Family planning, growth and income distribution in Rwanda: SAM multiplier and graph-theoretic path analysis

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Abstract

This paper examines the linkages among family planning, sectoral growth and income distribution in Rwanda. Drawing on the 2006 SAM accounting multipliers, macroeconomic effects of alternative income policies are evaluated. Furthermore, the high and low-income gain pathways are identified by applying the graph-theoretic path analysis. The following findings are noted. The rural income gain spreads over the entire economy, whereas the urban income gain largely remains within urban areas, suggesting relatively larger income multiplier effects of rural development policies. Second, investing in education, health and family planning promises a significant increase in agricultural production, which in turn creates considerable employment in rural areas. Targeted rural development policies thus seem to be the best strategy to bring growth and harmoniously improve income distribution. Third, a unit increase in the demand for family planning-health commodities generates 60% more income for the urban-Kigali households than rural households. Finally, a unit increase in the family planning-health demand raises agricultural production by 1.3 unit, which is followed by 1.2 unit increase in service production and by 0.74 unit increase in manufacturing production. To sum up, investing in family planning-health is a viable strategy to promote agricultural growth and reduce poverty through employment created in the rural sector.

1 Background and Introduction

Family planning has long been a central component of population policies and programs and is an integral part of reproductive health.[1] It provides individuals with an opportunity not only to reduce fertility and child mortality but also augment investment in child health and education. Since the 1960s, the use of family planning has been steadily increasing in the developing world, leading to a steady and much faster decline in fertility than the fertility decline the developed world. For Europe and the United States, for example, it took a century to reduce their average family size from around 6 to 3 children. For the developing world, however, a comparable decline in family size took only about four decades during the period of 1960 - 2000. Despite the convergence in family size across the two worlds, unmet need for family planning in the developing world still remains about one-fifth of the currently married women.

Rwanda is no exception to high unmet need for family planning. Relative to other African countries, it is leading with its low contraceptive prevalence (36% in 2008), high fertility rate (5 children in 2009) and high unmet need for family planning (32% in 2008).[2] Data obtained from the 2008 DHS paint a picture of a rather unstable pattern of contraceptive use among married women. The contraceptive use sharply fell after the 1994 genocide, from 21.2% in 1992 to only 13.2% in 2000 and thereafter it slowly increased until 2005 to the point where 17.4% of married women were using modern contraception. By 2008, the contraceptive use has leveled around 36% - which can largely be attributed to the recent surge of investment in family planning services and increasing flow of donor funds targeting population programs.[3] At present, the main concern is the adverse effect of high unmet need and high fertility on per child human capital investment (i.e., education, health and nutrition) and economic growth through declining productivity.

Recognizing the link between family planning and development outcomes, the Rwandan government views family planning as an instrument for targeting poverty and raising per child resource allocation at the household level. However, poverty is multi-sectoral in its cause, and decreasing family size can reduce it only partially. The creation of new employment opportunities is necessary for households to benefit from their investment in child quality because employment, sectoral productivity and household family size decision are interlinked at the meso level through economic and demographic policies. In this paper, therefore, we seek to analyze the linkages among family planning, sectoral

[1] Reproductive health services include family planning, maternal health, childbirth, infant care and other personal reproductive health services for women. Health interventions relating to these services include contraceptive use, maternal health, infant and child health, neonatal and maternal morbidity and mortality, infertility technologies among others. Family planning would lower fertility through reducing obstacles to contraceptive use and access to reproductive health services.

[2] The author's own collection of data from the World Bank, UNDP, IMF and WHO online databases.

[3] See Solo (2008) for a coprehensive review of the developments in family planning in Rwanda over the period 1992-2008.

growth and income distribution in Rwanda.[4] Based on the 2006 Social Accounting Matrix (SAM) accounting multipliers, macroeconomic effects of alternative income policies are assessed. Furthe applied is the graph-theoretic path analysis (GPA) aimed to identify the high and low-income pathways within the Rwandan economy.

Analysis of the economic effects of fertility usually focuses on an assessment of the rate of return to investment in human capital because high fertility puts mothers at risk, rises the dependency ratio and lowers per child investment in human capital, which in turn reduces productivity and income. Suggested by World Development Report (1990), providing family planning services is the most direct and effective way to reduce fertility, making other interventions more effective in improving overall welfare.[5] Many micro-econometric and demographic studies show that family size is negatively correlated with children's educational and health attainment.[6] Complementing micro studies are macroeconomic analyses which integrate household fertility behavior with the consumption/saving decision. The models presented by Becker and Barro (1988) and Barro and Becker (1989), for example, demonstrate that fertility is inversely related to growth. At low levels of education, a combination of low productivity and high fertility point to a Malthusian equilibrium. With a general equilibrium model, Becker, Murphy and Tamura (1990) derives the conditions under which a country may switch from the Malthusian to the "development" equilibrium in which high levels of human capital stock lead to high productivity and low fertility. Their analysis highlights that a country may reach a reasonably high development level if it has good policies that favor human capital investment. More recently, the focus switched towards models that discuss demographic transition and offer diverse explanations (e.g., Galor and Weil 1996, 2000). Azarnert (2004) introduced an analysis of interactions between income redistribution, fertility and growth in an open economy. The list can be extended at will.

To the best of our knowledge, analysis of family planning and reproductive health within SAM framework has not yet received much attention from development economists, although such analysis would provide important information on sectoral income multiplier effect of an investment in family planning. The current study intends to fill this gap in the literature. The SAM multiplier method has been widely applied to analyze: growth strategies in developing economies (Pyatt and Round, 1985; Robinson, 1988; Romeo, Rabinson and El-Said, 1999); income distribution and redistribution (Pyatt and Thorbecke, 1976; Pyatt and Roe, 1977a; Adelman and Robinson, 1978; Roland-Holst and Sancho, 1992); fiscal policies (Whalley and Hillaire, 1987), intersectoral linkages and poverty (Thorbecke, 1995) and the circular flow of income (Stone, 1981; Pyatt

[4]See Rosenzweig (1988) and Bloom, Canning and Sevilla (2001) for a through analysis of the linkages between population pressure and economic development.

[5]See Ross, Parker, Green and Cooke (1992) for a comprehensive assessment of family planning programs.

[6]The reader is referred to Schultz (2005) for a comprehensive survey of the literature on microeconomic linkages between fertility and growth.

and Round, 1979; Defourny and Thorbecke, 1984; Robinson and Roland-Holst, 1988; Tarp, Roland-Holst and Rand, 2002; Roberts, 2005) among many others.

The application of the so-called structural path analysis of Defourny and Thorbecke (1984) has also been a common practice in the literature. The structural path analysis aims to identify critical pathways of average expenditure propensities behind SAM multipliers, uncovering the actual sources of the multiplier effects concerned. The current paper departs from this tradition and applies instead the GPA to identify critical pathways of the multipliers with a view to accounting for the contribution to sectoral incomes (Cormen, Leiserson, and Rivest 1990; Hudson 1992; Pearl 1995; Richardson 1999). To our knowledge, the literature lacks research applying the graph-theoretic path analysis (GPA) to explore the role that family planning and reproductive health play in the transmission of economic influences in the Rwandan economy.

The following findings seem to emerge from our analysis. The rural income gain spreads over the entire economy, whereas the urban income gain largely remains within urban areas, suggesting relatively larger income multiplier effects of rural development policies. Second, investing in education, health and family planning promises a significant increase in agricultural production, which in turn creates considerable employment in rural areas. Targeted rural development policies thus seem to be the best strategy to bring growth and harmoniously improve income distribution. Third, a unit increase in the demand for family planning-health commodities generates 60% more income for the urban-Kigali households than rural households. Finally, a unit increase in the family planning-health demand raises agricultural production by 1.3 unit, which is followed by 1.2 unit increase in service production and by 0.74 unit increase in manufacturing production. To sum up, investing in family planning-health is a viable strategy to promote agricultural growth and reduce poverty through employment created in the rural sector.

The rest of the paper is organized as follows. The following section highlights the critical socio-economic developments in Rwanda, with an examination of historical trends in unmet need and contraceptive use, fertility, child mortality and growth. Section 3 presents the SAM multiplier and the GPA. Section 4 describes available data and the adjustment of the existing SAM to incorporate family planning into the analysis. Section 5 discusses the key findings and their policy implications. Section 6 concludes the paper.

2 The critical socio-economic developments

Figure 1 shows three distict family planning-fertility patterns in Rwanda. The first pattern, which emerged during 1983-1994, points to a significant progress in contraceptive use and a corresponding decline in fertility from about 8 to 6. During the same period though, unmet need for family planning remained unchanged at 37%. The second pattern, which emerged during 1994-2004, shows a mix picture, with contraceptive use first declining and then stabilizing around 15% and fertility rate about 5.6. Again, unmet need remained the same. Finally, significant changes are observed in all three indicators during the period of 2004-2009. Contraceptive use jumped up more than 100% from 17% in 2005 to 36% in 2008; fertility rate droped from 5.6 to almost 5; and unmet need droped from 38% to 32%, which is a very significant decline for the first time since 1983. The year 2005 marks the turning point in the family planning program because significant donor and government support began to systematically work towards implementation at a national scale (Solo, 2008). The role of the economic and political stability prevailing since early 2000 cannot be overlooked in the production of children as well as in the investment in education, health and nutrition of the existing children in a household.

Figures 2 and 3 show that by the year 2008 many health indicators had returned to pre-genocide levels, with infant mortality declining from 86 to 62 and under-five mortality, from 152 to 103. Mortality and fertility rates have also returned to pre-genocide levels in 1998 and thereafter continued to show a stable decline. The effects of family planning programs on mortality and fertility cannot be undermined. Better birth spacing promoted by these programs not only reduces fertility but also improves maternal and child health, which in turn reduces infant, under-five and maternal mortality.

Substantial evidence reveals that poverty cannot be reduced under high rates of population growth and that lowering fertility—in part through family planning—is essential. This link between poverty and fertility has been acknowledged for the first time in the 2008-2012 Economic Development and Poverty Reduction Strategy Paper (MINECOFIN, 2007), justifying family planning to be regarded not only as a health but also an economic intervention. **Figure 4** shows a negative relationship between GDP per capita and fertility since 2000. This can in part be attributed to the double-intervention role of family planning. Increasing GDP per capita provides necessary resources for the needed human capital investment in education, health, and nutrition of children and this puts forward a window of economic opportunity for Rwanda's young population.

Rwanda is approaching to high economic growth trajectory implied by sectoral value-added and employment figures. However, extreme land fragmentation and weak capital market slow down the transition process. As seen from **Figure 5**, from 2000 on, sectoral growth has been undergoing a structural change, with a declining agricultural value-added, a stable manufacturing value-added around 15% and a significantly growing service value-added. Furthermore, agricultural employment declined from 85 % in 2001/02 to 71% in 2006/06, while off-farm self-employment increased (MINECOFIN, 2007).

3 Methodology

3.1 Accounting multipliers

SAM is a matrix representation of the system of national accounts. In a SAM, column sums (i.e., expenditures) are equal to row sums (i.e., incomes). To analyze a policy change, some accounts in the SAM must be manipulable exogenously; therefore, in a modeling framework, the SAM is partitioned as endogenous and exogenous accounts. Production activities, commodities, factors, households and firms represent endogenous accounts, while the government, savings-investment and the rest of the world accounts are assumed to be exogenous.

Let $T_{(d,d)} = [t_{ij}]_{i=j=1,...,d}$ denote a SAM with $d = (n + x)$ where n and x denote the number of endogenous and exogenous accounts, respectively. An element, t_{ij}, represents account j's expenditure on the output from account i. Let $T_{(d,d)}$ be partitioned as:

$$\begin{aligned} T_{(d,d)} &= \begin{bmatrix} T_{nn} & T_{nx} \\ T_{xn} & T_{xx} \end{bmatrix} \qquad (1) \\ \text{where } T_{nn} &= \text{transactions among endogenous accounts} \\ T_{nx} &= \text{injections from exogenous into endogenous accounts} \\ T_{xn} &= \text{leakeges from endogenous into exogenous accounts} \\ T_{xx} &= \text{residuals arising from interactions among exogenous accounts} \\ (N, X, L, R) &= \text{vectors of row sums of } (T_{nn}, T_{nx}, T_{xn}, T_{xx}), \text{ respectively} \\ y &= (y_1, ..., y_d) \equiv ((y_n), (y_x)) = \text{vector of row sums of } T_{(d,d)} \\ y' &= (y'_1, ..., y'_d) \equiv ((y'_n), (y'_x)) = \text{vector of column sums of } T_{(d,d)} \end{aligned}$$

Let $A_{(d,d)} = [a_{ij}]_{i=j=1,...d}$ denote a matrix of average expenditure propensities (AEPs) where $a_{ij} = (t_{ij}/y'_j)$ and $\sum_{i=1}^{d} a_{ij} = \sum_{i=1}^{d} (t_{ij}/y'_j) = 1$ for $\forall_{j=1,2,...,d}$. Let $A_{(d,d)}$ be partitioned as:

$$A_{(d,d)} = \begin{bmatrix} A_{nn} & A_{nx} \\ A_{xn} & A_{xx} \end{bmatrix} \qquad (2)$$

where A_{nn} is a square matrix of AEPs across n endogenous accounts; A_{xn} is a matrix of leakages; that is, the proportions of n endogenous accounts that leak out as expenditure into x exogenous accounts; A_{nx} is a matrix of injections; that is, the proportions of expenditures of x exogenous accounts injected into n endogenous accounts; and A_{xx} is a matrix of residuals; that is, the proportions of expenditures circulated only among x exogenous accounts.

SAM accounting multiplier matrix, M_{nn}, follows from:

$$\begin{aligned} y_n &= N + X = A_{nn} y_n + X \\ &= (I - A_{nn})^{-1} X \\ &= M_{nn} X. \qquad (3) \end{aligned}$$

For notational convenience, from now on, we drop the subscript n from M_{nn}. The multiplier matrix $M = (dy_n/dX) = (I - A_{nn})^{-1}$ measures the impact of unit change in aggregate demand, X, on the incomes of endogenous accounts, y_n.[7]

[7] See Defourny and Thorbecke (1984) for the implication of unitary income elasticity and

There are two ways to conduct scenario analysis. The simplest and most commonly applied way is to deal with only one target ("sink": point of final effect) and one instrument ("source": point of injection). Eq. (3) represents the model used for the analysis of a single, aggregate injection. A more complex model given in Eq. (4) is used to deal with multiple targets and multiple instruments. Replacing X in Eq. (3) with T_{nx} allows us to disentagle the individual impacts of multiple injections originating from several exogenous accounts:

$$y_{nx} = MT_{nx} \tag{4}$$

where y_{nx} is a matrix of n rows and x columns. Each column in y_{nx} represents the vector of endogenous incomes associated with a single exogenous account such as the government.

3.2 Graph-theoretic path analysis

The Rwandan economy is characterized using shortest and longest paths in M', which represents a directed-graph. Following Cormen, Leiserson and Rivest (1990), we apply the *Dijkstra shortest path algorithm* to identify shortest (s) paths in M'.[8] For the identification of longest (l) paths, however, we apply the *Dijkstra algorithm* to M'_A defined as:

$$M'_A = [m^A_{ij}]_{i,j,=1,2,\ldots,n} = \left(\begin{array}{ll} m^A_{ij} = \frac{\overline{m}}{m_{ij}} & \text{if } m_{ij} \neq 0 \\ m^A_{ij} = 0 & \text{if } m_{ij} = 0 \end{array} \right)_{i,j=1,2,\ldots,n}.$$

with $\overline{m} \geq m_{ij} > 0$ where $\overline{m} = \max\{m_{ij} \mid m_{ij} \in M'\}$.

Two kinds of paths are relevant for our analysis: binary paths and multiple-account paths. The set of binary paths is given by $\{p^z_{ij} \in P^z_{(n,n)} \mid P^z_{(n,n)} =$ an n by n matrix of binary $type - z$ paths; $p^z_{ij} \equiv (i \to j)_z =$ a binary $type - z$ path from account i to account j where $i = j = 1, 2, ..., n$ and $z = s, l\}$. The set of multiple-account paths is given by $\{p^z_{ik...vj} \in P^z_{(n,n)} \mid P^z_{(n,n)} =$ an n by n matrix of multiple-account $type - z$ paths; $p^z_{ik...vj} \equiv (i \to k \to ... \to v \to j)_z =$ a multiple-account $type - z$ path from i to j through the intermediate accounts $k, ..., v$ where $i = k = ... = v = j = 1, 2, ..., n$ and $z = s, l\}$. There are also economic influence multipliers associated with the binary and multiple-account paths defined above. The set of binary economic influence multipliers is given by $\{e^z_{ij} \in E^z_{(n,n)} \mid E^z_{(n,n)} =$ an n by n matrix of binary $type - z$ influence multipliers; $e^z_{ij} \equiv m^z_{ij} =$ a binary $type - z$ influence multiplier associated with $p^z_{ij} \equiv (i \to j)_z$ where $i = j = 1, 2, ..., n$ and $z = s, l\}$. The set of multiple-account economic influence multipliers is given by $\{e^z_{ik...vj} \in E^z_{(n,n)} \mid E^z_{(n,n)} =$ an n by n matrix of multiple-account $type - z$ influence multipliers; $e^z_{ik...vj} = (m^z_{ik} * ... * m^z_{vj}) =$ a

for the linkages between accounting and fixed-price multipliers. The lack of data on expenditure (income) elasticities does not allow us to compute marginal expenditure propensities associated with the SAM of Rwanda.

