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DISCUSSION PAPER

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Adaptation and mitigation in long-term climate policies

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Abstract

The paper analytically explores the optimal policy mix between mitigation and environmental adaptation against climate change at a macroeconomic level. The constructed economic-environmental model is formulated as a social planner problem with the adaptation and abatement investments as separate decision variables. The authors prove the existence of a unique steady state and provide a comparative static analysis of the optimal investment. It leads to essential implications for associated long-term environmental policies. In particular, the dependence of the optimal ratio between abatement and adaptation investments on economic efficiency appears to have an inverted U-shape. Data calibration and numerical simulation are provided to illustrate theoretical outcomes.

Keywords: environmental adaptation, mitigation, optimal investment, long-term climate policies.

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1. Introduction

In the anticipation of forthcoming climate change, households, communities, and regulators need to implement measures that reduce the vulnerability of natural and human systems, which are known as *adaptation*. The IPCC Fourth Assessment Report defines adaptation to climate change as "an adjustment in natural or human systems in response to actual or expected climatic stimuli or their effects, which moderates harm or exploits beneficial opportunities". By contrast, *mitigation* is defined as "an anthropogenic intervention to reduce the sources or enhance the sinks of greenhouse gases" (IPCC, 2007, chap. 18, p. 750).[2] In fact, the single term *adaptation* covers a nebulous of actions such as investment in a coastal protection infrastructure, diversification of crops, implementation of warning systems, improvement in water resource management, development of new insurance instruments, modification of air cooling devices, etc. Hence, to deal with climate change the human society has two main long-term strategies: to mitigate greenhouse gas emissions or to adapt to global warming.[3] The economic and environmental science literature has scrupulously analyzed the cost and effectiveness of mitigation but has paid sufficiently less attention to adaptation.[4]

The cost of adaptation measures must not be underestimated. According to the World Bank, 2009, the studies available in the literature provide a wide range of estimates: adapting between 2010 and 2050 to a 2°C warmer world by 2050 would cost between 75 and 100 billion USD per year. This amount is of the same order of magnitude as the foreign aid that developed countries currently give developing countries each year, but it is still a very low percentage of the wealth of the developed countries as measured by their GDP. Another recent study conducted on behalf of the UNFCCC for the year 2030 on five sectors (water supply, human health, coastal zones, forestry, and fisheries) estimates the average cost of adaption between 28 and 67 billion USD per year in developing countries (UNFCCC, 2007). There also exists an extensive, but uneven and not exhaustive, literature on adaptation costs and benefits at the sector level, see, for example (Rosenzweig and Parry, 1994; Adams *et al*., 2003; Reilly *et al*, 2003) for the agricultural sector, (Morrison and Mendelsohn, 1999; Sailor and Pavlova, 2003; Mansur *et al.*, 2005) for energy demand, (Fankhauser, 1995; Yohe and Sclesinger, 1998; Nicholls and Tol, 2006) for sea-level rise. A little attention has been given to water resource management (EEA, 2007), transportation infrastructure (Kirshen *et al.*, 2007), health, or tourism. It follows from these examples that, in contrast to mitigation, adaptation more frequently occurs at each level of human activity. It starts nationwide and spreads to cities, municipalities, and households. Environmental adaptation possesses essential potential benefits that should be used wisely.

[2] There exists a difference between *mitigation* and *abatement*: the former refers to a reduction in *net emissions* of greenhouse gases while the latter refers to a reduction in *gross* emissions. Most theoretical models integrate only emission abatement opportunities (*i.e.* no sinks). It will also be the case in our paper.

[3] Two other options are also available, carbon sequestration and geo-engineering, but their contribution to cope with the problem of global warming is less important (see IPCC,2007).

[4] For a synthesis of current works devoted to abatement scenarios, see the Energy Modeling Forum 22 in the special issue of *Energy Economics*, 31, 2009.

For these reasons, adaptation has now reached the top of the policy agenda in most countries.[5] This contrasts with the spare economic literature devoted to it.

As stressed by (IPCC, 2007, chap. 18; UNFCCC, 2007; Agrawal and Fankhauser, 2008) adaptation offers many appealing and innovative characteristics as a policy instrument. Three of them deserve to be mentioned here. First, some adaptation measures are drawn by private agents' self-interest (*e.g.* air cooling in dwellings) while others have the property of a public good (*e.g.* dams). So the first policy issue about adaptation is to set the right incentives to reach an optimal level of adaptation. The second characteristic is that some developing countries do not have the financial capacity to address adequate adaptation measures. This can prevent such countries from implementing the optimal policy and impede their participation in international agreements.[6] However, the third and even more important issue is that, although if people can protect themselves from some adverse impacts of climate change, they cannot fully avoid them. Because adaptation does not tackle the causes of climate change, the world cannot afford to neglect abatement in greenhouse gas emissions. So, a key issue is to find the optimal balance between adaptation measures and emission abatement in order to implement an effective and efficient long-term climate policy.

While Shalizi and Lecocq, 2009 stress the need for an integrated portfolio of policy actions to minimize the climate bill, only a few studies explicitly consider adaptation and mitigation as policy responses to climate change. Some are descriptive (e.g. Kane and Yohe,2000; Smit et al., 2000; Agrawal and Fankhauser, 2008; EEA,2007; UNFCCC, 2007). Other papers use a game-theoretic framework, either static (Shalizi and Lecocq, 2009; Kane and Shogren, 2000) or dynamic (Buob and Stephan, 2010). Some others use computational integrated assessment models (Bosello et al., 2009; de Bruin et al., 2009). Among key questions addressed by these papers, the first one is to know whether mitigation and adaptation are substitutes or complementary policy instruments, and the second question is to know whether the country's stage of development influences the optimal policy mix between mitigation and adaptation. To date, the first question still remains open. Buod and Stephan (2010) answer the second question, arguing that high income countries should invest in both mitigation and adaptation, while low income countries should invest only in mitigation. In our paper, we propose a novel theoretical framework to address these two questions and provide original answers to each of them. In particular, we challenge the result of Buob and Stephan and show that the issue of substitutability between the two instruments depends on the stage of development.

In this paper we focus on a long-term dynamic analysis of a model with accumulation in physical capital, greenhouse gases, and adaptation capital. To do this, we rule out the strategic dimension of the question addressed by Buod and Stephan (2010) and the role of endogeneous risk addressed by Shalizi and Lecocq (2009) and Kane and Shogren (2000). In summary, we provide an analytic framework for studying optimal investment levels and associated long-term policies that encompass both mitigation and adaptation.

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[5] For a policy agenda update, see the UNFCCC web page devoted to adaptation: http://unfccc.int/adaptation.

[6] During the UNFCCC Copenhagen conference, one of the hottest policy questions was the funding of adaptation in developing countries and the required financial transfers from industrialized ones.

The goal of our paper is to analytically explore the optimal policy mix between mitigation and adaptation in response to climate changes at the macroeconomic level. The constructed economic-environmental model combines adaptation and emission abatement investments for the first time in economic literature. In formulating our model, we follow the mainstream of macroeconomic growth models with the environmental quality and investments. Gradus and Smulders (1993) and Smulders and Gradus (1996) are among the first to analyze long-term growth models with pollution, which involve spending on abatement activities. They model pollution as a flow rather than a stock. Stokey (1998) analyzes the optimal technology choice in several models with the pollution as both a flow and a stock, but without spending on abatement or pollution cleanup. Her models include the control of a technology index that linearly impacts the output and nonlinearly impacts the pollution. The abatement process is described in Byrne (1997) and Vellinga (1999) similarly to an environmental clean-up process. More recently, Economides and Philippopoulos (2008) examine optimal cleanup and public infrastructure policies through distorting tax in a general equilibrium model with renewable natural resources and compare them with a corresponding social planner problem. They classify their approach as close to Stokey (1998) but use renewable natural resources rather than pollution as the environmental quality indicator. Here we follow the commonly accepted abatement description of Gradus and Smulders (1993) and their followers (e.g., Chen et al., 2009).

The present paper suggests an aggregated economic-environmental model with both abatement and adaption investments and considers a related social planner problem. We choose the specifications of production and pollution processes and social preferences and justify the model in Section 2. Section 3 proves the existence of a unique steady state in the model without adaptation and derives some qualitative conclusions about optimal abatement policies. Here we employ perturbation techniques and obtain approximate analytic formulas for the steady state, which allow for a further comparative static analysis of the optimal policies. Section 4 analyzes the model with abatement and adaption and demonstrates essential economic implications of the obtained results. In particular, the dependence of the optimal policy mix between abatement and adaption investments on the economy scale is shown to be of an inverted U-shape. Data calibration and some numeric simulation of the optimal policies are provided in Section 5. Section 6 concludes and indicates possible extensions of research.

2. The model

We use the Solow-Swan one-sector growth framework in which the economy uses a Cobb-Douglas technology with constant returns to produce a single final good Y. The social planner allocates the final good across the consumption C, the investment I_K into the physical capital K, the investment I_D into the environmental adaptation D, and the emission abatement expenditures B in order to maximize the utility of the infinitely lived representative household:

$$\max_{I_K, I_D, C} \int_0^\infty e^{-\rho t} U(C(t), P(t), D(t)) dt \tag{1}$$

$I_K(t) \geq 0$, $\quad I_D(t) \geq 0$, $\quad C(t) \geq 0$, subject to the following constraints:

$$Y(t) = AK^{\alpha}(t) = I_K(t) + I_D(t) + B(t) + C(t), \tag{2}$$

$$K'(t) = I_K(t) - \delta_K K(t), \qquad K(0)=K_0, \tag{3}$$

$$D'(t) = I_D(t) - \delta_D D(t), \qquad D(0)=D_0, \tag{4}$$

where $\rho>0$ is the rate of time preference, $A>0$ and $0<\alpha<1$ are parameters of the Cobb-Douglas production function, $\delta_K\geq 0$, $\delta_D\geq 0$ are scrapping coefficients for physical capital and adaptation capital. The environmental quality is measured by P. The utility function (1) depends on the consumption C, the environmental quality P, and the environmental adaptation capital D.

