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Reliability of Public Private Partnership Projects under Assumptions of Cash Flow Volatility

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**Arbeitspapiere zur immobilienwirtschaftlichen Forschung und Praxis,
Band Nr. 11, Dezember 2007**

Zitierempfehlung:
Schöbener, H./ Schetter, C./ Pfnür, A. (2007): Reliability of Public Private Partnership Projects under Assumptions of Cash Flow Volatility. In: Andreas Pfnür (Hrsg.), Arbeitspapiere zur immobilienwirtschaftlichen Forschung und Praxis, Band Nr. 11.

Impressum (v.i.S.d.P.):

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ISSN Nr. 1862-2291

Abstract

This paper focuses on dynamic financial modelling of recurring cash flow items in PPP projects in operating stage and on risks associated with the volatility of these cash flows. As we concentrate on so-called government-pays schemes, only cash-outflows are considered, such as operating costs, repairs and maintenance expenses, and administration costs, whereas the revenue side is considered to be not at risk.

We show different approaches to modelling the uncertainty of recurring operating expenses and explain how to interpret the results. Our analysis is based on the mathematical framework of stochastic processes, which, in finance, are particularly used to describe price series evolutions in capital markets. We apply them to generate variable trajectories of operating costs and integrate them into a stochastic simulation of the financial model.

Key Words: Public Private Partnerships; Operating Costs; Risk Modelling; Stochastic Processes; Stochastic Simulation

Contents

Abbreviations

DSCR	Debt service coverage ratio
eqn.	equation
EUR	Euro
i. e.	id est
mn	million
NPV	Net present value
p.	page(s)
PPP	Public private partnership
prob.	probability
PSC	Public sector comparator
SPC	Special purpose company
sqm.	square meter

1 Introduction

We actually can determine a trend where real-estate investment decisions move away from initial investment costs as the most relevant decision parameter. Rather real-estate investment decisions are more frequently reached on the basis of life-cycle costs. Particularly in public private partnership (PPP) projects the life cycle cost approach is applied to increase the efficiency within the provision of public real estate.[1]

The operating costs represent a large part of all life cycle costs. In this working paper under the term „operating costs“ we subsume all operating costs including costs for day-to-day maintenance (but not heavy maintenance and replacements) as well as administration costs. Depending on the type of the real estate the net present value of the operating costs over the life-span corresponds approximately to the initial capital outlays.[2] Within the scope of PPP projects the cash flow has to be forecasted reliably right from the design or planning stage.

The cost evolution under time is usually projected as a constant growth path[3] in today's financial models. Consequently the uncertainty as to the varying operating cost items during the project's life cycle causes the following substantial risks for the project:

1. Public sector's risk: Risk of miscalculation of the public sector comparator. As a consequence the public sector may choose the wrong alternative to develop the infrastructure project.

2. Investor's risk: Generally the operating costs influence the investor's return. Moreover cost overruns may encumber liquidity reserves as well as debt service coverage ratios (DSCR).

[1] In the planing phase ca. 90 % of the life cycle costs of a real-estate project can be affected. Usually energy and operating costs can be reduced for example because of a higher investment for heating installation or an optimized layout planning for the building; see for example Pfnür (2002, p. 46)
[2] According to Pfnür (2002, p.47)
[3] The expenses grow exponentially with the inflation as the growth factor, thus: $C_t = C_0 \cdot (1 + i)^t$

3. Operator's risk: If consumption and price risks are conveyed to the operator and the operating costs will be higher than calculated in some periods, the profitability of the operator is compromised.

There are many approaches to forecast the operating costs and the associated risks, which the tenderer has to reflect in his bidding. Within this article we will show the status quo as well as enhanced approaches to modelling the uncertainty of recurring operating costs.

2 Operating expenses under time and consequences

As Public Private Partnerships are a relatively new approach in Germany, reliable data on operating costs is hardly available. As long as PPPs in public property management are considered, real estate data could be a suitable alternative. Just only to get an impression of the recurrent behaviour of operating expenses, we exemplarily analysed a sample of 14 office buildings, which are located across Germany. Figure 1 shows the annual changes of operating expenses per sqm. during the years 1997-2006 for these properties.