[8] For the mathematics and applications of the Dijkstra algorithm, the reader is reffered to Cormen, Leiserson, and Rivest (1990), Pearl (1995) among others.

multiple-account *type* $- z$ influence multiplier associated with $p^z_{ik...vj} \equiv (i \to k \to ... \to v \to j)_z$ where $i = k = ... = v = j = 1, 2, ..., n$ and $z = s, l\}$. For a given (i, j), the resulting set of paths and their associated influence multipliers, $\{(p^z_{ij},\ p^z_{ik...vj}) \in P^z_{(n,n)},\ (e^z_{ij},\ e^z_{ik...vj}) \in E^z_{(n,n)}$ where $z = s, l\}$, provides us with part of the information required for the characterization of the Rwandan economy.

Useful information about the structure of the Rwandan economy can also be derived by applying the principles of the systems methodology. For example, the principle of *"controllability"* of the economic system concerned requires the identification of dominant, sub-ordinate and interactive accounts in M'.[9] We apply this principle to detect those sectors which are critical from a policy intervention perspective. This principle calls for the derivation of cause-effect information in M'. Account i is said to cause account j if i purchases goods or services from j and that account i is said to be affected (or influenced) by account j if j purchases goods and services from i. This implies that the sum of the elements in row i and column i of M' would respectively represent the degree of "Cause" (C_i) and "Effect" (E_i). A coordinate $(C_i,\ E_i)$ shows the location of account i in a two-dimensional graph, measuring the extent to which account i causes the system and to which account i is influenced by the rest of the system. Based on this coordinate system, account i is classified as dominant if $C_i > E_i$, sub-ordinate if $C_i < E_i$ and interactive if $C_i \simeq E_i$. Such grouping of endogenous accounts in a SAM would provide policymakers with critical information about: (1) dominant accounts which act as the "source" of an exogenous income injection, (2) subordinate accounts which act as the "sink" of the final impact of that injection, and (3) interactive accounts which act as the intermediary poles for "effective transmission" of both causes and effects. The $(C^z_i,\ E^z_i)$ co-ordinates from $E^z_{(n,n)}$ would respectively represent lower and upper bounds for each account: $C^z_i \leq C_i \leq C^z_i$ and $E^z_i \leq E_i \leq E^z_i$ where $z = s, l$.

It should be noted that the size of an economic influence of account i on j depends on the magnitude of binary multiplier m_{ij}: (i) shortest (longest) paths imply minimum (maximum) influences if $m_{ij} > 1$ for all i and j; (ii) shortest (longest) paths imply maximum (minimum) influences if $m_{ij} < 1$ for all i and j; and (iii) shortest and longest paths and their corresponding influences should be analyzed case by case if $0 < m_{ij} \leq 1$ for some i and j. Case (iii) is relevant for our analysis.

[9] The systems methodology has been widely applied to examine agricultural, environmental, and cross-cutting issues. Examples include Goldsworthy and de Vries [5] presenting a collection of studies adopting the systems approach as a tool to assess opportunities in the developing country agriculture. Savory [6] and Gill [7] elaborate on the potential of the systems approach in sustainability planning in agro-ecological issues. The list can be extended at will.

4 Data description and integration

The Ministry of Health of Rwanda has so far compiled 5 consequtive National Health Accounts (NHAs). An important issue concerning the NHAs is that Demographic and Health Surveys (DHS) do not provide data on household health expenses, therefore, insurance companies' and service providers' revenues from households are used in the NHAs to approximate household health care expenses. The NHAs are further broken down to facilitate sub-analyses such as the analyses of reproductive health and family planning, malaria, HIV/AIDS and child health care and so on. The NHA reproductive health and family planning (NHA-RHFP) sub-analysis organizes data on health expenditures but does so only for RHFP sub-account.

Using the data obtained from the NHA-RHFP sub-analysis, the original SAM developed by Emini (2007) has been adjusted to create an additional SAM account, which is called "reproductive health and family planning (RHFP)" account **(see Table 1)**. Since the general health expenditure given in the original SAM includes the RHFP expenditure, we disentagle the RHFP expenses from the general health expenses. Two assumptions are made in the adjustment. First, the general health and RHFP production sectors are assumed to apply similar technologies and employ homogenous labor, capital, and intermediate commodities. Using data from the NHA (2006), we calculated the percentage of the RHFP expenditure in total health expenditure, which is equal to 0.0623. This is multiplied by the total cost of, say, labor (14333 million Rwf given in the original SAM 2006) employed by the health sector; that is, (0.0623 * 14333) = 892.96 million Rwf, which represents the total cost of labor used in the RHFP production sector. And (14333 - 892.96) = 13444 million Rwf would represent the net labor cost of general health production. The same procedure is applied, wherever needed, to derive the sectoral distribution of the RHFP revenue and expenditure. Second, household demand for general health care and RHFP services is assumed to be homogenous. This is a rather restrictive assumption as it implies a uniform expenditure elasticity of income across general health and reproductive health production. In order to integrate household-specific out-of-pocket (HOP) expenditure for RHFP into the original SAM, the following operations are carried out using data obtained from NHA (2006).

$$\begin{aligned}
\eta_r &= (H_r/H_g) = 0.024 \text{ where } H_r \text{ and } H_g\text{, respectively, denote HOP reproductive health-general health expenses.}\\
H_r^{SAM} &= (\eta_r * H_g^{SAM}) = 242 \text{ million Rwf, where } H_g^{SAM} = 10102 \text{ is the HOP general health expenditures given in the 2006 SAM; } H_r^{SAM} \text{ is the implied HOP reproductive health expenses.}\\
\hat{H}_g^{SAM} &= (H_g^{SAM} - H_r^{SAM}) = 9859 \text{ mil Rwf, net HOP health expense}\\
hs_i &= (hs_1, ..., hs_{20}) = (0.079, 0.049, 0.018, 0.028, 0.015, 0, 0.006, 0.002, 0.001, 0{,}205, 0.038, 0.004, 0.160, 0.040, 0.001, 0.118, 0.041, 0.004, 0.125, 0.065) \text{ is the given vector of proportions of households across 20 regions.}\\
H_{r,i}^{SAM} &= hs_i * H_r^{SAM} \text{ and } H_r^{SAM} = \sum_{i=1}^{20} H_{r,i}^{SAM} \text{ where } H_{r,i}^{SAM} \text{ and } H_{g,i}^{SAM}\text{, respectively, are the HOP reproductive health-gen. health expenses of households in region } i.\\
\hat{H}_{g,i}^{SAM} &= (H_{g,i}^{SAM} - H_{r,i}^{SAM})\text{, net HOP general health expenses of households in } i.
\end{aligned}$$

The set $\{(\hat{H}_{g,i}^{SAM}, H_{r,i}^{SAM})\}_{i=1}^{20}$ are used in the adjusted 2006 SAM. Following the above procedure, one can easily calculate $(\hat{H}_{g,1}^{SAM}, H_{r,1}^{SAM}) = (3146, 19)$ for households in urban-Kigali, $(\hat{H}_{g,2}^{SAM}, H_{r,2}^{SAM}) = (371, 12)$ for urban-South, $(\hat{H}_{g,3}^{SAM}, H_{r,3}^{SAM}) = (298, 4)$ for urban-West, etc.

5 Key findings and their policy implications

This section discusses the policy implications of the key findings obtained from three distinct analyses based on the models given in Eq. (3) and Eq. (4): the analysis of the estimated multiplier matrix M, the analysis of alternative scenarios concerning aggregate (X) and disaggregate (T_{nx}) exogenous injections, and the graph-theoretic analysis of paths in M'.

5.1 Multiplier analysis

For all the figures quoted in this section, the reader is referred to the multiplier matrix M given in **Table 2**. As seen from Eq. (5), M has 6 blocks of endogenous accounts: factor block (F) with labor and capital sub-accounts, household block (H) with 11 sub-accounts, firm (Fi) with one sub-account, production block (P) with 9 sub-accounts, commodity block (C) with 10 sub-accounts and export block (X) with 3 sub-accounts. For illustrative purposes, M is formated in blocks as:

$$M = \begin{bmatrix} M^{FF}_{2,2} & M^{FH}_{2,11} & M^{FF_i}_{2,1} & M^{FP}_{2,9} & M^{FC}_{2,10} & M^{FX}_{2,3} \\ M^{HF}_{11,2} & M^{HH}_{11,11} & M^{HF_i}_{11,1} & M^{HP}_{11,9} & M^{HC}_{11,10} & M^{HX}_{11,3} \\ M^{F_iF}_{1,2} & M^{F_iH}_{1,11} & M^{F_iF_I}_{1,1} & M^{F_iP}_{1,9} & M^{F_iC}_{1,10} & M^{F_iX}_{1,3} \\ M^{PF}_{9,2} & M^{PH}_{9,11} & M^{PF_i}_{9,1} & M^{PP}_{9,9} & M^{PC}_{9,10} & M^{PX}_{9,3} \\ M^{CF}_{10,2} & M^{CH}_{10,11} & M^{CF_i}_{10,1} & M^{CP}_{10,9} & M^{CC}_{10,10} & M^{CX}_{10,3} \\ M^{XF}_{3,2} & M^{XH}_{3,11} & M^{XF_i}_{3,1} & M^{XP}_{3,9} & M^{XC}_{3,10} & M^{XX}_{3,3} \end{bmatrix} = \begin{bmatrix} 7 & 29 & 0 & 30 & 31 & 10 \\ 7 & 41 & 0 & 29 & 31 & 10 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 7 & 39 & 0 & 40 & 42 & 13 \\ 9 & 52 & 0 & 42 & 55 & 14 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}. \quad (5)$$

A sub-matrix $M^{ij}_{s_i,s_j}$ maps the multipliers between block i and block j where s_i and s_j stand for the number of sub-accounts within block i and block j, respectively. For example, $M^{CP}_{10,9}$ is a sub-matrix mapping the multipliers between 9 production (P) and 10 commodity (C) sub-accounts. The sum of the individual multipliers in $M^{CP}_{10,9}$ is equal to 42, implying that one unit injection into production block P yields 42 unit increase in the income of commodity block C.

Table 2 shows that all of the diagonal elements in sub-matrix $M^{HH}_{11,11}$ are greater than one. This implies that a unit injection into a household group generates for the same household group an income larger than the initial injection. With a diagonal entry of 1.53, households in urban-Kigali (H_{uk}) are internally the most integrated within the household account, followed by $H_{rw} = 1.48$, $H_{re} = 1.47$, $H_{rs} = 1.42$ and so on. However, the H_{uk} is found to be least integrated with other household groups in $M^{HH}_{11,11}$, reflected by its relatively low column sum of 3.13. This finding suggests that any income transfer to the H_{uk} is more likely to create the lowest impact on the incomes of other household groups. On the contrary, rural households appear to be the most integrated with other household groups, reflected by column sums of $H_{rn} = 3.85$, $H_{rw} = 3.80$, and $H_{rs} = 3.79$. All together, these results confirm that public policies should aim to strengthen the urban-rural linkages for regional income gains to effectively spread over the entire economy.

The input-output multiplier sub-matrix, $M^{PP}_{9,9}$, highlights three important observations. First, agricultural production has the largest income multiplier (i.e., the sum of the elements in the 1^{st} row of $M^{PP}_{9,9} = 13$), followed by services (5^{th} row sum = 12) and manufacturing (4^{th} row sum = 8). That means, of one Rwf injection into production block, agriculture would benefit the most with an income gain of 32 cents (i.e. 13/40 = 0.33), followed by services with 29 cents (i.e., 12/40 = 0.3) and by manufacturing with 19 cents (i.e., 8/40 = 0.2). In other words, agricultural, service and manufacturing production all together account for 83 % of total multiplier (40) in $M^{PP}_{9,9}$; that is, 33/40 =

0.83. The rest 17 % is distributed across public sectors, including public administration, education, general health and RHFP. Second, the C-E coordinates of $M^{PP}_{9,9}$ show that agricultural, service and manufacturing production accounts are sub-ordinate, while public sectors are dominant accounts of the Rwandan economy. Third, agricultural production (P_a) is internally the most integrated, implied by a diagonal multiplier of 2.34, followed by services (P_s) with 2.16 and manufacturing (P_m) with 1.75. In conclusion, from the systems perspective, education, health and RHFP sectors should be considered viable policy intervention instruments. This justifies the investment in health and RHFP, which would promote agricultural production, which would in turn reduce poverty.

$M^{PH}_{9,11}$ maps the multipliers associated with the economic influence of households on production. The $(1,1)^{th}$ element of $M^{PH}_{9,11}$, for example, measures the income multiplier effect on agricultural production of a unit increase in the income of the H_{uk}. The sectoral analysis suggests that agriculture (40%), service (31%) and manufacturing (22%) production accounts for 93% of one Rwf injection into household block. The contribution of public sectors is about 7 %. The estimations also show that, on average, one Rwf increase in the transferable funds H_{tra} would yield an additional 3.5 Rwf for the entire production system. The findings further underline the presence of a strong rural-urban divide. Specifically, production effects of rural household income increase are much larger than the effects of an equivalent increase in urban household income. On average, of one Rwf injection into household block, rural households tend to spend higher proportions on education, general health and RHFP relative to urban households. To sum up, invesmenting in rural areas promises substantial improvement in health and productivity indicators.

$M^{HP}_{11,9}$ maps the multipliers associated with the influence of production on households. Of one Rwf injection into the production system, 86 % is absorbed only by 5 household groups: 28 % by H_{uk} (i.e., 8/29 where 8 is the sum of the multipliers associated with the H_{uk} and 29 is the sum of the individual multipliers in $M^{HP}_{11,9}$), 17 % by the H_{rw} and the H_{re} (i.e., 5/29), 14 % by the H_{rs} (i.e., 4/29) and 10 % by the H_{rn} (i.e., 3/29). Together, $M^{HP}_{11,9}$ and $M^{PH}_{9,11}$ present a consistent picture in which increasing GDP benefits rural areas more than urban areas. Urban areas outside Kigali deserve special attention in this respect.

Three pathways have important implications for sectoral growth, income distribution and poverty in Rwanda. Take, for example, a closed-loop pathway $\{H \rightarrow C \rightarrow P \rightarrow F \rightarrow H\}$ with 5 blocks and $4-$ linkages (i.e., edges). An income injection to household block induces demand for commodities, which trigger production, which in turn increases the derived demand for factors. The factor income earned is then distributed back to households. A second pathway $\{P \rightarrow F \rightarrow H \rightarrow C\}$ starts with an exogenous increase in production, which raises factor demand and the factor income earned is distributed across households. In turn, households raise the demand for commodities. The third pathway $\{F \rightarrow H \rightarrow C \rightarrow P\}$ should be interpreted similarly.

The first pathway $\{H \rightarrow C \rightarrow P \rightarrow F \rightarrow H\}$ is examined through the analysis of four sub-matrices $\{M_{10,11}^{CH},\ M_{9,10}^{PC},\ M_{2,9}^{FP},\ M_{11,2}^{HF}\}$. The most important observation from $M_{10,11}^{CH}$ is that one Rwf injection into household block induces a relatively more increase in rural housholds' demand for agricultural commodities. This suggests that relatively higher employment and hence income will be created in rural areas. As implied by $M_{9,10}^{PC}$, one Rwf increase in the demand for education, general health and RHFP stimulates agricultural growth the most, followed by the growth of services and manufacturing production. This suggests that investment in education-health-RHFP all together would pave the way for economic growth. Regarding the type of technological change, $M_{2,9}^{FP}$ shows that agricultural, service and manufacturing sectors operate under labor-saving technologies because one unit increase in production yields 0.44 unit increase in labor use as opposed to 0.56 unit increase in capital use. Increasing education-health-RHFP production, on the other hand, induces a neutral technological change as implied by an equivalent rise in labor and capital use. As a result, investing in education, health and RHFP significantly improves agricultural, service and manufacturing production. Sectoral production differs with respect to technology use: agriculture, service and manufacturing production employ labor-saving, while education, health and RHFP production employs neutral technologies.[10]

The second pathway $\{P \rightarrow F \rightarrow H \rightarrow C\}$ is examined by analyzing three sub-matrices $\{M_{2,9}^{FP},\ M_{11,2}^{HF},\ M_{10,11}^{CH}\}$. $M_{2,9}^{FP}$ and $M_{10,11}^{CH}$ have already been discussed above. As seen from $M_{11,2}^{HF}$, the H_{uk} receives 0.27 % and 0.24 % of the economy-wide labor and capital incomes, respectively; whereas rural households respectively receive 14% and 18%. This confirms that the H_{uk} relies more on labor income; on the contrary, housholds in the rural sector as a whole relies more on capital income. Furthermore, the urban sector earns a larger proportion of factor income relative to the rural sector.

Following Defourny and Thorbecke (1984), M is decomposed as transfer, open-loop and closed-loop multipliers.[11] The calculations show that the most important closed-loop interactions take place within commodity block, as implied by a net multiplier effect of 23. The important open-loop interactions take place between household and production blocks, with a net multiplier of 18. Then comes the interactions within production (16) and household (16) blocks. These figures demonstrate two fundamental characteristic of the Rwandan economy. First, exogenous income injections should be made through commodity and household blocks as they have very high income multiplier effects. Second, policies aimed to correct income inequality would stimulate significant demand

[10] See Khan and Thorbecke (1989) for the application of the multiplier method to quantify macroeconomic effects of technological change.

