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Information revelation in a security market:

The impact of uncertain participation

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Abstract. The paper analyzes how uncertainty on traders' participation affects a competitive security market in which there are some informed traders. We show that discontinuities, or "crashes", can arise at equilibrium, even when no investor posts a priori an increasing demand. Because of uncertain participation, the precision of the information brought by a price is endogenous, affected by the size of the trades. As a result, two prices with different volumes and information revelation may clear the market for the same values of the fundamentals. At one price, insurance motives drive the exchanges, noise is large and little information is revealed. At another price, uninformed trades are small, which makes the clearing price much more informative. This multiplicity of prices with different precision of information generates discontinuities.

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1 Introduction

It is widely recognized that differences in information play a crucial role in explaining trades and prices in security markets. How much these differences affect investors' behavior, and to which extent they destabilize the equilibrium process are very challenging questions.

Private information is not fully revealed to all participants in the market, even if investors are 'rational', except under strong conditions that relate the dimension of the signals to the dimension of prices. In a single stock market for instance, the price cannot perfectly reveal a private signal on the stock if the demand for the stock is also affected by an unobserved independent shock. The price may however transmit some information. Indeed, in an almost 'standard' model, the equilibrium price is explicitly computed as a smooth, in fact linear, noisy version of private information, both in competitive and monopolistic environments (Grossman and Stiglitz [1980] and Kyle [1985]). Private information has an impact on price formation but it is a smooth one and the value of information for an investor can be defined. The standard model however relies on strong assumptions, 'noise trading' (as made precise later), constant risk averse traders and normal variables. Our aim here is to study the robustness of the analysis. We investigate whether private information is likely to be smoothly transmitted without inducing some discontinuities or "crashes". Such a robustness analysis is worth performing given the large finance literature built on the standard model.

Our analysis is conducted in a fully competitive environment, thereby leaving aside phenomena that may be due to a strategic use of information (on this issue see for example Bhattacharya and Spiegel [1991]). We analyze a single stock market, to keep the model as simple as possible and to avoid cross effects between markets. This implies that, without private information on the stock future value, a unique price clears the market and that any difficulty in the equilibrium process stems from asymmetric information.

We consider a market with informed investors, and two classes of uninformed investors, those who try to infer information from the prices, called speculators, and those who do not, called Walrasian traders. Speculators may represent intermediaries who trade for the count of small investors. Each trader submits a demand schedule contingent on the price, and a clearing price is selected.[2] Demand schedules are affected by private information (for informed traders) or by some shock (for Walrasian traders). The speculators' behavior is more complex to analyze. When forming his demand contingent on a price p, a speculator *assumes* that p is a clearing price and

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[2]Every demand schedule, not necessarily decreasing, may be obtained as the sum (possibly infinite) of limit orders and stop orders. A limit order specifies the quantity a trader wants to buy or sell and the limit price he agrees to pay or receive for this quantity. A stop order specifies the quantity the trader wants to buy or sell and the minimum or maximum price that will initiate the transaction.

attempts to infer information on the private signal from this assumption. Since a clearing price depends on the demand of all investors, informed or not, the distribution of the shocks that affect uninformed demand plays a crucial role in the inference process. A shock can be 'noise trading', usually defined as a fixed exogenous random demand independent of the stock price. A traditional justification for noise trading is that small occasional investors have sudden amounts of cash to invest or sudden needs for cash, which are independent of the market conditions and the price level. A different type of shocks is equally realistic. The sensitivity of the demand to prices may be (perceived as) uncertain because the number, or the risk aversion or the risk perception of some traders are uncertain. In that case, that we shall refer to as uncertain participation- the trade volumes affect the 'noise' in the market. As a result, the precision of the information on the stock revealed by prices is endogenous and may vary significantly with volume.

To analyze price formation, two alternative equilibrium concepts are considered. One is the traditional rational expectations equilibrium (REE) based on a price function of the shocks, function supposed to be known and used by some traders, here the speculators, to infer information. The other equilibrium, called a 'market equilibrium', is given by a Nash equilibrium among (an infinite number of) speculators. We shall argue that markets where the two concepts lead to the same prediction are stable. This occurs in particular when there is a rational expectations equilibrium with a continuous price function. In that case, private information has not a dramatic impact and does not explain crashes.

We show that uncertainty on traders' participation has a large impact on stability. Specifically, in a model with noise trading but where participation is certain, there is a continuous REE under some mild conditions, much more general than the standard constant risk aversion gaussian setup. If instead the participation of some traders is uncertain, a continuous REE may not exist due to multiple clearing prices. This possibility is related with the various motives to trade, information and insurance. Consider a market where, in the absence of private information, the exchanges between Walrasian traders and speculators are large at the competitive price, motivated by insurance purpose. Under uncertain participation, these exchanges generate important 'noise'. As a result, the presence of insiders may have a moderate impact on exchanges and prices. Because of the amount of noise due to the large trades driven by insurance motives around the competitive price, little information is revealed, which in turn justifies large trades and a clearing price close to the competitive one. But, markets may also clear at a different, more informative, price for some values of the signal. At such a price, the volume of Walrasian trades is expected to be low, which makes the speculators reluctant to trade because they would trade mostly with informed traders: adverse selection considerations drive the exchanges with little volume. Hence, two prices can simultaneously clear the market for the same values of the shocks with drastically different

volume and information transmission.

Gennotte and Leland [1990] show that multiple clearing prices, and the associated discontinuities may arise if some traders post an increasing demand, as possibly generated by program trading. The source of discontinuity found in this paper differs from theirs, since no exogenous increasing demand is assumed. It is the fact that the precision of conveyed information is endogenous, affected by trade volume, that may generate nonmonotone, nonlinear demand schedules.

A famous result in Grossman and Stiglitz [1980] is that the value of acquiring information is decreasing with the number of agents who acquire information, which is at the root of the so-called Grossman and Stiglitz paradox. The robustness of this result is challenged by Barlevy and Veronesi [2000] and Chamley [2007] in a static and dynamic setting respectively. There may be complementarities in the acquisition of private information under some conditions in particular when noise trading is not normally distributed. This paper analyzes a different kind of robustness, and shows that even defining the value of information may be problematic because the transmission of information may not be uniquely defined nor continuous.

Finally our paper is also related to the more theoretical literature that is concerned with the concept and the existence of rational expectations equilibrium in (incomplete) markets of contingent claims (Green [1973], Jordan and Radner [1982]). Keeping a simple structure, close to the standard finance model, allows us to interpret equilibrium properties.

Section 2 presents the model. Section 3 defines and compares the equilibrium concepts and introduces the property of strategic substitutes trades. Section 4 analyzes the situation where participation is known -referred to as the 'pure noise trading' model- and Section 5 studies the impact of uncertain participation. Proofs are gathered in the final section.

2 The model

The model analyzes a stock market at a single date, $t = 0$. A unit of the stock yields a random payoff $\widetilde{v}$ at a future date[3] $t = 1$. Its variance is normalized to 1. There is also a risk-less security with return normalized to 1.

The stock price is determined through a call auction on an order driven market. First, each trader posts a demand schedule, which specifies the number of units demanded (possibly negative) at each possible price. Second an auctioneer selects a price and allocates quantities. The exchange price is set equal to a clearing price, if any, and traders receive the amount they demand at that price. All traders are competitive, meaning that they do not recognize the influence of their own demand on the clearing price.

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[3]In the sequel, a random variable is denoted by $\widetilde{a}$, and its realization by a.

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Traders Traders are divided into three groups:

– *Walrasian* or 'naive' traders who occasionally participate in the market. They do not infer information from prices and each one submits a standard demand function, continuous and non-increasing in the price p. Their aggregate demand is called the 'Walrasian demand'. It is affected by the traders' characteristics, possibly perceived as random. The demand will be denoted by $Z(p, \widetilde{\xi})$ in which $\widetilde{\xi}$ denotes the vector of random traders' characteristics.

Noise trading for example refers to a pure random term, $\widetilde{\omega}$, added to a standard deterministic demand, which yields the Walrasian demand $Z(p) + \widetilde{\omega}$. Noise trading is interpreted as unexpected hedging needs under normality and constant risk aversion assumptions. Specifically, a Walrasian trader i whose (constant) risk tolerance coefficient is τ_i and future endowments are $\widetilde{e}_i$ demands $[\tau_i(Ev - p) - cov(\widetilde{e}_i, \widetilde{v})]$ units of the asset in which $-cov(\widetilde{e}_i, \widetilde{v})$ is referred to as i's hedging need because it is the demand that minimizes the variance of i's final wealth.[4] Aggregating over traders, the Walrasian demand is

$$\tau_W(E\widetilde{v} - p) - cov(\widetilde{e}, \widetilde{v}) \tag{1}$$

where τ_W is the aggregate risk tolerance coefficient and $-cov(\widetilde{e}, \widetilde{v})$ is the aggregate hedging needs at the time of the trade. If the hedging needs are not publicly known, set $\widetilde{\omega}$ to be equal to their variation around their expected value. Assuming the aggregate risk tolerance coefficient τ_W to be known, noise trading is obtained, as in the standard finance model. This is a rather specific assumption. Owing to uncertain participation, or to uncertain assessment of the riskiness of the asset payoff by occasional traders, both the aggregate hedging needs and the risk tolerance coefficient may be perceived as random. In this case, the Walrasian demand is affected by the shock $\widetilde{\xi} = (\widetilde{\tau_W}, \widetilde{\omega})$.

– *informed* traders who receive an advanced private random signal $\widetilde{\theta}$, on the future realization of the asset payoff $\widetilde{v}$: $\widetilde{v} = \widetilde{\theta} + \widetilde{\varepsilon}_v$, where θ and ε_v are independent with $E\widetilde{\varepsilon}_v = 0$. Informed traders have nothing to learn from the price, so that their demand is derived from standard utility maximization. Their aggregate demand schedule[5] is described by a function Y of the net expected gain $\theta - p$, where Y is continuous, increasing, and satisfies $Y(0) = 0$. The analysis straightforwardly extends to the case where a random term, independent of information, affects informed traders' demand. The random term can simply be added to the Walrasian demand.