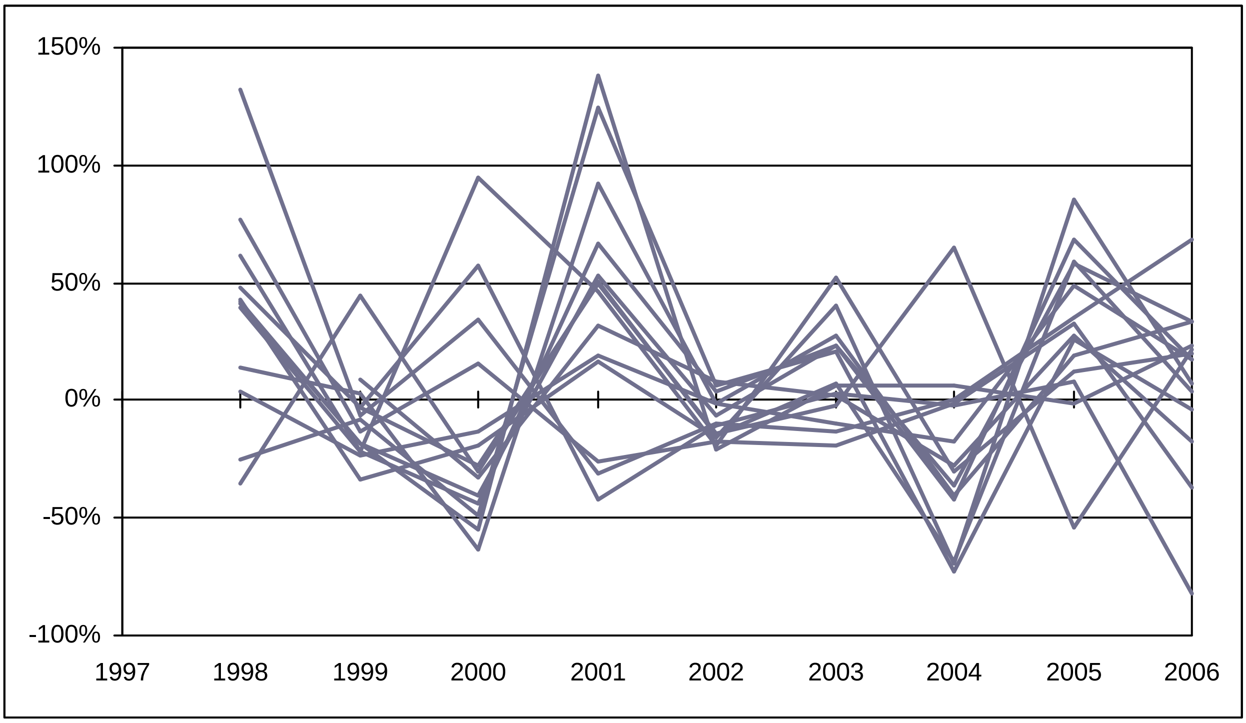

Figure 1: Annual changes of operating expenses of a sample of 14 office buildings (excluding maintenance and repairs) during 1997-2006 (indexed).
Data source: IPD, RREEF, TU Darmstadt, own illustration

We excluded all expenditures for maintenance and repairs since these usually occur irregularly. Even though the graph comprises only regular running and administrative costs, there seems to be a high volatility in these cost items for the analysed (but non-representative) set of buildings during the last years. Volatilities or variances are commonly known as a measure for risk to summarize the spread of possible outcomes.[4]

When we assume that operating expenses of public private partnerships in public property management are also subject to significant fluctuations, we have to ask how to reliably forecast there evolution under time and how to estimate major risks associated with the fluctuations.

While setting up, planning and budgeting a project, even the projection of operating costs at t_0 (starting point of the operating stage) is somehow a matter of uncertainty. Furthermore operating costs under time will be volatile to a certain amount rather than following a constant growth path. The reason may be found in externalities the project partners can neither influence nor forecast reliably (such as

[4] See Brealey/Myers (2000; p 161-164)

weather conditions) and therefore have random characteristics to them. The divergent energy consumption rates during the last two winter seasons, for example, have been a consequence of an atypical meteorological situation.[5] Other factors with (partially) random characteristics[6] are, among others, (energy) price levels, changes in market efficiencies,[7] the occurrence and implications of legal changes, vandalism, force majeure, or technical[8] and human failures.

Interestingly there seems to be a fairly good correlation between most of the properties presented above. This would rather support the assumption of high volatilities since if the data set was just irregular there wouldn't be significant correlations. And this would also mean that influencing factors on operating costs have similar effects to different properties. Considering the fact that all properties are located across western Germany this should be expected for most of the outlined random effects.

It remains to be said, however, that we have not analysed the random characteristics of the above data set. Even though the evolution of operating costs under time looks rather erratic, it could be determined somehow.[9]

The level of uncertainty concerning operating costs in PPP projects has a major influence on the overall project return for sponsors and investors, the return of the operating company, and also the public sector comparator. Moreover, in a PPP project, where annual net earnings are usually paid out to sponsors and investors, volatilities of costs may cause a financial distress, even though on average they do not exceed the calculated costs according to the projects' business plan, as is illustrated below.

[5] See, for example, Deutscher Wetterdienst (2007)
[6] We believe that it is not important whether these factors are genuine ore pseudo-random as long as the participants have no possibility to influence or forecast them reliably.
[7] See Moody's (2007, p. 14-15)
[8] See Anani/Madanat (2007, p. 15), who have analysed random effects on highway deterioration and their consequences to maintenance costs.
[9] See for further reference: Wüstefeld (2000, p. 64-81)

