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Optimal Capital Income Taxation, Investment Subsidies and Redistribution in a Neoclassical Growth Model*

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Abstract

In this paper I readdress the result that capital income taxes are bad instruments for pure redistribution and should be zero in the long run. In a neoclassical growth model a capital income cum investment subsidy tax, which is not distorting accumulation, is considered to investigate if net capital income taxes used for pure redistribution are zero in a long-run optimum. I find that capital income taxes may be nonzero, depending on the political power of those who receive redistributive transfers, the distribution of pre-tax factor incomes, and the intertemporal elasticity of substitution.

KEYWORDS: Growth, Redistribution, Investment Subsidies, Capital Income Taxes

JEL classification: O41, H21, D33

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1 Introduction

In influential papers Judd (1985) and Chamley (1986) have shown that capital income taxes are no good instruments for pure redistribution in a neoclassical growth framework. Their finding ist that optimally capital income taxes should be zero in the long run.[1]

The intuition for the result is intriguing. Even workers who may not own capital and may, therefore, not accumulate resources might benefit more from higher steady state wages resulting from nondistorted accumulation with zero taxes than having redistributive transfers now at the expense of a lower steady state capital stock and so wages in the long run.

The authors then contemplated other capital income policy packages, including consumption taxes, and basically found the same result as in, for instance, Judd (1999). However, that capital income taxers are not good instruments for redistribution need not always hold, as was shown by Lansing (1999). He found a counterexample for a world where agents have logarithmic utility.

Also, in an endogenous growth framework with productive government expenditure financed by a capital income tax, Rehme (1995) shows that zero capital income tax rates are not optimal. That the optimal capital income tax rate may be nonzero in growth contexts has, for instance, been shown by Uhlig and Yanagawa (1996) and others.

In this paper I relate to these finding in a simple neoclassical growth framework. Coupling capital income taxes with investment subsidies to finance pure redistributive transfers to the non-accumulated factor or production ("workers") may also imply a nondistortionary policy package, similar to a consumption tax on "capitalists". That governments redistribute resources but also subsidize in-

[1]Similar results have been obtained by many authors as, for example, Lucas (1990).

vestment from collected tax revenues appears to be a pervasive phenomenon in most countries. Hence, these realistic features may justify the policy package under consideration.

When taking governments - no matter which clientele a benevolent government represents - to pursue such a nondistortionary policy that seems to benefit everybody, it turns out that capital income taxes may not always be optimally zero in the long run. Rather, I find that capital income taxes may optimally be nonzero for redistribution. This depends on very intuitive conditions. As one might expect from actual taxation by governments the optimal choice of capital income taxes in the long run depends on the political power of those who receive redistributive transfers, the distribution of pre-tax income among individuals, and the intertemporal elasticity of substitution.

Complementing the counterexample of Lansing (1999), which is based on logarithmic utility functions, this paper's results may qualify the generality of the zero-capital-income result in other important and possibly quite realistic ways.

The paper is organized as follows: Section 2 presents the model. Section 3 analyzes the optimality for tax rates in long-run equilibrium. Section 4 provides concluding remarks.

2 The Model

The economy consists of a government, identical competitive firms and two types of infinitely-lived, equally patient and price taking individuals called workers and capitalists. All agents derive utility form the consumption of a homogenous, malleable good. The population is normalized so that the number of each type equals one. The model abstracts from uncertainty, technological progress, population growth and depreciation. The latter implies that aggregates are really defined in net terms which has no consequence for the price-taking, market clearing logic of the model. The workers supply one unit of unskilled labour inelastically and do not save or invest.[2] Thus, all the wealth is concentrated in the hands of the capitalists who do not work.

2.1 Capitalists

At each period the *capital owners* choose how much of their income to consume or invest, and they take prices and policy as given. Their instantaneous budget constraint is given by

$$c_t + i_t = (1 - \theta_t) r_t k_t + p_t i_t \quad \text{and} \quad i_t = \dot{k}_t.$$

Thus, the capitalists derive income from renting their capital[3], k_t, to competitive firms at the rate r_t. Gross rental income is taxed at the rate θ_t and a fraction p_t of investment undertaken, i_t, is subsidized by the government. Thus, investment subsidies are $p_t i_t$. The capitalists' consumption c_t depends on their after-tax capital income minus after-tax investment.[4]

[2]The assumption may be rationalized by imposing transaction costs on the workers when borrowing small amounts. Thus, the model uses the commonly used framework of Kaldor (1956) and Pasinetti (1962), which is also employed by Judd (1985) and Lansing (1999).

