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by Igor Vetlov[2], Tibor Hlédik[3], Magnus Jonsson[4], Henrik Kucsera[5] and Massimiliano Pisani[6]

In 2011 all ECB publications feature a motif taken from the €100 banknote.

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Abstract

In view of the increasing use of Dynamic Stochastic General Equilibrium (DSGE) models in the macroeconomic projections and the policy process, this paper examines, both conceptually and empirically, alternative notions of potential output within DSGE models. Furthermore, it provides historical estimates of potential output/output gaps on the basis of selected DSGE models developed by the European System of Central Banks' staff. These estimates are compared to the corresponding estimates obtained applying more traditional methods. Finally, the paper assesses the usefulness of the DSGE model-based output gaps for gauging inflationary pressures.

Keywords: potential output, simulation and forecasting models, monetary policy

JEL classification codes: E32, E37, E52

Non-technical summary

Over recent years, policymaking institutions, including the European Central Bank and the National Central Banks within the European System of Central Banks, have given increasing emphasis to measures of potential output and the associated output gaps, both in the macroeconomic projections and in the assessment of the monetary policy stance. Typically, these measures rely on the macro-economic production function approach, or relatively simple statistical filters. More recently, a new approach to estimating potential output has emerged, which is based on New-Keynesian Dynamic Stochastic General Equilibrium (NK DSGE) models. It allows estimating alternative model-based notions of potential output encompassing the level of output obtained under flexible prices and wages.

Against this background, this paper examines, both conceptually and empirically, alternative notions of potential output widely used in DSGE models. More specifically, it first defines three distinctive notions of potential output: efficient output, natural output, and trend output. Next, it discusses the main findings of the literature on DSGE-based estimation of potential output. It then presents examples of historical estimates of potential output, and the associated output gaps, obtained from DSGE models used for policy analysis in a number of central banks, compares them with estimates obtained using more traditional approaches, and discusses unconditional correlations between alternative notions of the output gap and several measures of inflation. Finally, it explores whether DSGE model-based output gaps are good indicators of inflationary pressures by evaluating their inflation forecast performance using a reduced-form Phillips-curve framework and by looking at conditional correlations between inflation and various model-based indicators of inflationary pressures.

The main findings of the paper are as follows:

- The model-consistent notions of efficient and natural output are obtained under the counterfactual assumption of full nominal flexibility and, in the case of efficient output, of perfect competition. Deviations from the model-consistent output level, i.e. the output gap, represent economic inefficiencies due to imperfect competition and/or price-setting frictions. Trend output corresponds more closely to estimates obtained with traditional approaches to filtering permanent changes in output. In this regard the trend output gap (deviation of actual output from trend output) measures business-cycle fluctuations.

- Given the conceptual differences across approaches, estimates of the flexible-price notions of potential output are expected to be more volatile and to imply smaller and less persistent output gaps than the corresponding estimates based on more traditional approaches. In terms of policy implications, explicit consideration of market inefficiencies and nominal rigidities make the model-consistent notions of potential output particularly relevant for the design of optimal monetary policy aimed at enhancing welfare and macroeconomic stabilization.

- The comparison of DSGE model-based estimates of the output gap with traditional measures reveals that the two approaches may deliver significantly different estimates of the output gap. Furthermore, like traditional estimates, DSGE model-based estimates

of potential output are subject to high uncertainty. This uncertainty reflects, inter alia, real-time uncertainty, parameter uncertainty, as well as critical assumptions underlying the identification of the models' structural shocks.

- Concerning the relationship between the output gap and inflation, there is no conclusive evidence yet proving that empirical estimates of model-consistent output gaps derived from larger (and more realistic) DSGE models are significantly better indicators of inflationary pressures than traditional measures.
- The effects of the output gap on inflation and the size of the trade-off between output and inflation stabilization depends on the type of shocks and other structural features (such as, e.g. degree of openness) of the analysed economy.

Arguably, there are numerous open issues faced by the DSGE approach to potential output estimation. However, this does not mean that DSGE model-based measures of potential output are useless. Alternative notions of potential output can be consistently analysed within a unified (DSGE) modelling framework. Importantly, it allows considering notions of potential output which are highly relevant for designing policies aimed at enhancing welfare and macroeconomic stabilization. Moreover, the joint estimation of potential output and structural shocks within a general equilibrium framework allows conducting a quantitative and coherent (internally consistent) assessment of inflationary pressures.

Admittedly, the empirical research on potential output estimation using the DSGE approach is relatively new and the contributions are relatively scarce. Yet looking forward, further advancement in DSGE modelling is expected to strengthen the case for using model-consistent measures of the output gap in policy discussions, albeit in a cautious way.

Introduction

Over recent years, policymaking institutions, including the European Central Bank (ECB) and the National Central Banks (NCBs) within the European System of Central Banks (ESCB), have given increasing emphasis to measures of potential output and the associated output gaps, both in the macroeconomic projections and in the assessment of the monetary policy stance. Traditionally, the measures of potential output rely on statistical filters or the estimation of the economy-wide production function. In the first case, the filters (for example, the Hodrick-Prescott, Christiano and Fitzgerald and band-pass filters) allow to identify permanent changes in observed output. The main advantage of the approach is simplicity. One possible disadvantage is little guidance from economic theory. As such, it is difficult, if not impossible, to provide any structural interpretation to the results. The production function approach takes a partial equilibrium perspective and estimates the relationship between factors of production and the maximum amount of output. The analysis of the contribution of each individual factor to potential production provides deeper insights into the structural sources of changes in potential output. Empirical literature on estimating traditional measures of potential output and the output gap emphasizes the high degree of uncertainty surrounding point estimates of potential output stemming from model uncertainty and real-time revisions.

A more recent approach to estimating potential output is based on New-Keynesian Dynamic Stochastic General Equilibrium (NK DSGE) models. The approach builds on three crucial elements. First, it rests on the advances in the theory of optimal monetary policy, that emphasize the role of model-consistent measures of potential output, and the related output gaps, for properly making monetary policy decisions and as a source of inflationary pressures. Second, it is supported by the advances in the estimation of DSGE models, that allow a quantitative and, owing to the discipline of the general equilibrium setup, internally consistent and fully structural interpretation of the dynamics of macroeconomic variables (in particular inflation, actual and potential output). Third, the DSGE framework allows using also more traditional concepts of potential output (not only the model-consistent concepts) for designing optimal monetary policy decisions in an internally-consistent model.

Against this background, and given the increasing use of DSGE models in policymaking institutions for projections exercises and policy analysis, the main objective of the paper is to examine, both conceptually and empirically, the alternative notions of potential output within DSGE models. In particular, the paper (a) reviews various concepts of potential output applied in DSGE models; (b) provides estimates of potential output and the associated output gaps on the basis of selected DSGE models actually used by central banks in comparison with more traditional measures; and (c) assesses the usefulness of the DSGE model-based output gaps for gauging inflationary pressures.

The main findings of the paper are largely drawn from empirical research on potential output estimation using the DSGE approach which is, admittedly, relatively new. The contributions are relatively scarce, and, therefore, preliminary. The paper points to numerous open issues faced by the DSGE approach to potential output estimation. However, it does not mean that DSGE model-based measures of potential output are useless. The joint estimation of potential output and structural shocks within a general equilibrium framework allows conducting a quantitative and internally consistent assessment of inflation pressures and a normative evaluation of alternative monetary policies. Further advancement in DSGE modelling, therefore, is expected to gradually strengthen the case for using model-consistent measures of the output gap in policy discussions.

The rest of the paper is structured as follows. The first part overviews conceptual issues related to the definition of potential output within the DSGE framework. On the basis of model-based impulse-response analysis it provides illustrations of how alternative notions of potential output and the implied output gaps react to various economic shocks. The second part presents estimates of selected definitions of potential output and the output gaps in actual DSGE models used in central banks and makes comparisons with the Hodrick-Prescott filter-based estimates. In the third part we focus on special issues which help to illustrate the usefulness of DSGE-based potential output estimates in providing valuable information on future inflationary developments. In the conclusions we summarize the key findings of the paper.

1. Conceptual issues in the DSGE framework

The current generation of NK DSGE models largely rests on a theoretical synthesis that assigns both "Keynesian" and "Real business cycle" theories a distinct and complementary role in the analysis of business cycle fluctuations and, hence, in defining actual and potential output.[7] Factors stressed in the (flexible-price) real business cycle theory explain the evolution of potential output over time. Factors stressed in Keynesian theory are related to delays in the adjustment of nominal wages and prices that result in transitory deviations of actual output from potential output. Both theories fall within the general equilibrium approach. Differently from more traditional approaches, where the Keynesian theory explains the short-run dynamics of the economy while classical theory explains the long run, the NK approach emphasizes that wages, prices and potential output do change in the short run and should be taken into account in the analysis of business cycles. Moreover, in the NK synthesis, fluctuations in economic activity are not necessarily desirable and monetary policy is not irrelevant for stabilization (keeping output close to the potential level). Because of distortions related to the delay in wage and price adjustment and associated time-varying profit mark-up fluctuations, the consequences of real disturbances can be inefficient, and their degree of inefficiency can be mitigated by the response of monetary policy.

1.1. Potential output notions

Empirical contributions that exploit the NK theoretical approach to estimating potential output are several. In line with the seminal paper by Smets and Wouters (2003) on the estimation of DSGE models, the contributions exploit Bayesian methods and a multiple shock approach. In Bayesian DSGE models, historical paths for unobserved structural shocks are estimated.[8] Consequently, it is possible to derive historical model-based estimates of potential output (and thereby the output gap), which, importantly, have structural interpretation. It is also possible to compare how model estimates of potential output and the output gap differ from those obtained using more conventional approaches.

This subsection, first, reports the theoretical definitions of potential output commonly used in NK DSGE models. Second, it discusses the main results of existing empirical contributions on estimating potential output in this class of models.

1.1.1. Trend, efficient and natural output

Within a NK DSGE model, three different notions of potential output can be analysed: trend output, efficient output, and natural output. Specifically:

(1) the *trend* level of output is equal to the sequence of permanent (unit-root) stochastic[9] technology shocks that characterize the stochastic balanced-growth path of the model;

[7] The synthesis is called "new neoclassical synthesis" (see Woodford (1999)).
[8] Estimates are obtained by using the Kalman filter.
[9] Trend output could also be equal to deterministic trend, e.g. in case only stationary structural shocks are modelled.

the corresponding output gap (equal to actual output less trend output) measures the business cycle component of output; thus, it is closely related to more traditional measures of the output gap;

(2) the *efficient* level of output is the level that would prevail if goods and labour markets were perfectly competitive (as such, prices and wages are fully flexible and both steady-state mark-ups and mark-up shocks are zero);[10] hence, the related gap (equal to actual output less efficient output) measures the relevance of imperfect competition and nominal rigidities;

(3) the *natural* level of output is the level of output that would prevail under flexible prices and wages and imperfectly competitive markets (so, differently from the efficient output, steady-state mark-up and mark-up shocks are different from zero); the related gap measures only the relevance of nominal rigidities.

The definition of trend output has a long-run dimension, as it is affected only by the unit root technology shocks and ignores fluctuations around the (steady-state) trend. It follows from the assumption of stochastic balanced-growth path of the model, which implies that the steady-state equilibrium level of the model is a stochastic trend. As reported later, its estimates are typically close to those obtained by using more conventional approaches (for example the Hodrick-Prescott filter). The efficient and natural levels, instead, have a business cycle dimension, related to the structural shocks that push the economy temporarily away from the steady state. They fluctuate around the balanced growth path, as they incorporate not only permanent shocks, but also transitory ones. In the NK models the efficient and natural output can deviate from the actual output because the latter, differently from the former two, is determined under the assumption of sticky prices and/or wages.

Furthermore, the relevance of business cycle fluctuations makes the definitions of efficient and natural outputs conceptually different not only from trend output, but also from the measure of potential output obtained using the more conventional production-function approach. The latter rests on choosing a technical relationship (for example, a Cobb-Douglas production function) representing the productive capacity of the economy, calibrating or estimating key parameters on the basis of the relevant data, determining the level of potential output by means of this function and modelling the resulting Solow residual using econometric techniques. As such, the production function-based measure of potential output is built up from smoothed values of multifactor productivity and production inputs. This approach implies that shocks affecting the economy at business cycle frequencies have no important effects on potential output. By contrast, in the DSGE approach the efficient and natural output levels can undergo swings and, hence, fluctuate over the business cycle.

As regards differences in efficient and natural outputs, these notions of potential output have a common business cycle dimension and both are based on the flexible-price assumption, however, they are not equivalent concepts. The natural output level does reflect imperfect

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[10] In NK models characterized only by imperfect competition and nominal rigidities, efficient output is the output associated with the competitive (flexible-price) equilibrium that yields the highest level of welfare to households (Ramsey equilibrium).

competition in the goods and labour market. By contrast, the efficient output level reflects perfect competition. This implies that efficient output is affected by the same shocks that affect natural output, except for (labour and goods) mark-up shocks (see section A in the Appendix for a model-based illustration of the relationship between the two concepts of potential output).

The concepts of efficient and natural output implied by DSGE models are relevant for the conduct of monetary policy (see Blanchard and Galí (2007)). In particular, under some simplifying assumptions (e.g., flexible wage stetting), the natural output gap is proportional to the real marginal cost which is the key driver of inflation in the NK Phillips Curve (NKPC). Therefore, natural output is relevant for inflation determination in DSGE models (see section B in the Appendix). From the welfare point of view, the relevant output gap (that policymakers should stabilize) is instead the difference between the actual and the efficient level of output. The relevance of natural and efficient output concepts for a normative assessment of monetary policy is discussed in details in the next section.

1.1.2. The design of optimal monetary policy

As said previously, the concepts of efficient and natural output implied by DSGE model are relevant for the optimal conduct of monetary policy. Depending on model structure, two cases can be distinguished. In the first case, shocks are such that there is a constant distance between natural and efficient output, e.g. in the basic NK model without mark-up shocks. This implies that stabilizing the efficient output gap is equivalent to stabilizing the natural output gap and inflation (the so called "divine coincidence"). In the second case, efficient and natural outputs are no longer proportional because of the presence of exogenous mark-up shocks that introduce a trade-off between stabilizing inflation and efficient output. As such, the "divine coincidence" does not hold anymore. Let's consider each case in turn.

The "divine coincidence"

The quantitative difference between natural and efficient output has crucial implications for the optimal conduct of monetary policy. In the basic NK model the difference is constant, invariant to shocks and proportional to the level of the steady-state mark-up. It implies that stabilizing the natural output gap is equivalent to stabilizing the efficient (welfare-relevant) output gap.[11] This equivalence is the source of the divine coincidence: the stabilization of inflation is equivalent to stabilization of output gap and it is optimal. To get some intuition, let's consider shocks to the aggregate demand for goods and services and to productivity.