The choice of a law of motion for the pollution P represents a major step in the problem under study (Toman and Withagen, 2000; Jones and Manuelli, 2001). Following Stokey (1998), Hritonenko and Yatsenko (2005), and some more recent works (e.g., Chen et al., 2009), and because of our interest in climate change, we shall assume that the pollution is accumulated as a stock.[7] The pollution inflow (net emission) is assumed to be proportional to the output Y. The abatement activity B is also a flow (Gradus and Smulders, 1993; Vellinga, 1999). The pollution stock grows as the net emission increases and declines as abatement expenditures B increase. Despite its simplicity, our specification captures the major qualitative features of the abatement activity B (see Gradus and Smulders, 1993).[8] Thus, the pollution motion is:

$$P'(t) = -\delta_P P(t) + \gamma Y(t)/B(t), \qquad P(0)=P_0. \tag{5}$$

The emission factor $\gamma>0$ in (5) characterizes the environmental dirtiness of the economy. To be more precise, it provides the net flow of pollution, that is, the flow resulting from productive activity net of abatement efforts. The pollution stock increases with this flow and deteriorates in time at a constant natural decay rate $\delta_P>0$.

The model (1)-(5) incorporates the key ingredients of the problem we are interested in. In particular, it will allow us to discuss the optimal policy mix (between emission abatement and adaptation) with respect to the stage of development of the economy. Hereafter, we shall discuss two polar cases. On one hand, a developing country is characterized by both a relatively small global factor productivity (small A) and a relatively high impatience degree (high ρ), and, on the other hand, a developed (industrialized) country possesses a high global productivity and smaller impatience. The question behind this comparison is the following: should the policy role of adaptation be different with the stage of development of a country?

[7] Thus, P represents the concentration of greenhouse gases in the atmosphere. For simplification, we shall consider this as a proxy to temperature increase, the latter is the real (though also approximate) causal factor for climate change welfare losses. This approximation is acceptable as temperature increase depends on the concentration of greenhouse gases, with some lags. See the fourth report of IPCC (2007).

[8] Smulders and Gradus (1996) also consider a more general polluting model as the flow $Y^{\varpi}B^{-\lambda}$, $\lambda>\varpi$.

Some discussions will be also done with respect to the pollution intensity of the economy γ. Empirically, the link between pollution intensity and global factor productivity is not that straightforward. Economic development may go with higher carbon intensity, but it may also lead to decarbonization (because of increased global productivity, energy or carbon saving technological progress, dematerialization, etc.).

The optimization problem (1)-(5) includes three decision variables I_K, I_D, C, four state variables K, D, B, P, and four constraints-equalities (2)-(5). The major novelty of the problem is the dependence of $U(C, P, D)$ on the adaptation expense D. In line with common specifications in the environmental literature (Gradus and Smulders, 1993; Stokey, 1998;[9] Byrne, 1997; Hritonenko and Yatsenko, 2005; Economides and Philippopoulos, 2008; and others), the utility function $U(C, P, D)$ is taken to be additively separable as

$$U(C, P, D) = U_1(C) - U_2(P, D) = \ln C - \eta(D)\frac{P^{1+\mu}}{1+\mu}, \tag{6}$$

where the factor $\eta(D)$ describes the *environmental vulnerability* of the economy to climate change. Being a key ingredient of our paper, the function $\eta(D)$ reflects the fact that the environmental vulnerability η can be reduced by investing in adaptation. In that sense, the function $\eta(D)$ can be also interpreted as the *efficiency of adaptation measures* to protect people from the damages of climate change. The parameter μ>0 reflects the negative increasing marginal utility of pollution, which is a common assumption in such models. The specific choice of the function $\eta(D)$ will be provided and discussed in Section 4 below. Many other specifications of the production and pollution dynamics are possible.

3. Benchmark model with abatement

In order to understand the basic dynamic properties of model (1)-(6), let us start with its benchmark version with pollution abatement, but without adaptation, *i.e.*, at η=const and D=0. In this case, model (1)-(6) becomes

$$\max_{I_K, I_M, C} \int_0^\infty e^{-\rho t}\left(\ln C - \eta\frac{P^{1+\mu}}{1+\mu}\right)dt, \qquad I_K(t)\geq 0,\ \ C(t)\geq 0, \tag{7}$$

$$AK^\alpha(t) = I_K(t) + B(t) + C(t), \tag{8}$$

$$K'(t) = I_K(t) - \delta K(t), \qquad K(0)=K_0, \tag{9}$$

$$P'(t) = -\delta_P P(t) + \gamma AK^\alpha(t)/B(t), \qquad P(0)=P_0, \tag{10}$$

where $\delta=\delta_K$. The current-value Hamiltonian for the problem (7)-(10) is given by

[9] Stokey (1998) considers a CRRA utility of consumption rather than the logarithmic one.

$$H = e^{-\rho t}(\ln C - \eta \frac{P^{1+\mu}}{1+\mu}) + \lambda_1(AK^\alpha - I_K - B - C) + \lambda_2(-\delta_P P + \gamma AK^\alpha / B)$$
$$+ \lambda_3(I_K - \delta K) + \mu_1 I_K + \mu_2 C \quad (11)$$

where the dual variables λ_1, λ_2, λ_3 are associated with equalities (8)-(10) and μ_1, μ_2 are related to the irreversibility constraints $I_K \geq 0$, $C \geq 0$. The first order extremum conditions for the two decision variables I_K and C are:

$$-\lambda_1 + \lambda_3 + \mu_1 = 0, \quad (12)$$

$$C^{-1}e^{-\rho t} - \lambda_1 + \mu_2 = 0, \quad (13)$$

or, in the case of an interior solution,

$$\lambda_1 = \lambda_3 = C^{-1}e^{-\rho t}. \quad (14)$$

The first order conditions for the state variables K, B and P are, respectively:

$$\alpha\lambda_1 AK^{\alpha-1} + \lambda_2 \frac{\gamma}{B}\alpha AK^{\alpha-1} - \delta\lambda_3 = -\lambda_3', \quad (15)$$

$$-\lambda_1 - \lambda_2 \frac{\gamma}{B^2} AK^\alpha = 0, \quad (16)$$

$$-\eta P^\mu e^{-\rho t} - \delta_P \lambda_2 = -\lambda_2', \quad (17)$$

and the transversality conditions take the form of $\lim_{t\to\infty}\lambda_2(t) = \lim_{t\to\infty}\lambda_3(t) = 0$. Excluding λ_1, λ_2, λ_3 from (15)-(17) and using (8)-(10), we obtain the system in K, B, C and P:

$$AK^\alpha = \delta K + B + C + K', \quad (18)$$

$$\alpha AK^{\alpha-1} - \alpha\frac{B}{K} - \frac{C'}{C} = \delta + \rho, \quad (19)$$

$$P' + \delta_P P = \gamma \frac{AK^\alpha}{B}, \quad (20)$$

$$\eta P^\mu = \frac{B}{\gamma AK^\alpha C}\left(B(\delta_P + \rho) + \alpha\frac{B}{K}K' + \frac{B}{C}C' - 2B' \right). \quad (21)$$

System (18)-(21) determines the interior optimal dynamics. This dynamics in the case of small values η should be close to the well-known neoclassical Solow-Swan model (Barro and Sala-i-Martin, 1995), which has an asymptotically stable steady–state equilibrium at $\alpha<1$. Similar result for our model is proven in Proposition 1.

Proposition 1. *The problem (7)-(10) possesses the unique steady state:*

$$\bar{B} = A\bar{K}^{\alpha} - \bar{K}(\delta + \rho)/\alpha, \tag{22}$$

$$\bar{C} = \delta\bar{K}(1-\alpha)/\alpha + \bar{K}\rho/\alpha, \tag{23}$$

$$\bar{P} = \frac{\gamma A}{\delta_P[A - \bar{K}^{1-\alpha}(\delta + \rho)/\alpha]}, \tag{24}$$

where $\bar{K}$, *0<* $\bar{K} < (\alpha A/(\delta + \rho))^{\frac{1}{1-\alpha}}$, *is found from the nonlinear equation*

$$\frac{[A - \bar{K}^{1-\alpha}(\delta + \rho)/\alpha]^{2+\mu}}{\bar{K}^{1-\alpha}} = \frac{\gamma^{1+\mu} A^{1+\mu}\eta[(\delta + \rho)/\alpha - \delta]}{(\delta_P + \rho)\delta_P{}^{\mu}}. \tag{25}$$

Proof. *See Appendix 1.*

As shown in Appendix 1, the nonlinear equation (25) always has a unique positive solution $\bar{K}$,

$$0 < \bar{K} < (\alpha A/(\delta + \rho))^{\frac{1}{1-\alpha}}. \tag{26}$$

The last term of this inequality is the maximal capital stock level (which will be defined below). As in the standard Solow-Swan model, the optimal trajectories $K(t)$, $B(t)$, $C(t)$, and $P(t)$ asymptotically converge to the steady state $\bar{K}$, $\bar{B}$, $\bar{C}$, and $\bar{P}$. Setting a maximum limit $\bar{P}$ for the pollution level P is commonly accepted in environmental economic literature to avoid a further degeneration of the environmental quality (see Smulders and Gradus, 1996; Bovenberg and Smulders, 1996; Elbasha and Roe, 1996; Hritonenko and Yatsenko, 2005). By Proposition 1, our model produces such dynamics.

The comparative statics analysis based on (22)-(25) can shed some light on qualitative properties of the economy and the relation between the optimal long-term abatement policy and model parameters. First, it appears that shifting to a lower pollution intensity γ increases the size of the economy $\bar{K}$. It also leads to smaller abatement expenses $\bar{B}$ and to a smaller pollution level $\bar{P}$. Thus, a cleaner technology is unambiguously good for the economy. If γ approaches 0, then $\bar{K}$ tends to its maximum level $(\alpha A/(\delta + \rho))^{\frac{1}{1-\alpha}}$, while $\bar{B}$ and $\bar{P}$ tend to zero. Second, if the vulnerability of the economy to climate change increases, then the whole economy downsizes (smaller $\bar{K}$), the pollution level decreases but abatement expenditures increase. If η tends to 0, then $\bar{K}$ tends to its maximum level, $\bar{B}$ tends to zero, and $\bar{P}$ tends to the infinity. Third, the economy also strongly depends on the natural decay rate of pollution δ_P. If the environmental decay rate δ_P becomes smaller, then the pollution level $\bar{P}$ increases and the size of the economy $\bar{K}$ shrinks. Indeed, it is more and more difficult to control a pollution stock when its decay rate becomes very small. If a pollution removal is negligible, that is, $\delta_P \to 0$, then $\bar{K}$ and $\bar{B}$ tend to zero while $\bar{P}$ tends to the infinity. This flows from the restricted abatement efficiency in the pollution motion (10), where abatement can only keep the pollution emission stable, but cannot decrease it to zero. It is also related to the negative increasing marginal utility of the pollution stock (at $\mu>0$), in particular, $\bar{K}$

remains positive at δ_P=0 if one assumes that μ=0.[10] So, the natural decay rate plays a critical role in our problem. In the mechanics of climate change, greenhouse gases that cause global warming remain for a very long time in the atmosphere, and the natural decay rate of this stock is extremely low.[11]

Let us now consider the use of the only policy instrument $\overline{B}$ available in this section to cope with climate change. An analysis of formula (22) and nonlinear equation (25) shows that both optimal steady state capital $\overline{K}$ and abatement $\overline{B}$ increase as $(A)^{\frac{1}{1-\alpha}}$ when the global productivity A increases.[12] Correspondingly, the optimal ratio $\overline{B}/\overline{K}$ appears to be independent of the productivity A (the derivative of B/K in A is zero).