[3]Capital may also be taken to be broadly defined to include human capital. See Mankiw, Romer, and Weil (1992).

[4]As $c_t = (1 - \theta_t) r_t k_t - \dot{k}_t + p_t \dot{k}_t$, the term $p_t \dot{k}_t$ may be interpreted as a form of politically determined capital depreciation allowance which is directly and positively related to the amount invested.

Rearranging the capital owners solve

$$\max_{c_t^k} \int_0^\infty u[c_t]\; e^{-\rho t} dt$$

$$s.t. \qquad \dot{k}_t = \left(\tfrac{1-\theta_t}{1-p_t}\right) r_t k_t - \tfrac{c_t}{1-p_t} \tag{1}$$

$$k(0) = \text{given}, \quad k(\infty) = \text{free}. \tag{2}$$

where ρ is the constant rate of time preference, common to all agents. The instantaneous utility function $u[c_t]$ satisfies the usual properties $u' \geq 0, u'' \leq 0$ and $\lim\limits_{c_t \to \infty} u' = 0$ and $\lim\limits_{c_t \to 0} u' = \infty$ where $u' = \frac{du[c_t]}{dc_t}$ and $u'' = \frac{d^2 u[c_t]}{dc_t^2}$. The current value Hamiltonian for this problem is

$$H = u[c_t] + \lambda_t \left(\left(\frac{1-\theta_t}{1-p_t} \right) r_t k_t - \frac{c_t}{1-p_t} \right)$$

and the necessary first order conditions for its maximization are

$$H_c: \qquad u' - \tfrac{\lambda_t}{1-\theta_t} = 0 \tag{3a}$$

$$H_k: \quad -\lambda_t \left(\tfrac{1-\theta_t}{1-p_t}\right) \; r_t + \rho\lambda_t = \dot{\lambda}_t \tag{3b}$$

plus the transversality condition $\lim\limits_{t \to \infty} k_t \lambda_t e^{-\rho t} dt = 0$ and the requirement that equation (1) holds.[5] The co-state variable λ_t represents the capital owners' shadow price of an additional unit of capital in terms of utility.

2.2 Workers

The (unskilled) *workers* do not invest and are not taxed by assumption.[6] They supply one unit of labour inelastically at each date and derive utility from con-

[5]As H is concave in c_t and k_t, the necessary conditions are also sufficient.

[6]The working population is normalized so that there is one worker and one capitalist.

suming their entire wage and transfer income. Their total income x_t depends on wage income and lump-sum transfers granted by the government,

$$x_t = w_t + TR_t. \tag{4}$$

Their intertemporal utility is given by $\int_0^\infty v[x_t]\, e^{-\rho t} dt$ where $v[x_t]$ need not be the same as that of the capitalists, but it is also assumed to satisfy $v' \geq 0, v'' \leq 0$ and the conditions $\lim_{x_t \to \infty} v' = 0$ and $\lim_{x_t \to 0} v' = \infty$ where $v' = \frac{dv[x_t]}{dx_t}$ and $v'' = \frac{d^2 v[x_t]}{dx_t^2}$.

2.3 Firms

The *firms* operate in a perfectly competitive environment and maximize profits. The capital owners rent capital to and demand shares of the firms, which are collateralized one-to-one by capital. The markets for assets, capital and labour clear at each point in time so that the firms face a path of uniform, market clearing rental rates for capital and labour. Given perfect competition the firms rent capital and hire labour in spot markets in each period. The price of output serves as numéraire and is set equal to 1 at each date, implying that the price of capital, k_t, in terms of overall consumption stays at unity.

Aggregate production is constant returns to scale in capital and labour inputs. Since the labour input equals one, k_t can also be interpreted as the capital labour-ratio. The production function $f(k_t)$ for the representative firm is assumed to be increasing and strictly concave in k_t with $\lim_{t \to \infty} f'(k_t) = 0$ and $\lim_{t \to 0} f'(k_t) = \infty$. Profit maximization implies

$$r_t = f'(k_t) \tag{5}$$

$$w_t = f(k_t) - f'(k_t)k_t \tag{6}$$

and perfect competition and the free entry and exit of firms means that profits, $f(k_t) - r_t k_t - w_t$, are zero.

2.4 Government

Following Judd (1985) and Lansing (1999), I rule out a market for government bonds and assume that the government can commit itself to following a tax-transfer policy announced at $t = 0$. The government chooses paths of θ_t, p_t and TR_t to maximize a weighted sum of the agents' lifetime utilities, subject to the optimal behaviour of the private sector in an equilibrium and the condition that its budget be balanced at each point in time

$$TR_t = \theta_t r_t k_t - p_t \dot{k}_t.$$

Thus, the government collects capital income taxes to grant an investment subsidy ($p_t \dot{k}_t$) to the capital owners and use the remaining resources for lump-sum transfers to the workers. Hence, we contemplate a capital-income-cum-investment-subsidy-tax (CICIST) scheme.

