(1-6)} = 0.625 - 0 = 0.625$ (see Table 2 for the PTDF values). The ATC between the two zones is then $200/0.625 = 320$ MWh. It is easy to verify that any net injection from North to South in configuration (3-3) that is smaller than 320 MWh will be feasible for the interconnection consisting of lines (1-6) and (2-5), at least if domestic transactions inside the zones do not entail further loading on these lines.

Consider now the case of the (4-2) configuration. The worst possible loading of the inter-connection is obtained by exporting from node 2 and importing in node 5. The flow put a maximal load on line (2-5) that has capacity of 250 MWh, while the other lines composing the inter-connection have infinite capacity. A unit export from N to S implies a flow on line (2-5) computed as $PTDF_{2,(2-5)} - PTDF_{5,(2-5)} = 0.5 + 0.125 = 0.625$. The ATC between the two zones is then $250/0.625 = 400$ MWh. Any net injection from North to South in configuration (4-2) that is smaller than 400 MWh will be feasible for the interconnection consisting of line (2-5), at least if domestic transactions inside the zones do not entail further loading on (2-5). Referring to the legal definition of congestion (Article 2 Paragraph 2 in Regulation No 714/2009 (European Commission [13]) any "international trade" lower than 320 and 400 MWh respectively will not exceed the "capacity of the inter-connectors and of the national transmission system concerned". This check applies to both configuration (3-3) and (4-2). As we shall see when discussing case (4-2), things may not be as simple as soon as the impact of domestic transactions needs to be taken on board, that is, as soon as one accounts for the "base case power flow".

4.3 Organizing counter-trading

The principle underlying the computation of transfer capacities is that a clearing of the energy market that satisfies the constraints on transfer capacities should dispense of counter-trading, at least as far as congestion on the interconnections are concerned. The implicit accompanying principle is that counter-trading efforts should thus be restricted to removing intra-zone congestions and the standard argument is that these should be minor. We abstract from this common wisdom and argue that transmission capacities computed on the basis of the worst case analysis unduly restrict the transmission possibilities offered to the energy market. We here introduce different organizations of counter-trading approaches that we analyze in terms of their economic implications in later sections. Recall that counter-trading always take the transactions resulting from the clearing of the energy market by the PXs as given: it does not change the transactions concluded on the energy market.

4.3.1 Perfectly coordinated counter-trading

We first consider a model of perfectly coordinated counter-trading where both TSOs cooperate in order to remove line overflows at minimal cost. The associated optimization problem is presented in Appendix E. Everything happens as if there were one global optimizer. Note that there is no indication in the literature that this is the way European TSOs operate. This is in fact an ideal situation in terms of the organization of the counter-trading market.

4.3.2 Uncoordinated Counter-Trading

Except for the recent horizontal integration resulting from mergers and acquisitions (TenneT and Elia respectively acquiring E.ON and Vattenfall grids), TSOs still operate on an independent basis. We model this uncoordinated behaviour as a Nash equilibrium: a TSO operates taking the actions of the other TSOs as given. This can be done under different assumptions: even though the two TSOs physically share the network because of Kirchhoff's law, this sharing may be organized through a market or left to non-market forces. In other words, there may be a market trading the services of the line capacities (recall that we resort to the flowgate language and hence to line services because this is the way European TSOs are now thinking when they go beyond transfer capacities); alternatively this market of line services may be missing. The outcome of a market of line capacities is rather clear: there is a single price for each line capacity that applies to each TSO. The situation is quite different when the market of line capacities is missing. Both TSOs still value the services of the line capacities. But there is no market for arbitraging valuation differences and arriving at a single price. Valuations of line capacities become parameters that identify particular equilibria in an incomplete market. Agent specific valuations of a common resource is a characteristic of an incomplete market. In economic terms, this can be modeled as a Generalized Nash Equilibrium (see Debreu [4], Arrow and Debreu [1], Rosen [27]).

From a mathematical point of view, a Generalized Nash Equilibrium (GNE) can be formulated as a Quasi-Variational Inequality (QVI) problem (see Facchinei and Pang [17], Harker [19]). This problem usually admits multiple solutions and Facchinei and Kanzow [16], Fukushima [18] and Nabetani et al. [22] give methods for exploring the range of these solutions. Oggioni and Smeers [24] and Oggioni et al. [25] apply that literature to the particular model of Market Coupling with a missing market of line capacities with the view of exploring the outcome of different key assumptions of market design. In Appendix F, we briefly present the developed models. Specifically we examine the impact of the horizontal integration of the market of counter-trading and the possible impact of the absence of a market of line capacities. The following introduces these different assumptions that we here explore in terms of their consequences on market efficiency.

Un-coordinated counter-trading Model 1 In this first case, we assume that each TSO has access to all counter-trading resources. This means that TSO^N can buy re-dispatch services from generators and consumers located both in the Northern and Southern areas. TSO^S can do the same. Each TSO trades these counter-trading services within the network constraints, taking the line utilization due to the other TSOs as given. In contrast with Section 4.3.1, there is no common management of the network constraints but two separate managements, each run by each TSO on the residual network resulting from the actions of the other TSO (the Nash assumption). This duality of grid constraints sets the difference between this model and that described in Section 4.3.1. Our numerical findings show that when both TSOs have access to all counter-trading resources at the same price (there is a internal market for these resources), and network characteristics are common knowledge for all TSOs, they also assign identical values to the congested lines and the solution of the problem is as in the coordinated case. This means that an internal market of counter-trading resources also implies an internal market of line capacities, irrespectively of whether one explicitly introduces this market of line services or not. This also implies a single equilibrium solution. The mathematical interpretation of these results is extensively explained in Oggioni et al. [25].

Un-coordinated counter-trading Model 2 Assume now that TSOs have a limited access (upper and lower bounds) to the counter-trading resources located outside of their control area. This suggests two outcomes. If TSOs do not hit their quota of counter-trading services in the other jurisdictions, the preceding reasoning (see "Un-coordinated counter-trading Model 1") applies and the problem turns out to be identical to the one of Section 4.3.1. In other words, one returns to the situation of perfect coordination whether one introduces a market of line services or not. When instead this quota is hit, then some TSOs face a scarcity rent on counter-trading resources with the consequences that their prices are no longer the same for the two TSOs. The consequence, discussed in [25], is that the preceding reasoning explained in the "Un-coordinated counter-trading Model 1" paragraph no longer applies. A partial integration of the counter-trading resources therefore does not imply an implicit market of line services. One thus needs to distinguish two cases depending on whether one explicitly introduces this market or not. TSOs identically value line capacities when this market exists; they may have different valuations when this market is missing; this latter case in turns leads to multiple Generalized Nash Equilibria as a function of the difference of line valuations by the TSOs. The mathematical fundamentals of these outcomes are illustrated in Oggioni et al. [25].

Un-coordinated counter-trading Model 3 Un-coordinated Model 3 can be considered as the most extreme case of our analysis. TSO can only access the counter-trading resources of their own area while taking the actions of the other as given. The preceding reasoning of an implicit market of line capacities does not apply by definition (see paragraph "Un-coordinated counter-trading Model 1"). One thus needs to distinguish whether one explicitly introduces a market of line services or not. In this latter case congested lines are priced differently, which, together with the segmentation of the market of re-dispatching resources, make the counter-trading market fully un-coordinated.

4.4 Combining Market Coupling with Counter-Trading

Counter-trading has a cost that must be paid by participants to the market. This cost is passed through the access charge to the grid in the European system. Because we do not identify the access charge in this study we instead assume that the cost of counter-trading is passed through a charge for accessing the PX. Market coupling therefore requires a combined action of PX and TSOs: while PXs always solve the same problem (see Appendix C and Oggioni et al. [25] for the mathematical formulation), the two TSOs operate counter-trading under different hypotheses of coordination as presented in the previous sections. Fully solving the market coupling problem requires finding an equilibrium between PX and TSOs whereby the cost incurred by TSOs is properly passed to the participants to the PXs. This means solving a fixed point problem where the average re-dispatching cost that we denote as α is added to the marginal cost of the generators. The computation of this fixed point is described in Appendix D.

5 Results of the nodal pricing model

Due the efficient integration of the energy and the transmission market, we take the nodal pricing model presented in the Introduction as reference. Results indicated in Tables 3 are obtained by solving the welfare maximization problem presented in Appendix B and applied to the network depicted in Figure 4.

Consumers in nodes 3, 5 and 6 require respectively 200, 300 and 300 MWh of electricity. The consumption in nodes 5 and 6 is higher than in node 3. Note that electricity demand in these nodes is mainly supplied by generators located in nodes 1 and 2 where power production is cheaper (compare prices in Table 3). The global export of 400 MWh congests line (1-6) in the positive direction[7], and its marginal value becomes 40 €/MWh. The situation is globally efficient and social welfare amounts to 23,000 €.

	Demand (MWh)	**Generation (MWh)**	**Prices (€/MWh)**
node 1		300	25
node 2		300	30
node 3	200		27.5
node 4		200	47.5
node 5	300		45
node 6	300		50

Table 3: Demand, Generation and Power Prices of the nodal pricing model

6 Results of market coupling models

The clearing of the energy market is the first step in the market coupling/counter-trading approach. We here show how the available transfer capacity of the inter-connectors can be computed and to which extend this affects the clearing of the energy market. Recall that PXs find the equilibrium on the energy market taking into account the available transfer capacity that TSOs provide them.

6.1 Market coupling: Computing transfer capacities

Table 4 and 5 report results of market coupling for different values of available transfer capacity (ATC) respectively computed for the (3-3) and (4-2) market configurations. Counter-trading, if necessary, is always performed in a perfectly coordinated way (as explained in Section 4.3.1), that is in the most efficient solution (global minimization of counter-trading costs). Both tables give the total welfare computed in the Market Coupling (MC) problem, the congested lines, the welfare at equilibrium when counter-trading is performed, the average re-dispatching cost and the welfare losses computed with respect to the nodal pricing model.