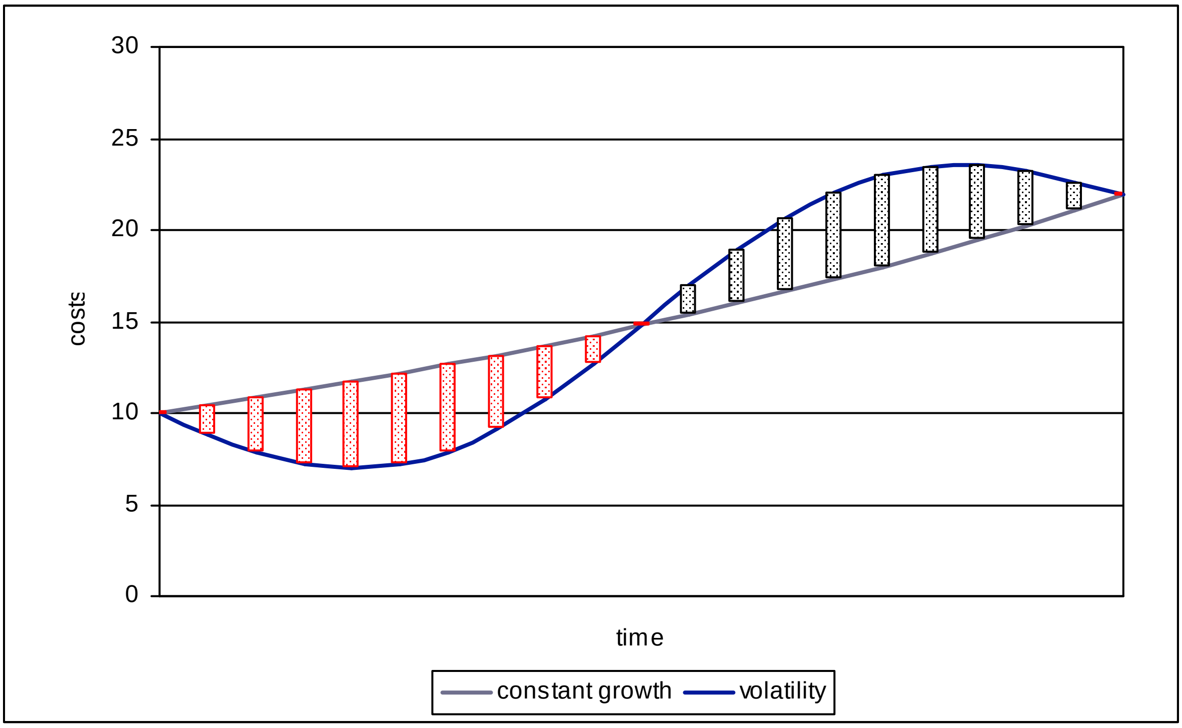

Figure 2: Evolution of operating costs under time: constant growth vs. volatility
Source: own illustration

In this abstract situation, the total operating costs amount to exactly the same in both the constant growth and the volatile case. Notwithstanding the fact that in terms of net present values the latter is even advantageous to the first one, it could compromise the financial stability of the SPC, if the assumption of a constant growth represented the original business case. During the "good" earlier years additional net cash flows resulting from lower costs than budgeted will be paid out as dividends to the project investors whereas in subsequent "poorer" years the cost overrun will encumber liquidity reserves as well as debt service coverage ratios.

3 Handling the volatility of operating costs

We will now present an abstract financial model of a PPP project, which was originally published and discussed by Pfnür/Eberhardt (2006),[10] and show different approaches to handle the uncertainty of operating costs under time. For the purposes of all our following analyses we assumed that any construction and operating risk, which is borne by the private sector, remains within the scope of the special

[10] See Pfnür/Eberhardt (2006, p. 177-184).

purpose company. Otherwise effects on the efficiency of public private partnerships as a result of a suitable risk transfer from the SPC down to its subcontractors could not be addressed.

3.1 The financial model

A public authority requires new office spaces and has to decide whether it should realise the new building either conventionally or through a public private partnership. The key data of the example project is given below.

	Public private partnership	**Conventional realisation**
Construction		
Time line	1 year	1 year
Construction costs	EUR 50,000,000	EUR 55,000,000
Operation		
Years of operation	19 years	19 years
Operating costs (1st year of operation)	EUR 4,500,000	EUR 5,000,000
Annual inflation	2.00%	2.00%
Financing		
Equity	0%	0%
Debt	100%	100%
Fixed interest rate	5.00%	4.90%
Initial amortization	1.00%	1.00%
Liquidity reserve (X month of debt service)	3 months	na
Special Purpose Company		
Unitary charge (1st year of operation)	EUR 7,950,000	na
Cash flow (1st year of operation)	EUR 450,000	na
Annual indexation	1.25%	na

Table 1: Key data of example project

During our further analyses we will focus both on the public side as well as the SPC. From the point of view of the public authority the PSC is of highest interest because it determines whether a PPP or a conventional realisation is the most efficient procurement process for the required office spaces. When below referring to the net present value with respect to the public authority, we mean the savings, which could be achieved (value for money), in terms of the difference between the net present values of the PSC and the costs of the PPP respectively.

From the point of view of the project company the expected return for sponsors and investors is crucial. As the PPP-project is assumed to be realised according to the forfeiting model, no equity financing is required. Consequently the return on equity or an internal rate of return cannot be determined and we look on the net present value of the projects distributions to sponsors and investors instead.

Additionally we have a look on the reliability of the project company over its expected life time. A situation of a financial distress is considered to be the case each time the liquidity reserve of the SPC falls below zero.

3.2 The case of certainty

The easiest way of forecasting the operating costs is to presume a situation of certainty. In this case the operating costs are forecasted for the period t_0. For any future period we assume constant growth in line with the expected price inflation.[11] Hence we imply a certain state of knowledge regarding the operating costs in period t_0 and a full determinacy in the future evolution of the costs.

In our base case the operating costs are implemented in the financial plan without assuming any uncertainties. The results of the example project as described above are as follows:

Parameter	**Special purpose company**	**Public authority**
Output variable	Net present value of investment	Net present value of savings
Expected value	EUR 6,067,000	EUR 4,934,000
Probability of liquidity shortfall	0%	na

Table 2: Exemplary project: output results of base case

The investors of the exemplary project would expect a net present value of their investment slightly above EUR 6mn whereas a liquidity shortfall could never happen since the financial plan rests on the assumption of positive cash flows during whole project life time. The public authority in turn could expect savings close to EUR 5mn or 4.8% of the total costs respectively when realising the new building through a PPP rather than the conventional way.

[11] The straight proportional slope is only influenced by the inflation of the prices for the operating costs, so that real prices remain constant over time.

Forecasting operating costs without considering any uncertainties, however, is the foundation of the existing economic comparisons for PPP projects.[12] In our point of view this kind of modelling represents a dissatisfying abstraction of the real circumstance as was exemplarily illustrated in chapter 2.