– traders, called *speculators*, who spend some effort to study the market so as to infer infor-

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[4]This obtains under the normality of $(\widetilde{e}_i, \widetilde{v})$. Aggregate demand is of the same form as (1) under heterogenity of assessments on risks, in which $E_i v$ and $\text{var}_i(\widetilde{v})$ may differ across traders and from the objective values.

[5]An informed trader who knows θ maximizes at p the expected utility $E[u(x(\theta + \widetilde{\varepsilon}_v - p) + \widetilde{W}]$ in which $\widetilde{W}$ is the trader's future endowment. Assuming $\widetilde{W}$ to be independent of the stock, demand is an increasing, continuous function of the expected net gain, $\theta - p$.

mation from the prices. They know both informed and competitive traders' behaviors. They are all identical, strictly risk averse with a continuous Von Neumann Morgenstern utility function u. They have no hedging needs. Their number, possibly not publicly known, is denoted by $\widetilde{\tau_S}$. Under constant risk aversion, $\widetilde{\tau_S}$ can be interpreted as the speculators' aggregate risk tolerance coefficient.

A speculator's behavior is not straightforwardly derived from primitives, in contrast with other traders. The reason is that an optimal speculator's demand depends on the inferred information, which in turn depends on the equilibrium process. The paper analyzes how this demand is formed and how the nature of the shocks affects the inference process. By symmetry, speculators post the same schedule, denoted by $x(.)$ (this is not an assumption at equilibrium, as explained in section 3).

To sum up, the shocks affecting the market are specified by the vector (θ, τ_S, ξ) where θ affects informed demand, τ_S affects speculators' participation, and ξ affects the Walrasian demand. The expected value of a participation parameter $\widetilde{\tau}$ will be denoted by a t (t is also used when the participation is certain).

Market clearing   Once demand schedules are posted, a clearing price, if any, is selected. Formally, given speculator's demand x and realization (θ, τ_S, ξ), a clearing price solves :

$$\tau_S x(p) + Y(\theta - p) + Z(p, \xi) = 0. \tag{2}$$

The set of clearing prices, possibly empty or multi-valued, is denoted by $C_x(\theta, \tau_S, \xi)$.

We take the following assumptions throughout the paper.

Assumptions *All shocks admit a continuous density.*

The distribution of (τ_S, ξ) is independent of that of θ.

The support of θ is $\Re$ and the range of Y is $\Re$.

The density assumption avoids some artificial discontinuities. Suppose for instance that noise trading ω takes discrete values and that the price is a combination of the true value θ and noise trading. An infinitesimal variation in the price may lead to a drastic revision in the distribution of the asset value. This generates a discontinuity in the demand of a trader who conditions on the price, even under standard assumptions on preferences, as shown by Kreps [1977]. The assumption on the independence of the distributions is mainly a simplifying assumption. Finally, assuming an unbounded support for the signal and that informed demand may be arbitrarily large, negative or positive, implies that the range of possible prices is the whole set $\Re$. As a result, demand schedules are defined on the whole set $\Re$ and must be 'rational' at every price, since each one is a priori reasonable.

3 Two equilibrium concepts under rational expectations

We introduce two equilibrium concepts under which speculator's demand is 'rationally' formed. The first concept is the standard rational expectations equilibrium.

Rational expectations equilibrium   The equilibrium is supported by the knowledge of a price function that relates (observed) clearing prices to the (unobserved) exogenous shocks. Such a function P assigns to each shock (θ, τ_S, ξ) a price $p = P(\theta, \tau_S, \xi)$. Thus, we exclude random price functions. This is in line with our concern to identify the situations in which information does not affect too much the equilibrium process in the sense that a continuous price function exists. An equilibrium is described as follows.

Definition. *A **rational expectations equilibrium** (REE) is a pair (x, P) where $x(.)$ is a speculator's demand schedule and P is a price function defined over the set of possible realizations of (θ, τ_S, ξ) that satisfy*

1. *for each p, $x(p)$ maximizes $E(u(a(\widetilde{v} - p))|p)$ over all trades a, where the prior distribution on $\widetilde{v}$ is revised using that $p = P(\theta, \tau_S, \xi)$*

2. *for each (θ, τ_S, ξ), market clearing (2) is satisfied at price $p = P(\theta, \tau_S, \xi)$.*

Condition *1* states that each speculator posts a demand that is 'rational' at each price p if he assumes that P is the price function and infers information accordingly. Condition *2* ensures that for each possible value for the shock the price predicted by the function P is a clearing price.

The REE concept raises several difficulties. Even in our simple model, a REE is not easy to handle with. An equilibrium price function is found as a fixed point on a functional set, which is computationally difficult. From the investors' point of view, it might be quite a challenge to "learn" a price function. In the standard finance model, the difficulty is solved by guessing a function P that is linear. But in general an equilibrium price function is unlikely to be linear. A second and more fundamental difficulty arises if several prices clear the market for some values of the shock. In that case the price function selects a particular clearing price among the possible ones. Investors behave as if they know the selection although it is not based on an explicit criterium. Imposing an explicit selection criterium, say to maximize the volume, does not help because it makes the existence of an equilibrium problematic. These difficulties motivate the *market equilibrium* approach that we introduce now. The underlying idea is to look for an equilibrium in which speculators infer information from market clearing prices only. The two approaches are related, and coincide in well behaved cases, as is made precise in Proposition 1.

Speculators and market equilibrium   In forming a demand at price p, each speculator assumes that p is a clearing price and infers information from that. Here, instead of using a price function as in the REE approach, inference is drawn by using the market clearing equation and the knowledge of others' behavior. Both informed and Walrasian demands are known (in distribution) but not the speculators' one. Let a speculator form some expectation $\widetilde{X}^e$ on speculators' aggregate demand at p. Taking his own trade as negligible he infers that (θ, τ_S, ξ) satisfies

$$\widetilde{X}^e + Y(\widetilde{\theta} - p) + Z(p, \widetilde{\xi}) = 0 \tag{3}$$

and revises the prior distribution on $\widetilde{\theta}$ accordingly. An optimal trade, say a, maximizes the speculator's expected utility conditional on this information (it is unique by strict concavity of u). At equilibrium, expectations are correct. This implies that all speculators expect an identical distribution for $\widetilde{X}^e$ because each one is small hence with no impact on the aggregate quantity. Hence, each speculator asks the same quantity a, which is the optimal one given their identical expectation $\widetilde{X}^e$. At equilibrium, expectations $\widetilde{X}^e$ are correct, given by $\widetilde{\tau_S}a$, which leads to the following definitions.

Definitions. *Let $D(p, a)$ be the trade that maximizes the speculator's utility $x \to E(u(x(\widetilde{v} - p))|p, a)$ where the distribution on $\widetilde{\theta}$ is revised by the market clearing equation given (p, a)*

$$\widetilde{\tau_S}a + Y(\widetilde{\theta} - p) + Z(p, \widetilde{\xi}) = 0. \tag{4}$$

*An **equilibrium trade** at price p is given by a trade a that satisfies $a = D(p, a)$. A **speculators' equilibrium** is a demand schedule x that selects for each price p an equilibrium trade at that price: $x(p) = D(p, x(p))$. A **market equilibrium** is a pair (x, C_x) formed with a speculators' equilibrium x and the price correspondence C_x that assigns to each shock the set of clearing prices.*

An equilibrium trade at a given price is a Nash equilibrium among infinitesimal speculators because each speculator optimally behaves given the others' behavior. A speculators' equilibrium is simply a demand schedule that picks up an equilibrium trade at each price. Hence, the information inferred from a given price is 'local' in the sense that it does not rely on the knowledge of a whole price function, as it is assumed to support a REE. The two equilibrium concepts lead to the same outcomes in some well behaved cases, as we study now.

Links between REE and market equilibrium   Market and rational expectations equilibria differ by the information used by a speculator to update his prior at a clearing price p. In a market equilibrium, inference is based on 'p is a clearing price given demand x', and in a REE on 'p is the clearing price *selected* by P'. If these two events coincide whatever the price level, speculators'

behaviors are identical, as stated by the following proposition which identify the situations in which this occurs.

Proposition 1

1. Let x be a speculators' equilibrium. If C_x is a function, that is if there is a unique market clearing price for each shock, then (x, C_x) is a REE.

   Let (x, P) be a REE. If $P(\theta, \tau_S, \xi)$ is the unique market clearing price for each shock (θ, τ_S, ξ), i.e. $C_x = P$, then (x, P) is a market equilibrium.

2. Let (x, P) be a REE with P continuous. Market clearing prices are unique and (x, P) is a market equilibrium.

Hence, under unique clearing prices, a market equilibrium is a REE and conversely. To see why a speculators' equilibrium with multiple clearing prices does not lead to a REE, let us consider noise trading $\widetilde{\omega}$ only (participation is certain). In Figure 1, an aggregate demand $t_S x(p) + Y(\theta - p) + Z(p)$ is drawn for a given value of θ. Given the realized value ω, clearing prices are found at the intersection of this demand with the vertical line of first coordinate $-\omega$. For a range of noise trading values, there are several clearing prices. Performing a selection of market clearing prices, keeping the speculator's demand unchanged, does not lead to a REE.[6] The reason is that *speculators are no longer fully rational if they know the selection.* The selection process affects the posterior distribution on θ, hence the speculators' behavior. If p' for example is the selected price for the realization of ω, the set of values of (θ, ω) for which p'' is a clearing price is reduced. The reduction changes the information conveyed by the fact that p'' is a clearing price, hence modifies optimal behavior at p''.

Proposition 1 leads us to consider as unstable a situation in which there is no equilibrium that is both a REE and a market equilibrium. On one hand, at a REE, the price function is surely discontinuous. and the speculator's schedule cannot be based on the clearing equation only, since it is not a speculators' equilibrium, but must also rely on a price selection rule supposed to be known. On the other hand, at a market equilibrium, there are surely multiple clearing prices, and the need to select a price creates an uncertainty that is not taken into account in demand formation (or, using the argument above, a known selection modifies behavior).

