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Patent Office in innovation policy:

Nobody's perfect

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Patent Office in Innovation Policy: Nobody's Perfect[*]

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Abstract

The number of patent applications and "bad" patents issued has been rising rapidly in recent years. Based on this trend, we study the overload problem within the Patent Office and its consequences on the firms' R&D incentives. We assume that the examination process of patent applications is imperfect, and that its quality is poorer under congestion. Depending on policy instruments such as submission fees and the toughness of the non-obviousness requirement, the system may result in a high-R&D equilibrium, in which firms self-select in their patent applications, or in an equilibrium with low R&D, opportunistic patent applications and the issuance of bad patents. Multiple equilibria often coexist, which deeply undermines the effectiveness of policy instruments. We investigate the robustness of our conclusions as to how the value of patent protection is formalized, taking into consideration the introduction of a penalty system for rejected patent applications, as well as the role of commitment to a given IP protection policy.

Keywords: Patent Office, patent quality, congestion, innovation.
JEL Classification Codes: D82, L50, O31, O34, L50.

1 Introduction

In January 2009, the U.S. Chamber of Commerce released a statement opening with the following assessment: "Today, the U.S.Patent and Trademark Office is an agency in crisis facing significant challenges".[1] Despite an aggressive hiring campaign and a growing annual budget in the last 10 years, the U.S. Patent and Trademark Office (USPTO) has not kept up with the increasing number of new applications.[2] The number of litigated patents is rising faster than patent grants, and half of the patents contested in court are invalidated.[3] As a result, there is now a widespread perception that patent quality has been undermined, as some critics accuse the USPTO of issuing low-quality patents that violate the patentability criteria of novelty and non-obviousness, such as Amazon.com's checkout cart for online shopping.[4]

Chief among the underlying causes behind a lower patent quality is the creation of the United States Court of Appeals for the Federal Circuit (CAFC) by Congress in 1982. Through a series of decisions, the CAFC has expanded the scope of patentable subject matter to include business methods, software, and some biotechnology products and processes.[5] This in turn encouraged patent applications: between 1990 and 2005, the yearly number of applications rose from 175,000 to 380,000.[6] With patent application filing rates projected to increase by 8% annually, this trend is unlikely to change in the near future.[7] As a result, the Patent Office is now experiencing workload pressures: the number of applications per examiner ratio increased about 25% in the last five years and the time the examiner spends on each application is shorter (Lemley and Shapiro, 2005).[8] In this paper, we study the overload problem of the Patent Office and its impact on firms' R&D incentives.

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[1]Source: U.S. Chamber of Commerce's recommendations for U.S. Patent and Trademark Office, *http : //www.theglobalipcenter.com/images/gipc_images/pdfs/usptopatentproject.pdf*

[2]The backlog of pending patents is now approaching a record 800,000, and average approval time has stretched to 31 months. Source: USPTO Performance and Accountability Report Fiscal Year 2008, *http : //www.uspto.gov/web/offices/com/annual/*2008/

[3]Since 1991, the compound annual growth rates of patents grants and patent infringement cases are respectively 3.8% and 5.8%. Source: 2008 Patent Litigation Study, PricewaterhouseCoopers Int. Ltd. According to Allison and Lemley (1998), only 54% of the patents contested in court are ruled valid.

[4]See for example the Wall Street Journal editorial "Patently Absurd" (A14, March 1, 2006).

[5]See Gallini (2002) for a review of those changes.

[6]Source: U.S. Patent Office annual reports.

[7]See *http : //www.uspto.gov/web/offices/com/strat*2007/*stratplan*2007 − 2012.*pdf*, Source: USPTO 2007-2012 Strategic Plan

[8]This is also an issue at the European Patent Office, where the number of hours spent examining each patent claim has more than halved since 1992, from 23.8 hours in 1992 to 11.8 hours in 2001. Source: Alison Abbott, "Pressured staff 'lose faith' in patent quality ", Nature 429 (3 June 2004).

For an invention to be patentable in the U.S., four requirements must be met: statutory, usefulness, novelty and non-obviousness.[9] In order to assess novelty and non-obviousness, an examiner of the Patent Office conducts a search of prior art, reviewing patents and non-patent literature (i.e. what has been used or described before). If he finds all the features of the invention in a single prior art reference, he rejects the application for not meeting the novelty requirement. If not, he goes on to verify that the invention is not obvious in view of the prior art, i.e. that it is "sufficiently different" from the prior art (as worded by the USPTO), above a minimum standard of inventiveness.[10] Therefore, if invalidating prior art exists but has not been found by the examiner, the patent is granted by mistake. Missing invalidating prior art turns out to be the main source of mistakes for examiners: according to Allison and Lemley (1998) and Cockburn and al. (2002), most patent invalidations are issued on the basis of obviousness or lack of novelty.[11] This is more likely to happen as examiners lack the time to review thoroughly all the relevant prior art for each application, especially in recent patentable areas such as software and business methods where most prior art is unpatented (and thus harder to find).

We introduce a model in which firms invest in risky R&D activity resulting in inventions of different qualities, which are their private information. Firms can seek protection for their projects by applying for a patent. The Patent Office then processes their claims, but examiners may not find out all the relevant prior art for a given patent application as they assess its degree of novelty or non-obviousness. The imperfect observability of these characteristics can lead to mistakenly granted patents. Such mistakes make it easier to obtain a patent, so more people file a patent application, therefore placing a heavier burden on the examiners, which further deteriorates the quality of examination. We formalize this overload or congestion phenomenon and analyze these strategic complementarities between the application strategies and the examination process.

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[9]In the U.S. code on Patents (35 U.S.C.), Usefulness is described in section 101, novelty in section 102 and non-obviousness in section 103. In Europe, the equivalent requirements are statutory subject matter, industrial application, novelty and inventive step.

[10]Hunt (1999) defines a non-obvious invention as "a nontrivial extension of what is already known". In his paper, discoveries are improvements to the quality of products, and the standard of non-obviousness sets the minimum extent of improvement which can result in a patent. Anything below the standard goes into the public domain. For an extensive survey of the economic analysis of the non-obviousness requirement, see Denicolo (2008).

[11]Cockburn and al. (2002) study 182 cases, half of which ended up in an invalidation by the CAFC, 37% of which on the basis of novelty (Section 102) and 47% for obviousness (Section 103). Allison and Lemley (1998) find that "sections 102 and 103 account for 138 out of 191 total determinations of invalidity".

We characterize equilibrium R&D and patent application strategies depending on the values of two policy instruments: submission fees, and the strength of the non-obviousness requirement. Within some range of values, the IP protection system is very effective, as it leads to a unique high-R&D equilibrium in which firms self-select in their decision to apply, and the imperfection of the examination process is therefore inconsequential. Outside of that range, however, the IP protection system is less effective: we show there exists an equilibrium in which some patents, that we call "bad patents", are issued, although they would not have been issued under a perfect examination process. In some case, the equilibrium may be unique and characterized by low R&D. In some other cases, there may be multiple equilibria in the number and the quality of patent applications, some unattractive equilibria being characterized by low R&D and yet many patent applications and the issuance of bad patents, and some other, more effective equilibria characterized by high R&D and only good patents issued. The unattractive equilibrium cannot be dismissed easily as it is preferred ex ante by firms. Moreover, the design of policy instruments to stimulate R&D would inevitably lead to a range of parameters where multiplicity of equilibria is prevalent. These results illustrate a weakening of standard control instruments when Patent Offices face serious congestion problems. Marginal changes in these instruments cannot solve the equilibrium selection problem faced by the Office and they have a limited impact on firms' strategies. In equilibrium, the Office must tolerate unwarranted patent applications and bad patents as a necessary evil.

We extend our basic setting in several directions and prove that our conclusions are by-and-large robust, in particular with respect to how the value of patent protection is formalized. We also consider the possibility of enlarging the set of instruments and introduce a penalty for rejected patent applications: such a penalty unambiguously improves the situation, although the characterization of equilibria remains similar in nature. Finally, we prove that the Patent Office's commitment power on the IP protection policy, in particular on the toughness of the non-obviousness criterion, is critical and we show that in the absence of such commitment, R&D incentives vanish entirely.

The theoretical economic literature on patents has paid little attention to the patent Office, and its examination process has not yet been formalized. Two exceptions are Langinier and Marcoul (2003) and Atal and Bar (2008), who address the issue of prior art search and disclosure

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by innovators. Langinier and Marcoul study agency problems that can arise between an examiner and an inventor: moral hazard on the inventor's search of prior art and adverse selection on whatever information he chooses to disclose in the application. Atal and Bar (2008) focus on innovators' incentives to search for prior art, before undertaking R&D and before applying for a patent. By contrast, we do not consider incentives to search for prior art (for the examiner); we elaborate on an idea outlined in Caillaud (2003) and Hall (2009), focusing on the impact of the Office on firms' incentives to innovate and to apply for patent protection. Our paper is close to Chiou (2008), who investigates the substitutability between the Patent Office examination effort and the effort made by other actors in the industry, to assess the quality and novelty of a patent application.

Many contributions in the literature analyze the impact of submission fees or of patentability standards on the innovation process.[12] A few papers focus on the optimal design of these instruments by a Patent Office. Lemley (2001) argues that it is efficient to implement a low examination standard, because the costs of improving the examination process and the quality of issued patents would outweigh the cost of the mistakes currently made (the Patent Office is "rationally ignorant"). Cornelli and Schankerman (1999) and Scotchmer (1999) study the design of renewal application fees and their impact on R&D. In those articles, patent life is endogenously chosen by firms via the renewal application fee. Scotchmer (1999) considers a model of asymmetric information where the Patent Office cannot observe the costs and benefits of firms' innovations. She shows that the only feasible incentive mechanisms are equivalent to patent renewal systems (where firms must pay back money to the Patent Office to extend their patents). Cornelli and Schankerman (1999) show that it can be welfare-improving to differentiate patent lives when firms have different R&D productivity, which is unobservable by the Patent Office. By contrast, we argue that these instruments may be of limited use to improve the performances of the IP protection system.