3.3 The case of uncertainty: Risk modelling for operating expenses

The operating expenses are afflicted with a risk potential, which is hard to estimate. Not only the actual amount, but also the time when they occur, is subject to uncertainties. In the following we show some approaches, which allow for a modelling of the operating expenses under uncertainties, and thus increase the awareness and the transparency with respect to risks.

3.3.1 Common approach

Within the calculation of the risk potential of PPP projects nowadays the "cost-element-percentage method"[13] is used, because it is advised by the relevant PPP guidelines[14] and frequently referred to as state of the art in contemporary academic literature.[15] Thereby for every individual risk a surcharge is calculated, which is determined from the probability of its occurrence as well as the extent of the occurrence in different scenarios.[16] Afterwards it is calculated as in a situation of certainty. For the operating expenses, which represent a yearly payment, a risk surcharge results, which is not changing during the life-time. The assumption of a constant growth path for the future evolution of the operating costs remains unchanged.

The surcharge represents in principle „the expected value of the economic disadvantage in all alternative situations, which are considered to possibly happen".[17] Hence, with this method we obtain an aggregation of the input variable to one value. That is there is no illustration of the risks, because neither

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[12] Uncertainties in today's economic comparisons are considered in separate risk positions, see below, chapter 3.3.1.
[13] In German: „Zuschlags- bzw. Korrekturverfahren".
[14] For example see Partnerships Victoria (2001. p 36), Treasury Taskforce (1999, p. 34), Finanzministerkonferenz (2006, p.63 - 65)
[15] See Boll (2007, p. 148-149)
[16] See Adam (2000, p. 353 f.)
[17] Pfnür/Eberhardt (2006, p. 168)

variances nor distributions are provided to quantify or measure the risk level. The surcharge, however, reduces but not excludes, the risk that the actual operating costs exceed the calculated.[18]

3.3.2 Simplified dynamic approach

Another alternative is to use simulative risk analyses in a singular stochastic experiment for any variable, which is subject to uncertainty. This is, with reservations, as we will see, more exact. Therefore a probability distribution for the operating expenses in the period t_0 has to be assumed, which reflects the risk and the chance respectively of a deviation from the expected value. Furthermore a constant evolution under time for the operating expenses with the inflation as the upward gradient parameter is assumed. Afterwards a value tuple for the operating cost is drawn and the respective final outcome for the output variables is calculated. After repeating this process n times a distribution function for any output variable results.

Here we refer to the essay of Pfnür/Eberhardt (2006), who described and discussed the risk analysis in connection with PPP projects by applying a stochastic simulation".[19] For our purposes we simplified the given financial model and assumed only construction costs as well as the operating costs to be at risk. The probability distributions are supposed to be discrete, where:

	Public private partnership				**Conventional realisation**		
Construction costs	47,200,000	50,000,000	53,000,000		50,000,000	55,000,000	60,500,000
Probability	30%	50%	20%		20%	50%	30%

Table 3: Discrete probability distribution for construction costs

	Public private partnership				**Conventional realisation**				
Operating costs	4,200,000	4,500,000	4,700,000		4,350,000	4,750,000	5,000,000	5,250,000	5,650,000
Probability	20%	50%	30%		5%	20%	20%	40%	15%

Table 4: Discrete probability distribution for operating costs in t_0

It has to be considered that due to the discrete probability distributions only nine possible outcomes exist with respect to the net present value of the PPP and 15 with respect to the conventional realisation. The results of the simulation of the project are given in the table below.

[18] See Hildenbrandt (1988, p. 23)
[19] See Pfnür/Eberhardt (2006, p 159-186)

Parameter	Special purpose company	Public authority
Output variable	Net present value of investment	Net present value of savings
Mean	EUR 6,252,000	EUR 7,030,000
Standard deviation	EUR 2,757,000	EUR 5,414,000
5% percentile	EUR 1,169,000	EUR -2,751,000
Lower Quartile	EUR 3,762,000	EUR 3,730,000
Upper Quartile	EUR 8,220,000	EUR 10,155,000
95% percentile	EUR 12,108,000	EUR 13,691,000
Probability of NPV < 0	0%	7.5%
Probability of liquidity shortfall	0%	na

Table 5: Exemplary project: output results of simulation in a simplified dynamic approach.

The probability distributions of the ingoing variables suggest that with respect to the conventional realisation a cost overrun is more likely than staying below the original budget. Consequently the expected value for the savings the public authority could achieve when choosing the PPP procurement increases significantly from EUR 4.9mn[20] to EUR 7.0mn or from 4.8% to 6.9% respectively. On the other hand the calculated savings are subject to significant variances, which is indicated by a standard deviation of EUR 5.4mn; and there is a chance of approximately 1 out of 13 that the conventional realisation will be more economic than the PPP (i. e. the NPV of savings is smaller than zero).

The conclusion regarding the SPC is that investors have to fear neither a negative return nor a liquidity shortfall since any possible combination according to the probability distributions as specified above results in positive cash flows.

By using the simulative risk analysis we refrain from the assumption of forecasting the operating expenses in period t_0 under certainty. Nevertheless the operating costs are not projected close to reality, because again we stick to the assumption of a constant growth path after period t_0. The surplus of this method, however, is the result of a statistic distribution for the output variables. The gain of transparency and risk awareness when compared to the cost-element-percentage approach is quite substantial.

[20] See above, chapter 3.2, Table 2

3.3.3 Enhanced dynamic approach

As shown above[21] there is evidence that a constant growth or, generally speaking, fully deterministic evolution of operating expenses under time is an over-simplified assumption. And, even if they were determined, it would at least be unrealistic for anyone to forecast their evolution exactly. If we assume that there is some indeterminacy in the future evolution of expenses (this means that there is more than one possibility the process might go to), the mathematical framework of stochastic processes can be an appropriate way to deal with the uncertainty.