[11] A detail analysis of the decomposition of M will be available upon request. Define M as:

$$M = \underbrace{I}_{\text{Initial inj.}} + \underbrace{(M_1 - I)}_{\text{Transfer multiplier}} + \underbrace{(M_2 - I)M_1}_{\text{Open-loop multiplier}} + \underbrace{(M_3 - I)M_2 M_1}_{\text{Closed-loop multiplier}}$$

where $A_0 =$ block diagonal matrix constructed from diagonal blocks of A_n; $M_1 = (I - A_0)^{-1}$; $A^* = M_1(A_n - A_0)$; $M_2 = (I + A^* + {A^*}^2 + ... + {A^*}^{(n-1)})$; and $M_3 = (I - {A^*}^n)^{-1}$.

for commodities and hence for production and factors. In conclusion, addressing income inequality should be the top priority for growth and family planning to take off.

5.2 Scenario analysis

Scenarios are analyzed using factual and counterfactual SAMs. The 2006 SAM represents the factual SAM since it reflects the already realized economic relations in Rwanda. The analysis based on the factual SAM therefore explores the implications of what has happened in 2006. A SAM with hypothetical parameters, on the other hand, represents the counterfactual SAM since it indicates the important but unrealized relations. The counterfactual SAM is used to assess "what if" scenarios and attempts to draw a picture of the implications of the postulated economic relations.

Factual scenario (1) assumes 10% increase in the RoW's transfer to households. The sum of the multipliers in sub-matrix $M_{11,11}^{HH}$, which is equal to 41, implies that, on average, an income transfer of 100 Rwf from the RoW into households is translated into an additional 4100 Rwf for households. According to this scenario, 10% increase, corresponding to an income transfer of 1192 mil Rwf, generates an additional income of 16551 mil Rwf for the entire economy. This national income gain is distributed across 4 accounts as follows: 5354 mil (33%) goes to commodity sectors; 4145 mil Rwf (25%), to households; 3982 mil (24%), to production sectors; and 2969 mil Rwf (18%), to factors. The key findings are three-fold. First, the transfer creates a significant rise in the commodity demand, which is followed by a considerable increase in the household income. Second, the distribution of household income gain shows a regional disparity. The transfer brings 1.84% increase in urban household income as opposed to 1.18% increase in rural household income. Third, the rise in the education, general health and RHFP commody demand is not negligible.

Factual scenario (2) assumes 10% increase in the manufacturing commodity demand for investment purposes. This yields 3.1% increase in the economy-wide income, which is distributed across sectors: 29 % goes to households (14.2% to urban and 14.8% to rural households); 25% to the commodity sectors; and 20%, to the production sectors. At the sub-sectoral level, agricultural export, manufacturing, services and agricultural commodity demand experience the largest income gain, while households experience moderate income gain. Education, general health and RHFP commodity sectors receive 2.6% of the economy-wide income gain.

Factual scenario (3) separately postulates 10% increase in the demand for education, health and RHFP commodities. First, 10% increase in the education demand generates 1.2% increase in the economy-wide income. This additional income is equally distributed among households (13% of 1.2), production (12%) and commodity (13%) accounts. Secondly, when the demand for health increases, the economy-wide income raises 0.36%, 19% of which goes to households, 27% to production and 34% to commodity accounts. Thirdly, a 10% increase in the family planning demand yields 0.03% increase in the economy-wide income, which follows the same distribution pattern as in the health demand. These findings suggest that the sectoral effect of education is homogenous as opposed to the heterogenous sectoral distribution of the national income gain under the health and family planning scenarios.

Factual scenario (4) assumes three simultaneous injections: (i) 5% increase in the investment in the manufacturing commodity sector, (ii) 5% increase in the provision of RHFP services and (iii) 5% increase in the income of urban-Kigali households through the transfer from RoW. The total amount injected into the economy is 678 mil Rwf, which yields net benefit of 127040 mil Rwf. Of this, 97% is attributed to the investment in manufacturing; 2% due to the *RoW's* transfers and 1% due to the provision of RHFP services. Of 127,040 mil Rwf, 25% goes to the agricultural, manufacturing and service sectors; 11%, to rural households; 9%, to factors of production; and 5%, to urban-Kigali households.

When factural scenario (4) is run with an income transfer to rural rather than urban-Kigali households, a higher level of national income is obtained compared to that under factural scenario (4). This justifies the implementation of policies that favor rural households and agricultural sector. Higher level of national income gain also suggests that income distribution policy should be an indispensible element of poverty reduction strategies.

Factual scenario (5) assumes an injection of 157 mil Rwf through two channles: (i) households receive a transfer of 100 mil Rwf from the RoW and (ii) the government increases its demand for family planning services by 57 mil Rwf. Of 2409 mil Rwf national income gain - which corresponds to 0.03% increase in national income - 23% goes to households; 25%, to production sectors and 33%, to commodity sectors. The family planning sector experiences 2.4% income gain and the rest 16.6% is distributed across other sectors in the economy.

Counterfactual scenario (1) compares a government subsidy of 400 mil Rwf to rural households with a subsidy to urban households. Rural subsidy results in a significantly larger income gain relative to urban subsidy: 6151 mil Rwf versus 5484 mil Rwf.

Counterfactual scenario (2a) assumes an injection of 157 mil Rwf through three channels: 55 mil Rwf from the RoW to H; 52 mil Rwf from the government to C_{fp}; and 51 mil Rwf from the S-I account to P_{fp}. Counterfactual scenario (2b) assumes the same amount of injection but the allocations across the three channels are adjusted: 55 mil Rwf from the RoW to H; 72 mil Rwf from the government to C_{fp}; and 32 mil Rwf from the S-I account to P_{fp}. Counterfactual scenario (2b) is welfare improving over Counterfactual scenario (2a), revealing that an increase in the public provision of family planning services can substantially improve national income in Rwanda. Comparing the efficiency of the three sources of injection, we find that the government injection yields the highest income gain (36% of the national income gain), followed by the injection through the S-I account (34%) and lastly by the injection through the RoW account (30%). This result suggests that the government activates those intermediate poles which are more effective in transmitting economic influence than the poles activated by the *RoW* and the S-I accounts.

Counterfactual scenario (3) assumes that the government and the S-I ac-

counts separately increase their demand for family planning production by 78 mil Rwf. The consumption of family planning commodities is not influenced much in spite of a significant rise in the supply of these commodities, which clearly points to the fact that households are more likely to raise their demand for family planning if they realize the full benefits (monetary and non-monetary) of family planning programs. Thus, the provision of family planning products needs to be coupled with policies aimed to raise awareness about the full benefits that households can obtain from the participation in the family planning programs concerned.

5.3 Path analysis

Drawing on M', the path analysis identifies the effective pathways of economic influences between family planning, sectoral growth and income distribution. This would provide information on the "effective instrument(s)" and the "effective target(s)" of a policy intervention. Additional useful information is also obtained by identifying the dominant, sub-ordinate and interactive sectors in the Rwandan economy.

Since our goal is to shed ligh on the interactions between family planning-general health, sectoral growth and income distribution, we limit our search for effective pathways of interactions between production, consumption and household accounts. The GPA addresses the following questions.

Which pathways from family planning-general health commodities to households (i.e., $C_{fp} \rightarrow H$ and $C_h \rightarrow H$) yield the maximum (minimum) income gain? **Table 3** shows that all the maximum-gain pathways from C_{fp} and C_h to households are direct without any intermediate sectors. Mapping the influence multipliers associated with these pathways, **Table 4** shows that one unit exogenous increase in the demand for family planning and general health commodities generates 0.84 unit income gain only for the H_{uk}. Income gain of rural households is significantly less than that of the urban-Kigali households, with 0.56 unit for the H_{re}, 0.55 unit for the H_{rw} and 0.48 unit for the H_{rs}. Regarding the minimum-gain pathways, **Table 5** shows that the H_{ue}'s and the P_{padm}'s demand for family planning commodities impede the income gain of households in urban-Kigali and urban-south regions.

Which pathways from households to family planning-general health consumption (i.e., $H \rightarrow C_{fp}$ and $H \rightarrow C_h$) yield the maximum (minimum) income gain? **Table 3** indicates that the majority of the maximum-gain pathways, $H \rightarrow C_{fp}$ and $H \rightarrow C_h$, involve F_C and H_{rs} as intermediate poles which have large multiplier effects on the consumption of family planning and health services. One unit exogenous increase in household income generates the maximum demand for family planning and health commodities when this additional household income raises the demand for capital owned by households in the rural-south region or is directly transferred to the same households. On the contrary, **Table 5** reveals that, of one unit increase in household income, a small portion is spent on family planning.

Does increasing demand for family planning-general health promote production (i.e., $C_{fp} \rightarrow P$ and $C_h \rightarrow P$)? The income multipliers in **Table 4** show that one unit exogenous injection to the C_{fp} and the C_h commodity sectors results in 1.3 unit increase in the agricultural production, followed by 1.21 unit increase in the production of the service sector and 0.74 unit increase in the manufacturing production. This finding suggests that investing in RHFP and general health can also be viewed as an investment in productivity improvement in the key sectors of the Rwandan economy through improved labor productivity. An interesting observation is that an equal amount of income injection to the service and manufacturing commodity sectors leads to an income gain in the agricultural production much smaller than the gain implied by the RHFP and health commodity demand. Specifically, this confirms that investing in family planning has a higher return relative to the investment in the service and manufacturing sectors.

Which intermediate poles most (least) effectively transmit economic influence from the source to the sink? **Table 3** reveals that labor, capital, households in the H_{uw}, the H_{rs}, the H_{rn} and the H_{re}, the manufacturing production and the agricultural consumption sectors would create significant income multiplier effects when any of them acts as an intermediate pole in between the source of income injection and the sink of the final impact of that injection. Two critical policy implications make themselves known: (1) the promotion of the manufacturing sector and agricultural development should go hand in hand to achieve economic growth and address poverty and (2) policies should target households in the urban-west, rural-south, rural-north and rural-east to stimulate not only family planning but also education and general health sectors. On the other hand, as seen from **Table 5**, households in the H_{ue} and the H_{tra}, the C_{fp}, the C_{padm}, and the P_{padm} suppress the existing direct path multipliers, which in turn reduce the multiplier effects of the pathways they are involved in.

Finally, household transfers point to a strong urban-rural divide. Except for the households in urban-Kigali, rural-west, rural-east, rural-south and rural-north receive a much higher proportion of one unit income transfer compared to households in other urban regions.

From the $C - E$ coordinates of M' presented in **Table 6**, households and public sector (including education, general health, RHFP and public administration) are found to be the *dominant*/accounts; agriculture and service production and commodity sectors, to be the *interactive* accounts; and the manufacturing commodity sector, to be the $sub - ordinate$ account. The dominant accounts represent the source of policy interventions, while the sub-ordinate accounts show the sinks of the final impact of these interventions. This means that the dominant accounts should be treated like exogenous factors in a modeling context. Our findings suggest that stimulating household and public sector demand is a viable option for policy makers to exert influence on other sectors of the economy and that the final impact should be first assessed in the manufacturing commodity sector and therafter in other sectors linked to the manufacturing sector.

6 Summary and Conclusions

This paper examines the linkages among family planning, sectoral growth and income distribution in Rwanda. Drawing on the 2006 SAM accounting multipliers, macroeconomic effects of alternative income policies are evaluated. Furthermore, the high and low-income gain pathways in the Rwandan economy are identified by applying the graph-theoretic path analysis.

Among the set of findings that seem to have emerged, particularly the following can be noted. As far as the economy-wide flow of the income generated by multiplier effects, the rural income gain spreads over the entire economy, whereas the urban income gain largely remains within urban areas. This suggests that the development policies targeting the rural sector would imply relatively larger income multiplier effects. Regarding the production and employment effects, investing in education, health and family planning seems to significantly promote agricultural production and create considerable employment in rural areas. The implementation of targeted rural development policies thus seems to be the best strategy to promote growth and harmoniously improve income distribution.

The household income-effect results confirm that an increase in household income creates a significant demand for family planning and health commodities if this additional income promotes the demand for capital owned by households in rural-south. The multiplier effects of family planning on household income growth further show that a unit increase in the demand for family planning-health commodities generates 0.84 unit increase in the income of the urban-Kigali households, while rural households as a whole experience an increase of 0.53 unit. Finally, the effect on production of increasing family planning demand shows that a unit increase in the family planning-health demand raises agricultural production by 1.3 unit, which is followed by 1.2 unit increase in service production and by 0.74 unit increase in manufacturing production. To sum up, investing in family planning-health paves the way for agricultural productivity improvement and that an equal investment in the services and manufacturing sectors does not create such productivity effect in agriculture. This means that investment in family planning-health is a viable strategy to promote agricultural growth and reduce poverty through employment created in the rural sector.

Some final remarks should be made on the limitation of the current study. First, the SAM data framework assumes that expenditure of an account represents the influece of that account on other accounts. In reality, the actual influence of one account on other accounts can be better approximated through a more detailed econometric causality estimation between the relevant accounts. Second, the multiplier analysis draws on average expenditure propensities obtained from the SAM, while marginal propensities are more reliable to depict non-linear structural relations. The implicit assumption of unitary expenditure elasticities may not show the actual behaviour of an account and hence the SAM multiplier analysis employing the average propensities may not reflect the real interactions among institutions.

Two critical issues warrant further research. First, there is the need for an indepth analysis of the family planning sector. The family planning production technology and the demand parameters should be approximated by an econometric estimation of the production and demand functions. The current paper postulates homogenous technology and demand parameters as in the health sector, while in reality family planning institutions significantly differ from health institutions. Health covers all the population, while family planning is limited to the choices made by parents. This difference needs to be reflected in the SAM including family planning as a separate account. Second, a SAM is a representation of aggregate economic identities, whereas family planning can only be characterized by a behavioral relation at the household level. The effects of family planning on economic indicators can be effectively analyzed by incorporating households and their familysize decisions in a general equilibrium model and such a model certainly calls for the construction of a SAM including socio-economic indicators.

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Figure 1: Contraceptive use, unmet need and fertility