Expansionary (contractionary) demand shocks tend to push prices up (down) and the output above (below) its natural and efficient levels, as the actual sticky-price output reacts to a greater extent than the natural level to the shock. Because in both cases the price level and the output gap are moving in the same direction, an increase (decrease) in the monetary policy rate stabilizes simultaneously the natural output gap and prices. Moreover, because of the constant proportionality between natural and efficient output gap, the monetary policy is able to stabilize the efficient output gap as well.

[11] In the basic NK model, given non-zero steady-state mark-ups, monetary policy stabilizes (or closes) the efficient output gap only up to a constant, while, in order to stabilize inflation, monetary policy stabilizes the natural output gap at the zero level (see Woodford (2003)).

A similar reasoning holds for shocks to productivity. A positive shock to productivity puts downward pressure on prices, as the shock would affect (flexible-price) natural and efficient output levels more than (sticky-price) actual output (hence, the output gap is negative). In this case, the optimal policy is expansionary, allowing an increase in actual output equal to that of natural and efficient output levels. The expansionary policy, by closing the output gap, would also counter-balance the decrease in inflation, stabilizing not only prices but also output at potential.

The above examples show that the monetary policy can stabilize inflation by stabilizing actual output at the natural level (in other terms, by closing the natural output gap) and that it is optimal to do so (as the efficient output gap is closed as well).

Trade-offs between inflation and output stabilization

Although the simplest sticky-price models imply that stabilizing sticky-price inflation and economic activity are two sides of the same coin, the presence of inefficient shocks or other (real) frictions besides sticky prices can generate trade-offs between stabilizing inflation and stabilizing the efficient output gap, consistent with more standard perceptions by central banks. To get such trade-off, it is necessary to allow for instances in which the difference between the natural output gap and the efficient output gap is not constant. In this case the "divine coincidence" would not hold and completely stabilizing sticky-price inflation would not imply stabilizing output around its efficient rate, introducing a trade-off for the optimal monetary policy.

For example, a temporary (inefficient) increase in the monopoly power that raises mark-ups would exert upward pressure on prices, by reducing the natural level of output. Differently from a negative technology shock, it would not simultaneously reduce the efficient level of output. As such, the distance between natural and efficient output would change, generating a trade-off between stabilizing inflation (by lowering actual output at the new natural level, below the efficient one) and stabilizing output at the efficient level (by allowing for an increase in the price level).[12]

In Section 1.2.3 empirical evidence suggests that the trade-off between inflation and output gap stabilization may depend upon the type of potential output concept used in policy considerations.

1.1.3. Conditional vs. unconditional potential output

Another relevant distinction in the DSGE literature on potential output is due to alternative treatment of the initial state of economy. The latter is represented by the value of the predetermined (state) variables (such as, for example, physical capital) at the beginning of the considered time period. In this regard, one can distinguish between *conditional* and *unconditional* notions of potential output.

[12] Alternatively, some contributions introduce the trade-off by inserting additional real imperfections in the NK model, such as real wage rigidities (real wages respond sluggishly to labour market conditions, as a result of some imperfection or friction in labour markets). Similarly to the case of introducing mark-up shocks, real wage rigidities imply that the gap between natural and efficient output is no longer constant, and is affected by shocks (see, for example, Blanchard and Galí (2007)). In this paper we consider the case of the trade-off introduced through mark-up shocks only.

Woodford (2003) argues that in the case of the NK model with physical capital the natural output level is the equilibrium output under (current and expected future) flexible prices which depends not only upon current and expected future exogenous disturbances, but also on the current stock of capital in the sticky-price economy. As such, this conditional (on sticky-price capital) notion of natural output is a function of past monetary policy decisions.

Alternatively, Neiss and Nelson (2003) define the natural rate of output as the unconditional equilibrium output that would hold if prices were not only currently flexible and expected always to be flexible in the future, but also had always been flexible in the past, so that what matters for the computation is not the capital stock that actually exists, but the one that would exist if prices had been flexible, given the actual history of exogenous disturbances.

According to Woodford, the conditional definition of potential output is more connected with equilibrium determination in the actual sticky-price economy, as it is the actual (sticky-price) capital stock and its effects on the economy's production capacity that are relevant for appropriately defining the natural and efficient level of activity and, hence, design of the optimal of monetary policy.

According to Neiss and Nelson, conditioning on the actual capital stock in defining potential output, as suggested by Woodford, can lead to some peculiar policy prescriptions, if the monetary policy reaction function is characterised by occasional unavoidable mistakes that can be corrected only in later periods. Neiss and Nelson consider, for example, a monetary policy mistake in the last period, that reduces the capital stock today and, hence, potential output today. The last period's mistake does not open the conditional output gap, as by definition in the current period the stock of capital is the same for both the actual and the conditional potential output. Therefore there is no need of a compensating policy response today (to correct last period's mistake). Instead, the last period's mistake does create the unconditional output gap, as, by definition, in the current period the stock of capital for producing actual output is affected by monetary policy while the stock of capital for producing the unconditional potential output is not. In this case last period's mistake creates an unconditional output gap and justifies a monetary policy easing today which aims at correcting the past mistake.

1.2. Estimates of potential output

The empirical literature on DSGE model-based estimates of potential output is scarce and the reported findings are still preliminary reflecting the fact that the literature is relatively new. Moreover, there are remaining issues regarding the robustness of the flexible-price gap estimates with respect to alternative model structures, shock identification schemes and data revisions.

1.2.1. Comparison of historical estimates

A large part of the literature compares historical estimates of potential output and the implied output gaps, derived using DSGE models, with more traditional measures.

There is evidence that estimates of efficient output share some features with conventional measures of potential output. For example, Justiniano and Primiceri (2008) estimate a NK DSGE model using the US data and find that the (unconditional) efficient output is similar to the HP filtered output or the Congressional Budget Office estimate. The reason is that the estimated efficient output has evolved quite smoothly in the post-war period. Sala et al. (2010), using a specification of the model similar to Smets and Wouters (2007) and Justiniano and Primiceri (2008), produce time paths for the US efficient output that follow actual output in booms and recessions, but tend to stay closer to the long-run trend. As such, these measures are similar to measures obtained using more conventional statistical techniques. In particular, the conditional efficient output follows actual output more closely than does the unconditional measure. Moreover, the unconditional measure is also more volatile than the conditional measure and actual output.

Other contributions report that there can be significant differences between the flexible-price output and more traditional potential output estimates. For example, using an estimated small open-economy DSGE model for Sweden, Adolfson et al. (2008) find that the estimated model-based (stochastic) trend output level resembles the HP filter-based estimates of potential output, while the conditional and unconditional efficient output are quite different. It is reported that in some periods the flexible-price output gaps can have different sign than the HP output gap and the trend output gap.

Justiniano and Primiceri (2008) also provide estimates of natural output for the US economy which is found to be extremely volatile and hence different from conventional measures of potential output. They find that the difference in volatility between natural and efficient output is almost exclusively due to the high variability of wage mark-ups shocks. The intuition they provide is the following. A positive wage mark-up shock represents a negative shift of labour supply. When wages are sticky, the labour supply schedule is relatively flat. Hence, the drop in equilibrium hours and output can be moderate. On the other hand, with flexible wages the labour supply curve is substantially steeper. Hence, a positive mark-up shock produces a severe contraction in economic activity; as a result, historical estimates of natural output are reported to be very volatile.

In an estimated two-sector DSGE model of the US economy, Edge et al. (2008) find that the model's estimated path of the unconditional natural output gap and other more traditional production function based estimates, such as those based on the FRB/US model, widen around NBER recession dates. Nonetheless, Edge et al. (2008) find that there are sharp differences between the FRB/US and DSGE model generated output gaps which partially reflect differences in the economic concept captured by the two series. The DSGE model's natural output gap is a driver of inflation, which implies that the path of inflation has an important bearing on the resulting output gap path. In their contribution, two instances illustrating this dependence are the early 1990s, when inflation continued to decline even though a slow recovery was underway and the late 1990s, when inflation remained contained despite very strong economic growth. These episodes are reflected in the DSGE model's output gap estimate, as this gap remains negative in the early 1990s and for much of the late 1990s.

Finally, Coenen et al. (2009), using an estimated version of the New Area Wide Model, find that euro area unconditional natural output is quite volatile. The relatively high variability is due to both wage and price mark-up shocks. Focusing on a flexible-price notion of potential output which is not affected by mark up shocks, they show, that, compared to traditional output gap measures, estimates of the euro area flexible-price output gap feature larger short-run volatility and, at times, display divergent tendencies. This seems largely caused by the influence of transitory technology shocks on the flexible-price output.

1.2.2. Robustness of DSGE model-based estimates

The historical estimates of efficient and natural output depend on the interpretation of the estimated shock. Justiniano and Primiceri (2008) show that high variable mark-up shocks favour smooth estimates of efficient output and extremely volatile estimates of natural output. However, Justiniano and Primiceri also argue that mark-up shocks do not have a clear structural interpretation.[13] When re-estimated with measurement errors in price and wage inflation instead of price and wage mark-up shock, the model produces a better fit and the estimated measurement errors are similar to the mark-up shocks. They find that under this alternative interpretation efficient and natural output move one-to-one (the natural output becomes less volatile).

Furthermore, Sala et al. (2010) emphasise that interpretation of the shock in the wage equation as a wage mark-up shock or labour (leisure) preference shock does have important implications for the estimation of the model-consistent potential output and hence monetary policy. As wage mark-up shocks do not affect the efficient allocation, they are "inefficient" shocks that drive output away from the efficient level and therefore should be counteracted by monetary policy. Labour supply shocks, on the other hand, are shocks to preferences which are efficient shocks that affect efficient output and therefore should be accommodated by monetary policy. These shocks shift the wedge between the marginal product of labour and the marginal rate of substitution between consumption and leisure. However, in the benchmark model (e.g. Smets and Wouters (2005, 2007)) the wage mark-up shock is observationally equivalent to shocks to the preference for leisure.

Sala et al. (2010) provide evidence showing that efficient output is more volatile when the shock is interpreted as a labour supply shock. On the quantitative side, the measure of efficient output that is conditional on the current state variables is sometimes up to twice as large as actual output. The output gap is therefore completely dominated by movements in efficient output. Hence the uncertainty on the source of fluctuations in the labour wedge leads to great uncertainty about the potential level of output, the output gap, and therefore about the appropriate design of monetary policy.[14]

1.2.3. Monetary policy design in empirical models

Some contributions consider alternative measures of potential output and the output gap for appropriately designing optimal monetary policy. For example, using an estimated medium-scale DSGE model of the Swedish economy Adolfson et al. (2008) computes the optimal monetary policy as the policy that minimizes an intertemporal loss function[15] (equal to a

[13] A similar criticism of wage mark-up shocks can be found in Chari et al. (2008).
[14] More recently, Galí et al. (2010) introduces the notion of unemployment into the model and uses the unemployment data in model estimation in an attempt to identify the labour supply and wage mark-up shocks.
[15] They choose a quadratic period loss function that corresponds to flexible inflation targeting and the Riksbank's monetary-policy objective. The function is the weighted sum of three terms: the squared

discounted sum of expected future period losses) under three alternative concepts of output gaps in the loss function: (1) the trend output gap, based on the trend (potential) output level growing stochastically because of the unit-root stochastic technology shock in the model; (2) the unconditional output gap, based on unconditional efficient output; (3) the conditional output gap, based on conditional efficient output. The authors compare optimal monetary policy projections for different output gaps in the loss function and contrast these with projections under the estimated instrument rule. They find that the optimal (loss function-based) projections for inflation, output, and the instrument rate differ substantially, depending on whether it is the flexible-price or trend output gap that enters the loss function. Moreover the monetary policy implied by the optimal policy is not always tighter than the policy implied by the simple instrument rule. In fact, the relative degree of tightness depends upon the initial state of the economy.

In a follow-up paper, Adolfson et al. (2009) use the same estimated model to study the trade-off between stabilizing CPI inflation and alternative definitions of the output gap. They find that the policy trade-off is more favourable for the unconditional and conditional output gap than for the commonly used trend output gap[16]. In the latter case, the variance trade-offs between inflation and the trend output gap is predominantly driven by the domestic technology shocks, as the stationary technology shock affects actual output but not trend output. Even if the shock is efficient (it lowers inflation pressures, natural and efficient output gaps), trend productivity is not affected and the related output gap therefore increases creating a trade-off between stabilizing inflation and the trend output gap. Thus, abandoning the trend output gap for either conditional or unconditional flexible-price output gap may be associated with lower inflation variability.

Galí (2010) uses a standard NK DSGE model calibrated to the US economy to analyze the implications for welfare of the output gap (defined as the difference between actual and efficient output level) and its fluctuations, by computing a measure of the associated utility losses and analyzing its changes over time. He finds that average welfare losses resulting from output gap fluctuations are small, while variations in the size of those losses over the cycle are shown to be substantial, with the losses experienced during recessions are not negligible.

1.2.4. The output gaps as predictors of inflation

In the basic NK DSGE model the natural output gap is an indicator of building inflation pressures. It is proportional to real marginal cost which directly affects inflation through the NKPC. This implies that empirical estimates of real marginal cost (which is not directly observable) can be used to assess inflation evolution. In particular, according to the theoretical restrictions the appropriate observable variable is the labour income share. Conditional on this theoretically-consistent measure of real marginal cost, several contributions estimate the NKPC using a Generalized Method of Moment approach.

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quarterly change in the Riksbank's instrument rate (the repo rate); the squared inflation gap between 4-quarter CPI inflation and the inflation target; the squared output gap between output and potential output.

[16] The trade-off remains significant in case of conditional and unconditional output gap used in the loss function because of the presence of mark-up shocks.

Results suggest that the marginal cost-based version of the NKPC can provide a reasonable fit of post-war inflation in the US (Sbordone (2002), Galí and Gertler (1999)) and in the euro area (Galí et al. (2001)). When the NKPC is estimated in a way consistent with the underlying theory it appears to fit the data much better than it had been concluded by the earlier literature that instead estimated the NKPC using conventional measures of the output gap. The reason, as emphasized by Fuhrer and Moore (1995), is the forward-looking nature of the NKPC, which implies that inflation should lead the output gap over the cycle, in the sense that a rise (decline) in current inflation should signal a subsequent rise (decline) in the output gap. This can be shown by looking at the NKPC (see section B in the Appendix for details):

(1) $\pi_t = \beta \cdot \pi_{t+1} + (\kappa/\eta) \cdot (y_t - y_{n,t})$.

Its forward solution shows that current period inflation is the discounted sum of future expected output gaps:

(2) $\pi_t = (\kappa / \eta) \cdot \sum_{s=0}^{\infty} \beta^s (y_{t+s} - y_{n,t+s})$.

Thus, an increase in current inflation should signal future increases in the output gap or, in other terms, inflation should lead the output gap over the business cycle. Yet, when the output gap is estimated by using some conventional measure of de-trended log GDP, exactly the opposite pattern can be found in the data (the current output gap co-moves positively with future inflation and negatively with lagged inflation).