To investigate stability, we proceed as follows. A speculators' equilibrium is obtained by searching for the equilibrium trades at each price, that is by solving $a = D(p, a)$ for each p. This can be significantly easier than finding a REE, which amounts to solve a fixed price on a functional set. Thus, when searching for an equilibrium that is both a REE and a market equilibrium, it is easier

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[6]The same remark applies if clearing prices are selected according to a known probability distribution.

to start by looking for speculators' equilibria. The property of strategic substitutes trades that we introduce now turns out to be useful.
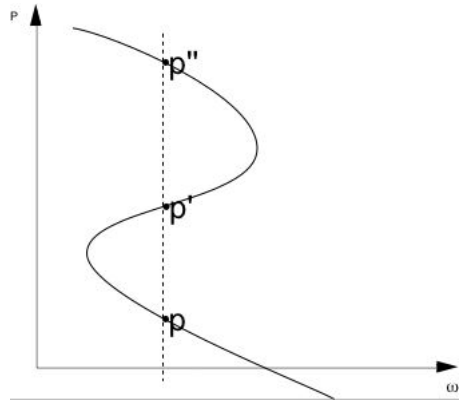

Figure 1: **Multiple equilibria**

Definition *Speculators' trades are said to be strategic substitutes at price p if the demand $D(p,a)$ decreases with speculators' trade a.*

The strategic substitutes property ensures a well behaved equilibrium schedule, as stated in next proposition.

Proposition 2 *If speculators' trades are strategic substitutes at each price, then there is a unique speculators' equilibrium, which is continuous.*

Corollary *If speculators' trades are strategic substitutes at each price, the speculators' equilibrium is the unique candidate for a continuous REE.*

To see whether trades have some chances to be substitutes, observe that given a price p the impact of a is purely informational: other speculators' trade affects optimal trade only through the information conveyed by market clearing (4). In well behaved cases, one might expect that the larger the expected trade a at a given price p, the 'worse' the information that p clears the market. Speculators' trades are then strategic substitutes. as studied in more details below.

That trades are substitutes is good news for stability. Not only the speculators' equilibrium is unique but also it can be reached through various learning or tatonnement processes (Milgrom and Roberts [1990]. See also Desgranges et al. [2003] who focus on the rationalizability of speculators' demand.) When there are several trade equilibria, a speculators' equilibrium selects one particular trade equilibrium at this price. Such a selection implicitly assumes some coordination in traders expectations, suggesting that the equilibrium is difficult to reach.

The corollary follows straightforwardly from Propositions 1 and 2. The market equilibrium is a candidate for a REE but may not be a REE because of the multiplicity of clearing prices. We give an example in Section 5 in which there is no continuous REE although trades are substitutes. Hence, the uniqueness of a speculators' equilibrium demand schedule has to be distinguished from the uniqueness of the clearing prices given a demand schedule.

4   The pure noise trading model

The pure noise trading model refers to the case in which no shock affects uninformed demand except noise trading. Thus, the participation of the speculators is known, denoted by t_S, the Walrasian demand writes as $Z(p) + \widetilde{\omega}$ with $E\widetilde{\omega} = 0$ where $Z(p)$ is a standard deterministic decreasing demand, and the vector of the shocks reduces to (θ, ω).

Before analyzing equilibrium, we study the impact of information on speculators' behavior.

Information and speculators' behavior   Market clearing writes as:

$$Y(\widetilde{\theta} - p) + \widetilde{\omega} = -t_S a - Z(p). \tag{5}$$

The information brought by the fact that p is a clearing price depends on the expectation on other speculators trade a. In the pure noise trading model, this impact is channelled through the quantity $s = -t_S a - Z(p)$, as can be seen from market clearing (5). In other words, knowing p, the information brought by s is the same as the information brought by a. It turns out to be useful to work with variable s instead of a.

Specifically, let us consider the auxiliary problem of a speculator who revises his prior on the net gain $\widetilde{g} = \widetilde{v} - p$ knowing and the value (p, s) and the relationship

$$Y(\widetilde{\theta} - p) + \widetilde{\omega} = s. \tag{6}$$

Let us denote by $f(g|p, s)$ the density of this revised distribution and by $A(p, s)$ the optimal trade conditional on it, i.e. the trade that maximizes expected utility $E[u(a(\widetilde{v} - p)|p, s]$. Demand D is derived from A as follows. Since given price p the information conveyed by a and market clearing (5) is the same as that conveyed by signal $s = -t_S a - Z(p)$ and market clearing (6), the optimal trade $D(p, a)$ is equal to $A(p, -t_S a - Z(p))$.

The benefit of working with the auxiliary program is that p or s have a monotone impact on information and on A as stated in next Lemma. The monotony properties of A depend on the impact of the signals p or s on the conditional distributions of the net gain. This impact is

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unambiguous under mild assumptions.[7] Specifically, increasing the value of one signal, p or s, results in a monotone change in the likelihood ratios. This holds true under the log-concavity of the density function of the shocks. A positive scalar function ψ is said to be *log concave* if $\psi(\epsilon - \epsilon_0)/\psi(\epsilon)$ is increasing with respect to ϵ for any $\epsilon_0 > 0$. Many density functions are log-concave, among them the normal one and the exponential one.

Lemma 1 *Consider the density $f(g|p,s)$ of the net gain $g = v - p$ conditional on (p,s) and the optimal speculator's trade $A(p,s)$ conditional on this distribution. Assume the density of ε_v to be log-concave.*

1. *If the density of ω is log-concave, then, for every p, $g > g'$ implies that the ratio $f(g|p,s)/f(g'|p,s)$ is increasing in s. Demand $A(p,s)$ is increasing in s.*

2. *If the density of θ is log-concave, then, for every s, $g > g'$ implies that the ratio $f(g|p,s)/f(g'|p,s)$ is decreasing in p. Demand $A(p,s)$ is decreasing in p.*

Point *1* is rather natural. Signal s is noisy version of the informed demand. A larger value of the signal is likely to be related with a larger value of informed demand, hence of the net gain. Monotony properties for the speculator's optimal trade A follow from the monotony of the likelihood ratios. (See Fishburn and Porter [1976], who also notice that an increase in the sense first-order stochastic dominance is not enough to guarantee an increase in the optimal demand.)

Point *2* is more tricky. Observe that, given a signal s, increasing the clearing price p has two effects on the posterior distribution of the net gain $\widetilde{v} - p$. There is a pure cost effect, which diminishes the distribution. There is an information effect on $\widetilde{v}$ (more precisely on $\widetilde{\theta}$) due to the knowledge that a higher price clears the market. The information effect is likely to increase the posterior distribution on $\widetilde{v}$. According to point *2*, the cost effect dominates the information effect under the log-concavity assumptions.

Equilibria Thanks to the above analysis on the impact of information and the relationship between A and D, we show that there is a unique market equilibrium, which furthermore leads to a (continuous) REE. This holds even though the speculators' equilibrium schedule may be increasing, as illustrated in example 1 below.

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[7]The impact is clearly not ambiguous in a linear gaussian set up, in which Y is linear and $(\widetilde{\varepsilon_v}, \widetilde{\theta}, \widetilde{\omega})$ is gaussian. Since the posterior distributions of the net gain associated with two different signals s and s' are gaussian with an identical variance, the two signals are ordered through their impact on the conditional expected value of the net gain. The importance of the log-concavity assumption to extend such results is well known and has been used by Milgrom (1981) and Desgranges et al. (2003) for instance.

Proposition 3 *Consider a pure noise trading model. Assume the densities of $\widetilde{\varepsilon}_v$ and $\widetilde{\omega}$ to be log-concave.*

1. *Speculators' trades are strategic substitutes, hence there is a unique speculators' equilibrium x, which is continuous.*

2. *Assume in addition the density of $\widetilde{\theta}$ to be log-concave. If speculators post x, then aggregate demand is strictly decreasing. Hence (x, C_x) is a REE and the only one to be continuous.*

Point *1* follows straightforwardly from the relationships between D and A. Since $D(p, a) = A(p, -t_S a - Z(p))$, trades are substitutes when $A(p, s)$ is increasing in s. This is ensured by the log-concavity assumptions (Lemma 1). The uniqueness and continuity of a speculators' equilibrium schedule then follow from Proposition 2. Observe however that the equilibrium schedule may not be decreasing. To clarify this point, let us assume the differentiability of demands D and A. The equilibrium schedule x satisfies $x(p) = D(p, x(p))$. Differentiation of this relation and of $D(p, a) = A(p, -t_S a - Z(p))$ gives

$$x'(p) = \frac{D_p}{1 + D_a}, D_p = A_p - Z'(p)A_s, \text{ and } D_a = -A_s t_S. \tag{7}$$

Under the assumptions, we have $A_p \leq 0$ and $A_s \geq 0$ (Lemma 1). Thus x is increasing if $D_p = A_p - Z'(p)A_s$ is positive, that is when the Walrasian demand is sufficiently sensitive to prices (this is possible because Z and A are defined independently). In that case, the positive informational effect due to a higher price signal $(-Z'(p)A_s)$ outweighs the negative cost effect (A_p). The reason is that the more responsive to prices the Walrasian demand is, the larger the revision on prior brought by a given variation of the market clearing price.

An increasing speculator's demand could generate a non monotone aggregate demand and multiple clearing prices. This is excluded according to point *2*, as can be seen easily under differentiability of A and D (the general proof is in the proofs section). The inequalities $A_s \geq 0$ and $A_p \leq 0$ imply $t_S x'(p) \leq -Z'(p)$, which ensures that the aggregate demand is decreasing. The uniqueness of a market clearing price easily follows, as well as existence and continuity.