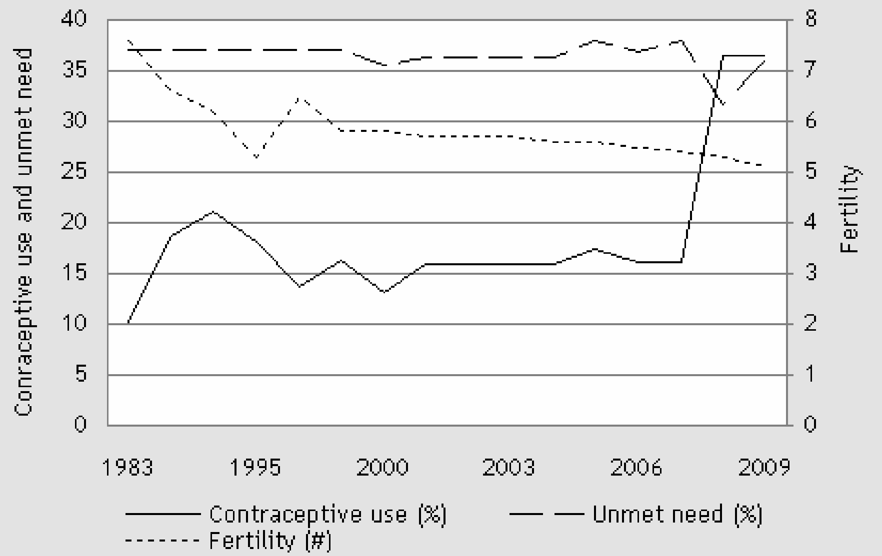

Figure 2: Mortality versus fertility

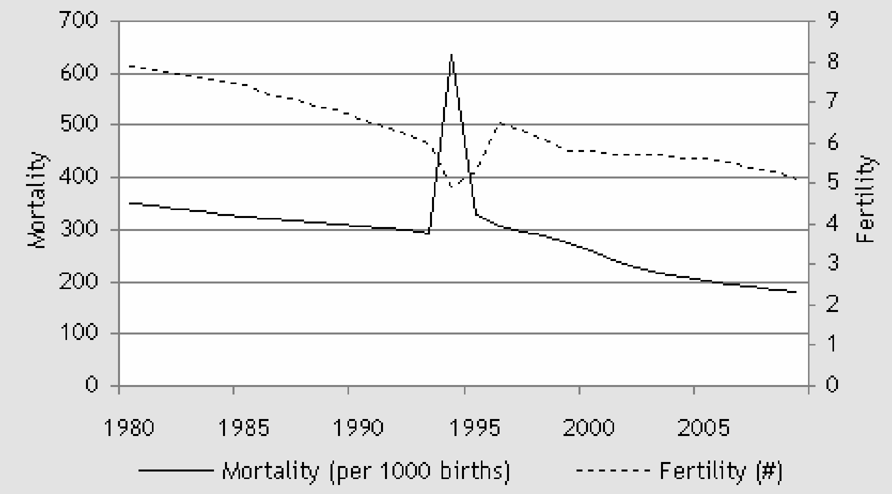

Figure 3. Infant versus under 5 mortality

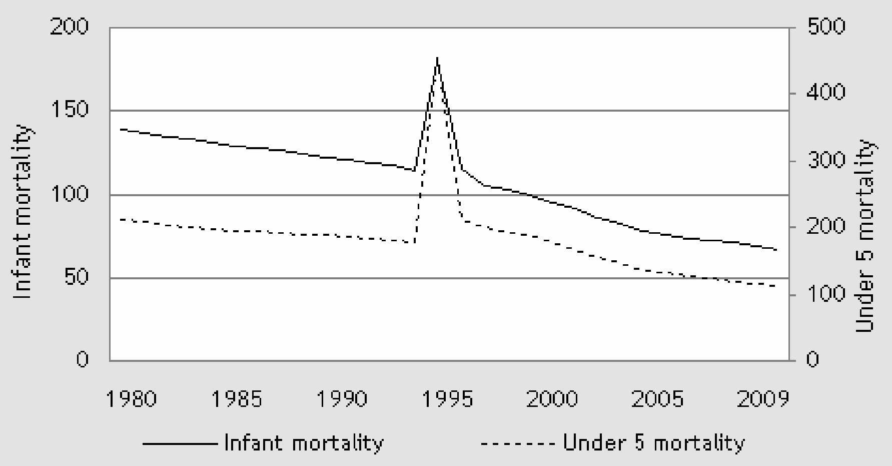

Figure 4. GDP per capita and fertility

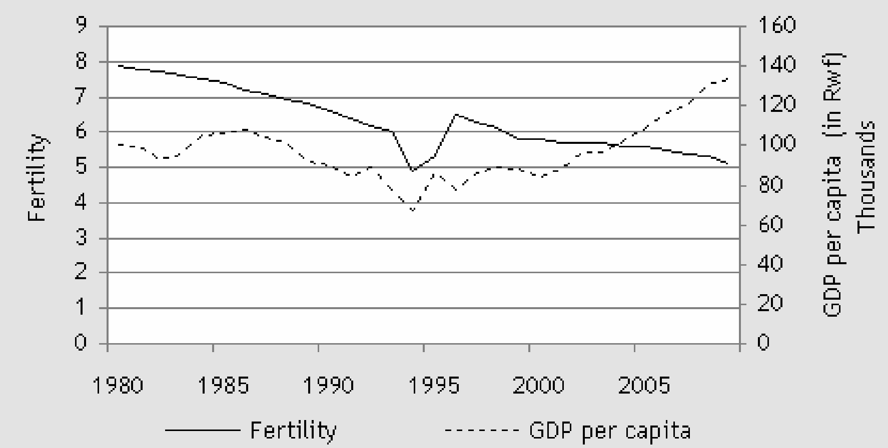

Figure 5. Sectoral Distribution of Value Added

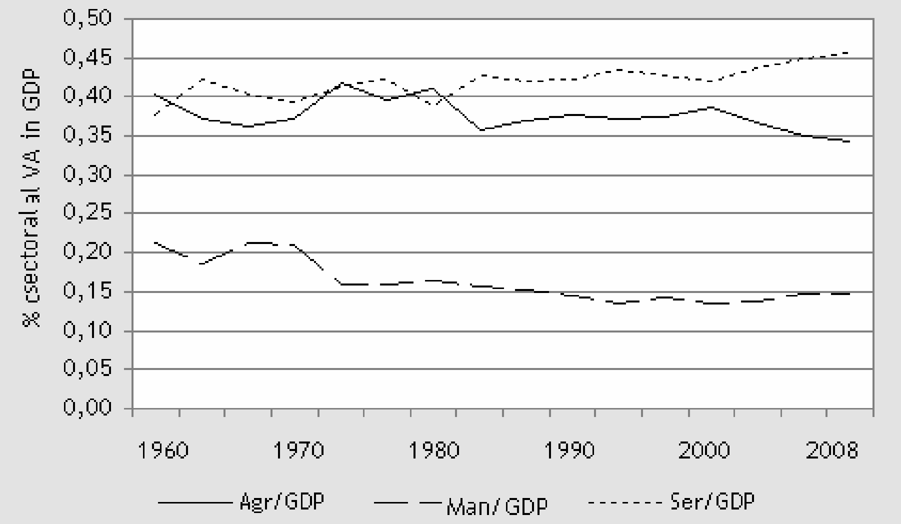

Table 1: Rwanda SAM 2006 (million Rwf)

		[1] F_L	[2] F_C	[3] H_{uk}	[4] H_{us}	[5] H_{uw}	[6] H_{un}	[7] H_{ue}	[8] H_{rk}	[9] H_{rs}	[10] H_{rw}	[11] H_{rn}	[12] H_{re}	[13] H_{tra}	[14] F_i	[15] P_a	[16] P_{ax}	[17] P_{fm}	[18] P_m	[19] P_s	[20] P_{padm}	[21] P_e	[22] P_{he}	[23] P_{fp}	[24] C_a	[25] C_{ax}	[26] C_{fm}	[27] C_m	[28] C_s	[29] C_{padm}	[30] C_e	[31] C_{he}	[32] C_{fp}	[33] C_{tm}	[34] X_a	[35] X_{mi}	[36] X_m	[37] G	[38] Td	[39] Ti	[40] SI	[41] RoW	TOTAL
[1]	F_L	0	0	0	0	0	0	0	0	0	0	0	0	0	0	262077	10021	25604	62785	196623	93331	44111	13440	893	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	708886
[2]	F_C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	325106	12439	30807	136884	248421	0	18339	5588	371	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	777956
[3]	H_{uk}	224466	147440	0	0	0	0	0	0	0	0	0	0	3330	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4171	379406
[4]	H_{us}	36294	30863	0	0	0	0	0	0	0	0	0	0	1034	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3026	71216
[5]	H_{uw}	19615	19247	0	0	0	0	0	0	0	0	0	0	377	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	254	39492
[6]	H_{un}	14817	12687	0	0	0	0	0	0	0	0	0	0	358	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	669	28532
[7]	H_{ue}	14761	13254	0	0	0	0	0	0	0	0	0	0	415	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	90	28520
[8]	H_{rk}	10528	11666	0	0	0	0	0	0	0	0	0	0	350	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12	22556
[9]	H_{rs}	92401	122499	0	0	0	0	0	0	0	0	0	0	2389	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1316	218605
[10]	H_{rw}	103357	139744	0	0	0	0	0	0	0	0	0	0	4181	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	795	248076
[11]	H_{rn}	71590	91724	0	0	0	0	0	0	0	0	0	0	3344	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	534	167193
[12]	H_{re}	106437	141670	0	0	0	0	0	0	0	0	0	0	3322	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1046	252475
[13]	H_{tra}	0	0	4177	1611	642	398	525	298	2430	3481	2657	2880	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19100
[14]	F_i	0	47162	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8576	0	0	0	30864	86602
[15]	P_a	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	620558	0	0	0	0	0	0	0	0	0	1483	0	0	0	0	0	0	0	622041
[16]	P_{ax}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	26781	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	26781
[17]	P_{fm}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	49610	0	0	0	0	0	0	0	0	18093	0	0	0	0	0	0	67703
[18]	P_m	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	467511	0	0	0	0	0	0	0	0	56095	0	0	0	0	0	523606
[19]	P_s	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	594376	0	0	0	0	0	0	0	0	0	0	0	0	0	594376
[20]	P_{padm}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	169890	0	0	0	0	0	0	0	0	0	0	0	0	169890
[21]	P_e	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	89214	0	0	0	0	0	0	0	0	0	0	0	89214
[22]	P_{he}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	27183	0	0	0	0	0	0	0	0	0	0	27183
[23]	P_{fp}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1806	0	0	0	0	0	0	0	0	0	1806
[24]	C_a	0	0	52629	23915	9980	8849	6338	8541	109590	124375	90411	105163	0	0	15364	0	0	90136	7644	11541	5083	964	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	670587
[25]	C_{ax}	0	0	2	0	0	1	0	0	32	44	35	67	0	0	0	0	0	27314	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	27495
[26]	C_{fm}	0	0	8888	2035	936	1092	667	1480	6073	7177	5300	13125	0	0	0	0	2214	4148	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	53143
[27]	C_m	0	0	139258	25018	15641	12167	12482	8710	70472	78808	47604	91173	0	0	16169	1196	4096	91602	58522	28397	6510	2246	149	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	233514	0	943733
[28]	C_s	0	0	86195	12148	7534	4196	5388	2690	18428	22958	14315	26320	0	0	3325	3125	4982	110737	83157	36621	15171	4945	329	0	0	0	0	0	0	0	0	0	150180	0	0	0	0	0	0	0	0	612745
[29]	C_{padm}	0	0	505	55	56	36	30	10	68	139	89	245	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	168658	0	0	0	0	169890
[30]	C_e	0	0	13110	1034	913	345	1263	324	5213	4182	2461	5474	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	54896	0	0	0	0	89214
[31]	C_{he}	0	0	3146	371	298	215	136	159	1574	1535	668	1757	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17339	0	0	0	0	27198
[32]	C_{fp}	0	0	19	12	4	7	4	2	59	49	39	47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1565	0	0	0	0	1807
[33]	C_{tm}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	48554	651	3533	97442	0	0	0	0	0	0	0	0	0	0	0	0	0	0	150180
[34]	X_a	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1483	1483
[35]	X_{mi}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	18093	18093
[36]	X_m	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	56095	56095
[37]	G	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	64536	129109	0	168989	362635
[38]	Td	0	0	25355	1842	2486	331	759	49	1244	1340	721	1589	0	28820	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	64536
[39]	Ti	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	5	0	110619	18369	0	0	16	1	0	0	0	0	0	0	0	0	0	129109
[40]	SI	0	0	44836	2927	944	893	923	288	2061	3261	2356	3789	0	38687	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2257	0	0	0	134805	233514
[41]	RoW	14620	0	1285	249	58	4	5	4	1362	727	537	844	0	19095	0	0	0	0	0	0	0	0	0	1375	58	0	268162	0	0	0	0	0	0	0	0	0	113858	0	0	0	0	422242
	TOTAL	708886	777955	379406	71216	39492	28532	28520	22556	218605	248076	167193	252475	19100	86602	622041	26781	67703	523606	594376	169890	89214	27183	1806	670587	27495	53143	943733	612745	169890	89214	27199	1807	150180	1483	18093	56095	362635	64536	129109	233514	422242	

Table 2: Multiplier Matrix M

	[1] F_L	[2] F_C	[3] H_{uk}	[4] H_{us}	[5] H_{uw}	[6] H_{un}	[7] H_{ue}	[8] H_{rk}	[9] H_{rs}	[10] H_{rw}	[11] H_{rn}	[12] H_{re}	[13] H_{tra}	[14] F_i	[15] P_a	[16] P_{ax}	[17] P_{fm}	[18] P_m
[1] F_L	2,098	1,088	0,940	1,128	1,071	1,121	1,087	1,194	1,237	1,237	1,260	1,201	1,168	0,000	1,518	1,490	1,489	1,320
[2] F_C	1,405	2,393	1,202	1,449	1,375	1,444	1,391	1,534	1,580	1,583	1,611	1,538	1,494	0,000	1,927	1,894	1,879	1,804
[3] H_{uk}	0,938	0,806	1,532	0,641	0,608	0,637	0,617	0,677	0,699	0,700	0,713	0,680	0,836	0,000	0,854	0,838	0,835	0,767
[4] H_{us}	0,166	0,153	0,098	1,118	0,112	0,117	0,114	0,125	0,129	0,129	0,131	0,125	0,176	0,000	0,157	0,154	0,153	0,141
[5] H_{uw}	0,094	0,090	0,057	0,068	1,065	0,068	0,065	0,072	0,074	0,074	0,076	0,072	0,090	0,000	0,091	0,089	0,089	0,082
[6] H_{un}	0,068	0,063	0,040	0,048	0,046	1,048	0,046	0,051	0,052	0,053	0,054	0,051	0,068	0,000	0,064	0,063	0,063	0,058
[7] H_{ue}	0,069	0,064	0,041	0,049	0,047	0,049	1,047	0,052	0,054	0,054	0,055	0,052	0,073	0,000	0,065	0,064	0,064	0,059
[8] H_{rk}	0,053	0,053	0,033	0,039	0,037	0,039	0,038	1,042	0,043	0,043	0,044	0,042	0,059	0,000	0,052	0,051	0,051	0,047
[9] H_{rs}	0,500	0,524	0,317	0,382	0,362	0,379	0,367	0,403	1,416	0,417	0,424	0,405	0,519	0,000	0,507	0,498	0,495	0,461
[10] H_{rw}	0,568	0,598	0,361	0,437	0,414	0,433	0,419	0,460	0,474	1,476	0,485	0,462	0,668	0,000	0,577	0,567	0,564	0,525
[11] H_{rn}	0,385	0,400	0,243	0,294	0,279	0,292	0,283	0,310	0,319	0,320	1,326	0,311	0,478	0,000	0,388	0,381	0,379	0,353
[12] H_{re}	0,579	0,607	0,367	0,443	0,419	0,440	0,425	0,467	0,482	0,483	0,492	1,469	0,630	0,000	0,587	0,576	0,573	0,534
[13] H_{tra}	0,045	0,044	0,038	0,056	0,048	0,047	0,050	0,048	0,047	0,050	0,053	0,046	1,048	0,000	0,044	0,043	0,043	0,040
[14] F_i	0,085	0,145	0,073	0,088	0,083	0,088	0,084	0,093	0,096	0,096	0,098	0,093	0,091	1,000	0,117	0,115	0,114	0,109
[15] P_a	1,321	1,337	0,996	1,349	1,223	1,324	1,213	1,446	1,589	1,591	1,645	1,483	1,442	0,000	2,338	1,296	1,290	1,377
[16] P_{ax}	0,037	0,036	0,034	0,038	0,038	0,040	0,040	0,041	0,040	0,040	0,040	0,040	0,039	0,000	0,037	1,037	0,037	0,090
[17] P_{fm}	0,111	0,110	0,092	0,111	0,103	0,120	0,104	0,152	0,118	0,119	0,123	0,139	0,118	0,000	0,109	0,107	1,138	0,107
[18] P_m	0,723	0,707	0,666	0,747	0,744	0,784	0,775	0,795	0,776	0,775	0,768	0,783	0,755	0,000	0,720	0,724	0,729	1,755
[19] P_s	1,045	1,009	1,024	1,100	1,089	1,087	1,110	1,103	1,078	1,087	1,088	1,085	1,076	0,000	1,027	1,140	1,096	1,235
[20] P_{padm}	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,000	0,003	0,003	0,003	0,003
[21] P_e	0,081	0,078	0,084	0,074	0,079	0,071	0,101	0,077	0,088	0,081	0,080	0,084	0,083	0,000	0,079	0,077	0,077	0,071
[22] P_{he}	0,023	0,023	0,022	0,022	0,024	0,024	0,021	0,025	0,026	0,025	0,023	0,025	0,024	0,000	0,023	0,022	0,022	0,020
[23] P_{fp}	0,001	0,001	0,000	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,000	0,001	0,001	0,001	0,001
[24] C_a	1,427	1,444	1,076	1,458	1,322	1,431	1,310	1,563	1,717	1,719	1,778	1,603	1,558	0,000	1,446	1,400	1,394	1,488
[25] C_{ax}	0,038	0,037	0,035	0,039	0,039	0,041	0,041	0,042	0,041	0,041	0,041	0,041	0,040	0,000	0,038	0,038	0,038	0,092
[26] C_{fm}	0,119	0,118	0,099	0,119	0,110	0,129	0,111	0,163	0,126	0,128	0,132	0,149	0,126	0,000	0,117	0,115	0,148	0,115
[27] C_m	1,460	1,427	1,344	1,509	1,503	1,582	1,565	1,605	1,567	1,565	1,549	1,580	1,523	0,000	1,454	1,462	1,471	1,523
[28] C_s	1,077	1,040	1,056	1,134	1,123	1,121	1,144	1,137	1,111	1,120	1,122	1,118	1,110	0,000	1,059	1,176	1,130	1,274
[29] C_{padm}	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,000	0,003	0,003	0,003	0,003
[30] C_e	0,081	0,078	0,084	0,074	0,079	0,071	0,101	0,077	0,088	0,081	0,080	0,084	0,083	0,000	0,079	0,077	0,077	0,071
[31] C_{he}	0,023	0,023	0,022	0,022	0,024	0,024	0,021	0,025	0,026	0,025	0,023	0,025	0,024	0,000	0,023	0,022	0,022	0,020
[32] C_{fp}	0,001	0,001	0,000	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,000	0,001	0,001	0,001	0,001
[33] C_{tm}	0,263	0,261	0,224	0,270	0,259	0,277	0,265	0,291	0,295	0,295	0,298	0,290	0,279	0,000	0,263	0,261	0,264	0,275
[34] X_a	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
[35] X_{mi}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
[36] X_m	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000