Overall, two points are worth stressing. First, the empirical research hasn't delivered satisfactory results yet. With quarterly data, it is often difficult to detect a statistically significant short-run effect of real activity on inflation using the theory-consistent structural relationship while at the same time measuring real activity by the output gap defined in terms of conventional de-trended output. Second, in recent years the basic NK model has been augmented with a number of real and nominal frictions in order to better match the data (see Smets and Wouters (2003, 2007), Christiano, Eichenbaum and Evans (2005), Adolfson et al. (2008)). Some key frictions that have been included are monopolistic competition in the labour market, wage stickiness, investment adjustment costs and habit formation. These frictions imply that the real wage rate no longer equals the marginal rate of substitution between consumption and leisure. As such, the equality between the real marginal cost and the flexible-price output gap is broken. The relationship between inflation and the flexible-price output gap is therefore generally quite complicated and will depend on the frictions, the type of shock that hit the economy and the conduct of policy. To what extent inflation is related to the flexible-price output gap in models with more frictions and shocks is a quantitative question. This point will be further illustrated in section 3.

1.3. Impulse-response analysis

To gain further insight about sources of possible differences in dynamics between alternative notions of potential output and the implied output gaps in what follows we present responses of the selected output measures to structural economic shocks identified in an estimated DSGE model. As a benchmark model[17] we utilise a closed-economy version of the New Area-Wide Model[18] (NAWM) which we estimate over 1985q1–2009q4 for the euro area data: growth in real GDP, growth in real private consumption, growth in real investment, level of employment, growth in nominal compensation per employee, growth in GDP deflator, nominal short-term interest rate. In terms of specification, the model closely resembles the Smets and Wouters (2007) model.

Similar to Christoffel et al. (2008), the steady-state parameters of the model are calibrated prior to estimation and a balanced growth path is imposed. Furthermore, the estimated monetary policy rule is specified in terms of (changes in) the trend output gap rather than the flexible–price measure of the output gap. Altogether eight structural shocks are considered: a permanent technology shock, a transitory technology shock, an investment-specific technology shock, an external risk premium shock, an autonomous (government) expenditure shock, a price mark-up shock, a wage mark-up shock, and an interest rate shock. Most shocks are modelled as autoregressive processes of order one, except for the interest rate shock which follows a white noise process and the permanent technology shock which follows a unit root process.

Responses of the following alternative notions of model-based potential output are discussed below: trend output, unconditional flexible-price output and conditional flexible-price output[19]. Given the difficulties in the identification of mark-up shocks, discussed above, in this section and in the empirical part of the paper (sections 0 and 3) we do not consider the natural output. Instead, similar to Smets and Wouters (2003), we apply the flexible-price notion of output which is driven only by efficient shocks, i.e. wage and price mark-up shocks do not affect potential output. This is, however, not the efficient output as in section 1.1.1 either since the flexible-price outputs are computed assuming positive steady-state mark-ups (as in the actual economy). More specifically:

- *Trend output* essentially represents a stochastic trend which contains a deterministic component given by constant growth in the labour force and labour productivity and a stochastic component represented by a unit-root shock to labour productivity (permanent technology shock). The latter implies a permanent shift in the level of trend output. The *trend output gap*, thus, measures the percentage deviation of actual output from trend output. A notion of stochastic trend output may also be viewed as a level of output obtained in the environment of no real or nominal rigidities and in the absence of all shocks except for shocks to labour productivity. Trend output most closely resembles the traditional approach to measuring potential output as a persistent, smooth

[17] Actual DSGE models used at central banks may deviate from the benchmark model applied in this section due to various modelling extensions, however, given that the core of these models typically closely corresponds to the Smets and Wouters model, the impulse-response results obtained using the benchmark model should be also useful for understanding dynamics of model-consistent output and output gaps in larger models. Still, the findings may reflect model-specific as well as estimation sample-specific results and, therefore, need to be taken with caution.

[18] For a full description of an open-economy version of the model see Christoffel et al. (2008). Estimation and simulation of a closed-economy version of the NAWM was carried out using YADA (Yet Another Dsge Application). YADA is a Matlab program developed and maintained by Anders Warne (ECB) (see Warne (2010) for details on the software).

[19] Similar to Smets and Wouters (2007), in estimating the flexible-price output the model is expanded with a flexible-price version of the model.

process. In this regard, it is a useful benchmark against which flexible-price measures of output can be compared within one (DSGE) model.

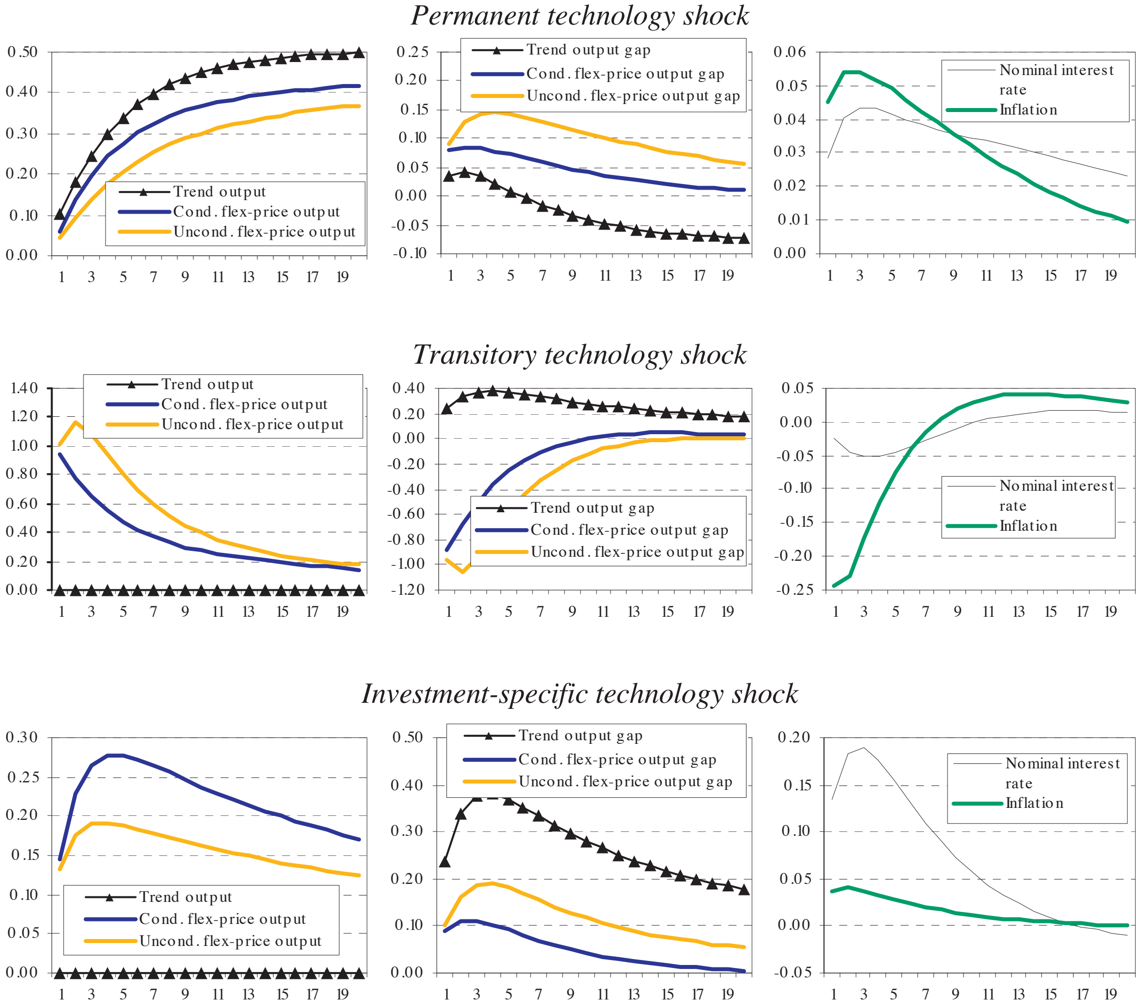
- *Unconditional flexible-price output* denotes the level of output under the assumption of flexible prices and wages and absence of shocks to the mark-ups. It assumes that the flexible-price economy has always featured flexible wages and prices but was subject to the same shocks as the actual economy. The unconditional potential output gap, thus, will not be affected by variation in wage and price mark-ups and as well as interest rate shocks. The *unconditional flexible-price output gap* measures the percentage deviation of actual output from the unconditional flexible-price output.

- *Conditional flexible-price output* also denotes the level of output under the assumption of flexible prices and wages and absence of shocks to the mark-ups. However, it takes the pre-determined variables as given and assumes that prices and wages suddenly become flexible in the current period and are expected to remain flexible in the future. Thus, while only a subset of shocks has a direct contemporaneous impact on the unconditional flexible-price output, all structural shocks may affect the conditional flexible-price output through their impact on state variables. The *conditional flexible-price output gap* measures the percentage deviation of actual output from the conditional flexible-price output.

Figures 1.1–1.3 display impulse-responses of alternative measures of potential output and the implied output gaps to eight structural shocks. In addition we also report the associated reaction of inflation and the nominal interest rate. Several observations are worthwhile highlighting.

First, alternative notions of potential output, on average, feature pro-cyclical responses which imply smaller estimates of the respective output gaps. Numerous exceptional cases in which potential output measures are unaffected by the shocks reflect considerations used in the construction of the potential output notions. In particular, by definition, trend output is affected only by the permanent technology shock. Also, besides the interest rate shock and the shocks to wage and price mark-ups, the unconditional flexible-price output does not react to a risk premium shock. The latter enters the Euler equation for consumption and measures the wedge between the interest rate controlled by the central bank and the return on assets held by the households. However, under full nominal flexibility the real interest rate will react instantaneously fully neutralizing the wedge induced by the risk premium shock, thus, leaving the inter-temporal allocation of consumption unchanged.

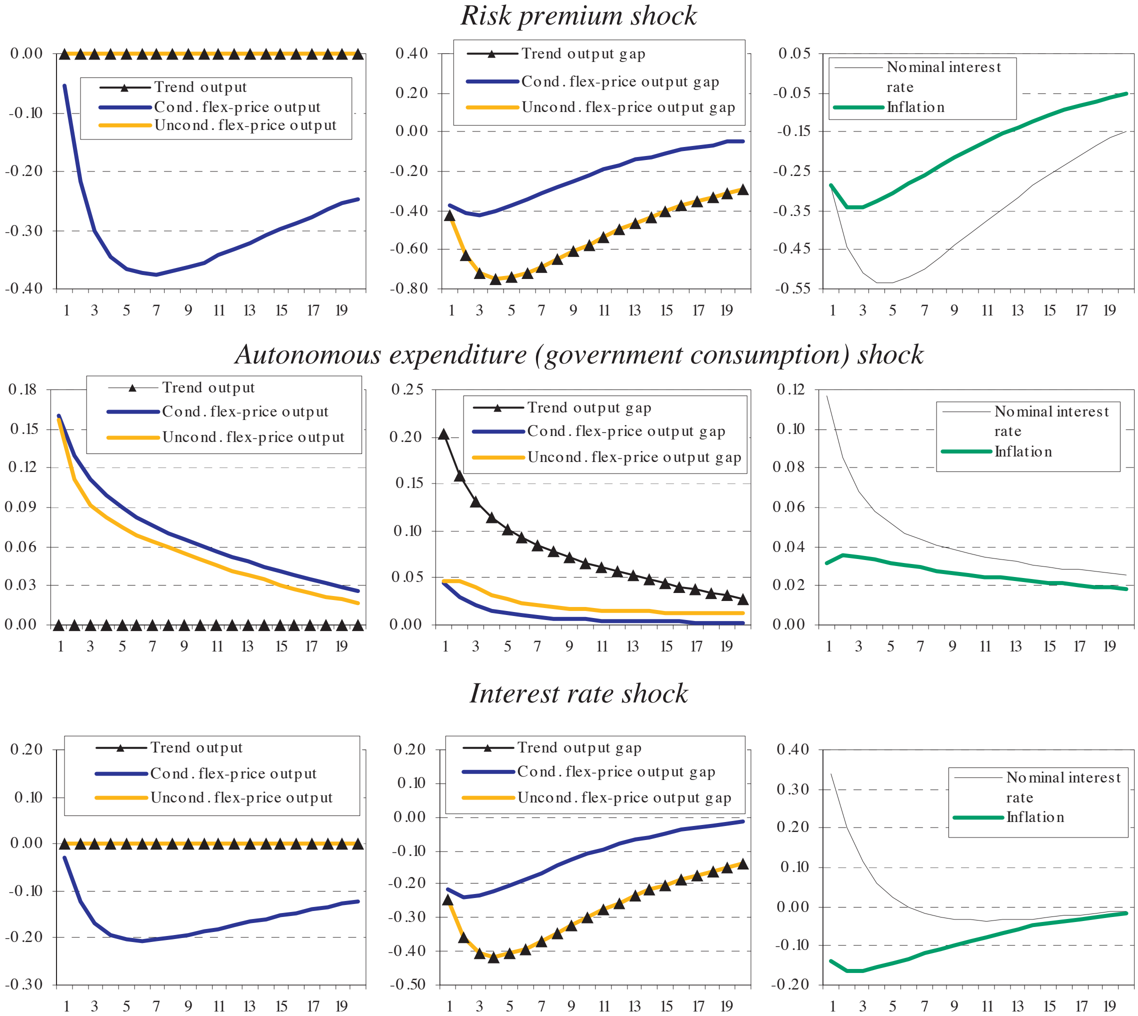
Figure 1.1. Impact of technology shocks



Note: All shocks are equal to one standard deviation. All responses are reported as percentage deviation from the model's non-stochastic steady state. The output gaps are defined as percentage deviation of actual output from the respective measure of potential output.

Second, the implied output gaps also tend to respond pro-cyclically to shocks reflecting the fact that the reaction of the potential output measures is considerably smaller than the response in actual output. Lower responsiveness of the model-based estimates of potential output to shocks may be largely attributed to flexibility of price and wage setting. There are also some exceptions in this regard. In particular, a transitory technology shock (essentially a shock to total factor productivity) has a considerably quicker and larger impact on the flexible-price output measures as compared to actual output for which the impact is significantly reduced and delayed by the presence of nominal rigidities. Nominal rigidities appear to limit the spill-over of the volatility in the marginal costs of production, induced by the shock, on the rest of the economy, thus, facilitating smoother overall macroeconomic dynamics. This apparent over-shooting in the flexible-price output measures implies negative output gaps following a positive shock to total factor productivity.