According to Proposition 3, a continuous REE exists in a noise trading model under fairly general assumptions, much less restrictive than the standard constant risk aversion gaussian assumptions. It is interesting to relate this result with the analysis of Gennotte and Leland (1990). In a constant risk aversion gaussian set up with certain participation, they show that multiple clearing prices may occur at a market equilibrium.[8] Contrary to our setup, they however assume

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[8]One can check that the "REE correspondence" they consider coincides with what we call a market equilibrium. Demand functions are computed by using the clearing price equation, and the correspondence of clearing price is derived.

a priori that some investors post a fixed *increasing* demand. As made clear by Proposition 3, this assumption drives their result. In the absence of an exogenous increasing demand, aggregate demand is well behaved. (The interesting point of their analysis is that even a "small" increasing demand may have a large impact on the inference process.)

We illustrate Proposition 3 in a constant risk aversion and gaussian set up. This setting will be extended in next section by introducing uncertain participation.

Example 1. Constant risk aversion and gaussian setup The constant tolerance coefficients, which are known, are denoted by a t, i.e. t_S, t_I, and t_W denote respectively the aggregate risk tolerance coefficient of speculators, informed traders, and Walrasian traders. The vector of the shocks, $(\widetilde{\theta}, \widetilde{\varepsilon}_v, \widetilde{\omega})$, is normally distributed. The standard finance model is an example in which the Walrasian demand is reduced to a pure noise, that is t_W is null.

Informed traders receiving signal θ demand

$$\widehat{t_I}.(\theta - p) \text{ with } \widehat{t_I} = \frac{t_I}{\text{var}\widetilde{\varepsilon}_v}. \tag{8}$$

The Walrasian demand is given by (1) with a known tolerance coefficient t_W. It is convenient for the sequel to introduce θ^* the price at which the demand is null on average so as to write the aggregate Walrasian demand as

$$Z(p) + \widetilde{\omega} \text{ with } Z(p) = t_W(\theta^* - p) \text{ and } E\widetilde{\omega} = 0. \tag{9}$$

To compute the speculators' equilibrium schedule, let us determine demand $D(p, a)$ at price p and speculators' trades a. The left hand side of the market clearing

$$\widehat{t_I}(\widetilde{\theta} - p) + \widetilde{\omega} = s = -[t_S a + t_W.(\theta^* - p)] \tag{10}$$

is a linear combination of $\widetilde{s} = \widehat{t_I}(\widetilde{\theta} - p) + \widetilde{\omega}$. Let ρ be its correlation coefficient with $\widetilde{\theta}$. Under normality assumptions and constant risk aversion, the speculator's demand is proportional to his risk tolerance coefficient so that one may work in terms of the demand 'per unit of risk tolerance'. The demand given (p, a) is $\frac{E[\widetilde{v}|p,a]-p}{var(\widetilde{v}|p,a)}$, which gives

$$D(p, a) = \frac{(1 - \rho)(E\widetilde{\theta} - p) - \rho[t_S a + t_W.(\theta^* - p)]/\widehat{t_I}}{(1 - \rho)\text{var}(\widetilde{\theta}) + \text{var}(\widetilde{\varepsilon}_v)} \text{ with } \rho = \frac{\text{var}(\widehat{t_I}\widetilde{\theta})}{\text{var}(\widehat{t_I}\widetilde{\theta}) + \text{var}(\widetilde{\omega})}. \tag{11}$$

An equilibrium schedule x solves $x(p) = D(p, x(p))$ (where $x(p)$ is the speculator's trade 'per unit of risk tolerance'). The larger the variation in uninformed demand relative to that due to informed traders is, the smaller ρ and the less precise the information are. Consider first the two polar cases that lead to the extreme values 0 or 1 for the correlation coefficient ρ.

Without private information, ρ is null, the equilibrium schedule is given by standard competitive behavior (recall that $\text{var}(\widetilde{v}) = 1$):

$$x^0(p) = E\widetilde{\theta} - p. \tag{12}$$

With private information and no uncertainty on uninformed demand, i.e., on ω, the correlation coefficient ρ is equal to 1. As can be seen from (11) letting ρ equal to 1, D is independent of the prior on θ so that a speculator only follows the market signal. So, the equilibrium schedule x^1 will be called the "herding" demand. Solving $x(p) = D(p, x(p))$, it is given by

$$x^1(p) = \frac{-Z(p)}{t_S + t_I} = \frac{t_W(p - \theta^*)}{t_S + t_I}. \tag{13}$$

The aggregate speculators' demand, $t_S x^1$ is proportional to the (certain) Walrasian *offer*, with a coefficient smaller than 1. Market clearing ensures that the speculator's trade per unit of risk tolerance is identical to that of the informed. To see this, plug $Z(p)$ as a function of $x^1(p)$ into the market clearing equation given θ, $t_S x^1(p) + \widehat{t_I}(\theta - p) + Z(p) = 0$. This yields $x^1(p) = (\widehat{t_I}/t_I)(\theta - p)$, which is equal to $(\theta - p)/var(\varepsilon_v)$, the trade per unit of risk tolerance when completely informed about the true θ.

When information is neither null nor perfect, the equilibrium schedule is explicitly obtained as a convex combination of the competitive demand x^0 and the herding demand x^1 at price p:

$$x(p) = (1 - \lambda)x^0(p) + \lambda x^1(p) \tag{14}$$

where

$$\lambda = \frac{t_S + t_I}{(1/\rho - 1)\widehat{t_I} + t_S + t_I}. \tag{15}$$

The weight on the herding demand, λ, increases with ρ, or equivalently with the precision of information brought by the market clearing price. One checks that the schedule is increasing if

$$t_W \widehat{t_I}\text{var}(\widetilde{\theta}) > \text{var}(\widetilde{\omega}) \text{ or equivalently } t_W t_I \frac{\text{var}(\widetilde{\theta})}{(1 - \text{var}(\widetilde{\theta}))} > \text{var}(\widetilde{\omega}) \tag{16}$$

Inequality (16) is satisfied if the Walrasian demand is sufficiently responsive to price (t_W is large enough), informed traders are sufficiently aggressive ($\widehat{t_I}$ is large enough) and are sufficiently well informed ($\text{var}(\widetilde{\theta})$ is large enough) relative to noise trading $\text{var}(\widetilde{\omega})$. This is in line with the comments following Proposition 3. In particular, the schedule cannot be increasing in the standard model as it assumes an inelastic Walrasian demand, $t_W = 0$.

In all situations, aggregate demand is decreasing in the price, as expected from Proposition 3: whatever the value of λ, the slope of the aggregate speculator's demand is smaller than that of aggregate herding demand ($t_S x^{1'}(p)$ is smaller than t_W). This yields a clearing price that is a linear function of the shock. Thus there is a linear REE. Furthermore it is the only continuous

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REE, since there is a unique market equilibrium. Surely, at another REE, if any, markets clear at several prices for some values of the shocks, which can occur only if a strong non-linearity effect is introduced by the price function.

5 The impact of uncertain participation on equilibrium

This section investigates the impact of uncertain traders' participation. We first analyze speculator's equilibrium. We then show that even if this equilibrium is unique and continuous, it may not lead to a continuous REE.

5.1 Existence and uniqueness of a speculator's equilibrium

To analyze the information conveyed by market clearing and to compare with the case of pure noise trading, it is convenient to write market clearing as

$$Y(\widetilde{\theta} - p) + \widetilde{\omega}(p, a) = -t_S a - E[Z(p, \widetilde{\xi})] \tag{17}$$

in which the 'noise'

$$\widetilde{\omega}(p, a) = (\widetilde{\tau_S} - t_S)a + (Z(p, \widetilde{\xi}) - E[Z(p, \widetilde{\xi})])$$

is the unexpected part of uninformed demand. This noise is affected by the price p (under uncertain Walrasian traders' participation) and by the speculators' trades a (under uncertain speculators' participation).

When the participation of speculators is certain, their trades have no impact on the noise. In that case, the information conveyed by a clearing price p and trade a can be analyzed as in the pure noise trading model, under the appropriate log-concavity assumption on the Walrasian demand. It follows that speculators' trades are substitutes. When the participation of speculators is uncertain, speculators' trades themselves modify the precision of the signal. Speculators' trades may not be substitutes, opening up the possibility of multiple equilibrium schedules as illustrated in example 2 in a mean-variance setting. Trades are however substitutes under some conditions as stated in next proposition.

We first need to consider the existence of a speculator's equilibrium. An equilibrium fails to exist if for some price p, $D(p, a) - a$ is of constant sign, meaning that a speculator has always an incentive to trade more or to trade less than his expectations on other speculators trades. This is excluded under the following weak form of substitutes trades property. Speculators' trades are said to be *weakly strategic substitutes at p if $D(p, a)$ is lower than $D(p, 0)$ for a is positive and larger for a negative*. The property is related to information and is interpreted as follows. $D(p, 0)$

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is the optimal response for a speculator who assumes that only informed and Walrasian traders exchange. If instead speculators are expected to buy, the fact that the same price p clears the market gives a worse signal, making the optimal response $D(p, a)$ lower than $D(p, 0)$. A similar argument can be used for negative a.

Proposition 4 *Assume the density of the Walrasian demand $Z(p, \widetilde{\xi})$ to be log-concave for each p. Then trades are weakly strategic substitutes at each p and there is a speculator's equilibrium.*

Assume in addition the density of $\log \widetilde{\tau_S}$ to be log-concave. Then speculators' trades are substitutes; hence there a unique speculators' equilibrium, which is continuous.

The existence of a speculator's equilibrium easily follows from the weak substitutes property (see lemma 2 in the the proofs section.). With a Walrasian demand given by $\widetilde{\tau_W} Z(p) + \widetilde{\omega}$, the density is log-concave if the density of each variable $\widetilde{\tau_W}$ and $\widetilde{\omega}$ is log-concave. This follows from a result in Miravate [2001] according to which the log-concavity of the density of two independent variables ensures the same property for any linear combination.

5.2 Uncertain Walrasian traders participation

In this section, the speculators' participation is certain and the Walrasian traders' participation is uncertain with a log-concave distribution. Hence, as we have just seen, trades are substitutes and there is a unique speculators' equilibrium, which is continuous. This equilibrium is the unique candidate for a continuous REE, and it is indeed a REE if clearing prices are unique (from Proposition 1). We show here that the uniqueness of clearing prices may fail under some situations that are related to the insurance needs of the Walrasian traders.

