



# Human Development Research Paper 2010/12

## **Improving the Measurement of Human Development**

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*Carmen Herrero*  
*Ricardo Martínez*  
*Antonio Villar*

Carmen Herrero is Professor of Economics at the University of Alicante and Senior Researcher at the Ivie. E-mail: [carmen.herrero@ua.es](mailto:carmen.herrero@ua.es).

Ricardo Martínez is Researcher at the University of Málaga. E-mail: [ricardo\\_martinez@uma.es](mailto:ricardo_martinez@uma.es).

Antonio Villar is Professor of Economics in Pablo de Olavide University and Senior Researcher at the Ivie. E-mail: [avillar@upo.es](mailto:avillar@upo.es).

Comments should be addressed by email to Antonio Villar.

# Abstract

We propose a new Human Development Index that involves a number of changes with respect to the present one, even though it keeps the basic structure of the index (namely, preserving “health”, “education” and “material wellbeing” as the three basic dimensions of human development). The first change refers to the substitution of the arithmetic mean by the geometric mean, as a way of aggregating the different dimensions in a more sensible way. The second one leads to the introduction of distributive considerations in the evaluation of material wellbeing. The last change consists of the introduction of new variables to approach health and education, looking for a higher sensitivity of the index with respect to the differences between countries. These new variables are specially indicated for the analysis of human development in highly developed countries. Besides the conceptual discussion, that includes a characterization of the chosen aggregation formula, we present a comparative analysis of this new index and the standard one, focusing on the OECD countries.

Keywords: Human Development, multiplicative indices, distributive concerns, highly developed countries, HDI(2).

JEL classification: O15, I31

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“A general goal for human development is to enhance the quality of human life. However, the concept “quality of human life” is not well defined. It is determined by a set of interrelated factors that cut across many disciplines with varied perspectives and paradigms. These include the prevailing culture, health status, economic performance, political and social conditions, the building of human capacity and capabilities, and institutional development...

However, these factors are not independent in their effects, nor do they act in harmony... (That speaks of) the complexity of measuring human development and achievement in the absence of a well-defined system of ranking.”

Ismail Sirageldin,

*Sustainable Human Development in the Twenty First Century*

## **1. Introduction**

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Improving human welfare and fostering economic development are two basic goals of most democratic Governments. Evaluating the achievements of policy measures from that perspective requires having sensible indicators for those concepts. This is not an easy task, as it implies making a number of theoretical and practical compromises concerning the dimensions involved and the way of approaching them in terms of the available data.

There is a long tradition of taking the dynamics of the GDP as the key reference measure for economic growth, which in turn is supposed to approximate the level of economic development of a society. National statistical offices provide regular information on that variable which is systematically used to evaluate the overall performance of the country (growth rate, relative position with respect to other countries, etc.). The limits of this indicator are well known: the GDP only computes market transactions, it ignores qualitative or distributive aspects, it only

provides a rough approximation of the cost of use of capital, it does not compute stocks of durables and infrastructures, etc. Yet we keep using this extremely simple indicator, partly because we are well aware of all those shortcomings and partly because it is positively correlated to several of the relevant aspects of economic development we would like to measure.

Using a single dimension to evaluate economic development appears as a weak methodological approach. A natural way of improving the analysis of economic development is, therefore, to build up multidimensional indicators that may account for several aspects related to human welfare and economic potential (environment, health, education, social integration, etc.). The construction of that type of indicators opens a whole line of research: which are the most relevant dimensions to be considered? How to approximate those dimensions by means of specific variables? How to aggregate those variables into a single indicator?

There has been a number of proposals in that direction, along different lines. Let us mention the United Nations 1954 report on the standards of living, the “basic needs approach” fostered by the International Labour Organization in 1974, the *Physical Quality of Life Index* (PQLI), due to Morris (1979) (reformulated by Ram (1982)), or that proposed by the Daj Hammarskjöld Foundation (Max-Neef (1984)). *Eurostat* has also set forth a protocol to approach sustainable development with a series of sensible indicators.<sup>1</sup> The need of multidimensional indicators for the assessment of economic development is already well established. The recent report by Stiglitz, Sen and Fitoussi (2009) is one of the last attempts to transform such a need into an institutional commitment that should lead to a change in our national accounting systems.

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<sup>1</sup> See [http://epp.eurostat.ec.europa.eu/portal/page/portal/product\\_details/publication?p\\_product\\_code=KS-68-05-551](http://epp.eurostat.ec.europa.eu/portal/page/portal/product_details/publication?p_product_code=KS-68-05-551)

The construction of those indicators requires covering three differentiated but closely related stages. First, to build up a wide agreement on the relevant dimensions to be considered. Second, to choose the variables that approximate those dimensions (variables that should be available on a regular basis through the existing statistical services). And third, to define the proper way of synthesizing those variables into an index that yields more operational the information involved and allows performing sensible comparisons.

The Human Development Index is probably the most successful multidimensional indicator nowadays. It was proposed by the United Nations in 1990 as a protocol to measure the countries' degree of development, based on Amartya Sen's idea of *functionings and capabilities* [see Sen (1985)]. This protocol identifies health, education, and material wellbeing as the key human functionings (first stage). The achievements in health, education and material wellbeing were associated with the variables *life expectancy at birth*, a mixture of *literacy rate* and *gross enrolment rate* (with weights of 2/3 and 1/3, respectively), and the *log of the standard per capita GDP*, respectively, suitably normalized (second stage). Finally, the *Human Development Index* (HDI, for short) consists of the *arithmetic mean* of the normalized values of those three variables (third stage).<sup>2</sup>

The HDI has been subject to a number of well-grounded criticisms, in spite of the improvement that it implies with respect to the mere comparison of per capita GDP values. The

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<sup>2</sup> The Human Development Index is complemented by other companion indices that focus on specific subjects, most notably gender and poverty. Two remarks are worth mentioning. One, that those indices use different types of mean in order to aggregate partial indicators. Two, that the poverty measure includes a specialized index for more developed countries. Both features are present in the proposal contained in this work.

main criticisms refer to:<sup>3</sup>

- (a) *The number and nature of the selected dimensions.* There are some relevant aspects of human development that are missing, such as social integration or sustainability.
- (b) *The choice of the variables that measure those dimensions.* Even though this is partly a practical matter (availability of data), it is not clear that the variables used to approximate health, education and material wellbeing are the most sensible ones.
- (c) *The lack of concern for distributive issues.* It is only natural to think that the level of human development should compute not only “the size of the cake”, but also the way in which it is distributed.
- (d) *The nature of the three variables involved.* This feature makes it difficult to interpret the HDI as an average value (a summary statistic of a representative agent).
- (e) *The additive structure of the index.* Aggregating the different components by the arithmetic mean has strong implications on their substitutability (linear indifference curves) and makes the index dependent on the normalization chosen for the different components.
- (f) *The lack of theoretical justification of the formula.* This makes it difficult to