Figure 1.2. Impact of demand and monetary shocks

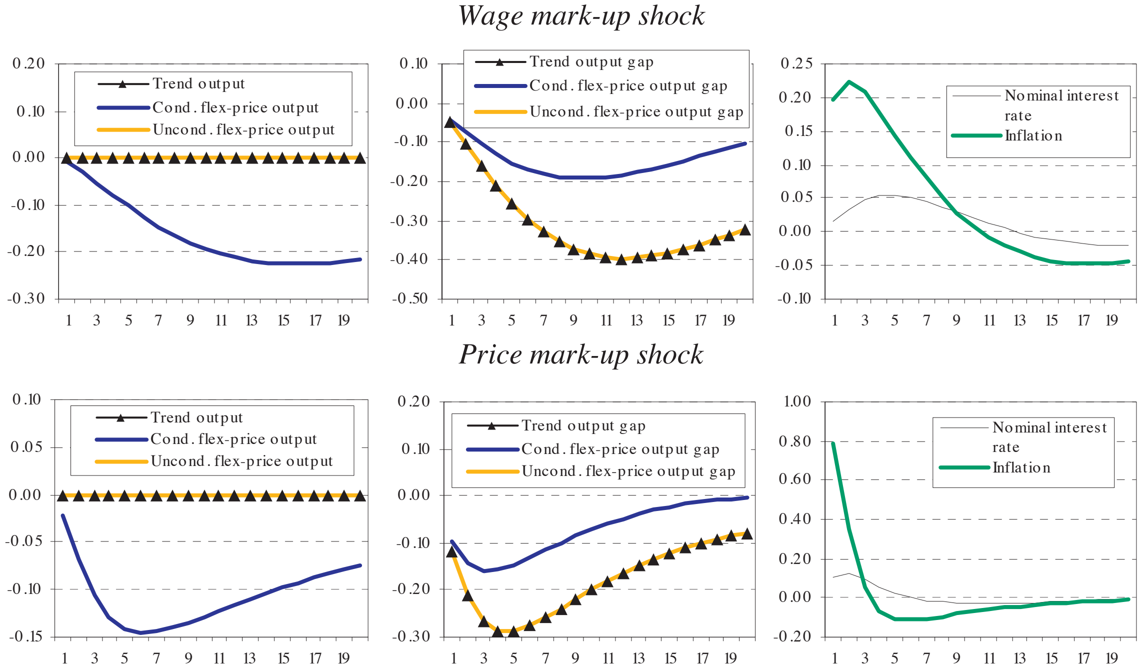


Note: All shocks are equal to one standard deviation. All responses are reported as percentage deviation from the model's non-stochastic steady state. The output gaps are defined as percentage deviation of actual output from the respective measure of potential output.

Third, the responses of the output gaps and actual inflation appear to be of similar sign in case of all shocks except for the mark-up shocks. The latter shocks are associated with an increase in market inefficiencies which dampen actual output while at the same time contributing to inflationary pressures. In addition, contrary to the flexible-price measures, following the transitory technology shock, the response in the trend output gap is found to be negatively correlated with the inflation reaction.

Fourth, in comparison to unconditional flexible-price output, with the exception of the response to a transitory technology shock, conditional flexible-price output features a stronger response to shocks, and therefore, a smaller reaction in the implied output gap.

Figure 1.3. Impact of wage and price mark-up shocks



Note: All shocks are equal to one standard deviation. All responses are reported as percentage deviation from the model's non-stochastic steady state. The output gaps are defined as percentage deviation of actual output from the respective measure of potential output.

Overall, the impulse-response analysis reveals that, in comparison to traditional measures of potential output and the output gap, flexible-price output measures display higher volatility while at the same time smaller implied output gaps. As discussed above, the response to a transitory technology shock may imply much higher volatility in the flexible-price output and hence the implied output gap. Co-movement of inflation and alternative notions of the output gap depends of the nature of the underlying shocks hitting the economy.

Furthermore, the trend output gap and the unconditional output gap share some common features. Both output notions react to efficient shocks (technology and government expenditure) and do not respond to the inefficient shocks (risk premium, interest rate, and mark-up). As a result, the associated output gaps react identically in case of inefficient shocks. These results vividly illustrate the importance of imperfect competition for the propagation of technology and demand shocks. Hall (1986) argues that imperfect competition is an important source of business cycles since it amplifies vulnerability of output to various demand and policy shocks: monopolistic competitive firms have little incentive to restore output to pre-shock levels facilitating persistent variation in output following a shock. As discussed in Rotemberg and Woodford (1999), in the environment of price stickiness (imperfect competition) the mark-up will move pro-cyclically in response to technology shocks and counter-cyclically in case of demand or policy shocks. Consequently, the output response to technology shocks will be larger and the response to demand or policy shocks will be smaller in case of a flexible-price (perfect competition) economy than in case of actual economy featuring nominal rigidity and imperfect competition.

2. Central bank DSGE model-based estimates

Let us now turn to some examples of potential output and the output gap estimates based on DSGE models used in central banks. In particular, we discuss estimates of potential output and the output gap based on the ECB's New Area-Wide Model (NAWM), the g3 model of the Czech National Bank and the PUSKAS model developed at the Central Bank of Hungary (MNB). While all three models feature small-open economy framework, degree of economic openness across the considered models differs substantially with the NAWM being relatively closed economy. There are also other important structural differences, including different monetary policy rule specifications[20], which are expected to have some impact on the derived estimates of potential output and the output gaps. Application of different models in the comparison exercise allows checking robustness of the main conclusions that follow from the analysis.

For the purposes of this empirical exercise carried out for the euro area, Czech Republic and Hungary the definitions of potential output and the output gap as well as the methodology of carrying out the simulations were unified. We estimate both unconditional and conditional flexible-price outputs and the corresponding output gaps under the assumption of flexible prices and wages and absence of shocks to the mark-ups[21]. In case of conditional flexible-price output the pre-determined variables are taken as given. In addition, we show historical estimates of trend output and Hodrick-Prescott (HP, with a smoothing parameter λ set to 1600) filtered output and the implied output gaps. These serve as natural benchmarks for comparison. Furthermore, in order to gauge possible interrelation between alternative measures of the output gap and inflation, we also report unconditional correlations (contemporaneous as well as at leads an lags) between the output gaps and the quarterly growth rate in selected price deflators, namely the GDP deflator, the consumer price index (CPI), the investment price deflator, and the import price deflator. The investigation sample is from 1999 1 quarter till 2010 1 quarter.[22]

Subsections 2.1-2.3 present the detailed results based on individual models whereas subsection 2.4 summarises the key findings on comparison across the models.

2.1. Potential output estimates in the NAWM

Alternative estimates of the euro area potential output and the implied output gaps over the EMU period are displayed in Figures 2.1 and 2.2. Besides the HP filter-based estimates, three

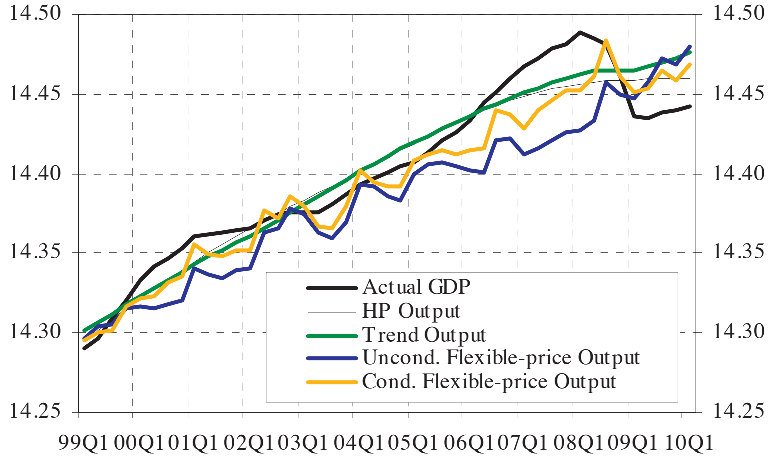
[20] More specifically, all three models feature interest rate smoothening and monetary authorities react to deviation of inflation from the target. In addition, in the NAWM interest rate systematically reacts to change in the trend output gap, whereas in PUSKAS it reacts to exchange rate. It is worth mentioning that policy rules in the NAWM and PUSKAS are backward-looking, while in g3 interest rate reacts to four-quarter-ahead forecast of inflation.

[21] Note, however, that the flexible-price outputs are computed assuming positive steady-state mark-ups (as in the actual economy).

[22] While the sample covers recent financial crisis (2008–2010) no attempts is made to assess in details policy implications of the estimated dynamics of alternative notions of potential output over the major financial turmoil of the post-WWII history. Our primary interest here is comparison of the model-based measures of potential output and the output gaps with more traditional measures both in terms of time series properties and possible relationship with various measures of inflation.

additional estimates of potential output using the NAWM are reported. In particular, trend output represents a stochastic trend associated with the stochastic balanced-growth path of the model. The evolution of trend output is driven by deterministic drift in the labor force and labor-augmenting productivity as well as a (unit-root) permanent technology shock. Unconditional and conditional flexible-price output measures correspond to the model-based estimates of potential output which is derived assuming constant steady-state mark-ups (no shocks to price and wage markups) and full flexibility of price and wage setting.

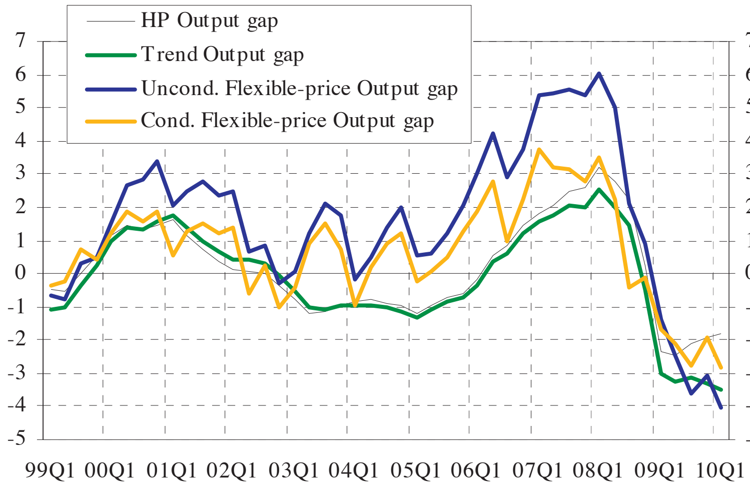
Figure 2.1. Alternative estimates of potential output for the euro area, on a logarithmic scale



Visually, both the HP and trend output measures feature much smoother estimates than the flexible-price measures of potential output. In terms of structural shock decomposition, the short-run fluctuations in the latter measures are mainly attributed to a relatively strong impact of stationary shocks to total factor productivity (transitory technology shocks) via the marginal cost channel. In addition, a persistent negative contribution of investment-specific and transitory technology shocks for most of the sample implies some divergence between the flexible-price output and more traditional measures of potential output. In this regard, the apparent differences between the conditional and unconditional potential output measures observed over the sample reflect a positive contribution of negative wage mark-up and interest rate shocks in 1999–2007 to the conditional flexible-price output via their impact on the pre-determined variables. Negative shocks to the wage mark-up reflect to a large extent a persistent weakness in real wage developments and the associated fall in the wage share in the euro area which has been observed since the mid-nineties. Negative shocks to the interest rate denote deviations of the actual policy rate from the interest rate implied by the estimated monetary policy rule. Substantial negative interest rate gaps are recorded over the period (2002–05) characterised by a historically low level of the policy rate.

The implied output gaps in Figure 2.2 reveal that all the measures seem to feature similar fluctuations at business cycle frequencies, although the flexible-price output gaps tend to fluctuate at somewhat higher level for most of the sample. The latter is in line with the strong dependence of the flexible-price measures of potential output on the negative shocks to technology discussed above. While displaying notable difference in levels over most of the sample, at the end of the sample the flexible-price and more traditional measures of output gap converge. In this regard the most striking example of convergence is due to the unconditional flexible-price and the HP filter-based output gaps. Both measures converge following a substantial deceleration of growth in the HP-based measure of potential output, whereas the flexible-price output displays some acceleration in potential output. The former may be largely attributed to the apparent sensitivity of the HP-based estimate to the collapse in actual output in 2008–09, whereas the latter tends to attribute the fall in output mostly to weaker demand conditions while at the same time pointing to importance of productivity gains in supporting the economic recovery at its initial stage.

Figure 2.2. Alternative output gap estimates for the euro area, in % of the respective potential output level

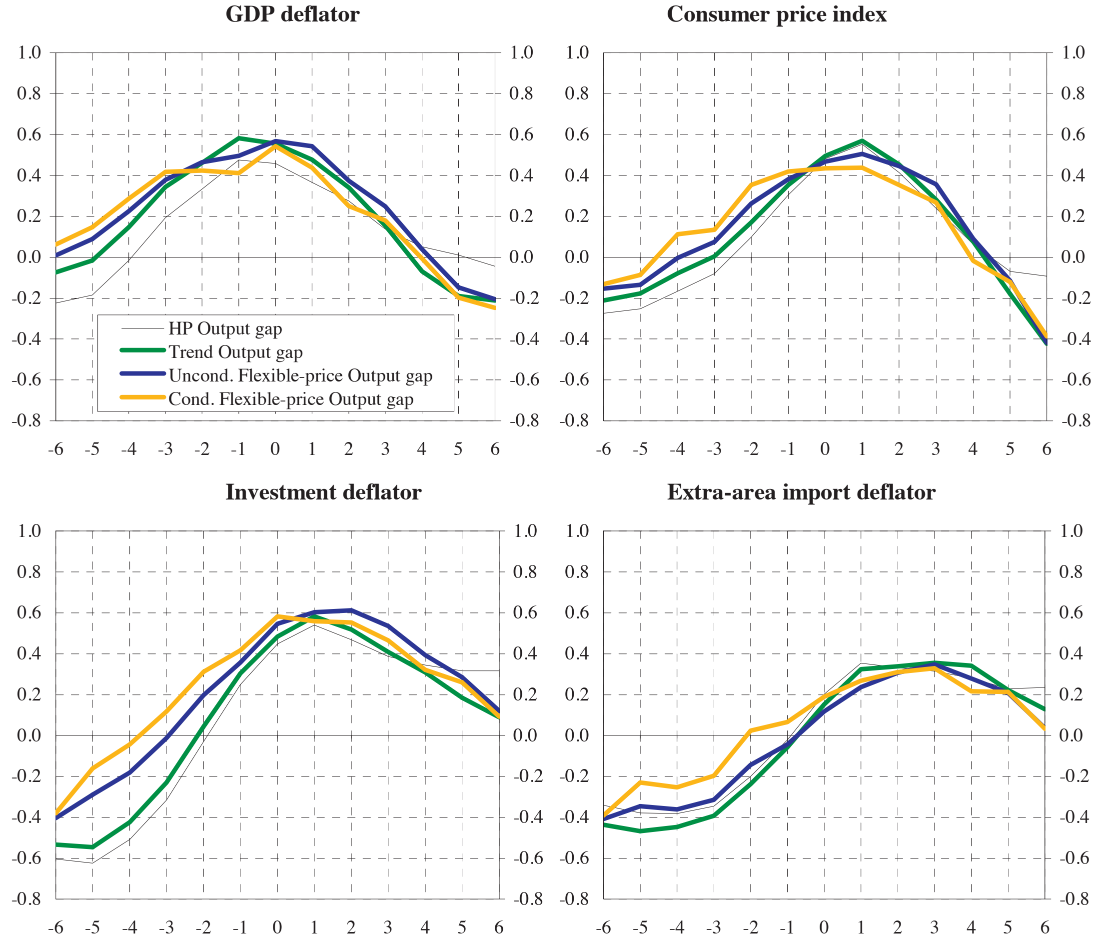


Unconditional correlations of various price indicators with contemporaneous values as well as lags and leads of alternative measures of output gaps are displayed in Figure 2.3. Overall, all output gap measures reveal substantial positive contemporaneous correlation. Furthermore, output gaps seem to lead fluctuations in domestic output prices (GDP deflator) pointing to their potential usefulness in gauging domestic inflationary pressures in a timely manner. In this regard, the model-based measures of output gap, in particular the conditional flexible-price output gap, visually[23] tend to outperform the HP filter-based output gap measure.