Description of method

A stochastic process is a sequence of random variables in chronological order. Regarding time and value of the variable, stochastic processes are distinguished in discrete and continuous processes.[22] The mapping of the variable under time is called trajectory or path of the process.[23]

In the field of finance the theory of stochastic processes is most often used for modelling price series on capital markets or determining prices of stock options.[24] But several attempts have been made to apply the theory also in the field of real estate finance: Wüstefeld used the concept to analyse and describe annual growth rates for office rents in Frankfurt[25] and Schaefer/Pfnür provided an approach to evaluate corporate real estate with real options, using Brownian motion to determine the option value.[26]

This now leads to the question of how to estimate the stochastic parameters of the respective process. It is common sense to derive the parameters from statistic properties of historic data.[27] With respect to PPP projects this is challenging due to the following reasons:

1. The extrapolation of historic parameters into the future is generally a controversially discussed issue in finance.[28]

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[21] See chapter 2
[22] See Betsch/Groh/Lohmann (2000, p. 44) and Paul/Baschnagel (p. 27)
[23] See Capasso/Bakstein (2005, p. 50)
[24] See Black/Scholes (1973, p 637-654)
[25] See Wüstefeld (2000, p. 72 and following, 103-108)
[26] See Schaefer/Pfnür (2001, p. 188-195)
[27] See Betsch/Groh/Lohmann (2000, p. 44)

2. As said earlier public private partnerships are a relatively new approach in Germany, and thus historic data is not available. The data set on real estate we have been presenting above,[29] however, does not meet the requirements in terms of sample size and time scale and is therefore not suitable to derive stochastic parameters.

3. Any PPP project will have its individual level of risk sharing in the operating stage,[30] where only risks transferred to the private partner are to be taken into consideration. From the point of view of the project company the risk transfer onto its subcontractors is also of importance.

In the following we will work with rather subjective estimations for the stochastic parameters and will pick up the issue later again.[31]

A typical random walk problem is looking for the statistic properties at the end of the process.[32] In our case of financial modelling of PPP-Projects, stochastic events at any given time *t* are of interest because the output variables, which we take into consideration, are continuously influenced by operating expenses over time. Consequently the output variables of our financial model are of high complexity and cannot easily be solved, for example, by calculus or any other analytical treatment.[33] That is why we try to describe them numerically and make use of stochastic simulations again.[34] We enhance the model as discussed before and integrate a stochastic path generator for operating expenses into the simulation.[35]

Geometric Brownian Motion

A Brownian motion (also: Wiener process) is a continuous-time stochastic process whose increments are normally distributed and independent from each other. In a more general way the process is a syn-

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[28] See Bernstein (1996, p. 121)
[29] See chapter 2
[30] See Arbeitskreis PPP im Management öffentlicher Immobilien im BPPP e.V. (2006, p. 13, 25-26)
[31] See chapter 4
[32] See Paul/Baschnagel (1999, p. 3)
[33] Generally speaking we are looking at a problem like: *What is the probability distribution of the net present value of a set of random variables* $(X_t)_{t \in N}$ *in a time discrete stochastic process?* See Capasso/Bakstein (2005) and Paul/Baschnagel (1999) for an analytical framework of stochastic processes in finance.
[34] See also Moody's (2006, p. 29-31)
[35] The numerical evaluation of statistical properties for general cases of stochastic processes is described, for exemple, by Paul/Baschnagel (1999, p. 74-76; 160-161) and Betsch/Groh/Lohman (2000, p. 53-54)

thesis of two components: the deterministic drift and the stochastic volatility. Let W_t be a standard Brownian motion then:

$$S_t = \mu t + \sigma W_t \qquad \text{(eqn. 3.1)}$$

is called a Brownian motion with drift μ and infinitesimal variance σ^2. In finance not absolute but rather fractional changes of the variable are of interest,[36] which is taken into account in a variant, the so-called *geometric Brownian motion*:[37]

$$\frac{dS}{S(t)} = \mu \cdot dt + \sigma \cdot dW(t) \qquad \text{(eqn. 3.2)}$$

The equation also suggests that a quantity that follows a geometric Brownian motion may take any value strictly greater than zero. In practise this applies to stock prices as well as operating costs in PPP projects.

The modelling of the dynamical behaviour of prices as geometric Brownian motion rests on the assumption of continuous market activity, i. e. the time interval between successive quotations tends to zero.[38] Of course this is an idealization for capital markets and even more for operating costs in Public Private Partnerships, where data is available on a monthly basis at best. This implies that the evolution should rather be modelled as a time-discrete stochastic process as follows:

$$\frac{dS}{S(t)} = \mu \cdot dt + \sigma \cdot \varepsilon \cdot \sqrt{dt} \qquad \text{(eqn. 3.3)}$$

where: ε = standard-normal distributed random variable

Such a discrete-time approximation of a Brownian motion is sometimes referred to as Gaussian random walk.[39] The use of the square root of *dt* allows for the assumption of the increment as a change over a discrete time interval, for example two successive days or years.[40]

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[36] This is sensible because a change or EUR 10 in a certain time interval is much more relevant for a capital of EUR 100 than for one of EUR 1.000. See Paul/Baschnagel (1999, p. 137).
[37] See Paul/Baschnagel (1999, p. 136)
[38] See Betsch/Groh/Lohman (2000, p. 44)
[39] See Friedmann/Sanddorf-Köhle (2000, p. 28)

In our model we assumed the annual drift μ for the operating costs is 2.0% and the annual volatility σ is 4.0%. Since a normal distribution implies that 95.5% of all stochastic events occur within the interval of plus/minus two standard deviations around the expected value, an annual volatility of 4.0% means that if in t_0 operating costs were at a level of EUR 100, in t_1 they would be within the interval of EUR 92-108 by a chance of 95.5%. This seems quite sensible, if we take into consideration that changes both of price levels and consumption rates are usually at risk of the private partner up to a certain amount.[41] The discrete probability distributions for the construction costs and the initial operating costs remain unchanged.[42]

Fed with these parameters the simulation of our exemplary project computed the following results after a number of 10,000 runs, where each run generated a new trajectory for the operating costs.