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<sup>3</sup> See the contributions in Anand & Sen (1994 a, b), Hicks (1997), Sagar & Najam (1999), Osberg & Sharpe (2002), Philipson & Soares (2001), Pinilla & Goerlich (2003), Foster, López-Calva & Székely (2003), Becker, Philipson & Soares (2005), Stiglitz, Sen & Fitoussi (2009), or Herrero, Soler & Villar (2010).



analyze the suitability of this index vis a vis other alternatives. Moreover, it induces the use of the HDI as an ordinal measure (a criterion to produce a ranking) and not as a cardinal measure that would help evaluating the *size* of the differences between countries.<sup>4</sup>

Applied work has also pointed out the scarce sensitivity of the HDI when applied to developed countries. The reasons are clear. On the one hand, the type of variables chosen to approximate the three selected dimensions: life expectancy at birth tends to overweight the health component of those countries with a less dynamic demography; the index of education hardly reflects the existing human capital due to the excessive weight given to the literacy rate; and the use of logs flattens income differences. On the other hand, the aggregation formula: the arithmetic mean pays no attention to the dispersion of the values of the three components.

There are different ways to modify and improve the HDI and its companion indices, in order to provide a better picture of the degree of development of a society. Let us comment on some of them as a way of clarifying the nature of our proposal.

Adding new dimensions is clearly one of those ways of improvement. In particular dimensions related to the sustainability of the society, the availability of infrastructures, the presence of social conflicts, the degree of social integration, or the basic rights of the citizens, to name a few. This is an important venue that requires reaching a consensus not only on the

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<sup>4</sup> This feature has raised some scepticism. Some researchers argue that the ranking produced by the HDI is not very different from that stemming from the per capita GDP, so that there is not a great need of such a multidimensional indicator (see for instance: Justin Wolfers, What does Human Development Measures? <http://freakonomics.blogs.nytimes.com/2009/05/22/what-does-the-human-development-index-measure/>).

additional dimensions to be considered, but also on the variables that measure them and the weights with which they should enter the final formula. Note also that some of the data needed to implement an index of this sort are not provided by the standard statistical offices. Therefore, a structural change of this nature, that is certainly needed, calls for a sound and presumably long term work involving changes in the national accounting systems.

This is *not* the nature of the alternative formulation presented here. We rather focus on some *modifications of the present index* that try to make it a more suitable measure of human development. Our proposal stems from three different considerations. First, that changes in the way of measuring human development should not be too drastic in order to keep at least part of the achievements obtained so far in providing a measure that goes beyond the GDP (this is partly a cultural asset that is to be protected and partly an operational interest in keeping track of the former work). Second, that the additive structure of the index involves relevant shortcomings, as it makes the resulting ranking dependent on the normalization process. And third, that some of the deficiencies of the HDI mentioned above are specially relevant when we apply this methodology to highly developed countries, which is an incentive not to use them as a relevant source of information on a regular basis.

We therefore propose a new approach to define the human development index that may actually result in two different indicators, one of them specialized for highly developed countries. The improvements we propose refer to the following elements:

- (i) The use of the geometric mean of the components, rather than the arithmetic mean, as a way of aggregating the three selected indicators, under a suitable theoretical justification.
- (ii) The introduction of distributive considerations, as we believe distributional aspects

are part of the basic features of the socio-economic performance.<sup>5</sup>

- (iii) A change of the variables that measure the dimensions concerning health and education, bearing in mind the availability of data. We look for a better way of capturing the differences between developed countries.

We understand that the first two modifications are applicable to the HDI in general, whereas the change of variables proposed here is specially needed for the case of highly developed countries. The idea of designing an index specific for highly developed countries is conceptually a parallel exercise to that of the poverty index for some selected OECD countries, already set up by the United Nations. That is why we propose to call this special index the HDI(2).

This is not the first proposal that tries to provide a better approximation to the measurement of human development within developed countries. The *American Human Development Index*<sup>6</sup> is an alternative indicator designed to face some of the shortcoming already mentioned when dealing with highly developed countries. It keeps life expectancy at birth as the variable that measures health, modifies the education variable in order to give more weight to the upper level

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<sup>5</sup> Taking care of distributional aspects may require a different definition of the variables that measure the achievements in health, education and material wellbeing. This is so because the standard way of introducing distributive considerations refers to the dispersion of the variable with respect to some average that can be regarded as the value of a representative agent [see however the proposal in Grim et al. (2008)]. The present way of measuring those variables, does not allow a clear interpretation of the index as some average value. As an illustration, consider the case of income. The use of logs in the income indicator makes it difficult to introduce distributive considerations. It is true that taking the log of the income permits one to interpret the resulting value as a welfare measure. Yet this principle is not applied to other variables and, from a descriptive viewpoint, hides again part of the existing differences between countries. This is especially arguable when we compare countries with a similar degree of development.

<sup>6</sup> See Burd-Sharps (2008), and <http://www.measureofamerica.org/>

of studies (much in line with our approach), and substitutes the log of the per capita GDP by the log of the median income, applying their index to compare the States in the US. Herrero, Soler & Villar (2010), analyze the evolution of human development in Spain and its regions between 1980 and 2007. They provide, besides the standard analysis, an alternative HDI close to our proposal here.

The rest of the paper goes as follows. Section 2 is devoted to the analysis of the alternative aggregation formula, the geometric mean, including an easy characterization that conveys theoretical support to this formula and permits its use in a cardinal sense. Section 3 introduces a new measure of material wellbeing that takes into account explicitly the income distribution (the inequality adjusted per capita GDP). Section 4 presents our proposals to approximate health (*life potential*) and education (*expected years of schooling over compulsory education*). Those variables are specifically designed for highly developed countries. Sections 3 and 4 include an empirical illustration of the impact of the proposed changes in 26 OECD countries with data of 2007.<sup>7</sup> Section 5 further extends the empirical analysis and provides a comparison of the results derived of using the HDI(2) with respect to the traditional HDI for the selected OECD countries. Section 6 gathers some final comments and recommendations.

## **2. The new aggregation formula**

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<sup>7</sup> We have excluded some countries for missing data (México and Turkey) and also Luxembourg for a different reason (it is a very small country with an extremely high per capita GDP).

## ***2.1. The additive structure of the HDI***

The use of the arithmetic mean as a way of aggregating the three partial indices chosen to approximate health, education and material wellbeing, is a notorious source of discomfort. On the one hand, the additive structure of the index implies a very peculiar trade-off between the different components. More specifically, such a structure amounts to assuming full substitutability between all components (linear indifference curves). On the other hand, the index generates a ranking that is sensitive to the normalization of the different variables (this is so because changing the normalization amounts to modifying the weights with which those variables enter the index).

From a different perspective, both shortcomings (the lack of theoretical justification of this formula and the sensitivity to the normalization) make it difficult to use it as a cardinal measure, a point always made clear by the Human Development Report. Yet having an idea of the relative distances between countries would clearly improve the informative content of the index.