In order to provide a further comparative static analysis, an explicit formula for the solution $\overline{K}$ of equation (25) would be highly desirable. To obtain such approximate analytic formulas, we employ perturbation techniques (small parameter methods) well known in such applied sciences as physics and engineering but relatively less common in economics. Some economic examples are provided in (Araujo and Scheinkman, 1977; Boucekkine et al., 2008; Cosimano, 2008; Gaspar and Judd, 1997; Hritonenko and Yatsenko, 2005; Judd and Guu, 1997; Khan and Rashid, 1982; Santos, 1994; or Wagener, 2006). The general idea of such methods is to reduce a problem under study to a simpler problem with a known solution (or a solution algorithm). In economics, such techniques are usually followed by a numeric solution, but they can also lead to approximate analytic solutions.

One of key challenges of the perturbation techniques is to identify a model parameter such that the general problem is reduced to a simpler one when the parameter is zero. As shown in Appendix 1, the convenient choice for such a parameter in our model is

$$\kappa = \eta\left(\frac{\gamma}{\delta_P}\right)^{\mu+1}\frac{1-\alpha\delta/(\rho+\delta)}{1+\rho/\delta_P} \qquad (27)$$

This parameter κ has some appealing economic interpretation directly related to the major contribution of our paper, namely, it encompasses the net pressure of human activity on the environment, that is, the pollution intensity γ of economic activity compared to the natural decay rate δ_P of the pollution stock. It is weighted by the degree η of vulnerability of the economy. These properties of the parameter κ lead us to interpret it as a *κ–indicator of environmental pressure*, which combines both the pressure γ/δ_P on the environment and the pressure η of the environment on welfare. The third factor in (27) involves other parameters but, if one considers that δ=0, as it will be done later in the paper, then this factor would be close to one.

[10] Similar qualitative dynamics of economic and environmental parameters are common in the economic-environmental models available in the literature, even when a much more detailed description of pollution accumulation and assimilation is considered (see, *e.g.*, Toman and Withagen, 2000).

[11] The three main greenhouse gases have the following lifetime in the atmosphere: carbon dioxide (CO_2): 100 to 150 years, methane (CH_4): 10 years, nitrous oxide (N_2O): 100 years (IPCC, 2007).

[12] It matches the result of Smulders and Gradus (1996) that "…an increase in environmental care is associated with an increase in growth".

Using perturbation techniques, we can show that the following approximate formulas are valid[13] for $\overline{K}$ (see Appendix 1 for the proof):

$$\overline{K} \cong \left[\frac{\alpha A}{(\rho+\delta)\kappa}\right]^{\frac{1}{1-\alpha}} \qquad \text{at} \quad \kappa >> 1, \tag{28}$$

$$\overline{K} \cong \left[\frac{\alpha A}{\rho+\delta}\left(1-\kappa^{\frac{1}{2+\mu}}\right)\right]^{\frac{1}{1-\alpha}} \qquad \text{at} \quad \kappa << 1\,. \tag{29}$$

As it may be expected, the optimal size of an economy is directly related to the environmental pressure it exerts on the environment, as expressed by the κ-indicator. More interestingly, this dependence is weak when the environmental pressure is small (κ-indicator << 1), but the dependence becomes stronger as the pressure increases (κ-indicator >> 1).

In the following sections we will focus on the case (28) (large κ-indicator) because it captures the situation faced by the world today: the environmental self–cleaning capability δ_P is negligible compared to the emission impact factor γ and the environmental vulnerability η. This fact will be formally expressed as an assumption in the next section and empirically discussed in Section 5. As already mentioned, the decay rate for atmospheric greenhouse gases concentrations is extremely small. In addition, the current vulnerability to climate change turns out to be more important than expected a few years ago, notably because of a faster pace for global warming and more local extreme events than predicted (hurricanes, droughts, floods…).[14] Finally, current power generation and transportation systems are mainly based on fossil fuel technologies. Because a revolutionary carbon-free technology does not come out yet, our economies are bound to spend money in order to reduce their carbon dependence (*i.e.*, abatement expenditures) which is also captured by our choice of (28).

Now let us introduce the possibility of reducing the damages of climate change, that is, adaptation.

4. A model with abatement and adaptation

Let us return to the original problem (1)-(6) and analyze how the possibility of the adaptation D affects the qualitative results obtained in Section 3. Let $\delta_K=\delta_D=\delta$. The current-value Hamiltonian for the problem (1)-(6) is given by

[13] Here and thereafter, the notation $f(\varepsilon) \cong g(\varepsilon)$ means that $f(\varepsilon)=g(\varepsilon)[1+o(\varepsilon)]$ for some small parameter $0<\varepsilon<<1$ and $f(\varepsilon)\to g(\varepsilon)$ when $\varepsilon\to 0$.

[14] As IPCC (2007) states, extreme events are becoming more frequent and temperature increase is going faster than expected.

$$H = e^{-\rho t}(\ln C - \eta(D)\frac{P^{1+\mu}}{1+\mu}) + \lambda_1(AK^\alpha - I_K - I_D - B - C) + \lambda_2(-\delta_P P + \gamma AK^\alpha / B)$$
$$+ \lambda_3(I_K - \delta K) + \lambda_4(I_D - \delta D) + \mu_1 I_K + \mu_3 I_D + \mu_2 C, \quad (30)$$

where the dual variables λ_1, λ_2, λ_3, λ_4, are associated with equalities (2)-(5) and μ_1, μ_2, μ_3 reflect the irreversibility constraints. The first order conditions for I_K, I_D and C are:

$$-\lambda_1 + \lambda_3 + \mu_1 = 0, \quad (31)$$
$$-\lambda_1 + \lambda_4 + \mu_3 = 0, \quad (32)$$
$$C^{-1}e^{-\rho t} - \lambda_1 + \mu_2 = 0, \quad (33)$$

which, in the case of interior solutions, leads to

$$\lambda_1 = \lambda_3 = \lambda_4 = C^{-1}e^{-\rho t} \quad (34)$$

The first order conditions for the state variables K, B, P and D are, respectively:

$$\alpha\lambda_1 AK^{\alpha-1} + \lambda_2 \frac{\gamma}{B}\alpha AK^{\alpha-1} - \delta\lambda_3 = -\lambda_3', \quad (35)$$
$$-\lambda_1 - \lambda_2 \frac{\gamma}{B^2} AK^\alpha = 0, \quad (36)$$
$$-\eta(D)P^\mu e^{-\rho t} - \delta_P \lambda_2 = -\lambda_2', \quad (37)$$
$$-\eta'(D)P^{\mu+1}e^{-\rho t}/(\mu+1) - \delta\lambda_4 = -\lambda_4'. \quad (38)$$

Excluding λ_1, λ_2, λ_3 and λ_4 from (35)-(38) and using (2)-(5), we obtain the system

$$AK^\alpha = \delta K + K' + \delta D + D' + B + C, \quad (39)$$
$$\alpha AK^{\alpha-1} - \alpha\frac{B}{K} - \frac{C'}{C} = \delta + \rho, \quad (40)$$
$$P' + \delta_P P = \gamma\frac{AK^\alpha}{B}, \quad (41)$$
$$\eta(D)P^\mu = \frac{B}{\gamma AK^\alpha C}\left(B(\delta_P + \rho) + \alpha\frac{B}{K}K' + \frac{B}{C}C' - 2B'\right), \quad (42)$$
$$-\eta'(D)\frac{P^{\mu+1}}{\mu+1} = \frac{1}{C}\left(\rho + \delta + \frac{C'}{C}\right). \quad (43)$$

for the optimal interior solution K, B, C, D, and P.

To proceed further with our analysis, we need a more specific form for the function $\eta(D)$. A good choice that possesses realistic features will be the following exponential function:

$$\eta(D) = \underline{\eta} + (\overline{\eta} - \underline{\eta})e^{-aD}, \quad \text{with } \overline{\eta} > \underline{\eta} > 0, \quad a > 0. \quad (44)$$

Indeed, the function (44) is monotonically decreasing in adaptation efforts and goes down from a maximum value $\eta(0) = \overline{\eta} > \underline{\eta}$ of environmental vulnerability, reached when there is no adaptation at all, to a minimum value $\eta(\infty) = \underline{\eta} > 0$, reached when adaptation efforts tend to infinity. The function η is illustrated in Figure 1. The first derivative of η, $\eta'(D) = -a(\overline{\eta} - \underline{\eta})e^{-aD}$, suggests us to introduce the parameter

$$\mathrm{M}_{\eta} = a(\overline{\eta} - \underline{\eta}) \tag{45}$$

as the *technological potential for adaptation* of the economy under consideration. In (45), $(\overline{\eta} - \underline{\eta})$ represents the range of physical adaptation opportunities, *i.e.* the benefits in terms of vulnerability reduction associated with adaptation measures. Depending on the physical characteristics of the economy (altitude, importance of coastal areas, etc.), the range of adaptation measures can be more or less wide. As a consequence, the potential welfare gain between no adaptation and full adaptation can vary depending on the country, which is captured by the difference $(\overline{\eta} - \underline{\eta})$ in (45). The exponential form of (44) reflects the assumption of decreasing returns of the adaptation investment *D*, which is natural by technological and economic applications. Indeed, initial adaptation measures are supposed to be the most efficient in terms of vulnerability reduction. In other words, the parameter *a* represents the *marginal efficiency* of adaptation, which is higher for the first adaptation measures and then decreases gradually with the amount of investment (see Figure 1). For instance, the first adaptation projects (*e.g.*, dams) can significantly decrease the environmental vulnerability of a specific country/region while the further (smaller) vulnerability decrease would require much larger investments.[15]

4.1. The existence and properties of the steady state

To keep the analytic complexity feasible, we will restrict ourselves with the long-term dynamics (steady state) and no capital depreciation, δ=0.