[7]Hereafter the directions 1 to 6 and 2 to 5 are assumed to be positive.

6.1.1 Configuration (3-3)

Recalling that the cross border flow from North to South in the nodal system amounts to 400 MWh, one notes that this is also the ATC value for which line (1-6) gets congested in Market Coupling. It is also the value leading to the smallest loss of welfare. Note that this is only a numerical finding, we do not know of any property that would prove that an ATC equal to the flow computed in nodal pricing is optimal for reducing welfare losses in a system like Market Coupling. We reasoned before that the approach derived in Section 4.2, which reflects the method followed by TSOs, leads to a transmission capacity of 320 MWh. This would not congest the interconnection lines, but would lead to a welfare loss of 1,519.167 € with respect to the nodal pricing model. Last, we also note that an ATC of 450 MWh, which is the sum of the capacities of the two lines, leads to a small welfare loss of 30.164 € when counter-trading is efficient. Taking stock of these findings, we conduct our analysis for ATC respectively equal to 320, 400 and 450 MWh. An ATC of 320 MWh does not require counter-trading as one can see from Table 4 and hence needs no further elaboration. ATCs of 400 and 450 MWh require moderate counter-trading when efficiently performed, that is, in a fully coordinated way. In Section 7, we examine the impact of both perfect and imperfect counter-trading models on these two cases.

ATC	**Welfare MC**	**Congested line**	**Welfare at equilibrium**	**Average Re-dispatching**	**Welfare loss**
0	12,520.833	-	-	-	10,479.167
50	14,145.833	-	-	-	8,854.167
100	15,687.500	-	-	-	7,312.500
150	17,145.833	-	-	-	5,854.167
200	18,520.833	-	-	-	4,479.167
250	19,812.500	-	-	-	3,187.500
300	21,020.833	-	-	-	1,979.167
320	21,480.833	-	-	-	1,519.167
350	22,145.833	-	-	-	854.167
400	23,187.500	(1-6)	22,999.268	0.2362	0.732
450	24,145.833	(1-6)	22,969.836	1.5081	30.164
550	25,812.500	(1-6) and (2-5)	22,779.499	4.0669	220.501
850	28,437.500	(1-6) and (2-5)	22,992.370	6.5130	7.630
∞	28,437.500	(1-6) and (2-5)	22,992.370	6.5130	7.630

Table 4: Market coupling process of configuration (3-3) under different transmission capacity scenarios

6.1.2 Configuration (4-2)

One shall observe that, differently from configuration (3-3), line (1-6) is always congested even when the transfer capacity between the two zones is zero. Note also that line (1-6) is now domestic (inside the Northern zone) and hence not accounted for in the coupling between the two zones. The most striking phenomenon is that the interconnection, which now only consists of line (2-5) becomes congested for an ATC higher than 39 MWh, notwithstanding the fact that we found an ATC of 400 MWh for configuration (4-2) as indicated in Section 4.2. The situation can be explained as follows: recall from that section that Rious et al. [26] explain that "*The TSO knows only the difference between total generation and total load on its control area*" and "*must anticipate the nodal sharing of total generation and load to forecast the base case power flows*". In the absence of ex ante knowledge on this base case power flow, we assumed it to be zero. Simulation reveals that it can effectively be quite high. As an example, the analysis of the 50 ATC case reveals injections of 400, 300 and 220 MWh in nodes 1, 2 and 4 with withdrawals of 150, 270 and 500 MWh at nodes 3, 5 and 6. Because node 6 is now in the Northern zone, this means that domestic transactions in that zone imply a "base case power flow" of 350 on line (2-5) leaving only 50 MWh for import/export. This reveals the difficulty of computing reliable ATC: TSOs must figure out the impact of domestic transactions in all zones on interconnection lines in order to determine the "base case power flow" that underlies the computation of the ATC. This may explain that published ATCs are "indicative" and "non binding". Last, another remarkable result is that the "welfare at equilibrium" remains quite close to the 23,000 € found with nodal pricing as soon as the transfer capacity is equal to or exceeds 100 MWh. These patterns are quite at variance with those obtained in configuration (3-3) suggesting that the decomposition of the market into zones in non nodal systems can have somewhat unpredictable effects.

ATC	**Welfare MC**	**Congested line**	**Welfare at equilibrium**	**Average Re-dispatching**	**Welfare loss**
0	23,519.643	(1-6)	22,769.215	0.8146	230.795
39	24,262.426	(1-6)	22,868.604	1.5434	131.396
50	24,462.500	(1-6) and (2-5)	22,891.642	1.7493	108.358
100	25,319.643	(1-6) and (2-5)	22,968.489	2.6877	31.511
150	26,091.071	(1-6) and (2-5)	22,999.423	3.6309	0.557
200	26,776.786	(1-6) and (2-5)	22,983.749	4.5802	16.251
250	27,376.786	(1-6) and (2-5)	22,920.922	5.5375	79.078
300	27,891.071	(1-6) and (2-5)	22,921.889	6.2652	78.111
350	28,294.643	(1-6) and (2-5)	22,980.697	6.5108	19.303
400	28,437.500	(1-6) and (2-5)	22,992.370	6.5130	7.630
450	28,437.500	(1-6) and (2-5)	22,992.370	6.5130	7.630
∞	28,437.500	(1-6) and (2-5)	22,992.370	6.5130	7.630

Table 5: Market coupling process of configuration (4-2) under different transmission capacity scenarios

6.2 Market coupling: clearing the energy market

6.2.1 Configuration (3-3)

Before examining the impact of imperfect coordination in the 400 and 450 MWh ATC cases, we first report additional results of the clearance of the energy market before congestion management. Consider first the case where the ATC is 400 MWh. The clearance of the energy market leads to a social welfare of 23,187.500 € (see Table 4). Generation in the Northern zone exceeds local consumption and 400 MWh are exported to the South. This saturates the zonal interconnection that takes a marginal value of 20 €/MWh. Because of congestion, the electricity price in the North is 27.50 €/MWh and 47.50 €/MWh in the South.

The situation is similar for an ATC of 450 MWh. Social welfare raises to 24,145.833 € (see Table 4). The Northern export to the South of 450 MWh congests the inter-connector whose marginal value becomes 18.333 €/MWh. This implies zonal electricity prices of 28.333 €/MWh and 46.667 €/MWh respectively in the Northern and in the Southern zones.

6.2.2 Configuration (4-2)

Reasoning as for the (3-3) zonal decomposition, we consider different values of the available North-South transmission capacity for the (4-2) configuration. As shown in Table 5 an ATC of 39 MWh is a natural reference: it does not congest line (2-5) and hence does not require cross border counter-trading (but it still requires "domestic" counter-trading to relieve congestion on line (1-6)). We also consider ATCs of 150 and 200 MWh, the first one giving the smallest welfare loss compared to nodal pricing; the second one corresponding to the use of line (2-5) (here the ATC between the two zones) in the nodal system. Last, we look at an ATC of 400 MWh whose results are identical to the case with infinite transfer capacity obtained by adding the capacities of the three lines ((2-5), (6-4), (6-5)) linking the Northern and Southern zones (see Table 5). We justify working with a high value of the transfer capacity by reference because, as we will see in the following, an ATC in excess of the flow found in the nodal solution increases counter-trading and degrades welfare. An infinite ATC is clearly the most extreme assumption. Recall that in contrast with the (3-3) configuration, all the situations in Table 5 imply domestic congestion of line (1-6).

The clearing of the energy market, that is before counter-trading, leads to the following figures. For an ATC of 39 MWh, welfare before re-dispatch amounts to 24,262.426 € (see column "Welfare MC" in Table 5). Northern zone exports 39 MWh to the South; the interconnection is congested and electricity prices are equal to 29.843 €/MWh in the North and 48.220 €/MWh in the South. Welfare before re-dispatch amounts to 26,091.071 € when the ATC is 150 MWh, with electricity prices respectively equal to 31.429 and 46.000 €/MWh in the Northern and Southern regions. The evolution towards higher welfare and trade increase when the ATC is 200 MWh: welfare before re-dispatch is now 26,776.786 € with electricity prices respectively equal to 32.143 and 45.000 €/MWh in the North and South. Last, we observe that there is no congestion of the interconnection when the ATC is equal to or higher than 400 MWh. The Northern zone exports 400 MWh to the South; electricity price is equal to 35 €/MWh in both zones and welfare before re-dispatch is 28,437.500 €.

7 Results of counter-trading models

We now discuss how re-dispatching costs and the different degrees of coordination between the two TSOs modify these values and present the results of the combined action of PXs and TSOs along the lines discussed in Section 4.4. In particular, we analyze the results of the three counter-trading organizations under the assumptions that re-dispatching resources are provided by both producers and consumers ("Prod/Cons") or by producers only ("Prod"). Tables report welfare losses that are all computed with respect to the level of nodal pricing (see Section 5); total and average re-dispatching costs respectively indicated by "TRC" and "ARC", net welfare at equilibrium after re-dispatching and finally the marginal value "MV" of lines (1-6) and (2-5) after re-dispatching.

7.1 Coordinated counter-trading

7.1.1 Configuration (3-3)

Table 6 compares the results of an efficient (integrated) counter-trading model. We report results for ATCs of 400 and 450 MWh.