**Example 1:** Suppose that we are willing to change the present binding caps (e.g. \$100 and \$40,000 for income; 99% for adult literacy and 100% for gross enrolment), because an increasing number of countries exceed the upper limits. Assume that, in order to keep all indices within the interval  $[0,1]$  the actual maximum and minimum values are chosen. Imagine that the country with the highest income value in (t-1), let us call it A, significantly increases its income

at time  $t$ , while the rest of the world stays the same. To be precise, suppose that the difference between max and min doubles. The effect on the rest of the countries is that the contribution of the income in  $t$  dimension is one half of that in  $(t-1)$ . That may change the relative order of the countries whose original values have not changed at all. That is the case of countries B and C whose partial indices at  $(t-1)$  were:  $B(t-1) = (0.1, 0.2, 0.3)$  and  $C(t-1) = (0.2, 0.2, 0.22)$ . The HDI tells us that country B is worse off than country A. Now, due to the change in the scale induced by the income increase of country A, we find the following values:  $B(t) = (0.05, 0.2, 0.3)$ ,  $C(t-1) = (0.1, 0.2, 0.22)$ . Now country B happens to appear above country C in the ranking, without none of them having experienced any change. This is a major drawback of the proposed normalization when linked to the additive formula.

We propose a twofold modification of the aggregation procedure for the HDI. On the one hand, normalizing the variables that measure health, education and material wellbeing in terms of the percentage of a maximum value (which amounts to setting all min values equal to zero). On the other hand, and this is the major change, substituting the arithmetic mean of the normalized variables by the geometric mean. The geometric mean can be characterized in terms of reasonable axioms (e.g. Herrero, Martínez & Villar (2010)) and exhibits much better properties than the arithmetic mean in this context.

We obtain in this way an index that solves the drawbacks mentioned above as it produces a ranking that does not depend on the choice of units of the different dimensions and, moreover, exhibits a decreasing rate of substitution between the variables, as the standard theory suggests.

Observe that the geometric mean is nothing else than the generalized mean of order 0, whereas the arithmetic mean is that of order 1. This is in line with the construction of other indicators of human development that use generalized mean of order -1 (the harmonic mean, to

measure gender discrimination) or that of order -3 (to measure poverty).<sup>8</sup> The geometric mean (and all generalized means of a smaller order) penalizes the differences in the values of the constituent variables. That is, this type of index takes into account negatively the dispersion of the partial indices that are being aggregated.<sup>9</sup> This implies that, in order to have a high position in the ranking produced by this indicator, one has to have high marks in all constituent variables and not only in some of them.

We present in section 2.2 a formal characterization of the geometric mean in terms of simple axioms. We show that this aggregation formula can be characterized by means of three properties: *Neutrality* (all normalized characteristics are equally important), *Scale* (if all variables are equal then the index takes on that very same value), and *Ratio Consistency* (a common change in the value of a variable keeps constant the ratio of the initial values).

Assuming those properties amounts, therefore, to evaluate a vector of three components  $(\alpha, \beta, \gamma)$  referring to normalized values of health, education and material wellbeing, as follows:

$$I(\alpha, \beta, \gamma) = (\alpha)^{1/3} (\beta)^{1/3} (\gamma)^{1/3}$$

The axiomatic support of the aggregation formula not only suppresses the arbitrary nature of the index but also conveys a cardinal dimension. As a consequence, we can perform quantitative comparisons and not only to generate and ordinal ranking.

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<sup>8</sup> There are several contributions that suggest the use of generalized means in this context, including alternative characterizations. See Foster, López-Calva & Székely (2005), Seth (2009), (2010), Villar (2009).

<sup>9</sup> See the discussion in Seth (2009), (2010).

## ***2.2. A pinch of theory: Axiomatization of the new index***

There are different ways of choosing an aggregation formula, in order to synthesize several indicators into a single number. One may recur to persuasion or invoke tradition to defend an intuitive and sensible aggregation function. For instance, one may think of the arithmetic mean, the geometric mean, the harmonic mean, or any other generalized mean. They are standard aggregators, simple to compute, well known, and are widely used (also in the Human Development Reports). The problem is, of course, how to choose among them, why one and not the other. This requires a comparative analysis of the implications of their use prior to selecting the formula that fits better our purposes.

An alternative way, with a long tradition in economics, is to choose solutions to economic problems on the basis of the normative and/or operational properties of those solutions. This is the so-called *axiomatic method*<sup>10</sup> that aims at identifying a solution function with a unique set of intuitive properties or *axioms*. In that case, choosing a given set of axioms turns out to be equivalent to choosing a given formula. We apply here this methodological approach in order to identify the geometric mean as the only way to aggregate the achievements in health, education and well-being that satisfies three intuitive requirements: *neutrality*, *scale*, and *ratio consistency*.

Let us formalize these ideas.

Suppose that we want to define an evaluation index for a given society that aggregates

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<sup>10</sup> See Thomson (2001) for a wide exposition of the advantages and disadvantages of the axiomatic method.



three (normalized) variables.<sup>11</sup> We define a **social state** as a vector  $(\alpha, \beta, \gamma)$  with 3 components, each of which belongs to the interval  $[0,1]$  (that is, the values those characteristics are already normalized so that the differences in their mean values have been cancelled). Therefore,  $\Omega = [0,1]^3$  is the space of admissible social states.

A **Social Evaluation Index** is a continuous single-valued mapping  $I: \Omega \rightarrow \mathbf{R}$  that provides a numerical evaluation of social states.<sup>12</sup>

We first introduce two basic requirements on the social evaluation index: neutrality and scale. *Neutrality* makes it explicit that all characteristics enter the evaluation function on an equal foot. That can be formalized by requiring that a permutation of the characteristics does not affect the social evaluation (recall that all variables vary in the interval  $[0, 1]$ , so that the differences in the units of measurement have already been neutralized). *Scale* fixes the value of the index when the social state is uniform (i.e. all entries are identical), by choosing precisely that very same value. Formally:

- **Neutrality.** For each point  $(\alpha, \beta, \gamma) \in \Omega$ , if  $\pi(\alpha, \beta, \gamma)$  denotes a permutation of its elements, then:  $I[\pi(\alpha, \beta, \gamma)] = I(\alpha, \beta, \gamma)$ .

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<sup>11</sup> Let us remark that the argument below can be extended to any arbitrary (finite) number of components (at the cost of making the proof much more cumbersome). For a more general approach see Herrero, Martínez & Villar (2010) or Villar (2009).

<sup>12</sup> Note that we introduce the requirement of continuity in the very definition of the index. That is, we focus our discussion on those mappings for which small changes in the variables imply small changes in the index.

- **Scale.** Let  $p \in [0,1]$ . Then,  $I(p, p, p) = p$ .