2.4.1 Non-Distortion of Accumulation

One important consequence of the result that capital income taxes be optimally zero is that the capital accumulation process will not be disturbed by political interference. Therefore, I assume that the government, no matter what clientele is represents, wishes to minimize its distortionary impact on accumulation.

The impact of accumulation distortion can be inferred from the Euler equation in (3b). It shows how agents evaluate the evolution of the state variable k_t in terms of their welfare. This then leads them to a particular accumulation programme. Policy would in general distort this evaluation which is captured by the term $\frac{1-\theta_t}{1-p_t}$.

The government does *not* distort this evaluation in a long-run equilibrium with $\dot{\lambda} = 0$ in (3b) when $\theta_t = 0, p_t = 0, \forall t$. This is basically what the result in Judd (1985) implies. But another nondistortionary policy is possible, namely when $\theta_t = p_t$. This is the one we contemplate from now on. Whether nondistortionry $\theta_t = p_t$ implies zero tax rates will be the focus of the analysis below.[7]

The nondistortion assumption $\theta_t = p_t$ has the following implications: When the factor input and goods markets are in equilibrium the workers' income is given by[8]

$$x = w + TR = f(k) - rk + \theta rk - \theta \dot{k} \tag{7}$$

In equilibrium the overall resource constraint is such that the agents satisfy their budget constraints. Substitution of (1) into (7) one then obtains

$$x = f(k) - rk + \frac{\theta c}{1 - \theta} \tag{8}$$

Thus, the equilibrium income of the workers is increasing in the consumption of the capital owners and in θ, because that raises tax revenues that can be transferred to the workers raising their total income.

[7] This assumption nests a setup with just θ and no p_t and perhaps finding that the optimal θ_t is then zero in the long run. Notice that this policy package is tantamount to a tax on the capitalists' consumption. However, it is implemented as an income tax scheme and, thus, different.

[8] From now on time subscripts are dropped for convenience whenever it is clear that a particular variable depends on time.

3 The Long-Run Optimal Capital Income Tax

A benevolent government respects the private sector optimality conditions, keeps the agents on their respective supply and demand curves, and chooses a policy that can be realized as a competitive equilibrium.[9] The government minimizes distortions for accumulation by setting $\theta_t = p_t$ and solves

$$\max_{k,c,\theta,\lambda} \int_0^\infty \left[\gamma\, v[f(k) - rk + \tfrac{\theta c}{1-\theta}] + u[c]\right] e^{-\rho t} dt \quad \text{s.t.}$$

$$u'(c) - \tfrac{\lambda}{1-\theta} = 0 \tag{9a}$$

$$-\lambda r + \rho\lambda = \dot{\lambda} \tag{9b}$$

$$rk - \tfrac{c}{1-\theta} = \dot{k} \tag{9c}$$

$$\theta \geq 0 \quad \text{and} \quad \lim_{t\to\infty} \lambda k e^{-\rho t} = 0 \tag{9d}$$

where $\gamma \in (0, \infty)$ represents the social weight attached to the welfare of the workers. If $\gamma \to 0$, the government is only concerned about the capitalists, whereas it only cares about the workers when $\gamma \to \infty$. The current value Hamiltonian for this problem is given by

$$H^g = \gamma v[\cdot] + u[c] + \mu_1(u' - \tfrac{\lambda}{1-\theta}) + q_1\lambda(-r + \rho) + q_2(r\, k - \tfrac{c}{1-\theta})$$

where q_1 is the *social* marginal value of the *private* marginal value λ which measures how valuable more capital is in terms of utility. Furthermore, q_2 is the

[9] Similar setups are used by Judd (1985), Judd (1999), and Lansing (1999).

