Testing Weak Form Efficiency on the Toronto Stock Exchange*

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Abstract

We believe that in order to test for weak form efficiency in the market a vast pool of individual stocks must be analyzed rather than a stock market index. In this paper, we use a model-based bootstrap to generate a series of simulated trials and apply a modified chart pattern recognition algorithm to all stocks listed on the Toronto Stock Exchange (TSX). We compare the number of patterns detected in the original price series with the number of patterns found in the simulated series. By simulating the price path we eliminate specific time dependencies present in real data, making price changes purely random. Patterns, if consistently identified, carry information which adds value to the investment process, however, this *informativeness* does not guarantee profitability. We draw conclusions on the relative efficiency of some sectors of the economy. Although, we fail to reject the null hypothesis of weak form efficiency on the TSX, some sectors of the Canadian economy appear to be less efficient than others. In addition, we find negative dependency of pattern frequencies on the two moments of return distributions, variance and kurtosis.

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Abstract

We believe that in order to test for weak form efficiency in the market a vast pool of individual stocks must be analyzed rather than a stock market index. In this paper, we use a model-based bootstrap to generate a series of simulated trials and apply a modified chart pattern recognition algorithm to all stocks listed on the Toronto Stock Exchange (TSX). We compare the number of patterns detected in the original price series with the number of patterns found in the simulated series. By simulating the price path we eliminate specific time dependencies present in real data, making price changes purely random. Patterns, if consistently identified, carry information which adds value to the investment process, however, this *informativeness* does not guarantee profitability. We draw conclusions on the relative efficiency of some sectors of the economy. Although, we fail to reject the null hypothesis of weak form efficiency on the TSX, some sectors of the Canadian economy appear to be less efficient than others. In addition, we find negative dependency of pattern frequencies on the two moments of return distributions, variance and kurtosis.

I Introduction and Literature Review

Technical analysis is a financial market technique that claims the ability to forecast the future direction of security prices through the study of past market data, primarily price and volume. Technical analysts may employ models and trading rules based on price transformations, moving averages, regressions, inter-market and intra-market price correlations, cycles or recognition of chart patterns.

The patterns in market prices are assumed to recur, and thus, these patterns can be used to predict future price movements. Critics argue that these patterns are simply random effects on which analysts

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impose causation, and bear no useful information, especially in the long term. Nonetheless, about 30 to 40 percent of practitioners appear to believe that technical analysis is important in determining price movements in shorter time horizons which last up to six months[1].

Taylor and Allen (1992), based on a survey among foreign exchange dealers in London, found that at least 90 per cent of respondents place some weight on technical analysis. In addition, the results of this survey revealed preference for technical, rather than fundamental, analysis at shorter time horizons. Lui and Mole (1998) report the results of a similar survey conducted in 1995 among foreign exchange dealers in Hong Kong. They found that over 85 per cent of respondents rely on both fundamental and technical analysis and, again, technical analysis was more popular at shorter time horizons.

Technical analysis relies on past market data to predict future movements and, thus, contradicts the weak form of the efficient market hypothesis. If historical price (and volume) data may be used to predict future movements of market prices, the market is said to be weak form inefficient.

The Efficient Market Hypothesis is one of the most important and widely disputed propositions in finance. The claim is that prices fully reflect all available information in the market and any forecasting of future price changes therefore is purely speculative. There is what Lo and MacKinlay (Lo and MacKinlay (1999), p.4) call "a wonderfully counter-intuitive and seemingly contradictory flavor" to the idea of informationally efficient markets: the greater the number of participants, the better their training and knowledge and the faster the dissemination of information, the more efficient a market should be; and "the more efficient the market, the more random the sequence of price changes generated by such a market, and the most efficient market of all is one in which price changes are completely random and unpredictable".

If everyone believes the market is efficient, it will no longer be efficient since no one will invest actively. In effect, efficient markets depend on investors believing that the market is inefficient and trying to beat it. In reality, markets should neither be strictly efficient nor strictly inefficient. The question is one of a degree - some markets are *relatively* more efficient than others.

We believe that in order to test for weak form efficiency in the market a vast pool of individual stocks must be analyzed rather than a stock market index. In this paper, we use a model-based bootstrap to

[1] Park and Irwin (2004) pp 1-2.

generate a series of simulated trials and apply a modified chart pattern recognition algorithm to stocks listed on the Toronto Stock Exchange (TSX), Canada's largest stock market. We compare the number of patterns detected in the original price series with the number of patterns found in the simulated series. By simulating the price path we eliminate specific time dependencies present in real data and price changes are thus purely random. Patterns, if consistently identified, carry information which adds value to the investment process, however, this *informativeness* does not guarantee profitability. We draw conclusions on the relative efficiency of particular sectors of the economy. If the number of patterns identified in the simulated series is the same as in the real price data, technical analysis cannot be gainfully applied and the weak form of the efficient market hypothesis cannot be rejected.

Park and Irwin (2007) provide a comprehensive review of technical analysis studies. They categorize the empirical literature into two groups, 'early' (1960-1987) studies and 'modern' (1988-2004) studies, based on an overall evaluation of each study in terms of the number of technical trading systems considered, treatment of transaction costs, risk, data snooping problems, parameter optimization, out-of-sample verification, and statistical tests adopted. 'Modern' studies are further classified into seven groups on the basis of differences in testing procedures. Park and Irwin (2004, 2007) provide general information about each of these groups. '*Standard*' refers to studies that include parameter optimization and out-of-sample tests, adjustment for transaction costs and risk, and statistical tests. '*Model-based bootstrap*' represents studies that conduct statistical tests for trading returns using the model-based bootstrap approach introduced by Brock *et al.* (1992). '*Reality check*' and '*genetic programming*' indicate studies that attempt to solve data snooping problems using White (2000)'s bootstrap reality check methodology and the genetic programming technique introduced by Koza (1992). '*Non-linear*' indicates studies that apply non-linear methods such as feed-forward neural networks or nearest neighbour regressions to recognize patterns in prices or estimate the profitability of technical trading rules. '*Chart patterns*' refers to studies that develop and apply recognition algorithms for chart patterns as in Lo *et al.* (2000); Dawson and Steeley (2003). Finally, '*other*' refers to studies that do not fit neatly in any of the previous categories.

In general, the 'early' studies showed limited evidence of the profitability of technical trading rules when applied to stock markets and thus supported market efficiency. In contrast, among a total of 95 'modern' studies reviewed in Park and Irwin (2007), 56 studies find positive results regarding technical trading strategies, while 39 studies indicate mixed or negative results[2].

[2] For a complete annotated summary of all studies, see Park and Irwin (2004).

In an influential study, Brock *et al.* (1992) use a very long price history (from 1897 to 1986) and, for the first time, apply the model-based bootstrap approach to draw statistical inference on technical trading profits. In their approach, returns conditional on buy or sell signals from the original series are compared to conditional returns from simulated return series generated by widely used models for stock returns. Results indicate that buy (sell) signals from the technical trading rules generate positive (negative) returns across all 26 rules and four sub-periods tested. All the buy–sell differences are positive and outperform the buy-and-hold strategy. It should be noted, however, that their results have recently been challenged by Sullivan *et al.* (1999). These authors argue that trading rules are subject to selection bias as only those that have been perceived to perform well continue to be examined. If these trading rules are only a small subset of all trading rules available, then almost certainly some trading rules will appear to outperform.

The paper by Lo *et al.* (2000) is one of the first papers to automate a process of chart pattern recognition. The authors identify 10 reversal patterns based on a set of consecutive local extrema points that would fit a particular geometrical form. These authors apply their methodology to a large set of stocks traded on the NYSE/AMEX and NASDAQ during the 1962-1996 period as well as the market indices on these U.S. exchanges. To support claim that the technical patterns do provide incremental information, the authors perform a goodness-of-fit test to compare the quantiles of returns conditioned on these technical patterns with those of unconditioned returns.

Dawson and Steeley (2003) replicate and extend the work of Lo *et al.* (2000) using data on the UK stock market and the same set of reversal patterns. In addition, when comparing whether the same patterns found for the US market also exist in the UK market and whether returns distributions are influenced by them, they find that, overall, the frequency of patterns in both markets is very similar. However, different patterns occur with different frequencies within the UK market and in different relativities to the frequencies found in the US market. Similar to Lo *et al.* (2000), they find that the distributions of returns conditioned on these technical patterns can be significantly different from unconditional returns distributions. It should be noted, however, that the means of conditional and unconditional returns are not significantly different from each other, yet the distributions are statistically different from each other. This may be due to differences in higher order moments of these distributions.

Successful pattern identification relies on past price performance. Technical analysis suggests that a particular price pattern can be recognized and a future price can be predicted based on past price(s). Thus, by simulating series using a model-based bootstrap we eliminate specific time dependencies present in real data and any pattern identified is thus purely random. If the number of patterns identified in the simulated series is the same as in real price data, then technical analysis cannot be gainfully applied and the weak form of the efficient market hypothesis cannot be rejected.

Our methodology is to find the number of reversal patterns in the price time series of a chosen asset. We then generate a number of random time series (in particular we construct an asset price path with the same distribution characteristics as the underlying asset) and find the number of reversal patterns in the simulated data. Comparing the results from the original and simulated series enables us to draw inferences on weak form efficiency in the market and its sectors. The weak form efficiency hypothesis will be rejected in the event of significantly larger number of reversal patterns in the real price series than in the simulated series.