The last property, *Ratio Consistency*, requires that the relative value of the indices of two social states with a common component does not depend on the value of that common component. Formally:

- **Ratio Consistency.** Let  $(\alpha, \beta, \gamma), (\alpha, \beta', \gamma') \in \Omega$  be two strictly positive social state vectors, with the same first component  $\alpha$ . If that common component changes to a different one,  $\alpha'$ , the ratio of the associated indices does not change.

That is,

$$\frac{I(\alpha, \beta, \gamma)}{I(\alpha, \beta', \gamma')} = \frac{I(\alpha', \beta, \gamma)}{I(\alpha', \beta', \gamma')}$$

This property says the following. Suppose that countries A and B have the same values concerning the health variable and different values with respect to education and material wellbeing, all positive, so that the overall index of country A is twice that of country B. Now both countries experience an improvement in health that changes the corresponding variable by exactly the same amount. That change obviously alters the associated development indices. Ratio Consistency implies that the new index of country A is still twice the new index of country B. That is to say, the relative value of the index is not affected by an equal change of a common value of a given variable.

**Remark.-** We ask this property to hold just for one component (the first one, in our definition)

for the sake of parsimony. When combined with “neutrality” this property actually applies to any common value.

Note that this consistency requirement is cardinal in nature and involves a separability feature in the evaluation index.

The following result is obtained:

**Theorem:** A social evaluation index  $I(\cdot)$  satisfies neutrality, scale and ratio consistency, if and only if it takes the form:

$$I(\alpha, \beta, \gamma) = \alpha^{1/3} \beta^{1/3} \gamma^{1/3}$$

Moreover, those properties are independent.

Proof:-

First note that scale implies  $I(\alpha, \alpha, \alpha) = \alpha$ , for all  $\alpha \in [0, 1]$ . By neutrality, the property of ratio consistency can be applied to any component of vector  $(\alpha, \beta, \gamma)$ .

Take now a vector  $(\alpha, \beta, \gamma) \gg 0$ . By ratio consistency and neutrality we can write:

$$\frac{I(\alpha, \alpha, \alpha)}{I(\alpha, \alpha, \beta)} = \frac{I(\alpha, \alpha, \alpha)}{I(\beta, \alpha, \alpha)} = \frac{I(\alpha, \beta, \gamma)}{I(\beta, \beta, \gamma)}$$

And thus,

$$I(\alpha, \alpha, \beta) = \alpha \frac{I(\beta, \beta, \gamma)}{I(\alpha, \beta, \gamma)}$$

In a similar way,  $\frac{I(\beta, \beta, \beta)}{I(\beta, \beta, \gamma)} = \frac{I(\alpha, \beta, \gamma)}{I(\alpha, \gamma, \gamma)}$ , so that  $I(\beta, \beta, \gamma) = \beta \frac{I(\gamma, \gamma, \alpha)}{I(\alpha, \beta, \gamma)}$ , and

$$I(\alpha, \alpha, \beta) = \alpha \beta \frac{I(\gamma, \gamma, \alpha)}{[I(\alpha, \beta, \gamma)]^2}$$

Finally,  $I(\gamma, \gamma, \beta) = \gamma \frac{I(\alpha, \alpha, \beta)}{I(\alpha, \beta, \gamma)}$ . Substituting,  $I(\alpha, \alpha, \beta) = \alpha \beta \gamma \frac{I(\alpha, \alpha, \beta)}{[I(\alpha, \beta, \gamma)]^3}$ .

Therefore, we conclude:

$$I(\alpha, \beta, \gamma) = \alpha^{1/3} \beta^{1/3} \gamma^{1/3}$$

(ii) To separate the properties let us consider the following indices:

(1)  $I(\alpha, \beta, \gamma) = \frac{1}{3}(\alpha + \beta + \gamma)$ . It satisfies neutrality, scale but not ratio consistency.

(2)  $I(\alpha, \beta, \gamma) = \frac{\alpha\beta\gamma}{3}$ . It satisfies neutrality, ratio consistency and not scale.

(3)  $I(\alpha, \beta, \gamma) = \alpha^a \beta^b \gamma^c$ , with  $a + b + c = 1$  and not all of them are equal. It satisfies ratio consistency and scale but not neutrality. **Q.e.d.**

This theorem says that, among those indices that satisfy neutrality and scale, ratio consistency determines that the evaluation formula is given by the geometric mean of the

corresponding normalized values of the chosen variables. Note that this aggregator is a special case of the family of the generalized means of order  $q \in \mathbf{R}$ ,  $\mu_q = \left[ \frac{1}{k} \sum_{j \in K} x_j^q \right]^{\frac{1}{q}}$ , for  $q \neq 0$ .

It is worth noting that if we change the units of any dimension in the normalization of its component, not only the ranking of the countries is preserved, but also the relative size of the indices. Thus, the new HDI allows for ordinal and cardinal comparisons among the countries.

**Remark.-** *This Theorem differs from the characterization in Herrero, Martínez & Villar (2010) in two respects. One, that it refers to normalized values at the population level, rather than individual values (there a social state is a matrix that describes the distribution of each characteristic within the population). And two, that it uses a stronger separability property (ratio consistency) but it requires neither “minimal lower boundedness” nor “monotonicity”.*<sup>13</sup>

### **2.3. To log or not to log? That is (not) the question**

There is some discussion about the use of logs in all variables, as a way of recognizing that the effect on human development of one additional unit of a variable does depend on the present level of the variable. The concavity that the log function introduces would capture naturally this fact (even though it flattens the differences between countries).

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<sup>13</sup> Minimal lower boundedness says that if a column is zero, then the index is zero. This property is obviously implied by the combination of our axioms. Monotonicity, however, is neither implied nor required, which gives us more flexibility in the definition of the egalitarian equivalent value.

The rationale for using logs or not is linked to the notion of development we try to capture with our index. If we think of the HDI more as a *welfare measure*, it might be reasonable to keep measuring the three variables in terms of logs (or in terms of any increasing and concave function) as those values can be interpreted as utility measures of an average citizen.<sup>14</sup> If we rather think of the HDI as a capability indicator that provides a reasonable description of the ability of a country to grow, compete and enhance material wellbeing, as we actually do, the use of logs does not seem justified.

One may argue that the *HDI* with logs is ordinally equivalent to our proposed index (letting aside the differences in the chosen variables). This not quite so and the difference is relevant. Taking logs of our index yields the following expression:

$$\log I(\alpha, \beta, \gamma) = \frac{1}{3} \log(\alpha) + \frac{1}{3} \log(\beta) + \frac{1}{3} \log(\gamma)$$

where  $\alpha, \beta, \gamma$  are the normalized variables that measure health, education and material wellbeing. Note that here *we first normalize and then take logs* of the normalized variables. In that way the properties of the index concerning the robustness with respect to changes in units is preserved.