Our paper is organized as follows. Section 2 outlines the model. Section 3 characterizes the different equilibria. In Section 4 we discuss the issue of multiplicity and we derive the comparative statics of the different equilibria, including the impact of submission fees and the

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[12]See e.g. O'Donoghue (1998), Hunt (1999 and 2004), and for a general view, Scotchmer (2004), on the impact of patentability requirements on the R&D process.

non-obviousness standard; finally, we introduce new possible instruments, such as a penalty in case of patent denial. In Section 5 we study and discuss various extensions of the model, while Section 6 concludes.

2   Model

We consider an industry consisting of a continuum of firms of mass 1.[13] Firms invest in risky R&D activities that generate more or less innovative new technologies or products that we call "projects". They can seek protection for their projects by applying for a patent to the Patent Office. The Patent Office processes claims and grants or denies protection, based on the results of an examination process. R&D activities are formalized in a rather standard way and we focus on the nature and imperfections of the examination process to investigate its impact on the pace of technical progress within the industry.

Each firm chooses an R&D effort level π in $[0,1)$ at cost $\gamma(\pi)$. The R&D outcome is a project characterized by its innovativeness θ, $\theta \in [0,1)$. $\theta = 0$ stands for a *non-novel* project that is already part of the existing prior art; a larger θ in $(0,1)$ refers to a project that is more innovative and farther from the frontier of existing prior art. R&D effort π determines the stochastic distribution of the project innovativeness, and a higher effort generates stochastically higher innovativeness (according to first-order stochastic dominance). We assume a specific distribution for θ to capture these elements: with probability $1 - \pi$, $\theta = 0$ and with probability π, θ is drawn from the uniform distribution on $[0, \pi]$. So, the c.d.f. equals $1 - \pi$ at $\theta = 0$ and $1 - \pi + \theta$ for $\theta \in (0, \pi]$.

A firm can either seek protection of a patent for its project, or forgo institutional protection. We assume the project has value v under patent protection and 0 otherwise. v should be interpreted as the difference between the discounted expected profits from the project when it is protected by a patent and the discounted expected profits of the best alternative strategy for the firm. Depending on the industry, this strategy can take several forms: e.g. letting the innovation

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[13]It is not clear whether the market for patent applications is concentrated or not: two dozen firms account for one fifth of all patents granted by the USPTO (Source: USPTO website and IFI Patent Intelligence 2008 report, see http://www.ificlaims.com/IFIPatents010909.htm). However, what matters is whether firms take into account the impact of their applications on the Office's congestion or, as we assume here, if they are "congestion-takers". It seems that even a large innovative company does not decide upon its IP protection strategy on the basis of how much congestion it will induce at the Patent Office.

fall into the public domain and secure short-term monopoly profits before imitation, or keeping the innovation secret and getting the associated monopoly rent as long as there has not been informational leakage or imitation. v may therefore be large for product innovations that cannot be kept secret and can be easily duplicated, while it may be small when secrecy is an effective mode of protection. The normalization of non-protected profits to 0 has the consequence that firms would engage in 0-R&D in the absence of any patent protection; given the previous interpretation, $\pi = 0$ should not be interpreted literally as no R&D but rather as the natural level of R&D that prevails in the industry in the absence of an institutional IP protection system.

That the value of a patent does not depend on the project innovativeness is admittedly a restrictive assumption. We relax it in an extension and show that our analysis is qualitatively robust. Another restriction is that the value of a patent does not depend on the number of patents granted. Hence, we rule out market interactions among potential innovations and market structure issues, assuming basically that projects can be marketed independently on isolated markets. We relax this assumption as well and show how this introduces a force counter-acting the congestion phenomena.

A firm applying for a patent has to pay an application fee $f \geq 0$. We focus on two standards of patentability: novelty and non-obviousness, and we formalize the patent examination process as follows. Investigating novelty and non-obviousness mostly relies on the examiners searching for existing prior art (existing patents, non-patented literature such as publications and non-protected inventions that already cover the claims contained in the application).[14] The search of prior art yields the examiner an imperfect signal σ, whose stochastic distribution depends on the true innovativeness as well as on the efficiency of the examination process. It can be viewed as an assessment of the frontier of knowledge in the field and we assume that the major source of imperfection rests on the risk of not identifying a recent or more advanced step in the technology, i.e. of not finding all relevant prior art, so that the innovativeness of a project can only be *over-estimated* by examiners.

To formalize these elements, we assume that σ is distributed on $\{\theta, 1\}$ with $\Pr\{\sigma = \theta\} =$

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[14]Note that in a dynamic setting, an increase in the number of patents makes the future search of examiners easier since the prior art will mostly be patented. For example, searching prior art and determining novelty on software patent applications becomes easier now that more software is patented than in the early 1980's when most of the prior art was not patented.

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$1 - h = 1 - \Pr\{\sigma = 1\}$. The efficiency of the examination process is affected by congestion, which is captured by supposing $h = h(n)$, where $h(.)$ is increasing within $[0, 1]$: the larger the number of applications, the more difficult for an examiner to study the field of each application, the less likely he is to find all the relevant prior art.

>From this formalization, it follows immediately that $\sigma = 0$ means that an identical invention already exists in the prior art, so the application does not pass the *novelty test*. If $\sigma > 0$, the application passes the novelty test. Then two situations can arise, depending on a non-obviousness requirement $\sigma^* > 0$ that characterizes the mandate of the Patent Office: the application passes the *non-obviousness test* (and therefore is patented) if and only if $\sigma \geq \sigma^*$. It follows that the probability of obtaining a patent for a project θ is equal to 1 if $\theta \geq \sigma^*$ and to h if $\theta < \sigma^*$.

We take the non-obviousness standard σ^* and the submission fee f as given: the Patent Office is an agent for the Congress, or a regulation authority or more generally for a social planner, and these policy instruments are fixed and well established, perhaps as a result of an optimal design, of congressional decisions, of legal judgements or of the reputation of the Office. Firms simultaneously engage in R&D programs, develop projects and choose to apply for a patent or not. Then, the Patent Office evaluates all applications and grants or denies patent protection, depending on the outcome of the examination process. We therefore adopt a positive view of the IP protection system and characterize the equilibrium in terms of R&D and patent applications in a given institutional framework.

To address the normative question of the optimal design of the IP protection system, in particular of these instruments (f, σ^*), we should consider explicit social objectives that incorporate the social welfare value of innovations, the cost of R&D and the deadweight losses associated with the monopoly positions created by patents, and we should then determine the system that maximizes these objectives ex ante. The analysis would be immediate if the examination technology is perfect (i.e. if $h(n) = 0$ for all n). But with an imperfect examination technology, we will argue that equilibrium multiplicity becomes a major issue for most values of (f, σ^*), thereby preventing a meaningful analysis of the issue of the optimal design of the IP protection system. Therefore, we content ourselves with a positive analysis; we will simply refer to one normative aspect: reducing the number of patents in equilibrium, while maintaining R&D constant, is

socially beneficial as it reduces the overall deadweight loss.[15]

The subjectivity and the lack of measurability of the non-obviousness standard might make it difficult for the Patent Office to commit to a rigorous examination (see Kahin (2001)).[16] Moreover, in a dynamic perspective, the policy of IP protection may evolve over time. So, as an extension, we investigate another polar case in which σ^* is determined ex post, once R&D has already been done and firms' applications have been submitted.

A related remark is that the Patent Office is not a sophisticated player; in fact, it is not even strategic in most of the analysis except in the extension in which σ^* is determined ex post. In real life, Patent Offices receive new applications and take new decisions every year. They can rely on other various instruments that could be changed over time, such as the number of examiners employed, or the backlog of unexamined applications (which may signal a high quality examination, where examiners spend more time on each application). Our simple model, assuming either full commitment to an exogenous examination process or assuming a simple second-mover situation in which σ^* is determined ex post, is a first preliminary step in analyzing the role of the Patent Office, before developing a fully dynamic analysis in which the issue of defining the Patent Office's objective function should be addressed.

Finally, we make the following assumption:

Assumption 1. $\gamma(.)$ *is increasing, convex from* $[0,1)$ *to* $[0,+\infty)$*,* $\gamma(0) = \gamma'(0) = 0$ *and* $\lim_{\pi \to 1^-} \left(\frac{\gamma'(\pi)}{\gamma(\pi)} \right) = +\infty$*. We let* $\phi(.)$ *denote the inverse marginal R&D cost function:* $\phi(.) = \gamma'^{-1}(.)$*.*

This leads to the following technical lemma, whose proof is immediate, hence omitted.

Lemma 1. *Let* $A_\sigma(.)$ *be defined for* $x \in [0,+\infty)$ *by:*

$$A_\sigma(x) = x(\phi(x) - \sigma) - \gamma(\phi(x));$$

$A_\sigma(.)$ *is convex,* $A_\sigma(0) = 0$*,* $\lim_{x \to \infty} A_\sigma(x) = +\infty$*; it has a unique minimum attained at* $\gamma'(\sigma)$ *and, for* $\sigma > 0$*, the equation* $A_\sigma(x) = 0$ *has a unique positive root* $a(\sigma)$ *and* $a(\sigma)$ *increases in* σ*.*

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[15]Note that in a dynamic setting, it could be argued that patents have also a beneficial impact per se as they speed up the diffusion of knowledge in the economy.

[16]Moreover, to analyze a repeated game we would have to introduce a litigation stage and the possibility of patent invalidation after issuance, or an opposition procedure like in the European Patent Office. This goes beyond the scope of the paper.