social marginal value of more capital k. The necessary first order conditions are

$$H_k^g: \qquad \gamma\, v'[\cdot](f'-r)+q_2 r=\rho q_2-\dot{q}_2 \tag{10a}$$

$$H_c^g: \quad \gamma\, v'[\cdot]\tfrac{\theta}{1-\theta}+u'[\cdot]+\mu_1 u''[\cdot]-q_2\tfrac{1}{1-\theta}=0 \tag{10b}$$

$$H_\theta^g: \ \theta\left(\gamma v'[\cdot]\tfrac{c}{(1-\theta)^2}-q_2\tfrac{c}{(1-\theta)^2}-\mu_1\tfrac{\lambda}{(1-\theta)^2}\right)=0 \tag{10c}$$

$$H_\lambda^g: \qquad -\tfrac{\mu_1}{1-\theta}+q_1(-r+\rho)=\rho q_1-\dot{q}_1 \tag{10d}$$

where (10c) has to hold with complementary slackness due to the requirement that θ cannot be negative.[10] Furthermore, the equations (9a), (9b) and (9c) and the transversality conditions $\lim_{t\to\infty} q_1\lambda e^{-\rho t}=0$ and $\lim_{t\to\infty} q_2 k e^{-\rho t}=0$ have to hold.

The analysis is restricted to the long-run when the economy is at a steady state, balanced growth position with $\dot{k}=\dot{\lambda}=\dot{c}=\dot{q}_1=\dot{q}_2=0$.

Suppose the government attaches some positive weight on the workers' welfare, $\gamma>0$. Then (10a) holds if $f'=r=\rho$. This pins down the capital stock to $\tilde{k}$ in steady state.[11] Equation (9c) implies $c=(1-\theta)\rho\tilde{k}$ in steady state and (10d) is only satisfied when $q_1=\mu_1=0$.[12] Then $q_2=\gamma\, v'[\cdot]$ by (10c) for an interior equilibrium and substitution of this into (10b) establishes that $\gamma v'=u'$ must hold. As the capital stock is fixed at $\tilde{k}$, which depends on ρ, and as $c=(1-\theta)\tilde{k}$, the latter condition boils down to finding θ such that

$$\gamma\, v'[f(\tilde{k})-\rho\tilde{k}+\theta\rho\tilde{k}]=u'[(1-\theta)\rho\tilde{k}]. \tag{11}$$

[10] One might argue that negative θ is a form of wage tax and should not be ruled out a priori. However, as can be verified from (10c) negative θ is only possible in the model when $\gamma=0$ and the government would not really be that benevolent anymore.

[11] Thus, as $t\to\infty$ the capital stock k_t approaches some time invariant constant $\tilde{k}$. From now on the tilde will denote variables in long-run steady state equilibrium.

[12] Lansing (1999) uses logarithmic utility for his counterexample to the zero capital income taxation result in Judd (1985). Notice here that if $\theta=0$ then $c=\rho k$ which would correspond to the optimal consumption rule of agents with logarithmic utility. However, in this paper the result is obtained by imposing optimality conditions in a steady state and for a very general class of utility functions.

Clearly as $\gamma \to \infty$ and the government is entirely pro-labour, the LHS becomes infinite and as a consequence $\theta = 1$ would be optimal, since $\lim_{c_t \to 0} u'[\cdot] = \infty$.[13]

Lemma 1 *If the workers and the capitalists have different utility functions and the government represents the workers only, then the optimal capital income tax under a capital-income-cum-investment-subsidy-tax (CICIST) scheme is nonzero in the long run and redistribution from capital to labour is maximal.*

Notice that this result does not depend on production externalities or any other things, the capital income taxes may be used for, except for using part of the revenue for investment subsidies.

Of course, the government does not always place so much weight on the workers. In fact, by implicit differentiation one verifies that the optimal tax rate, if it exists, is increasing in γ. Thus, as the workers get more political power they would choose higher capital income taxes under the capital-income-cum-investment-subsidy-tax (CICIST) scheme.[14]

It is not entirely clear why workers should evaluate a consumption good any differently than a capital owner. For that reason it is now assumed that $v[x] = u[c]$ for any $x = c$ so that the two groups have the same utility function. As I am only interested in conditions under which the capital income tax is zero in the long-run let us assume that the utility functions are of the constant relative risk aversion (CRRA) type: $u[c] = \frac{c^{1-\beta}-1}{1-\beta}$ and $v[x] = \frac{x^{1-\beta}-1}{1-\beta}$. Then (11) would be

[13] Rehme (1995) and Rehme (2002) obtain a similar result in an endogenous growth framework where redistribution occurs via productive government input financed by a capital income tax cum investment subsidy scheme.