The *HDI* with variables in logs, however, adopts the following expression:

$$HDI^{\log} = \frac{1}{3}(\alpha') + \frac{1}{3}(\beta') + \frac{1}{3}(\gamma')$$

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<sup>14</sup> That interpretation would also suggest substituting life expectancy at Barth by some index of quality adjusted life years, much in the tradition of health economics.

where  $\lambda' = \frac{\log \lambda - \log(\min \lambda)}{\log(\max \lambda) - \log(\min \lambda)}$ , for  $\lambda = \alpha, \beta, \gamma$ . Here the effect of the normalization on the

ranking does not disappear, because *we first take logs and then normalize*.

### **3. The concern for equality: Inequality-adjusted income**

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The lack of concern for distributive issues in the income dimension of human welfare is perhaps one of the most surprising features of the HDI. There are statistics that approximate inequality for most of the countries and we have a well-established theory that permits one to link the evaluation of the size and distribution of income simultaneously. Moreover, there are already a number of contributions that suggest ways of introducing equality concerns in this particular context.<sup>15</sup>

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<sup>15</sup> See, for instance, Anand & Sen (1994b), Hicks (1997), Foster, López-Calva & Székely (2005), Herrero, C., Martínez, R. & Villar, A. (2010), Seth (2009), (2010), Villar (2009).

**Example 2:**<sup>16</sup> South Africa exhibits a per capita GDP that is about 1% higher than that of Panama (5.914 US\$ and 5.833 US\$, respectively). Yet, the Gini index in South Africa is 37% higher than in Panama (0.74 and 0.54, respectively). Where a newborn has *ex ante* better chances in life? We strongly believe that ignoring distributive aspects does not help to assess the level of human development. Needless to say, the same reasoning applies to highly developed countries. Take the case of United States and Austria, for instance. Here again USA has a per capita GDP 1% higher than Austria (45.592 US\$ and 44.879 US\$, respectively), while the inequality is 37% higher (0.4 and 0.29, respectively).

Material wellbeing is to be measured, according to our proposal, by the per capita GDP suitably adjusted by the income distribution. As in the UN traditional methodology, we assume that the standard per capita GDP, expressed in terms of PPP US dollars, is the basic variable. We propose, however, to suppress the use of logs in order to fully capture the differences among countries in that aspect. Moreover, and this is also a relevant part of the proposal, we deflate that figure by the corresponding inequality index.

The standard way of conveying a normative content to an inequality measure is that of interpreting inequality as a welfare loss, in the tradition of Dalton, Atkinson, Sen and Kolm, to name a few representative thinkers. To do so let  $\mathbf{y} = (y_1, y_2, \dots, y_n)$  denote the income distribution of a society and let  $W(\mathbf{y})$  a social welfare measure of that distribution. Then, define the *egalitarian equivalent income*,  $y^e$ , as that amount of income that equally distributed would yield the same social welfare than the current income distribution. That is,  $y^e$  is the value that

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<sup>16</sup> Data from the *Human Development Report 2009*.



satisfies the following equation:

$$W(y_1, y_2, \dots, y_n) = W(y^e, y^e, \dots, y^e)$$

This value  $y^e$  always exists, provided  $W$  is a continuous function defined on a compact domain. Moreover, under reasonable hypothesis (quasi-concavity) the egalitarian equivalent income is always below per capita income. We can, therefore, define an inequality measure as follows:

$$I(\mathbf{y}) = 1 - \frac{y^e}{\mu(\mathbf{y})}$$

where  $\mu(\mathbf{y})$  is the mean income. This formula tells us that inequality can be understood as the welfare loss due to the difference between the egalitarian equivalent income and the mean value.

This can be rewritten as follows:

$$y^e = \mu(\mathbf{y})[1 - I(\mathbf{y})]$$

This is the type of indicator we propose for the measurement of material wellbeing. The choice of the right inequality index can be done making use of the properties we deem relevant (e.g. decomposability, degree of preference for equality, etc.).<sup>17</sup>

For the sake of the empirical application presented below, we propose the use of the Gini coefficient as a sensible way of measuring inequality. That is, we shall measure material wellbeing in terms of the following inequality adjusted income:

$$IAI_G = GDP_{pc}[1 - G]$$

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<sup>17</sup> See, for instance, Cowell (1995), Sen & Foster (1997), Goerlich & Villar (2009).

$GDP_{pc}$  is the per capita Gross Domestic Product (as an approximation of the mean income) and  $G$  the Gini coefficient that measures income dispersion. By so doing, we deflate the  $GDP_{pc}$  by inequality, measured with the Gini coefficient. Therefore, if we find two societies with identical  $GDP_{pc}$ , we consider more developed the one which is more egalitarian.

The Gini coefficient has some well-known shortcomings, such as the lack of additive decomposability or the insensitivity to the size of the income differences (a property sometimes called “homothetic distributivity”). And it has also many advantages, derived from the multiple ways of writing and interpreting this index, easily derived from the Lorenz curve. Be as it may, it is the most frequently used inequality measure in empirical work and therefore has become an index supplied regularly by most statistical offices.

The data required to construct this variable can be obtained from United Nations (Human Development Report 2009)<sup>18</sup> and the OECD website.<sup>19</sup>

Table 1: The Gini Coefficient, the per capita GDP and the Inequality Adjusted Income in OECD countries (2007)

	Gini	pcGDP	Ranking pcGDP	IAI(G)	Ranking IAI(G)	Difference
Norway	0.28	53433	1	38471.76	1	0
Ireland	0.33	44613	3	29890.71	2	1
Switzerland	0.28	40658	4	29273.76	3	1
Sweden	0.23	36712	7	28268.24	4	3
United States	0.38	45592	2	28267.04	5	-3
Netherlands	0.27	38694	5	28246.62	6	-1
Denmark	0.23	36130	8	27820.1	7	1
Austria	0.27	37370	6	27280.1	8	-2

<sup>18</sup> Data on per capita GDP, expressed in terms of PPP 2005 dollars.

<sup>19</sup> Dataset: Income distribution, Inequality, Income and population measures, Gini coefficient after taxes.

Iceland	0.28	35742	10	25734.24	9	1
Belgium	0.27	34935	12	25502.55	10	2
Finland	0.27	34526	14	25203.98	11	3
Australia	0.3	34923	13	24446.1	12	1
Canada	0.32	34812	9	24352.16	13	-4
France	0.28	33674	16	24245.28	14	2
Germany	0.3	34401	15	24080.7	15	0
U. Kingdom	0.34	35130	11	23185.8	16	-5
Japan	0.32	33632	17	22869.76	17	0
Spain	0.32	31560	18	21460.8	18	0
Italy	0.35	30353	19	19729.45	19	0
Greece	0.32	28517	20	19391.56	20	0
New Zealand	0.34	27336	21	18041.76	21	0
Czech Rep.	0.27	24144	22	17625.12	22	0
Slovak Rep.	0.27	20076	24	14655.48	23	1
Portugal	0.38	22765	23	14114.3	24	-1
Hungary	0.29	18755	25	13316.05	25	0
Poland	0.37	15987	26	10071.81	26	0
Average	0.31	33248.85		23290.20		
Coef. Variation	0.1307	0.2485		0.2613		

Table 1 provides the basic data for most OECD countries. Introducing the inequality deflator increases the coefficient of variation in some 5 % and produces small changes in the ranking.<sup>20</sup> Note, however, that the UN index of material wellbeing involves taking logs of that variable, which produces an enormous reduction in the variability (as we shall see later, the coefficient of variation of that index in the HDI is around eight times smaller than the per capita GDP).