As regards the investment deflator and consumer prices (HICP), the observed correlation with lagged output gaps is on average lower than in case of GDP. This may reflect the impact of imported goods in the investment and consumers' basket. Changes to price of imported goods tend to lead the output gap measures which may be attributed to possible positive spillovers of gains to the price competitiveness on aggregate demand.

[23] Formal statistical testing is required to evaluate the significance of the observed differences.

Figure 2.3. Unconditional correlation of quarterly growth rate in prices with lags and leads of alternative measures of the output gap in the euro area



2.2. Potential output estimates in the CNB's g3 model

The alternative estimates of potential output for the Czech Republic using the Czech National Bank's DSGE model[24] g3 are shown in Figure 2.4. Visually, the flexible-price notions of output are more volatile than the trend output (which is given by a linear deterministic trend) and HP filter-based potential output estimates. This is especially true for the pre-crisis period of 2007–8, when model-consistent flexible-price potential output estimates indicate higher potential growth rates than the HP-based and trend output estimates.

The implied output gaps are displayed in Figure 2.5. All approaches produced relatively smooth estimates and identified a negative output gap over the financial crisis which hit Czech economy in the last quarter of 2008. The most significant difference between the HP and the flexible-price output gaps is in the period from mid-2006 till end 2008, when the HP and trend output gaps indicate a more pronounced overheating of the Czech economy than the corresponding flexible-price estimates. For the last two quarters, the HP output gap appears to be more intuitive than the unconditional flexible-price output gap. Furthermore, in this period,

[24] See Andrle et al. (2009) for documentation.

the conditional output gap is closer to the HP gap rather than to the unconditional gaps (something which is not always true in history).

Figure 2.4. Alternative estimates of potential output for the Czech Republic, on a logarithmic scale

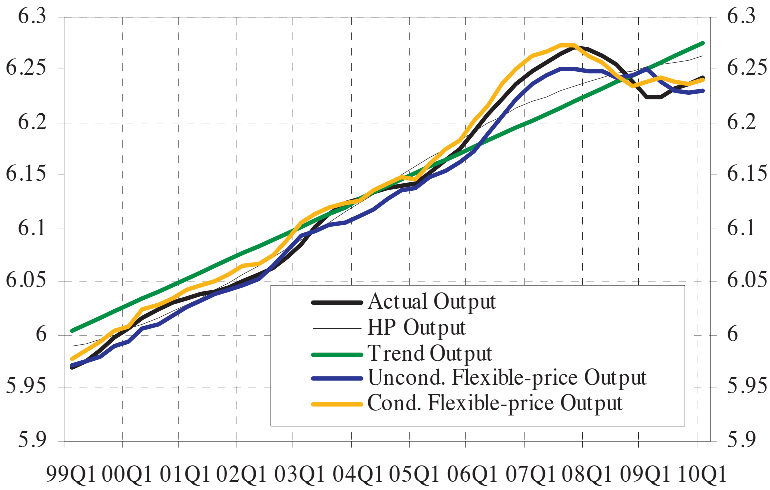


Figure 2.5. Alternative output gap estimates for the Czech economy, in % of the respective potential output level

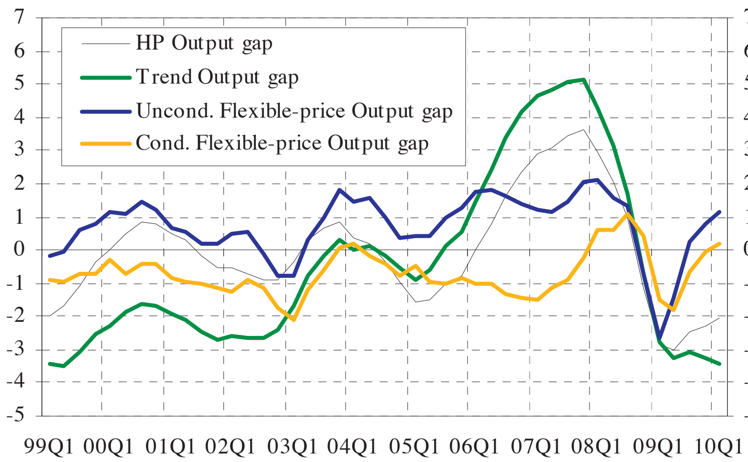
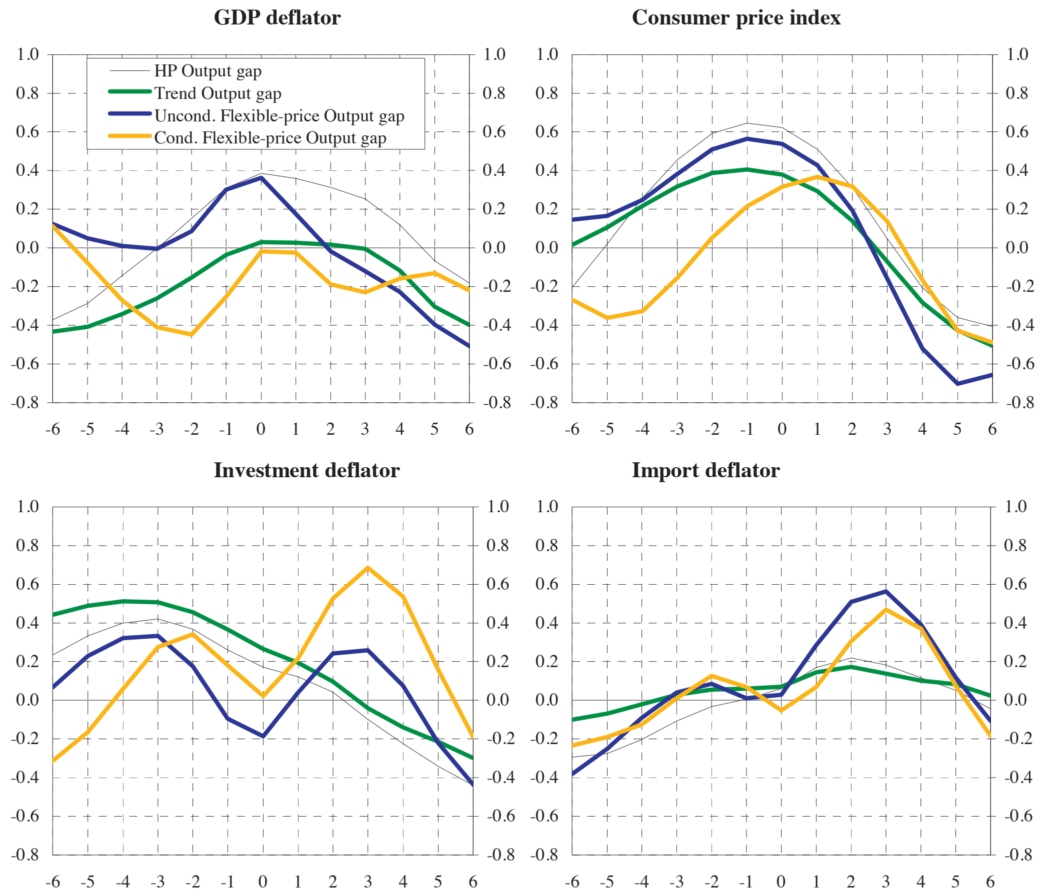


Figure 2.6 displays the unconditional correlations between the output gaps and four measures of inflation. The correlation coefficients are very sensitive with respect to the sample period, especially to the inclusion of the period 1996-1998[25]. The sensitivity to this period may be caused by a set of various factors, such as the exchange-rate crisis in 1997, the change in monetary policy in that period (the beginning of inflation targeting in 1998), or a strong and surprising disinflation in 1999. Therefore, we report the sample correlations for the period since the beginning of the year 2000. The results show that the correlation is high for the HP output

[25] On the other hand, the correlations are quite robust with respect to the sample period corresponding to the current financial crisis.

gap and unconditional flexible-price output gap with the GDP deflator and CPI. For the GDP deflator the strongest correlation is contemporaneous, while the two gaps seem to lead the CPI by one quarter. Weaker correlation of the gap measures with the GDP deflator than with the CPI is likely caused by the fact that the Czech economy is small and open with an important component of foreign goods in the Czech intermediate production.

Figure 2.6. Unconditional correlation of quarterly growth rate in prices with lags and leads of alternative measures of the output gap in the Czech Republic



The output gaps lagged the import deflator by about two or three quarters. This can be attributed to the fact that the Czech import deflator reacts relatively quickly to the Euro area prices, while the cyclical position of the Czech output reacts more slowly to its European counterpart. Finally, the HP and the trend output gaps seem to lead the investment deflator inflation, while the correlation with the flexible-price measures does not appear to have a clear pattern.

2.3. Potential output estimates in the MNB's Puskas model

In the following section we present estimates of flexible-price output based on an augmented version[26] of DSGE model PUSKAS in Jakab and Világi (2008). Figure 2.7 shows levels of alternative notions of potential output. The flexible-price notions of potential output appear to be more volatile than both HP filtered potential output and trend output. Conditional flexible-price output follows more closely actual output than unconditional flexible-price output. Large differences after 2008q4 are due to a more significant impact of depreciation of the national currency on the flexible-price output than on the sticky-price (actual) output: the depreciation brought a relatively sizeable surplus in the foreign trade balance in the flexible-price model.

Figure 2.7 Alternative estimates of potential output for Hungary, logarithmic scale

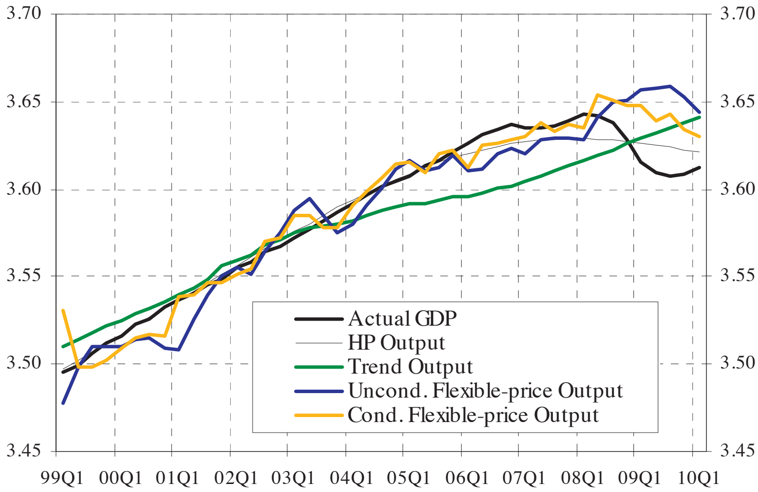


Figure 2.8 shows estimates of the alternative output gap notions. The series are similar, though the trend output gap is significantly higher than the other estimates during 2004q1–2008q3. This difference is attributable to the somewhat inflexible estimate of trend output as can be seen in Figure 2.7. There is also large difference between values of the model-based output gaps and the HP output gap at the end of the sample. In case of the flexible-price output gaps this difference can be explained by a (relative) boom in the export sector as mentioned earlier. Similarly, the sizeable negative flexible-price output gaps around 2001 reflect export sector responses to negative foreign demand shocks.

Figure 2.9 depicts the correlations between the various notions of the output gap and selected measures of inflation. Overall the correlations are quite modest in size. The largest correlations are those of various output gaps and the import deflator amounting to about 0.4 (except for the trend output gap). The output gaps seem to lag behind import price changes by 1-2 quarters.

[26] To carry out the exercise the Jakab and Világi (2008) model was modified: a balanced growth path was imposed on the original model with a deterministic component of TFP growth and a stochastic trend of labour augmenting technology change.

Figure 2.8 Alternative output gap estimates for the Hungarian economy, in % of the respective potential output level

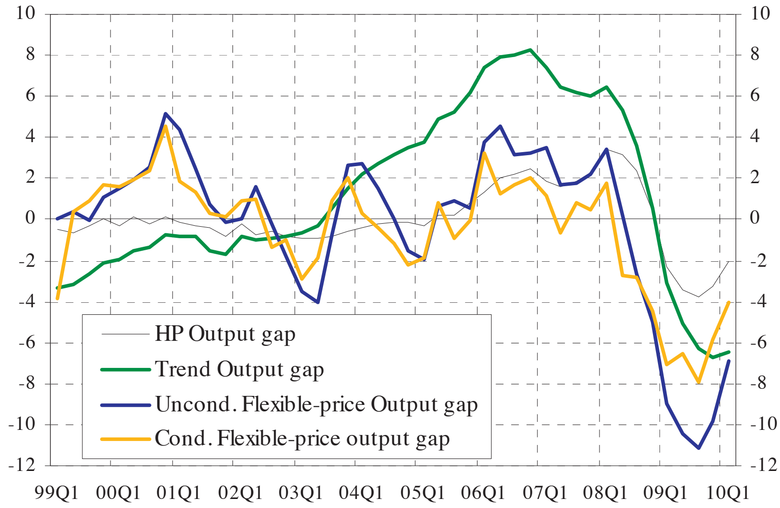
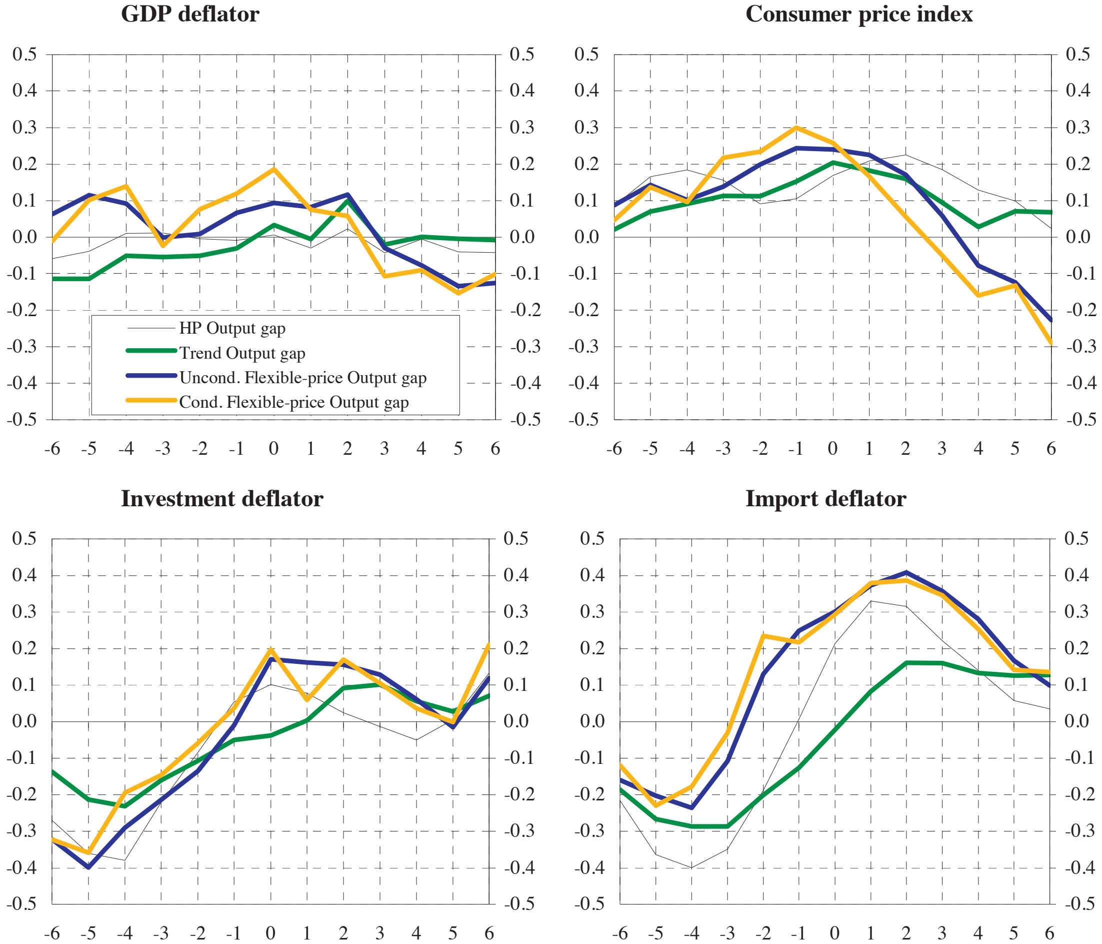