	[19] P_s	[20] P_{padm}	[21] P_e	[22] P_{he}	[23] P_{fp}	[24] C_a	[25] C_{ax}	[26] C_{fm}	[27] C_m	[28] C_s	[29] C_{padm}	[30] C_e	[31] C_{he}	[32] C_{fp}	[33] C_{tm}	[34] X_a	[35] X_{mi}	[36] X_m
[1] F_L	1,443	1,690	1,643	1,635	1,635	1,506	1,484	1,483	0,799	1,400	1,690	1,643	1,634	1,634	1,400	1,518	1,489	1,320
[2] F_C	1,846	1,468	1,679	1,669	1,669	1,913	1,888	1,873	1,079	1,791	1,468	1,679	1,668	1,668	1,791	1,927	1,879	1,804
[3] H_{uk}	0,814	0,820	0,846	0,841	0,841	0,847	0,835	0,832	0,461	0,790	0,820	0,846	0,841	0,841	0,790	0,854	0,835	0,767
[4] H_{us}	0,149	0,147	0,153	0,152	0,152	0,155	0,153	0,153	0,085	0,145	0,147	0,153	0,152	0,152	0,145	0,157	0,153	0,141
[5] H_{uw}	0,086	0,084	0,088	0,087	0,087	0,090	0,089	0,088	0,049	0,084	0,084	0,088	0,087	0,087	0,084	0,091	0,089	0,082
[6] H_{un}	0,061	0,060	0,063	0,062	0,062	0,064	0,063	0,062	0,035	0,059	0,060	0,063	0,062	0,062	0,059	0,064	0,063	0,058
[7] H_{ue}	0,062	0,061	0,064	0,063	0,063	0,065	0,064	0,064	0,036	0,061	0,061	0,064	0,063	0,063	0,061	0,065	0,064	0,059
[8] H_{rk}	0,050	0,048	0,050	0,050	0,050	0,052	0,051	0,051	0,028	0,048	0,048	0,050	0,050	0,050	0,048	0,052	0,051	0,047
[9] H_{rs}	0,484	0,457	0,484	0,481	0,481	0,503	0,496	0,493	0,277	0,470	0,457	0,484	0,481	0,481	0,470	0,507	0,495	0,461
[10] H_{rw}	0,551	0,519	0,550	0,547	0,548	0,573	0,565	0,562	0,315	0,535	0,519	0,550	0,547	0,547	0,535	0,577	0,564	0,525
[11] H_{rn}	0,371	0,351	0,371	0,369	0,369	0,385	0,380	0,378	0,212	0,360	0,351	0,371	0,369	0,369	0,360	0,388	0,379	0,353
[12] H_{re}	0,560	0,528	0,560	0,557	0,557	0,582	0,574	0,571	0,320	0,543	0,528	0,560	0,557	0,557	0,543	0,587	0,573	0,534
[13] H_{tra}	0,042	0,040	0,042	0,042	0,042	0,043	0,043	0,043	0,024	0,041	0,040	0,042	0,042	0,042	0,041	0,044	0,043	0,040
[14] F_i	0,112	0,089	0,102	0,101	0,101	0,116	0,114	0,114	0,065	0,109	0,089	0,102	0,101	0,101	0,109	0,117	0,114	0,109
[15] P_a	1,278	1,281	1,326	1,300	1,300	2,254	1,292	1,287	0,810	1,239	1,281	1,326	1,299	1,299	1,239	2,338	1,290	1,377
[16] P_{ax}	0,038	0,039	0,038	0,038	0,038	0,037	1,011	0,037	0,048	0,037	0,039	0,038	0,038	0,038	0,037	0,037	0,037	0,090
[17] P_{fm}	0,105	0,101	0,105	0,105	0,105	0,108	0,107	1,069	0,064	0,101	0,101	0,105	0,105	0,105	0,101	0,109	1,138	0,107
[18] P_m	0,737	0,758	0,734	0,736	0,736	0,718	0,722	0,728	0,943	0,715	0,758	0,734	0,736	0,736	0,715	0,720	0,729	1,755
[19] P_s	2,155	1,238	1,203	1,212	1,212	1,102	1,160	1,162	0,828	2,091	1,238	1,203	1,211	1,211	2,091	1,027	1,096	1,235
[20] P_{padm}	0,003	1,003	0,003	0,003	0,003	0,003	0,003	0,003	0,002	0,003	1,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003
[21] P_e	0,075	0,073	1,076	0,076	0,076	0,078	0,077	0,076	0,043	0,073	0,073	1,076	0,076	0,076	0,073	0,079	0,077	0,071
[22] P_{he}	0,022	0,021	0,022	1,022	0,022	0,022	0,022	0,022	0,012	0,021	0,021	0,022	1,021	0,022	0,021	0,023	0,022	0,020
[23] P_{fp}	0,001	0,001	0,001	0,001	1,001	0,001	0,001	0,001	0,000	0,001	0,001	0,001	0,001	1,000	0,001	0,001	0,001	0,001
[24] C_a	1,381	1,385	1,433	1,405	1,405	2,435	1,396	1,390	0,876	1,339	1,385	1,433	1,404	1,404	1,339	1,446	1,394	1,488
[25] C_{ax}	0,039	0,040	0,039	0,039	0,039	0,038	1,038	0,038	0,049	0,038	0,040	0,039	0,039	0,039	0,038	0,038	0,038	0,092
[26] C_{fm}	0,112	0,108	0,113	0,112	0,112	0,116	0,115	1,146	0,068	0,109	0,108	0,113	0,112	0,112	0,109	0,117	0,148	0,115
[27] C_m	1,487	1,530	1,482	1,486	1,486	1,450	1,458	1,469	1,904	1,443	1,530	1,482	1,485	1,485	1,443	1,454	1,471	1,523
[28] C_s	1,191	1,276	1,240	1,249	1,249	1,136	1,196	1,198	0,853	2,155	1,276	1,240	1,248	1,248	2,155	1,059	1,130	1,274
[29] C_{padm}	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003	0,002	0,003	1,003	0,003	0,003	0,003	0,003	0,003	0,003	0,003
[30] C_e	0,075	0,073	0,076	0,076	0,076	0,078	0,077	0,076	0,043	0,073	0,073	1,076	0,076	0,076	0,073	0,079	0,077	0,071
[31] C_{he}	0,022	0,021	0,022	0,022	0,022	0,022	0,022	0,022	0,012	0,021	0,021	0,022	1,022	0,022	0,021	0,023	0,022	0,020
[32] C_{fp}	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,000	0,001	0,001	0,001	0,001	1,001	0,001	0,001	0,001	0,001
[33] C_{tm}	0,262	0,266	0,265	0,264	0,264	0,335	0,284	0,329	0,266	0,254	0,266	0,265	0,263	0,263	1,254	0,263	0,264	0,275
[34] X_a	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	1,000	0,000	0,000
[35] X_{mi}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	1,000	0,000
[36] X_m	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	1,000

Table 3: Longest paths in M'

	[1] F_L	[2] F_C	[3] H_{uk}	[4] H_{us}	[5] H_{uw}	[6] H_{un}	[7] H_{ue}	[8] H_{rk}	[9] H_{rs}	[10] H_{rw}	[11] H_{rn}	[12] H_{re}	[13] H_{tra}
[1] F_L	{1}	{1,2}	{1,3}	{1,4}	{1,5}	{1,6}	{1,7}	{1,8}	{1,9}	{1,10}	{1,11}	{1,12}	{1,11,13}
[2] F_C	{2,1}	{2}	{2,3}	{2,4}	{2,5}	{2,1,6}	{2,1,7}	{2,8}	{2,9}	{2,10}	{2,11}	{2,12}	{2,11,13}
[3] H_{uk}	{3,1}	{3,2}	{3}	{3,1,4}	{3,1,5}	{3,1,6}	{3,1,7}	{3,2,8}	{3,2,9}	{3,2,10}	{3,2,11}	{3,2,12}	{3,2,11,13}
[4] H_{us}	{4,1}	{4,2}	{4,3}	{4}	{4,1,5}	{4,1,6}	{4,1,7}	{4,2,8}	{4,2,9}	{4,10}	{4,2,11}	{4,12}	{4,13}
[5] H_{uw}	{5,1}	{5,2}	{5,3}	{5,1,4}	{5}	{5,1,6}	{5,1,7}	{5,2,8}	{5,2,9}	{5,2,10}	{5,2,11}	{5,2,12}	{5,13}
[6] H_{un}	{6,1}	{6,2}	{6,3}	{6,1,4}	{6,1,5}	{6}	{6,1,7}	{6,2,8}	{6,2,9}	{6,10}	{6,2,11}	{6,12}	{6,13}
[7] H_{ue}	{7,1}	{7,2}	{7,3}	{7,1,4}	{7,1,5}	{7,1,6}	{7}	{7,2,8}	{7,2,9}	{7,10}	{7,2,11}	{7,12}	{7,13}
[8] H_{rk}	{8,1}	{8,2}	{8,3}	{8,1,4}	{8,1,5}	{8,1,6}	{8,1,7}	{8}	{8,9}	{8,10}	{8,2,11}	{8,12}	{8,13}
[9] H_{rs}	{9,1}	{9,2}	{9,3}	{9,1,4}	{9,1,5}	{9,1,6}	{9,1,7}	{9,2,8}	{9}	{9,10}	{9,11}	{9,12}	{9,13}
[10] H_{rw}	{10,1}	{10,2}	{10,3}	{10,1,4}	{10,1,5}	{10,1,6}	{10,1,7}	{10,2,8}	{10,9}	{10}	{10,11}	{10,12}	{10,13}
[11] H_{rn}	{11,1}	{11,2}	{11,3}	{11,1,4}	{11,1,5}	{11,1,6}	{11,1,7}	{11,2,8}	{11,9}	{11,10}	{11}	{11,12}	{11,13}
[12] H_{re}	{12,1}	{12,2}	{12,3}	{12,1,4}	{12,1,5}	{12,1,6}	{12,1,7}	{12,2,8}	{12,9}	{12,10}	{12,2,11}	{12}	{12,13}
[13] H_{tra}	{13,1}	{13,2}	{13,3}	{13,4}	{13,5}	{13,6}	{13,7}	{13,8}	{13,9}	{13,10}	{13,11}	{13,12}	{13}
[14] F_i	{1}	{2}	{3}	{4}	{5}	{6}	{7}	{8}	{9}	{10}	{11}	{12}	{13}
[15] P_a	{15,1}	{15,2}	{15,3}	{15,4}	{15,5}	{15,1,6}	{15,1,7}	{15,8}	{15,9}	{15,10}	{15,11}	{15,12}	{15,11,13}
[16] P_{ax}	{16,1}	{16,2}	{16,3}	{16,4}	{16,5}	{16,1,6}	{16,1,7}	{16,2,8}	{16,9}	{16,10}	{16,11}	{16,12}	{16,11,13}
[17] P_{fm}	{17,1}	{17,2}	{17,3}	{17,4}	{17,5}	{17,1,6}	{17,1,7}	{17,2,8}	{17,9}	{17,10}	{17,11}	{17,12}	{17,11,13}
[18] P_m	{18,1}	{18,2}	{18,3}	{18,1,4}	{18,1,5}	{18,1,6}	{18,1,7}	{18,2,8}	{18,9}	{18,10}	{18,11}	{18,12}	{18,11,13}
[19] P_s	{19,1}	{19,2}	{19,3}	{19,4}	{19,1,5}	{19,1,6}	{19,1,7}	{19,2,8}	{19,9}	{19,10}	{19,11}	{19,12}	{19,11,13}
[20] P_{padm}	{20,1}	{20,2}	{20,3}	{20,1,4}	{20,1,5}	{20,1,6}	{20,1,7}	{20,1,8}	{20,9}	{20,10}	{20,11}	{20,12}	{20,11,13}
[21] P_e	{21,1}	{21,2}	{21,3}	{21,4}	{21,1,5}	{21,1,6}	{21,1,7}	{21,1,8}	{21,9}	{21,10}	{21,11}	{21,12}	{21,11,13}
[22] P_{he}	{22,1}	{22,2}	{22,3}	{22,4}	{22,1,5}	{22,1,6}	{22,1,7}	{22,1,8}	{22,9}	{22,10}	{22,11}	{22,12}	{22,11,13}
[23] P_{fp}	{23,1}	{23,2}	{23,3}	{23,4}	{23,1,5}	{23,1,6}	{23,1,7}	{23,1,8}	{23,9}	{23,10}	{23,11}	{23,12}	{23,11,13}
[24] C_a	{24,1}	{24,2}	{24,3}	{24,4}	{24,5}	{24,1,6}	{24,1,7}	{24,8}	{24,9}	{24,10}	{24,11}	{24,12}	{24,11,13}
[25] C_{ax}	{25,1}	{25,2}	{25,3}	{25,4}	{25,5}	{25,1,6}	{25,1,7}	{25,2,8}	{25,9}	{25,10}	{25,11}	{25,12}	{25,11,13}
[26] C_{fm}	{26,1}	{26,2}	{26,3}	{26,4}	{26,5}	{26,1,6}	{26,1,7}	{26,2,8}	{26,9}	{26,10}	{26,11}	{26,12}	{26,11,13}
[27] C_m	{27,1}	{27,2}	{27,3}	{27,1,4}	{27,1,5}	{27,1,6}	{27,1,7}	{27,2,8}	{27,2,9}	{27,2,10}	{27,2,11}	{27,2,12}	{27,2,11,13}
[28] C_s	{28,1}	{28,2}	{28,3}	{28,1,4}	{28,1,5}	{28,1,6}	{28,1,7}	{28,2,8}	{28,9}	{28,10}	{28,11}	{28,12}	{28,11,13}
[29] C_{padm}	{29,1}	{29,2}	{29,3}	{29,1,4}	{29,1,5}	{29,1,6}	{29,1,7}	{29,1,8}	{29,9}	{29,10}	{29,11}	{29,12}	{29,11,13}
[30] C_e	{30,1}	{30,2}	{30,3}	{30,4}	{30,1,5}	{30,1,6}	{30,1,7}	{30,1,8}	{30,9}	{30,10}	{30,11}	{30,12}	{30,11,13}
[31] C_{he}	{31,1}	{31,2}	{31,3}	{31,4}	{31,1,5}	{31,1,6}	{31,1,7}	{31,1,8}	{31,9}	{31,10}	{31,11}	{31,12}	{31,11,13}
[32] C_{fp}	{32,1}	{32,2}	{32,3}	{32,4}	{32,1,5}	{32,1,6}	{32,1,7}	{32,1,8}	{32,9}	{32,10}	{32,11}	{32,12}	{32,11,13}
[33] C_{tm}	{33,1}	{33,2}	{33,3}	{33,1,4}	{33,1,5}	{33,1,6}	{33,1,7}	{33,2,8}	{33,9}	{33,10}	{33,11}	{33,12}	{33,11,13}
[34] X_a	{34,1}	{34,2}	{34,3}	{34,4}	{34,5}	{34,1,6}	{34,1,7}	{34,8}	{34,9}	{34,10}	{34,11}	{34,12}	{34,11,13}
[35] X_{mi}	{35,1}	{35,2}	{35,3}	{35,4}	{35,5}	{35,1,6}	{35,1,7}	{35,2,8}	{35,9}	{35,10}	{35,11}	{35,12}	{35,11,13}
[36] X_m	{36,1}	{36,2}	{36,3}	{36,1,4}	{36,1,5}	{36,1,6}	{36,1,7}	{36,2,8}	{36,9}	{36,10}	{36,11}	{36,12}	{36,11,13}