<sup>20</sup> Let us recall that the coefficient of variation is a dispersion measure consisting of the ratio between the standard deviation and the average, that is unit-free.

## 4. The new variables (specially fit for highly developed countries)

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We propose here a new set of variables that allows for a more accurate approximation of the health and education dimensions and are intended to improve the sensitivity of those partial indicators (most specially in highly developed countries). Besides, we aim at using a set of variables that permits one to interpret the resulting HDI as an average (the value of a representative agent of the society under consideration).<sup>21</sup>

Needless to say, there are several alternative ways of modifying the existing variables in order to achieve those goals. What the best alternative indicator is depends on data availability, always a limiting factor, and on the “domain” of application, that is, the universe of societies on which we want to apply the indicator. Bearing in mind the restriction on data availability, we focus here on the domain consisting of highly developed countries. We believe that a multidimensional index of this nature requires some adjustments depending on the level of development of different groups of countries (as it is already acknowledged when analyzing poverty measures and also in the presentation of the HDI figures). This is important because the adherence to new measurement standards requires those new indicators to provide a better description of the reality they refer to. This is not the case so far for highly developed countries

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<sup>21</sup> Note that the standard HDI is a composition of three variables that are very different in nature. The per capita GDP (without logs!) can be interpreted in terms of the expected value of an individual picked at random in this society. Life expectancy at birth may be interpreted this way only with respect to the newborn, but it tells very little about the whole population. The combination of the literacy index and the gross enrolment rates generates a variable of still a different nature that cannot be nailed down to any sensible expected value. So the aggregation of those three variables cannot be interpreted in terms of a representative individual.

with the present HDI, for reasons already discussed.

#### ***4.1. Health: Life potential***

Life expectancy (at birth) is a variable constructed in such a way that it turns out to be independent on the demographic structure. Besides, it tends to over-weight the health component of those countries with a higher share of old people. This last aspect is most arguable in the context of evaluating development capabilities for it ignores the differences in the present and future working age population.

Most developed countries exhibit very high values of life expectancy at birth, with a small variance, while they exhibit more relevant differences in the demographic structure (in particular in the share of young people in the population, that in some cases is linked to the arrival of new immigrants in late years). We believe that those differences are actually more important in developed countries than those corresponding to life expectancy at birth, when we come to assess human development possibilities. We therefore propose to substitute this variable for that of “life potential”.

*Life Potential* measures the life expectancy of a representative individual in the population (Goerlich & Pinilla (2005)). To define this variable we first consider the number of years that individuals of age  $x$  are expected to live at time  $t$  (typically the present) and aggregate them:

$$B = \sum_{x=0}^{\infty} N_x e_x$$

Here,  $N_x$  is the number of people of age  $x$  and  $e_x$  is the expected number of years that people of that age will live. Life potential obtains from taking the per capita value of this variable. That is,

$$b = \frac{1}{N} \sum_{x=0}^{\infty} N_x e_x ,$$

$N$  being the population size. This variable provides a measure of the average life expectancy of the population, taking into account its demographic structure.

The data required to calculate the life potential index can be obtained from the Human Mortality Database that provides both the life tables and the distribution of the population by age for almost all the countries in the OECD.<sup>22</sup>

One may argue that life potential is still further away from approaching the health condition of a population than life expectancy. True as that may be, we understand that the capability approach that informs the construction of human development indices is better served by this variable which incorporates an indirect estimate of potential economic growth in terms of the structure of the labour force.

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<sup>22</sup> See <http://www.mortality.org/>. See also the Health Database of the OECD, or the Life Tables of the World Health.

Table 2: Life Expectancy and Life Potential per capita in OECD countries (2007)

	<b>LE (years)</b>	<b>Ranking LE (RLE)</b>	<b>LP (years)</b>	<b>Ranking LP (RLP)</b>	<b>Difference (RLE-RLP)</b>
Australia	81.4	4	45.96	2	2
Austria	79.9	12	41.68	14	-2
Belgium	79.48	16	41.47	15	1
Canada	80.62	9	44.31	5	4
Czech Republic	76.36	23	39.25	25	-2
Denmark	78.22	22	40.98	19	3
Finland	79.48	17	41.35	17	0
France	80.98	6	43.82	7	-1
Germany	79.78	14	39.81	23	-9
Greece	79.12	19	40.15	21	-2
Hungary	73.3	26	36.80	26	0
Iceland	81.76	2	46.75	1	1
Ireland	79.66	15	45.92	3	12
Italy	81.1	5	41.14	18	-13
Japan	82.66	1	41.38	16	-15
Netherlands	79.84	13	42.79	10	3
New Zealand	80.14	11	45.68	4	7
Norway	80.5	10	43.69	8	2
Poland	75.52	24	40.10	22	2
Portugal	78.58	21	40.87	20	1
Slovak Republic	74.62	25	39.64	24	1
Spain	80.74	8	42.62	11	-3
Sweden	80.8	7	42.20	13	-6
Switzerland	81.7	3	43.09	9	-6
U. Kingdom	79.36	18	42.32	12	6
United States	79.12	20	44.02	6	14
Average	79.4131		42.2230		
Coef. Of variation	0.0276		0.0550		

Table 2 compares the data on life expectancy at birth and life potential in 26 selected OECD countries. The picture we get with one or the other variable is rather different. To start with, the coefficient of variation of life potential is twice that of life expectancy (i.e. life potential

discriminates much more than life expectancy). The rankings produced by both measures are rather different as well. The last column of the table tells us about the changes in the ranking, to be interpreted as the number of positions that a country advances in the ranking of life potential with respect to that of life expectancy. Note that Japan and Italy lose 15 and 13 positions, respectively, whereas the United States advances 14 and Ireland 12.

#### ***4.2. Education: Expected years of schooling***

The index of education used in the HDI does not reflect the differences in human capital in developed countries. That is mostly due to the excessive weight given to the literacy rate. In countries with a well-established compulsory education system (that in many cases involves more than 9 years of school attendance), the present index of education hides most of the relevant differences. It is basically non-compulsory education what makes the difference, as it reflects the society's investment in human capital beyond what is legally required. Moreover, this variable exhibits much larger differences between developed countries than that of compulsory education, which is rather uniform.