	ATC=400		**ATC=450**	
	Prod/Cons	**Prod**	**Prod/Cons**	**Prod**
Welfare loss (€)	0.74	64.76	30.16	124.65
TRC (€)	187.50	250.95	1,145.83	1,234.95
ARC (€/MWh)	0.2362	0.32	1.508	1.633
Net welfare (€)	**22,999.26**	**22,935.24**	**22,969.84**	**22,875.35**
MV line (1-6) (€/MWh)	40.00	40.61	40.00	42.22
MV line (2-5) (€/MWh)	0.00	0.00	0.00	0.00

Table 6: Results of the coordinated counter-trading Model 1 under 400 and 450 MWh ATC scenarios for configuration (3-3)

Table 6 shows that welfare losses and re-dispatching costs are lower when both producers and consumers contribute to counter-trading. This is intuitively reasonable. Counter-trading on both producers and consumers for an ATC of 400 MWh minimizes the welfare loss compared to nodal pricing. This amounts to 0.74 €, leading to a net welfare of 22,999.26 €. The re-dispatched energy is 0.0000100 MWh with an average counter-trading cost of 0.2362 €/MWh. Line (1-6) is still congested in the direction North-South and its marginal value remains 40 €/MWh. This suggests (but does not prove) that a good adjustment of the available transfer capacity in market coupling may reduce overflows and hence re-dispatching needs without sacrificing much on welfare. When the re-dispatch is only conducted on producers (see the third column of Table 6), welfare losses and re-dispatching costs both increase. The corresponding re-dispatched electricity is around 6 MWh, still considerably lower than in the 450 MWh ATC scenarios where it amounts to 50 MWh and 55.55 MWh respectively in the "Prod/Cons" and the "Prod" cases. This degradation of efficiency for a higher ATC is also reflected in the re-dispatching costs that, in average, respectively reach the value of 1.508 €/MWh and 1.633 €/MWh as well as in welfare losses and net welfares. Netting out counter-trading flows, line (1-6) remains congested in the North-South direction with flows from the energy market in that direction and counter-trading flows in the opposite direction. As expected the marginal value of this line capacity is higher in the "Prod" than in the "Prod/Cons" case.

7.1.2 Configuration (4-2)

Table 7 reports the results of the "Prod/Cons" and the "Prod" cases for the two extreme values of the ATC for configuration (4-2).

	ATC=39		ATC=400	
	Prod/Cons	**Prod**	**Prod/Cons**	**Prod**
Welfare loss (€)	131.40	2,453.01	7.84	2,320.15
TRC (€)	1,363.88	3,485.51	5,078.98	7,065.17
ARC (€/MWh)	1.54	4.28	6.51	9.56
Net welfare (€)	**22,868.60**	**20,546.99**	**22,992.16**	**20,679.85**
MV line (1-6) (€/MWh)	40.00	52.96	40.00	57.05
MV line (2-5) (€/MWh)	0.00	0.00	0.00	0.00

Table 7: Results of the coordinated counter-trading Model 1 under the 39 and 400 MWh ATC scenarios for configuration (4-2)

Compared to the (3-3) configuration, the (4-2) market organization implies both higher re-dispatching costs and higher net welfare when TSOs can access both production and demand resources. As in the (3-3) case, limiting these resources to the sole generators increases counter-trading costs and reduces net welfare. Also expected, counter-trading costs increase with the interconnection capacity. This is particularly evident in the "Prod" scenario with a transmission capacity of 400 MWh[8], where the average re-dispatching cost is 9.557 €/MWh and the marginal costs of congested line (1-6) reaches 57.05 €/MWh.

Table 8 reports results for the intermediate ATC values. They are mostly unsurprising in the sense that they lie between those found in the two extreme cases. As already seen and in contrast with the configuration (3-3), assuming an ATC equal to the flows found in the nodal system no longer maximizes net welfare, which is now optimized for an ATC equal to 150 MWh. Consider now the average re-dispatching cost computed for this ATC of 150 MWh. It is still relatively high, particularly so when counter-trading resources are limited to producers. An incentive regulation that induces TSOs to reduce counter-trading costs would also induce them to first limit transfer capacity in order to reduce counter-trading costs. This would lead to a suboptimal ATC and hence to a welfare decrease.

7.2 Un-coordinated counter-trading Model 1

Suppose first that TSO^N and TSO^S can access to all counter-trading resources in both zones. Recall that we referred to this case as an internal market of counter-trading resources and, as

[8]Recall that this also corresponds to the case with infinite capacity. See Table 5.

	ATC=150		ATC=200	
	Prod/Cons	**Prod**	**Prod/Cons**	**Prod**
Welfare loss (€)	0.57	2,447.13	16.25	2,586.782
TRC (€)	2,925.99	4,952.85	3,529.33	5,535.00
ARC (€/MWh)	3.63	6.83	4.58	8.12
Net welfare (€)	**22,999.42**	**20,552.87**	**22,983.90**	**20,413.22**
MV line (1-6) (€/MWh)	40.00	54.134	40.00	54.650
MV line (2-5) (€/MWh)	0.00	0.00	0.00	0.00

Table 8: Results of the coordinated counter-trading Model 1 under the 150 and 200 MWh ATC scenarios for configuration (4-2)

explained in Section 4.3.2, that an internal market of counter-trading resources also implies an internal market of line capacities (see the paragraph "Un-coordinated counter-trading Model 1" of Section 4.3.2). Numerical results confirm this reasoning in the sense that we effectively fall back on the solution of the optimized (coordinated) counter-trading problem (see Section 7.1). This holds for both market configurations. Numerically, this case amounts to an allocation of the counter-trading actions obtained in the coordinated counter-trading case between the two TSOs. This allocation is arbitrary as different sharing of the counter-trading flows between the two TSOs give the same total re-dispatching cost.

7.3 Un-coordinated counter-trading Model 2

Suppose that we now restrict the access of TSO^N and TSO^S to counter-trading resources in non-domestic jurisdictions and impose the re-dispatching limits indicated in Table 9 respectively for configurations (3-3) and (4-2). These bounds are obtained by halving the re-dispatched

configuration (3-3)					
$\overline{\Delta q}_1^S$	$\overline{\Delta q}_2^S$	$\overline{\Delta q}_3^S$	$\overline{\Delta q}_4^N$	$\overline{\Delta q}_5^N$	$\overline{\Delta q}_6^N$
33.333	16.666	8.333	16.666	8.333	16.666
configuration (4-2)					
$\overline{\Delta q}_1^S$	$\overline{\Delta q}_2^S$	$\overline{\Delta q}_3^S$	$\overline{\Delta q}_4^N$	$\overline{\Delta q}_5^N$	$\overline{\Delta q}_6^S$
72.478	22.478	94.957	94.957	34.978	59.978

Table 9: Limits on TSOs' actions in the (3-3) and (4-2) configurations

quantities obtained in the respective coordinated counter-trading model when the available transmission capacity is set at 450 MWh[9].

We successively treat the case where there is a market of line capacity and the case where this market is missing.

[9]In both configurations, we choose the results of the 450 MWh ATC case with coordinated counter-trading to compute the re-dispatching limits. Note that in the (4-2) configuration the results of this ATC case are identical to those of 400 MWh up to infinite ATC scenarios when counter-trading is coordinately performed.

7.3.1 Configuration (3-3)

We still consider the "Prod/Cons" and "Prod" cases that we analyze for both the 400 and 450 MWh ATC values.

A market of line capacity Suppose first that there is a market of line capacity as discussed in the paragraph "Un-coordinated counter-trading Model 2" of Section 4.3.2. Each line has a single price seen by both TSO^N and TSO^S. We find that neither the upper nor the lower bounds on the TSOs' actions in the other zones are binding with the result that the solution to the problem is still identical to the one of Table 6 in Section 7.1.1, where both TSOs operate in a coordinated way.

No market of line capacity The situation changes when the market of line capacity is missing. We explained in the paragraph "Un-coordinated counter-trading Model 2" of Section 4.3.2 that this creates an incomplete market that can have different equilibria depending on the implicit valuation of the lines by the two TSOs. Assume, as illustration, that TSO^N values line (1-6) higher than TSO^S by an amount of 60 €/MWh. Results in the two ATC scenarios are reported in Table 10.

	ATC=400		**ATC=450**	
	Prod/Cons	**Prod**	**Prod/Cons**	**Prod**
Welfare loss (€)	236.18	276.66	396.30	493.90
TRC (€)	420.01	459.75	1,489.59	1,580.12
ARC (€/MWh)	0.53	0.59	1.995	2.126
Net welfare (€)	**22,763.82**	**22,723.34**	**22,603.70**	**22,506.10**
MV line (1-6) TSO^N (€/MWh)	69.45	71.52	82.67	87.68
MV line (2-5) TSO^N (€/MWh)	0.00	0.00	0.00	0.00
MV line (1-6) TSO^S (€/MWh)	9.45	11.52	22.67	27.68
MV line (2-5) TSO^S (€/MWh)	0.00	0.00	0.00	0.00

Table 10: Results of the un-coordinated counter-trading Model 2 with configuration (3-3) when ATCs equal 400 and 450 MWh and the (1-6) line wedge is 60 €/MWh

Note that line (1-6) is still congested in the direction North-South with line marginal values now depending on the TSO (TSO^N and TSO^S seeing values of 69.45 €/MWh and 9.45 €/MWh for line (1-6) respectively). In the 400 MWh ATC scenario, re-dispatch amounts to 14 MWh and 10 MWh respectively in the "Prod/Cons" and in the "Prod" cases; when the ATC is 450 MWh, re-dispatched quantities are respectively of 30 MWh and 35.10 MWh. Again inefficiency increases when the ATC is selected too high. Comparing Tables 10 and 6, we observe that counter-trading costs increase compared to the coordinated counter-trading model both in the "Prod/Cons" and in the "Prod" cases. This implies a decrease of the net welfare.

7.3.2 Configuration (4-2)

A market of line capacity Both TSOs see the same prices for the congested lines. We here have different situations. As in the (3-3) organization, all results of the 39 MWh and the results of the Prod/Cons 400 MWh ATC extreme cases are identical to those of the perfectly coordinated counter-trading model (see Table 7 for the respective values). The reason is that the restrictions to access in the other jurisdiction are not binding.