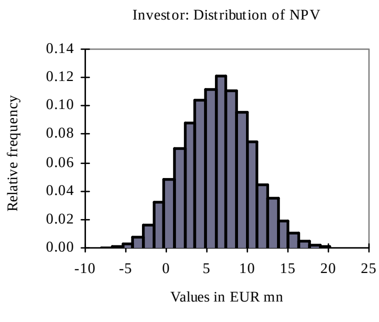
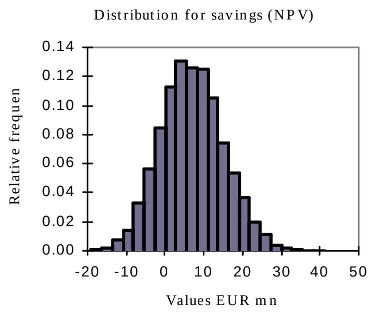
Parameter	**Special purpose company**	**Public authority**
Output variable	Net present value of investment	Net present value of savings
Mean	EUR 6,271,000	EUR 7,016,000
Standard deviation	EUR 4,173,000	EUR 8,154,000
5% percentile	EUR -596,000	EUR -5,935,000
Lower Quartile	EUR 3,330,000	EUR 1,330,000
Upper Quartile	EUR 9,117,000	EUR 12,343,000
95% percentile	EUR 13,220,000	EUR 20,857,000
Probability of NPV < 0	6.9%	19.5%
Distribution graphs		
Probability of liquidity shortfall	7.4%	na

Table 6: Operating costs as geometric Brownian motion: results of simulation

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40 See Hull (1997, p. 211)

41 See Arbeitskreis PPP im Management öffentlicher Immobilien im BPPP e.V. (2006, p. 25) and Sächsisches Staatsministerium der Justiz/Sächsisches Staatsministerium der Finanzen (2006, p. 17)

42 See above, chapter 3.3.2

When compared to the situation above (see chapter 3.3.2), the data points out the additional risks for both the special purpose company as well as the public authority, which is resulting from the volatility of operating costs under time only. Whereas the expected values (means) remain almost unchanged the standard deviations relative to the expected values increase significantly from 44.1% to 66.5% (investor's return) and from 77.0% to 116.2% respectively (public savings). By a quite substantial chance of almost 20% the public authority would rather waste money than save money when choosing the PPP procurement. The investors of the project company in turn would have to bear a negative NPV by a chance of 6.9%, whereas the probability of a liquidity shortfall is calculated with 7.4%. The difference between both figures indicates those cases where the overall project return is positive but the liquidity falls short sometime during the operating stage.[43]

Extension 1: Conditional stochastic behaviour

In an early stage of a PPP project the sponsors can generally influence the operating costs for later years. Typically operating costs will be lower when investing more at the beginning, for example in additional lagging or low-current technical equipment. In terms of risk measures such issues can be comprehended as a negative correlation between construction and operating costs.

To take into account for such effects we modified our Gaussian random walk model, this time no longer using a standard-normal distribution for the random component *(*ε, see eqn. 3.3*)*, but instead applied conditional probabilities, expressed through either a left-skewed, right-skewed or symmetric Pert distribution, depending on the stochastic occurrence of the construction costs.

	PPP realisation			**Conventional realisation (PSC)**		
Construction Costs	47,200,000	50,000,000	53,000,000	50,000,000	55,000,000	60,500,000
Minimum	-2.5	-2,5	-2.5	-2.5	-2,5	-2.5
Expected Value	**0.5**	**0,0**	**-0.5**	**0.5**	**0,0**	**-0.5**
Maximum	2.5	2,5	2.5	2.5	2,5	2.5

Table 7: Conditional pert distribution parameters depending on construction costs

[43] See above, chapter 2

μ and σ we left at an unchanged level of 2.0% and 4.0% per year respectively. The results of the simulation are as follows:

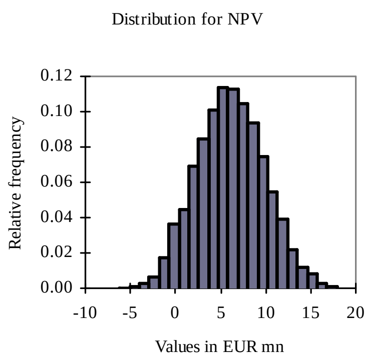
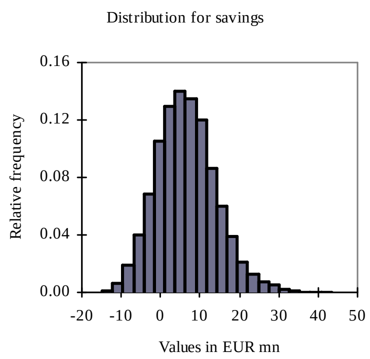
Parameter	**Special purpose company**	**Public authority**
Output variable	Net present value of investment	Net present value of savings
Mean	EUR 6,139,000	EUR 6,512,000
Standard deviation	EUR 3,711,000	EUR 7,559,000
5% percentile	EUR 34,000	EUR -5,227,000
Lower Quartile	EUR 3,551,000	EUR 1,203,000
Upper Quartile	EUR 8,721,000	EUR 11,216,000
95% percentile	EUR 12,313,000	EUR 19,466,000
Probability of NPV < 0	4.9%	19.7%
Distribution graphs		
Probability of liquidity shortfall	5.7%	na

Table 8: Operating costs as asymmetric, conditionally pert-distributed random walk: results of simulation

When assuming a negative correlation between construction costs and operating costs under time we would expect an overall decrease in risks. As to the SPC this is clearly the case, since the distribution of the output variable fairly narrows in comparison to the Brownian motion situation before. Also the probability for a negative return or a liquidity shortfall decrease significantly.