Proposition 2. *The economy given by equations (39)-(43) possesses a unique steady state:*

$$\overline{B} = A\overline{K}^{\alpha} - \overline{K}\rho/\alpha, \tag{46}$$

$$\overline{C} = \overline{K}\rho/\alpha, \tag{47}$$

$$\overline{P} = \frac{\gamma A}{\delta_P (A - \overline{K}^{1-\alpha}\rho/\alpha)}, \tag{48}$$

where the steady state components $\overline{K}$, $0 < \overline{K} < (\alpha A/\rho)^{1/(1-\alpha)}$, *and* $\overline{D} \geq 0$.
If $\overline{K}$ *is small, then optimal* $\overline{D}=0$. *Otherwise,* $\overline{K}>0$ *and* $\overline{D}>0$ *are determined by the following system of two equations :*

$$\frac{(A - \overline{K}^{1-\alpha}\rho/\alpha)^{2+\mu}}{\overline{K}^{1-\alpha}} = \frac{\rho\gamma^{1+\mu}A^{1+\mu}}{\alpha(\delta_P + \rho)\delta_P{}^{\mu}}[\underline{\eta} + (\overline{\eta} - \underline{\eta})e^{-a\overline{D}}], \tag{49}$$

$$a(\overline{\eta} - \underline{\eta})e^{-a\overline{D}}\,\frac{\gamma^{\mu+1}A^{\mu+1}}{\alpha\delta_P{}^{\mu+1}(\mu+1)} = \frac{(A - \overline{K}^{1-\alpha}\rho/\alpha)^{\mu+1}}{\overline{K}} \tag{50}$$

Proof. *See Appendix 2.*

[15] Alternative hypotheses on the function $\eta(D)$ are possible and are of obvious interest.

By Proposition 2, the optimal adaptation level is $\overline{D}$=0 and the vulnerability is maximal, $\eta(D)=\eta(0)=\overline{\eta}$, for small values of $\overline{K}$ from some interval $[0,\overline{K}_c]$. In this case, the optimal capital has the unique solution (28) as in the model without adaptation described by Proposition 1. Proposition 2 expresses the range for positive adaptation in terms of the endogenous variable $\overline{K}$ and is not specific enough in answering whether and when the optimal adaptation can be positive, $\overline{D}$>0.

4.2. On the range of positive optimal adaptation

To express the condition for the positive optimal adaptation $\overline{D}$>0 in terms of model parameters, we shall consider the case where the value of the κ-indicator is high when the environmental vulnerability is minimal, *i.e.* when no adaptation measures are implemented ($\eta(D)=\eta(0)=\overline{\eta}$). This leads to the following assumption.

Assumption A1. $\overline{\kappa}=\left(\dfrac{\gamma}{\delta_P}\right)^{\mu+1}\dfrac{\overline{\eta}}{(1+\rho/\delta_P)}>>1$. (51)

Empirical evidence to support this assumption for the current world economy will be provided in Section 5 below. Then the following result is valid.

Proposition 3. *Under Assumption A1 and the restriction*

$$\frac{A}{\rho}a^{1-\alpha}\left(\overline{\eta}-\underline{\eta}\right)^{1-\alpha}>\overline{\eta}\left(\frac{\gamma}{\delta_P}\right)^{(\mu+1)\alpha}\frac{(\mu+1)^{1-\alpha}}{(1+\rho/\delta_P)\alpha^{\alpha}}\ , \tag{52}$$

the system (49)-(50) has a unique solution ($\overline{K}$, $\overline{D}$) with a positive optimal adaptation level given by

$$\overline{D}=\frac{1}{a}\ln\frac{a\left(\overline{\eta}-\underline{\eta}\right)\gamma^{\mu+1}\overline{K}}{\alpha(\mu+1)\delta_P{}^{\mu+1}[1-\overline{K}^{1-\alpha}\rho/(A\alpha)]^{\mu+1}}>0. \tag{53}$$

If condition (52) does not hold, then the optimal adaptation $\overline{D}$ is the corner (zero) solution and the optimal $\overline{K}$ is determined from (49) at $\overline{D}$ =0 as:

$$\overline{K}\cong\left[\frac{\alpha A(\rho+\delta_P)\delta_P{}^{\mu}}{\rho\overline{\eta}\gamma^{\mu+1}}\right]^{\frac{1}{1-\alpha}}. \tag{54}$$

Proof. *See Appendix 2.*

Formula (54) directly follows from (27) and (28) where η is replaced by $\overline{\eta}$ reached in the absence of adaptation. It appears that the adaptation policy $\overline{D}$>0 is positive only under some restrictions on the model parameters. This restriction suggests a balance between the *technological potential for adaptation*, $a\,(\overline{\eta}-\underline{\eta})$ and the factor productivity A, on one side, and the limit of environmental vulnerability $\overline{\eta}$, the pollution intensity γ, and the natural pollution assimilation rate δ_P, on the other.

If the adaptation opportunities or their marginal efficiency are too small in the economy, then (52) does not hold and the optimal adaptation level $\overline{D}$ is zero. In other words, there exists a minimal level of the adaptation potential below which (52) does not hold. Let us introduce the critical value

$$\mathrm{M}_{\eta cr} = [a(\overline{\eta} - \underline{\eta})]_{cr} = \left[\left(\frac{\gamma}{\delta_P} \right)^{(\mu+1)\alpha} \frac{\rho \overline{\eta} (\mu+1)^{1-\alpha}}{A(1+\rho/\delta_P)\alpha^{\alpha}} \right]^{\frac{1}{1-\alpha}} \tag{55}$$

of the adaptation potential under which optimal adaptation is zero. The formula (55) is determined from (52) with the equality sign instead of inequality. The following corollary provides the main properties of this critical value.

Corollary 1 (on the critical potential adaptation value). *The critical value* $\mathrm{M}_{\eta cr} = [a(\overline{\eta} - \underline{\eta})]_{cr}$ *for the adaptation potential is smaller for a greater global productivity of the economy A and a smaller discount factor ρ, ceteris paribus. If* $\mathrm{M}_{\eta} > \mathrm{M}_{\eta cr}$, *i.e., (52) holds, then the optimal* $\overline{D}>0$ *is larger for a greater productivity factor A and a smaller discount factor ρ.*

Let us interpret this corollary. When the economy is very productive, it can support adaptation measures of smaller efficiency and, on the other side, the opportunity cost of adaptation (which is consumption) is less important compared to the environment quality in the utility function. Furthermore, it also appears that the threshold value $\mathrm{M}_{\eta cr}$ is smaller for smaller discount factors ρ. Hence, because of these two effects, it can be expected that a developed country (A large, ρ small) will engage itself sooner in adaptation measures than a developing country (A small, ρ large) *ceteris paribus*.

Two other effects of Corollary 1 are also deserved to be mentioned. First, the threshold value given by (55) for the adaptation marginal efficiency is smaller for a smaller pollution intensity γ.[16] Indeed, a smaller γ also means less efficient abatement activities, which opens a room for the adaptation sooner. Second, the threshold (55) increases for smaller natural pollution depreciation factors δ_P because the adaptation measures are accumulated as a stock, while abatement measures last just one time period. So, when the stock effect of pollution is very important (δ_P is small), then the relative benefits of investing in long-lasting measures increase. When the pollution does not accumulate or is self-cleaned rapidly, then flow-abatement measures are more efficient. Thus, the optimal policy arbitrage between spending the resources of the economy in a stock (adaptation measures) or in a flow (abatement measures) depends on the nature of the pollutant. The relative importance of environment-related investments increases as the lifetime of the pollutant gets longer.

[16] The optimal adaptation D is *always* positive at γ=0 when the abatement B does not impact the pollution level.

If the restriction (52) is not valid, then the optimal adaptation level $\overline{D}$=0, and the capital $\overline{K}$ has the unique solution (54). If (52) holds, then, as established by Proposition 2, there exist a unique optimal capital level $\overline{K}$, 0<$\overline{K}$<$(A/\rho)^{1/(1-\alpha)}$, and the corresponding unique optimal adaptation level $\overline{D}$>0 found from equations (49) and (50).

4.3. On the optimal adaptation - abatement policy mix

Solving the nonlinear equations (49)-(50) enables us to provide specific policy recommendations and find the optimal policy mix between adaptation and abatement. We can obtain such an *explicit* approximate solution under assumptions on the economic and adaptation efficiency stricter than (51):

Assumption A2. $$\underline{\kappa} = \left(\frac{\gamma}{\delta_P}\right)^{\mu+1} \frac{\underline{\eta}}{(1+\rho/\delta_P)} >> 1 \qquad (56)$$

Assumption B. $$\left(\frac{A}{\rho}a^{1-\alpha}\right)^{1/\alpha} >> \underline{\kappa}. \qquad (57)$$

Assumption A2 implies that even when the economy has reached its minimal level of vulnerability, the value of its *κ–indicator of environmental pressure* remains high. So, Assumption A2 is stricter than Assumption A1 because it includes the minimal possible vulnerability $\underline{\eta}$ rather than the maximal vulnerability $\overline{\eta}$ as in (51). This means that the economy cannot fully avoid the adverse effects of pollution, even when all the adaptation measures are implemented. Again, it seems to be a relevant assumption for climate change. Assumption B indicates that the ratio of the global productivity A and the adaptation efficiency a to the discount factor ρ is much larger than the previous ratio $\underline{\kappa}$ (which is already large). The feasibility of Assumptions A2 and B for the current world economy is discussed in Section 5. Under Assumptions A2 and B, the optimal steady state capital $\overline{K}$ and adaptation $\overline{D}$ levels are determined by the approximate formulas (see Appendix 2 for the proof):

$$\overline{K} \cong \left[\frac{\alpha A(\rho+\delta_P)\delta_P{}^{\mu}}{\underline{\eta}\rho\gamma^{\mu+1}}\right]^{\frac{1}{1-\alpha}}, \qquad (58)$$

$$\overline{D} \cong \frac{1}{a(1-\alpha)}\ln\left[\left(\frac{a(\overline{\eta}-\underline{\eta})}{\mu+1}\right)^{1-\alpha} \frac{A(\rho+\delta_P)\alpha^{\alpha}}{\underline{\eta}\rho\gamma^{\alpha(\mu+1)}\delta_P{}^{1-\alpha(\mu+1)}}\right] >> 1 . \qquad (59)$$

The only difference between the approximate formulas (58) and (54) is that (58) includes the vulnerability $\underline{\eta}$ reached at large $\overline{D}$>>1 and (54) has the vulnerability $\overline{\eta}$

reached at $\overline{D}$=0. To demonstrate the benefits of adaptation and abatement management versus the case with no adaptation, let us denote the optimal steady state solutions (58), (46)-(48) as ($\overline{K}_D$, $\overline{C}_D$, $\overline{B}_D$, $\overline{P}_D$) and the corresponding optimal steady state solutions (28), (22)-(24) in the model with no adaptation as ($\overline{K}_{ND}$, $\overline{C}_{ND}$, $\overline{B}_{ND}$, $\overline{P}_{ND}$).