The situation changes when the ATC is 400 MWh and re-dispatch is limited to producers: the problem becomes infeasible meaning that counter-trading is impossible because of lack of resources! This situation has been encountered in practice. TSOs have a strong and obvious incentive to avoid that outcome, something that they can do by reducing the ATC offered to the energy market. This is what the Swedish TSO is reputed to have done (Norwegian Electricity Association [23]).

Table 11 reports the results for the intermediate ATC of 150 and 200 MWh. These are again intermediate compared to those of the extreme cases, but with some interesting features. The 150 MWh ATC with access to both producers and consumers is again extremely efficient and identical to that of the perfectly coordinated counter-trading scenario (compare Tables 11 and 8). But the degradation is severe when counter-trading resources are limited to producers. When increasing the ATC to 200 MWh (again going beyond the optimal 150 MWh), there is a slight degradation (still identical to that of the perfectly coordinated counter-trading scenario) in the Prod/Cons case that becomes dramatic when counter-trading is limited to producers as the problem is infeasible again!

	ATC=150		**ATC=200**	
	Prod/Cons	**Prod**	**Prod/Cons**	**Prod**
Welfare loss (€)	0.57	3,010.42	16.25	infeasible
TRC (€)	2,925.99	5,373.05	3,529.33	infeasible
ARC (€/MWh)	3.63	7.61	4.58	infeasible
Net welfare (€)	**22,999.42**	**19,989.58**	**22,983.90**	infeasible
MV line (1-6) (€/MWh)	40.00	54.13	132.57	infeasible
MV line (2-5) (€/MWh)	0.00	0.00	0.00	infeasible

Table 11: Results of the un-coordinated counter-trading Model 2 with configuration (4-2) when ATCs equal 150 and 200 MWh and the (1-6) line wedge is 0 €/MWh

No market of line capacity Table 12 gives the results for the two extreme ATC values. We first observe that limiting counter-trading to producers remains infeasible when the ATC is 400 MWh. This is not surprising: eliminating the market of line capacity cannot help counter-trading; what was not possible with a market of line capacity remains impossible without it. Other elements of Table 12 are more unusual and deserve additional interpretation. The table shows a column where there is "No equilibrium". In order to understand this situation, recall that both TSOs always warrant a zero value for line (2-5) with or without capacity market. But line (1-6) is always congested and hence susceptible of different valuations by the TSOs. This has implications that we now discuss.

	ATC=39		ATC=400	
	Prod/Cons	**Prod**	**Prod/Cons**	**Prod**
Welfare loss (€)	No equilibrium	2,798.54	2,472.40	infeasible
TRC (€)	No equilibrium	3,783.14	7,190.99	infeasible
ARC (€/MWh)	No equilibrium	4.703	9.764	infeasible
Net welfare (€)	No equilibrium	**20,201.46**	**20,527.60**	infeasible
MV line (1-6) TSO^N (€/MWh)	No equilibrium.	118.38	78.16	infeasible
MV line (2-5) TSO^N (€/MWh)	No equilibrium	0.00	0.00	infeasible
MV line (1-6) TSO^S (€/MWh)	No equilibrium	58.38	18.16	infeasible
MV line (2-5) TSO^S (€/MWh)	No equilibrium	0.00	0.00	infeasible

Table 12: Results of the un-coordinated counter-trading Model 2 with configuration (4-2) when ATCs equal 39 and 400 MWh and the (1-6) line wedge is 60 €/MWh

The missing market of line capacity has a particularly realistic interpretation in this (4-2) configuration. Line (1-6) is now a congested domestic line in the Northern zone. Market coupling does not foresee today any cooperation mechanism for the two TSOs to remove this congestion. A possible approach would be to include line (1-6) in an internal market of line capacity. TSOs have indeed realized the importance of the problem and the potential of this approach; in TenneT [31], they envisage including so-called "critical infrastructures" (in former US parlance "commercially significant flowgates") in their flow-based model. At this stage, this has not been implemented.

	ATC=150		ATC=200	
	Prod/Cons	**Prod**	**Prod/Cons**	**Prod**
Welfare loss (€)	No equilibrium	3,010.08	1,028.40	infeasible
TRC (€)	No equilibrium	5,373.18	4,364.02	infeasible
ARC (€/MWh)	No equilibrium	7.61	5.92	infeasible
Net welfare (€)	No equilibrium	**19,989.92**	**21,971.61**	infeasible
MV line (1-6) TSO^N (€/MWh)	No equilibrium	132.60	62.07	infeasible
MV line (2-5) TSO^N (€/MWh)	No equilibrium	0.00	0.000	infeasible
MV line (1-6) TSO^S (€/MWh)	No equilibrium	72.60	2.07	infeasible
MV line (2-5) TSO^S (€/MWh)	No equilibrium	0.00	0.000	infeasible

Table 13: Results of the un-coordinated counter-trading Model 2 with configuration (4-2) when ATCs equal 150 and 200 MWh and the (1-6) line wedge is 60 €/MWh

A wedge between the valuations of line (1-6) by the two TSOs reflects the lack of attention of some TSOs (in this case the Southern TSO) for a domestic congested infrastructure in another jurisdiction (here the Northern zone) in a zonal system. We computed the equilibrium when this wedge is zero for line (1-6) in the previous paragraph. We now consider non zero wedges for the valuation of line (1-6), for which we test values 30 €/MWh and 60 €/MWh.

Assuming different valuations of line (1-6) can dramatically affect results for the very moderate 39 ATC threshold (see Table 12) as well as for the optimal (in terms of welfare deduction) 150 MWh case (Table 13). Specifically, there is no equilibrium in the "Prod/Cons" case for a wedge of 60 €/MWh and these two ATC values. In contrast, there is an equilibrium for a wedge of 30 €/MWh (given in Table 14).

Because we found an equilibrium for a zero wedge in the preceding paragraph, one can show that there is a whole range of equilibria for wedges between 0 and 30 €/MWh. Additional equilibria also exist for wedges higher than 30 €/MWh, but the situation suddenly turns to non existence of the equilibrium somewhere below 60 €/MWh. We are thus in the embarrassing situation where the market can end-up in any of several equilibria or not find an equilibrium at all. It all depends on the different valuations of a domestic congested line by two TSOs that, under the current market design, have no incentive to jointly work to remove that congestion. This signals a badly posed problem or, in other terms, a flawed market design. Possibly adding to the confusion, we find an equilibrium when counter-trading resources are limited to the producers, whether the wedge is 60 or 30 €/MWh.

	ATC=39	**ATC=150**
	Prod/Cons	**Prod/Cons**
Welfare loss (€)	172,45	560.40
TRC (€)	1,403.29	3,414.89
ARC (€/MWh)	1.59	4.33
Net welfare (€)	**22,827.55**	**22,439.60**
MV line (1-6) TSO^N (€/MWh)	44.61	56.32
MV line (2-5) TSO^N (€/MWh)	0.00	0.00
MV line (1-6) TSO^S (€/MWh)	14.61	26.32
MV line (2-5) TSO^S (€/MWh)	0.00	0.00

Table 14: Results of the un-coordinated counter-trading Model 2 with configuration (4-2) when ATCs equal 39 and 150 MWh and the (1-6) line wedge is 30 €/MWh

The tables also show that there is always an equilibrium for these two wedges for the higher ATCs of 200 and 400 MWh when the problem is feasible (sufficient counter-trading resources). These look like chaotic results and that is what they probably are! But this should not come as a surprise: a missing market of line capacity is a market failure that economics tell us very little about: we are in unexplored waters and many things can indeed happen. Last, note that the tables also show that equilibria, when they exist, can become quite inefficient as a result of both the restriction of counter-trading resources and the missing transmission market. Existing equilibria are indeed associated with dramatic welfare decreases.

This analysis suggests an important policy conclusion. An integrated market of counter-trading resources implicitly creates a market of line capacities (in the same way as a nodal energy market creates a transmission market). This ensures an efficient counter-trading. That property can be lost when the market of counter-trading is only partially integrated. It is then important to explicitly create a market of line capacities in order to complete the market and avoid it to become at best inefficient, at worst erratic and ambiguous. As argued before, the principle is that this should not be too difficult: even if one reasons that nodal pricing is too complicated for traders on the energy market, this argument should not apply to TSO that by nature should master the technology and economics of the grid.

7.4 Un-coordinated counter-trading Model 3

The un-coordinated counter-trading Model 3 describes an extreme case where TSO^N and TSO^S manage congestion by only procuring counter-trading resources in their own jurisdiction. The market of counter-trading resources is thus entirely segmented. This means that the re-dispatched quantities sum up to zero inside of each zone. As is now standard, we successively consider the cases where there is a market of line capacity and when this market is missing for both (3-3) and (4-2) configurations.

7.4.1 Configuration (3-3)

A market of line capacity Table 15 gives the results of this case where by construction, both TSOs see the same price for the lines. The 450 MWh ATC scenario reveals a highly inefficient counter-trading. In contrast, inefficiency is again minimal when the ATC is optimally determined at 400 MWh. The results are thus quite sensitive to the choice of the ATC (recall that the ATC is an ill-defined notion and hence the choice of a good value is controversial). In both cases, re-dispatching costs are higher when counter-trading is applied only to producers.

	ATC=400		**ATC=450**	
	Prod/Cons	**Prod**	**Prod/Cons**	**Prod**
Welfare loss (€)	0.37	95.40	1,454.21	3,095.56
TRC (€)	187.13	281.25	2,441.56	3,781.25
ARC (€/MWh)	0.2358	0.3557	3.448	5.878
Net welfare (€)	**22,999.63**	**22,904.60**	**21,545.79**	**19,904.44**
MV line (1-6) (€/MWh)	40.00	60.00	146.67	220.00
MV line (2-5) (€/MWh)	0.00	0.00	0.00	0.00

Table 15: Results of the un-coordinated counter-trading Model 3 with configuration (3-3) when ATCs equal 400 and 450 MWh and the (1-6) line wedge is 0 €/MWh

No market of line capacity Table 16 gives the results when we again assume that the prices of line (1-6) seen by the two TSOs differ by 60 €/MWh. For both "Prod" cases, results are identical to those of Table 15, except for the marginal values of line (1-6). In the "Prod/Cons" scenarios, instead, market incompleteness degrades the situation leading to a reduction of social welfare with respect to the corresponding cases with a market for line capacities (compare Table 15).