II The Data

Our data consist of daily closing prices (adjusted for splits and dividends) for 1336 Toronto Stock Exchange (TSX) securities. The time period covered is June 1983 (where available) through June 2008 - a span of 25 years.

Data were obtained from Datastream, Thomson-Reuters' financial statistical database. Each of the security listed is categorized into one of the thirty eight sectors of the Canadian economy. We construct ten additional categories of securities: stocks listed on the TSX Composite Index[3]; stocks listed on each of the nine iShares ETFs. Our assertion is that securities listed in these indices are more likely to be followed by a large number of market participants, resulting in the faster dissemination of information, thus increasing efficiency.

We adopt the following notation throughout the report:
$P_{i,t}^{(R)}, t = 0..T_i$ - adjusted daily close prices for stocks listed on the TSX, where i refers to each individual

[3]The S&P/TSX Composite Index is composed of the largest companies on the Toronto Stock Exchange as measured by market capitalization. The Toronto Stock Exchange-listed companies in this index comprise about 71% of the market capitalization for all Canadian-based companies listed on the TSX. The number of securities listed in the S&P/TSX Composite Index as of June 28, 2008 was 251. The total number of stocks listed on the Toronto Stock Exchange on June 28, 2008 was 1336.

security, and *(R)* denotes original data series.

$P_{i,t}^{(S)}, t = 0..T_i$ - adjusted daily close prices for stocks listed on the TSX, where i refers to each individual security, and *(S)* denotes simulated data series.

$P_{i,0}^{(R)}$ - beginning or base price

$r_{i,t}^{(R)}, t = 0..T_i$ - log return series for security i where T_i is the number of return observations for security i.

III Methodology

A Null Models: Data Generating Process

In this study we aim to answer the following question: is the number of reversal patterns identified in the real time series of stock prices significantly greater than the number of patterns in the simulated series?

The general goal of technical analysis is to identify regularities in the time series of prices by extracting nonlinear patterns from noisy data. In order to perform a statistical inference, we would have to compare the number of patterns identified in the original series with the simulated ones, based on a particular model for generating the simulated series. One could argue that the results of this comparison depend on the data generation process used in simulating the price series. Patterns uncovered by technical rules might not be explained by autocorrelation or changes in volatility. Thus, in this study, we use three null data generating models, each tailored to capture specific data characteristics present in financial returns. Well known deviations from assumed distributions of returns (leptokurtosis, autocorrelation, conditional heteroskedasticity, changing conditional mean) will be addressed by generating simulated series from the following null models for stock returns: (i) random walk with a drift; (ii) ARMA(p,q); and (iii) EGARCH(p,q).

Random walk with a drift This model is particularly popular in the finance literature. With the random walk with drift model time series are simulated by taking the returns from each of the 1336 stock return series and sampling them with replacement. Simulated samples will have the same drift in prices and the same volatility as the original series. Returns will be independent and identically distributed by construction. However, one of the downsides of this conventional approach is that it assumes identical and independently distributed returns across time. It is evident, however, that such distributions tend to underestimate serial correlation and volatility clustering normally present in financial time series data.

The model-based bootstrap methodology following Brock *et al.* (1992) is detailed below:

$$p_{i,t} - p_{i,t-1} = \mu_i + \varepsilon_{i,t}$$

$$r_{i,t} = \mu_i + \varepsilon_{i,t}$$

$$\widehat{\varepsilon}_{i,t} = r_{i,t} - \widehat{\mu}_i$$

$$r_{i,t}^{(S)} = \widehat{\mu}_i + \hat{e}_{i,t}$$

where $p_{i,t} = \ log\,(P_{i,t})$, and $\hat{e}_{i,t}$ is sampled with replacement from $\widehat{\varepsilon}_{i,t}$.

ARMA(p,q) Our data reveal the presence of significant moving average, and, in some instances, autoregressive component. Due to the presence of these effects we apply ARMA(p,q) as a null model to obtain simulated series. In this procedure a model is fitted to each of the 1336 original return series to obtain estimated parameters and residuals. The estimated residuals are then re-sampled with replacement and used with the estimated parameters to form a new representative series. Applying this procedure, the residuals are not restricted to a particular distribution, and at the same time the data generating process preserves autocorrelation and moving average properties of the underlying series.

$$r_t = \alpha + \sum_{i=1}^{p} \phi_i r_{t-i} + + \sum_{j=1}^{q} \theta_i \varepsilon_{t-j} + \varepsilon_t$$

The parameters $p,\ q,\ \phi,\ \theta$ are chosen to minimize Akaike information criterion with a second order correction for small sample sizes (AICc).

EGARCH(p,q) In addition, to account for heteroskedasticity present in the financial return series, we need to model the conditional variance equation to account for volatility clustering. The most popular models in the finance literature are ARCH and GARCH models[4]. However, both ARCH and GARCH models do not address the leverage effect (or asymmetry) present in the return data and first discovered by Black (1976). This effect occurs when an unexpected drop in price (bad news) increases predictable volatility more than an unexpected increase in price (good news) of similar magnitude. This makes a symmetric, constrained on conditional variance, function in past error terms inappropriate.

[4]Bollerslev (1986); Bollerslev *et al.* (1992)

Exponential GARCH has been proposed to capture this negative correlation between asset returns and volatility (Nelson (1991)). EGARCH is able to capture most of the asymmetry (Engle and Ng (1993)), however, the parametric nature of GARCH models makes it hard to capture highly irregular phenomena such as market crashes, subsequent rebounds and other structural changes.

The mean equation used in our null model is:

$$r_t = \alpha + \gamma g(\sigma_t) + \theta\varepsilon_{t-1} + \varepsilon_t$$

$$g(\sigma_t) = e^{h_t}, \quad \varepsilon_t = e^{1/2h_t} z_t, \quad h_t = \log\sigma_t^2$$

and the conditional variance equation:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \frac{|z_{t-i}| + \gamma_i z_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q} \beta_j h_{t-j}$$

$$z_t \sim N(0,1)$$

if $z_{t-i} > 0$ the total effect is $(1+\gamma_i) z_{t-i}$ and if $z_{t-i} < 0$ the total effect is $(1-\gamma_i)|z_{t-i}|$, meaning that bad news will have a bigger impact on volatility than good news. Parameters p, q are selected using Bayesian information criterion (BIC).

B Model-based bootstrap

Using the bootstrap method allows us to estimate confidence intervals for the means and standard deviations of chart pattern frequencies, which in turn, enables us to obtain a more rigorous insight into the riskiness of these chart patterns. We use the following approach for our bootstrap resampling:

1. Model parameters and the residuals are estimated from original return series.

2. The residuals are then re-sampled with replacement

3. The null model is then used to generate the simulated return series using parameter estimates and scrambled residuals to obtain $r_{i,t}^{(S)}$, where *(S)* denotes the simulated series .

4. The base price is then used $P_{i,0}^{(S)} \equiv P_{i,0}^{(R)}$ together with $r_{i,t}^{(S)}$ to derive the simulated price series $P_{i,t}^{(S)}$.

C Prices: Pattern Recognition

Following Lo *et al.* (2000) we study 10 chart patterns, namely head-and-shoulders (HS), inverse head-and-shoulders (IHS), broadening tops (BT), broadening bottoms (BB), triangle tops (TT) triangle bottoms (TB), rectangle tops (RT), rectangle bottoms (RB), falling wedge (FW) and rising wedge (RW).

These reversal patterns can be recognized based on five consecutive local extrema in the price path and conditioned on the volume information corresponding to these 5 local extrema[5]. Blume *et al.* (1994) show that volume provides some insight on information quality that otherwise cannot be deduced from price alone. However, according to Lo *et al.* (2000), volume trends appear to provide little incremental information with only a few exceptions. These authors found that the difference between the conditional distributions of increasing and decreasing volume trends was statistically insignificant for most patterns in both NYSE/AMEX and NASDAQ markets.

D Data Smoothing

To be able to successfully identify chart patterns we require a vector of local extrema for each price series. Since the price series are not differentiable functions we use the cubic B-spline method to smooth the data to locate the vector of local extrema. We model the observed price time series as $P_{it} = f_i(t) + \varepsilon_{it}$ where $f_i(t)$ is a smooth function and $\varepsilon_{it} \sim N\left(0, \sigma_i^2\right)$. The estimator is found by minimizing:

$$\hat{f}_i(t) = \arg \min_{f \in C^2[1,T_i]} \left(\sum_{t=1}^{T_i} \left(P_{it} - f_i(t)\right)^2 + \lambda_i \int_1^{T_i} \left(f^{''}(x)\right)^2 dx \right)$$

The solution $f_i(.)$ has an explicit, finite-dimensional, unique minimizer which is a natural cubic spline with knots at the unique data points[6].

The smoothness of the function $f_i(.)$ is controlled through the coefficient of the integrated second squared derivative penalty function, $\lambda_i \geq 0$.

The optimal parameter λ_i (or equivalently **df**) can be obtained through ordinary cross validation,

[5] Please refer to Appendix for formal conditions and restrictions on pattern identification.

[6] de Boor (1978)

however such an approach results in a highly under-smoothed estimate.