The situation of the public authority, however, looks slightly different, since the data suggests not only a narrowing but also a left shift of the distribution, because the mean decreases from EUR 7.0mn to EUR 6.5mn. The question remains whether the public authority is even in a position to reach a deliberate decision about higher investments in the beginning with the purpose to save operating costs later on.

Extension 2: Jump model

So far our random walk models have the assumption of constant drift and volatility over time in common. But we may also think of discontinuities in the time evolution of operating costs. Unexpected and abrupt changes (shocks), for example, can be described by jump models or by using stochastic volatilities. The following jump diffusion model comprises a superimposition of a continuous Brownian motion with discrete jumps:[44]

$$\frac{dS}{S(t)} = (\mu - n \cdot S) \cdot dt + \sigma \cdot dW(t) + dN(t) \qquad \text{(eqn. 3.4)}$$

where: *n* = jump rate

S = expected amplitude of jumps (jump size)

N(t) = Poisson process with rate *n*

The drift in this compound model is reduced by the average jump amplitude per period. The second term in eqn. 3.4 describes a geometric Brownian motion while the third contribution is resulting from a Poisson-distributed random variable with rate *n*. For our purposes we again approximated the Brownian motion by a Gaussian random walk as above (see eqn. 3.3, μ = 2.0% and σ = 4.0% per year) while we assumed the jump rate *n* to be 20% (meaning one jump every five years) and the amplitude *S* to be 5.0%.[45] The simulation of the project with these parameters gives the following results:

Parameter	**Special purpose company**	**Public authority**
Output variable	Net present value of investment	Net present value of savings
Mean	EUR 6,285,000	EUR 7,021,000
Standard deviation	EUR 4,366,000	EUR 8,824,000
5% percentile	EUR -991,000	EUR -6,827,000
Lower Quartile	EUR 3,240,000	EUR 955,000
Upper Quartile	EUR 9,340,000	EUR 12,682,000
95% percentile	EUR 13,362,000	EUR 22,367,000
Probability of NPV < 0	7.9%	21.7%

[44] See Betsch/Groh/Lohmann (2000, p. 50-51) and Merton (1990, p. 145)

[45] For the sake of simplicity we left the amplitude *S* constant and did not assume a stochastic distribution for jump sizes, as is usually suggested, see Kou (2002, p. 1,087)

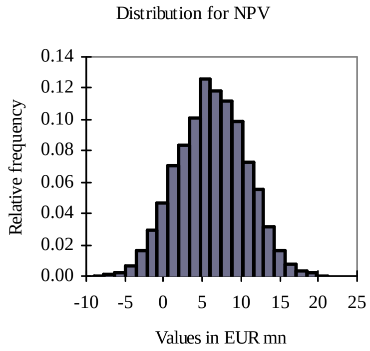
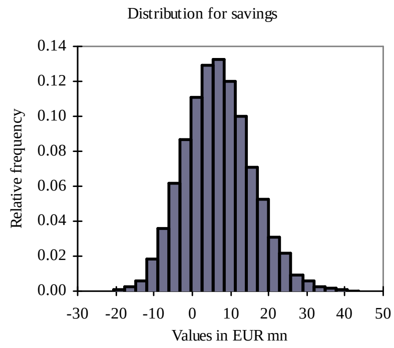
Distribution graphs		
Probability of liquidity shortfall	8.9%	na

Table 9: Operating costs as a compound of geometric Brownian motion and Poisson process: results of simulation

Discontinuities or shocks within the timely process increase risks. Within the operating period of the project of 19 years we have to expect 4.75 shocks but de facto a lot more or less can happen.[46] Consequently the distributions for both output variables widen, whereas the expected values compared to the sole Brownian motion model remain almost unchanged.

3.4 Comparison of results

3.4.1 Special purpose company

The following table provides a comparison of the results we found trough the diverse variations of our financial model with respect to the special purpose company.

Parameter	Base Case (page 9)	Simplified dynamic approach (page 12)	Geometric Brownian motion (page 16)	Conditional stochastic behaviour (page 18)	Jump model (page 20)
Mean	6,067,000	6,252,000	6,271,000	6,139,000	6,285,000
Standard deviation	na	2,757,000	4,173,000	3,711,000	4,366,000
Probability of NPV < 0	na	0%	6.9%	4.9%	7.9%
Prob. of liquidity shortfall	na	0%	7.4%	5.7%	8.9%

Table 10: Comparison of results: special purpose company

3.4.2 Public authority

With respect to the public authority the results are as follows:

[46] A number of exactly 4.75 shocks during the project's life time is impossible anyway.