	ATC=400		ATC=450	
	Prod/Cons	**Prod**	**Prod/Cons**	**Prod**
Welfare loss (€)	94.57	95.40	1,554.62	3,095.56
TRC (€)	280.41	281.25	2,528.64	3,781.25
ARC (€/MWh)	0.3547	0.3557	3.591	5.878
Net welfare (€)	**22,905.43**	**22,904.60**	**21,445.38**	**19,904.44**
MV line (1-6) TSO^N (€/MWh)	60.0000051	60.0000001	166.67	220.000
MV line (2-5) TSO^N (€/MWh)	0.00	0.00	0.00	0.00
MV line (1-6) TSO^S (€/MWh)	0.0000051	0.0000001	106.67	160.00
MV line (2-5) TSO^S (€/MWh)	0.00	0.00	0.00	0.00

Table 16: Results of the un-coordinated counter-trading Model 3 with configuration (3-3) when ATCs equal 400 and 450 MWh and the (1-6) line wedge is 60 €/MWh

7.4.2 Configuration (4-2)

We report the tables of this scenario in Appendix G.

A market of line capacity Results are not fundamentally surprising. The limitation of counter-trading resources to the sole producers makes counter-trading impossible because of lack of resources. When resources are sufficient as in the "Prod/Cons" cases, their restriction to the sole domestic zone entails drastic welfare decreases compared to the nodal pricing model. Average re-dispatching costs are also quite high for high ATC with the already mentioned result that an incentive regulation would in fact induce TSOs to restrict the ATC to 39 MWh in order to reduce re-dispatching costs to 1.868 €/MWh and appear more efficient (see Tables 17 and 18 in Appendix G).

No market of line capacity The combination of the segmentation of the counter-trading resources and the absence of a transmission market makes the situation very unstable as one oscillates between "no equilibrium" and "infeasible", with some cases of inefficient equilibria. The interpretation of these situations derives from explanations given above and will not be repeated here. We report the results in Tables 19 and 20 in Appendix G.

8 Conclusion

This paper only offers an illustrative analysis. The discussion is conducted on a simple six node example taken from the literature. This six node market is split into two zone systems of which we consider two versions. They both take the form of a North-South decomposition to which we refer as the (3-3) and (4-2) configurations (the first and second figures are respectively the number of nodes in the "Northern" and "Southern" zones). We then look at different organizations of cross border trade in these two configurations. Even with this simple set up we find interesting phenomena.

We first apply nodal pricing that we take as the first best, reference, solution with respect to which we measure welfare losses of other organizations. By definition, nodal pricing gives the same results for the two zonal configurations and maximizes welfare. This is well known and needs no additional comment. Various considerations have hampered the introduction of nodal pricing so far in Europe and the so-called Market Coupling organization is currently the state of the art. This architecture relies on a partial integration of the energy and transmission markets. Energy clears on the basis of a simplified representation of the grid that currently relies on "transfer capacities", but should move to a "flow-based" (flowgate in US parlance) type approach where the grid is represented by a set of "critical infrastructures". Whatever the final description of the grid, the inevitable simplifications for representing it may require counter-trading in order to match the clearing of the energy market with the real capacities of the grid. The Regulation 714/2009 [13] requires TSOs to maximize transmission capacities. This obligation is not implementable in general because increasing the transmission capacity between two zones in a meshed grid may at the same time reduce transmission capacity between two other zones. The example used in this paper bypasses this logical impossibility by addressing a simpler two zone system for which it is then effectively possible to "maximize" the transfer capacity. Even with this restriction, the exercise reveals that Market Coupling still raises questions.

Counter-trading has a cost that it is reasonable to try to minimize. Integration of counter-trading operations achieves that result. We thus take, as second best solution, a Market Coupling organization where possible overflows resulting from the energy market are eliminated by an efficient, minimal cost, counter-trading. This happens when all TSOs operate in a coordinated way. Because it is impossible to find the transfer capacity between two zones in a univocal way, we conduct the analysis by assuming different ATC values. A first finding is that the outcome of this second best solution is quite sensible to the zonal decomposition. Welfare losses, as a function of the ATC, show quite different evolutions in the two zonal configurations (3-3) and (4-2). They can be important in the (3-3) configuration while they always remain modest in the (4-2) arrangement. We also find that, barring strategic behaviours, it may be possible for the second best to get close to the first best provided we find the "right" ATC and sufficient counter-trading resources are available. But we have no clue to offer on the finding of this right value. One possible idea would be to try to take advantage of nodal simulations to do so. We indeed find that an ATC of 400 MWh that is the export from North to South in the nodal pricing system minimizes welfare losses in the second best organization of configuration (3-3). But this finding does not carry through to the (4-2) configuration, where an ATC of 150 MWh (and then lower than nodal line use) minimizes welfare losses!! Recalling how simplistic our six node network is, we conclude that it will be very very difficult in general to derive ATCs from simulations of optimal dispatch. In short, the second best can get us quite close to the first best, but this depends on finding the right ATC that remains an unsolved (or most likely unsolvable) problem.

Suppose however that welfare loss minimizing ATC has been properly defined in the sense that an integrated counter-trading minimizes welfare losses compared to nodal pricing. We have seen that the corresponding ATC are 400 and 150 MWh respectively in the (3-3) and (4-2) configurations. An important question is how it will be possible to achieve this integrated counter-trading. Numerical experiments confirm previous methodological derivations that show that this may not require the horizontal integration of the TSOs (even though this is probably the most efficient solution). We observe that the creation of an internal market of counter-trading resources is a perfect substitute for the horizontal integration of TSOs: the arbitrage taking place through access to these resources results in the same counter-trading operations as the integrated TSOs. Needless to say the broader the access to counter-trading resources, the better the result: limiting counter-trading resources to producers impairs efficiency. But whatever the extent of the counter-trading resources, making them available through an internal market has a dramatic beneficial impact on efficiency.

Experience has shown that implementing an internal market, whether for energy or ancillary services such as reserve or balancing is difficult in European electricity market. The same is likely to be true for counter-trading resources as some TSOs may want to reserve some resources for their own actions. This is where the vulnerability of market coupling to variations of market design shows up. We consider two different features of market design on which we conduct numerical experiments. One is the extent of access by one TSO to counter-trading resources in the other jurisdiction. This is a particular case of the more general question of horizontal integration of ancillary resources such as reserves or balancing in Europe. Specifically we consider both partial and no access to resources in the other jurisdiction. The other market design feature is the existence of a market of line capacity at the counter-trading level that deals with the real characteristics of the grid. One argument used for justifying market coupling is to simplify the intricacies of the grid for energy traders; this argument obviously does not apply for TSO that should be conversant with grid technology and economics and hence able to deal with a complete transmission market. We thus consider two scenarios that differ by whether a market of line capacity exists or not.

Assume first a market of line capacity and consider the case of a restricted internal market of counter-trading resources. The results are as expected and (methodologically) reassuring: the more one restricts access to counter-trading resources in the other jurisdiction (from partial to no access) the more inefficient the system becomes and the higher the incurred welfare losses. It may even be impossible to counter-trade, something that has indeed been observed in practice. This may have a very counter productive effect on the "maximization" of transmission capacities. TSOs that face the impossibility to counter-trade because of insufficient resources or are subject to an incentive regulation to reduce counter-trading costs will be induced to restrict ATC. This has also been observed in practice. The basic recommendation coming from these numerical experiments is thus straightforward: one should have an integrated market of counter-trading resources. One even observes that a full horizontal integration of these resources allow one to reestablish the second best efficiency (albeit probably at higher transaction costs).

The situation is quite different when there is no market of line capacity. The market begins to behave in a chaotic way and this is also not surprising: a missing market of line capacity is a market failure, the outcome of which is largely unknown. We observe both a possible lack of equilibria and a multiplicity of equilibria; these signal a bad market design. When equilibria exist, they are accompanied by further degradations of welfare, which also signal a very inefficient market design. Needless to say, all this can be compounded by the restricted access to counter-trading resources.

All this analysis can be summarized in a simple recommendation: if one persists in not installing a proper transmission market at the energy level, at least one should install it at the counter-trading level. The compound of inadequate ATC computations and inefficient counter-trading can create havoc in the market.

References

[1] K.J. Arrow, G. Debreu, Existence of an equilibrium for a competitive economy, Econometrica. 22 (1954) 265-893.

[2] H.P. Chao, S.C. Peck, Reliability management in competitive electricity markets, Journal of Regulatory Economics. 14 (1998) 198-200.

[3] D-cision and The Brattle Group, A system for congestion management in the Netherlands. (2009) Available at http://www.rijksoverheid.nl/documenten-en-publicaties/rapporten/2009/11/16/rapport-d-cision-en-brattle-group-a-system-for-congestion-management-in-the-netherlands.html

[4] G. Debreu, A social equilibrium existence theorem, Proc. nat. Acad. Sci. 38 (1952) 886-290.

[5] B. Den Ouden, The Trilateral Market Coupling in the Dutch-Belgian-French market and beyond. (2007) Presentation at the APEX conference, Paris, October 2007. Available at http://www.theapex.org/Documentos/Apex07/Sesion2/2-1a_Trilateral_Market.ppt

[6] DERA (Danish Energy Regulatory Authority), Regulators' 2009 National Report to the European Commission. (2009) Available at http://www.energy-regulators.eu/portal/page/portal/EER_HOME/EER_PUBLICATIONS/NATIONAL_REPORTS/National%20Reporting%202009

[7] E. Dijkgraaf, M.C.W. Janssen, Price convergence in the European electricity market Erasmus Competition and Regulation institute, Erasmus Competition and Regulation institute. (2007) Available at http://people.few.eur.nl/dijkgraaf/Epubs/Price%20convergence%20August7%202007.pdf

[8] C. Duthaler, M. Emery, G. Andersson, M. Kurzidem, Analysis of the use of the PTDF in the UCTE Transmission Grid, Power System Computation Conference, Glasgow. (2008).