[14] Notice that $\tilde{k}$ would be the same under any other capital income tax scheme for which it is shown that the long-run capital income tax should be zero. This is an important point, because overall welfare (sum of utilities) may be higher under CICIST in comparison to those other capital income tax schemes.

given by

$$\begin{aligned} \gamma \left(f(\tilde{k}) - (1-\theta)\rho\tilde{k} \right)^{-\beta} &= \left((1-\theta)\rho\tilde{k} \right)^{-\beta} \\ \frac{f(\tilde{k})}{(1-\theta)\rho\tilde{k}} &= \gamma^{\frac{1}{\beta}} + 1. \end{aligned}$$

As $r = \rho = f'$ the fraction $\frac{\rho\tilde{k}}{f(\tilde{k})} \equiv \alpha$ corresponds to the capital share in production. Hence, the optimal θ is determined by

$$\tilde{\theta} = \frac{\alpha(\gamma^{\frac{1}{\beta}} + 1) - 1}{\alpha(\gamma^{\frac{1}{\beta}} + 1)} \tag{12}$$

and is increasing in the share of capital so that distribution matters. Furthermore, the optimal long-run capital income tax rate is positive as long as

$$\gamma > \left(\frac{1-\alpha}{\alpha} \right)^{\beta}. \tag{13}$$

In the macroeconomics literature it is common to argue that α is less than one half.[15] Furthermore, there is evidence that β, that is, the *inverse* of the intertemporal elasticity of substitution of consumption between different dates is quite large. That would imply that one would need a sufficiently large γ to obtain the result that $\tilde{\theta}$ is positive in the long run.

However, if $0 \leq \gamma < \left(\frac{1-\alpha}{\alpha}\right)^{\beta}$, then $\tilde{\theta} = 0$ would follow. This is so because $\tilde{\theta} = 0$ implies $u' = q_2 = \lambda$ by (10b) and (9a). Thus, as long as $\tilde{k}$ satisfies $r = f' = \rho$ and as long as $\gamma < \left(\frac{1-\alpha}{\alpha}\right)^{\beta}$ we have $\gamma v' < u'$ so that indeed $\tilde{\theta} = 0$ is optimal in

[15] The capital share is less than one half in standard neoclassical growth models. If one assumes that capital is broadly defined as in Mankiw, Romer, and Weil (1992), then the share of capital is usually bigger than one half. In that case the condition on γ would be less demanding. Thus, if $\alpha > \frac{1}{2}$, then less political weight going to the workers would be needed to obtain the result that the capital income tax rate is nonzero in the optimum. However, in what follows I will implicitly concentrate on the more conventional case of a capital share that is less than one half.

those circumstances.

But then the result that zero capital income taxes are optimal in the long run depends on 1. the social weight attached to the workers, 2. the income share of capital in production and 3. the intertemporal elasticity of substitution.

Proposition 1 *Let the agents possess the same constant relative risk aversion utility functions. Under a capital-income-cum-investment-subsidy-tax (CICIST) scheme the optimal capital income tax rate $\tilde{\theta}$ is non-zero if the social planner attaches sufficient weight on the welfare of the workers $\gamma > \left(\frac{1-\alpha}{\alpha}\right)^{\beta}$. In contrast, if $\gamma < \left(\frac{1-\alpha}{\alpha}\right)^{\beta}$, then $\tilde{\theta} = 0$ is optimal. Hence, under CICIST the income distribution, preferences and the political weight of the workers determine whether the optimal capital income taxes are zero in the long run.*

Thus, under the (nondistorting) capital income tax scheme under consideration (CICIST) distributional and preference parameters matter and that may provide another counterexample to Judd (1985) and Chamley (1986). Importantly, the proposition establishes that there may be instances when capital income taxes are optimally non-zero in the long run. For the result here one does not need an explicit decision rule of the private sector to obtain the result. Lansing (1999) bases his counterexample on the fact that solving for the optimal private sector decision rule first may subsequently pin down the choice of consumption for a benevolent social planner. Capital income taxes may then be non-zero when the special case of logarithmic utility is considered. But here the non-zero tax result may hold even though the agents have very different utility functions or all utility functions are of the general CRRS type which includes the logarithmic one as a special case. Thus, even though the social planner only concentrates on the first order conditions of the private sector and does not explicitly know the agents' final decision rules, and even though he has freedom to choose consumption and capital independently, capital income taxes may optimally be non-zero in the long-run.

4 Conclusion

In this paper I readdress the result that capital income taxes are bad instruments for pure redistribution and should be zero in the long run.

Coupling capital income taxes with investment subsidies for financing pure redistribution may imply nonzero capital income taxes. I consider a policy package that is nondistortionary for accumulation and find that whether or not capital income taxes are optimally zero in the long run depends on probably quite realistic conditions for taxation policy. The most important conditions identified in this paper are: (a) the political power of those who receive redistributive transfers, (b) the distribution and so inequality in pre-tax factor incomes, and (c) the intertemporal elasticity of substitution.

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