For that purpose, let us extend example 1 to incorporate uncertain Walrasian participation $\widetilde{\tau_W}$. Individuals exhibit constant risk aversion and the vector $(\widetilde{\theta}, \widetilde{\varepsilon_v}, \widetilde{\tau_W})$ is normally distributed (the law on τ_W is an approximation since τ_W must be positive; the approximation is however valid if the standard error is small compared to the mean). Under these assumptions, informed traders' demand is given by (8) and the Walrasian demand by $\widetilde{\tau_W}(\theta^* - p) + \widetilde{\omega}$, hence they are both linear in the price. The demand D per unit of risk tolerance is given by the same expression as (11), but with a correlation between the private and market signals that depends on the price p as the (price-dependent) volume of Walrasian trades generate noise. It follows that the speculator's equilibrium schedule is given by the same expression (14), a combination of the competitive demand x^0 and the herding demand x^1, but with a weight λ that varies with the price:

$$x(p) = (1 - \lambda(p))x^0(p) + \lambda(p)x^1(p), \text{ with } \lambda(p) = \frac{t_S + t_I}{(1/\rho(p) - 1)\widehat{t_I} + t_S + t_I}. \tag{18}$$

The equilibrium schedule is explicit but not linear in p, and possibly non monotone. Since the

herding demand is increasing and the competitive one is decreasing, the monotony depends on the behavior of λ, that is on how Walrasian trades affect the precision of information. In the top graph of Figure 2, the speculator equilibrium schedule x (the plain line), the competitive demand (dashed and decreasing) and the herding demand (dashed and increasing) are drawn in the plan (p, a) for the following parameter values: $E[\widetilde{v}] = E[\widetilde{\theta}] = 5$, $\text{var}(\widetilde{\theta}) = 0.4$, $\text{var}(\widetilde{\omega}) = 0$ (no noise trading), $\theta^* = 4.37$, $t_S = t_W = 1$, $\widehat{t_I} = 0.066$. The value θ^* is at one standard error of $E[\widetilde{\theta}]$ and $4\text{var}(\widetilde{\tau_W}) = t_W^2$ (hence the variable τ_W is almost surely positive). Speculator's demand goes through the intersection of the herding and competitive demands (p^c, x^c) and is null at the price θ^* where the Walrasian demand is null. It is useful to consider in more detail these two 'focal' points. They are important in our analysis because multiple clearing prices are possible only when the two points are not too close.

The first focal point is given by the intersection of the competitive and herding demands, (p^c, x^c). Observe that x^c is an equilibrium trade at price p^c whatever value for λ since (p^c, x^c) satisfies $x^c = x^0(p^c) = x^1(p^c)$, hence (18). We show that p^c is the competitive equilibrium price and x^c the competitive speculator's trade per unit of risk tolerance in a market without private information nor shocks. To see this, observe that equation $x^0(p) = x^1(p)$ writes as $E[\widetilde{\theta}] - p = \frac{-Z(p)}{t_S + t_I}$, or equivalently as $(t_S + t_I)(E[\widetilde{\theta}] - p) + Z(p) = 0$, which is the clearing equation in a market without any shock. The values of p^c and x^c are

$$p^c = E[\widetilde{\theta}] - \frac{t_W(E[\widetilde{\theta}] - \theta^*)}{t_S + t_I + t_W}, x^c = \frac{t_W(E[\widetilde{\theta}] - \theta^*)}{t_S + t_I + t_W}.$$

In the absence of insiders, the insurance motives drive the exchanges and determine the risk premium. Here Walrasian traders are insured by other traders who ask for a premium to bear this risk. When Walrasian endowments are positively correlated with the asset for instance, one has $\theta^* < E[\widetilde{\theta}]$ and the price is below the expected value $E[\widetilde{\theta}]$. A market in which the speculator's trade per unit of risk tolerance $|x^c|$ is large is one in which there are large benefits to insure Walrasian traders.

The second focal point is given by $(\theta^*, 0)$ where θ^* is the price at which the Walrasian demand is expected to be null. To simplify the presentation, assume hedging needs to be known, that is $\widetilde{\omega}$ is nil with certainty. Let informed traders receive the signal θ^*. We claim that at $p = \theta^*$ there is a *no trade equilibrium* in which information is completely revealed. The argument follows a standard adverse selection argument. By construction, Walrasian traders do not exchange at that price, so that all transactions take place between informed traders and speculators. None of them have insurance motives. Hence speculators do not trade because otherwise they take the opposite side of informed traders, which is clearly suboptimal. Now, if neither speculators nor Walrasian traders trade, markets clear at $p = \theta^*$ only if informed traders demand is null, which reveals that

they have received the signal θ^*. Information is perfectly revealed.

We show that markets clear at other prices than θ^* under some conditions as stated in next proposition. The intuition is as follows. Speculators' demand is driven by the benefits of trading with Walrasian traders, since as just seen above, they lose against informed traders. Trades with the Walrasian traders are profitable because of their hedging needs and the resulting risk premium. When these motives are large (the volume x^c is large), and the Walrasian traders' participation is uncertain, not much information is transmitted. This explains why multiple clearing prices at possible for θ around θ^* when the two focal points are not too close and the Walrasian traders participation is sufficiently variable.[9] Let us introduce the condition

$$\frac{\text{var}(\widetilde{\tau_W})(\theta^* - E[\widetilde{\theta}])^2}{\widehat{t_I}^2 \text{var}(\widetilde{\theta})} > 4(1 + \frac{t_W + \widehat{t_I}}{t_S})^2. \tag{19}$$

Proposition 5 *In a model with uncertain Walrasian participation, no noise trading (null ω), constant risk aversion and normal variables, consider the unique market equilibrium. There is a unique market clearing price for values of the shocks all equal to their expected values.*

If (19) holds, then there are three clearing prices for signal θ close to the no trade price θ^* and participation τ_W close to its expected value t_W. One price is close to θ^*, trades are small and information is almost revealed to the market. In the others, the price is between θ^* and the prior $E[\widetilde{\theta}]$, speculators and Walrasian trades are large and less information is revealed.

We can thus derive from Proposition 1 that there is no continuous REE when (19) holds.

To understand the condition (19), observe that multiple clearing prices require the aggregate demand to be non monotone, which can happen only if the speculators' demand is increasing, and with a sensitive slope. This can precisely occur for prices around θ^* because of the changing precision of information effect and its impact on the weight $\lambda(p)$. As the price is raised starting from θ^*, information becomes less precise because Walrasian traders start to sell. As a result, the speculators' demand becomes closer to the competitive demand. This induces a sharp increase if (1) the competitive demand is sufficiently large compared to the herding demand around prices close to θ^*, and (2) the precision of the information decreases rapidly enough with the price. These conditions are more likely to be satisfied if the insurance motives are large enough and the Walrasian traders participation is sufficiently variable, i.e. for large $\text{var}(\widetilde{\tau_W})(\theta^* - E[\widetilde{\theta}])^2$, and if informed traders are not too aggressive, i.e. for small $\widehat{t_I}$. Finally, to result in multiple clearing prices, speculators must be sufficiently active, that is t_S must be large enough.

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[9]We look for a robust result, in which multiplicity occurs for plausible values of the shocks, in particular when τ_W is not too far from its mean. Otherwise, when x is not monotone between θ^* and $\bar{\theta}$, one can find a positive value τ_W such that p clears the market if $\theta = \theta^*$. This yields multiplicity since θ^* also clears the market, whatever value for τ_W.
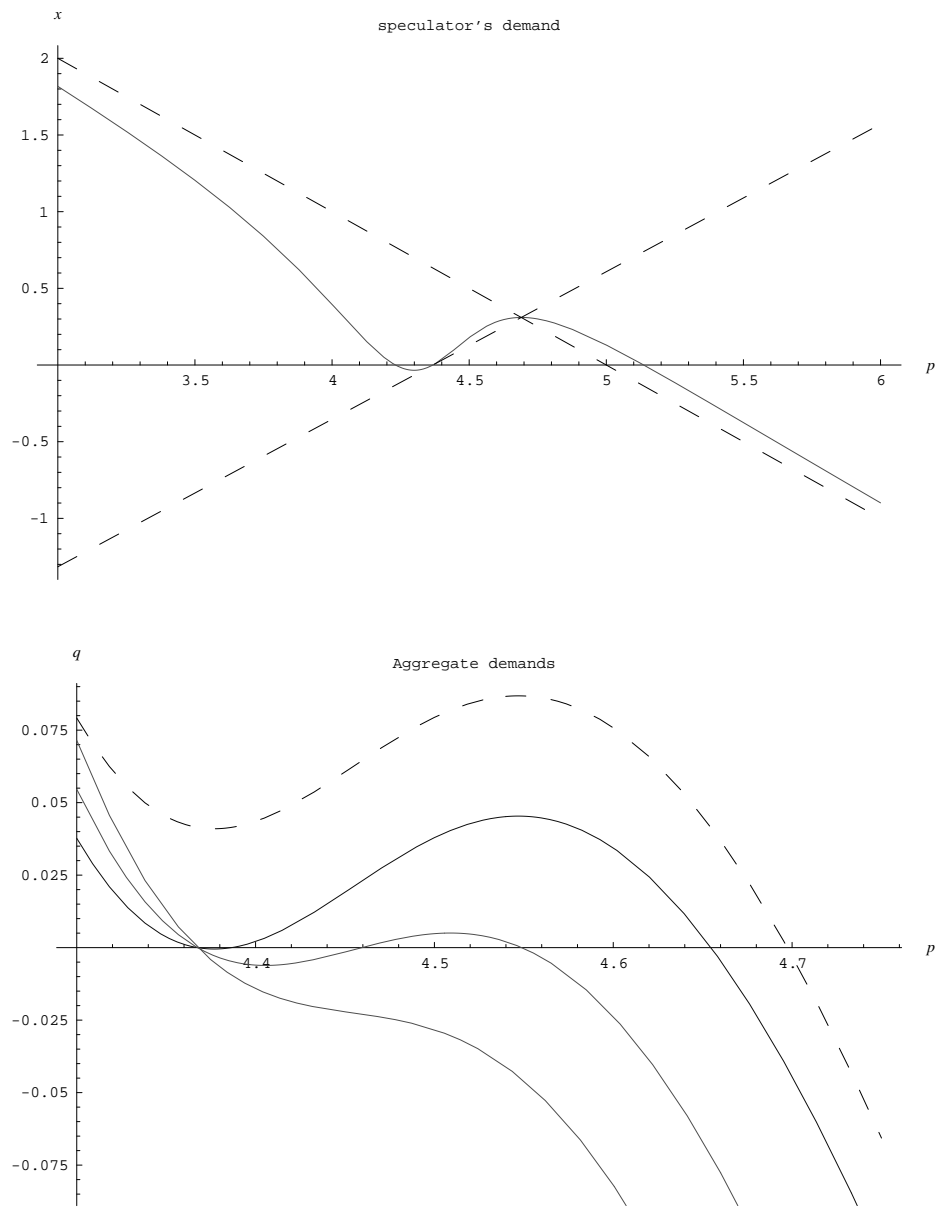
Figure 2: **Speculator's and aggregate demands**