	[14] F_i	[15] P_a	[16] P_{ax}	[17] P_{fm}	[18] P_m	[19] P_s	[20] P_{padm}	[21] P_e	[22] P_{he}	[23] P_{fp}
[1] F_L	{1,2,14}	{1,15}	{1,18,16}	{1,12,17}	{1,18}	{1,19}	{1,5,20}	{1,21}	{1,9,22}	{1,9,23}
[2] F_C	{2,14}	{2,15}	{2,18,16}	{2,12,17}	{2,18}	{2,19}	{2,5,20}	{2,21}	{2,9,22}	{2,9,23}
[3] H_{uk}	{3,2,14}	{3,15}	{3,18,16}	{3,2,12,17}	{3,18}	{3,19}	{3,1,5,20}	{3,21}	{3,2,9,22}	{3,2,9,23}
[4] H_{us}	{4,2,14}	{4,15}	{4,18,16}	{4,17}	{4,18}	{4,19}	{4,1,5,20}	{4,1,21}	{4,2,9,22}	{4,2,9,23}
[5] H_{uw}	{5,2,14}	{5,15}	{5,18,16}	{5,2,12,17}	{5,18}	{5,19}	{5,20}	{5,21}	{5,2,9,22}	{5,2,9,23}
[6] H_{un}	{6,2,14}	{6,15}	{6,18,16}	{6,17}	{6,18}	{6,19}	{6,20}	{6,1,21}	{6,22}	{6,2,9,23}
[7] H_{ue}	{7,2,14}	{7,15}	{7,18,16}	{7,12,17}	{7,18}	{7,19}	{7,1,5,20}	{7,21}	{7,2,9,22}	{7,2,9,23}
[8] H_{rk}	{8,2,14}	{8,15}	{8,18,16}	{8,17}	{8,18}	{8,19}	{8,1,5,20}	{8,21}	{8,22}	{8,9,23}
[9] H_{rs}	{9,2,14}	{9,15}	{9,18,16}	{9,17}	{9,18}	{9,19}	{9,1,5,20}	{9,21}	{9,22}	{9,23}
[10] H_{rw}	{10,2,14}	{10,15}	{10,18,16}	{10,17}	{10,18}	{10,19}	{10,1,5,20}	{10,21}	{10,22}	{10,9,23}
[11] H_{rn}	{11,2,14}	{11,15}	{11,18,16}	{11,17}	{11,18}	{11,19}	{11,1,5,20}	{11,21}	{11,9,22}	{11,9,23}
[12] H_{re}	{12,2,14}	{12,15}	{12,18,16}	{12,17}	{12,18}	{12,19}	{12,1,5,20}	{12,21}	{12,22}	{12,9,23}
[13] H_{tra}	{13,2,14}	{13,15}	{13,18,16}	{13,17}	{13,18}	{13,19}	{13,5,20}	{13,21}	{13,9,22}	{13,9,23}
[14] F_i	{14}	{15}	{16}	{17}	{18}	{19}	{20}	{21}	{22}	{23}
[15] P_a	{15,2,14}	{15}	{15,18,16}	{15,12,17}	{15,18}	{15,19}	{15,5,20}	{15,21}	{15,9,22}	{15,9,23}
[16] P_{ax}	{16,2,14}	{16,15}	{16}	{16,12,17}	{16,18}	{16,19}	{16,5,20}	{16,1,21}	{16,9,22}	{16,9,23}
[17] P_{fm}	{17,2,14}	{17,15}	{17,18,16}	{17}	{17,18}	{17,19}	{17,5,20}	{17,1,21}	{17,9,22}	{17,9,23}
[18] P_m	{18,2,14}	{18,15}	{18,16}	{18,12,17}	{18}	{18,19}	{18,1,5,20}	{18,1,21}	{18,9,22}	{18,9,23}
[19] P_s	{19,2,14}	{19,15}	{19,18,16}	{19,12,17}	{19,18}	{19}	{19,1,5,20}	{19,1,21}	{19,9,22}	{19,9,23}
[20] P_{padm}	{20,2,14}	{20,15}	{20,18,16}	{20,12,17}	{20,18}	{20,19}	{20}	{20,1,21}	{20,9,22}	{20,9,23}
[21] P_e	{21,2,14}	{21,15}	{21,18,16}	{21,12,17}	{21,18}	{21,19}	{21,1,5,20}	{21}	{21,9,22}	{21,9,23}
[22] P_{he}	{22,2,14}	{22,15}	{22,18,16}	{22,12,17}	{22,18}	{22,19}	{22,1,5,20}	{22,1,21}	{22}	{22,9,23}
[23] P_{fp}	{23,2,14}	{23,15}	{23,18,16}	{23,12,17}	{23,18}	{23,19}	{23,1,5,20}	{23,1,21}	{23,9,22}	{23}
[24] C_a	{24,2,14}	{24,15}	{24,18,16}	{24,12,17}	{24,18}	{24,19}	{24,5,20}	{24,21}	{24,9,22}	{24,9,23}
[25] C_{ax}	{25,2,14}	{25,15}	{25,16}	{25,12,17}	{25,18}	{25,19}	{25,5,20}	{25,1,21}	{25,9,22}	{25,9,23}
[26] C_{fm}	{26,2,14}	{26,15}	{26,18,16}	{26,17}	{26,18}	{26,19}	{26,5,20}	{26,1,21}	{26,9,22}	{26,9,23}
[27] C_m	{27,2,14}	{27,15}	{27,18,16}	{27,2,12,17}	{27,18}	{27,19}	{27,1,5,20}	{27,1,21}	{27,2,9,22}	{27,2,9,23}
[28] C_s	{28,2,14}	{28,15}	{28,18,16}	{28,12,17}	{28,18}	{28,19}	{28,1,5,20}	{28,1,21}	{28,9,22}	{28,9,23}
[29] C_{padm}	{29,2,14}	{29,15}	{29,18,16}	{29,12,17}	{29,18}	{29,19}	{29,20}	{29,1,21}	{29,9,22}	{29,9,23}
[30] C_e	{30,2,14}	{30,15}	{30,18,16}	{30,12,17}	{30,18}	{30,19}	{30,1,5,20}	{30,21}	{30,9,22}	{30,9,23}
[31] C_{he}	{31,2,14}	{31,15}	{31,18,16}	{31,12,17}	{31,18}	{31,19}	{31,1,5,20}	{31,1,21}	{31,22}	{31,9,23}
[32] C_{fp}	{32,2,14}	{32,15}	{32,18,16}	{32,12,17}	{32,18}	{32,19}	{32,1,5,20}	{32,1,21}	{32,9,22}	{32,23}
[33] C_{tm}	{33,2,14}	{33,15}	{33,18,16}	{33,12,17}	{33,18}	{33,19}	{33,1,5,20}	{33,1,21}	{33,9,22}	{33,9,23}
[34] X_a	{34,2,14}	{34,15}	{34,18,16}	{34,12,17}	{34,18}	{34,19}	{34,5,20}	{34,21}	{34,9,22}	{34,9,23}
[35] X_{mi}	{35,2,14}	{35,15}	{35,18,16}	{35,17}	{35,18}	{35,19}	{35,5,20}	{35,1,21}	{35,9,22}	{35,9,23}
[36] X_m	{36,2,14}	{36,15}	{36,16}	{36,12,17}	{36,18}	{36,19}	{36,1,5,20}	{36,1,21}	{36,9,22}	{36,9,23}

	[24] C_a	[25] C_{ax}	[26] C_{fm}	[27] C_m	[28] C_s	[29] C_{padm}	[30] C_e	[31] C_{he}	[32] C_{fp}	[33] C_{tm}	[34] X_a	[35] X_{mi}	[36] X_m
[1] F_L	{1,24}	{1,18,25}	{1,26}	{1,27}	{1,28}	{1,5,29}	{1,30}	{1,9,31}	{1,9,32}	{1,24,33}	{34}	{35}	{36}
[2] F_C	{2,24}	{2,18,25}	{2,12,26}	{2,27}	{2,28}	{2,5,29}	{2,30}	{2,9,31}	{2,9,32}	{2,24,33}	{34}	{35}	{36}
[3] H_{uk}	{3,24}	{3,18,25}	{3,2,12,26}	{3,27}	{3,28}	{3,1,5,29}	{3,30}	{3,2,9,31}	{3,2,9,32}	{3,24,33}	{34}	{35}	{36}
[4] H_{us}	{4,24}	{4,18,25}	{4,26}	{4,27}	{4,28}	{4,1,5,29}	{4,1,30}	{4,2,9,31}	{4,2,9,32}	{4,24,33}	{34}	{35}	{36}
[5] H_{uw}	{5,24}	{5,18,25}	{5,26}	{5,27}	{5,28}	{5,29}	{5,30}	{5,2,9,31}	{5,2,9,32}	{5,24,33}	{34}	{35}	{36}
[6] H_{un}	{6,24}	{6,18,25}	{6,26}	{6,27}	{6,28}	{6,29}	{6,1,30}	{6,31}	{6,2,9,32}	{6,33}	{34}	{35}	{36}
[7] H_{ue}	{7,24}	{7,18,25}	{7,26}	{7,27}	{7,28}	{7,1,5,29}	{7,30}	{7,2,9,31}	{7,2,9,32}	{7,24,33}	{34}	{35}	{36}
[8] H_{rk}	{8,24}	{8,18,25}	{8,26}	{8,27}	{8,28}	{8,1,5,29}	{8,30}	{8,31}	{8,9,32}	{8,33}	{34}	{35}	{36}
[9] H_{rs}	{9,24}	{9,18,25}	{9,26}	{9,27}	{9,28}	{9,1,5,29}	{9,30}	{9,31}	{9,32}	{9,33}	{34}	{35}	{36}
[10] H_{rw}	{10,24}	{10,18,25}	{10,26}	{10,27}	{10,28}	{10,1,5,29}	{10,30}	{10,31}	{10,9,32}	{10,33}	{34}	{35}	{36}
[11] H_{rn}	{11,24}	{11,18,25}	{11,26}	{11,27}	{11,28}	{11,1,5,29}	{11,30}	{11,9,31}	{11,9,32}	{11,33}	{34}	{35}	{36}
[12] H_{re}	{12,24}	{12,18,25}	{12,26}	{12,27}	{12,28}	{12,1,5,29}	{12,30}	{12,31}	{12,9,32}	{12,33}	{34}	{35}	{36}
[13] H_{tra}	{13,24}	{13,18,25}	{13,26}	{13,27}	{13,28}	{13,5,29}	{13,30}	{13,9,31}	{13,9,32}	{13,33}	{34}	{35}	{36}
[14] F_i	{24}	{25}	{26}	{27}	{28}	{29}	{30}	{31}	{32}	{33}	{34}	{35}	{36}
[15] P_a	{15,24}	{15,18,25}	{15,12,26}	{15,27}	{15,28}	{15,5,29}	{15,30}	{15,9,31}	{15,9,32}	{15,24,33}	{34}	{35}	{36}
[16] P_{ax}	{16,24}	{16,18,25}	{16,12,26}	{16,27}	{16,28}	{16,5,29}	{16,1,30}	{16,9,31}	{16,9,32}	{16,24,33}	{34}	{35}	{36}
[17] P_{fm}	{17,24}	{17,18,25}	{17,26}	{17,27}	{17,28}	{17,5,29}	{17,1,30}	{17,9,31}	{17,9,32}	{17,24,33}	{34}	{35}	{36}
[18] P_m	{18,24}	{18,25}	{18,12,26}	{18,27}	{18,28}	{18,1,5,29}	{18,1,30}	{18,9,31}	{18,9,32}	{18,33}	{34}	{35}	{36}
[19] P_s	{19,24}	{19,18,25}	{19,12,26}	{19,27}	{19,28}	{19,1,5,29}	{19,1,30}	{19,9,31}	{19,9,32}	{19,24,33}	{34}	{35}	{36}
[20] P_{padm}	{20,24}	{20,18,25}	{20,12,26}	{20,27}	{20,28}	{20,1,5,29}	{20,1,30}	{20,9,31}	{20,9,32}	{20,24,33}	{34}	{35}	{36}
[21] P_e	{21,24}	{21,18,25}	{21,12,26}	{21,27}	{21,28}	{21,1,5,29}	{21,1,30}	{21,9,31}	{21,9,32}	{21,24,33}	{34}	{35}	{36}
[22] P_{he}	{22,24}	{22,18,25}	{22,12,26}	{22,27}	{22,28}	{22,1,5,29}	{22,1,30}	{22,9,31}	{22,9,32}	{22,24,33}	{34}	{35}	{36}
[23] P_{fp}	{23,24}	{23,18,25}	{23,12,26}	{23,27}	{23,28}	{23,1,5,29}	{23,1,30}	{23,9,31}	{23,9,32}	{23,24,33}	{34}	{35}	{36}
[24] C_a	{24}	{24,18,25}	{24,12,26}	{24,27}	{24,28}	{24,5,29}	{24,30}	{24,9,31}	{24,9,32}	{24,33}	{34}	{35}	{36}
[25] C_{ax}	{25,24}	{25}	{25,12,26}	{25,27}	{25,28}	{25,5,29}	{25,1,30}	{25,9,31}	{25,9,32}	{25,33}	{34}	{35}	{36}
[26] C_{fm}	{26,24}	{26,18,25}	{26}	{26,27}	{26,28}	{26,5,29}	{26,1,30}	{26,9,31}	{26,9,32}	{26,33}	{34}	{35}	{36}
[27] C_m	{27,24}	{27,18,25}	{27,2,12,26}	{27}	{27,28}	{27,1,5,29}	{27,1,30}	{27,2,9,31}	{27,2,9,32}	{27,33}	{34}	{35}	{36}
[28] C_s	{28,24}	{28,18,25}	{28,12,26}	{28,27}	{28}	{28,1,5,29}	{28,1,30}	{28,9,31}	{28,9,32}	{28,24,33}	{34}	{35}	{36}
[29] C_{padm}	{29,24}	{29,18,25}	{29,12,26}	{29,27}	{29,28}	{29}	{29,1,30}	{29,9,31}	{29,9,32}	{29,24,33}	{34}	{35}	{36}
[30] C_e	{30,24}	{30,18,25}	{30,12,26}	{30,27}	{30,28}	{30,1,5,29}	{30}	{30,9,31}	{30,9,32}	{30,24,33}	{34}	{35}	{36}
[31] C_{he}	{31,24}	{31,18,25}	{31,12,26}	{31,27}	{31,28}	{31,1,5,29}	{31,1,30}	{31}	{31,9,32}	{31,24,33}	{34}	{35}	{36}
[32] C_{fp}	{32,24}	{32,18,25}	{32,12,26}	{32,27}	{32,28}	{32,1,5,29}	{32,1,30}	{32,9,31}	{32}	{32,24,33}	{34}	{35}	{36}
[33] C_{tm}	{33,24}	{33,18,25}	{33,12,26}	{33,27}	{33,28}	{33,1,5,29}	{33,1,30}	{33,9,31}	{33,9,32}	{33}	{34}	{35}	{36}
[34] X_a	{34,24}	{34,18,25}	{34,12,26}	{34,27}	{34,28}	{34,5,29}	{34,30}	{34,9,31}	{34,9,32}	{34,24,33}	{34}	{35}	{36}
[35] X_{mi}	{35,24}	{35,18,25}	{35,26}	{35,27}	{35,28}	{35,5,29}	{35,1,30}	{35,9,31}	{35,9,32}	{35,24,33}	{34}	{35}	{36}
[36] X_m	{36,24}	{36,25}	{36,12,26}	{36,27}	{36,28}	{36,1,5,29}	{36,1,30}	{36,9,31}	{36,9,32}	{36,33}	{34}	{35}	{36}

Table 4: Longest path influence in M'