E Choosing optimal smoothing parameter

For a given original price series data we will:

1. Use a broad range of degrees of freedom (**df** or equivalently λ_i) to obtain vectors of smoothed price series, $\{P_{\mathrm{df}_i}\}_{i=1}^{T}$
2. Run the pattern identification algorithm on every vector in step 1 and calculate the number of patterns identified for every vector.
3. Plot degrees of freedom vs. number of patterns identified (refer to Appendix D on page 26).
4. The optimal **df** is then chosen based on one of the criteria:
   (a) assuming investors/technical analysts have rational expectations they will choose the smoothing parameter which will result in the largest number of patterns identified[7]. One could argue that this might pose a data mining problem. We propose yet another alternative,
   (b) the number of reversal patterns identified in any price series will be zero for *df=1* (i.e. zero patterns for a straight line) and will tend to zero for *df=T*. Thus we would expect the plot in step 3 to be a concave function which might exhibit periods of local stability (e.g. the number of successfully identified reversal patterns is approximately constant for several consecutive degrees of freedom).

Once the optimal smoothing parameter is chosen for an individual security, the same parameter(s) are used in smoothing the simulated price series for this security.

F Reversal Patterns

We run an iterative algorithm to check whether any of the five consecutive local extrema fits a particular reversal pattern. For each of the three null models in this study we generate 999 random return

[7] A similar situation probably occurred in Lo *et al.* (2000)when the "chartists" were presented several graphs with different smoothed series and were asked to choose the "most appropriate" ones. It is unnecessary to point out that the most appropriate ones will be the ones with the seemingly largest number of patterns. Note, that identification conditions for reversal patterns will prevent or severely restrict successful pattern identification in un-smoothed, lightly-smoothed or under-smoothed series due to the particular geometrical restrictions on each pattern.

samples of the same sample size as the underlying original return series[8]. The number of each of the reversal patterns is then calculated for each of these simulations.

Together with rank statistics, order statistics are among the most fundamental tools in non-parametric statistics and inference. We use order statistics to derive empirical distribution functions (EDFs) and density histograms for every reversal pattern for every stock we study. We then calculate the percentile of the number of patterns identified in the real price data of every stock. One can then infer whether the number of patterns identified in the real data is significantly larger than the number of patterns found in the randomly generated data.

Let M_{ij} be a number of patterns identified for security i in simulation j, let M_{i0} denote the number of patterns identified in the original price series $\forall j = 1..n$ $[n = 999]$. The weak form efficient market hypothesis can then be stated as follows:

$$H_0 : \; M_{i0} \leq \; \overline{M}_i$$

$$H_1 : \; M_{i0} > \; \overline{M}_i$$

where $\overline{M}_i = \sum_{j=0}^{n} M_{ij}$.

The amount of incremental information obtained through reversal patterns can be used to study the relative efficiency of markets or sectors within a market.

IV Discussion of Results

We believe that the efficiency of the market (or a sector withing the market) can be evaluated by the proportion of securities with significantly higher number of reversal patterns identified in the original price series than the average number of patterns found in simulated data. If this proportion is high, one would be able to make a judgment on efficiency in this market (or sector).

We perform analysis of 34 sectors of Canadian market[9]. In addition we perform analysis on all 1336 stocks listed on the TSX, as well as 251 stocks listed under the TSX Composite index and nine iShares

[8] Davidson and MacKinnon (2000)

[9] Sectors with the number of stocks less than or equal to three where excluded from the analysis. Excluded sectors: Alternative energy (3) , Mobile telecommunications (2), Personal goods (2), Tobacco (1).

ETFs[10]. We evaluate each of these subsets of stocks based on the proportion of securities with a significantly high number of reversal patterns. We calculate these proportions for each of the 10 reversal patterns, however, BT, BB, RW and FW were excluded from the set of evaluation criteria due to the lack of variability in the proportions for these chart patterns.

Tables 1-3 in the Appendix contain the summary of the results. It is apparent from these results that accounting for short-term dependencies in the returns, in our case through ARMA(p,q) and EGARCH(p,q) models, we find fewer securities with a number of patterns significantly larger than the average number of patterns identified in the simulated series.

The presence of a large percentage of stocks within a sector or a particular subgroup where the number of reversal patterns is persistently higher than the number of patterns identified on average in the random series would point to inefficiency within this sector or subgroup. Although these results do not necessarily imply that technical analysis can be used to generate excess trading profits in some sectors, they do raise the possibility that technical analysis may add value to the investment process in particular sectors of the economy.

We use a total order ranking method for interpretation of our results[11]. Total order ranking methods are multicriteria decision making techniques used for the ranking of various alternatives on the basis of more than one criterion. Let us consider a K-dimensional system, with an associated $(S \times K)$ data matrix. To each of the S sectors (or subsets) of securities a set of K evaluation criteria is associated.

As one can see from results in Tables 1-3, the criteria are not always in agreement, they are, at times, conflicting, motivating the need to find an overall optimum that can deviate from the optima of one or more of the single criterion. Total order ranking methods are based on an aggregation of the criteria in a scalar function, i.e. an order or ranking index, which allows us to sort elements according to their numerical values. Several evaluation methods which define a ranking parameter generating a total order ranking are used: *desirability functions*, *utility functions*, *dominance functions* and *absolute reference method*.

[10] Namely *iShares XEG* (45), *XFN* (23), *XGP* (15), *XTR* (48), *XMA* (43), *XRE* (11), *XIT* (5), *XCG* (57), *XCV* (65). Note: figures in parentheses signify the number of stocks listed under each index.

[11] Full description of total order ranking methods can be found in Pavan and Todeschini (2008).

Various sectors of the Canadian market and several commonly referred indices have been analyzed and compared on the basis of multiple criteria, the aim being to find out the most (in)efficient set of securities among all the sectors analyzed. To illustrate, Table 5 shows the ranking results for the EGARCH(p,q) null model based on the dominance function as a reference. Most "efficient" sectors are assigned high ranking, and most "inefficient" sectors are the ones with low ranking. We use a set of all securities (*All 1336*) as a reference for the market efficiency on average and compare the rest of the sectors to this benchmark. As expected, most of the iShares listed securities are more efficient on average than the market. The banking sector, stocks listed under the TSX composite index, as well as the Care Equipment and Services, Gas Water and Utilities, Oil Equipment and Services, Life Insurance, Oil and Gas producers are among the most efficient sectors. On the other hand, Real Estate, and stocks listed under iShares Real Estate index, as well as Media, Industrial Engineering, Travel and Leisure, Electricity, Financial Services, Food Producers and Aerospace&Defense sectors are among the most inefficient sectors of the Canadian economy.

Another interesting fact is that we find a strong negative dependency between the average number of pattern occurrence per year and two moments of the underlying return series, namely variance and kurtosis (refer to Appendix C). The first relation seems to be intuitive: as the variance of a stock increases, it becomes increasingly difficult to forecast the future price path or to fit a particular reversal pattern to the price series. The negative relation between the number of reversal patterns and kurtosis is harder to interpret. Higher kurtosis means more of the variance is due to infrequent extreme deviations, as opposed to frequent modestly-sized deviations. Thus, patterns with a relatively large number of extreme observations tend to have few reversal patterns in their price paths.

Although, we fail to reject the null hypothesis of weak form efficiency on the TSX, some sectors of the Canadian economy appear to be less efficient than others. A further breakdown of data into 5 year periods and subsequent analysis of each of these periods might reveal a different result. The data we collected on the number of reversal patterns identified through a pattern recognition algorithm were aggregated over all 25 years. However, over this period, economic conditions as well as the technological advances which enable today's markets to share information instantly and across several trading floors have changed. Thus, analysis of the market efficiency would be more complete if done within smaller sub periods.

A Appendix

A Data Smoothing

The estimator is found by minimizing:

$$\hat{f}_i(t) = \arg \min_{f \in C^2[1,T_i]} \left(\sum_{t=1}^{T_i} (P_{it} - f_i(t))^2 + \lambda_i \int_1^{T_i} \left(f''(x) \right)^2 dx \right)$$

The solution $f_i(.)$ has an explicit, finite-dimensional, unique minimizer which is a natural cubic spline with knots at the unique data points[12]. The smoothness of the function $f_i(.)$ is controlled through the coefficient of the integrated second squared derivative penalty function, $\lambda_i \geq 0$.

However, for our purpose we chose instead to control for degrees of freedom. There is a direct relationship between the two, as was shown by Hastie *et al.* (2001):

The above criterion is reduced to

$$RSS(\theta, \lambda) = (\mathbf{y} - \mathbf{N}\theta)^T (\mathbf{y} - \mathbf{N}\theta) + \lambda \theta^T \Omega_N \theta$$

Where $\{\mathbf{N}\}_{ij} = N_j(x)$, $\{\Omega_N\}_{jk} = \int N_j''(t) N_k''(t)\, dt$, and $N_j(x)$ are an N-dimensional set of basis functions for representing the family of natural splines and $f(x) = \sum_{j=1}^{N} N_j(x)\theta_j$.

The solution can then be written in the following form:

$$\widehat{\theta} = \left(\mathbf{N}^T\mathbf{N} + \lambda\Omega_N\right)^{-1} \mathbf{N}^T\mathbf{y}$$

And

$$\hat{f}(x) = \sum_{j=1}^{N} N_j(x)\widehat{\theta}_j$$

Or

$$\widehat{\mathbf{f}} = \mathbf{N}\left(\mathbf{N}^T\mathbf{N} + \lambda\Omega_N\right)^{-1} \mathbf{N}^T\mathbf{y} = \mathbf{S}_\lambda \mathbf{y}$$

Where $\widehat{\mathbf{f}}$ is the N-vector of fitted values $\hat{f}(x_i)$ and $\mathbf{S}_\lambda$ is a smoothing matrix. Then the corresponding effective degrees of freedom is given by:

[12] de Boor (1978)

$$df_{\lambda} = trace\left(\mathbf{S}_{\lambda}\right)$$

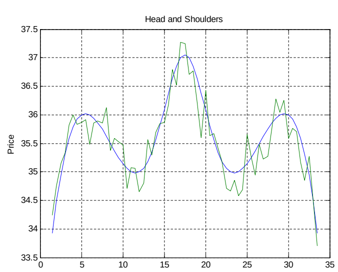
Since $df_{\lambda} = trace\left(\mathbf{S}_{\lambda}\right)$ is monotone in λ for smoothing splines[13], we can invert the relationship and specify λ by fixing **df**. Using **df** in this way provides a uniform approach to compare many different smoothed prices series for different levels of smoothing parameters.