Condition (19) is satisfied at plausible levels of the parameters. For example choose θ^* sufficiently close to $E[\widetilde{\theta}]$ to have some chance to be observed, say take $(\theta^* - E[\widetilde{\theta}])^2 = \text{var}(\widetilde{\theta})$, and the level of the variance $\text{var}(\widetilde{\tau_W})$ low enough for the variable τ_W to be almost surely positive, say $4\text{var}(\widetilde{\tau_W}) = t_W^2$. Then condition (19) writes $t_W^2 > 16\,\widehat{t_I}^2(1 + \frac{t_W + \widehat{t_I}}{t_S})^2$.

In the bottom graph of Figure 2, the aggregate demand schedule is represented for the same parameters as in the top graph and for different values of the shocks. The dashed line is obtained for shocks equal to their expected values $(E[\widetilde{\theta}], t_W)$. The unique clearing price is 4.697. The dark line corresponds to shocks (θ^*, t_W). There are three clearing prices : $\theta^* = 4.37$, 4.386, and 4.655 (the two first equilibrium prices are too close to be distinguished). The third clearing price is very close to the price that clears the market when all shocks are equal to their expected values (4.697): not much information is conveyed because Walrasian trades are large.

The two other lines represent aggregate demand for (θ^*, τ_W) where τ_W is respectively equal to $t_W + 0.5\sqrt{\text{var}(\widetilde{\tau_W})}$ and $t_W + \sqrt{\text{var}(\widetilde{\tau_W})}$. In both cases, Walrasian participation is larger than the expected one, which implies that for $p > \theta^*$ Walrasian traders sell more than expected. This should 'favor' the transmission that the signal is not good. There are still two other clearing prices in addition the no trade equilibrium $\theta^* = 4.37$ when $\theta = \theta^*$ for $\tau_W = t_W + 0.5\text{var}(\widetilde{\tau_W})$ but they disappear for $\tau_W = t_W + \sqrt{\text{var}(\widetilde{\tau_W})}$.

Example 2. Non strategic substitutes trades and multiple equilibrium trades This example is made as simple as possible to show that multiple equilibrium trades are possible at some prices. This implies that trades are not strategic substitutes.

Walrasian demand is certain, denoted simply $Z(p)$. Speculators follow a mean-variance criterion so that the demand per unit of risk tolerance $D(p, a)$ is given by the ratio of the net gain over its variance:

$$D(p,a) \quad = \quad \frac{E[\widetilde{\theta} - p|p, a]}{\text{var}(\widetilde{\varepsilon_v}) + var(\widetilde{\theta} - p|p, a)} \tag{20}$$

in which the conditional moments are determined through the market clearing equation. Given a price p, we analyze the impact of a on the monotonicity of D. To understand better these effects, let us write the relationships between the conditional moments of the net gain and the speculators' participation obtained by the market clearing equation where $\widehat{t_I}$ is equal to 1.

$$E[\widetilde{\theta} - p|p, a] = -Z(p) - E[\widetilde{\tau_S}a|p, a] \text{ and } var(\widetilde{\theta} - p|p, a) = var(\widetilde{\tau_S}a|p, a) \tag{21}$$

Let us assume that τ_S takes two values $\tau_1 > \tau_2$ with equal probability. Given (p, a), the net gain can take on two values, given by $\theta_k - p = -[Z(p) + \tau_k a]$, each with probability $\pi^k(p, a)$ proportional to $\psi_\theta(p - Z(p) - \tau_k a)$ $k = 1, 2$ in which ψ_θ denotes the density of θ. We analyze first the impact of
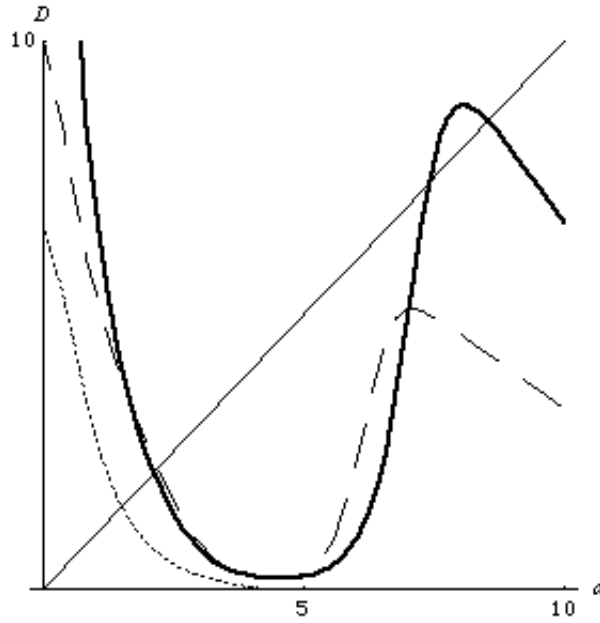
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Figure 3: **Multiple equilibrium trades**

a on the conditional moments of expected aggregate speculators' trade, and then on the demand D by using (20) and (21).

An increase in a starting from 0 decreases the probability π^1 of the high value from $1/2$ to 0. It follows that both moments of the speculators' participation decrease. The expected participation $E[\widetilde{\tau_S}|p, a]$ decreases because the low participation level τ_2 becomes more likely. The variance $var(\widetilde{\tau_S}|p, a)$ decreases as information becomes more precise. These decreases translate into ambiguous effects on the two conditional moments of the aggregate speculators' trade $a\tau_S$. An increase in a has two effects in opposite directions on the expected aggregate trade $aE[\widetilde{\tau_S}|p, a]$: a positive direct effect (reflected by the term a) and a negative information effect on the expected speculators participation. An increase in a has also two effects in opposite directions on the variance of the trade $var(a\widetilde{\tau_S}|p, a) = a^2 var(\widetilde{\tau_S}|p, a)$: an increase due to the larger noise resulting from a larger volume (reflected by the term a^2) and a decreasing effect due to a better information on the speculators' participation.

These effects translate directly on the two conditional moments of the net gain by using (21). As a result, the monotony of demand D is unclear (except that it can be shown that D decreases for a small enough). Figure 3 displays the demand $D(p, .)$ with a positive on the first axis obtained for different parameter values. They are all drawn for $E[\widetilde{v}] = E[\widetilde{\theta}] = 5$, and $p = 4$, at one standard error of the expected value of $\widetilde{v}$. Recall that trades are substitutes at a given price when D is decreasing, and equilibrium trades are found at the intersection with the 45 degree line (so only

positive values of D are drawn). The dotted line represents D when the speculators participation is certain, $t_S = 0.45$. As expected it is decreasing with a and trades are substitutes. The dashed line represents D when τ_S takes the values 0.2, and 0.7 and var$(\widetilde{\theta})$ is equal to 0.8. Trades are not substitutes but there is a unique equilibrium trade. Finally the thick line is obtained by increasing var$(\widetilde{\theta})$ up to 0.9. Now, not only trades are not substitutes but there are multiple equilibria.

5.3 Conclusion

Our analysis suggests that an important ingredient of the equilibrium process in the presence of private information is the knowledge of the sensitivity of demand to prices. In particular, uncertain participation makes information much more difficult to infer than what the standard finance model suggests. Even if a speculators trades are substitutes and their equilibrium behavior is uniquely determined, discontinuities in the selected market clearing price may occur thereby suggesting some form of instability.

6 Proofs

PROOF OF PROPOSITION 1. Given a speculator's demand x, let F be the aggregate excess demand:

$$F(p, \theta, \tau_S, \xi) = \tau_S x(p) + Y(\theta - p) + Z(p, \xi).$$

1. Assume x to be a speculators' equilibrium. By definition $x(p)$ is optimal if the prior on $\widetilde{v}$ is revised conditional on market clearing, i.e. on the set $\{(\theta, \tau_S, \xi), F(p, \theta, \tau_S, \xi) = 0 \}$ or equivalently on $\{(\theta, \tau_S, \xi), p \in C_x(\theta, \tau_S, \xi)\}$. If for every (θ, τ_S, ξ) there is a unique clearing price, this is equivalent to conditioning on $\{(\theta, \tau_S, \xi), p = C_x(\theta, \tau_S, \xi)\}$. Thus (x, C_x) is a REE.

Let (x, P) be a REE for which the market clears at a unique price for each possible value of the shock, $P = C_x$. Then for each p the set $\{(\theta, \tau_S, \xi), p = P(\theta, \tau_S, \xi)\}$ is identical to the set $\{(\theta, \tau_S, \xi), F(p, \theta, \tau_S, \xi) = 0\}$. Thus, the speculator's optimal trade obtained under conditioning on the latter set, $D(p, x(p))$, is equal to the optimal trade obtained under conditioning on the former set, $x(p)$, which proves that x is a speculators' equilibrium.

2. Let (x, P) be a REE with P continuous. By standard arguments, x is continuous as well. We show that $P(\theta, \tau_S, \xi)$ is the unique clearing price for each (θ, τ_S, ξ).