Corollary 2 (on the comparison with and without adaptation). *Under Proposition 3, if adaptation is positive, then*

(i) the size of the economy is larger, $\overline{K}_D > \overline{K}_{ND}$ *;*

(ii) the pollution level is higher, $\overline{P}_D > \overline{P}_{ND}$ *;*

(iii) the relative abatement efforts are smaller, $(\overline{B}/\overline{K})_D < (\overline{B}/\overline{K})_{ND}$.

Proof. Dividing formulas (58) and (46)-(48) by corresponding (28),(22)-(24), we obtain

$$\frac{\overline{K}_D}{\overline{K}_{ND}} = \frac{\overline{C}_D}{\overline{C}_{ND}} \cong \left(\frac{\overline{\eta}}{\underline{\eta}}\right)^{\frac{1}{1-\alpha}}, \quad \frac{\overline{B}_D}{\overline{B}_{ND}} \cong \left(\frac{\overline{\eta}}{\underline{\eta}}\right)^{\frac{\alpha}{1-\alpha}}, \quad \frac{(\overline{B}_D/\overline{K}_D)}{(\overline{B}_{ND}/\overline{K}_{ND})} \cong \frac{\underline{\eta}}{\overline{\eta}} < 1. \qquad \square$$

Since the adaptation enhances the flexibility of the economy and allows it to suffer less from a given level of pollution, a suitable level of adaptation is beneficial for the economy as a whole. When the economy protects itself with adaptation, the optimal abatement effort can be smaller and the pollution level can be larger. Because the size of the economy is not the same, we compare abatement efforts as expressed per unit of capital $\overline{B}/\overline{K}$. When comparing the two economies, abatement and adaptation appear as substitutable policy instruments: a positive adaptation level reduces emission abatement efforts. Actually, the interaction between adaptation and abatement is not that straightforward for it depends on the country characteristics. The following corollary describes the optimal policy mix between adaptation and abatement. It shows how the optimal policy mix changes when the global factor productivity increases. The relationship is not monotonic, and the optimal adaptation level can also be zero.

Corollary 3 (on the optimal adaptation policy). *Under Proposition 3,*

(i) the optimal abatement effort $\overline{B}/\overline{K}$ *is independent of the productivity level A;*

(ii) the optimal policy mix $\overline{D}/\overline{B}$ *is zero when* $0 < A < A_c$, $(A_c>0)$, *it is increasing in A until a critical value* $A_{cr} > A_c$, *and then it is decreasing in A when* $A > A_{cr}$.

The two critical values on global productivity are

$$A_c = \frac{\eta\rho(\mu+1)^{1-\alpha}\gamma^{\alpha(\mu+1)}}{M_\eta^{\ 1-\alpha}(1+\rho/\delta_P)\alpha^\alpha\delta_P^{\ \alpha(\mu+1)}}, \qquad A_{cr} = e^{1-\alpha}A_c. \tag{60}$$

Proof. *See Appendix 2.*

The first statement of this corollary has been already demonstrated for the model without adaptation in Section 3. The statement (*ii*) is new and has major policy implications. It states that, when the global productivity of the economy is weak, *i.e.* lower than a critical level A_c, then it is optimal to focus on abatement and not to spend money on adaptation. It is worth noting that the critical productivity level A_c depends negatively on the adaptation potential M_η and positively on the minimal vulnerability level $\underline{\eta}$. Put differently, when the adaptation opportunities are wide (large M_η and small $\underline{\eta}$), then the critical value for the global productivity above which the optimal adaptation is positive tends to zero. Let us consider, as an example, two countries with the same adaptation opportunities M_η and $\underline{\eta}$ but with two different global productivities. Then, the optimal policy may be no adaptation for the country with the low productivity and a positive adaptation for the other country (depending on their relative position with respect to the critical value A_c). A graphical illustration is provided in Figure 2. Above the critical value A_c, adaptation is always positive, but its relative contribution to the optimal policy mix (adaptation/abatement ratio) first increases when the global productivity of the country increases, and then decreases. The turning point is A_{cr}. When the global productivity is larger than A_{cr}, the optimal policy mix decreases with the further productivity increase. Does this mean that the country must increase abatement efforts when its global productivity is high? No, because B/K is independent of A by Corollary 3-(*i*). This means that adaptation efforts D/K are reduced after A_{cr}. The rationale behind this result is that abatement has a constant marginal efficiency ($1/\gamma$) while adaptation has a decreasing marginal efficiency ($\eta'(D) = -a(\overline{\eta} - \underline{\eta})e^{-aD}$).

Now we compare the optimal values W_D and W_{ND} of the objective function (1) in two economies *with* and *without* adaptation and check which economic component causes the difference. Let us assume that the initial conditions K_0, D_0, P_0 in (3)-(5) coincide with the optimal solutions at the initial moment, $K_0=\overline{K}$, $D_0=\overline{D}$ and $P_0=\overline{P}$. Then, the optimization problem has no transition dynamics. Substituting equations (47) and (48) into (1)-(6) and using (58), we obtain the exact values of the objective functions

$$W_D = \frac{1}{\rho}\left[\ln\frac{K_D\rho}{\alpha} - \frac{\alpha}{aK_D} - \underline{\eta}\frac{P_D^{\ \mu+1}}{\mu+1}\right] \tag{61}$$

$$W_{ND} = \frac{1}{\rho}\left[\ln\frac{K_{ND}\rho}{\alpha} - \overline{\eta}\frac{P_{ND}^{\ \mu+1}}{\mu+1}\right] \tag{62}$$

In these two equations, the first term represents the contribution of consumption to welfare while the last term reflects the impact of the pollution.

There are two differences between (61) and (62). First, the second (additional) term in (61) represents the opportunity cost of adaptation (crowding effect). The magnitude of this effect is represented by the ratio α/a that can be interpreted as follows. Spending one dollar in adaptation contributes positively to welfare with a marginal technical efficiency weight a, but it has also an opportunity cost due to a lower capital accumulation because of the budget constraint (2) on the economy. A lower capital level induces a loss in output, weighted by α. So, the smaller the technical efficiency of adaptation with respect to α, the higher the opportunity cost of adaptation on welfare. Second, the pollution impact on welfare is weighted in (61) by the minimal vulnerability level $\underline{\eta}$ with adaptation, while it is weighted by the maximal vulnerability level $\overline{\eta}$ when adaptation is not an option in (62).

We know that the pollution level is larger under adaptation, but this effect is offset by the difference between $\overline{\eta}$ and $\underline{\eta}$. When the country can reach a very high protection level (very small $\underline{\eta}$), then the only positive effect of adaptation is to increase the consumption level. When the adaptation is available and optimally used, then the resources that are not spent for the pollution abatement can be used for capital accumulation, thus leading to a higher consumption level in the long run.

5. A numerical illustration

One of the goals of this paper is to interpret the optimal solution for countries with different characteristics. So it is natural to check that our key assumptions are empirically relevant that will be done in this section.

Our model involves both economic and environmental variables, so a dimensional analysis of measurement units is a pre-requisite for a more meaningful model calibration. The dimensional analysis is rarely used in environmental economics while it is quite common in applied mathematicians, physics, and engineering for better understanding the relation among different quantities and checking the plausibility of derived equations (see, *e.g.*, Kasprzak *et al.*, 1990). In financial economics, the dimensional analysis is also common in interpreting various financial, economic, and accounting ratios. In this section we shall use the common notation $[x]$ for the unit of measurement (UOM) of a variable x. We shall first present the dimensional analysis and then discuss parameters value and assumptions.

5.1. Dimensional analysis of the model

It is convenient to choose a common UOM for all economic variables. In our case we will choose $[B] = [C] = [D] = [K] = [Y] = 10^9$ \$ (billion US dollars). Following the common practice, the UOM for pollution will be $[P] = 10^{12}$ tC (trillion tons of carbon)[17]. Let us now determine the measurement units for model parameters. It is obvious that α in (2) and μ in (6) are dimensionless (they have no UOM), and that $[\rho] = (\text{time})^{-1}$. Rewriting the pollution equation (5) as $(\gamma/\delta_P) = (P+P'/\delta_P)B/Y$, one can see that $[\delta_P] = (\text{time})^{-1}$ and $[\gamma/\delta_P] = [P] = 10^{12}$ tC. Hence, $[\gamma] = 10^{12}$ tC/time. The parameter η needs to be consistent with the objective function (6), which involves the logarithmic utility $ln(C)$. Following the general rule of the dimensional analysis of transcendental functions such as logarithm, exponent, etc., we consider $ln(C)$ as dimensionless (for the fixed UOM of the consumption C). It is natural from an economic viewpoint, because the utility U is based on customer preferences. So, we have that $[\eta] = [P]^{(-1-\mu)} = (10^{12}\text{ tC})^{(-1-\mu)}$. Finally, $[A] = [Y]/[K]^{\alpha} = (10^9\text{ \$})^{1-\alpha}$ by (2), and $[a] = (10^9\text{ \$})^{-1}$ by (44).

Now we are ready to check the empirical relevance of our model, in particular assumptions A1, A2 and B.

[17] Or Giga tons of carbon.

5.2. Estimation of model parameters and regulation ranges

To set up some plausible values for the model parameters ρ, α, A, δ_P, γ, μ, η, and a, we use the most recent world data and parameters gathered by W.D. Nordhaus.[18] World GDP in billion US dollars in 2005 was 61,100, and capital stock was 137,000. Using $\alpha = 0.66$, we get a global factor productivity of 24.9. Let us also assume that $\rho = 0.01$. The natural decay rate for carbon concentration in the atmosphere is known to be very small. Some authors even consider it to be zero, which we cannot accept in our model. We shall take $\delta_P = 0.00001$ as a basis (decay rate per year). The current carbon concentration is $[P] = 808.9$ (10^{12} tC) and the yearly rate of increase is $P'/P = 0.3$ percent. Then, the pollution equation (5) in the form $(\gamma/\delta_P) = P[1+P'/(P/\delta_P)]B/Y$ gives us the estimate $\gamma/\delta_P = P(1+0.003/0.00001)B/Y = 808.9 \cdot B/Y$ (10^9 tC/year). By assuming that $B/Y = 0.1$, we end up with $\gamma/\delta_P = 24{,}347$.