The optimal parameter λ_i (or equivalently **df**) can be obtained through ordinary cross validation[14], however such an approach results in a slightly under-smoothed estimate. For a number of knots equal to the unique data points[15] the solution exhibits more smoothing but less accuracy in satisfying $P_{it} = f_i\left(t\right)$ when *df* are set smaller then the optimal CV value.

B Pattern identification conditions

Based on the five consecutive local extrema points, $E_1, ..., E_5$, we identify a set of ten reversal patterns. In our algorithm we have chosen the following parameters:$C = 0.03$; $S = 0.03$; $F = 0.015$; $R = 0.0075$. These values are consistent with the original simulation in Lo *et al.* (2000). However, the additional restriction on the distance between the two consecutive extrema has been added to concentrate on short-term reversal patterns. This condition prevents pattern identification based on two consecutive local extrema longer than eight trading days apart.

Head-and-Shoulders (HS)



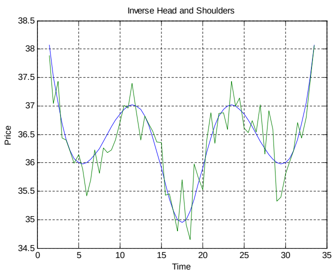
$E_1 > E_2$	ensures that E_1 is a local maximum
$E_3 > E_1$	head is larger than left shoulder
$E_3 > E_5$	head is larger than right shoulder
$\frac{\lvert E_1 - E_5 \rvert}{(E_1 + E_5)/2} \leq C$	E_1 and E_5 are within C% of their average
$\frac{\lvert E_2 - E_4 \rvert}{(E_2 + E_4)/2} \leq C$	E_2 and E_4 are within C% of their average
$\frac{E_3 - E_1}{E_3} \geq S$	ensures that the head of the pattern is significantly larger than the shoulders

[13]Hastie *et al.* (2001) p.134

[14]One can also look at the integrated square predictor error (ISPE) function, but overall the CV is approximately unbiased as an estimate of the ISPE function.

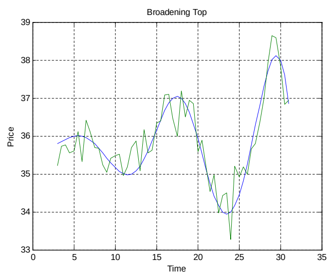
[15]In our procedure we choose to set the number of knots equal to the length of the time series.

Inverse Head-and-Shoulders (IHS)



$$E_1 < E_2 \quad \text{ensures that } E_1 \text{ is a local minimum}$$
$$E_3 < E_1 \quad \text{head is smaller than left shoulder}$$
$$E_3 < E_5 \quad \text{head is smaller than right shoulder}$$
$$\frac{|E_1 - E_5|}{(E_1+E_5)/2} \leq C \quad E_1 \text{ and } E_5 \text{ are within C\% of their average}$$
$$\frac{|E_2 - E_4|}{(E_2+E_4)/2} \leq C \quad E_2 \text{ and } E_4 \text{ are within C\% of their average}$$
$$\frac{E_1 - E_3}{E_3} \geq S \quad \text{ensures that the head of the pattern is significantly smaller than the shoulders}$$

Broadening Top (BT)



$E_1 > E_2$ ensures that E_1 is a local maximum

$E_1 < E_3$

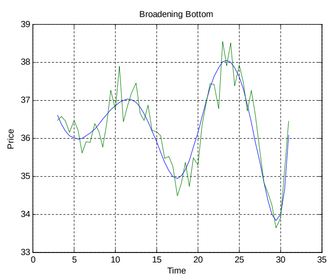
$E_3 < E_5$

$E_2 > E_4$

Conditions below make sure that fluctuations are significant and most probably not just noise :

$$\frac{|E_1 - E_2|}{E_1+E_2} \geq F,\ \frac{|E_2 - E_3|}{E_2+E_3} \geq F,\ \frac{|E_3 - E_4|}{E_3+E_4} \geq F,\ \frac{|E_4 - E_5|}{E_4+E_5} \geq F$$

Broadening Bottom (BB)



$E_1 < E_2$ ensures that E_1 is a local minimum

$E_1 > E_3$

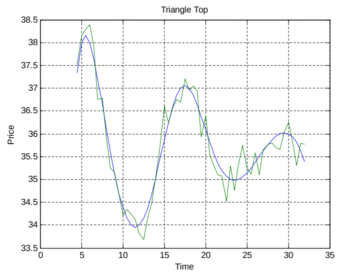
$E_3 > E_5$

$E_2 < E_4$

Conditions below make sure that fluctuations are significant and most probably not just noise :

$$\frac{|E_1 - E_2|}{E_1+E_2} \geq F,\ \frac{|E_2 - E_3|}{E_2+E_3} \geq F,\ \frac{|E_3 - E_4|}{E_3+E_4} \geq F,\ \frac{|E_4 - E_5|}{E_4+E_5} \geq F$$

Triangle Top (TT)



$E_1 > E_2$ ensures that E_1 is a local maximum

$E_1 > E_3$

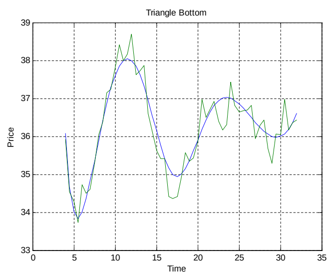
$E_3 > E_5$

$E_2 < E_4$

Conditions below make sure that fluctuations are significant and most probably not just noise :

$\frac{|E_1-E_2|}{E_1+E_2} \geq F$, $\frac{|E_2-E_3|}{E_2+E_3} \geq F$, $\frac{|E_3-E_4|}{E_3+E_4} \geq F$, $\frac{|E_4-E_5|}{E_4+E_5} \geq F$

Triangle Bottom (TB)



$E_1 < E_2$ ensures that E_1 is a local minimum

$E_1 < E_3$

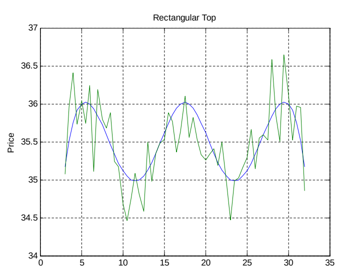
$E_3 < E_5$

$E_2 > E_4$

Conditions below make sure that fluctuations are significant and most probably not just noise :

$\frac{|E_1-E_2|}{E_1+E_2} \geq F$, $\frac{|E_2-E_3|}{E_2+E_3} \geq F$, $\frac{|E_3-E_4|}{E_3+E_4} \geq F$, $\frac{|E_4-E_5|}{E_4+E_5} \geq F$

Rectangular Top (RT)



$E_1 > E_2$ ensures that E_1 is a local maximum

Conditions below ensure that E_1, E_3, E_5 and E_2, E_4 are within R of their average :

$\frac{\left|E_i - \frac{E_1+E_3+E_5}{3}\right|}{(E_1+E_3+E_5)/3} \leq R$, for $i = 1, 3, 5$

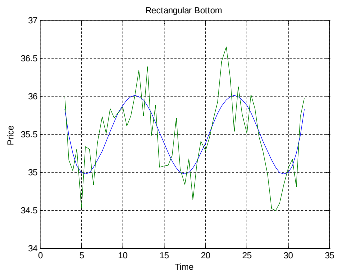
$\frac{\left|E_i - \frac{E_2+E_4}{2}\right|}{(E_2+E_4)/2} \leq R$, for $i = 2, 4$

Conditions below make sure that fluctuations are significant and most probably not just a noise :

$\frac{|E_1-E_2|}{E_1+E_2} \geq F$, $\frac{|E_2-E_3|}{E_2+E_3} \geq F$, $\frac{|E_3-E_4|}{E_3+E_4} \geq F$, $\frac{|E_4-E_5|}{E_4+E_5} \geq F$

$\min(E_1, E_3, E_5) > \max(E_2, E_4)$

Rectangular Bottom (RB)



$E_1 < E_2$ ensures that E_1 is a local minimum

Conditions below ensure that E_1, E_3, E_5 and E_2, E_4 are within R of their average :

$\frac{\left|E_i - \frac{E_1+E_3+E_5}{3}\right|}{(E_1+E_3+E_5)/3} \leq R,$ for $i = 1, 3, 5$

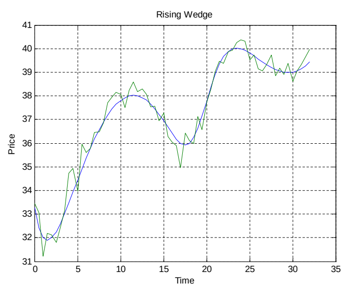
$\frac{\left|E_i - \frac{E_2+E_4}{2}\right|}{(E_2+E_4)/2} \leq R,$ for $i = 2, 4$

Conditions below make sure that fluctuations are significant and most probably not just a noise :