It turns out that in the following analysis, $a(\sigma^*)$ is the appropriate way to measure how tough the non-obviousness standard σ^* is.

3   Equilibrium analysis

In this section, we provide the complete equilibrium analysis in our setting.[17]   In each case, we posit an equilibrium R&D effort π^* and the set of values of θ for which firms apply for patent protection in equilibrium. We find necessary conditions for this application strategy to be a best response for firms. We then deduce the optimal application decision of a firm that deviates by investing $\pi \neq \pi^*$ in R&D and compute the expected profits from such a deviation. These expected profits have potentially two local extrema, one at 0 and one within $(\sigma^*, 1)$; the detailed analysis, relegated in the appendix, consists in comparing these two extrema. This gives us necessary conditions for the posited strategies to actually sustain an equilibrium, these conditions being sufficient in all cases.

Given the nature of the examination process, it is immediate that a project θ, with $\theta \geq \sigma^*$, applies for a patent if and only if $v \geq f$, while a project θ, with $\theta < \sigma^*$, applies if and only if $h(n)v \geq f$, where n is the expected number of applications. The following lemma is then immediate.

Lemma 2. *In any equilibrium, if a firm of type $\theta < \sigma^*$ applies with positive probability, then all firms with $\theta > \sigma^*$ apply with probability 1; and if a firm of type $\theta > \sigma^*$ does not apply, no firm with $\theta < \sigma^*$ applies with positive probability.*

This lemma rules out equilibria where the Patent Office only receives applications from the most obvious projects.[18]   It shows that, besides π^*, equilibria can be characterized by m_+^* and m_-^*, the masses of applications for patent coming from projects with $\theta \in (\sigma^*, \pi^*]$ and with $\theta \in [0, \sigma^*]$ respectively.[19]   The imperfect examination technology will lead to patents being granted to projects that would not be patented under perfect examination, i.e. to projects such

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[17]More precisely, we provide the analysis for equilibria under pure strategies with respect to the choice of R&D effort. In the basic setting this is without loss of generality for almost all values of the parameters of the model (see footnote 21 below).

[18]We do not consider dynamic issues where the backlog of pending applications could discourage the best innovators to apply. Instead, we focus on situations where the patent system still serves its initial purpose of attracting the best innovators.

[19]When $\pi^* < \sigma^*$, $m_+^* = 0$ and only m_-^* matters.

that $\theta < \sigma^*$; for this reason, we will call these "bad patents", although ex post all patents are costly in the economy because of the monopoly deadweight loss they create.

For ease of presentation, we distinguish cases according to the level of equilibrium R&D: high-R&D equilibria, and low-R&D equilibria or no-R&D equilibria, depending on whether the equilibrium R&D effort is above σ^* or within $[0, \sigma^*]$.

3.1 High-R&D equilibria

We explore first the existence of a high-R&D equilibrium, i.e. with positive R&D such that $\sigma^* < \pi^*$. There must exist some firms applying for patent protection otherwise R&D would just be a pure waste. So, necessarily $v \geq f$ and at least highly-innovative projects with $\theta > \sigma^*$ apply for protection, hence $m_+^* > 0$. Lemma 2 implies that: $m_-^* > 0 \Rightarrow m_+^* = \pi^* - \sigma^*$ and $m_+^* < \pi^* - \sigma^* \Rightarrow m_-^* = 0$.

Suppose that $m_-^* = 0$ and $0 < m_+^* < \pi^* - \sigma^*$. Since firms $\theta > \sigma^*$ face the same problem ex post and do not all apply, they must be indifferent so that necessarily: $v = f$. But then, the expected profit function is simply equal to the cost $-\gamma(\pi)$, hence decreasing. Such an equilibrium with $\pi^* > \sigma^*$ cannot exist. Therefore, $m_+^* = \pi^* - \sigma^*$, i.e. in any high-R&D equilibrium, all projects with $\theta \geq \sigma^*$ apply for patent protection. Let us call "separating", an equilibrium with $m_-^* = 0$, i.e. in which projects with $\theta < \sigma^*$ refrain from applying for patent protection, "pooling", an equilibrium with $m_-^* = 1 + \sigma^* - \pi^*$, i.e. in which all projects apply, and "mixed", an equilibrium with $0 < m_-^* < 1 + \sigma^* - \pi^*$, i.e. in which some projects with $\theta < \sigma^*$ apply. We obtain the following characterization:

Proposition 1. *If $v < f$, no high-R&D equilibrium exists. When $v \geq f$:*

- *there exists a high-R&D separating equilibrium if and only if $f + a(\sigma^*) \leq v$ and*

$$h(\phi(v - f) - \sigma^*)v \leq f; \tag{1}$$

  then, $\pi^* = \phi(v - f)$ and $\phi(v - f) - \sigma^*$ patents are issued;

- *there exists a high-R&D mixed equilibrium if and only if $f + a(\sigma^*) \leq v$, $f \leq h(1)v$ and (1) holds as a strict inequality; then, $\pi^* = \phi(v - f)$, a mass $m_-^* = \sigma^* - \phi(v - f) + h^{-1}(\frac{f}{v})$ of firms with $\theta < \sigma^*$ applies for protection and $(\phi(v - f) - \sigma^*)\frac{v - f}{v} + h^{-1}(\frac{f}{v})\frac{f}{v}$ patents are issued;*

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- *there exists a high-R&D pooling equilibrium if and only if $a(\sigma^*) \leq v(1-h(1))$ and $f \leq h(1)v$; then, $\pi^* = \phi(v-h(1)v)$ and $(\phi(v-h(1)v)-\sigma^*)(1-h(1))+h(1)$ patents are issued.*

It is immediate to write the key condition (1) in the following more intuitive way:

Lemma 3. *For $v > f$, there exists a decreasing function $F(.;v) : [0,(1-h(0)v) \longrightarrow (h(0)v, h(1)v)$ such that within $\{(\sigma^*, f), \sigma^* > 0, 0 < f < v - a(\sigma^*)\}$, the condition $h(\phi(v-f)-\sigma^*)v \leq f$ is equivalent to $f \geq F(\sigma^*, v)$.*

High-R&D separating equilibria are of particular interest: they are characterized by significant R&D activity and they exhibit self-screening in the application process as only non-obvious projects apply for patent protection. Therefore, only "good" patents are issued, i.e. patents that would be granted under a perfect examination technology.[20]

In our model, a marginal increase in R&D investment stochastically increases the innovativeness of the project, which is valuable only to the extent that it increases the probability of obtaining a patent, i.e. only to the extent that it increases the probability of non-obvious projects $\theta > \sigma^*$. Therefore, R&D investment below σ^* is useless; only at high level of R&D, hence for high R&D marginal cost $\gamma'(\pi)$, does a marginal increase in R&D generate a positive marginal profit, equal to $v - f$ the net value of a patent. A high application fee reduces the marginal value of a patent and therefore reduces or even annihilates incentives to R&D. Similarly, a tough non-obviousness standard implies that firms have to invest high levels of R&D to make a difference, which might be too costly. This is why separating equilibria exist only if $f + a(\sigma^*) \leq v$, i.e. if the application fee and the non-obviousness standard are low enough.

The other condition for the existence of separating equilibria ($f \geq F(\sigma^*, v)$) guarantees that the number of non-obvious projects applying for patent protection is not too high, so that the Patent Office is effective enough in identifying poorly innovative projects, thereby deterring applications by obvious innovations. This is the case when the application fee or the non-obviousness standard are high enough (as $F(.,,v)$ is decreasing), as it limits the number of non-obvious projects. We let $\mathcal{D}_S \equiv \{(\sigma^*, f) \in \mathbb{R}_+^2; f + a(\sigma^*) \leq v, f \geq F(\sigma^*, v)\}$ denote the domain of parameter values for which separating equilibria exist; Figure 1 illustrates our discussion.

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[20]Remember that we take σ^* as given here and do not address the question of whether the non-obviousness standard is too tough or too permissive.
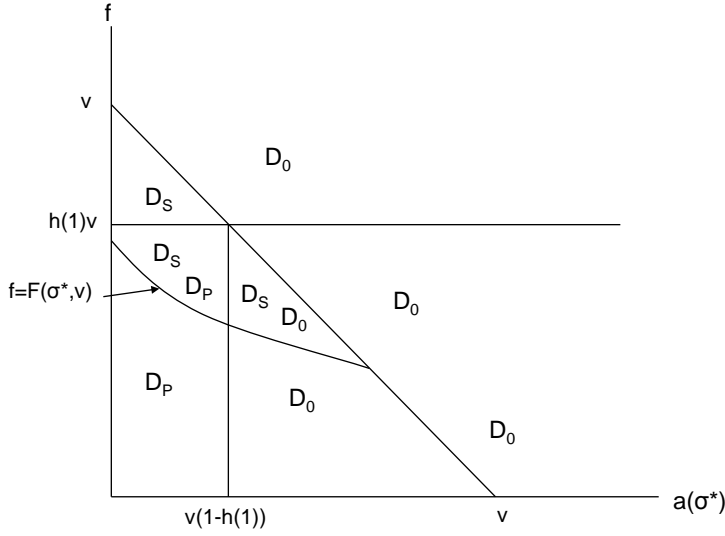
Figure 1: Equilibria

High-R&D pooling equilibria are also characterized by a significant R&D activity, but the process of granting patents cannot rely on self-screening and only rests on the imperfect examination. So, some obvious projects obtain unwarranted patent protection that is, they would not be patented given the standard σ^* if the examination technology were perfect. A marginal increase in R&D induces a shift in the probability of getting a patent from $h(1)$ for obvious projects to 1 for non-obvious ones. In equilibrium, marginal benefit equals marginal cost of R&D: $\gamma'(\pi_P) = v(1 - h(1))$, provided the non-obviousness standard is low enough for such an effort to be more profitable than simply not investing at all in R&D. Moreover, the application fee has to be small as well, so that applying for protection is attractive even for obvious projects. Therefore, pooling equilibria exist within the domain $\mathcal{D}_P \equiv \{(\sigma^*, f) \in \mathbb{R}_+^2; a(\sigma^*) \leq v(1 - h(1)), f \leq h(1)v)\}$ of low application fees and low non-obviousness standard (see Figure 1).