Figure 2.9 Unconditional correlation of quarterly growth rate in prices with lags and leads of alternative measures of the output gap in Hungary



Concerning the correlation of CPI and the alternative output gaps, there is some correlation between CPI and the flexible-price output gaps at lag 1, but this correlation reverses at high leads. There is also a small positive correlation between CPI and the HP filter-based output gap and the trend output gap at all lags and leads. Interestingly, there seems to be very low correlation between the GDP deflator and the output gap measures at all lags. This can be attributed to the fact that Hungary is a small open economy where external shocks seem to play an important role in shaping macroeconomic fluctuations.

2.4 Comparison across the models

The volatility of the flexible-price output gaps relative to the HP output gap, measured as a ratio of standard deviations, is reported in Table 2.1. The unconditional flexible-price output gap is about 50% more volatile than the HP output gap in the NAWM, whereas it is almost three times less volatile in the Czech model, and has similar volatility in the Hungarian model. The conditional flexible-price output gap is less (about 2/3) volatile than the HP output gap in all models. Relatively low volatility of the unconditional flexible-price gap in case of the Czech model is outstanding indeed; however, it is not a robust feature, since the relative volatility significantly depends on the investigated period.

Table 2.1 Volatility of the model-based output gap measures relative to the HP output gap

	NAWM(ECB)	g3(CNB)	PUSKAS(MNB)
Unconditional flexible-price output gap	1.51	0.35	1.01
Conditional flexible-price output gap	0.66	0.71	0.70

Table 2.2 Autocorrelation function

	0	1	2	3	4	5
ECB HP output gap	1.000	0.913	0.715	0.469	0.202	-0.060
ECB trend output gap	1.000	0.927	0.749	0.505	0.208	-0.125
ECB unconditional output gap	1.000	0.895	0.723	0.490	0.229	-0.058
ECB conditional output gap	1.000	0.785	0.611	0.427	0.258	0.031
CNB HP output gap	1.000	0.924	0.723	0.440	0.110	-0.232
CNB trend output gap	1.000	0.964	0.865	0.719	0.540	0.347
CNB unconditional output gap	1.000	0.748	0.288	-0.083	-0.287	-0.337
CNB conditional output gap	1.000	0.708	0.236	-0.103	-0.229	-0.199
MNB HP output gap	1.000	0.908	0.708	0.431	0.150	-0.055
MNB trend output gap	1.000	0.971	0.890	0.762	0.600	0.427
MNB unconditional output gap	1.000	0.891	0.691	0.492	0.278	0.084
MNB conditional output gap	1.000	0.721	0.550	0.389	0.230	0.075

Table 2.2 reports the autocorrelation properties of the output gap measures. The pattern of autocorrelations appears to be more robust. First, the HP and trend output gaps display the highest autocorrelation for all models. Also, the conditional flexible-price output gap is the least autocorrelated in all three cases. This can be expected as the initial conditions for its computation are given by the actual predetermined variables. The autocorrelation of the unconditional flexible-price output gap is lowest for the Czech model, while the respective estimates are similar in the ECB and Hungarian models.

As regards analysis of correlation between the output gap and inflation across the considered models, it appears that openness of economy may play a role in the detected differences. In particular, using the Czech and Hungarian models we find a rather low correlation between the output gaps and inflation whereas in the NAWM it is much higher. Relative importance of external shocks can corroborate this finding.

Overall, in comparison to traditional measures of the output gap, the flexible-price output gaps seem to correlate well with expected inflation, though the observed co-movement in prices and the output gaps should not be interpreted as evidence of a structural economic relationship. Arguably, the observed correlation over some specified sample will depend on the nature of structural shocks underlying economic developments. We turn next to a more detailed analysis of usefulness of model-consistent output gaps in gauging inflationary pressures.

3. Is the flexible-price output gap a good indicator of inflation?

This section focuses on a specific issue related to the usefulness of the model-based output gaps in the context of conduct of monetary policy. In particular, we evaluate whether the derived output gaps are good indicators of inflationary pressures. First, we evaluate the inflation forecast performance of alternative notions of the output gap using the Phillips curve framework. Next, we look at conditional correlations between inflation and the model-consistent output gap measures as well as other model-based indicators, such as real marginal cost, real interest rate gap, etc.

3.1. Phillips curve-based analysis

As argued in the literature, within the NK framework flexible-price output gaps should be good indicators of inflationary or deflationary pressures. Since the seminal work of Phillips (1958) reduced-form relationships between real activity and prices have frequently been exploited by modellers for forecasting future inflation. While forecast accuracy of early versions of the Phillips curve largely deteriorated in the seventies, the search for a proper specification of Phillips curves continues as output and/or unemployment gaps remain some of the key indicators considered by many policymaking institutions.[27]

In this section, we explore the performance of the NAWM-based output gaps for forecasting inflation in the private consumption deflator in the euro area at various horizons.

Forecast Evaluation Procedure

Our forecast evaluation procedure is based on an approach similar to the one applied by Fischer, Lenza, Pill, and Reichlin (2009) in studying the performance of money-based inflation forecasts in the euro area. In particular, the pseudo (the exercise is based on unrevised data) real-time out-of-sample forecast of inflation is obtained on the basis of bivariate models which are estimated using rolling samples of 40 quarters. The bivariate models of inflation features lagged inflation and some measure of inflationary pressure (alternative output gap estimates or real marginal costs). The model-based measures of inflationary pressures are derived from the NAWM recursively estimated in pseudo real-time: starting with the initial sample spanning the period 1985q1–1998q4, a set of recursive estimates of the model-based indicators of inflationary pressure is obtained extending the sample forward by one quarter. The NAWM parameters are re-estimated over 1998–2009 annually once data for the fourth quarter of the year becomes available.

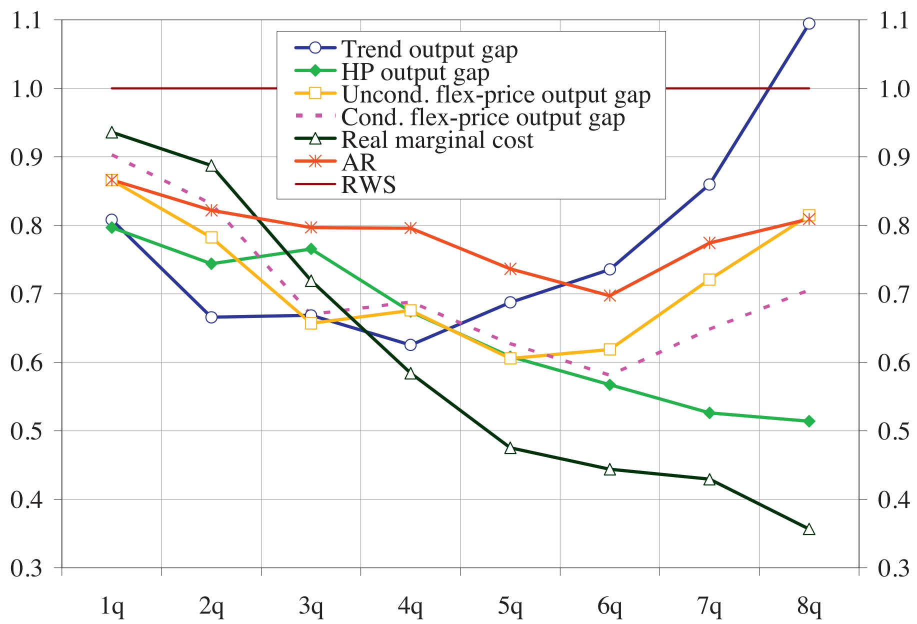
[27] Stock and Watson (2008) review the recent literature on pseudo out-of-sample evaluation of Phillips curve-based inflation forecast models in the United States. An important benchmark in this regard is Atkeson and Ohanian (2001), who analyze US GDP deflator growth over the period 1984–1999 and show that, in terms of forecast accuracy, naïve benchmarks, such as a smooth random walk, can easily outperform Phillips curve-based models that rely on output gaps or other measures of economic slack. Stock and Watson (2008) show that relative forecast performance of the Phillips curve may be episodic. In periods of a stable macroeconomic environment the Phillips curve-based forecasts are outperformed by naïve models, whereas in the face of large business cycle swings forecast accuracy of the former improves considerably over the latter.

Altogether, 42 samples of quarterly data (with the initial sample spanning 1989q1–1998q4 and the final sample covering 1999q2–2009q1)[28] are used. The forecast evaluation is done over the period from 1999q1 until 2010q1. The mean squared forecast errors (MSFE) of the bivariate models are then compared to the MSFE of benchmark models, which in our case are limited to a smooth random walk and univariate autoregressive models of inflation. A formal description of the forecast evaluation procedure is provided in the Appendix (see section C).

Forecast Evaluation Results

Overall, seven models are compared: the random walk (RW), the autoregressive model (AR), five bivariate models of inflation. The first bivariate model is specified in terms of the trend output gap. The second one utilizes the HP-based estimates of the output gap. The third and fourth bivariate models are based on the unconditional and conditional flexible-price output gaps respectively. Lastly, the fifth bivariate model is specified in terms of the real marginal cost indicator derived from the NAWM. Forecast accuracy is evaluated at forecast horizons of 1 to 8 quarters ahead. Detailed results of the forecast evaluation exercise are reported in Table B.1 in the Appendix. The first column in the table reports the MSFE for each model. The second and third columns show the relative MSFE: the MSFE of a given model relative to the MSFE of the RW and the AR models. The fourth column reports the bias of the forecast and the last two columns decompose the MSFE into contributions by the forecast error variance and the bias. Figure 3.2 summarises the forecast accuracy of the rival model vis-à-vis random walk.

Figure 3.2. The MSFE of alternative models relative to random walk



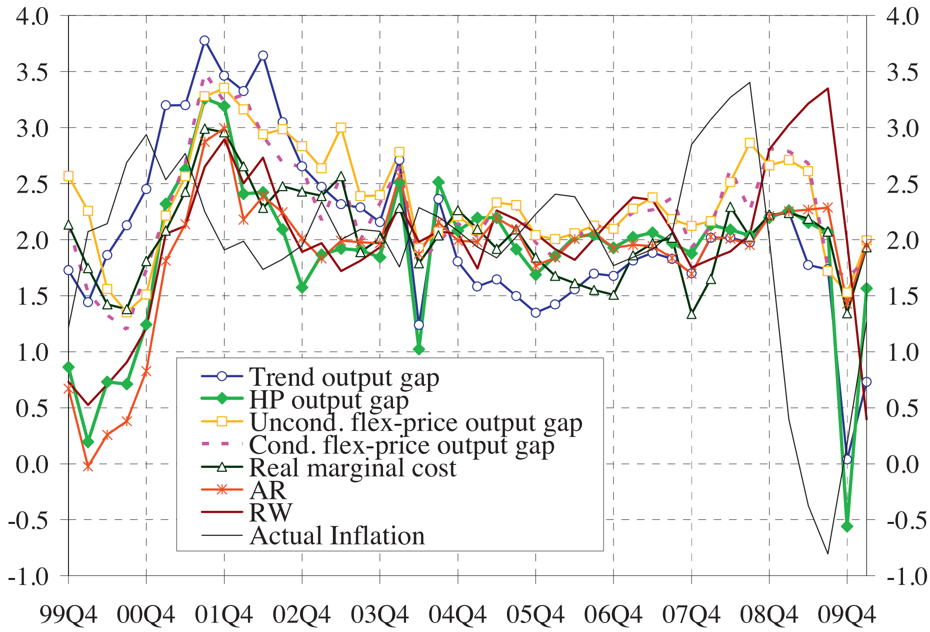
Overall, judging by the MSFE statistics, the bivariate models of inflation do feature considerably better forecast accuracy than the random walk model over most of the forecast

[28] We chose rolling sample estimates to put the rival forecasting models on a more equal footing, since under recursive estimation method the RW may have advantage over alternative models by using the most recent inflation data.

horizons. This contrasts sharply with the previous findings based on the evaluation period up to 2006 which show relatively strong forecasting performance of the random walk model (Coenen et al. 2009). This may, in particular, be attributed to forecast failure of the random walk model when applied to the recent economic crisis featuring large swings in euro area inflation (see Figure 3.3).

Up to the four-quarter horizon, the trend output gap-based model of inflation provides a relatively accurate inflation forecast, though over longer horizon its forecast performance deteriorates substantially. Beyond the three-quarter horizon, the real marginal cost indicator seems to provide the best projection of inflation. The HP output gap-based projection compares well across all forecast horizons.