Let us now turn to the parameters of the utility function. First, the parameter μ describes the marginal disutility of pollution; we shall take the conservative value of $\mu = 0.5$. To define the value of the environmental vulnerability parameter η, the only meaningful constraint is that utility should be non-negative. If we assume that consumption takes 65 percent of GDP, then we have

$$\eta \le \ln C \frac{1+\mu}{P^{1+\mu}} = \ln (0.65 \cdot 61.100 \cdot 10^9) \cdot 1.5 \cdot (808.9)^{-1.5} = 0.002$$

By taking $\eta = 0.002$ and $\delta = 0$, the value of the κ-indicator of environmental pressure is 7.75, which is larger than 1 and supports our Assumption A1.

Finally, the adaptation efficiency function (45) in Section 4 includes two parameters: the adaptation range $\overline{\eta} - \underline{\eta}$ and the marginal efficiency a. For testing purposes, we shall assume that the maximal vulnerability (the one without adaptation) is the current one, so $\overline{\eta} = 0.002$, and the minimal vulnerability is twice as small: $\underline{\eta} = \overline{\eta}/2$. Thus, the adaptation range is $\overline{\eta} - \underline{\eta} = 0.001$. Let us further assume that $a = 0.00001$.[19] Then the maximal marginal adaptation efficiency is $\mathrm{M}_\eta = a(\overline{\eta} - \underline{\eta}) = 0.1 \cdot 10^{-6}$. Hence, Assumption A2 holds (namely, $\underline{\kappa} = 3.87 >> 1$), as well as Assumption B: $(Aa^{1-\alpha}/\rho)^{1/\alpha} = 371.02 >> 1$.

Under this set of the given model parameters, the optimal policy mix between adaptation and abatement D/B turns out to be 0.63. The optimal abatement effort, expressed as the ratio B/K, amounts to 0.10 in the absence of adaptation, and it drops to 0.04 with adaptation. This numerical calibration also provides an illustration for our Corollary 2. In particular, it shows that the increase in the pollution level due to the presence of adaptation is rather small ($P_D/P_{ND} = 1.17$) in comparison with the increase in the size of the economy ($P_D/P_{ND} = 7.68$).

[18] The Nordhaus' documentation is available on his web site: *http://nordhaus.econ.yale.edu*.

[19] Let us recall that $[a] = (10^9\,\$)^{-1}$.

This rough calibration shows that our model, although stylized, has a strong empirical relevance. It further shows that three assumptions A1, A2, and B are supported by the data. Sensitivity analyzes can easily be carried out for any economically relevant range of parameters value. For instance, when changing the abatement effort in the benchmark (*B/Y*) from 0.1 to 0.05, the three assumptions still hold and the optimal policy mix *D/B* goes down from 0.63 to 0.26.[20]

6. Conclusion

Combining economic-environmental growth models and comparative static analysis with perturbation techniques, we have derived analytical expressions for the optimal policy mix between emission abatement and environmental adaptation at macroeconomic level. It allows us to investigate how the economy size shapes its optimal climate policy. It is shown that the importance of adaptation depends on the stage of country development. Specifically, the optimal policy mix between abatement and adaption investments (the ratio *D/B*) depends on the country economic potential. One of the essential findings here is the inverted U-shape dependence of the optimal ratio *D/B* on the economy. If the economic efficiency is weak (lower than a certain critical level), then it is not optimal to invest into adaptation. So, in the case of a poor country, the optimal policy may be no adaptation at all. In the case of a developed country, this ratio remains rather low. The maximum adaptation efforts (in terms of *D/B*) should be done by medium-developed countries.

Our theoretical model can be extended in several directions. Adding an exponential population growth will not change the structure of obtained results. Then the endogenous variables I_K, I_D, C, K, D, B, P will be *per capita*, and the actual parameters will grow exponentially. However, adding an (exogenous or endogenous) technological change to the production function (3) can alter the results significantly and make them more optimistic. The authors are going to exploit this issue later, following classical works (Gradus and Smulders, 1993; Stokey, 1998). An interesting approach is outlined by Bovenberg and Smulders (1996) who explore the link between environmental quality and economic growth in an endogenous growth model with pollution-augmenting technological change and examine sustainable growth.

Our model settings hold for a world economy, or a closed economy. By 'closed' we mean not only the absence of external trade but also a closed interaction between the economy and the environment. In other words, the environment is not a public good in our setting, because all costs and benefits of environmental degradation accrue to the country. It is well recognized that climate change has the nature of a global public good and that its international dimension constitutes one of its cornerstones. Of particular interest for us would be to understand how the optimal policy of a given country depends, first, on its stage of development and, second, on its position in the international area. Other cornerstones for the climate change problem are its long term perspective, the existence of large uncertainties on climate change impacts, and huge distributional effects among

[20] Naturally, a sound calibration would be necessary if one wants to draw really accurate policy recommendations from our model, which was not the purpose of this short section.

activity sectors and countries. Extending our model to a n-country model with strategic behaviors would be highly desirable to address this issue, but it raises new mathematical challenges and is out of the scope of the current paper.

Appendix 1. Analysis of model (7)-(10)

Proof of Proposition 1.

Let us analyze the possibility of a steady state

$$K(t)=\overline{K}, \quad C(t)=\overline{C}, \quad B(t)=\overline{B}, \quad P(t)=\overline{P}. \tag{A1}$$

in the model (7)-(10). The substitution of (A1) into (18)-(21) leads to

$$A\overline{K}^{\alpha}=\delta\overline{K}+\overline{B}+\overline{C}, \qquad \alpha A\overline{K}^{\alpha-1}-\alpha\frac{\overline{B}}{\overline{K}}=\delta+\rho, \qquad \delta_P\overline{P}=\gamma A\frac{\overline{K}^{\alpha}}{\overline{B}}, \tag{A2}$$

$$\eta\overline{P}^{\mu}=\frac{\overline{B}^{2}(\rho+\delta_P)}{\gamma A\overline{K}^{\alpha}\overline{C}}. \tag{A3}$$

Using (A2) we express the steady state $\overline{B}$, $\overline{C}$, and $\overline{P}$ in the terms of $\overline{K}$ as (22)-(24). Substituting these formulas into (A3), we obtain the following equation for $\overline{K}$:

$$\eta\frac{\gamma^{\mu+1}A^{\mu+1}}{\delta_P{}^{\mu}}\overline{K}^{1-\alpha}[\rho+\delta(1-\alpha)]=\alpha(\rho+\delta_P)[A-\overline{K}^{1-\alpha}(\delta+\rho)/\alpha]^{2+\mu}. \tag{A4}$$

To show that (A4) has a unique solution, let us introduce the new unknown variable $x=\frac{\delta+\rho}{\alpha A}\overline{K}^{1-\alpha}$ and use notation (27) for the parameter κ, Then equation (A4) takes the following dimensionless form:

$$\frac{(1-x)^{2+\mu}}{x}=\kappa. \tag{A5}$$

It is easy to see that equation (A5) has a unique solution x^*, $0< x^* < 1$. Indeed, the left-hand side $F(x)=\frac{(1-x)^{2+\mu}}{x}$ of (A5) strictly decreases from ∞ at x=0 to $F(1)=0$ and intersects the horizontal line $G(x)=\kappa>0$ at some point x^* (see Figure 3). Then, equation (A4), or (25) in the theorem, has a unique solution $\overline{K}$, $0<\overline{K}<\left(\alpha A/(\delta+\rho)\right)^{\frac{1}{1-\alpha}}$.❑

Proof of formulas (28)-(29)

For the purposes of our future analysis, we need an approximate analytic solution of (A4). Let us consider two situations when it is possible.

Case 1: the parameter κ is large, κ>>1. Then, presenting (A5) as $\kappa x = (1-x)^{2+\mu}$, we see that 0<x<<1. Using the Taylor expansion of $(1-x)^{2+\mu}$, we obtain $\kappa x = 1-(\mu+2)x+o(x)$ or $x = \dfrac{1}{\kappa+\mu+2}+o(x) = \dfrac{1}{\kappa}+o(x,\kappa^{-1})$ or $x \cong \dfrac{1}{\kappa}$, which justifies (28). The condition κ>>1 can be replaced with $\eta\gamma^{\mu+1} >> (\rho+\delta_P)\delta_P{}^{\mu}$ because the parameter α is fixed and α<1 (in real economies, α≈0.8).

Case 2: the parameter κ is small, κ<<1. Then, rewriting (A5) as $\dfrac{z^{2+\mu}}{1-z} = \kappa$ with respect to the unknown z=1-x, we see that z<<1. Using the Taylor expansion of $(1-z)^{-1}$, we obtain that $z^{\mu+2}[1+O(z)] = \kappa$ or $z \cong \kappa^{1/(\mu+2)}$, which leads to (29). ❑

Appendix 2: Analysis of model (1)-(6) with adaptation

Proof of Proposition 2.

With no capital depreciation, δ=0, equalities (39)-(43) produce the following equations

$$A\overline{K}^{\alpha} = \overline{B} + \overline{C}, \qquad \alpha A\overline{K}^{\alpha-1} - \alpha\frac{\overline{B}}{\overline{K}} = \rho, \qquad \delta_P\overline{P} = \gamma\frac{A\overline{K}^{\alpha}}{\overline{B}}, \tag{A6}$$

$$\eta(\overline{D})\overline{P}^{\mu} = \frac{\overline{B}^{2}}{\gamma A\overline{K}^{\alpha}\overline{C}}(\delta_P+\rho), \tag{A7}$$

$$-\eta'(\overline{D})\frac{\overline{P}^{\mu+1}}{\mu+1} = \frac{\rho}{\overline{C}}. \tag{A8}$$

for the steady state $K(t)=\overline{K}$, $C(t)=\overline{C}$, $B(t)=\overline{B}$, $P(t)=\overline{P}$, $D(t)=\overline{D}$. The explicit formulas (46)-(48) are obtained from (A6). Combining (46)-(48) and (A7), we can write the following nonlinear equation

$$\frac{(A-\overline{K}^{1-\alpha}\rho/\alpha)^{2+\mu}}{\overline{K}^{1-\alpha}} = \frac{\rho\gamma^{1+\mu}A^{1+\mu}}{\alpha(\delta_P+\rho)\delta_P{}^{\mu}}\eta(\overline{D}), \tag{A9}$$

where $\overline{D}$ should be found from the nonlinear equation (A8). Combination of (44) and (A9) gives (49).Then, differentiating (44) and using (A8) and (48) we obtain (50)

Let us notice that $\overline{C}\to$0 by (47) and $\overline{P}\to\gamma/\delta_P$ by (48) as $\overline{K}\to$0. Since $-b \le \eta'(D) < 0$ by (44), the equation (A8) cannot have a solution $\overline{D}$>0 for small values of $\overline{K}$. It means that the extremum condition (43) for the interior optimal $\overline{D}$>0 is not satisfied and the optimal $\overline{D}$ is boundary, that is, $\overline{D}$=0, for small $\overline{K}$. Hence, there is no adaptation ($\overline{D}$=0) and $\eta(D) = \eta(0) = \overline{\eta}$ in (A9) for some small $\overline{K}>0$. In this case, equation (A9) has the unique positive solution $\overline{K}$, which satisfies the approximate formulas (28)-(29) at $\eta = \overline{\eta}$ (as the similar equation (25)). ❑

Proof of Proposition 3.