Finally, the proposition proves that within a subset of $\mathcal{D}_S$, there also exists equilibria in which some obvious innovations ask for patent protection. This multiplicity is not surprising as the game of application for patent protection exhibits strategic complementarities, as is usual for congestion models: when more firms apply, the congestion problem becomes more severe, it becomes more likely for an obvious project to get patented, which translates into more numerous

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applications for obvious projects. Note that the mixed equilibrium is unstable in the following sense: if a mass $m < m_-^*$ of firms apply for protection, $h(\phi(v - f) - \sigma^* + m)v < f$ so that all obvious projects would strictly prefer not to ask for protection, while if $m > m_-^*$, all obvious projects would be strictly prefer to ask for protection. Hence, we omit the mixed equilibrium in our later discussion.[21]

3.2   Low-R&D and no-R&D equilibria

It is quite immediate to analyze low-R&D equilibria, i.e. equilibria with positive equilibrium R&D such that $0 < \pi^* \leq \sigma^*$. The intuition provided in the previous subsection shows that a low R&D activity generates obvious projects so that a marginal increase in R&D in this range has no value. This clearly suggests that zero R&D is preferred to low R&D. This is formalized in the following proposition.

Proposition 2. *There cannot exist equilibria with low R&D, i.e. such that $\pi^* \in (0, \sigma^*]$.*

So we focus on no-R&D equilibria, $\pi^* = 0$, for which a mass $m_-^* \in [0, 1]$ of firms applies for protection in equilibrium. Unsurprisingly, the set of no-R&D equilibria exhibits the same kind of multiplicity due to strategic complementarity as in the previous subsection.

Proposition 3. *If $f > v$, there exists a no-R&D equilibrium in which no firm applies for patent protection. If $f \leq v$:*

- *there exists a no-R&D equilibrium in which all firms apply for patent if and only if $h(1)v \geq f$ and $a(\sigma^*) \geq v(1 - h(1))$; then, $h(1)$ firms obtain protection;*

- *there exists a no-R&D equilibrium in which no firm applies for protection if and only if $h(0)v \leq f < v$ and $f + a(\sigma^*) \geq v$;*

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[21] For completeness, it can be shown that:

- Within $\{(\sigma^*, f) \in \mathbf{R}_+^2; f + a(\sigma^*) = v, f \geq h(0)v\}$, which is a zero-measure set, there exist mixed-strategy equilibria in which firms randomize between $\pi = \phi(v - f)$ and $\pi = 0$ and ask for patent protection according to a separating or a mixed equilibrium in the application sub-game.

- Within $\mathcal{D}_P$, there exist mixed-strategy equilibria in which firms randomize between $\pi = \phi(a(\sigma^*))$ (with probability $\frac{a(\sigma^*)}{v(1-h(1))}$) and $\pi = 0$ and ask all for patent protection; these equilibria are similar to the pooling equilibria described in the text.

We omit these equilibria from our discussion, but see Proposition 5 in subsection 5.2.

- *there exists a no-R&D equilibrium in which some but not all firms apply for patent if and only if $h(0)v \leq f < h(1)v$ and $f + a(\sigma^*) \geq v$; then, a mass $m_-^* = h^{-1}(\frac{f}{v})$ of firms applies and $\frac{f}{v}h^{-1}(\frac{f}{v})$ patents are delivered.*

Corollary 1. *There exists a no-R&D equilibrium if and only if $(\sigma^*, f) \in \mathcal{D}_0$, with:*

$$\mathcal{D}_0 \equiv \{(\sigma^*, f) \in \mathbb{R}_+^2; a(\sigma^*) \geq \min\{v - h(1)v, v - f\}\}.$$

This proposition characterizes all cases in which the IP protection system is ineffective, i.e. does not stimulate R&D at all.[22] The intuition is similar to that of the previous subsection. First, if the application fee is so high that a patent has negative net value ($f > v$), then no firm asks for patent protection and there is no incentives to R&D at all. Second, the system may still be ineffective even when the application fee is moderate ($f < v$): if the standard of non-obviousness σ^* is high enough, the probability that a firm comes up with a non-obvious project is too low so that firms prefer not to invest in R&D. Within this domain, there are multiple equilibria in terms of application strategies, depending on whether f is indeed low or not: if f is low, firms can count on congestion in the examination process to obtain a patent, even though they have obvious applications. Note again that equilibria with randomization are unstable and we omit them in our discussion.

Equilibria without any patent application reflect a situation where the IP protection system plays no role, neither positive nor negative, and could be removed. No-R&D equilibria in which all firms apply for patent protection are however clearly quite unattractive: in such an equilibrium with congestion of the Patent Office, the IP protection system does not stimulate R&D at all and some patents are granted whereas no project would pass the novelty test if the examination technology was perfect.

Finally, the corollary indicates that if congestion is a major impediment to the examination process so that $h(1)$ is close to 1, then there are only very specific values of the instruments of IP protection (f, σ^*) that guarantee that R&D is positive in any equilibrium, i.e. that the IP protection system serves its purpose. In other words, when congestion is a major problem, a no-R&D equilibrium is likely to exist and the IP protection system may then be quite ineffective.

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[22]See again Figure 1.

4   Equilibrium properties

4.1   Multiplicity and comparative statics

In this subsection, we discuss the issue of multiplicity and compare the properties of the various equilibria that can emerge. As a preliminary remark, we summarize in Corollary 2 the properties of R&D efforts in high-R&D equilibria.

Corollary 2. *The R&D effort in the pooling equilibrium increases as v increases, but does not depend on f and σ^*. The R&D effort in the separating (and mixed) equilibrium is an increasing function of v, a decreasing function of f, and does not depend on σ^*.*

In the separating equilibrium, the submission fee f is spent only for non-obvious projects and therefore discourages R&D. In the pooling equilibrium, however, f is spent for all types of projects; it acts as a fixed cost and so, has no impact on R&D.[23] The impact of v on R&D efforts is straightforward. Finally, the non-obviousness requirement has no impact on R&D efforts: in the pooling equilibrium all firms apply for a patent, whether their application is below or above the requirement, and in the separating equilibrium the requirement does not affect the firms' marginal benefit of investing in R&D. Therefore, the threshold σ^* determines equilibrium submission strategies but has no direct impact on R&D incentives.

The separating equilibrium exists within the domain $\mathcal{D}_S$. In $\mathcal{D}_S^* \equiv \{(\sigma^*, f) \in \mathbb{R}_+^2; f + a(\sigma^*) < v, f > h(1)v\} \subset \mathcal{D}_S$, it is moreover the unique equilibrium. In this case, the IP protection system works perfectly, although the examination technology is noisy, because firms self-select in their application strategies and invest in intense R&D programs. Note that when congestion is a serious problem, i.e. when $h(1)$ is close to 1, $\mathcal{D}_S^*$ shrinks: multiplicity is most likely and with it, the presence of no-R&D equilibria.

In $\mathcal{D}_M \equiv \{(\sigma^*, f) \in \mathbb{R}_+^2; f + a(\sigma^*) \leq v, F(\sigma^*, v) \leq f \leq h(1)v\} \subset \mathcal{D}_S$, Propositions 1 and 3 show that the separating equilibrium coexists with an equilibrium where all firms apply for a patent with either some R&D effort or none. In this range of values, there are multiple equilibria and the Patent Office is unable to ensure that only good patents will be issued. We compare the properties of the different types of equilibria within this domain $\mathcal{D}_M$.

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[23]Note however that renewal fees (maintenance fees due every four years after the patent grant in the U.S.) may have an impact on the R&D effort. This analysis would require a dynamic model.

Corollary 3. *Within $\mathcal{D}_M$, firms' profits are lower, R&D efforts are higher and good patents are more numerous in the separating equilibrium than in the pooling equilibria.*

When multiple equilibria exist, there is a pooling equilibrium with some or no R&D activity and a high volume of applications, and a separating equilibrium with higher R&D activity, fewer applications and more good patents. If indeed the pooling equilibrium prevails, congestion has a clear negative impact on R&D: as examiners receive too many demands, the probability to obtain a patent on a bad project increases, so incentives to innovate decrease and firms make a weaker R&D effort. Consequently, more patents are issued on obvious applications and there are fewer non-obvious patents in the industry; in the end, issuance of obvious patents slows down the pace of innovation.

The pooling equilibria are preferred ex ante by firms. Selecting among multiple equilibria is problematic and we do not want to argue necessarily that the pooling equilibrium should prevail because it is preferred by the firms. But conversely, the pooling equilibrium cannot be dismissed as being implausible and the tension between social and industry valuation should raise at least serious concerns about the functioning of the IP protection system in this range of parameter values.

Corollary 4 compares the expected volume of patents delivered in the separating equilibrium and the pooling equilibria (with low or no R&D) when multiple equilibria co-exist.

Corollary 4. *Within $\mathcal{D}_M$, if $\phi(v - f) - \sigma^* < h(1)$, the expected volume of patents delivered is lower in the separating equilibrium than in the pooling equilibrium.*

Therefore, if the submission fee and / or the degree of non-obviousness requirement are high, the Patent Office delivers more patents in the pooling equilibrium (with or with no R&D) than in the separating equilibrium (with high R&D and fewer applications). This result shows that there is not necessarily a correlation between the volume of patents granted and firms' R&D activity.