Figure 3.3. The four-quarter inflation forecast of alternative models



Although the flexible-price output gaps do improve over autoregressive models of inflation over the medium-run, they are found to be largely outperformed by other bivariate models. Relatively weak forecast performance of the flexible-price output gaps are related to positive forecast bias (Table D.1 in the Appendix): bivariate models based on the flexible-price output gaps tend to over-predict inflation. Another reason is due to relatively large (as compared to actual inflation series) high frequency volatility of the flexible-price output gap estimates which results in a large variance of the forecast errors over the short-term horizon.

In conclusion, although the results of our forecasting exercise above provides some favourable evidence on the predicting power of the flexible-price output gap in gauging medium-term inflationary pressures, there seem to be better indicators of inflation. In particular, also in line with the conceptual discussion above, real marginal costs could serve as a better basis for projecting future inflation developments. Beyond one-year forecast horizon, the real marginal cost-based models produce the most efficient and unbiased forecast of inflation.

Needless to say, given the short out-of-sample forecast evaluation period, it is important to bear in mind that it is not clear to what extent the differences in forecast performance of alternative measures of output gap are robust and significant; therefore the findings need to be taken with caution.

3.2 Conditional correlation analysis

In the basic NK model, inflation is fundamentally a function of current and future real marginal costs, which in turn is proportional to the flexible-price output gap. The real marginal cost and the flexible-price output gap are determined by current and future real interest rate gaps. Thus, the real interest rate gap and the flexible-price output gap should therefore be good indicators of current and future inflation pressure.[29] In this section we describe in terms of conditional correlations to what extent inflation is related to the flexible-price output gap, the real marginal cost, the real interest rate gap, expected inflation and the trend output gap in a version of the basic NK model that is augmented with a large number of frictions and shocks.[30] To this end we use the Riksbank's forecast and policy model Ramses, which is described in Adolfson et al. (2008).

Ramses is an open-economy version of the NK model augmented with the following frictions: monopolistic competition in the labour market, real wage stickiness, habit formation, investment adjustment costs, a cash-in-advance constraint on wages and distortionary taxation. Correlation between inflation and the model variables is therefore expected to differ from the basic NK model.[31] The unconditional correlation between the real marginal cost and the flexible price output gap is about 0.4 in Ramses. This suggests that the relationship between inflation and the flexible-price output gap will be weak. In fact, the point estimate of the unconditional correlation coefficient is close to zero. Furthermore, more conventional measure of the output gap, actual output minus trend output, is also found to be only weakly related to inflation: point estimate is close to zero. The variable that has the strongest correlation with current inflation is expected inflation. Also the real interest rate gap has a relatively strong correlation.

The conditional correlation between variables depends in general on both the frictions and the shocks in the model. This subsection illustrates how different shocks affect the correlations. To this end we calculate correlations that are conditioned on one specific shock at the time. We report results from thirteen different shocks: ε a stationary technology shock, Υ an investment-specific technology shock, μ a non-stationary technology shock, z^* an asymmetric technology shock, ζ^c a consumption preference shock, ζ^h a labour supply preference shock, ς a monetary policy shock, π^c an inflation target shock, ϕ a risk premium shock, λ_d a domestic mark-up shock, λ_{mc} an imported consumption mark-up shock, λ_{mi} an

[29] Moreover, Neiss and Nelson (2003) note: "In this framework, the key variable for the analysis of 'inflationary or deflationary' pressures is the gap between the current level of the natural rate of interest rate and the interest controlled by the central bank".

[30] The results in this section can also be found in Jonsson, Laséen and Walentin (2006).

[31] The Riksbank has recently introduced a new version of Ramses that includes search and matching frictions as well as a financial friction, see Christiano, Trabandt and Walentin (2007) for a description. In addition to other frictions, this may weaken the relationship between inflation and the flexible price output gap even more.

imported investment mark-up shock, λ_x an export mark-up shock. The variables are inflation (CPI) denoted π_t^{cpi}, expected inflation denoted π_{t+1}^{cpi}, real marginal cost denoted mc_t, flexible-price real interest rate gap denoted r_t^{fp}, flexible price output gap denoted $\hat{y}_t^{fp}$ and the trend output gap denoted $\hat{y}_t$. Table 3.1 shows the conditional correlations. Note that the first column shows the unconditional correlations, i.e. correlations when all shocks hit the economy.

Table 3.1 Conditional correlations

Moment	All	ϵ_t	$\hat{\Upsilon}_t$	$\mu_{z,t}$	$\tilde{z}_t^*$	$\hat{\varsigma}_t^c$	$\hat{\varsigma}_t^h$
$\rho(\pi_t^{cpi}, mc_{t-1})$	0.20	0.62	0.12	0.69	0.41	0.76	0.79
$\rho(\pi_t^{cpi}, \pi_{t+1\|t-1}^{cpi})$	0.92	0.95	0.99	0.98	0.97	0.99	0.85
$\rho(\pi_t^{cpi}, \hat{r}_{t-1}^{fp})$	−0.42	−0.51	−0.33	−0.52	−0.66	−0.61	−0.60
$\rho(\pi_t^{cpi}, \hat{y}_{t-1}^{fp})$	0.04	0.54	−0.10	0.37	0.67	0.79	−0.40
$\rho(\pi_t^{cpi}, \hat{y}_{t-1})$	−0.11	−0.83	−0.62	−0.61	−0.75	−0.95	−0.54
	ϱ_t	$\hat{\bar{\pi}}_t^c$	$\hat{\phi}_t$	$\lambda_{d,t}$	$\lambda_{mc,t}$	$\lambda_{mi,t}$	$\lambda_{x,t}$
$\rho(\pi_t^{cpi}, mc_{t-1})$	0.36	−0.45	0.50	0.07	−0.22	−0.12	−0.50
$\rho(\pi_t^{cpi}, \pi_{t+1\|t-1}^{cpi})$	0.84	0.99	0.94	0.22	0.53	1.00	0.98
$\rho(\pi_t^{cpi}, \hat{r}_{t-1}^{fp})$	−0.77	−0.55	−0.71	−0.20	−0.43	−0.12	−0.94
$\rho(\pi_t^{cpi}, \hat{y}_{t-1}^{fp})$	0.71	−0.22	0.11	0.13	0.31	−0.44	−0.56
$\rho(\pi_t^{cpi}, \hat{y}_{t-1})$	0.67	−0.65	0.64	0.11	0.30	−0.51	−0.68

The results illustrate that the correlation between any two variables depends on which shock that hits the economy. For example, the correlation between inflation and the flexible-price output gap lagged one period is 0.04 when all shocks hit the economy. On the other hand, if only technology shocks hit the economy the correlation is 0.54 and if only labour supply shocks hit the economy the correlation is -0.40. The four technology shocks give rise to clear and positive relationships while the labour supply shock, the inflation target shock, the risk premium shock and the mark-up shock on imported investment goods all give rise to a clear negative relationship. This means that when accounting for all shocks the relationship becomes close to zero. Hence, the correlation between any two variables can shift between being positive, negative or zero depending on which shock that hit the economy. A similar story also holds for the correlation between inflation and the trend output gap. It is close to zero when accounting for all shocks but, for example, it is strongly positive for monetary policy shocks and strongly negative for technology shocks.[32]

The unconditional correlation between the lagged real marginal cost and inflation is higher than in case of the output gaps, however, the correlation also appears to be highly sensitive to the type of shocks hitting the economy. Expected inflation is in general a good indicator of inflation. The unconditional correlation is 0.92. A reason for this is that this relationship is not

[32] As regards the technology shocks, note that the differences in sign of coefficients of the inflation-output gap correlation in case of the flexible-price and the trend output gaps are consistent with findings of impulse-response analysis reported in section 1.3.

very sensitive to which shock that hit the economy. The correlation varies between 0.22 and 1.00 depending on the shock. There is also a negative relationship between inflation and the real interest rate gap for all shocks, although these correlations are weaker in quantitative terms than those between inflation and expected inflation.

The performed analysis has shown that, the expected inflation and the real interest rate gap are in general good indicators of inflation pressures. The effects of alternative measures of the output gap on inflation strongly depend on the type of shock hitting the economy. It is therefore unlikely that the output gap as a single measure of inflation pressures will work in all times.

Conclusions

Micro-founded dynamic stochastic general equilibrium models offer an alternative approach to estimating potential output and the output gap. As compared to more traditional approaches, the DSGE approach allows for a deeper structural interpretation of measures of potential output and the output gap derived from such models. Within the DSGE framework two distinctive notions of model-consistent potential output are typically utilized: efficient output and natural output. Efficient output is referred to the hypothetical level of output prevailing under flexible prices (and wages) and perfect competition. Natural output is defined as the hypothetical level of output obtained under full nominal flexibility while retaining assumption of monopolistic competition. Thus, the difference between the two notions will be driven by both static (non-zero average price and wage mark-ups) and dynamic (exogenous price and wage mark-up shocks) inefficiencies in the goods and labour markets due to imperfect competition. Theoretical literature assigns an important role to flexible-price output gaps in informing the policymakers about the need for welfare improving policies. Moreover, under some simplifying assumption the model-consistent notion of the output gap is also considered to be a good indicator of inflationary pressures in the economy.

While the DSGE model-consistent notions of potential output are defined in terms of flexible-price output, more traditional notions of potential output, such as trend output, can also be analysed within the DSGE framework. Conceptual differences in defining potential output have important implications for the time-series properties of output gap estimates implied by different approaches. Compared to the traditional approaches, which in a DSGE model is equivalent to assuming that potential output is driven solely by permanent (unit-root) technology shocks, the DSGE approach in defining model-consistent notions of potential output assumes that other shocks, for example fiscal policy shocks, consumer preference shocks and terms-of-trade shocks, can also affect potential (i.e., flexible-price) output dynamics over the business cycle. As a result, application of the DSGE approach is expected to produce more volatile estimates of potential output, and smaller and less persistent estimates of the output gap, when compared to the corresponding estimates obtained using the traditional approaches.

Overall, theoretical predictions about the behaviour of the model-consistent potential output and the output gap as well as their usefulness for the policy conduct are subject to ongoing testing. Empirical literature on potential output in DSGE models is relatively scarce and its findings in many cases are preliminary ones. As regards the comparison with more traditional measures, the evidence is mixed. In particular, estimates of efficient output share some similarity with conventional measures of potential output, whereas estimates of natural output are instead far away from conventional measures of potential output. Moreover, estimates of model-consistent potential output can be sensitive to the interpretation of structural shocks, in particular, reflecting poor identification of wage mark-up and labour preference shocks. As regards implications for the design of optimal monetary policy, by focusing on minimizing the flexible-price output gap, instead of the trend output gap, a central bank faces a more favourable trade-off between inflation and the output gap stabilization and is able to deliver lower inflation volatility.

Concerning the relationship between the output gap and inflation, there is no conclusive evidence yet proving that empirical estimates of the model-consistent output gaps derived from larger (and more realistic) DSGE models are significantly better indicators of inflation than the traditional measures. The empirical research, however, has shown that the real unit labour cost series (which is a measure of the model-consistent natural output gap within the basic NK model) is empirically more relevant to inflation determination contrary to conventional measures of output gaps. Alternative measures of the output gap are found to be sensitive to types of shocks driving the economy; therefore, they are unlikely to provide the best measure of inflationary pressures at all time. While, presumably, policymakers often ask for simple measures of economic activity that can predict inflation pressure, in order to understand the effect of the output gap on inflation it is, instead, important to understand which shocks drive the economy. The DSGE framework is well-tailored to conduct conditional analysis emphasizing the importance of structural shocks for the observed correlation between various policy-relevant macroeconomic aggregates.

Arguably, there are numerous open issues faced by the DSGE approach to potential output estimation. However, it does not mean that DSGE-based measures of potential output are useless. The joint estimation of potential output and structural shocks within the general equilibrium framework allows conducting a quantitative and coherent (internally consistent) assessment of inflation pressures and a normative evaluation of alternative monetary measures. Looking forward, further advancement in DSGE modelling, especially tackling shock identification issues, is expected to strengthen further the case for a more active use of the model-consistent measures of the output gap in policy considerations.

References

Adolfson, M., S. Laseén, J. Lindé, and L. E. Svensson (2008): “Optimal Monetary Policy in an Operational Medium-Sized DSGE Model,” Working Paper 6907, CEPR.

Atkeson, A., and L. E. Ohanian (2001): “Are Phillips Curves Useful for Forecasting Inflation?,” *Federal Reserve Bank of Minneapolis Quarterly Review*, 25(1), 2–11.

Adolfson, M., S. Laséen, J. Lindé, and M. Villani (2008): “Evaluating an Estimated New Keynesian Small Open Economy Model,” *Journal of Economic Dynamics and Control*, 32, 2690–2721.

Adolfson, M., S. Laseén, J. Lindé, and L. E. Svensson (2009): “Monetary Policy Trade-offs in an Estimated Open-Economy DSGE Model,” Working Paper 232, Sveriges Riksbank.

Andrle, M., Hlédik, T., Kameník, O., Vlček, J. (2009): “Implementing the New Structural Model of the Czech National Bank,” Czech National Bank Working Paper 2/2009.

Bean, C., (2005): “Comments on: Separating the Business Cycle from other Economic Fluctuations,” In: *The Greenspan Era: Lessons for the Future*, Federal Reserve Bank of Kansas City.

Blanchard, O., and J. Galí (2007): “Real Wage Rigidities and the New Keynesian Model,” *Journal of Money, Credit, and Banking*, supplement to vol. 39, 1, 35–66.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2008): “New Keynesian Models: Not Yet Usefull for Policy Analysis,” Working Paper 664, Federal Reserve Bank of Minneapolis.

Christiano, L., M. Eichenbaum, and Ch. Evans (2005): “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,” *Journal of Political Economy*, 113, 1–45.

Christiano, L., M. Trabandt, and K. Walentin (2007): “Introducing Financial Frictions and Unemployment into a Small Open Economy Model,” Working Paper Series 214, Sveriges Riksbank.

Christoffel, K., G. Coenen, and A. Warne (2008): “The New Area-Wide Model of the Euro Area: A Micro-Founded Open-Economy Model for Forecasting and Policy Analysis,” Working Paper 944, European Central Bank.

Coenen, G., F. Smets and I. Vetlov (2009): “Estimation of the Euro Area Output Gap Using the NAWM”, Working Paper 5, Lietuvos Bankas.

Edge, R., M. T. Kiley, and J.P. Laforte (2008): “Natural Rate Measures in an Estimated DSGE Model of the U.S. Economy,” *Journal of Economic Dynamics and Control*, 32, 2512–2535.

Fischer, B., M. Lenza, H. Pill, and L. Reichlin (2009): “Monetary Analysis and Monetary Policy in the Euro area 1999–2006,” *Journal of International Money and Finance*, 28, 1138–1164.

Fuhrer, J. C., and G. R. Moore (1995): “Inflation Persistence,” *Quarterly Journal of Economics*, 440, February, 127–159.

Galí, J. (2010): Unemployment Fluctuations and Stabilization Policies: A New Keynesian Perspective, mimeo.

Galí, J. (2002): “New Perspectives on Monetary Policy, Inflation and the Business Cycle,” Discussion Paper 3210, CEPR.