[9] ELIA, Annual report. (2009) Available at http://www.elia.be

[10] E.ON Benelux, Position paper in response to the report "A system for congestion management in the Netherlands" by D-Cision and The Brattle Group. (2009). Available at www.rijksoverheid.nl

[11] EPEX Spot, Press Release, Launch of CWE Market Coupling proposed on 7 September. (2010a) Available at http://static.epexspot.com/document/8447/20100407_CWE_MC_final_EN.pdf

[12] EPEX Spot, Press Release, Cooperation of 6 Power Exchanges on European Price Coupling Solution. (2010b). Available at http://static.epexspot.com/document/8225/20100318_EPEX_6_px_pcr_final.pdf

[13] European Commission, Regulation No 714/2009 of the European Parliament and of the Council of 13 July 2009 on conditions for access to the network for cross-border exchanges in electricity and repealing Regulation (EC) No 1228/2003 Text with EEA relevance. (2009). Available at http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2009:211:0015:01:EN:HTML.

[14] EWIS, Towards a successful integration of large scale wind power into European electricity grids. (2010). Available at
http://www.wind-integration.eu/downloads/library/EWIS_Final_Report.pdf

[15] EXAA, Company News, TGE, EXAA and PXE have signed a Memorandum of Understanding (MoU) concerning strategic cooperation - spot electricity markets integration based on market coupling. (2010) Available at
http://www.exaa.at/service/press/items/news2010_03.html.

[16] F. Facchinei, C. Kanzow, Generalized Nash equilibrium,problems, 4OR. 5 (2007) 173-210.

[17] F. Facchinei, J-S. Pang, Finite-Dimensional Variational Inequalities and Complementarity Problems, Springer, Vol. I and II. (2003).

[18] M. Fukushima, Restricted generalized Nash equilibria and controlled penalty method, Computational Management Sciences, DOI: 10.1007/s10287-009-0097-4 (2008).

[19] P.T. Harker, Generalized Nash games and quasi-variational inequalities, European Journal of Operation Research. 54 (1991) 81-94.

[20] W.W. Hogan, Contract networks for electricity power transmission. Journal of Regulatory Economics. 4(3) (1992) 211-242.

[21] W.W. Hogan, Electricity Market Restructuring: successful market design, NARUC Summer Meetings, Electricity Committee, Austin TX. (2005). Presentation Available at http://www.hks.harvard.edu/fs/whogan/hogan_naruc_072605.pdf

[22] K. Nabetani, P. Tseng, M. Fukushima, Parametrized variational inequality approaches to generalized Nash equilibrium problems with shared constraints. Comput Optim Appl, DOI 10.1007/s10589-009-9256-3 (2009).

[23] Norwegian Electricity Association, Congestion management – Dansk Energi‘s complaint regarding Swedish TSO practice. (2006). Available at http://www.danskenergi.dk/upload/filer_til_nyheder/brev_til_kommissionen_fra_ebl.pdf

[24] G. Oggioni, Y. Smeers, Degree of coordination in market coupling and counter-trading, *CORE Discussion Paper.* 01 (2010a).

[25] G. Oggioni, Y. Smeers, E. Allevi, S. Schaible, Generalized Nash Equilibrium and Market Coupling in the European power system, Accepted in the EPRG Working Paper Series and submitted. (2010b).

[26] V. Rious, J. Usaola, M. Saguan, J.M. Glachant, P. Dessante, Assessing Available Transfer Capacity on a Realistic European Network: Impact of Assumptions on Wind Power Generation. Author manuscript, published in “1st International scientific conference *Building Networks for a Brighter Future*”, Rotterdam, Netherlands. (2008). Available at http://hal-supelec.archives-ouvertes.fr/docs/00/33/87/49/PDF/WindPowerATC_VR-MS-JU-JMG-PD4.pdf

[27] J.B. Rosen, Existence and uniqueness of equilibrium points for concave N-person games, Econometrica. 33 (1965) 520-534.

[28] RTE, Activity report. (2008). Available at: http://www.rte-france.com/uploads/media/rapport-d-activite/2008/RTE_RapportAnnuel_2008_EN.pdf

[29] F.P. Sioshansi, W. Pfaffenberger, Electricity Market Reform: An International Perspective, Elsevier Science & Technology. (2006).

[30] S. Stoft, Power System Economics, IEEE/Wiley. (2002).

[31] TenneT, A report for the MoU signatories on the design of the market coupling solution in the Central West European (CWE) region, by the CWE MC Project. (2008). Available at http://www.tennet.org

[32] TenneT, Launch of CWE Market Coupling proposed on 7 September. (2008). Available at http://www.tennet.org

A Non convexities in generation systems

The discussion in the paper is conducted assuming increasing marginal costs of generation stations and decreasing linear demand functions. These are standard economic assumptions. Reality differs from this convex economy to the extent that machines have several non-convex characteristics. Start up cost and minimal operations levels are two major properties that invalidate standard convexity assumptions: start up costs need to be incurred before the plant can operate at any positive level. When operating, this should be above some given percentage of its rated capacity. Other characteristics such as minimum up and down times add to these non-convexities. Non-convexities are not amenable increasing marginal cost curves and hence to the usual increasing supply curves supposed in this paper. The problem is well recognized and addressed in two different ways in restructured electricity systems. One approach, which is implemented in the Pool systems of the United States, is to allow bidders to communicate information on start up costs, technical minima, minimal and up and down times to the ISO. This takes place in the bidding process and influences its outcome. For given efficiently and fuel cost parameters, being more flexible plant is a competitive advantage. The European approach is different; it casts these machine characteristics in modifications of the bids. For instance bidders may require that their machine, if selected, operate for a certain number of hours in order to recoup the fixed start up cost. This gives rise to the bloc bids implemented in both CWE and the coupling between Denmark and Germany. We do not discuss these matters here. First, they are non standard in economics and hence would require getting into considerable additional complications. Second, they are quite often neglected in analysis of market design.

B Nodal pricing model

B.1 Notation

We here list the sets, parameters and variables that are common to all models.

Sets

- $l = (1-6);(2-5)$: Set of lines with limited transmission capacity;
- $n = 1,2,3,4,5,6$: Nodes
- $i(n) = 1,2,4$: Subset of production nodes;

- $j(n) = 3, 5, 6$: Subset of consumption nodes.

  Parameters

- $PTDF_{l,n}$: Power Transfer Distribution Factor (PTDF) matrix of node n on line l;
- $\bar{F}_l$: Limit of flow through lines $l = (1-6); (2-5)$.

  Variables

- q_n: Power traded (bought or sold) in node n in MWh. These are determined in the market coupling problem and are taken as data in the counter-trading model.

B.2 Model

The nodal pricing (or flowgate) model describes the full integration of the energy and transmission markets into a pool like organization. This is one of the possible formulations of that problem when one abstracts away from reliability issues. The problem is formulated as a welfare maximization problem where the energy and transmission markets are coordinated and clear simultaneously. Congestion costs directly affect electricity prices that in case of a grid saturation differ by node. Because there is currently an attempt to implement such approach in Europe, we adopt this formulation as reference. The application of the nodal pricing model to the six node network leads to following formulation where the objective function (1) is subject to transmission (2)-(3), energy balance (4) and non-negativity constraints (5).

$$\mathbf{Max_{q_n}} \qquad \sum_{j=3,5,6} \int_0^{q_j} w_j(\xi) d\xi - \sum_{i=1,2,4} \int_0^{q_i} c_i(\xi) d\xi \tag{1}$$

s.t.

$$\overline{F}_l - [\sum_{i=1,2,4} PTDF_{i,l}(q_i) - \sum_{j=3,5,6} PTDF_{j,l}(q_j)] \geq 0 \qquad (\lambda_l^+) \tag{2}$$

$$\overline{F}_l + [\sum_{i=1,2,4} PTDF_{i,l}(q_i) - \sum_{j=3,5,6} PTDF_{j,l}(q_j)] \geq 0 \qquad (\lambda_l^-) \tag{3}$$

where $l = (1-6), (2-5)$

$$\sum_{i=1,2,4} q_i - \sum_{j=3,5,6} q_j = 0 \qquad (\phi) \tag{4}$$

$$q_n \geq 0 \qquad \forall n \qquad (\nu_n) \tag{5}$$

C Market coupling model

Market coupling problem defines the equilibrium on the energy market. We assume that PXs coordinately operate in the Northern and the Southern zones taking into account a simplified representation of the grid. This may generate overflows that are re-adjusted by TSOs with counter-trading operations. In our models, counter-trading costs are charged to producers only. These constitute the link between the PXs and TSOs' problems. The average re-dispatching costs α are included in the objective function of the PXs' problem as indicates below. We now present the market coupling problem of the (3-3) and (4-2) market configurations. For a more detailed discussion of the model see Oggioni et al. [25].