There are several ways of giving more weight to non-compulsory education. One is to add up all students that reach some reference level, as this implies that those with higher level of studies are computed several times (that is the approach followed by the American Human Development Index). An alternative way is computing the average number of years of school attendance or the percentage of working age population with non-compulsory studies (as in Herrero, Soler & Villar (2010)). Here we propose to use the variable "expected years of education between 15 and 29" used by the OECD when assessing education and the labour



market.<sup>23</sup> This variable has several advantages: it is already used as an international standard, it captures precisely the extent of non-compulsory education,<sup>24</sup> and it has the nature of an average.

The expected years of education at 15, in summary, is a variable that permits to capture much better the differences in human capital among developed countries. Let us mention that, on average, a person living in an OECD country who is 15 years-old in 2007 can expect to remain in school for an additional 6.9 years. Yet the differences range from four years (e.g. Turkey) to more than eight (e.g. Finland, Iceland).

The data on that variable are available from the OECD.<sup>25</sup>

Table 3 below shows the differences between both forms of measuring the educational achievements. We present the data corresponding to the “gross enrolment rate” and the “UN index of education” to illustrate the effect of the type of index chosen (i.e. the weight of the literacy rate on the overall education measure). Differences are outstanding: the coefficient of variation of the expected years of schooling (EYS) is 50 % higher than that of the gross enrolment rate and six times higher than that corresponding to the index of education. The last column gives us, as in the former table, the number of positions that a country gains or loses when using the variable expected years of schooling rather than the gross enrolment rate.<sup>26</sup> Differences are simply outstanding: nine countries out of 26 move more than ten positions upwards or downwards!

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<sup>23</sup> See <http://www.oecd.org/dataoecd/41/25/43636332.pdf>

<sup>24</sup> Compulsory education in most developed countries ends at 15 years.

<sup>25</sup> See <http://stats.oecd.org/index.aspx>, <http://dx.doi.org/10.1787/664770480457>.

<sup>26</sup> We think it more sensible the comparison of the EYS variable with the GER variable as the index of education involves an elaboration that distorts the meaning of the comparison.

Table 3: Expected Years of Schooling (EYS), Gross Enrolment Rates (GER) and UN Education Index (EI) in OECD countries (2007)

	GER	EI	Ranking GER	EYS	Ranking EYS	Difference
Australia	114.2	0.993	1	6.8	10	-9
Austria	90.5	0.962	17	6.5	18	-1
Belgium	94.3	0.974	13	6.8	11	2
Canada	99.3	0.991	6	6.5	19	-13
Czech Republic	83.4	0.938	24	6.7	14	10
Denmark	101.3	0.993	5	7.8	6	-1
Finland	101.4	0.993	4	8.5	1	3
France	95.4	0.978	12	7.6	7	5
Germany	88.1	0.954	21	7.9	5	16
Greece	101.6	0.981	3	6.4	20	-17
Hungary	90.2	0.96	18	7.3	9	9
Iceland	96	0.98	11	8.5	2	9
Ireland	97.6	0.985	8	5	26	-18
Italy	91.8	0.965	16	6.7	15	1
Japan	86.6	0.949	23	5.8	24	-1
Netherlands	97.5	0.985	9	8	3	6
New Zealand	107.5	0.993	2	6.8	12	-10
Norway	98.6	0.989	7	6.7	16	-9
Poland	87.7	0.952	22	8	4	18
Portugal	88.8	0.929	20	5.9	23	-3
Slovak Republic	80.5	0.928	26	6.1	21	5
Spain	96.5	0.975	10	5.4	25	-15
Sweden	94.3	0.974	14	7.5	8	6
Switzerland	82.7	0.936	25	6.8	13	12
United Kingdom	89.2	0.957	19	6	22	-3
United States	92.4	0.968	15	6.7	17	-2
Average	94.1308	0.9685		6.8731		
Coef. of variation	0.08010	0.0209		0.1296		

Source: OECD

## 5. The HDI(2) vis a vis the HDI in the OECD

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In former sections we have introduced and discussed the alternative variables we propose to measure the achievements in health, education and material wellbeing. We have also provided a comparison between those new variables and the ones traditionally used by UN.

We call HDI(2) to the index that uses all the new variables and the new aggregation formula.

We present here an empirical illustration on the ways in which the HDI(2) measures human development, relative to the standard HDI, with respect to the 26 selected OECD countries (data corresponding to 2007). The comparative analysis is made not only with respect to the final indices, but also with respect to their three different components in order to underline the effects of the normalization choices. We analyze the changes in the values as well as the changes in the ranking.

Following the approach of the construction of the HDI, we first normalize the partial indices so that all they range within the interval  $[0,1]$ . In order to do so we choose a high enough value for each variable and express each measure as the fraction of that reference value (i.e. we simply divide the actual value by the reference one, which amounts to set the minimum value of each variable equal to zero). In this way our indicator turns out to be robust with respect to the choice of the reference values: both the ranking obtained and the relative values of any pair of countries are independent on the normalization chosen. We have, therefore, a measure that

allows us to perform cardinal comparisons besides comparing the ranking of the countries. Be as it may, we select normalization values that seem reasonable for each variable, in the sense that they could be kept for some years to come, even if societies progress substantively.

The normalization values proposed are the following: 50 years for the life potential, 10 years for the expected years of schooling, and 60,000 PPP 2005 \$ deflated by the OECD average inequality rate as measured by the Gini coefficient. That is, we define:

$$lp = \frac{LP}{50}, \quad eys = \frac{EYS}{10}, \quad iai_G = \frac{IAI_G}{60,000(1 - 0.31)}$$

The precise formula for the HDI(2) is therefore:

$$HDI(2) = (lp)^{1/3} (eys)^{1/3} (iai_G)^{1/3}$$

The components and values of the HDI and the HDI(2) are presented in Tables 4 and 5 below. They show that the HDI(2) is much more sensitive to the differences between countries than the HDI, which results in a much larger variability. Measuring the variability of the data in terms of the coefficient of variation, yields a clear cut outcome: the HDI(2) has a dispersion three times that of the HDI. Looking at the different components we observe that the change in the dispersion of the health indicator is very small (an increase of 15 %), whereas that of education is much larger (the coefficient of variation of the “expected years of schooling” is three times that of the UN index of education). The largest increase variability corresponds, not surprisingly, to the material wellbeing component, mostly as a result of removing the log transformation. Table 6 below, obtained from the data in Tables 4 and 5, summarizes those outcomes.