While an IP protection system within $\mathcal{D}_S^*$ works effectively, there are strong arguments suggesting that the range of multiplicity $\mathcal{D}_M$ is also relevant. Indeed, recall that the R&D effort in the separating equilibrium is a decreasing function of f. For any normative viewpoint on how f should be designed, f should be reduced as much as possible within $\mathcal{D}_S^*$ to stimulate

R&D. This would lead to fixing $f = h(1)v$, a boundary on which another less appealing equilibrium exists, with applications by obvious projects and therefore bad patents being delivered with positive probability. In other words, the existing IP protection instruments (σ^*, f) are not sufficient to solve unambiguously the trade-off between selecting only non-obvious applications and encouraging R&D.

Therefore, when the Patent Office is subject to overload and congestion, the whole process of patent application and evaluation may go through discontinuous changes of regime and may not function properly due to a fundamental problem of strategic complementarity across firms and multiplicity of equilibria that cannot be fixed by setting appropriate levels for the classical instruments of the IP protection system. Observing large swings in the number of applications or of issued patents may be due to a shift from one equilibrium to the other, and therefore not be related at all to small changes in the environment, e.g. to a change in the cost of R&D as coming from technical change.

4.2   Introducing a penalty for rejected applications

The IP protection instruments (f, σ^*) are of limited efficiency to induce self-selection by non-obvious projects and to stimulate R&D. Adding other instruments could help the Patent Office select the separating equilibrium. In our setting, fees could be contingent on the Office's observable decision; this amounts to introducing a penalty that a firm would have to pay if its projects is proven to lack novelty or to be obvious, i.e. if the patent is rejected.[24] This is not the case for the moment, neither in the US nor in Europe or in Japan. While it has no impact on the submission strategy for non-obvious projects, this penalty can modify the submission strategy for other projects as well as the R&D effort in equilibrium.

Let us therefore incorporate in the previous model a penalty b to be paid when the Patent Office finds out a signal σ with $\sigma < \sigma^*$. It is immediate to see that the situation is the same as if the submission fee were equal to $f + b$ and the value of the patent were equal to $v + b$. Hence, the following characterization of equilibria:

Proposition 4. *In the model with penalty for patent rejection, the characterization of equilibria*

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[24] In a legal discussion paper, Thomas (2001) suggests the similar idea of a "patent bounty", that would combine a fine to the applicant of a bad patent with a reward to the third party that would prove its invalidity (the latter relates to "post grant reviews" used in the European Patent Office.

follows that of Propositions 1-2-3 with separating equilibria with R&D effort $\pi^* = \phi(v - f)$ *in*

$$\mathcal{D}_S \equiv \{(\sigma^*, f) \in \mathbb{R}_+^2; f + a(\sigma^*) \leq v, f \geq F(\sigma^*, v + b) - b\},$$

pooling equilibria with R&D effort $\pi^* = \phi((v + b)(1 - h(1)))$ *in*

$$\mathcal{D}_P \equiv \{(\sigma^*, f) \in \mathbb{R}_+^2; a(\sigma^*) \leq (v + b)(1 - h(1)), f \leq h(1)(v + b) - b\},$$

and no-R&D equilibria in

$$\mathcal{D}_0 \equiv \{(\sigma^*, f) \in \mathbb{R}_+^2; a(\sigma^*) \geq \min\{v + b - h(1)v, v - f\}\};$$

within $\mathcal{D}_S^* \equiv \{(\sigma^*, f) \in \mathbb{R}_+^2; f + a(\sigma^*) < v, f + b > h(1)(v + b)\}$, *the separating equilibrium is the unique equilibrium.*

As shown on figure 2, if b increases, the domain of existence of no-R&D equilibria shrinks (it is reduced by C+D), and the domain of existence of high-R&D separating equilibria $\mathcal{D}_S$ expands (by A) and the domain $\mathcal{D}_S^*$ where the only equilibrium is the separating equilibrium also expands (by B+C). The domain of existence of high-R&D pooling equilibria increases by D and decreases by B.[25]

In the separating equilibrium, firms only apply for non-obvious patents, so none spends the penalty; the penalty has no impact on R&D activity. In the pooling equilibrium, only firms with obvious projects may have to incur the penalty b, so an increase in b increases incentives to have a non-obvious project and therefore stimulates R&D.

Therefore the penalty would be a powerful instrument, as it would unambiguously encourage R&D: in addition to reducing the domain of no-R&D and expanding that of separating equilibria, it increases the R&D effort in the pooling equilibrium.

However, the trade-off between encouraging R&D and selecting only non-obvious applications still remains. Indeed, R&D activity is maximized with the smallest possible submission fee f that induces the separating equilibrium, or the largest possible penalty b that induces the pooling equilibrium. Reducing f or increasing b lead to the domain of multiple equilibria with positive R&D. In that domain, the R&D effort is higher in the separating equilibrium than in the pooling equilibrium, but firms' profit is higher in the latter, so the Patent Office may still be unable to select only non-obvious applications.

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[25]Overall, it expands if b is small compared to v (if $b \leq \frac{v(2h(1) - 1)}{2(1 - h(1))}$) and shrinks otherwise. In a pooling equilibrium, an increase in b increases the R&D effort, as well as the number of patent grants.
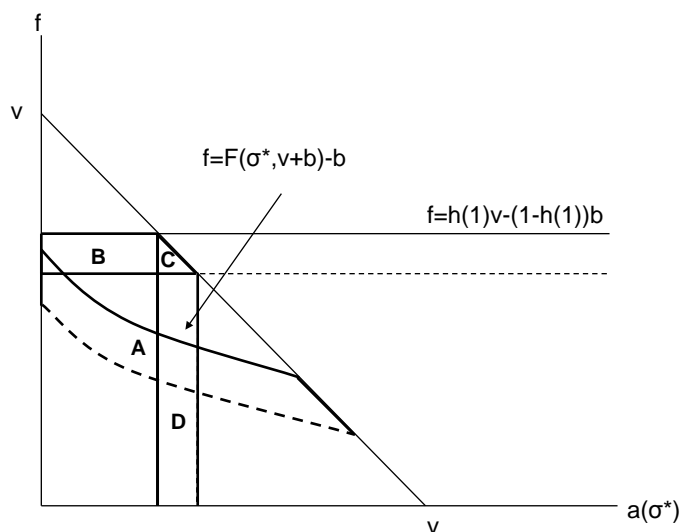
Figure 2: Impact of a penalty for rejected applications

5 Extensions

5.1 Imperfect commitment by the Patent Office

Throughout the model we have assumed that the standard σ^* is determined ex ante, so firms know it as they choose their R&D and submission strategies. However, since patent examination is done by individuals (examiners), it can be considered as a rather subjective process, and the Patent Office may not be able to commit to a standard. In that case, the threshold for non-obviousness is determined ex post, once R&D has already been sunk and the Office has received patent applications.

Ex post, the impact of granting patents on social welfare is unambiguously negative: the monopoly power they grant leads to a deadweight loss and is therefore socially costly. Note that the dynamic impact is ambiguous: when innovation is sequential and the first generation is patented, it can discourage R&D by subsequent innovators, but at the same time patents imply disclosure of knowledge (relative to secrecy), which can enable subsequent innovations.[26] However, in our static model we rule out dynamic considerations and interactions between innovations; so, *ceteris paribus,* granting patents is socially costly.

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[26]In addition, granting obvious patents creates a social cost of litigation when the patent is challenged in court.

Therefore, if its mandate is aligned with social welfare, the Patent Office should reject all applications ex post, i.e. $\sigma^* = 1$. Anticipating this, firms have no incentives to invest in R&D. Therefore, the Patent Office's ability to commit to a standard of non-obviousness $\sigma^* < 1$ is critical for the incentives to innovate provided by the patent system.

Another facet of the commitment problem is related to the fact that, even if the Patent Office could perfectly commit to the standard σ^*, examination is performed by examiners whose incentives may not be perfect. To illustrate this point, suppose an examiner in charge of n applications must exert unobservable effort e at cost $\psi(e)$ (increasing convex) to assess the existing prior art on these applications with an outcome given by the probability $\tilde{h}(n,e)$ of not finding prior art on an application $\theta < \sigma^*$. As before, $\tilde{h}(.,.)$ increases in n but it decreases in e. Suppose that no examination effort leads to a zero probability of finding prior art, i.e. $\tilde{h}(n,0) = 1$. In this moral hazard situation, a Patent Office examiner can be rewarded only on the basis of the observable variables at the level of the Office that is, on the basis of the number of applications and the number of those rejected because prior art has been found.

In such a setting, the IP protection system cannot be perfectly effective because there cannot exist high-R&D separating equilibria. For, suppose there exists one. Then all applications would be non-obvious and the examiner would have no incentives whatsoever to exert effort as he could not possibly reject one of the applications. Formally, condition (1) with $e = 0$ and $\tilde{h}(\pi^* - \sigma^*, 0) = 1$ would amount to $v \le f$. Equilibria in this extended framework must therefore imply bad patents (if only a high-R&D pooling equilibrium exists) or low R&D (if a no-R&D equilibrium prevails). Our emphasis on this weakness of the IP protection system is therefore reinforced in such a setting.[27]

5.2 The value of patent protection depends on the number of patents

In the model, we assumed that the profitability of a patent does not depend on the number of patents granted in the industry, so we ruled out interactions between innovations. However it would be more realistic to assume that there is competition between innovations. One possible reason is the overlap between patents: as more patents are granted, the monopoly power of a patent holder is eroded by the overlapping patents (Hunt, 2004). Another reason is dynamic

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[27]Solving completely the model with examiner's effort is feasible, but it would not add to our understanding of the forces at work here.

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competition: as more patents are granted, the duration of monopoly for a current patent holder is smaller, so the value associated with obtaining a patent is lower (Hunt, 2004).[28] In this subsection, we analyze the situation where the number of patents granted has a negative impact on the value of patent protection.