$\frac{|E_1-E_2|}{E_1+E_2} \geq F, \frac{|E_2-E_3|}{E_2+E_3} \geq F, \frac{|E_3-E_4|}{E_3+E_4} \geq F, \frac{|E_4-E_5|}{E_4+E_5} \geq F$

$\max(E_1, E_3, E_5) < \min(E_2, E_4)$

Rising Wedge (RW)



$E_1 < E_3$

$E_3 < E_5$

$E_2 < E_4$

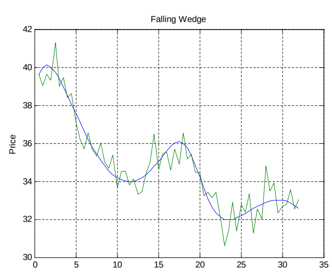
$|E_1 - E_2| > |E_3 - E_4|$

$|E_5 - E_4| < |E_4 - E_3|$

Conditions below make sure that fluctuations are significant and most probably not just noise :

$\frac{|E_1-E_2|}{E_1+E_2} \geq F, \frac{|E_2-E_3|}{E_2+E_3} \geq F, \frac{|E_3-E_4|}{E_3+E_4} \geq F, \frac{|E_4-E_5|}{E_4+E_5} \geq F$

Falling Wedge (FW)



$E_1 > E_3$

$E_3 > E_5$

$E_2 > E_4$

$|E_1 - E_2| > |E_3 - E_4|$

$|E_5 - E_4| < |E_4 - E_3|$

Conditions below make sure that fluctuations are significant and most probably not just noise :

$\frac{|E_1-E_2|}{E_1+E_2} \geq F, \frac{|E_2-E_3|}{E_2+E_3} \geq F, \frac{|E_3-E_4|}{E_3+E_4} \geq F, \frac{|E_4-E_5|}{E_4+E_5} \geq F$

Table 1. Random walk model: Proportions of securities with significantly large number of chart patterns.

This table shows the percentages of stocks whose number of patterns identified in the original price series is significantly[16] larger than the average number of patterns identified in the simulated price series, based on the random walk data generation null model. N is the total number of securities within each subcategory or sector.

	N	HS	IHS	BT	BB	TT	TB	RW	FW	RT	RB
All 1336 securities	**1336**	**0.50**	**0.48**	**0.01**	**0.01**	**0.19**	**0.18**	**0.01**	**0.03**	**0.63**	**0.64**
TSX Composite members	**251**	**0.35**	**0.29**	**0.00**	**0.00**	**0.10**	**0.11**	**0.00**	**0.01**	**0.67**	**0.63**
iShares XEG members	*45*	*0.31*	*0.33*	*0.00*	*0.00*	*0.09*	*0.13*	*0.00*	*0.00*	*0.53*	*0.53*
iShares XFN members	*23*	*0.30*	*0.09*	*0.00*	*0.00*	*0.09*	*0.04*	*0.00*	*0.00*	*0.65*	*0.65*
iShares XGP members	*15*	*0.47*	*0.40*	*0.00*	*0.00*	*0.07*	*0.20*	*0.00*	*0.00*	*0.47*	*0.47*
iShares XTR members	*48*	*0.29*	*0.44*	*0.00*	*0.02*	*0.15*	*0.25*	*0.00*	*0.00*	*0.65*	*0.60*
iShares XMA members	*43*	*0.44*	*0.37*	*0.00*	*0.00*	*0.07*	*0.09*	*0.00*	*0.00*	*0.56*	*0.58*
iShares XRE members	*11*	*0.27*	*0.55*	*0.00*	*0.00*	*0.09*	*0.27*	*0.00*	*0.00*	*0.91*	*0.73*
iShares XIT members	*5*	*0.40*	*0.40*	*0.00*	*0.00*	*0.20*	*0.40*	*0.00*	*0.00*	*0.60*	*0.40*
iShares XCG members	*57*	*0.30*	*0.23*	*0.00*	*0.00*	*0.09*	*0.09*	*0.00*	*0.04*	*0.65*	*0.65*
iShares XCV members	*65*	*0.34*	*0.28*	*0.00*	*0.00*	*0.11*	*0.15*	*0.00*	*0.00*	*0.68*	*0.60*
Aerospace & Defense	7	0.86	0.57	0.00	0.00	0.00	0.29	0.00	0.00	0.86	0.86
Alternative Energy	3	0.67	0.67	0.00	0.00	0.67	1.00	0.00	0.00	0.33	0.33
Automobiles & Parts	10	0.60	0.60	0.00	0.00	0.10	0.30	0.00	0.00	0.70	0.60
Banks	15	0.33	0.33	0.00	0.00	0.13	0.13	0.00	0.00	0.67	0.47
Chemicals	15	0.33	0.47	0.00	0.00	0.13	0.13	0.00	0.07	0.40	0.47
Construction & Materials	22	0.45	0.55	0.05	0.00	0.23	0.18	0.00	0.00	0.73	0.73
Electricity	24	0.50	0.33	0.04	0.00	0.08	0.08	0.00	0.00	0.71	0.71
Electronic & Electrical Equipm	27	0.52	0.44	0.00	0.00	0.15	0.19	0.00	0.00	0.70	0.67
Financial Services (Sector)	178	0.55	0.52	0.05	0.03	0.23	0.34	0.07	0.02	0.67	0.62
Fixed Line Telecommunications	8	0.75	0.75	0.00	0.00	0.38	0.13	0.00	0.00	0.88	0.88
Food & Drug Retailers	8	0.50	0.38	0.00	0.00	0.13	0.38	0.00	0.13	1.00	0.88
Food Producers	19	0.68	0.58	0.05	0.00	0.21	0.26	0.05	0.00	0.74	0.68
Forestry & Paper	17	0.29	0.24	0.00	0.00	0.06	0.18	0.00	0.00	0.65	0.71
Gas Water & Multiutilities	7	0.29	0.43	0.00	0.00	0.00	0.14	0.00	0.00	0.71	0.71
General Industrials	20	0.50	0.65	0.05	0.00	0.10	0.15	0.00	0.00	0.80	0.90
General Retailers	31	0.61	0.55	0.00	0.00	0.13	0.23	0.03	0.00	0.77	0.84
Health Care Equipment & Servic	24	0.63	0.33	0.00	0.00	0.42	0.13	0.00	0.00	0.46	0.54
Household Goods & Home Constru	9	0.67	0.56	0.00	0.00	0.33	0.11	0.00	0.00	0.67	0.78
Industrial Engineering	19	0.42	0.47	0.00	0.00	0.26	0.21	0.00	0.05	0.74	0.63
Industrial Metals & Mining	61	0.34	0.33	0.00	0.00	0.11	0.16	0.00	0.02	0.54	0.61
Industrial Transportation	16	0.56	0.31	0.00	0.00	0.19	0.31	0.00	0.06	0.81	0.63
Leisure Goods	5	0.40	0.60	0.00	0.00	0.20	0.20	0.00	0.00	0.60	0.60
Life Insurance	8	0.25	0.13	0.00	0.13	0.38	0.13	0.00	0.00	0.50	0.75
Media	31	0.55	0.35	0.00	0.00	0.19	0.16	0.00	0.03	0.81	0.74
Mining	264	0.57	0.45	0.01	0.00	0.16	0.09	0.01	0.03	0.60	0.62
Mobile Telecommunications	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.50
Nonlife Insurance	6	0.00	0.17	0.00	0.00	0.33	0.33	0.00	0.00	0.83	0.83
Oil & Gas Producers	145	0.37	0.39	0.01	0.00	0.17	0.14	0.00	0.03	0.51	0.53
Oil Equipment & Services	60	0.38	0.38	0.02	0.02	0.27	0.15	0.00	0.03	0.47	0.50
Personal Goods	2	0.50	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.50	1.00
Pharmaceuticals & Biotechnolog	57	0.60	0.56	0.00	0.02	0.14	0.09	0.02	0.12	0.56	0.56
Real Estate Investment & Servi	23	0.74	0.74	0.00	0.00	0.17	0.17	0.00	0.04	0.83	0.87
Real Estate Investment Trusts	28	0.32	0.61	0.11	0.00	0.21	0.32	0.00	0.00	0.86	0.82
Software & Computer Services	51	0.53	0.63	0.00	0.00	0.24	0.16	0.00	0.06	0.61	0.61
Support Services	51	0.49	0.69	0.00	0.00	0.24	0.16	0.04	0.00	0.67	0.71
Technology Hardware & Equipmen	29	0.52	0.62	0.00	0.00	0.28	0.17	0.00	0.00	0.52	0.69
Tobacco	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
Travel & Leisure	23	0.57	0.52	0.00	0.00	0.30	0.30	0.00	0.04	0.74	0.74

[16] The bootstrap method enables us to construct confidence intervals for chart pattern counts. If the number of patterns identified in the original data is outside of this confidence interval, we conclude that the number of patterns identified in the original price series is significantly different than the average number of patterns identified in the simulated price series.

Table 2. EGARCH(p,q) model: Proportions of securities with significantly large number of chart patterns.

This table shows the percentages of stocks whose number of patterns identified in the original price series is significantly[17] larger than the average number of patterns identified in the simulated price series, based on the EGARCH data generation null model. N is the total number of securities within each subcategory or sector.