		[1] F_L	[2] F_C	[3] H_{uk}	[4] H_{us}	[5] H_{uw}	[6] H_{un}	[7] H_{ue}	[8] H_{rk}	[9] H_{rs}	[10] H_{rw}	[11] H_{rn}	[12] H_{re}	[13] H_{tra}	[14] F_i	[15] P_a	[16] P_{ax}	[17] P_{fm}	[18] P_m	[19] P_s	[20] P_{padm}	[21] P_e	[22] P_{he}	[23] P_{fp}	[24] C_a	[25] C_{ax}	[26] C_{fm}	[27] C_m	[28] C_s	[29] C_{padm}	[30] C_e	[31] C_{he}	[32] C_{fp}	[33] C_{tm}	[34] X_a	[35] X_{mi}	[36] X_m
[1]	F_L	1,00	1,41	0,94	0,17	0,09	0,07	0,07	0,05	0,50	0,57	0,39	0,58	0,02	0,20	1,32	0,06	0,08	0,72	1,04	0,00	0,08	0,01	0,00	1,43	0,07	0,12	1,46	1,08	0,00	0,08	0,01	0,00	0,48	1,00	1,00	1,00
[2]	F_C	1,09	1,00	0,81	0,15	0,09	0,07	0,07	0,05	0,52	0,60	0,40	0,61	0,02	0,15	1,34	0,06	0,08	0,71	1,01	0,00	0,08	0,01	0,00	1,44	0,06	0,09	1,43	1,04	0,00	0,08	0,01	0,00	0,48	1,00	1,00	1,00
[3]	H_{uk}	0,94	1,20	1,00	0,16	0,09	0,06	0,06	0,06	0,63	0,72	0,48	0,73	0,03	0,17	1,00	0,06	0,10	0,67	1,02	0,00	0,08	0,02	0,00	1,08	0,06	0,11	1,34	1,06	0,00	0,08	0,02	0,00	0,36	1,00	1,00	1,00
[4]	H_{us}	1,13	1,45	0,64	1,00	0,11	0,08	0,08	0,08	0,76	0,44	0,58	0,44	0,06	0,21	1,35	0,07	0,11	0,75	1,10	0,00	0,09	0,02	0,00	1,46	0,07	0,12	1,51	1,13	0,00	0,09	0,02	0,00	0,49	1,00	1,00	1,00
[5]	H_{uw}	1,07	1,37	0,61	0,18	1,00	0,07	0,07	0,07	0,72	0,82	0,55	0,83	0,05	0,20	1,22	0,07	0,12	0,74	1,09	0,00	0,08	0,02	0,00	1,32	0,07	0,11	1,50	1,12	0,00	0,08	0,02	0,00	0,44	1,00	1,00	1,00
[6]	H_{un}	1,12	1,44	0,64	0,19	0,11	1,00	0,08	0,08	0,76	0,43	0,58	0,44	0,05	0,21	1,32	0,07	0,12	0,78	1,09	0,00	0,09	0,02	0,00	1,43	0,07	0,13	1,58	1,12	0,00	0,09	0,02	0,00	0,28	1,00	1,00	1,00
[7]	H_{ue}	1,09	1,39	0,62	0,18	0,10	0,07	1,00	0,07	0,73	0,42	0,56	0,43	0,05	0,20	1,21	0,07	0,06	0,78	1,11	0,00	0,10	0,02	0,00	1,31	0,07	0,11	1,56	1,14	0,00	0,10	0,02	0,00	0,44	1,00	1,00	1,00
[8]	H_{rk}	1,19	1,53	0,68	0,20	0,11	0,08	0,08	1,00	0,40	0,46	0,61	0,47	0,05	0,22	1,45	0,07	0,15	0,80	1,10	0,00	0,08	0,03	0,00	1,56	0,07	0,16	1,61	1,14	0,00	0,08	0,03	0,00	0,29	1,00	1,00	1,00
[9]	H_{rs}	1,24	1,58	0,70	0,20	0,12	0,08	0,08	0,08	1,00	0,47	0,32	0,48	0,05	0,23	1,59	0,07	0,12	0,78	1,08	0,00	0,09	0,03	0,00	1,72	0,07	0,13	1,57	1,11	0,00	0,09	0,03	0,00	0,30	1,00	1,00	1,00
[10]	H_{rw}	1,24	1,58	0,70	0,20	0,12	0,08	0,08	0,08	0,42	1,00	0,32	0,48	0,05	0,23	1,59	0,07	0,12	0,78	1,09	0,00	0,08	0,02	0,00	1,72	0,07	0,13	1,56	1,12	0,00	0,08	0,02	0,00	0,30	1,00	1,00	1,00
[11]	H_{rn}	1,26	1,61	0,71	0,21	0,12	0,09	0,09	0,09	0,42	0,48	1,00	0,49	0,05	0,23	1,65	0,07	0,12	0,77	1,09	0,00	0,08	0,01	0,00	1,78	0,07	0,13	1,55	1,12	0,00	0,08	0,01	0,00	0,30	1,00	1,00	1,00
[12]	H_{re}	1,20	1,54	0,68	0,20	0,11	0,08	0,08	0,08	0,40	0,46	0,61	1,00	0,05	0,22	1,48	0,07	0,14	0,78	1,08	0,00	0,08	0,02	0,00	1,60	0,07	0,15	1,58	1,12	0,00	0,08	0,02	0,00	0,29	1,00	1,00	1,00
[13]	H_{tra}	1,17	1,49	0,84	0,18	0,09	0,07	0,07	0,06	0,52	0,67	0,48	0,63	1,00	0,22	1,44	0,07	0,12	0,75	1,08	0,00	0,08	0,01	0,00	1,56	0,07	0,13	1,52	1,11	0,00	0,08	0,01	0,00	0,28	1,00	1,00	1,00
[14]	F_i	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
[15]	P_a	1,52	1,93	0,85	0,16	0,09	0,10	0,10	0,05	0,51	0,58	0,39	0,59	0,02	0,28	1,00	0,06	0,08	0,72	1,03	0,00	0,08	0,01	0,00	1,45	0,07	0,09	1,45	1,06	0,00	0,08	0,01	0,00	0,48	1,00	1,00	1,00
[16]	P_{ax}	1,49	1,89	0,84	0,15	0,09	0,10	0,10	0,10	0,50	0,57	0,38	0,58	0,02	0,27	1,30	1,00	0,08	0,72	1,14	0,00	0,12	0,01	0,00	1,40	0,07	0,09	1,46	1,18	0,00	0,12	0,01	0,00	0,47	1,00	1,00	1,00
[17]	P_{fm}	1,49	1,88	0,83	0,15	0,09	0,10	0,10	0,10	0,50	0,56	0,38	0,57	0,02	0,27	1,29	0,07	1,00	0,73	1,10	0,00	0,12	0,01	0,00	1,39	0,07	0,15	1,47	1,13	0,00	0,12	0,01	0,00	0,47	1,00	1,00	1,00
[18]	P_m	1,32	1,80	0,77	0,22	0,12	0,09	0,09	0,10	0,46	0,53	0,35	0,53	0,02	0,26	1,38	0,09	0,07	1,00	1,24	0,00	0,11	0,01	0,00	1,49	0,09	0,08	1,52	1,27	0,00	0,11	0,01	0,00	0,27	1,00	1,00	1,00
[19]	P_s	1,44	1,85	0,81	0,15	0,14	0,10	0,10	0,10	0,48	0,55	0,37	0,56	0,02	0,27	1,28	0,07	0,08	0,74	1,00	0,00	0,12	0,01	0,00	1,38	0,07	0,08	1,49	1,19	0,00	0,12	0,01	0,00	0,46	1,00	1,00	1,00
[20]	P_{padm}	1,69	1,47	0,82	0,28	0,16	0,11	0,12	0,09	0,46	0,52	0,35	0,53	0,02	0,21	1,28	0,07	0,07	0,76	1,24	1,00	0,14	0,01	0,00	1,38	0,07	0,08	1,53	1,28	0,00	0,14	0,01	0,00	0,46	1,00	1,00	1,00
[21]	P_e	1,64	1,68	0,85	0,15	0,15	0,11	0,11	0,09	0,48	0,55	0,37	0,56	0,02	0,24	1,33	0,07	0,08	0,73	1,20	0,00	1,00	0,01	0,00	1,43	0,07	0,08	1,48	1,24	0,00	0,13	0,01	0,00	0,48	1,00	1,00	1,00
[22]	P_{he}	1,64	1,67	0,84	0,15	0,15	0,11	0,11	0,09	0,48	0,55	0,37	0,56	0,02	0,24	1,30	0,07	0,08	0,74	1,21	0,00	0,13	1,00	0,00	1,40	0,07	0,08	1,49	1,25	0,00	0,13	0,01	0,00	0,47	1,00	1,00	1,00
[23]	P_{fp}	1,64	1,67	0,84	0,15	0,15	0,11	0,11	0,09	0,48	0,55	0,37	0,56	0,02	0,24	1,30	0,07	0,08	0,74	1,21	0,00	0,13	0,01	1,00	1,40	0,07	0,08	1,49	1,25	0,00	0,13	0,01	0,00	0,47	1,00	1,00	1,00
[24]	C_a	1,51	1,91	0,85	0,16	0,09	0,10	0,10	0,05	0,50	0,57	0,39	0,58	0,02	0,28	2,25	0,06	0,08	0,72	1,10	0,00	0,08	0,01	0,00	1,00	0,07	0,09	1,45	1,14	0,00	0,08	0,01	0,00	0,33	1,00	1,00	1,00
[25]	C_{ax}	1,48	1,89	0,84	0,15	0,09	0,10	0,10	0,10	0,50	0,56	0,38	0,57	0,02	0,27	1,29	1,01	0,08	0,72	1,16	0,00	0,12	0,01	0,00	1,40	1,00	0,09	1,46	1,20	0,00	0,12	0,01	0,00	0,28	1,00	1,00	1,00
[26]	C_{fm}	1,48	1,87	0,83	0,15	0,09	0,10	0,10	0,10	0,49	0,56	0,38	0,57	0,02	0,27	1,29	0,07	1,07	0,73	1,16	0,00	0,12	0,01	0,00	1,39	0,07	1,00	1,47	1,20	0,00	0,12	0,01	0,00	0,33	1,00	1,00	1,00
[27]	C_m	0,80	1,08	0,46	0,13	0,07	0,05	0,05	0,06	0,57	0,65	0,43	0,65	0,02	0,16	0,81	0,08	0,09	0,94	0,83	0,00	0,06	0,01	0,00	0,88	0,09	0,10	1,00	0,85	0,00	0,06	0,01	0,00	0,27	1,00	1,00	1,00
[28]	C_s	1,40	1,79	0,79	0,23	0,13	0,09	0,10	0,09	0,47	0,53	0,36	0,54	0,02	0,26	1,24	0,06	0,08	0,71	2,09	0,00	0,11	0,01	0,00	1,34	0,07	0,08	1,44	1,00	0,00	0,11	0,01	0,00	0,45	1,00	1,00	1,00
[29]	C_{padm}	1,69	1,47	0,82	0,28	0,16	0,11	0,12	0,09	0,46	0,52	0,35	0,53	0,02	0,21	1,28	0,07	0,07	0,76	1,24	1,00	0,14	0,01	0,00	1,38	0,07	0,08	1,53	1,28	1,00	0,14	0,01	0,00	0,46	1,00	1,00	1,00
[30]	C_e	1,64	1,68	0,85	0,15	0,15	0,11	0,11	0,09	0,48	0,55	0,37	0,56	0,02	0,24	1,33	0,07	0,08	0,73	1,20	0,00	1,08	0,01	0,00	1,43	0,07	0,08	1,48	1,24	0,00	1,00	0,01	0,00	0,48	1,00	1,00	1,00
[31]	C_{he}	1,63	1,67	0,84	0,15	0,15	0,11	0,11	0,09	0,48	0,55	0,37	0,56	0,02	0,24	1,30	0,07	0,08	0,74	1,21	0,00	0,13	1,02	0,00	1,40	0,07	0,08	1,49	1,25	0,00	0,13	1,00	0,00	0,47	1,00	1,00	1,00
[32]	C_{fp}	1,63	1,67	0,84	0,15	0,15	0,11	0,11	0,09	0,48	0,55	0,37	0,56	0,02	0,24	1,30	0,07	0,08	0,74	1,21	0,00	0,13	0,01	1,00	1,40	0,07	0,08	1,49	1,25	0,00	0,13	0,01	1,00	0,47	1,00	1,00	1,00
[33]	C_{tm}	1,40	1,79	0,79	0,23	0,13	0,09	0,10	0,09	0,47	0,53	0,36	0,54	0,02	0,26	1,24	0,06	0,08	0,71	2,09	0,00	0,11	0,01	0,00	1,34	0,07	0,08	1,44	2,16	0,00	0,11	0,01	0,00	1,00	1,00	1,00	1,00
[34]	X_a	1,52	1,93	0,85	0,16	0,09	0,10	0,10	0,05	0,51	0,58	0,39	0,59	0,02	0,28	2,34	0,06	0,08	0,72	1,03	0,00	0,08	0,01	0,00	1,45	0,07	0,09	1,45	1,06	0,00	0,08	0,01	0,00	0,48	1,00	1,00	1,00
[35]	X_{mi}	1,49	1,88	0,83	0,15	0,09	0,10	0,10	0,10	0,50	0,56	0,38	0,57	0,02	0,27	1,29	0,07	1,14	0,73	1,10	0,00	0,12	0,01	0,00	1,39	0,07	0,15	1,47	1,13	0,00	0,12	0,01	0,00	0,47	1,00	1,00	1,00
[36]	X_m	1,32	1,80	0,77	0,22	0,12	0,09	0,09	0,10	0,46	0,53	0,35	0,53	0,02	0,26	1,38	0,09	0,07	1,75	1,24	0,00	0,11	0,01	0,00	1,49	0,09	0,08	1,52	1,27	0,00	0,11	0,01	0,00	0,27	1,00	1,00	1,00

Table 5: Shortest paths in M'