C.1 Configuration (3-3)

PXs maximize the social welfare under balance and available transmission capacity constraints. In particular, the objective function (6) includes the average re-dispatching costs α that PXs charge to generators. We assume that α is proportional to the quantity injected in the energy market. Note that in the market coupling problem α is considered as a parameter that comes from the solution of the counter-trading problem. Conditions (7) and (8) are the energy balances of the Northern and Southern zones respectively. These account for the free variable I that represents the import/export between the two zones. The shadow variables $\phi^{N,S}$ are the marginal electricity prices of the two zones. Constraints (9) and (10) respectively impose an upper and a lower bound $\overline{I}$ on the flow I. The absolute value of $\overline{I}$ corresponds to the available transmission capacity of the zonal interconnecting line. The dual variables δ^1 and δ^2 are the marginal cost of utilization of this zonal link. Finally, condition (11) defines the non-negativity of variables q_n.

$$\mathbf{Max}_{\mathbf{q_n}} \quad \sum_{j=3,5,6} \int_0^{q_j} w_j(\xi)d\xi - \sum_{i=1,2,4} \int_0^{q_i} c_i(\xi)d\xi - \alpha \cdot (q_1 + q_2 + q_4) \tag{6}$$

s.t.

$$q_1 + q_2 - q_3 - I = 0 \qquad (\phi^N) \tag{7}$$

$$q_4 - q_5 - q_6 + I = 0 \qquad (\phi^S) \tag{8}$$

$$\overline{I} - I \geq 0 \qquad (\delta^1) \tag{9}$$

$$\overline{I} + I \geq 0 \qquad (\delta^2) \tag{10}$$

$$q_n \geq 0 \qquad \forall n \tag{11}$$

C.2 Configuration (4-2)

The comparison with the (3-3) market coupling problem shows that the objective function (12) is identical to (6), while zonal energy balances (13) and (14) are modified according to the disposition of the nodes in the (4-2) market organization. Finally, according to our input data, constraints (15) and (16) are not binding for ATC levels ($\overline{I}$) equal or above 400 MWh.

$$\mathbf{Max}_{\mathbf{q_i}} \quad \sum_{j=3,5,6} \int_0^{q_j} w_j(\xi)d\xi - \sum_{i=1,2,4} \int_0^{q_i} c_i(\xi)d\xi - \alpha \cdot (q_1 + q_2 + q_4) \tag{12}$$

s.t.

$$q_1 + q_2 - q_3 - q_6 - I = 0 \qquad (\phi^N) \tag{13}$$

$$q_4 - q_5 + I = 0 \qquad (\phi^S) \tag{14}$$

$$\overline{I} - I \geq 0 \qquad (\delta^1) \tag{15}$$

$$\overline{I} + I \geq 0 \qquad (\delta^2) \tag{16}$$

$$q_n \geq 0 \qquad \forall n \tag{17}$$

D Equilibrium computation

Finding the equilibrium means solving a fixed point problem where the average re-dispatching cost is equal to the parameter α used in the formulation of the market coupling model. This condition links the two models and guarantees that participants in the energy market remunerate the TSOs for the counter-trading costs that they impose on the grid. We apply a simple contraction mapping type iterative process in order to find this α. The iterative process starts with the solution to the market coupling problem without counter-trading as described in Section 6.2 respectively for configuration (3-3) and (4-2).

E Coordinated counter-trading model

The coordinated counter-trading model presented below is suitable for both market configurations. A more in depth discussion of the model is presented in Oggioni et al. [25].

E.1 Notation

We here introduce the counter-trading variables Δq_n that represent the incremental or decremental electricity changes with respect to q_n (MWh) that TSO^N and TSO^S require in order to restore the network feasibility. Note that in all counter-trading problem, TSOs take the power quantities q_n coming from the market coupling problem as given.

E.2 Model

TSOs minimize the global re-dispatching costs stated in the objective function (18). The problem is subject to the energy balances (19) and (20) that impose that the sum of variations in production ($\Delta q_{i=1,2,4}$) and consumption ($\Delta q_{j=3,5,6}$) imposed by TSOs equals zero[10] and to the transmission constraints (21) and (22) whose dual variables $\lambda_l^{\pm}$ indicate the marginal values of the congested lines. By model construction, the two TSOs see identical values for the transmission lines. Finally, constraints (23) and (24) state that the quantities of electricity demanded and produced in the Northern and the Southern zones plus the incremental and decremental injections of the TSO^N and TSO^S have to be non-negative.

$$\mathbf{Min}_{\mathbf{\Delta q_n}} \quad \sum_{i=1,2,4} \int_{q_i}^{q_i+\Delta q_i} c_i(\xi) d\xi - \sum_{j=3,5,6} \int_{q_j}^{q_j+\Delta q_j} w_j(\xi) d\xi \tag{18}$$

s.t.

$$\sum_{i=1,2,4} \Delta q_i + \sum_{j=3,5,6} \Delta q_j = 0 \qquad (\mu^1) \tag{19}$$

$$\sum_{i=1,2,4} \Delta q_i - \sum_{j=3,5,6} \Delta q_j = 0 \qquad (\mu^2) \tag{20}$$

$$\overline{F}_l - [\sum_{i=1,2,4} PTDF_{i,l}(q_i + \Delta q_i) - \sum_{j=3,5,6} PTDF_{j,l}(q_j + \Delta q_j)] \geq 0 \qquad (\lambda_l^+) \tag{21}$$

[10]This also imply that counter-trading does not affect the quantities q_n defined by the PXs on the energy market.

$$\overline{F}_l + [\sum_{i=1,2,4} PTDF_{i,l}(q_i + \Delta q_i) - \sum_{j=3,5,6} PTDF_{j,l}(q_j + \Delta q_j)] \geq 0 \qquad (\lambda_l^-) \tag{22}$$

$$q_i + \Delta q_i \geq 0 \qquad i = 1,2,4 \tag{23}$$

$$q_j + \Delta q_j \geq 0 \qquad j = 3,5,6 \tag{24}$$

The average re-dispatching cost value α is obtained by dividing the total re-dispatching costs (18) by $\sum_{i=1,2,4} q_i$

F Un-coordinated counter-trading models

We distinguish three types of un-coordinated counter-trading models that we describe in the following. For a more detailed discussion of these models see Oggioni et al. [25].

F.1 Un-coordinated counter-trading Model 1

We here denote the counter-trading variables of the Northern and Southern TSOs respectively as $\Delta q^N_{n=1,...,6}$ and $\Delta q^S_{n=1,...,6}$. The formulation of this model is the same for both the (3-3) and (4-2) market configurations. We present the model solved by each TSO. Note that in this case, we assume that the two TSOs have full access to all re-dispatching resources.

Problem of TSO^N

As in the coordinated counter-trading problem, TSO^N minimizes its re-dispatching costs (25) accounting for the transmission constraints (28)-(29); the energy variation balance constraints (26)-(27) and the overall non-negativity constraint (30) on generation and consumption. However, it also considers the actions of the TSO^S as highlighted by the transmission and the non-negativity constraints where the re-dispatching variables of TSO^S appear.

$$\mathbf{Min}_{\mathbf{\Delta q_n^N}} \quad \sum_{i=1,2,4} \int_{q_i+\Delta q_i^S}^{q_i+\Delta q_i^S+\Delta q_i^N} c_i(\xi)d\xi - \sum_{j=3,5,6} \int_{q_j+\Delta q_j^S}^{q_j+\Delta q_j^S+\Delta q_j^N} w_j(\xi)d\xi \tag{25}$$

s.t.

$$\sum_{i=1,2,4} \Delta q_i^N + \sum_{j=3,5,6} \Delta q_j^N = 0 \qquad (\mu^{N,1}) \tag{26}$$

$$\sum_{j=3,5,6} \Delta q_j^N - \sum_{i=1,2,4} \Delta q_i^N = 0 \qquad (\mu^{N,2}) \tag{27}$$

$$\overline{F}_l - [\sum_{i=1,2,4} PTDF_{i,l}(q_i + \Delta q_i^N + \Delta q_i^S) - \sum_{j=3,5,6} PTDF_{j,l}(q_j + \Delta q_j^S + \Delta q_j^N)] \geq 0 \qquad (\lambda_l^{N,+}) \tag{28}$$

$$\overline{F}_l + [\sum_{i=1,2,4} PTDF_{i,l}(q_i + \Delta q_i^N + \Delta q_i^S) - \sum_{j=3,5,6} PTDF_{j,l}(q_j + \Delta q_j^S + \Delta q_j^N)] \geq 0 \qquad (\lambda_l^{N,-}) \tag{29}$$

$$q_n + \Delta q_n^N + \Delta q_n^S \geq 0 \qquad n = 1,...,6 \qquad (\nu_n^N) \tag{30}$$

Problem of the TSO^S

Problem (31)-(36) solved by TSO^S is similar to that of the TSO^N.

$$\mathbf{Min}_{\mathbf{\Delta q_n^S}} \quad \sum_{i=1,2,4} \int_{q_i+\Delta q_i^N}^{q_i+\Delta q_i^S+\Delta q_i^N} c_i(\xi)d\xi - \sum_{j=3,5,6} \int_{q_j+\Delta q_j^N}^{q_j+\Delta q_j^S+\Delta q_j^N} w_j(\xi)d\xi \tag{31}$$

s.t.

$$\sum_{i=1,2,4} \Delta q_i^S + \sum_{j=3,5,6} \Delta q_j^S = 0 \qquad (\mu^{S,1}) \tag{32}$$

$$\sum_{j=3,5,6} \Delta q_j^S - \sum_{i=1,2,4} \Delta q_i^S = 0 \qquad (\mu^{S,2}) \tag{33}$$

$$\overline{F}_l - [\sum_{i=1,2,4} PTDF_{i,l}(q_i + \Delta q_i^N + \Delta q_i^S) - \sum_{j=3,5,6} PTDF_{j,l}(q_j + \Delta q_j^N + \Delta q_j^S)] \geq 0 \qquad (\lambda_l^{S,+}) \tag{34}$$

$$\overline{F}_l + [\sum_{i=1,2,4} PTDF_{i,l}(q_i + \Delta q_i^N + \Delta q_i^S) - \sum_{j=3,5,6} PTDF_{j,l}(q_j + \Delta q_j^N + \Delta q_j^S)] \geq 0 \qquad (\lambda_l^{S,-}) \tag{35}$$

$$q_n + \Delta q_n^N + \Delta q_n^S \geq 0 \qquad n = 1, ..., 6 \qquad (\nu_n^S) \tag{36}$$

In this case, the average re-dispatching cost results from the division of the combination of the total re-dispatching costs of the two TSOs by $\sum_{i=1,2,4} q_i$.