Write the market clearing equation given the demand x as $Y(\theta - p) = -[\tau_S x(p) + Z(p, \xi)]$. Taking the reciprocal of Y (which exists since informed traders' demand Y is strictly decreasing and its range is $\Re$) market clearing can be written as

$$\theta = G(p, \tau_S, \xi)$$

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for a function G that is continuous in p over $\Re$. Since P assigns clearing prices for each shock, we have $\theta = G(P(\theta, \tau_S, \xi), \tau_S, \xi)$ for each (θ, τ_S, ξ). Fix (τ_S, ξ) and consider $P_{|\tau_S, \xi}$ the restriction of P on prices p, and similarly $G_{|\tau_S, \xi}$ the restriction of G on signals θ. Both functions are continuous on $\Re$ and they satisfy $\theta = G_{|\tau_S, \xi}(P_{|\tau_S, \xi}(\theta))$ for each θ in $\Re$. Applying Lemma A1 of the appendix to $g = G_{|\tau_S, \xi}$ and $f = P_{|\tau_S, \xi}$ gives that $G_{|\tau_S, \xi}$ is strictly monotone. This implies that for each τ_S, ξ and each θ there is a unique solution p to $\theta = G_{|\tau_S, \xi}(p)$, that is a unique p for which $\theta = G(p, \tau_S, \xi)$. This proves the uniqueness of clearing prices. ∎

PROOF OF LEMMA 1. The impact of information on v can be derived through that on θ by using standard technics in the statistical literature (see Lemma A2 in Section ??). Recall that $\widetilde{v} = \widetilde{\theta} + \widetilde{\varepsilon_v}$ where $\widetilde{\varepsilon_v}$ is independent of all other variables. Hence the conditional density of v (say conditional on p, s) is obtained by integration of the conditional density of θ. The log-concavity of the density of ε_v ensures that the monotony property of the likelihood ratios are preserved by this integration. From this property it suffices to show the lemma for the likelihood ratios of variable θ instead of v. To avoid introducing new notation we denote $\widetilde{\theta} - p$ by g and its density conditional on (p, s) and market clearing $Y(\widetilde{\theta} - p) + \widetilde{\omega} = s$ by f. Let ψ_θ denote the density of $\widetilde{\theta}$ and similarly ψ_ω that of $\widetilde{\omega}$.

Point 1. Fix p and g, g', $g > g'$. We show that the ratio $f(g|p, s)/f(g'|p, s)$ is increasing in s. This is equivalent to show the same property for the density of s knowing g. We have $f(s|g, p) = \psi_\omega(s - Y(g))$. Since function Y is increasing, the logconcavity of ψ_ω implies that the likelihood ratio $\psi_\omega(s - Y(g))/\psi_\omega(s - Y(g'))$ is increasing in s.

The monotony of demand $A(p, s)$ follows from the monotony of the likelihood ratio property. The result is well known. The first order condition $E[u'(a\widetilde{g})\widetilde{g}|p, s] = 0$ is necessary and sufficient $a = D(p, s)$. We have $g[f(g|s) - f(g|s')\frac{f(g|s)}{f(g|s)}] \geq 0$ for any g if $s' < s$. Multiplying by $u' > 0$ and integrating gives $E[u'(a\widetilde{g})\widetilde{g}|p, s'] \leq E[u'(a\widetilde{g})\widetilde{g}|p, s] = 0$ at $a = D(p, s)$. ∎

Point 2. Let us consider the impact of p on the distribution of $\widetilde{\theta} - p$ knowing (p, s). The density of θ knowing price p and signal s is

$$\frac{\psi_\omega(s - Y(\theta - p))\psi_\theta(\theta)}{\int \psi_\omega(s - Y(\theta - p)\psi_\theta(\theta))d\theta}$$

which yields the density of the net gain $g = \theta - p$ given p, s:

$$f(g|p, s) = \frac{\psi_\omega(s - Y(g))\psi_\theta(g + p)}{\int \psi_\omega(s - Y(g))\psi_\theta(g + p)dg}.$$

Hence the likelihood ratio $f(g|p, s)/f(g'|p, s)$ decreases in p for any $g > g'$ if and only if $\psi_\theta(g + p)/\psi_\theta(g' + p)$ decreases in p, which follows from the log concavity of ψ_θ. ∎

We shall use several times that the weak substitutes property is sufficient to guarantee the existence of a speculator's equilibrium. We state it as a separate lemma.

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Lemma 2 *Assume weak strategic substitutes trades:* $D(p, a) \leq D(p, 0) \leq D(p, a')$ *holds for each* a, a' *with* $a' \leq 0 \leq a$. *Then there exists a speculator's equilibrium. Furthermore, at an equilibrium* x, *the trade* $x(p)$ *is of the same sign as* $D(p, 0)$.

PROOF OF LEMMA 2. D is continuous thanks to assumption A. Under the assumption of Lemma 2, we have $a - D(p, a) \geq a - D(p, 0)$ for $a > 0$ so that $a - D(p, a) > 0$ for a large enough. Similarly $a - D(p, a) \leq a - D(p, 0) < 0$ for $a < 0$ small enough. By continuity of D with respect to a, an equilibrium trade solution to equation $a = D(p, a)$ exists for each p, hence there is a speculator's equilibrium.

We now show that any equilibrium trade at p is of the same sign as that of $D(p, 0)$. Such a trade satisfies $x(p) = D(p, x(p))$. Hence if $x(p) > 0$, then $D(p, 0) \geq D(p, x(p))$ by the assumption on D, which implies $D(p, 0) \geq x(p) > 0$. Similarly $x(p) < 0$ implies $D(p, 0) \leq x(p) < 0$. ■

PROOF OF PROPOSITION 2. If speculators' trades are strategic substitutes, then the inequalities in Lemma 2 are met. Hence there is a speculators' equilibrium. Uniqueness is obvious.

To show continuity, let (p_n) be a sequence converging to p^0 and $a_n = x(p_n)$. Any finite limit point a of the sequence (a_n) satisfies $a = D(p^0, a)$, by continuity of D. So a is equal to $x(p^0)$. It remains to show that no (sub)sequence converges to $+$ or $-\infty$, i.e. that (a_n) is bounded.

Let K be a bound on $|D(p, 0)|$ for p in an interval around p^0 (the bound exists by continuity of D). Since D decreases in a, $a - D(p, a) \geq a - D(p, 0)$ for $a > 0$ hence is positive for $a > K$. Similarly $a - D(p, a) \leq a - D(p, 0)$ is negative for $a < -K$. Thus the unique solution $x(p)$ to $a = D(p, a)$ lies in the interval $[-K, K]$. This proves that the sequence $a_n = x(p_n)$ is bounded. ■

PROOF OF PROPOSITION 3.

Point *1.* The property that trades are strategic substitutes follows directly from Lemma 1. Since A increases in s, the relationship $D(p, a) = A(p, -\tau_S a - Z(p))$ implies that D decreases in a. Applying Proposition 2 gives then that there is a unique speculators' equilibrium, which is continuous.

Point *2.* We first show that the aggregate expected uninformed demand, $t_S x + Z$, is strictly decreasing. The speculator's equilibrium x satisfies $x(p) = A(p, s)$ where $s = -[t_S x(p) + Z(p)]$. Assume by contradiction that for some $p > p'$, $t_S x(p) + Z(p) \geq t_S x(p') + Z(p')$. In that case the two associated signals s and s' satisfy $s \leq s'$. Under the assumptions of log-concavity, $p > p'$ and $s \leq s'$ imply $A(p, s) < d(p', s')$, or $x(p) < x(p')$. Since competitive demand Z is not increasing in p, we get $t_S x(p) + Z(p) < t_S x(p') + Z(p')$, which gives the contradiction.

The monotonicity of aggregate demand implies that there is at most one clearing price. Hence C_x is a continuous function and (x, C_x) is a REE. ■

PROOF OF PROPOSITION 4. Let p be given. To simplify notation denote by $\widetilde{o}$ the Walrasian offer

$-Z(p, \widetilde{\xi})$ at that price. We consider the distribution of θ knowing the relationship

$$Y(\widetilde{\theta} - p) + \widetilde{\tau_S}a = \widetilde{o}$$

when a varies. Denote by ψ_θ the density of $\widetilde{\theta}$, by ψ_τ that of τ_S, and by ψ_o that of $\widetilde{o}$. The density of θ conditional on a is given by

$$f(\theta|a) = \frac{1}{K_a} \int \psi_\tau(\tau)\psi_o(Y(\theta - p) + \tau a))d\tau, \ K_a = \int \int \psi_\tau(\tau)\psi_o(Y(\theta - p) + \tau a))\psi_\theta(\theta)d\tau d\theta, \ (22)$$

(a) Assume that the distribution of $\widetilde{o}$ is log concave. We first show that the ratio $\frac{f(\theta|a)}{f(\theta|0)}$ decreases in θ for $a > 0$. From (22) we have

$$\frac{f(\theta|a)}{f(\theta|0)} = K_0/K_a \int \psi_\tau(\tau)[\frac{\psi_o(Y(\theta - p) + \tau a))}{\psi_o(Y(\theta - p))}]d\tau.$$

For each $\tau > 0$ the function inside the square brackets decreases in θ because ψ_o is log-concave and $Y(\theta - p)$ increases. Hence the ratio $\frac{f(\theta|a)}{f(\theta|0)}$ decreases in θ. This implies the inequality $D(p, a) \leq D(p, 0)$ for $a > 0$. A similar argument yields that the ratio increases for $a < 0$, hence $D(p, a) \geq D(p, 0)$. Thus trades are weakly strategic substitutes. A speculator's equilibrium exists by Lemma 2.