Parameter	Base Case (page 9)	Simplified dynamic approach (page 12)	Geometric Brownian motion (page 16)	Conditional stochastic behaviour (page 18)	Jump model (page 20)
Mean	4,934,000	7,030,000	7,016,000	6,512,000	7,021,000
Standard deviation	na	5,414,000	8,154,000	7,559,000	8,824,000
Probability of NPV < 0	na	7.5%	19.5%	19.7%	21.7%

Table 11: Comparison of results: public authority

4 Discussion and conclusions

We have seen that incorrect estimations and the volatile behaviour of operating expenses represent substantial risks. Moreover these risks imply a threat of opportunistic behaviour of the participants. Hence it is important to forecast the operational costs as reliably as possible. Because of indeterminacies, which the participants can neither influence nor forecast, financial models should rather ensure an awareness and transparency towards the risk situation than aiming to forecast the timely evolution of operating costs exactly.

At this point the problem of the most commonly used cost-element-percentage approach becomes clear: This approach provides de facto no analytical treatment of risks, because the surcharge aggregates all possible variances to one single expected value. But the ignorance of variances results in a systemic disregard of risks. We would therefore assume that the application of overall risk surcharges is, below the line, inefficient, because during the tender the individual surcharges are subject to competition whereas later on, when the project is in operation and risks occur in reality, the surcharges may be inadequate.

The treatment of variances (i. e. risks) can be facilitated with simulative analyses. In a simplified approach, operating costs at t_0 as well as other input variables are subject to uncertainties. As the timely evolution will be erratic somehow, stochastic processes may provide a further enhancement, because they prompt the participants to analyse the structure of the uncertain factors and their implications and therefore create a substantial risk-awareness.[47] This general procedures can also be implemented both under forcasting the incomes of a PPP project and under transacting real estate investment appraisals.

Regarding the calculation of the public sector comparator our findings suggest to take into account for different possible outcomes. Rather than yielding an absolute number for the expected savings, such justifications of the value for money of a project should have a more risk related attitude. This could be achieved by calculating ranges instead of absolute numbers[48] and by outlining the probabilities for positive outcomes.

In our opinion there are two important fields to deal adequately and transparently with the risks during the project life-cycle: the financing of the project and a suitable risk control system.

An adequate structuring of the financing can help to mitigate risks in the operating stage substantially. At first, sufficiently covered saving accounts and liquidity reserves should be applied, to which the investors only can access up to a determined maximum. Moreover the financing structure can positively influence the problem of opportunistic behaviour. If, for example, operating risks were significantly high in comparison to the project's equity, it could be advisable to hold the project company harmless by keeping the respective financial liability out of it. For that operating risks would have to be transferred adequately onto the subcontractors of the SPC.

But the downward risk transfer becomes important also from a very other perspective that is the general efficiency of PPP projects. If we accept the assumption that the private sector is more efficient than the public sector it follows that the cost range for infrastructure projects under PPP procurement is lower than under conventional procurement. Over a portfolio of several hundred projects we have to assume that the public sector would on average realise these projects on the expected cost level (i. e. mean value). The private sector participants in turn may be forced to place their biddings on a cost estimate that they are 95% confident will not be exceeded, for example in order to achieve an invest-

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[47] See Betsch/Groh/Lohmann (2000, p. 44) and Deutsche Bundesbank (1998, p. 77)
[48] See Effciency Unit (2003, p. 32)

ment grade credit rating.[49] This would generally mean that, unless the SPCs cannot place most of the construction and operating risks with their diverse subcontractors, private sector cost estimates around the 95% confidence level had to outperform mean cost estimates for conventional realisations.[50]

Our calculations under chapter 3.3.3 suggest in all three variants a probability for a liquidity crunch of more than 5% up to 8.9%. Consequently the SPC had to raise the unitary charges if an adequate downward risk transfer could not be achieved and investors were to be 95% confident with their investment.

Among others we believe that the downward risk transfer especially in so-called one-stop-shop-projects, where a single private corporation is the sole investor of the project company and rendering most of the PPP-related services on its own accounts, requires special attention and for that is an interesting field for further academic research.

Another important issue in our opinion, which is addicted to the financing, is the opportunity of diversifying risks by pooling several projects – or maybe some components of their cash flows only – in portfolios. Since not all individual risks during the operating stage occur equally to diverse projects, a diversification effect will mitigate such risks as long as the financial liability is anchored on the portfolio level rather than on the project level.

As regards the risk controlling from our point of view a "risk-control system" should be established and applied during the entire operating phase, because it helps to get a sufficient awareness for all risks and their priority, which are resulting from the project.

In case of economic erroneous trends or shocks, which hit especially one of the partners, opportunistic behaviour is the normal consequence. For this reason it is very important to know all about the related risks as early as possible, i. e. the risk controlling has to fulfil the function of an "early warning system". Hence the partners have all information to react "in partnership" towards a risk-reallocation.

[49] See for PPP credit ratings Moody's (2006) and Moody's (2007)
[50] These issues were raised by Colin Ratcliffe during a discussion of an earlier version of this paper with the National Audit Office in the UK.

Since a shock in reality may be a more or less identifiable situation it is feasible, that project contracts allow for adequate compensations in such circumstances.

A risk control system might furthermore point out the management effects on the cash flow impacts of risk occurrences during the whole project life time. While, for example, assuming a genuine geometric Brownian motion for the evolution of operating costs over time, we neglect active management interventions and related mean reversion tendencies. Such interventions might, if they were adequate and successful, push back excessive operating costs towards their originally budgeted path or even below.

On the other hand a PPP project's income is usually linked to inflation indices but a significant proportion of its cost base is labour-related, were labour costs over a long time horizon usually grow faster than common price indices. It followed from discussions of our ideas with experts and practitioners that these differentials can destroy economic value in a project.

In this respect the risk control system can be used as data pool for future projects. If there is more information about the different risk situations, the initial parameters for any kind of risk modelling can be better determined.

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