		[1] F_L	[2] F_C	[3] H_{uk}	[4] H_{us}	[5] H_{uw}	[6] H_{un}	[7] H_{ue}	[8] H_{rk}	[9] H_{rs}	[10] H_{rw}	[11] H_{rn}	[12] H_{re}	[13] H_{tra}	[14] F_i	[15] P_a	[16] P_{ax}	[17] P_{fm}	[18] P_m	[19] P_s	[20] P_{padm}	[21] P_e	[22] P_{he}	[23] P_{fp}	[24] C_a	[25] C_{ax}	[26] C_{fm}	[27] C_m	[28] C_s	[29] C_{padm}	[30] C_e	[31] C_{he}	[32] C_{fp}	[33] C_{tm}	[34] X_a	[35] X_{mi}	[36] X_m
[1]	F_L	{1}	{1,2}	{1,32,7,3}	{1,20,4}	{1,20,5}	{1,32,6}	{1,32,7}	{1,32,8}	{1,32,7,9}	{1,32,7,10}	{1,32,7,11}	{1,32,7,12}	{1,32,13}	{1,14}	{1,32,7,15}	{1,16}	{1,29,17}	{1,18}	{1,19}	{1,20}	{1,20,21}	{1,32,22}	{1,23}	{1,32,7,24}	{1,25}	{1,20,26}	{1,27}	{1,28}	{1,29}	{1,29,30}	{1,32,31}	{1,32}	{1,33}	{34}	{35}	{36}
[2]	F_C	{2,1}	{2}	{2,20,7,3}	{2,20,4}	{2,20,5}	{2,6}	{2,20,7}	{2,32,8}	{2,20,7,9}	{2,20,7,10}	{2,20,7,11}	{2,20,7,12}	{2,32,13}	{2,20,14}	{2,20,7,15}	{2,16}	{2,29,17}	{2,18}	{2,19}	{2,20}	{2,20,21}	{2,32,22}	{2,23}	{2,20,7,24}	{2,25}	{2,20,26}	{2,27}	{2,28}	{2,29}	{2,29,30}	{2,32,31}	{2,32}	{2,33}	{34}	{35}	{36}
[3]	H_{uk}	{3,1}	{3,2}	{3}	{3,4}	{3,5}	{3,6}	{3,7}	{3,8}	{3,9}	{3,10}	{3,11}	{3,12}	{3,13}	{3,14}	{3,15}	{3,16}	{3,17}	{3,18}	{3,19}	{3,20}	{3,20,21}	{3,32,22}	{3,23}	{3,24}	{3,25}	{3,26}	{3,27}	{3,28}	{3,29}	{3,29,30}	{3,32,31}	{3,32}	{3,33}	{34}	{35}	{36}
[4]	H_{us}	{4,1}	{4,7,2}	{4,3}	{4}	{4,5}	{4,6}	{4,7}	{4,8}	{4,9}	{4,10}	{4,11}	{4,12}	{4,32,13}	{4,14}	{4,7,15}	{4,16}	{4,29,17}	{4,32,18}	{4,19}	{4,20}	{4,21}	{4,22}	{4,23}	{4,7,24}	{4,25}	{4,20,26}	{4,32,27}	{4,28}	{4,29}	{4,30}	{4,31}	{4,32}	{4,32,33}	{34}	{35}	{36}
[5]	H_{uw}	{5,1}	{5,2}	{5,3}	{5,4}	{5}	{5,6}	{5,7}	{5,8}	{5,9}	{5,10}	{5,11}	{5,12}	{5,32,13}	{5,14}	{5,15}	{5,16}	{5,17}	{5,32,18}	{5,19}	{5,32,20}	{5,32,20,21}	{5,32,22}	{5,23}	{5,24}	{5,25}	{5,26}	{5,32,27}	{5,28}	{5,32,29	{5,32,29,30}	{5,32,31}	{5,32}	{5,33}	{34}	{35}	{36}
[6]	H_{un}	{6,1}	{6,7,2}	{6,3}	{6,4}	{6,5}	{6}	{6,7}	{6,8}	{6,9}	{6,10}	{6,11}	{6,12}	{6,32,13}	{6,14}	{6,7,15}	{6,32,16}	{6,29,17}	{6,32,18}	{6,19}	{6,20}	{6,21}	{6,32,22}	{6,23}	{6,7,24}	{6,32,25}	{6,20,26}	{6,32,27}	{6,28}	{6,29}	{6,30}	{6,32,31}	{6,32}	{6,32,33}	{34}	{35}	{36}
[7]	H_{ue}	{7,1}	{7,2}	{7,3}	{7,4}	{7,5}	{7,6}	{7}	{7,8}	{7,9}	{7,10}	{7,11}	{7,12}	{7,32,13}	{7,14}	{7,15}	{7,32,16}	{7,17}	{7,32,18}	{7,19}	{7,20}	{7,20,21}	{7,22}	{7,23}	{7,24}	{7,32,25}	{7,26}	{7,32,27}	{7,28}	{7,29}	{7,20,30}	{7,31}	{7,32}	{7,32,33}	{34}	{35}	{36}
[8]	H_{rk}	{8,7,1}	{8,7,2}	{8,7,3}	{8,4}	{8,5}	{8,6}	{8,7}	{8}	{8,9}	{8,10}	{8,11}	{8,12}	{8,32,13}	{8,20,14}	{8,7,15}	{8,32,16}	{8,29,17}	{8,32,18}	{8,19}	{8,20}	{8,20,21}	{8,32,22}	{8,23}	{8,7,24}	{8,32,25}	{8,20,26}	{8,32,27}	{8,28}	{8,29}	{8,29,30}	{8,32,31}	{8,32}	{8,32,33}	{34}	{35}	{36}
[9]	H_{rs}	{9,7,1}	{9,7,2}	{9,7,3}	{9,4}	{9,5}	{9,6}	{9,7}	{9,8}	{9}	{9,7,10}	{9,11}	{9,7,12}	{9,32,13}	{9,20,14}	{9,7,15}	{9,32,16}	{9,29,17}	{9,32,18}	{9,19}	{9,20}	{9,20,21}	{9,32,22}	{9,23}	{9,7,24}	{9,32,25}	{9,20,26}	{9,32,27}	{9,28}	{9,29}	{9,29,30}	{9,32,31}	{9,32}	{9,32,33}	{34}	{35}	{36}
[10]	H_{rw}	{10,7,1}	{10,7,2}	{10,7,3}	{10,4}	{10,5}	{10,6}	{10,7}	{10,8}	{10,9}	{10}	{10,11}	{10,7,12}	{10,32,13}	{10,20,14}	{10,7,15}	{10,32,16}	{10,29,17}	{10,32,18}	{10,19}	{10,20}	{10,20,21}	{10,32,22}	{10,23}	{10,7,24}	{10,32,25}	{10,20,26}	{10,32,27}	{10,28}	{10,29}	{10,29,30}	{10,32,31}	{10,32}	{10,32,33}	{34}	{35}	{36}
[11]	H_{rn}	{11,7,1}	{11,7,2}	{11,7,3}	{11,4}	{11,5}	{11,6}	{11,7}	{11,8}	{11,7,9}	{11,7,10}	{11}	{11,7,12}	{11,32,13}	{11,20,14}	{11,7,15}	{11,32,16}	{11,29,17}	{11,32,18}	{11,19}	{11,20}	{11,20,21}	{11,32,22}	{11,23}	{11,7,24}	{11,32,25}	{11,20,26}	{11,32,27}	{11,28}	{11,29}	{11,29,30}	{11,32,31}	{11,32}	{11,32,33}	{34}	{35}	{36}
[12]	H_{re}	{12,7,1}	{12,7,2}	{12,7,3}	{12,4}	{12,5}	{12,6}	{12,7}	{12,8}	{12,9}	{12,10}	{12,11}	{12}	{12,32,13}	{12,20,14}	{12,7,15}	{12,32,16}	{12,29,17}	{12,32,18}	{12,19}	{12,20}	{12,20,21}	{12,32,22}	{12,23}	{12,7,24}	{12,32,25}	{12,20,26}	{12,32,27}	{12,28}	{12,29}	{12,29,30}	{12,32,31}	{12,32}	{12,32,33}	{34}	{35}	{36}
[13]	H_{tra}	{13,32,7,1}	{13,32,7,2}	{13,32,7,3}	{13,20,4}	{13,20,5}	{13,32,6}	{13,32,7}	{13,32,8}	{13,32,7,9}	{13,32,7,10}	{13,32,7,11}	{13,32,7,12}	{13}	{13,14}	{13,32,7,15}	{13,32,16}	{13,29,17}	{13,32,18}	{13,19}	{13,20}	{13,20,21}	{13,32,22}	{13,23}	{13,32,7,24}	{13,32,25}	{13,20,26}	{13,32,27}	{13,28}	{13,29}	{13,29,30}	{13,32,31}	{13,32}	{13,32,33}	{34}	{35}	{36}
[14]	F_i	{1}	{2}	{3}	{4}	{5}	{6}	{7}	{8}	{9}	{10}	{11}	{12}	{13}	{14}	{15}	{16}	{17}	{18}	{19}	{20}	{21}	{22}	{23}	{24}	{25}	{26}	{27}	{28}	{29}	{30}	{31}	{32}	{33}	{34}	{35}	{36}
[15]	P_a	{15,20,7,1}	{15,20,7,2}	{15,20,7,3}	{15,20,4}	{15,20,5}	{15,32,6}	{15,20,7}	{15,32,8}	{15,20,7,9}	{15,20,7,10}	{15,20,7,11}	{15,20,7,12}	{15,32,13}	{15,20,14}	{15}	{15,16}	{15,29,17}	{15,18}	{15,19}	{15,20}	{15,20,21}	{15,32,22}	{15,23}	{15,20,7,24}	{15,25}	{15,20,26}	{15,27}	{15,28}	{15,29}	{15,29,30}	{15,32,31}	{15,32}	{15,33}	{34}	{35}	{36}
[16]	P_{ax}	{16,20,7,1}	{16,20,7,2}	{16,20,7,3}	{16,20,4}	{16,20,5}	{16,32,6}	{16,20,7}	{16,32,8}	{16,20,7,9}	{16,20,7,10}	{16,20,7,11}	{16,20,7,12}	{16,32,13}	{16,20,14}	{16,20,7,15}	{16}	{16,29,17}	{16,18}	{16,32,13,19}	{16,20}	{16,20,21}	{16,22}	{16,23}	{16,20,7,24}	{16,25}	{16,20,26}	{16,27}	{16,32,13,28}	{16,29}	{16,29,30}	{16,31}	{16,32}	{16,33}	{34}	{35}	{36}
[17]	P_{fm}	{17,20,7,1}	{17,20,7,2}	{17,20,7,3}	{17,20,4}	{17,20,5}	{17,6}	{17,20,7}	{17,32,8}	{17,20,7,9}	{17,20,7,10}	{17,20,7,11}	{17,20,7,12}	{17,32,13}	{17,20,14}	{17,20,7,15}	{17,16}	{17}	{17,18}	{17,19}	{17,20}	{17,20,21}	{17,22}	{17,23}	{17,20,7,24}	{17,25}	{17,20,26}	{17,27}	{17,28}	{17,29}	{17,29,30}	{17,31}	{17,32}	{17,33}	{34}	{35}	{36}
[18]	P_m	{18,7,1}	{18,7,2}	{18,7,3}	{18,4}	{18,5}	{18,6}	{18,7}	{18,8}	{18,7,9}	{18,7,10}	{18,7,11}	{18,7,12}	{18,13}	{18,20,14}	{18,7,15}	{18,32,16}	{18,29,17}	{18}	{18,13,19}	{18,20}	{18,21}	{18,22}	{18,23}	{18,7,24}	{18,32,25}	{18,20,26}	{18,32,27}	{18,13,28}	{18,29}	{18,30}	{18,31}	{18,32}	{18,32,33}	{34}	{35}	{36}
[19]	P_s	{19,7,1}	{19,7,2}	{19,7,3}	{19,4}	{19,5}	{19,6}	{19,7}	{19,8}	{19,7,9}	{19,7,10}	{19,7,11}	{19,7,12}	{19,13}	{19,29,14}	{19,7,15}	{19,16}	{19,29,17}	{19,32,18}	{19}	{19,20}	{19,21}	{19,22}	{19,23}	{19,7,24}	{19,25}	{19,29,26}	{19,32,27}	{19,13,28}	{19,29}	{19,30}	{19,31}	{19,32}	{19,33}	{34}	{35}	{36}
[20]	P_{padm}	{20,7,1}	{20,7,2}	{20,7,3}	{20,4}	{20,5}	{20,6}	{20,7}	{20,8}	{20,7,9}	{20,7,10}	{20,7,11}	{20,7,12}	{20,13}	{20,14}	{20,7,15}	{20,32,16}	{20,17}	{20,32,18}	{20,13,19}	{20}	{20,21}	{20,22}	{20,23}	{20,7,24}	{20,32,25}	{20,26}	{20,32,27}	{20,13,28}	{20,29}	{20,30}	{20,31}	{20,32}	{20,32,33}	{34}	{35}	{36}
[21]	P_e	{21,7,1}	{21,7,2}	{21,7,3}	{21,20,4}	{21,20,5}	{21,6}	{21,7}	{21,8}	{21,7,9}	{21,7,10}	{21,7,11}	{21,7,12}	{21,13}	{21,20,14}	{21,7,15}	{21,16}	{21,29,17}	{21,18}	{21,13,19}	{21,20}	{21}	{21,22}	{21,23}	{21,7,24}	{21,25}	{21,20,26}	{21,27}	{21,13,28}	{21,29}	{21,29,30}	{21,31}	{21,32}	{21,32,33}	{34}	{35}	{36}
[22]	P_{he}	{22,7,1}	{22,7,2}	{22,7,3}	{22,20,4}	{22,20,5}	{22,6}	{22,7}	{22,8}	{22,7,9}	{22,7,10}	{22,7,11}	{22,7,12}	{22,13}	{22,20,14}	{22,7,15}	{22,16}	{22,29,17}	{22,18}	{22,13,19}	{22,20}	{22,20,21}	{22}	{22,23}	{22,7,24}	{22,25}	{22,20,26}	{22,32,27}	{22,13,28}	{22,29}	{22,29,30}	{22,31}	{22,32}	{22,33}	{34}	{35}	{36}
[23]	P_{fp}	{23,7,1}	{23,7,2}	{23,7,3}	{23,20,4}	{23,20,5}	{23,6}	{23,7}	{23,8}	{23,7,9}	{23,7,10}	{23,7,11}	{23,7,12}	{23,13}	{23,20,14}	{23,7,15}	{23,16}	{23,29,17}	{23,18}	{23,13,19}	{23,20}	{23,20,21}	{23,22}	{23}	{23,7,24}	{23,25}	{23,20,26}	{23,32,27}	{23,13,28}	{23,29}	{23,29,30}	{23,31}	{23,32}	{23,33}	{34}	{35}	{36}
[24]	C_a	{24,20,7,1}	{24,20,7,2}	{24,20,7,3}	{24,20,4}	{24,20,5}	{24,32,6}	{24,20,7}	{24,32,8}	{24,20,7,9}	{24,20,7,10}	{24,20,7,11}	{24,20,7,12}	{24,32,13}	{24,20,14}	{24,20,7,15}	{24,16}	{24,29,17}	{24,18}	{24,19}	{24,20}	{24,20,21}	{24,32,22}	{24,23}	{24}	{24,25}	{24,20,26}	{24,27}	{24,28}	{24,29}	{24,29,30}	{24,32,31}	{24,32}	{24,32,33}	{34}	{35}	{36}
[25]	C_{ax}	{25,20,7,1}	{25,20,7,2}	{25,20,7,3}	{25,20,4}	{25,20,5}	{25,6}	{25,20,7}	{25,32,8}	{25,20,7,9}	{25,20,7,10}	{25,20,7,11}	{25,20,7,12}	{25,32,13}	{25,20,14}	{25,20,7,15}	{25,32,16}	{25,29,17}	{25,18}	{25,32,13,19}	{25,20}	{25,20,21}	{25,22}	{25,23}	{25,20,7,24}	{25}	{25,20,26}	{25,27}	{25,32,13,28}	{25,29}	{25,29,30}	{25,31}	{25,32}	{25,32,33}	{34}	{35}	{36}
[26]	C_{fm}	{26,7,1}	{26,7,2}	{26,7,3}	{26,20,4}	{26,20,5}	{26,6}	{26,7}	{26,20,8}	{26,7,9}	{26,7,10}	{26,7,11}	{26,7,12}	{26,32,13}	{26,20,14}	{26,7,15}	{26,16}	{26,29,17}	{26,18}	{26,32,13,19}	{26,20}	{26,20,21}	{26,22}	{26,23}	{26,7,24}	{26,25}	{26}	{26,27}	{26,32,13,28}	{26,29}	{26,29,30}	{26,31}	{26,32}	{26,32,33}	{34}	{35}	{36}
[27]	C_m	{27,1}	{27,2}	{27,3}	{27,4}	{27,5}	{27,6}	{27,7}	{27,8}	{27,9}	{27,10}	{27,11}	{27,12}	{27,13}	{27,14}	{27,15}	{27,32,16}	{27,17}	{27,32,18}	{27,19}	{27,20}	{27,21}	{27,22}	{27,23}	{27,24}	{27,32,25}	{27,26}	{27}	{27,28}	{27,29}	{27,30}	{27,31}	{27,32}	{27,32,33}	{34}	{35}	{36}
[28]	C_s	{28,7,1}	{28,7,2}	{28,7,3}	{28,4}	{28,5}	{28,6}	{28,7}	{28,8}	{28,7,9}	{28,7,10}	{28,7,11}	{28,7,12}	{28,13}	{28,20,14}	{28,15}	{28,16}	{28,17}	{28,18}	{28,13,19}	{28,20}	{28,21}	{28,22}	{28,23}	{28,24}	{28,25}	{28,26}	{28,27}	{28}	{28,29}	{28,30}	{28,31}	{28,32}	{28,33}	{34}	{35}	{36}
[29]	C_{padm}	{29,7,1}	{29,7,2}	{29,7,3}	{29,4}	{29,5}	{29,6}	{29,7}	{29,8}	{29,7,9}	{29,7,10}	{29,7,11}	{29,7,12}	{29,13}	{29,14}	{29,7,15}	{29,32,16}	{29,17}	{29,32,18}	{29,13,19}	{29,32,20}	{29,21}	{29,22}	{29,23}	{29,7,24}	{29,32,25}	{29,26}	{29,32,27}	{29,13,28}	{29}	{29,30}	{29,31}	{29,32}	{29,32,33}	{34}	{35}	{36}
[30]	C_e	{30,7,1}	{30,7,2}	{30,7,3}	{30,20,4}	{30,20,5}	{30,6}	{30,7}	{30,8}	{30,7,9}	{30,7,10}	{30,7,11}	{30,7,12}	{30,13}	{30,20,14}	{30,7,15}	{30,16}	{30,29,17}	{30,18}	{30,13,19}	{30,20}	{30,20,21}	{30,22}	{30,23}	{30,7,24}	{30,25}	{30,20,26}	{30,27}	{30,13,28}	{30,29}	{30}	{30,31}	{30,32}	{30,32,33}	{34}	{35}	{36}
[31]	C_{he}	{31,7,1}	{31,7,2}	{31,7,3}	{31,20,4}	{31,20,5}	{31,6}	{31,7}	{31,8}	{31,7,9}	{31,7,10}	{31,7,11}	{31,7,12}	{31,13}	{31,20,14}	{31,7,15}	{31,16}	{31,29,17}	{31,18}	{31,13,19}	{31,20}	{31,20,21}	{31,32,22}	{31,23}	{31,7,24}	{31,25}	{31,20,26}	{31,27}	{31,13,28}	{31,29}	{31,29,30}	{31}	{31,32}	{31,33}	{34}	{35}	{36}
[32]	C_{fp}	{32,7,1}	{32,7,2}	{32,7,3}	{32,20,4}	{32,20,5}	{32,6}	{32,7}	{32,8}	{32,7,9}	{32,7,10}	{32,7,11}	{32,7,12}	{32,13}	{32,20,14}	{32,7,15}	{32,16}	{32,29,17}	{32,18}	{32,13,19}	{32,20}	{32,20,21}	{32,22}	{32,20,23}	{32,7,24}	{32,25}	{32,20,26}	{32,27}	{32,13,28}	{32,29}	{32,29,30}	{32,31}	{32}	{32,33}	{34}	{35}	{36}
[33]	C_{tm}	{33,7,1}	{33,7,2}	{33,7,3}	{33,4}	{33,5}	{33,6}	{33,7}	{33,8}	{33,7,9}	{33,7,10}	{33,7,11}	{33,7,12}	{33,13}	{33,20,14}	{33,15}	{33,16}	{33,17}	{33,18}	{33,13,19}	{33,20}	{33,21}	{33,22}	{33,23}	{33,24}	{33,25}	{33,26}	{33,27}	{33,13,28}	{33,29}	{33,30}	{33,31}	{33,32}	{33}	{34}	{35}	{36}
[34]	X_a	{34,20,7,1}	{34,20,7,2}	{34,20,7,3}	{34,20,4}	{34,20,5}	{34,32,6}	{34,20,7}	{34,32,8}	{34,20,7,9}	{34,20,7,10}	{34,20,7,11}	{34,20,7,12}	{34,32,13}	{34,20,14}	{34,20,7,15}	{34,16}	{34,29,17}	{34,18}	{34,19}	{34,20}	{34,20,21}	{34,32,22}	{34,23}	{34,20,7,24}	{34,25}	{34,20,26}	{34,27}	{34,28}	{34,29}	{34,29,30}	{34,32,31}	{34,32}	{34,33}	{34}	{35}	{36}
[35]	X_{mi}	{35,20,7,1}	{35,20,7,2}	{35,20,7,3}	{35,20,4}	{35,20,5}	{35,6}	{35,20,7}	{35,32,8}	{35,20,7,9}	{35,20,7,10}	{35,20,7,11}	{35,20,7,12}	{35,32,13}	{35,20,14}	{35,20,7,15}	{35,16}	{35,29,17}	{35,18}	{35,19}	{35,20}	{35,20,21}	{35,22}	{35,23}	{35,20,7,24}	{35,25}	{35,20,26}	{35,27}	{35,28}	{35,29}	{35,29,30}	{35,31}	{35,32}	{35,33}	{34}	{35}	{36}
[36]	X_m	{36,7,1}	{36,7,2}	{36,7,3}	{36,4}	{36,5}	{36,6}	{36,7}	{36,8}	{36,7,9}	{36,7,10}	{36,7,11}	{36,7,12}	{36,13}	{36,20,14}	{36,7,15}	{36,32,16}	{36,29,17}	{36,32,18}	{36,13,19}	{36,20}	{36,21}	{36,22}	{36,23}	{36,7,24}	{36,32,25}	{36,20,26}	{36,32,27}	{36,13,28}	{36,29}	{36,30}	{36,31}	{36,32}	{36,32,33}	{34}	{35}	{36}

Table 6: Cause-Effect coordinates of M'

	Minimum influence		Optimal influence		Maximum influence	
	Cause	**Effect**	**Cause**	**Effect**	**Cause**	**Effect**
F_L	10	10	15	49	17	49
F_C	10	10	15	58	17	58
H_{uk}	14	6	12	28	16	28
H_{us}	10	4	14	6	18	8
H_{uw}	14	3	14	4	19	6
H_{un}	10	4	14	3	18	5
H_{ue}	14	3	14	3	18	5
H_{rk}	8	3	15	3	19	5
H_{rs}	7	6	15	17	18	19
H_{rw}	8	6	15	19	18	21
H_{rn}	7	5	16	13	19	16
H_{re}	8	5	15	19	18	21
H_{tra}	6	3	16	3	19	3
F_i	36	2	1	5	36	9
P_a	9	10	16	49	18	49
P_{ax}	7	3	16	3	19	5
P_{fm}	9	3	16	7	19	7
P_m	5	16	16	28	18	28
P_s	5	23	16	42	18	42
P_{padm}	5	2	16	2	19	3
P_e	7	3	16	5	19	6
P_{he}	6	2	16	3	19	4
P_{fp}	6	2	16	2	19	3
C_a	8	11	17	51	19	50
C_{ax}	6	3	17	2	20	4
C_{fm}	7	3	17	5	20	5
C_m	11	27	11	53	14	53
C_s	10	23	16	42	19	42
C_{padm}	5	2	17	1	20	2
C_e	7	3	17	4	20	6
C_{he}	7	2	17	2	20	3
C_{fp}	7	2	17	1	20	2
C_{tm}	10	6	17	11	20	16
X_a	8	36	17	1	19	36
X_{mi}	8	36	17	1	19	36
X_m	4	36	17	1	19	36