	N	HS	IHS	BT	BB	TT	TB	RW	FW	RT	RB
All 1336 securities	**1336**	**0.29**	**0.27**	**0.03**	**0.03**	**0.14**	**0.15**	**0.05**	**0.10**	**0.43**	**0.45**
TSX Composite members	**251**	**0.13**	**0.12**	**0.02**	**0.02**	**0.08**	**0.09**	**0.04**	**0.05**	**0.35**	**0.34**
iShares XEG members	*45*	*0.13*	*0.13*	*0.00*	*0.00*	*0.07*	*0.04*	*0.00*	*0.04*	*0.27*	*0.18*
iShares XFN members	*23*	*0.09*	*0.04*	*0.00*	*0.00*	*0.09*	*0.00*	*0.00*	*0.00*	*0.30*	*0.22*
iShares XGP members	*15*	*0.20*	*0.07*	*0.00*	*0.00*	*0.00*	*0.20*	*0.00*	*0.00*	*0.27*	*0.33*
iShares XTR members	*48*	*0.17*	*0.19*	*0.00*	*0.02*	*0.10*	*0.17*	*0.02*	*0.02*	*0.33*	*0.23*
iShares XMA members	*43*	*0.14*	*0.09*	*0.00*	*0.00*	*0.00*	*0.07*	*0.00*	*0.00*	*0.28*	*0.35*
iShares XRE members	*11*	*0.27*	*0.18*	*0.00*	*0.00*	*0.00*	*0.27*	*0.09*	*0.00*	*0.64*	*0.45*
iShares XIT members	*5*	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.20*	*0.00*	*0.00*	*0.40*	*0.20*
iShares XCG members	*57*	*0.12*	*0.09*	*0.02*	*0.02*	*0.11*	*0.05*	*0.04*	*0.09*	*0.37*	*0.39*
iShares XCV members	65	0.12	0.12	0.00	0.02	0.08	0.09	0.02	0.03	0.29	0.28
Aerospace & Defense	7	0.57	0.29	0.00	0.00	0.14	0.00	0.14	0.14	0.71	0.71
Alternative Energy	3	0.33	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.33
Automobiles & Parts	10	0.30	0.60	0.00	0.10	0.20	0.20	0.10	0.10	0.50	0.50
Banks	15	0.20	0.00	0.00	0.00	0.07	0.13	0.00	0.00	0.20	0.20
Chemicals	15	0.27	0.27	0.00	0.00	0.13	0.07	0.00	0.07	0.27	0.40
Construction & Materials	22	0.27	0.23	0.09	0.00	0.18	0.05	0.05	0.00	0.50	0.45
Electricity	24	0.33	0.29	0.00	0.00	0.25	0.21	0.08	0.08	0.50	0.58
Electronic & Electrical Equipm	27	0.26	0.19	0.07	0.04	0.19	0.22	0.07	0.07	0.41	0.44
Financial Services (Sector)	178	0.43	0.38	0.03	0.04	0.17	0.21	0.06	0.08	0.47	0.48
Fixed Line Telecommunications	8	0.13	0.25	0.00	0.00	0.13	0.13	0.00	0.00	0.50	0.38
Food & Drug Retailers	8	0.38	0.13	0.00	0.00	0.13	0.13	0.00	0.13	0.88	0.88
Food Producers	19	0.32	0.42	0.00	0.00	0.16	0.16	0.05	0.05	0.63	0.53
Forestry & Paper	17	0.18	0.18	0.00	0.00	0.00	0.18	0.00	0.00	0.06	0.41
Gas Water & Multiutilities	7	0.14	0.14	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.43
General Industrials	20	0.20	0.45	0.05	0.05	0.05	0.10	0.00	0.10	0.50	0.65
General Retailers	31	0.19	0.16	0.03	0.03	0.06	0.13	0.03	0.03	0.65	0.68
Health Care Equipment & Servic	24	0.13	0.21	0.00	0.00	0.17	0.04	0.00	0.00	0.17	0.17
Household Goods & Home Constru	9	0.22	0.33	0.00	0.00	0.00	0.11	0.00	0.00	0.44	0.56
Industrial Engineering	19	0.32	0.42	0.00	0.05	0.21	0.21	0.11	0.16	0.58	0.53
Industrial Metals & Mining	61	0.28	0.25	0.05	0.07	0.13	0.21	0.10	0.16	0.31	0.46
Industrial Transportation	16	0.31	0.06	0.00	0.00	0.06	0.13	0.00	0.06	0.50	0.44
Leisure Goods	5	0.20	0.40	0.00	0.00	0.00	0.20	0.00	0.00	0.40	0.40
Life Insurance	8	0.13	0.13	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.25
Media	31	0.42	0.35	0.06	0.06	0.23	0.16	0.10	0.10	0.61	0.55
Mining	264	0.26	0.21	0.05	0.05	0.14	0.14	0.06	0.14	0.42	0.42
Mobile Telecommunications	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Nonlife Insurance	6	0.00	0.00	0.00	0.00	0.17	0.33	0.00	0.00	0.33	0.50
Oil & Gas Producers	145	0.19	0.14	0.04	0.01	0.14	0.10	0.04	0.09	0.29	0.32
Oil Equipment & Services	60	0.18	0.22	0.03	0.03	0.12	0.12	0.03	0.12	0.25	0.22
Personal Goods	2	0.50	0.50	0.00	0.50	0.50	0.50	0.00	0.50	0.50	0.50
Pharmaceuticals & Biotechnolog	57	0.30	0.28	0.00	0.02	0.11	0.04	0.04	0.16	0.39	0.42
Real Estate Investment & Servi	23	0.61	0.57	0.00	0.00	0.17	0.30	0.09	0.17	0.61	0.70
Real Estate Investment Trusts	28	0.32	0.32	0.07	0.00	0.14	0.21	0.07	0.04	0.68	0.68
Software & Computer Services	51	0.31	0.33	0.02	0.04	0.12	0.08	0.04	0.10	0.45	0.53
Support Services	51	0.25	0.45	0.02	0.04	0.24	0.14	0.06	0.08	0.47	0.47
Technology Hardware & Equipmen	29	0.21	0.24	0.03	0.03	0.07	0.17	0.03	0.03	0.38	0.41
Tobacco	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
Travel & Leisure	23	0.30	0.35	0.00	0.00	0.17	0.26	0.00	0.09	0.61	0.61

[17] The bootstrap method enables us to construct confidence intervals for chart pattern counts. If the number of patterns identified in the original data is outside of this confidence interval, we conclude that the number of patterns identified in the original price series is significantly different than the average number of patterns identified in the simulated price series.

Table 3. ARMA(p,q) model: Proportions of securities with significantly large number of chart patterns.

This table shows the percentages of stocks whose number of patterns identified in the original price series is significantly[18] larger than the average number of patterns identified in the simulated price series, based on the ARMA data generation null model. N is the total number of securities within each subcategory or sector.