F.2 Un-coordinated counter-trading Model 2

In this un-coordinated counter-trading model, TSOs have limited access to counter-trading resources that are not located in their control area. This means that TSO^N face restrictions in operating re-dispatch in the Southern zone and conversely for TSO^S. Starting from the model presented in Appendix F.1, this assumption can be easily modelled by adding constraint (37) to the TSO^N problem and (38) to the TSO^S problem:

$$-\overline{\Delta q_n^N} \leq \Delta q_n^N \leq \overline{\Delta q_n^N} \qquad n = 4, 5, 6 \qquad (\eta_n^{N,\pm}) \tag{37}$$

$$-\overline{\Delta q_n^S} \leq \Delta q_n^S \leq \overline{\Delta q_n^S} \qquad n = 1, 2, 3 \qquad (\eta_n^{S,\pm}) \tag{38}$$

where $\overline{\Delta q_n^N}$ and $\overline{\Delta q_n^S}$ are the bounds, taken in absolute values, on re-dispatching variables $\Delta q_n^{N,S}$. Conditions (37) and (38) refer to configuration (3-3). When considering configuration (4-2), they become:

$$-\overline{\Delta q_n^N} \leq \Delta q_n^N \leq \overline{\Delta q_n^N} \qquad n = 4, 5 \qquad (\eta_n^{N,\pm}) \tag{39}$$

$$-\overline{\Delta q_n^S} \leq \Delta q_n^S \leq \overline{\Delta q_n^S} \qquad n = 1, 2, 3, 6 \qquad (\eta_n^{S,\pm}) \tag{40}$$

The average re-dispatching cost α is computed as in Appendix F.1.

F.3 Un-coordinated counter-trading Model 3

The third and last un-coordinated counter-trading case assumes that TSOs operate counter-trading in their control area only. We here present this model for the market configuration (4-2). For configuration (3-3) see Oggioni et al. [25].

F.3.1 Problem of TSO^N

TSO^N minimizes its total re-dispatching costs (41) considering energy variation balances (42)-(43), transmission (44)-(45) and non-negativity constraints (46). It operates only in the Northern zone without interactions with TSO^S.

$$\mathbf{Min}_{\mathbf{\Delta q_n^N}} \quad \sum_{i=1,2} \int_{q_i^t}^{q_i+\Delta q_i^N} c_i(\xi) d\xi - \sum_{j=3,6} \int_{q_j}^{q_j+\Delta q_j^N} w_j(\xi) d\xi \tag{41}$$

s.t.

$$\Delta q_6^N + \Delta q_3^N + \Delta q_1^N + \Delta q_2^N = 0 \qquad (\mu^{N,1}) \tag{42}$$

$$\Delta q_6^N + \Delta q_3^N - \Delta q_1^N - \Delta q_2^N = 0 \qquad (\mu^{N,2}) \tag{43}$$

$$\overline{F}_l - [\sum_{i=1,2} PTDF_{i,l}(q_i + \Delta q_i^N) + \sum_{i=4} PTDF_{i,l}(q_i + \Delta q_i^S) - \sum_{j=3,6} PTDF_{j,l}(q_j + \Delta q_j^N) \tag{44}$$

$$- \sum_{j=5} PTDF_{j,l}(q_j + \Delta q_j^S)] \geq 0 \qquad (\lambda_l^{N,+})$$

$$\overline{F}_l + [\sum_{i=1,2} PTDF_{i,l}(q_i + \Delta q_i^N) + \sum_{i=4} PTDF_{i,l}(q_i + \Delta q_i^S) - \sum_{j=3,6} PTDF_{j,l}(q_j + \Delta q_j^N) \tag{45}$$

$$- \sum_{j=5} PTDF_{j,l}(q_j + \Delta q_j^S)] \geq 0 \qquad (\lambda_l^{N,-})$$

$$q_n + \Delta q_n^N \geq 0 \qquad n = 1,2,3,6 \qquad (\nu_n^N) \tag{46}$$

F.3.2 Problem of TSO^S

TSO^S operates counter-trading only in nodes 4 and 5. Its problem accounts for this assumption.

$$\mathbf{Min}_{\mathbf{\Delta q_n^S}} \quad \sum_{i=4} \int_{q_i}^{q_i+\Delta q_i^S} c_i(\xi) d\xi - \sum_{j=5} \int_{q_j}^{q_j+\Delta q_j^S} w_j(\xi) d\xi \tag{47}$$

s.t.

$$\Delta q_5^S + \Delta q_4^S = 0 \qquad (\mu^{S,1}) \tag{48}$$

$$\Delta q_5^S - \Delta q_4^S = 0 \qquad (\mu^{S,2}) \tag{49}$$

$$\overline{F}_l - [\sum_{i=1,2} PTDF_{i,l}(q_i + \Delta q_i^N) + \sum_{i=4} PTDF_{i,l}(q_i + \Delta q_i^S) - \sum_{j=3,6} PTDF_{j,l}(q_j + \Delta q_j^N) \tag{50}$$

$$- \sum_{j=5} PTDF_{j,l}(q_j + \Delta q_j^S)] \geq 0 \qquad (\lambda_l^{S,+})$$

$$\overline{F}_l + [\sum_{i=1,2} PTDF_{i,l}(q_i + \Delta q_i^N) + \sum_{i=4} PTDF_{i,l}(q_i + \Delta q_i^S) - \sum_{j=3,6} PTDF_{j,l}(q_j + \Delta q_j^N) \tag{51}$$

$$- \sum_{j=5} PTDF_{j,l}(q_j + \Delta q_j^S)] \geq 0 \qquad (\lambda_l^{S,-})$$

$$q_n + \Delta q_n^S \geq 0 \qquad n = 4,5 \qquad (\nu_n^S) \tag{52}$$

Parallel to the methodology applied in the other sections, the "global" average-dispatching cost is determined dividing the sum of objective functions (41) and (47) by $\sum_{i=1,2,4} q_i$.

G Results of the Un-coordinated counter-trading Model 3 of configuration (4-2)

	ATC=39		ATC=400	
	Prod/Cons	**Prod**	**Prod/Cons**	**Prod**
Welfare loss (€)	417.01	infeasible	3,008.64	infeasible
TRC (€)	1,635.11	infeasible	7,628.92	infeasible
ARC (€/MWh)	1.868	infeasible	10.498	infeasible
Net welfare (€)	**22,582.990**	infeasible	**19,991.36**	infeasible
MV line (1-6) (€/MWh)	37.99	infeasible	82.30	infeasible
MV line (2-5) (€/MWh)	0.00	infeasible	0.00	infeasible

Table 17: Results of the un-coordinated counter-trading Model 3 with configuration (4-2) when ATCs equal 39 and 400 MWh and the (1-6) line wedge is 0 €/MWh

	ATC=150		ATC=200	
	Prod/Cons	**Prod**	**Prod/Cons**	**Prod**
Welfare loss (€)	569.99	infeasible	1,028.24	infeasible
TRC (€)	3,423.20	infeasible	4,363.902	infeasible
ARC (€/MWh)	4.34	infeasible	5.923	infeasible
Net welfare (€)	**22,430.01**	infeasible	**21,971.76**	infeasible
MV line (1-6) (€/MWh)	54.97	infeasible	62.07	infeasible
MV line (2-5) (€/MWh)	0.00	infeasible	0.00	infeasible

Table 18: Results of the un-coordinated counter-trading Model 3 with configuration (4-2) when ATCs equal 150 and 200 MWh and the (1-6) line wedge is 0 €/MWh

	ATC=39		ATC=400	
	Prod/Cons	**Prod**	**Prod/Cons**	**Prod**
Welfare loss (€)	No equilibrium	infeasible	3,008.64	infeasible
TRC (€)	No equilibrium	infeasible	7,628.92	infeasible
ARC (€/MWh)	No equilibrium	infeasible	10.498	infeasible
Net welfare (€)	No equilibrium	infeasible	**19,991.36**	infeasible
MV line (1-6) TSO^N (€/MWh)	No equilibrium	infeasible	142.30	infeasible
MV line (2-5) TSO^N (€/MWh)	No equilibrium	infeasible	0.00	infeasible
MV line (1-6) TSO^S (€/MWh)	No equilibrium	infeasible	82.30	infeasible
MV line (2-5) TSO^S (€/MWh)	No equilibrium	infeasible	0.00	infeasible

Table 19: Results of the un-coordinated counter-trading Model 3 with configuration (4-2) when ATCs equal 39 and 400 MWh and the (1-6) line wedge is 60 €/MWh

	ATC=150		ATC=200	
	Prod/Cons	**Prod**	**Prod/Cons**	**Prod**
Welfare loss (€)	No equilibrium	infeasible	1,028.24	infeasible
TRC (€)	No equilibrium	infeasible	4,363.90	infeasible
ARC (€/MWh)	No equilibrium	infeasible	5.92	infeasible
Net welfare	No equilibrium	infeasible	**21,971.76**	infeasible
MV line (1-6) TSO^N (€/MWh)	No equilibrium	infeasible	62.07	infeasible
MV line (2-5) TSO^N (€/MWh)	No equilibrium	infeasible	0.00	infeasible
MV line (1-6) TSO^S (€/MWh)	No equilibrium	infeasible	2.07	infeasible
MV line (2-5) TSO^S (€/MWh)	No equilibrium	infeasible	0.00	infeasible

Table 20: Results of the un-coordinated counter-trading Model 3 with configuration (4-2) when ATCs equal 150 and 200 MWh and the (1-6) line wedge is 60 €/MWh

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