Let $v(x)$ denote the expected value of obtaining a patent when x patents are granted, and assume it decreases with the total number of patents: $v'(x) < 0$. There is now a force counteracting the fact that congestion increases the probability of obtaining a patent and therefore induces firms to apply for patent protection: the larger the number of expected patents delivered, the lower the value of obtaining a patent, and therefore, the lower the incentives to apply for patent protection. We will assume that $xv(x)$ is increasing in x, so that the congestion effect remains dominant.

A complete analysis would be tedious. So, we focus on emphasizing the robustness of our findings and illustrating new interesting phenomena. Moreover, we mostly limit our investigation to the following case: $h(n) = n$, so that congestion effects are easily tractable. In this special case, Propositions 1, 2 and 3 indicate that for some policy instruments, there exists a (pure strategy) high-R&D equilibrium, and it is separating ($\mathcal{D}_S \neq \varnothing$); yet, there also always exists a no-R&D equilibrium ($\mathcal{D}_0 = \mathbb{R}_+^2$) so that the high-R&D separating equilibrium is never the unique equilibrium ($\mathcal{D}_S^* = \varnothing$); finally, there never exists a high-R&D pooling equilibrium ($\mathcal{D}_P = \varnothing$). The following proposition should be contrasted with these results.

Proposition 5. *Consider the model in which the value of patent protection depends on the number of granted patents and $h(n) = n$:*

- *there exists a no-R&D equilibrium with no patent applications if and only if $f + a(\sigma^*) \geq v(0)$; there exists a no-R&D equilibrium with all firms applying for patent protection if and only if $f \leq v(1)$; therefore, there exists a no-R&D equilibrium except in the domain $\mathcal{D}_1 \equiv \{(\sigma^*, f) \in \mathbb{R}_+^2; f + a(\sigma^*) \leq v(0), f \geq v(1)\}$;*

- *there never exists a high-R&D pooling equilibrium;*

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[28]Hunt (2004) studies a model with sequential innovation, where a patent is replaced by the next generation's innovation, and the patent holder's profit is driven to zero. A stronger patentability reduces the probability that the next generation's innovation will be patentable, so it prolongs the expected duration of monopoly of a patent holder.

- *there exists a non-empty domain $\mathcal{D}_S = \{(\sigma^*, f) \in \mathbb{R}_+^2; f + a(\sigma^*) \leq v(x_S(f, \sigma^*)), f \geq x_S(f, \sigma^*)v(x_S(f, \sigma^*))\}$ in which there exists a high-R&D separating equilibrium; then, R&D effort $\pi_S(f, \sigma^*)$ uniquely solves: $\pi = \phi(v(\pi - \sigma^*) - f)$, and it is decreasing in f and increasing in σ^*;*

- *within the domain $\mathcal{D}_S^{**} \equiv \{(\sigma^*, f) \in \mathbb{R}_+^2; f + a(\sigma^*) \leq v(x_S(f, \sigma^*)), f \geq v(1)\} \subset \mathcal{D}_1$, the unique equilibrium is the separating high-R&D equilibrium with $\pi_S(f, \sigma^*)$;*

- *for some values of the policy instruments, e.g. within the domain $\{(\sigma^*, f) \in \mathbb{R}_+^2; v(x_S(f, \sigma^*)) < f + a(\sigma^*) < v(0), f \geq v(1)\} \subset \mathcal{D}_1$, the equilibrium involves strictly mixed strategies in R&D activity, i.e. firms randomize between R&D effort $\pi_S(f, \sigma^*)$ and 0.*

The negative impact of the number of patents granted on the value of patents induces a counter-active force on the congestion phenomenon. This translates into the fact that it is possible to design policy instruments so that there is some R&D activity in the unique equilibrium. As a new result, R&D effort may be random in some other range of the parameters, with some firms ending up exerting R&D effort and others no exerting R&D effort at all. Yet, the domain $\mathcal{D}_1$, in which there exists no no-R&D equilibrium, is rather limited: in the neighborhood of the model of section 2, i.e. when $v(x) = v + \varepsilon w(x)$ with ε small, $\mathcal{D}_1$ shrinks as ε goes to 0.

As expected, a stricter standard of non-obviousness and a lower submission fee lead to more R&D in the separating equilibrium. However, the maximum σ^* and the minimum f such that the separating equilibrium exists are such that there exists an equilibrium with no R&D, as in the basic model. Therefore, there is still a trade-off between the level of innovative activity and the number of applications in equilibrium.

Finally it is also interesting to mention the possibility of high-R&D pooling equilibria in the model with a general congestion function $h(.)$. Then, R&D effort is the unique solution of:

$$\pi_P = \phi(v[h(1) + (1 - h(1))(\pi_P - \sigma^*)][1 - h(1)]),$$

so that π_P is increasing in the non-obviousness standard σ^*. As in Proposition 1, the submission still has a negative impact on the R&D effort in the separating equilibrium and has no impact on the R&D effort in the pooling equilibrium; but now, both π_S and π_P increase with the non-obviousness standard σ^*. Echoing Corollary 3, it is still the case that when separating

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and pooling high-R&D equilibria co-exist, R&D is higher in the separating equilibrium.[29] The difference between firms' expected profit in the pooling equilibrium and firms' expected profits in the separating equilibrium is not unambiguously positive, however; it is a decreasing function of the submission fee f, but it is negative for values of f close to $h(1)v(x_P)$.[30]

5.3 Innovativeness affects the value of patent protection

The model so far makes the restrictive assumption that the value of patent protection does not depend on the degree of inventiveness θ of the innovation. One may argue that more innovative innovations, if protected, generate higher marketing profits, so that v should be an increasing function of θ. It is not entirely convincing, though, as more inventive innovations do not necessarily meet more profitable demands and, at the same time, a standard technology (low θ) that is patented may induce full monopolization of a profitable market segment and therefore may generate high profits. So, there is a weak case for considering that the value of patent protection is increasing in innovativeness and we show that our model predictions are qualitatively robust to this extension.

More precisely, let us suppose that a project of innovativeness θ generates profits $v(\theta)$ if protected by a patent, with $v(.) : [0, 1] \rightarrow \mathbb{R}_+$ non decreasing and let us assume that $v(.) - \gamma'(.)$ is decreasing: the impact of innovativeness on the value of patent protection is limited compared to how fast the marginal cost of R&D increases with effort. This assumption guarantees a minimal concavity property in the programs that follow and is in line with the absence of compelling

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[29]The pooling equilibrium exists only if $f \leq h(1)v(x_P)$. Given that $v(.)$ is decreasing,

$$
\begin{aligned}
v(\pi_P - \sigma^*) - f &> v(\pi_P - \sigma^* + (1 - h(1))(1 - (\pi_P - \sigma^*))) - f \\
&= v(x_P) - f \geq v(x_P)(1 - h(1)).
\end{aligned}
$$

Therefore, $\phi(v(\pi_P - \sigma^*) - f) > \phi(v(x_P)(1 - h(1))) = \pi_P$. If $\pi_S \leq \pi_P$, $\pi_S = \phi(v(\pi_S - \sigma^*) - f) > \phi(v(\pi_P - \sigma^*) - f) \geq \pi_P$, a contradiction. Therefore, $\pi_S > \pi_P$.

[30]The difference between the ex ante profits in the pooling and in the separating is:

$$
A_{\sigma^*}(v(x_P)(1 - h(1))) - A_{\sigma^*}(v(x_S) - f) + h(1)v(x_P) - f.
$$

Given that $\frac{\partial x_S}{\partial f} = \frac{-\phi'}{1 - \phi'v'}$ as shown in the appendix, the derivative of this difference with respect to f is equal to:

$$
-1 + \frac{1}{1 - \phi'v'}[\phi(v(x_S) - f) - \sigma^*] < 0.
$$

The pooling exists only if $f \leq h(1)v(x_P)$ and for this value, the difference equals:

$$
A_{\sigma^*}(v(x_P) - h(1)v(x_P)) - A_{\sigma^*}(v(x_S) - h(1)v(x_P)).
$$

But $\pi_S > \pi_P$ is equivalent to $\phi(v(x_S) - f) > \phi(v(x_P) - h(1)v(x_P))$ and so, $v(x_S) - f > v(x_P) - h(1)v(x_P)$, which means that the difference is negative for $f = h(1)v(x_P)$.

arguments for a steep $v(.)$ function. To simplify the analysis, let us moreover assume that congestion takes the extreme form: $h(n) = n$, as in the previous subsection.

Although the complete analysis is tedious, the following proposition shows that the main conclusions of the basic model (with $h(n) = n$) are robust to this modeling change.

Proposition 6. *Consider the model in which innovativeness affects the value of patents and $h(n) = n$; then, as in the basic model,*

- *there exists a no-R&D equilibrium for all values of the policy instruments (f, σ^*);*

- *for some values of the policy instruments (f, σ^*), there exists high-R&D equilibria;*

- *in all equilibria with non-trivial R&D, R&D effort is given by $\hat{\pi}(f)$, that uniquely solves*

$$v(\hat{\pi}) - f - \gamma'(\hat{\pi}) = 0,$$

  some obvious projects refrain from applying for patent protection, but there may be bad patents in equilibrium.

Policy instruments cannot avoid the existence of no-R&D equilibria. When well designed, the policy instruments can lead to the existence of high-R&D equilibria as well, in which R&D effort is a decreasing function of the application fee and in which at least some poorly innovative projects refrain from applying for a patent: such an equilibrium generalizes the high-R&D separating equilibrium of the basic model.

6   Conclusion

This paper has proposed a model of the IP protection system, in particular of the patent issuance process. We formalized the overload problem within Patent Offices, echoing the well-documented evidence of a worldwide congestion. In a basically static model focusing on a simple representation of the current patent system, we analyzed the consequences on R&D activity and on the firms' strategies when applying for patent protection. Our conclusions point toward the possibility of a systemic malfunctioning as a result of opportunistic patent applications in an overload situation. Consequently, more bad patents are being issued, with the possibility of multiple equilibria. These problems cannot be resolved by standard control instruments only.