	N	HS	IHS	BT	BB	TT	TB	RW	FW	RT	RB
All 1336 securities	**1336**	**0.30**	**0.29**	**0.01**	**0.00**	**0.08**	**0.09**	**0.01**	**0.02**	**0.57**	**0.59**
TSX Composite members	**251**	**0.20**	**0.20**	**0.00**	**0.01**	**0.07**	**0.09**	**0.01**	**0.01**	**0.62**	**0.60**
iShares XEG members	*45*	*0.24*	*0.29*	*0.00*	*0.00*	*0.04*	*0.09*	*0.00*	*0.00*	*0.51*	*0.44*
iShares XFN members	*23*	*0.22*	*0.13*	*0.00*	*0.00*	*0.04*	*0.04*	*0.00*	*0.00*	*0.70*	*0.65*
iShares XGP members	*15*	*0.33*	*0.20*	*0.00*	*0.00*	*0.07*	*0.13*	*0.00*	*0.00*	*0.47*	*0.47*
iShares XTR members	*48*	*0.10*	*0.31*	*0.00*	*0.02*	*0.08*	*0.21*	*0.00*	*0.00*	*0.58*	*0.54*
iShares XMA members	*43*	*0.28*	*0.21*	*0.00*	*0.00*	*0.05*	*0.07*	*0.00*	*0.00*	*0.51*	*0.56*
iShares XRE members	*11*	*0.09*	*0.27*	*0.00*	*0.00*	*0.00*	*0.18*	*0.00*	*0.00*	*0.82*	*0.73*
iShares XIT members	*5*	*0.40*	*0.40*	*0.00*	*0.00*	*0.00*	*0.40*	*0.00*	*0.00*	*0.40*	*0.20*
iShares XCG members	*57*	*0.18*	*0.14*	*0.00*	*0.00*	*0.07*	*0.07*	*0.00*	*0.02*	*0.63*	*0.61*
iShares XCV members	*65*	*0.18*	*0.18*	*0.00*	*0.00*	*0.08*	*0.14*	*0.00*	*0.00*	*0.65*	*0.55*
Aerospace & Defense	7	0.71	0.14	0.00	0.00	0.00	0.00	0.00	0.00	0.86	0.71
Alternative Energy	3	0.67	0.67	0.00	0.00	0.33	0.67	0.00	0.00	0.33	0.33
Automobiles & Parts	10	0.30	0.30	0.00	0.00	0.10	0.20	0.00	0.00	0.70	0.60
Banks	15	0.33	0.27	0.00	0.00	0.07	0.13	0.00	0.00	0.73	0.53
Chemicals	15	0.33	0.33	0.00	0.00	0.13	0.07	0.00	0.07	0.33	0.53
Construction & Materials	22	0.27	0.36	0.05	0.00	0.09	0.05	0.00	0.00	0.64	0.73
Electricity	24	0.25	0.17	0.04	0.00	0.04	0.08	0.00	0.00	0.63	0.71
Electronic & Electrical Equipm	27	0.30	0.22	0.00	0.00	0.04	0.07	0.00	0.00	0.63	0.63
Financial Services (Sector)	178	0.31	0.24	0.01	0.02	0.08	0.16	0.05	0.02	0.54	0.51
Fixed Line Telecommunications	8	0.50	0.50	0.00	0.00	0.38	0.13	0.00	0.00	0.75	0.88
Food & Drug Retailers	8	0.25	0.00	0.00	0.00	0.13	0.13	0.00	0.13	1.00	0.88
Food Producers	19	0.32	0.26	0.05	0.00	0.11	0.05	0.05	0.00	0.74	0.63
Forestry & Paper	17	0.18	0.12	0.00	0.00	0.00	0.12	0.00	0.00	0.65	0.65
Gas Water & Multiutilities	7	0.29	0.29	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.43
General Industrials	20	0.20	0.35	0.05	0.00	0.00	0.05	0.00	0.00	0.70	0.85
General Retailers	31	0.45	0.23	0.00	0.00	0.03	0.13	0.03	0.00	0.77	0.84
Health Care Equipment & Servic	24	0.29	0.21	0.00	0.00	0.21	0.04	0.00	0.00	0.38	0.42
Household Goods & Home Constru	9	0.22	0.44	0.00	0.00	0.11	0.00	0.00	0.00	0.67	0.78
Industrial Engineering	19	0.21	0.26	0.00	0.00	0.11	0.11	0.00	0.05	0.74	0.63
Industrial Metals & Mining	61	0.23	0.20	0.00	0.00	0.07	0.10	0.02	0.02	0.48	0.57
Industrial Transportation	16	0.25	0.06	0.00	0.00	0.06	0.06	0.00	0.00	0.69	0.50
Leisure Goods	5	0.40	0.40	0.00	0.00	0.00	0.20	0.00	0.00	0.60	0.60
Life Insurance	8	0.13	0.00	0.00	0.00	0.13	0.00	0.00	0.00	0.50	0.63
Media	31	0.26	0.32	0.03	0.03	0.16	0.10	0.03	0.03	0.71	0.65
Mining	264	0.33	0.28	0.00	0.00	0.06	0.05	0.01	0.02	0.57	0.59
Mobile Telecommunications	2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.50
Nonlife Insurance	6	0.00	0.17	0.00	0.00	0.33	0.33	0.00	0.00	0.83	0.83
Oil & Gas Producers	145	0.23	0.30	0.01	0.00	0.07	0.07	0.00	0.01	0.45	0.49
Oil Equipment & Services	60	0.23	0.25	0.00	0.02	0.12	0.08	0.00	0.03	0.45	0.47
Personal Goods	2	0.50	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.50	1.00
Pharmaceuticals & Biotechnolog	57	0.35	0.33	0.00	0.00	0.05	0.02	0.00	0.07	0.46	0.53
Real Estate Investment & Servi	23	0.48	0.39	0.00	0.00	0.00	0.04	0.00	0.04	0.78	0.83
Real Estate Investment Trusts	28	0.21	0.29	0.04	0.00	0.07	0.07	0.00	0.00	0.82	0.75
Software & Computer Services	51	0.37	0.37	0.00	0.00	0.06	0.04	0.00	0.04	0.57	0.61
Support Services	51	0.27	0.47	0.00	0.00	0.18	0.10	0.02	0.00	0.59	0.63
Technology Hardware & Equipmen	29	0.38	0.41	0.00	0.00	0.07	0.14	0.00	0.00	0.48	0.66
Tobacco	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
Travel & Leisure	23	0.30	0.35	0.00	0.00	0.13	0.26	0.00	0.04	0.61	0.61

[18] The bootstrap method enables us to construct confidence intervals for chart pattern counts. If the number of patterns identified in the original data is outside of this confidence interval, we conclude that the number of patterns identified in the original price series is significantly different than the average number of patterns identified in the simulated price series.

Table 4. Random walk model: Total ranking report.

In this table dominance was used as a sorting function, however, results based on desirability, utility and absolute reference (reference object: *Chemicals*) are also reported[19]. Ranking order is from more efficient to less efficient.

Rank	Objects	Desirability	Utility	Dominance	Abs.Ref.
1	*iShares XEG*	*0.655*	*0.678*	*0.718*	*0.916*
2	*iShares XFN*	*0.637*	*0.696*	*0.687*	*0.795*
3	*iShares XMA*	*0.616*	*0.647*	*0.671*	*0.896*
4	*iShares XCG*	*0.618*	*0.667*	*0.668*	*0.838*
5	Chemicals	0.664	0.678	0.641	1
6	Banks	0.625	0.656	0.607	0.878
7	**TSX Composite**	**0.597**	**0.641**	**0.606**	**0.853**
8	Oil & Gas Producers	0.633	0.651	0.596	0.936
9	Industrial Metals & Mining	0.624	0.65	0.592	0.9
10	*iShares XGP*	*0.639*	*0.656*	*0.58*	*0.923*
11	*iShares XCV*	*0.6*	*0.641*	*0.574*	*0.852*
12	Forestry & Paper	0.593	0.647	0.564	0.826
13	Oil Equipment & Services	0.631	0.642	0.517	0.926
14	Gas Water & Multiutilities	0.553	0.619	0.504	0.826
15	Electricity	0.536	0.597	0.501	0.816
16	Life Insurance	0.596	0.646	0.493	0.787
17	*iShares XTR*	*0.574*	*0.604*	*0.467*	*0.874*
18	Mining	0.549	0.585	0.453	0.857
19	Pharmaceuticals & Biotechnolog	0.547	0.582	0.449	0.861
20	Health Care Equipment & Servic	0.563	0.583	0.423	0.821
21	*iShares XIT*	*0.588*	*0.6*	*0.403*	*0.853*
22	Leisure Goods	0.539	0.567	0.362	0.878
23	**All 1336**	**0.533**	**0.563**	**0.353**	**0.86**
24	Electronic & Electrical Equipm	0.514	0.556	0.322	0.832
25	Software & Computer Services	0.509	0.539	0.296	0.848
26	*iShares XRE*	*0.419*	*0.53*	*0.294*	*0.756*
27	Industrial Engineering	0.508	0.544	0.282	0.831
28	Automobiles & Parts	0.479	0.517	0.273	0.806
29	Media	0.463	0.532	0.272	0.775
30	Industrial Transportation	0.477	0.531	0.27	0.774
31	Technology Hardware & Equipmen	0.504	0.534	0.269	0.846
32	Nonlife Insurance	0.466	0.583	0.253	0.683
33	Construction & Materials	0.476	0.523	0.252	0.814
34	Support Services	0.465	0.51	0.251	0.812
35	Financial Services (Sector)	0.489	0.511	0.23	0.818
36	General Industrials	0.373	0.483	0.227	0.738
37	Household Goods & Home Constru	0.432	0.481	0.223	0.766
38	Food & Drug Retailers	0	0.458	0.191	0.678
39	General Retailers	0.403	0.478	0.189	0.751
40	Food Producers	0.431	0.474	0.174	0.769
41	Real Estate Investment Trusts	0.392	0.476	0.164	0.743
42	Aerospace & Defense	0.31	0.429	0.159	0.661
43	Travel & Leisure	0.436	0.471	0.152	0.775
44	Real Estate Investment & Servi	0.319	0.413	0.123	0.687
45	Fixed Line Telecommunications	0.285	0.375	0.086	0.657
	No. of levels	40	36	42	38
	Degeneracy k (%)	0.51	1.11	0.3	0.91
	Degeneracy D(%)	11.36	20.45	6.82	15.91
	Discrimination power (%)	99.72	99.43	99.84	99.58
	Stability (%)	1.66	1.52	1.76	1.58

[19]Desirability, utility, dominance and absolute reference ranking indices are evaluated based on degeneracy, discrimination power and stability. (Pavan and Todeschini (2008)).

Table 5. EGARCH(p,q) model: Total ranking report.

In this table dominance was used as a sorting function, however, results based on desirability, utility and absolute reference (reference object: *iShares XFN*) are also reported[20]. Ranking order is from more efficient to less efficient.