(b) Assume in addition that the density ψ_t of $t = \log \tau$ is log concave. To show that trades are strategic substitutes if the ratio $f(\theta|a')/f(\theta|a)$ decreases with θ for $0 < a < a'$ or $a < a' < 0$. Operating the change of variable $\tau'a = \tau a'$ in the integral (22) defining $f(\theta|a')$ gives

$$f(\theta|a') = a'/(aK_{a'}) \int \psi_\tau((\tau'a/a')\psi_o(Y(\theta - p) + \tau'a))d\tau'.$$

Thus, setting $K = aK_a/(a'K'_a)$

$$\frac{f(\theta|a')}{f(\theta|a)} = K \int R(\tau) \left[\frac{\psi_\tau(\tau)\psi_o(Y(\theta - p) + \tau a)}{\int \psi_\tau(\tau)\psi_o(Y(\theta - p) + \tau a)d\tau}\right] d\tau \ \text{where} \ R(\tau) = \frac{\psi_\tau(\tau a/a')}{\psi_\tau(\tau)} \quad (23)$$

Observe first that function R is an increasing function for $\lambda = a/a' < 1$. To see this, note that the densities of τ and $\log(\tau)$ are related by $\psi_\tau(\tau) = \psi_{\log \tau}(log(\tau))/\tau$. Hence the ratio $\psi_\tau(\lambda\tau)/\psi_\tau(\tau)$ varies as increases with τ under the assumption that $\psi_{\log \tau}$ is log-concave and λ is smaller than 1.

Consider first the case $0 < a < a'$. From the just above property, the function R is an increasing function. The integral in (23) is the expectation of R under a distribution whose density is the term inside the square brackets. Denote this density $h(\tau|\theta)$. The expectation is decreasing in θ if the ratio $h(\tau|\theta)/h(\tau|\theta')$ decreases in τ for $\theta > \theta'$. The ratio is proportional to $\psi_o(Y(\theta - p) + \tau a))/\psi_o(Y(\theta' - p) + \tau a))$. It is indeed decreasing in τ for $a > 0$ because ψ_o is log-concave and $Y(\theta - p)$ is larger than $Y(\theta' - p)$.

In the case $a < a' < 0$, monotony properties for R and the likelihood ratio are reversed: the function R decreases because $\lambda = a/a' > 1$and the ratio is increasing in τ because $a < 0$. This gives again that the expectation of R hence the left hand side of (23) decreases with θ. ∎

PROOF OF PROPOSITION 5. For realized shocks equal to their expected values, a market clearing price p does not modify the expected value on θ but reduces its variance. Thus speculators' demand is proportional to $E[\widetilde{\theta}] - p$ as that of the informed traders (because by assumption they have received the signal $E[\widetilde{\theta}]$). Since both types of traders are on the same side of the market, they exchange with Walrasian traders at a price p which must be between $E[\widetilde{\theta}]$ and θ^*. Take for example $E[\widetilde{\theta}] > \theta^*$. Let us assume by contradiction that there are two distinct market clearing prices p and p', say with $p > p'$. Then $E[\widetilde{\theta}] > p > p' > \theta^*$ implies that the expected gain at p is smaller than at p' and that the uncertainty is larger because the volume of the Walrasian trades is larger at p than at p': surely $x(p) < x(p')$. But then all demands are smaller at p than at p': both prices cannot clear the market.

Noise trading ω is assumed to be null so that $Z(p, \tau_W) = \tau_W(\theta^* - p)$. We find conditions under which there are three clearing prices for the realizations $\theta = \theta^*$ and $\tau_W = t_W$. By continuity, there are also three solutions for shocks close enough to these values. A speculator's equilibrium trade $a = x(p)$ at p satisfies

$$a = (1 - \lambda(p, a))(E[\widetilde{\theta}] - p) + \lambda(p, a)\frac{t_W(p - \theta^*)}{t_S + t_I}, \tag{24}$$

with

$$\lambda(p, a) = \frac{t_S + t_I}{r_w(p - \theta^*)^2\widehat{t_I} + t_S + \tau_y}, r_w = \frac{\operatorname{var}(\widetilde{\tau_W})}{\widehat{t_I}^2\operatorname{var}(\widetilde{\theta})}. \tag{25}$$

The clearing equation for $\theta = \theta^*$ and $\tau_W = t_W$ writes as

$$(p - \theta^*) = \gamma x(p), \text{with } \gamma = \frac{t_S}{t_W + \widehat{t_I}} \tag{26}$$

Replacing $p - \theta^*$ by γa into (24) gives the equation to be solved by $a = x(p)$:

$$a = (1 - \Lambda(a))(E[\widetilde{\theta}] - \theta^* - \gamma a) + \Lambda(a)\frac{t_W\gamma a}{t_S + t_I}, \tag{27}$$

where $\Lambda(a)$ denotes the value of $\lambda(p, a)$ when $p - \theta^*$ is equal to γa:

$$\Lambda(a) = \frac{t_S + t_I}{\widehat{t_I}r\gamma^2a^2 + t_S + t_I}. \tag{28}$$

Since $\Lambda(0) = 1$, the no trade equilibrium $a = 0$ is a solution to (27), as expected. Dividing (27) by a, multiplying by $\widehat{t_I}r_w\gamma^2a^2 + t_S + t_I$ and rearranging terms gives a second degree equation in a with coefficients respectively equal to

$$A = r_w\gamma^2\widehat{t_I}(1 + \gamma), B = -r\widehat{t_I}(E[\widetilde{\theta}] - \theta^*), C = t_S + t_I - t_W\gamma = \widehat{t_I}(1 + \gamma var(\varepsilon_v))$$

The equation admits two solutions if $r_w\gamma^2(E[\widetilde{\theta}] - \theta^*)^2 \geq 4(1 + \gamma)(1 + \gamma var(\varepsilon_v))$. This condition is surely satisfied if $r_w\gamma^2(E[\widetilde{\theta}] - \theta^*)^2 \geq 4(1 + \gamma)^2$.. Dividing by γ^2, using the expression of r_w given

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in (25) and $\gamma = \frac{t_S}{t_W + \widehat{t_I}}$ yields

$$\frac{\mathrm{var}(\widetilde{\tau_W})(\theta^* - E[\widetilde{\theta}])^2}{\widehat{t_I}^2 \mathrm{var}(\widetilde{\theta})} > 4(1 + \frac{t_W + \widehat{t_I}}{t_S})^2$$

which is condition (19). ∎

Appendix: Lemma A1 and A2

Lemma A1 *Let $g(.)$ be a continuous function on $\Re$. If there is f continuous on $\Re$ such that $\theta = g(f(\theta))$ for each θ, then g is strictly monotone over $\Re$.*

PROOF. (This Lemma must be well known, but I did not find a reference.) The identity $\theta = g(f(\theta))$ for each θ in $\Re$ implies that f is one-to-one on $\Re$. Hence f is strictly monotone by using a well known result on continuous scalar functions. Let $\mathcal{I} = f(\Re)$ be the range of f and $g_{|I}$ be the restriction of g on the interval $\mathcal{I}$. The identity $\theta = g_{|I}f(\theta)$ holds for each θ on $\Re$ and composition with f gives $p = f(g_{|I})(p)$ for each $p \in \mathcal{I}$. Thus $g_{|I}$ is one-to-one and is the reciprocal of f. Since $g_{|\mathcal{I}}$ is continuous (because g is assumed to be continuous) and defined on the interval $\mathcal{I}$, $g_{|\mathcal{I}}$ is strictly monotone using the same result as mentioned above. Hence it suffices to show that $\mathcal{I}$ is the whole set $\Re$ in order to prove that g is strictly monotone over $\Re$. By contradiction, let us assume $\mathcal{I}$ to be bounded, from above or below, by a value m. Then the inequality $g_{|\mathcal{I}}(\theta) \leq g(m)$ or the reverse one holds for each θ because $g_{|\mathcal{I}}$ is monotone and g is continuous over $\Re$. Hence the range $g_{|\mathcal{I}}(\mathcal{I})$ is bounded. But then the identity $\theta = g_{|I}f(\theta)$ cannot hold, which gives the contradiction. ∎

The next lemma follows from a general result in the statistical literature (Ahlswede and Daykin [1979]). I give a direct proof here. It allows us to derive monotonicity properties of the likelihood ratio of v from those of θ.

Lemma A2 *Let f and h denote the densities of θ and v conditional on some (possibly multidimensional) signal α independent on ε_v.*

$$f(\theta|\alpha)f(\theta'|\alpha') \geq f(\theta'|\alpha)f(\theta|\alpha') \ \text{for all } \theta > \theta', \alpha > \alpha' \tag{29}$$

$$\text{imply } h(v|\alpha)h(v'|\alpha') \geq h(v'|\alpha)h(v|\alpha') \ \text{for all } v > v', \alpha > \alpha' \tag{30}$$

PROOF. We have $h(v|\alpha) = \int f(\theta|\alpha)\psi_{\varepsilon_v}(v - \theta)d\theta$. The inequality (30) may be written as

$$\int \int f(\theta|\alpha)f(\theta'|\alpha')\psi_{\varepsilon_v}(v - \theta)\psi_{\varepsilon_v}(v' - \theta')d\theta d\theta' \geq \int \int f(\theta'|\alpha)f(\theta|\alpha')\psi_{\varepsilon_v}(v' - \theta)\psi_{\varepsilon_v}(v - \theta')d\theta d\theta'$$

Consider separately the values $\theta > \theta'$ and $\theta < \theta'$. Exchanging the variable of integration for $\theta < \theta'$, the first integral over $\theta < \theta'$ writes as $\int \int_{\theta > \theta'} f(\theta'|\alpha)f(\theta|\alpha')\psi_{\varepsilon_v}(v - \theta')\psi_{\varepsilon_v}(v' - \theta)d\theta d\theta'$. Operating the same change in the second integral and rearranging, (30) can be written as

$$\int \int_{\theta > \theta'} [f(\theta|\alpha)f(\theta'|\alpha') - f(\theta'|\alpha)f(\theta|\alpha')][\psi_{\varepsilon_v}(v - \theta)\psi_{\varepsilon_v}(v' - \theta') - \psi_{\varepsilon_v}(v - \theta')\psi_{\varepsilon_v}(v' - \theta)]d\theta d\theta' \geq 0$$

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The first term inside square brackets is nonnegative by (29) and the second one is nonnegative as well by logconcavity of ψ_{ε_v}. ∎

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