Our model obviously suffers from several limitations. First, R&D is a continuous and often cumulative process, with patent applications landing in the Patent Office's mailbox relentlessly. The Office's policy may evolve over time, and the examination process may become more or less efficient according to the knowledge accumulated in previous patents; in the long run, it is also likely that reputation will play a more prominent role than the mere commitment to some given criteria.

Second, the Office's actual toolkit is more complex and more sophisticated than formalized here. Patent applications are examined sequentially, in the order in which they entered the Office, and the Office can adjust the processing speed or prioritize applications. Following up on an idea introduced by Lichtman and Lemley (2007) which was recently carried on by the Obama administration,[31] the USPTO is thinking about offering a menu of procedures, i.e. a two-tiered patent system, that would allow applicants to pay a higher fee for tighter Patent Office scrutiny, resulting in a "gold-plated" patent. Such a screening process would give patents the benefit of a stronger presumption of validity in court, which would make it more valuable.[32] Overall, it would be interesting to address the issue of the optimal design for the IP protection system, taking into account the possibility of congestion.

Third, the IP protection system relies not just on the ex ante control performed by Patent Offices, but also on ex post invalidation procedures arbitrated by courts. A few economic contributions have started to devise a theoretical framework for this dual regulatory institution (Chiou, 2008), and it would be interesting to investigate how the overload problem that we analyzed would be affected in a wider context. The main effect on our settings would be to make the value of patent protection endogenous, as it would then depend on the competitors' incentives to contest the patent, based on their beliefs about the average quality of patents.

Finally, the Patent Office itself could be viewed as an Agent acting on behalf of Congress, but with its own agenda. The Office has a number of examiners on its payroll, and their incentives should also be more precisely formalized. An overall approach in terms of optimal regulation within an agency framework could then be developed.

These limitations, along with other fascinating issues, provide material for future research.

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[31]See $http://www.barackobama.com/pdf/issues/technology/Fact\_sheet\_innovation\_and\_technology.pdf$

[32]In a recent paper, Atal and Bar (2009) formalize this idea and study its impact on the volume and quality of patent applications and grants.

7 References

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8 Appendix

8.1 Proof of Proposition 1

In a separating equilibrium, the application strategy is a best response for firms only if $h(\pi^* - \sigma^*)v \leq f \leq v$. The expected profits of a firm choosing its R&D are then given by:[33]

$$V(\pi) = (\pi - \sigma^*)^+(v - f) - \gamma(\pi). \tag{2}$$

$V(.)$ is decreasing concave within $[0, \sigma^*]$, concave within $(\sigma^*, 1]$ with an upward kink at σ^*. A positive global maximum arises at $\pi^* = \phi(v - f)$ only if $A_{\sigma^*}(v - f) \geq 0$ (which implies that $\phi(v - f) > \sigma^*$). Conversely, under these conditions, the posited strategy consisting in $\pi^* = \phi(v - f)$ and in application for protection whenever $\theta \geq \sigma^*$ obviously sustains an equilibrium.

In a mixed equilibrium, firms $\theta < \sigma^*$ must be indifferent between applying for protection or not, i.e. $h(\pi^* - \sigma^* + m_-^*)v = f$. Apart from this, the analysis of expected profits and of the equilibrium R&D is similar to the previous case. So, a mixed equilibrium exists if and only if $A_{\sigma^*}(v - f) \geq 0$, (1) and $f \leq h(1)v$ hold, with (1) as a strict inequality; $\pi^* = \phi(v - f)$ and m_-^* is determined by $h(\phi(v - f) - \sigma^* + m_-^*)v = f$. Note that several type of mixed equilibria may exist: to achieve m_-^*, it can be that either all firms $\theta < \sigma^*$ randomize (applying with probability $\frac{m_-^*}{\sigma^* + 1 - \phi(v - f)}$), or that firms apply above a threshold θ^* (with $\theta^* = \sigma^* - m_-^*$), or any combination.

In a pooling equilibrium, all firms apply for protection so that $h(1)v \geq f$ necessarily holds. The expected profits of a firm choosing its R&D are given by:

$$V(\pi) = vh(1) + v(1 - h(1))(\pi - \sigma^*)^+ - \gamma(\pi) - f. \tag{3}$$

A positive maximum arises at $\pi_P = \phi(v - h(1)v)$ only if $A_{\sigma^*}(v - h(1)v) \geq 0$. Conversely, under this condition, such a pooling equilibrium trivially exists.

Finally, from Lemma 1, it is immediate that $A_{\sigma^*}(v - f) \geq 0$ is equivalent to $f \leq v - a(\sigma^*)$, for $\sigma^* \in [0, a^{-1}(v)]$ and $A_{\sigma^*}(v - h(1)v) \geq 0$ is equivalent to $a(\sigma^*) \leq v(1 - h(1))$.

8.2 Proof of Proposition 2

On such an equilibrium path, (almost) all firms end up with $\theta < \sigma^*$. If firms do not expect to get strictly valuable protection with positive probability, the expected profit function when

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[33]We use the notation: $x^+ = \sup\{0, x\}$.

a firm chooses its R&D is strictly decreasing for $\pi \in [0, \sigma^*]$. Therefore, it is necessary that in equilibrium a positive mass applies for protection and that $h(1)v > f$. But then, $v > f$ and firms would also apply if they turned out to get $\theta > \sigma^*$. The expected profits function is therefore:

$$V(\pi) = vh(1) + v(1 - h(1))(\pi - \sigma^*)^+ - \gamma(\pi) - f,$$

whose maximum can only be equal to 0 or to $\pi_P = \phi((1 - h(1))v) > \sigma^*$.

8.3 Proof of Proposition 3

All firms apply for a patent ($m_-^* = 1$) only if $h(1)v \geq f$; the expected profit of a firm choosing its R&D ex ante is given by (3). If the firm makes no R&D effort ($\pi = 0$), $V(0) = h(1)v - f \geq 0$. Therefore, an equilibrium with zero R&D and all firms applying for patent protection exists only if $h(1)v \geq f$ and $A_{\sigma^*}(v - h(1)v) \leq 0$. These conditions are obviously sufficient.

No firm applies for a patent ($m_-^* = 0$) if $h(0)v \leq f$; if, in addition, $v \leq f$, the expected profit function of a firm choosing its R&D is equal to $-\gamma(\pi)$, and therefore leads to $\pi^* = 0$. If $h(0)v \leq f < v$, the expected profit function of a firm choosing its R&D is given by (2). The equilibrium exists only if $h(0)v \leq f$ and either $v \leq f$ or $A_{\sigma^*}(v - f) \leq 0$. Sufficiency is obvious.

If $m_-^* \in (0, 1)$, then necessarily $h(m_-^*)v = f$. The analysis is then similar to the previous case when $v > f$.

8.4 Proof of Corollary 3

Firms' expected profit is $V^S = A_{\sigma^*}(v - f)$ in the separating equilibrium, $V^P = A_{\sigma^*}(v(1 - h(1)) + vh(1) - f$ in the high R&D pooling equilibrium and $V^0 = vh(1) - f$ in the no-R&D equilibrium where all firms apply. A necessary condition for multiple equilibria to arise is $f \leq vh(1)$. First, $V^0 - V^S = vh(1) - f - A_{\sigma^*}(v - f)$ is a decreasing function of f (its derivative is $-1 + \phi(v - f) - \sigma^* \leq 0$); when the no-R&D equilibrium exists with application by all projects, $a(\sigma^* \geq v(1 - h(1))$ and so, $V^0 - V^S$ is positive for $f = vh(1)$. Therefore, when the separating equilibrium co-exists with the no-R&D equilibrium, $V^0 \geq V^S$. Second, $V^P - V^S = vh(1) - f - A_{\sigma^*}(v - f) + A_{\sigma^*}(v(1 - h(1))$ is also a decreasing function of f, and is null when $f = vh(1)$. Therefore, when the separating equilibrium co-exists with the high R&D pooling equilibrium, $V^P \geq V^S$.

Moreover, the R&D effort is higher in the separating equilibrium than in the pooling equilibrium if and only if $\phi(v - f) \geq \phi(v(1 - h(1))) \Leftrightarrow v - f \geq v(1 - h(1)) \Leftrightarrow vh(1) \geq f$. Hence the

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conclusion.

8.5 Proof of Proposition 5

In a separating equilibrium with high R&D π_S, there are $\pi_S - \sigma^*$ applications and $x_S = \pi_S - \sigma^*$ patents are granted. The firms' R&D effort at the equilibrium is determined implicitly by: $\pi_S = \phi(v(x_S) - f)$. Note that the equation:

$$\pi_S = \phi(v(\pi_S - \sigma^*) - f)$$

is such that the left-hand side is larger than the right-hand side when π_S goes to 1, while it is smaller when π_S goes to σ^*, since $f + a(\sigma^*) \leq v(0)$ implies that $v(0) - f \geq a(\sigma^*) > \gamma'(\sigma^*)$. This proves the existence of (π_S, x_S). Uniqueness follows from the fact that the right-hand side is decreasing in π_S. Total differentiation then gives:

$$
\begin{aligned}
(1 - \phi'v')d\pi_S &= -\phi'v'd\sigma^* - \phi'df \\
(1 - \phi'v')dx_S &= -d\sigma^* - \phi'df,
\end{aligned}
$$

hence the variations of π_S. As in the basic analysis, a separating equilibrium exists if and only if $A_{\sigma^*}(v(x_S) - f) \geq 0$ and $h(\pi_S)v(x_S) \leq f \leq v(x_S)$. Note that there also exist mixed strategy (in applications) equilibria within this range. The non-existence of pooling equilibria follows from the same analysis as in the basic model.