Rank	Objects	Desirability	Utility	Dominance	Abs.Ref.
1	*iShares XFN*	*0.87*	*0.877*	*0.784*	*1*
2	*iShares XEG*	*0.86*	*0.863*	*0.748*	*0.949*
3	*iShares XMA*	*0.836*	*0.845*	*0.736*	*0.923*
4	*iShares XCV*	*0.831*	*0.836*	*0.691*	*0.942*
5	Banks	0.863	0.867	0.684	0.914
6	*iShares XCG*	*0.8*	*0.813*	*0.662*	*0.92*
7	*iShares XIT*	*0.853*	*0.867*	*0.661*	*0.895*
8	**TSX Composite**	**0.805**	**0.813**	**0.646**	**0.925**
9	Health Care Equipment & Servic	0.853	0.854	0.639	0.901
10	Gas Water & Multiutilities	0.788	0.81	0.629	0.884
11	*iShares XGP*	*0.815*	*0.822*	*0.605*	*0.888*
12	Oil Equipment & Services	0.815	0.817	0.596	0.903
13	Life Insurance	0.828	0.833	0.596	0.92
14	Forestry & Paper	0.822	0.833	0.589	0.835
15	Oil & Gas Producers	0.799	0.803	0.575	0.915
16	*iShares XTR*	*0.799*	*0.802*	*0.53*	*0.903*
17	Chemicals	0.759	0.767	0.486	0.857
18	Pharmaceuticals & Biotechnolog	0.733	0.746	0.452	0.842
19	Fixed Line Telecommunications	0.734	0.75	0.446	0.856
20	Technology Hardware & Equipmen	0.743	0.753	0.444	0.854
21	Industrial Transportation	0.728	0.75	0.431	0.84
22	Leisure Goods	0.719	0.733	0.398	0.804
23	Mining	0.722	0.732	0.397	0.847
24	Nonlife Insurance	0.755	0.778	0.387	0.814
25	Household Goods & Home Constru	0.696	0.722	0.366	0.793
26	General Retailers	0.631	0.688	0.33	0.752
27	Industrial Metals & Mining	0.719	0.727	0.327	0.825
28	Construction & Materials	0.702	0.72	0.325	0.83
29	**All 1336**	**0.702**	**0.712**	**0.323**	**0.824**
30	General Industrials	0.635	0.675	0.299	0.737
31	Software & Computer Services	0.677	0.696	0.296	0.791
32	*iShares XRE*	*0.664*	*0.697*	*0.285*	*0.776*
33	Electronic & Electrical Equipm	0.708	0.716	0.279	0.831
34	Support Services	0.65	0.663	0.204	0.767
35	Food & Drug Retailers	0.432	0.583	0.204	0.62
36	Aerospace & Defense	0.527	0.595	0.174	0.656
37	Food Producers	0.605	0.632	0.15	0.733
38	Automobiles & Parts	0.596	0.617	0.14	0.704
39	Financial Services (Sector)	0.631	0.641	0.139	0.747
40	Electricity	0.623	0.639	0.117	0.754
41	Travel & Leisure	0.591	0.616	0.116	0.723
42	Industrial Engineering	0.605	0.623	0.112	0.734
43	Media	0.591	0.613	0.108	0.724
44	Real Estate Investment Trusts	0.564	0.607	0.106	0.702
45	Real Estate Investment & Servi	0.476	0.507	0.044	0.599
	No. of levels	36	38	41	39
	Degeneracy k (%)	1.01	0.71	0.4	0.61
	Degeneracy D(%)	20.45	15.91	9.09	13.64
	Discrimination power (%)	99.43	99.58	99.78	99.65
	Stability (%)	1.52	1.58	1.7	1.62

[20] Desirability, utility, dominance and absolute reference ranking indices are evaluated based on degeneracy, discrimination power and stability. (Pavan and Todeschini (2008)).

Table 6. ARMA(p,q) model: Total ranking report.

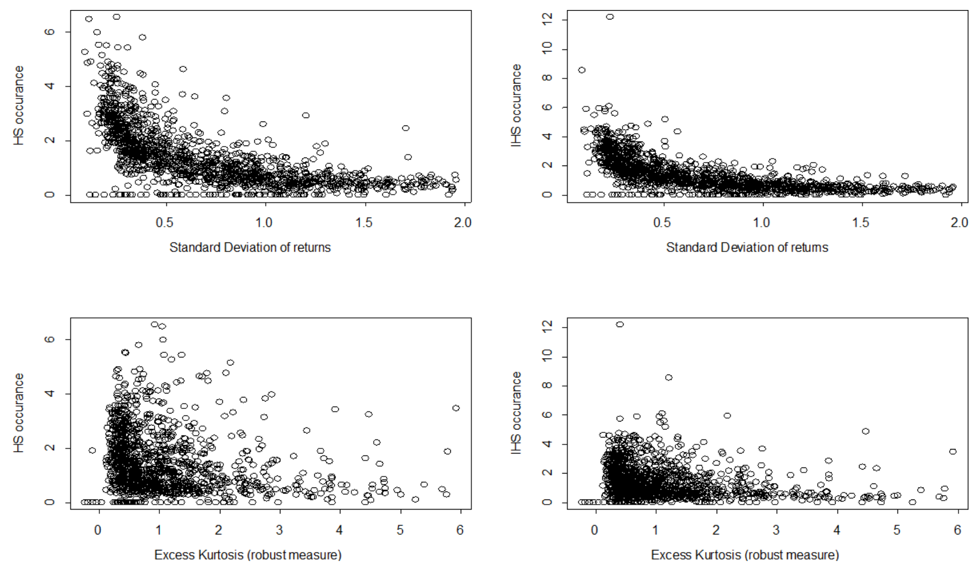
In this table dominance was used as a sorting function, however, results based on desirability, utility and absolute reference (reference object: *Health Care Equipment & Services*) are also reported[21]. Ranking order is from more efficient to less efficient.

Rank	Objects	Desirability	Utility	Dominance	Abs.Ref.
1	Gas Water & Multiutilities	0.707	0.738	0.597	0.878
2	Life Insurance	0.724	0.771	0.581	0.848
3	Industrial Transportation	0.677	0.729	0.542	0.842
4	*iShares XMA*	*0.691*	*0.721*	*0.522*	*0.895*
5	Industrial Metals & Mining	0.699	0.727	0.522	0.898
6	Oil & Gas Producers	0.712	0.731	0.519	0.915
7	Health Care Equipment & Servic	0.733	0.743	0.518	1
8	*iShares XEG*	*0.709*	*0.73*	*0.516*	*0.902*
9	*iShares XCG*	*0.666*	*0.716*	*0.497*	*0.846*
10	*iShares XFN*	*0.636*	*0.703*	*0.496*	*0.819*
11	Oil Equipment & Services	0.718	0.733	0.494	0.938
12	Forestry & Paper	0.656	0.716	0.487	0.818
13	**TSX Composite**	**0.661**	**0.705**	**0.478**	**0.856**
14	Pharmaceuticals & Biotechnolog	0.685	0.711	0.475	0.898
15	*iShares XGP*	*0.705*	*0.722*	*0.441*	*0.917*
16	Mining	0.652	0.688	0.439	0.875
17	Electricity	0.626	0.688	0.43	0.827
18	*iShares XCV*	*0.661*	*0.703*	*0.413*	*0.852*
19	Electronic & Electrical Equipm	0.637	0.685	0.41	0.847
20	Software & Computer Services	0.626	0.663	0.369	0.853
21	**All 1336**	**0.648**	**0.681**	**0.369**	**0.875**
22	*iShares XTR*	*0.664*	*0.694*	*0.367*	*0.843*
23	Chemicals	0.694	0.711	0.359	0.919
24	Financial Services (Sector)	0.671	0.694	0.357	0.895
25	General Industrials	0.53	0.642	0.353	0.753
26	*iShares XIT*	*0.685*	*0.7*	*0.35*	*0.789*
27	Aerospace & Defense	0.464	0.595	0.339	0.697
28	*iShares XRE*	*0.547*	*0.652*	*0.319*	*0.742*
29	Industrial Engineering	0.597	0.658	0.295	0.817
30	Banks	0.605	0.656	0.289	0.829
31	Household Goods & Home Constru	0.553	0.63	0.277	0.781
32	Construction & Materials	0.584	0.644	0.276	0.816
33	Real Estate Investment Trusts	0.528	0.631	0.275	0.761
34	Food Producers	0.588	0.649	0.274	0.821
35	Leisure Goods	0.599	0.633	0.26	0.817
36	Real Estate Investment & Servi	0.475	0.58	0.227	0.728
37	Support Services	0.594	0.627	0.223	0.835
38	Automobiles & Parts	0.59	0.633	0.222	0.825
39	Media	0.583	0.634	0.22	0.825
40	Food & Drug Retailers	0	0.604	0.218	0.668
41	Technology Hardware & Equipmen	0.611	0.644	0.216	0.843
42	General Retailers	0.485	0.591	0.208	0.741
43	Nonlife Insurance	0.466	0.583	0.205	0.691
44	Travel & Leisure	0.596	0.623	0.194	0.834
45	Fixed Line Telecommunications	0.403	0.479	0.047	0.707
	No. of levels	40	34	41	32
	Degeneracy k (%)	0.51	1.21	0.4	1.52
	Degeneracy D(%)	11.36	25	9.09	29.55
	Discrimination power (%)	99.72	99.26	99.78	99.08
	Stability (%)	1.66	1.47	1.7	1.43

[21]Desirability, utility, dominance and absolute reference ranking indices are evaluated based on degeneracy, discrimination power and stability. (Pavan and Todeschini (2008)).

C Dependency of pattern frequencies on the moments of return distribution

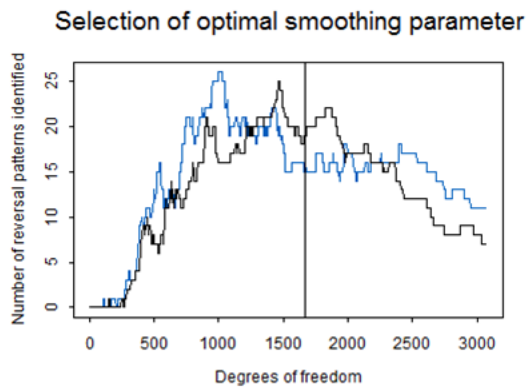
We find a strong dependency of a frequency of technical pattern occurrence on the second and fourth moments of return distributions[22]. It is difficult to interpret these higher order moments in terms of market efficiency.



[22] We use robust measures of skewness and kurtosis as suggested in Kim and White (2004)

D Selection of optimal smoothing parameter

Number of Head-And-Shoulders patterns identified for Bank of Nova Scotia and plotted vs. degrees of freedom of the bi-cubic spline. (for illustrative purposes only)



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