Let us analyze the possibility whether equation (A8) can have a solution $\overline{D}>0$. Let (A8) hold *a priori*. Then from equations (44), (46)-(48), and (A8) we get

$$be^{-a\overline{D}}\frac{\gamma^{\mu+1}}{\alpha\delta_P{}^{\mu+1}(\mu+1)} = \frac{(1-\overline{K}^{1-\alpha}\rho/(A\alpha))^{\mu+1}}{\overline{K}}. \tag{A10}$$

and, substituting (A10) into (45),

$$\eta(\overline{D}) \overset{def}{=} \hat{\eta}(\overline{K}) = \underline{\eta} + \frac{\alpha\delta_P{}^{\mu+1}(\mu+1)(1-\overline{K}^{1-\alpha}\rho/(A\alpha))^{1+\mu}}{a\gamma^{\mu+1}\overline{K}} \le \overline{\eta}\ . \tag{A11}$$

As in Appendix 1, we use dimensionless variables to simplify the further analysis. Substituting (A11) into (A9) and using the unknown $x = \dfrac{\rho}{\alpha A}\overline{K}^{1-\alpha}$ and the parameter

$$\underline{\kappa} = \frac{\underline{\eta}\gamma^{1+\mu}}{(\delta_P+\rho)\delta_P{}^{\mu}}, \tag{A12}$$

we obtain one dimensionless equation

$$\frac{(1-x)^{2+\mu}}{x} = \underline{\kappa} + \frac{\alpha\delta_P(\mu+1)}{a(\delta_P+\rho)(\alpha A/\rho)^{\frac{1}{1-\alpha}}}\frac{(1-x)^{1+\mu}}{x^{\frac{1}{1-\alpha}}} \tag{A13}$$

with respect to the optimal value x^*, $0<x^*<1$. The left-hand function $F(x) = \dfrac{(1-x)^{2+\mu}}{x}$ strictly decreases from ∞ at x=0 to $F(1)=0$ and is the same as in equation (A5). The right-hand function

$$G(x) = \underline{\kappa} + \frac{\alpha\delta_P(\mu+1)}{a(\delta_P+\rho)(\alpha A/\rho)^{\frac{1}{1-\alpha}}}\frac{(1-x)^{1+\mu}}{x^{\frac{1}{1-\alpha}}} \tag{A14}$$

strictly decreases from ∞ at x=0 to $\underline{\kappa}$ at x=1 (see Figure 4). Moreover, $G(x)\le\overline{\kappa}$ by inequality (A11), where $\overline{\kappa} = \dfrac{\overline{\eta}\gamma^{1+\mu}}{(\delta_P+\rho)\delta_P{}^{\mu}}$. Therefore, we are interested only in solutions x^* from the interval $[x_{cr},1]$, where $x_{cr}>0$ is such that $G(x_{cr})=F(x_{cr})=\overline{\kappa}$. So, the value x_{cr} is the solution of the equation

$$\frac{(1-x)^{1+\mu}}{x^{\frac{1}{1-\alpha}}} = \bar{\kappa}\,. \tag{A15}$$

It is easy to see that the functions $F(x)$ and $G(x)$ intersect at $x^*\geq x_{cr}$ and the equation (A13) has a unique solution $x_{cr}\leq x^*<1$, *if and only if* $G(x_{cr}) \leq \bar{\kappa}$ (see Figure 4) or

$$a(\bar{\kappa}-\underline{\kappa}) \geq \frac{\alpha\delta_P(\mu+1)}{(\delta_P+\rho)(\alpha A/\rho)^{\frac{1}{1-\alpha}}}\frac{(1-x_{cr})^{1+\mu}}{x_{cr}^{\frac{1}{1-\alpha}}} \tag{A16}$$

To obtain *a priori* condition for the solvability of equation (A13) in the terms of given model parameters, let us consider the special case $\bar{\kappa}$>>1 (Case 1 of Section 3). Then, the equation (A15) has the approximate solution $x_{cr} \approx \bar{\kappa}^{-1}$. Its substitution into (A16) leads to

$a(\bar{\kappa}-\underline{\kappa}) \geq \dfrac{\alpha\delta_P(\mu+1)\bar{\kappa}^{\frac{1}{1-\alpha}}}{(\delta_P+\rho)(\alpha A/\rho)^{\frac{1}{1-\alpha}}}$, and, after routine transformations, to

$$\frac{A}{\rho}a^{1-\alpha}\left(1-\frac{\underline{\kappa}}{\bar{\kappa}}\right)^{1-\alpha} \geq \bar{\kappa}^{\alpha}\frac{(\mu+1)^{1-\alpha}}{(1+\rho/\delta_P)^{1-\alpha}\alpha^{\alpha}}\,.$$

which gives the formula (52) in the terms of the original parameters $\bar{\eta}$, A, γ and δ_P. This concludes the proof. ❑

Proof of formulas (58)-(59)

To find an approximate explicit formula for x and $\bar{K}$, we assume that

$$\frac{\alpha\delta_P(\mu+1)}{a(\delta_P+\rho)(\alpha A/\rho)^{\frac{1}{1-\alpha}}}\frac{(1-x^*)^{1+\mu}}{x^{*\frac{1}{1-\alpha}}} << \underline{\kappa}\,. \tag{A17}$$

that is, $G(x^*) = \underline{\kappa} + o(\underline{\kappa})$ (see Figure 4). Then, equation (A13) becomes

$$\frac{(1-x)^{2+\mu}}{x} + o(x^{-1}) = \underline{\kappa} \tag{A18}$$

which is equation (A5). To obtain its approximate solution, let us assume additionally that $\underline{\kappa} >> 1$. Then, as shown in Section 3, equation (A18) has the unique positive solution $x^* \cong \underline{\kappa}^{-1}$, that leads to formula (58). Finally, substituting $x^* \cong \underline{\kappa}^{-1}$ and (A12) into the inequality (A17) and combining the obtained result with $\underline{\kappa} >> 1$, we get the condition (56). The approximate formula (59) for $\bar{D}$ follows from substituting (58) into (53) and the condition $x^*<<1$. ❑

Proof of Corollary 3.

Using (46) and (58), we represent the ratio $\bar{B}/\bar{K}$ as

$$\frac{\overline{B}}{\overline{K}} = A\overline{K}^{\alpha-1} - \rho/\alpha \cong A\left[\frac{\alpha A(\rho+\delta_P)\delta_P{}^{\mu}}{\underline{\eta}\rho\gamma^{\mu+1}}\right]^{\frac{\alpha-1}{1-\alpha}} - \rho/\alpha = \frac{\underline{\eta}\rho\gamma^{\mu+1}}{\alpha(\rho+\delta_P)\delta_P{}^{\mu}} - \rho/\alpha\,,$$

that justify the first part of Corollary 3.

In order to prove part (ii) of Corollary 3, we analyze the ratio $\overline{D}/\overline{B}$ obtained from (46), (58), and (59). First of all, (59) is valid if $\left(\frac{M_\eta}{\mu+1}\right)^{1-\alpha}\frac{A(\rho+\delta_P)\alpha^{\alpha}}{\underline{\eta}\rho\gamma^{\alpha(\mu+1)}\delta_P{}^{1-\alpha(\mu+1)}} > 1$, that is,

$$A > \frac{\underline{\eta}\rho}{(1+\rho/\delta_P)\alpha^{\alpha}}\left(\frac{\mu+1}{M_\eta}\right)^{1-\alpha}\left(\frac{\gamma}{\delta_P}\right)^{\alpha(\mu+1)} = A_c\,. \tag{A19}$$

The last relation is the first formula (60). If (A19) is not satisfied, then $\overline{D}$ =0. Otherwise,

$$\frac{\overline{D}}{\overline{B}} \cong \frac{(\underline{\eta}\rho\gamma^{\mu+1})^{\frac{\alpha}{1-\alpha}}}{a(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}((\rho+\delta_P)\delta_P{}^{\mu})^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}}\ln\left[\left(\frac{M_\eta}{\mu+1}\right)^{1-\alpha}\frac{(1+\rho/\delta_P)\delta_P{}^{\alpha(\mu+1)}\alpha^{\alpha}}{\underline{\eta}\rho\gamma^{\alpha(\mu+1)}}A\right]$$

To investigate the monotonicity of the ratio $\overline{D}/\overline{B}$, let us look at its first derivative in A:

$$\left(\frac{\overline{D}}{\overline{B}}\right)'_A \cong \frac{(\underline{\eta}\rho\gamma^{\mu+1})^{\frac{\alpha}{1-\alpha}}}{a(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}((\rho+\delta_P)\delta_P{}^{\mu})^{\frac{\alpha}{1-\alpha}}}\frac{A^{\frac{\alpha}{1-\alpha}}(1-\frac{1}{1-\alpha}\ln\left[\left(\frac{M_\eta}{\mu+1}\right)^{1-\alpha}\left(\frac{\delta_P}{\gamma}\right)^{\alpha(\mu+1)}\frac{(1+\rho/\delta_P)\alpha^{\alpha}}{\underline{\eta}\rho}A\right]}{A^{\frac{2}{1-\alpha}}}$$

.

The ratio $\overline{D}/\overline{B}$ increases if $\left(\overline{D}/\overline{B}\right)'_A > 0$ or, in terms of (A19), if

$$0 < A < e^{1-\alpha}A_c = A_{cr}\,, \tag{A20}$$

and decreases if $A > A_{cr}$, which proves statement (ii) of Corollary 3.