**Table 4. The HDI and its components. International comparison. 2007**

	Life expectancy index	Education index	GDP index	HDI
Norway	0.925	0.989	1.000	0.971
Australia	0.940	0.993	0.977	0.970
Iceland	0.946	0.980	0.981	0.969
Canada	0.927	0.991	0.982	0.966
Ireland	0.911	0.985	1.000	0.965
Netherlands	0.914	0.985	0.994	0.964
Sweden	0.930	0.974	0.986	0.963
France	0.933	0.978	0.971	0.961
Japan	0.961	0.949	0.971	0.960
Switzerland	0.945	0.936	1.000	0.960
Finland	0.908	0.993	0.975	0.959
United States	0.902	0.968	1.000	0.956
Austria	0.915	0.962	0.989	0.955
Denmark	0.887	0.993	0.983	0.955
Spain	0.929	0.975	0.960	0.955
Belgium	0.908	0.974	0.977	0.953
Italy	0.935	0.965	0.954	0.951
New Zealand	0.919	0.993	0.936	0.950
Germany	0.913	0.954	0.975	0.947
United Kingdom	0.906	0.957	0.978	0.947

Greece	0.902	0.981	0.944	0.942
Portugal	0.893	0.929	0.906	0.909
Czech Republic	0.856	0.938	0.916	0.903
Poland	0.842	0.952	0.847	0.880
Slovak Republic	0.827	0.928	0.885	0.880
Hungary	0.805	0.960	0.874	0.879
Source: UN				

**Table 5. The HDI(2) and its components. International comparison. 2007**

	Life potential index	Expected years of education index	Inequality adjusted income index	HDI(2)
Norway	0.8738	0.67	0.9293	0.816
Iceland	0.9350	0.85	0.6216	0.791
Netherlands	0.8557	0.80	0.6823	0.776
Sweden	0.8441	0.75	0.6828	0.756
Denmark	0.8196	0.78	0.6720	0.755
Finland	0.8271	0.85	0.6088	0.754
Switzerland	0.8619	0.68	0.7071	0.746
United States	0.8805	0.67	0.6828	0.739
France	0.8765	0.76	0.5856	0.731
Australia	0.9193	0.68	0.5905	0.717
Germany	0.7962	0.79	0.5817	0.715
Austria	0.8336	0.65	0.6589	0.709

Belgium	0.8293	0.68	0.6160	0.703
Canada	0.8863	0.65	0.5882	0.697
Ireland	0.9183	0.50	0.7220	0.692
United Kingdom	0.8463	0.60	0.5600	0.658
New Zealand	0.9136	0.68	0.4358	0.647
Japan	0.8276	0.58	0.5524	0.642
Italy	0.8229	0.67	0.4766	0.640
Greece	0.8030	0.64	0.4684	0.622
Spain	0.8524	0.54	0.5184	0.620
Czech Republic	0.7849	0.67	0.4257	0.607
Hungary	0.7359	0.73	0.3216	0.557
Slovak Republic	0.7928	0.61	0.3540	0.555
Portugal	0.8175	0.59	0.3409	0.548
Poland	0.8019	0.80	0.2433	0.538
Source: OECD, UN, Human Mortality Database and Eurostat.				

<b>Table 6: Coefficients of Variation (HDI, HDI(2) and its components, 2007)</b>				
	<b>Health</b>	<b>Education</b>	<b>Income</b>	<b>Global Index</b>
United Nations' variables	0.0479	0.0415	0.0339	0.0381
New Variables	0.0550	0.1296	0.2613	0.1139
% of the new variables w.r.t. United Nations' variables	115	312	772	299

One may argue against that type of comparison because the HDI only intends to provide an ordinal measure of human development in order to generate a ranking (computing the coefficient of variation would not be meaningful in that case). So let us consider the *changes in the ranking* which are derived from our approach to measuring human development in highly developed countries. Table 7 below gives us that information. A mere ocular inspection tells us that the HDI(2) is really a different way of approaching human development. There are many and large changes in the global index (up to ten positions out of 26). The analysis of partial indices is quite informative, as already pointed out. Note that most of the changes in the ranking occur within the Education component, followed by the Health component. Material wellbeing does not change very much and those changes are obviously due to the differences in inequality (see Table 3 above).<sup>27</sup> This is interesting because, in spite of the large differences in the coefficient of variation with respect to the income dimension, the induced changes in the ranking are small. The opposite happens for the health dimension.

**Table 7.<sup>28</sup> Ranking differences between the HDI(2) and the HDI, and its components. 2007**

	Health	Education	Material wellbeing	Human development
Denmark	3	-4	1	9
Germany	-9	15	0	8

<sup>27</sup> Removing the log transformation does not affect the ranking, as the logarithmic function is a positive monotone transformation.

<sup>28</sup> Each number in the table tells us the positions a country gains (when positive) or losses (when negative) when using the new variables and the HDI(2) with respect to the standard ones.



Finland	0	2	3	5
United Kingdom	6	-3	-5	4
United States	14	0	-1	4
Belgium	1	1	3	3
Hungary	0	9	0	3
Netherlands	3	5	-1	3
Sweden	-6	6	3	3
Switzerland	-6	11	0	3
Austria	-2	-2	-2	1
Czech Republic	-2	6	0	1
Greece	-2	-11	0	1
Iceland	1	8	1	1
New Zealand	7	-7	0	1
Slovak Republic	1	5	1	1
Norway	2	-8	1	0
France	-1	4	2	-1
Italy	-13	0	0	-2
Poland	2	17	0	-2
Portugal	1	2	-1	-3
Spain	-3	-13	0	-6
Australia	2	-9	0	-8
Japan	-15	-2	0	-9
Canada	4	-13	-4	-10
Ireland	12	-19	-1	-10
Source: OECD, UN, Human Mortality Database and Eurostat				

A simple way of evaluating the extent of the changes in the ranking is by calculating Spearman's coefficient of correlation. Values close to 1 (resp. -1) indicate a highly positive (resp. a highly negative) correlation, whereas values close to zero indicate lack of correlation. That index, when applied to the ranking generated by the HDI and the HDI(2) yields a result smaller than 0.77, which tells us that we are actually measuring in a different way. Looking for the source of that change in the ranking we find that education is the most important variable in explaining the differences (the Spearman's coefficient tells us that there is practically no correlation whatsoever between both ways of estimating the education index). Material wellbeing is ranked much in the same way, in spite of the huge differences in the coefficients of variation.

<b>Table 8. Spearman's coefficients of correlation. 2007</b>				
	Health	Education	Material wellbeing	Human development
Coefficient	0.643	0.175	0.970	0.777

## **6. Conclusion**

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There is a general agreement on the need of revising the HDI, after 20 years of good service.

Let us summarize very briefly the main conclusions that derive of this study, formulated in terms of recommendations.

1. It is worth keeping, for the time being, the three traditional dimensions of human development (health, education and material wellbeing). Introducing new dimensions should be part of the research agenda for the immediate future.
2. Using the geometric mean instead of the arithmetic mean, as a way of aggregating the three dimensions into a single indicator, is a clear improvement of the HDI because it makes the index independent on the normalization. Moreover, it is better to normalize the primary variables as a percentage of a maximum value (thus setting the min value equal to zero in all dimensions).
3. Introducing distributive considerations is also conceptually important, even if it does not imply large changes in the ranking.
4. Defining a specific index for developed countries would improve the descriptive power of the HDI and stimulate the adherence to this standard in OECD countries. On that respect the variables that approach health and education should be modified along the lines proposed here.

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