In an equilibrium where firms make no R&D effort and all apply for a patent, $x = 1$ patents are granted. Such an equilibrium exists only only if $v(1) \geq f$ and this condition is obviously sufficient. In an equilibrium with no-R&D and no firms applying for patent protection, 0 patents are granted, so this equilibrium exists only if $0 \leq f$ and $A_{\sigma^*}(v(0) - f) \leq 0$, which are also sufficient conditions. The characterization of $\mathcal{D}_1$ and $\mathcal{D}_S^{**}$ follows straightforwardly.

Finally, let us consider the domain $\{(\sigma^*, f) \in \mathbb{R}_+^2; v(x_S(f, \sigma^*)) < f + a(\sigma^*) < v(0), f \geq v(1)\}$. From our analysis, no equilibrium with pure strategies in R&D effort exists. Consider the following system:

$$
\begin{aligned}
\pi^* &= \phi(v(x^*) - f) \\
x^* &= \alpha(\pi^* - \sigma^*);
\end{aligned}
$$

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for the same reasons as above, there exists a unique solution (π^*, x^*) as a function of (α, f, σ^*). Let us then examine the equation:

$$v(x^*) = a(\sigma^*) + f. \tag{4}$$

When α goes to 0, x^* goes to 0 and the left-hand side is larger than the right-hand side; when α goes to 1, x^* goes to x_S and, from the assumption, the left-hand side is smaller than the right-hand side. Hence there exists α^* that solves the equation.

Consider now the following strategies for firms: randomize R&D effort by choosing $\pi^*(\alpha^*, f, \sigma^*)$ with probability α^* and 0 with probability $1 - \alpha^*$; then, apply for patent protection whenever $\theta > \sigma^*$ and refrain otherwise. Firms expect $\alpha^*(\pi^* - \sigma^*)$ applications that will all be accepted. Their application strategy is an equilibrium strategy because $v(1) \leq f \leq v(x^*)$ and since by assumption $v(1) \geq x^* v(x^*)$, this implies: $x^* v(x^*) \leq f \leq v(x^*)$. Moreover, firms' expected profits ex ante have two maxima: one at $\pi = 0$ and one at π^*; hence, randomization is a best response for firms. It follows that the postulated strategies sustain an equilibrium.

8.6 Proof of Proposition 6

Suppose that $v(0) \geq f$ and consider the following strategies: no firm exerts R&D effort and firms always apply for patent protection, whatever their project. Since $v(0) \geq f$, for any θ, $v(\theta) \geq f$. Given that all firms apply, a firm with project θ therefore applies for protection as a best response. The expected profits of a particular firm investing π in R&D equals:

$$v(0) + \int_0^\pi [v(\theta) - v(0)]d\theta - \gamma(\pi) - f,$$

which is decreasing in π.[34] Therefore, all firms choose $\pi = 0$. The proposed strategies therefore sustain an equilibrium.

Suppose instead that $f > v(0)$ and let $\bar{\theta}(f) \equiv \inf\{v^{-1}(f), 1\} > 0$. Consider the following strategies: no R&D effort and patent application for $\theta \geq \sup\{\sigma^*, \bar{\theta}(f)\}$. If all firms follow these strategies, there is no application in equilibrium (since all projects are non-novel) and therefore a firm applies if and only if $v(\theta) \geq f$ and $\theta \geq \sigma^*$. The expected profits of a particular firm investing $\pi \geq \sup\{\sigma^*, \bar{\theta}(f)\}$ in R&D equals:

$$\int_{\sup\{\sigma^*, \bar{\theta}(f)\}}^\pi [v(\theta) - f]d\theta - \gamma(\pi)$$

---

[34]The derivative in π equals $v(\pi) - v(0) - \gamma'(\pi)$; it is decreasing in π and non-positive for $\pi = 0$.

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and they are equal to $-\gamma(\pi)$ otherwise. The derivative of this expected profit function for $\pi \geq \sup\{\sigma^*, \bar{\theta}(f)\}$ is equal to $v(\pi) - \gamma'(\pi) - f$; since it is decreasing in π, $v(\pi) - \gamma'(\pi) - f \leq v(0) - f - \gamma'(0) < 0$. So, the best R&D choice for a firm is $\pi = 0$.

Therefore, whatever (f, σ^*), there exists a no-R&D equilibrium, which proves the first part of the proposition.

It is immediate to prove that there cannot exist low-R&D equilibria in this setting either.

There cannot exist a high-R&D equilibrium where all firms apply, i.e. a full pooling equilibrium, irrespective of their innovativeness, since all would have probability 1 of being patented $(h(1) = 1)$ and therefore none would have any incentives to invest in R&D. Provided it exists, a non-zero-R&D equilibrium is therefore characterized by $\pi^* > \sigma^*$ and by θ^*, the critical value of innovativeness such that firms in equilibrium apply if and only if $\theta \geq \theta^*$ (a immediate version of Lemma 2 applies). Different cases must be investigated depending on the relative position of θ^* and σ^*.

There cannot exist a high-R&D equilibrium such that $\sigma^* < \theta^* \leq \pi^*$. For, if it existed, $v(\theta^*) = f$. The expected profits of a firm investing π would equal $-\gamma(\pi)$ if $\pi < \theta^*$ and

$$\int_{\theta^*}^{\pi} (v(\theta) - f)d\theta - \gamma(\pi)$$

if $\pi > \theta^*$. Within the range $\pi > \theta^*$, the slope of the expected profits is equal to $v(\pi) - \gamma'(\pi) - f$; it is decreasing and negative for $\pi = \theta^*$ since $v(\theta^*) = f$. A firm would then invest $\pi = 0$, a contradiction.

Suppose an equilibrium exists such that $0 < \theta^* \leq \sigma^* < \pi^*$. Then, patent application for $\theta \geq \theta^*$ is a best response only if $(\pi^* - \sigma^*)v(\theta^*) \leq f \leq v(\sigma^*)$ and, if $\theta^* < \sigma^*$, it must be that $(\pi^* - \theta^*)v(\theta^*) = f$. The expected profits of a firm investing π are given by $-\gamma(\pi)$ for $\pi < \theta^*$, $\int_{\theta^*}^{\pi}[(\pi^* - \sigma^*)v(\theta) - f]d\theta - \gamma(\pi)$ if $\theta^* \leq \pi < \sigma^*$, and

$$\int_{\theta^*}^{\sigma^*} [(\pi^* - \sigma^*)v(\theta) - f]d\theta + \int_{\sigma^*}^{\pi} (v(\theta) - f)d\theta - \gamma(\pi)$$

if $\pi \geq \sigma^*$. On the range (θ^*, σ^*), if indeed $\theta^* < \sigma^*$, the derivative of this expression equals $(\pi^* - \sigma^*)v(\pi) - f - \gamma'(\pi)$. As $v(.) - \gamma'(.)$ is decreasing, $(\pi^* - \sigma^*)v(\pi) - f - \gamma'(\pi) \leq (\pi^* - \sigma^*)v(\theta^*) - f - \gamma'(\theta^*) = -\gamma'(\theta^*) < 0$. Expected profits are therefore decreasing within this range. For $\pi > \sigma^*$, the derivative equals $v(\pi) - f - \gamma'(\pi)$. For such a high-R&D equilibrium to exist,

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it is necessary that $v(\sigma^*) - f - \gamma'(\sigma^*) > 0$. Then, there exists a unique $\hat{\pi}(f)$ (continuity and monotonicity of $v - \gamma'$), such that:

$$v(\hat{\pi}) - f - \gamma'(\hat{\pi}) = 0.$$

It must then be that:

$$\int_{\theta^*}^{\sigma^*} [(\pi^* - \sigma^*)v(\theta) - f]d\theta + \int_{\sigma^*}^{\hat{\pi}(f)} (v(\theta) - f)d\theta - \gamma(\hat{\pi}(f)) \geq 0.$$

The necessary conditions found out are sufficient for strategies with R&D effort $\hat{\pi}(f)$ to constitute an equilibrium.

It follows that the set of necessary and sufficient conditions for the existence of a high-R&D equilibrium with $\theta^* = \sigma^*$ are: $(\hat{\pi}(f) - \sigma^*)v(\sigma^*) \leq f < v(\sigma^*) - \gamma'(\sigma^*)$ and $\int_{\sigma^*}^{\hat{\pi}(f)}(v(\theta) - f)d\theta - \gamma(\hat{\pi}(f)) \geq 0$. Suppose $f \in (\hat{\pi}(v(0))v(0), v(0))$. Then, there exists $\varepsilon > 0$ such that for $\sigma^* < \varepsilon$, the double inequality holds. Moreover, for $\sigma^* = 0$, the integral condition writes down as:

$$\int_0^{\hat{\pi}(f)} (v(\theta) - f - \gamma'(\theta))d\theta > 0,$$

holding as a strict inequality. Therefore, there exists an open neighborhood of $\{(f, \sigma^*), \hat{\pi}(v(0))v(0) < f < v(0), \sigma^* = 0\}$ in which there exists a high-R&D equilibrium with $\theta^* = \sigma^*$. Note that such high R&D equilibrium is the only equilibrium for which there are only good patents, i.e. only projects with $\theta \geq \sigma^*$ get patent protection.

For the sake of completeness, there may also exist high-R&D equilibria with $0 < \theta^* < \sigma^* < \pi^*$ and high-R&D equilibria with $\theta^* = 0$, in which case firms with no innovation randomize and apply with probability m. In this latter case, we know that $m < 1$, as there cannot exists a high-R&D equilibrium in which all firms apply (see above). And in these equilibria, there are projects with $\theta \in (\theta^*, \sigma^*)$ that get patented, i.e. there are bad patents.