(A20) gives us the second formula of (60) and concludes the proof of Corollary 3. ❑

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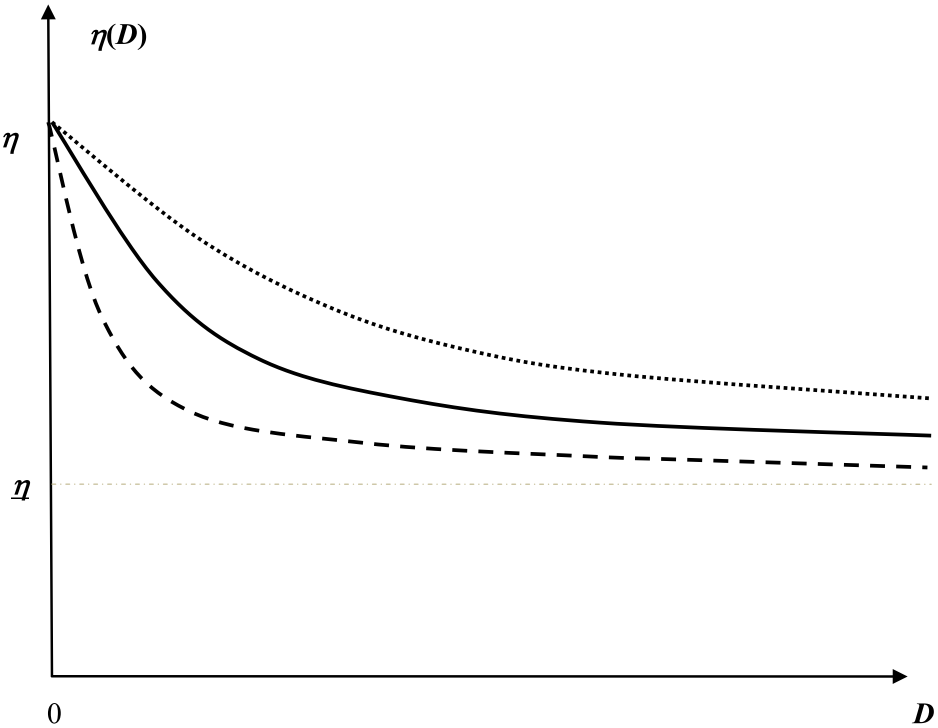

Figure 1. The dependence of the environmental vulnerability η on the adaptation expense D. It tends to the horizontal asymptote $\eta = \underline{\eta} > 0$ when D grows indefinitely. The dashed curve has a larger adaptation efficiency parameter a than the solid curve. The dotted curve has a smaller parameter a than the solid one.

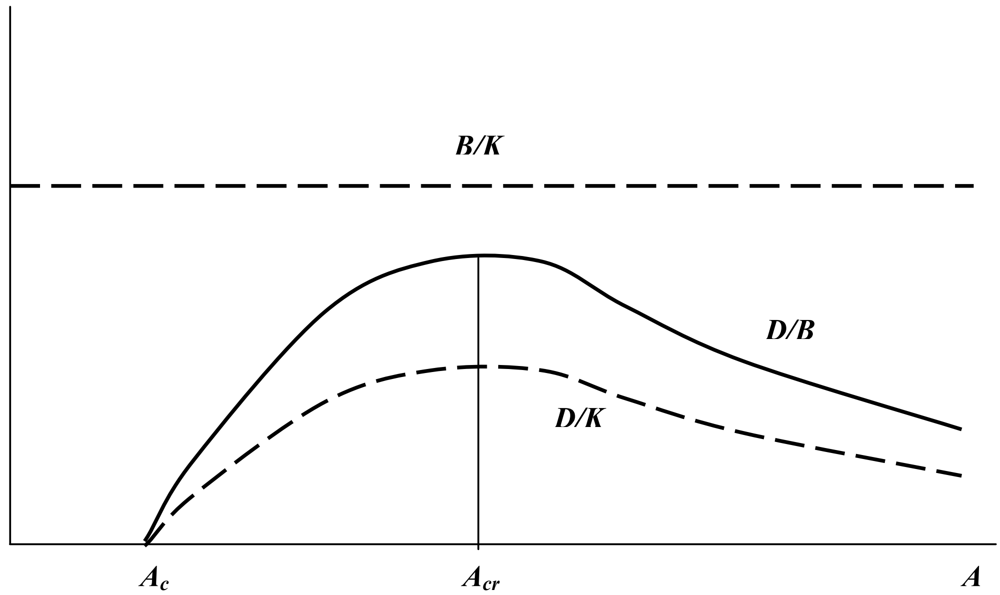

Figure 2. The dynamics of the optimal ratios *B/K*, *D/K*, and *D/B* in *A*.

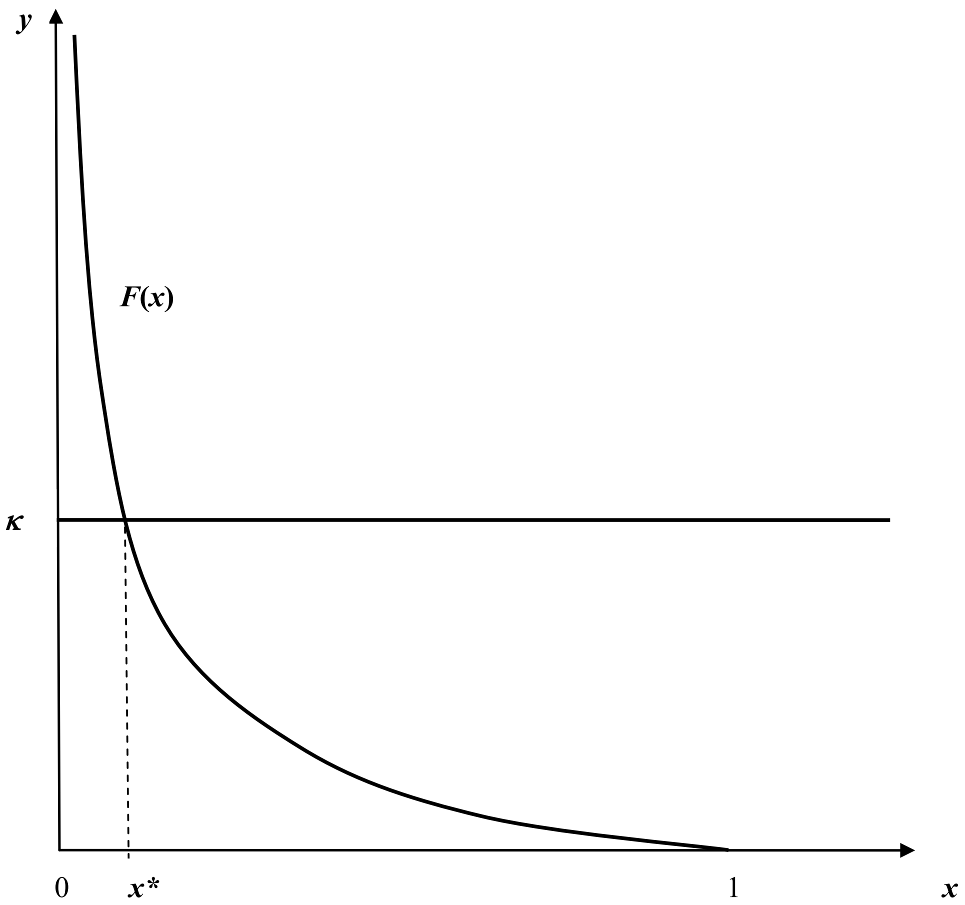

Figure 3. The point x^* is the unique solution of the nonlinear equation (A5). The strictly decreasing function y=F(x) represents the left-hand side of (A5) and the horizontal line $y = \kappa > 0$ is its right-hand side.

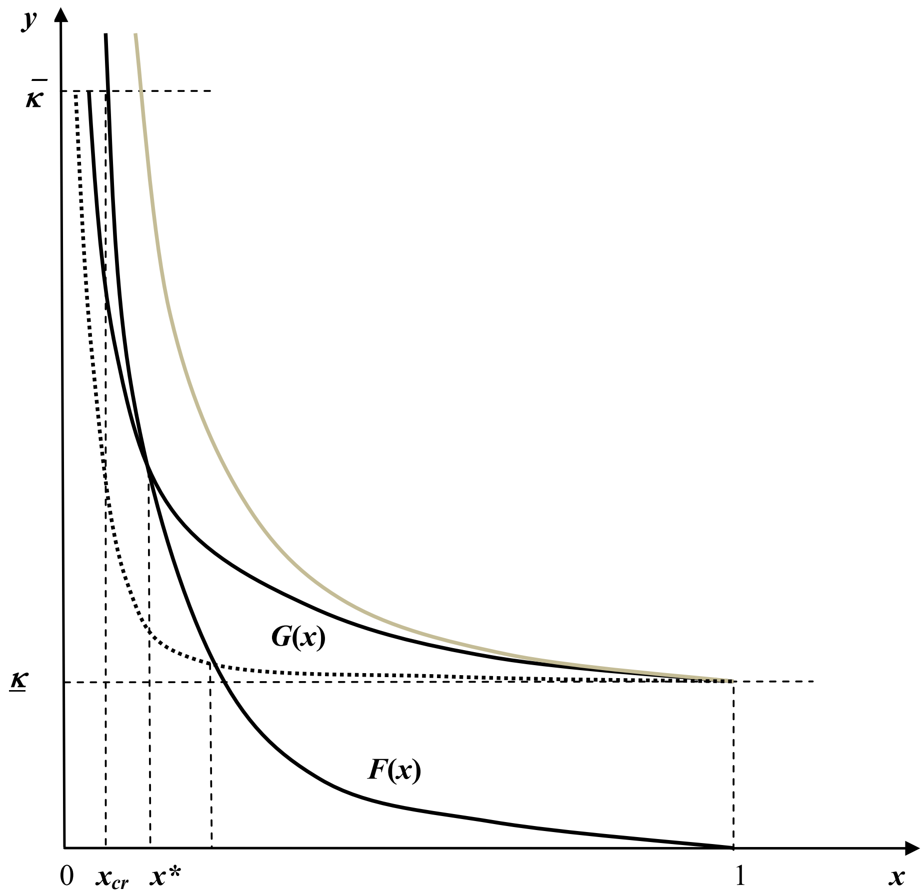

Figure 4. The decreasing function $y=F(x)$ represents the left-hand side of the nonlinear equation (A13) and the decreasing function $y=G(x)$ represents its right-hand side (A14). Their intersection point x^* is the unique solution of the equation under condition (A16). The dotted curve show the case when $G(x)$ is close to $\underline{\kappa}$ near x^* (then x^* is given by the approximate formula (58)). The gray function $y=G(x)$ demonstrates the situation when condition (A16) fails.

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