Galí, J., M. Gertler, and J. D. López-Salido (2007): “Markups, Gaps, and the Welfare Costs of Business Fluctuations,” *The Review of Economics and Statistics*, 81(1), 44–59.

Galí, J., M. Gertler, and J. D. López-Salido (2001): “European Inflation Dynamics,” *European Economic Review*, 45, 1237-1270.

Galí, J. and M. Gertler (1999): “Inflation Dynamics: A Structural Econometric Analysis,” *Journal of Monetary Economics*, 44(2), 195-222.

Galí, J., F. Smets, and R. Wouters (2010): “Unemployment in an Estimated New Keynesian Model,” Unpublished manuscript.

Hall, R. (2005): “Separating the Business Cycle from Other Economic Fluctuations,” in *The Greenspan Era*, Federal Reserve Bank of Kansas City.

Hall, R. (1986): “Market Structure and Macroeconomic Fluctuations,” *Brookings Papers on Economic Activity*, 2, 285–322.

Jakab, M. Z. and B. Világi, B. (2008): “An Estimated DSGE Model of the Hungarian Economy,” Working Paper 2008/9, Magyar Nemzeti Bank.

Jonsson, M., S, Laséen, and K. Walentin (2006): “Inflation Indicators in a Sticky Price Framework,” Sveriges Riksbank, mimeo.

Justiniano, A., and G. Primiceri (2008): “Potential and Natural Output,” Unpublished manuscript, Federal Reserve Bank of Chicago.

Kydland, F., and E. Prescott (1982): “Time-to-build and Aggregate Fluctuations,” *Econometrica*, 50, 1345–1370.

Neiss, K. S., and E. Nelson (2003): “The Real Interest Rate Gap as an Inflation Indicator,” *Macroeconomic Dynamics*, 7, 239–262.

Phillips, A. (1958): “The Relationship between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957,” *Economica*, 25(100), 283–299.

Rotemberg, J. J., and M. Woodford (1999): “The Cyclical Behaviour of Prices and Costs,” in *Handbook of Macroeconomics*, ed. by J. Taylor, and M. Woodford, Elsevier, 1051–1135.

Rotemberg, J. J., and M. Woodford (1997): “An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy,” in *NBER Macroeconomic Annual*, ed. by B. Bernanke, and J. Rotemberg, Cambridge, MIT Press, 297–346.

Sala, L. U. Söderström and A. Trigari (2010): “The Output Gap, the Labor Wedge, and the Dynamic Behaviour of Hours,” Working Paper 365, Innocenzo Gasparini Institute for Economic Research, University of Bocconi.

Sbordone A. (2002): “Prices and Unit Labor Costs: A New Test of Price Stickiness” Journal of Monetary Economics Vol. 49 (2), March 2002.

Smets, F., and R. Wouters (2003): “An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area,” *Journal of the European Economic Association*, 1(5), 1123–1175.

Smets, F., and R. Wouters (2005): “Comparing Shocks and Frictions in US and Euro Area Business Cycles: A Bayesian DSGE Approach,” *Journal of Applied Econometrics*, 20(2), 161–183.

Smets, F., and R. Wouters (2007): “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” *The American Economic Review*, 97(3), 586–606.

Stock, J. H., and M. W. Watson (2008): “Phillips Curve Inflation Forecasts,” Working Paper 14322, National Bureau of Economic Research.

Warne, A. (2010): *YADA Manual – Computational Details*, Manuscript, June 9.

Woodford, M. (1999): “Revolution and Evolution in Twentieth-Century Macroeconomics,” mimeo.

Woodford, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton.

Appendix

A. Natural vs. efficient output

Some intuition concerning the relationship between the natural and the efficient notion of output can be gained by analysing the flexible-price (natural) equilibrium of the basic NK DSGE model (as in Justiniano and Primiceri (2008)). More specifically, the basic model represents a closed economy without capital and habit formation in consumption, and featuring monopolistic competition on both goods and labour markets where respectively firms set prices to maximize profits given demand for the produced goods and households set wages to maximize utility given demand for the labour by firms. Provided that prices and wages are flexible, firms would optimally set prices equal to a mark-up over marginal costs (equation A.1) and, similarly, households would optimally set the real wage equal to a mark-up over the marginal rate of substitution between consumption and hours worked (equation A.2).

(A.1) $P_{n,t} = (1+\Lambda_{p,t})\cdot\dfrac{W_{n,t}}{\alpha\cdot A_t\cdot(L_{n,t})^{\alpha-1}}$, [price mark-up equation]

(A.2) $\dfrac{W_{n,t}}{P_{n,t}} = (1+\Lambda_{w,t})\cdot C_{n,t}\cdot(L_{n,t})^{v}$, [wage mark-up equation]

(A.3) $Y_{n,t} = A_t\cdot(L_{n,t})^{\alpha}$, [production function]

(A.4) $Y_{n,t} = C_{n,t}$, [goods market equilibrium]

where subscript n denotes natural equilibrium values, $P_{n,t}$ denotes the output price, $(1+\Lambda_{p,t})$ and $(1+\Lambda_{w,t})$ are respectively the (exogenous) price and wage gross mark-ups, $W_{n,t}$ is the nominal wage, A_t is a technology shock, $L_{n,t}$ is employment, $C_{n,t}$ is consumption, $Y_{n,t}$ is output, v is a parameter measuring disutility from labour, and α is a technology parameter.

The above four equations characterize the natural (flexible-price) equilibrium of the model. Once log-linearised, the above equations can be rewritten as:

(A.1') $0 = \lambda_{p,t} + (w_{n,t} - p_{n,t}) - (1/\alpha)\cdot a_t + (1/\alpha - 1)\cdot y_{n,t} - log(\alpha)$,

(A.2') $(w_{n,t} - p_{n,t}) = v\cdot l_{n,t} + c_{n,t} + \lambda_{w,t}$,

(A.3') $y_{n,t} = \alpha\cdot l_{n,t} + a_t$,

(A.4') $y_{n,t} = c_{n,t}$,

where variables in lower-case letters denote the logarithm of the corresponding upper case letters, except for $\lambda_{p,t} = log(1+\Lambda_{p,t})$ and $\lambda_{w,t} = log(1+\Lambda_{w,t})$.

The combination of equations (A.1')–(A.4') yields:

(A.5) $y_{n,t} = (\alpha/(1+v))\cdot log(\alpha) + a_t - (\alpha/(1+v))\cdot(\lambda_{p,t} + \lambda_{w,t})$.

According to equation (A.5), the evolution of the natural level of output is a function of two components. The first component is composed of the first two terms on the right-hand side and corresponds to efficient output, which is related to the path of productivity shocks. The second component is the third term on the right–hand side and corresponds to the output distortion due to imperfect competition, captured by the mark-up variables. It implies that natural output is more volatile than efficient output because the exogenous mark-up shocks make this distortion time-varying.

B. Natural output, real marginal cost and inflation

In the basic NK DSGE model the inflation rate is determined according to the NKPC:

(B.1) $\pi_t = \beta \cdot \pi_{t+1} + \kappa \cdot rmc_t,$

where π_t and π_{t+1} are respectively the current and the expected next period inflation rate, rmc_t denotes real marginal cost, κ is a positive coefficient inversely related to degree of price rigidities, and β is the households' discount rate.

It can be shown that in the basic NK model the natural output gap (the difference between the actual and natural level of output) is proportional to real marginal cost (Rotemberg and Woodford, 1997):

(B.2) $y_t - y_{n,t} = \eta \cdot rmc_t,$

where η is a parameter depending on households' preferences and firms' technology.

Substituting equation (B.2) in the Phillips curve (B.1) we get:

(B.3) $\pi_t = \beta \cdot \pi_{t+1} + (\kappa/\eta) \cdot (y_t - y_{n,t}).$

Hence the natural output does affect inflation through the (natural) output gap $y_t - y_{n,t}$. From this point of view, the notion of natural output in the NK model corresponds to the older Keynesian notion that natural output is the level of output at which there is no pressure for inflation to either increase or decrease. Note also that the equation (B.1) or, equivalently, equation (B.3) can be solved forward. As such, inflation at time t is the expected sum of future discounted marginal costs or, equivalently, of the future discounted natural output gaps.

C. Forecast evaluation procedure

We consider forecasting the annualized h-period change in the private consumption deflator, π^h_{t+h}:

(C.1) $$\pi^h_{t+h} = 100 * \left(\left(\frac{P_{t+h}}{P_t} \right)^{\frac{4}{h}} - 1 \right),$$

where P_t is the price level at t, h is the forecast horizon in quarters.

The general specification of the bivariate models for each estimation sample v is as follows:

(C.2) $\pi^{h}_{v,t+h} = a_v + b_v(L)\pi_{v,t} + c_v(L)x_{v,t} + \varepsilon^{h,x}_{v,t+h}$,

where $\pi_{v,t} = 400 * \left(\frac{P_t}{P_{t-1}} - 1\right)$ is the annualized one-period change in the private consumption deflator, $x_{v,t}$ is the exogenous variable (output gap or real marginal cost), $b_v(L)$ and $c_v(L)$ are finite polynomials of order p and q.

The forecasting models are estimated by Ordinary Least Squares (OLS). Starting with a general specification of four lags for both inflation and output gap/real marginal cost, lags for the dependent and exogenous variables are then selected using the Schwartz information criterion. For each estimation sample, based on the final specification in equation (C.2) a forecast of inflation is obtained:

(C.3) $\pi^{h,x}_{v,t+h} = a^{OLS}_v + b^{OLS}_v(L)\pi_{v,t} + c^{OLS}_v(L)x_{v,t}$.

The autoregressive models of inflation are estimated following the same lag-selection procedure described above. The random walk forecast of inflation h period ahead is given by the Atkeson and Ohanian (2001) random walk model where the inflation forecast is given by the average rate of inflation over the previous four quarters available for a given sample[33]:

(C.4) $\pi^{h,RW}_{v,t+h} = 100 * \left(\frac{P_t}{P_{t-4}} - 1\right)$.

Forecast errors e_t for the generic forecast from model M are defined as:

(C.5) $e^{h,M}_{t+h} = \pi^{h,M}_{v,t+h} - \pi^{h}_{t+h}$,

where π^{h}_{t+h} is the realized inflation rate in the last available sample.

Having computed forecast errors we then estimate the bias (*bias*) and the variance σ^2 of the forecast errors for each model:

(C.6) $bias^M = \frac{1}{T}\sum_{t=1}^{T} e^{h,M}_{t+h}$,

(C.7) $\left(\sigma^M\right)^2 = \frac{1}{T}\sum_{t=1}^{T}\left(e^{h,M}_{t+h} - \frac{1}{T}\sum_{t=1}^{T}\left(e^{h,M}_{t+h}\right)\right)^2$,

where T is number of forecast points.

The sum of the variance (C.7) and squared bias (C.6) gives us the MSFE:

(C.8) $MSFE^M = \left(\sigma^M\right)^2 + \left(bias^M\right)^2$.

[33] This implies that the random walk forecast of inflation will be common for all forecast horizons considered.

D. Tables

Table D.1. Analysis of Forecast Accuracy: Rolling Regressions

Model	MSFE	MSFE/RW	MSFE/AR	*bias*	σ^2	*bias*^{2}
Horizon 1q						
Trend output gap	1.49	0.81	0.93	0.03	1.49	0.00
HP output gap	1.47	0.80	0.92	-0.09	1.47	0.01
Uncond. flex-price output gap	1.60	0.87	1.00	0.23	1.55	0.05
Cond. flex-price output gap	1.67	0.90	1.04	0.15	1.65	0.02
Real marginal cost	1.73	0.94	1.08	0.00	1.73	0.00
AR	1.60	0.87	1.00	0.00	1.60	0.00
RW	1.85	1.00	1.15	0.06	1.84	0.00
Horizon 2q						
Trend output gap	1.12	0.67	0.81	0.11	1.11	0.01
HP output gap	1.25	0.74	0.90	-0.16	1.23	0.02
Uncond. flex-price output gap	1.32	0.78	0.95	0.28	1.24	0.08
Cond. flex-price output gap	1.40	0.83	1.01	0.22	1.35	0.05
Real marginal cost	1.49	0.89	1.08	0.08	1.49	0.01
AR	1.38	0.82	1.00	0.01	1.38	0.00
RW	1.68	1.00	1.22	0.05	1.68	0.00
Horizon 3q						
Trend output gap	1.09	0.67	0.84	0.12	1.07	0.01
HP output gap	1.24	0.77	0.96	-0.10	1.23	0.01
Uncond. flex-price output gap	1.07	0.66	0.82	0.34	0.95	0.11
Cond. flex-price output gap	1.09	0.67	0.84	0.26	1.02	0.07
Real marginal cost	1.17	0.72	0.90	0.10	1.16	0.01
AR	1.30	0.80	1.00	-0.08	1.29	0.01
RW	1.63	1.00	1.25	0.04	1.62	0.00
Horizon 4q						
Trend output gap	0.96	0.63	0.79	0.12	0.95	0.01
HP output gap	1.04	0.67	0.85	-0.09	1.03	0.01
Uncond. flex-price output gap	1.04	0.68	0.85	0.38	0.89	0.15
Cond. flex-price output gap	1.06	0.69	0.86	0.29	0.97	0.08
Real marginal cost	0.90	0.58	0.73	0.07	0.89	0.00
AR	1.22	0.80	1.00	-0.09	1.21	0.01
RW	1.54	1.00	1.26	0.04	1.53	0.00

Table D.1. Analysis of Forecast Accuracy: Rolling Regressions (cond.)

Model	MSFE	MSFE/RW	MSFE/AR	*bias*	σ^2	*bias*^{2}
Horizon 5q						
Trend output gap	0.96	0.69	0.93	0.18	0.93	0.03
HP output gap	0.85	0.61	0.83	-0.04	0.85	0.00
Uncond. flex-price output gap	0.85	0.61	0.82	0.43	0.66	0.18
Cond. flex-price output gap	0.88	0.63	0.85	0.29	0.79	0.09
Real marginal cost	0.67	0.48	0.65	0.06	0.66	0.00
AR	1.03	0.74	1.00	-0.12	1.02	0.02
RW	1.40	1.00	1.36	0.06	1.40	0.00
Horizon 6q						
Trend output gap	0.91	0.74	1.05	0.15	0.88	0.02
HP output gap	0.70	0.57	0.81	-0.05	0.69	0.00
Uncond. flex-price output gap	0.76	0.62	0.89	0.47	0.54	0.22
Cond. flex-price output gap	0.72	0.58	0.83	0.34	0.60	0.11
Real marginal cost	0.55	0.44	0.64	0.03	0.55	0.00
AR	0.86	0.70	1.00	-0.16	0.83	0.03
RW	1.23	1.00	1.43	0.04	1.23	0.00
Horizon 7q						
Trend output gap	0.77	0.86	1.11	0.13	0.75	0.02
HP output gap	0.47	0.53	0.68	-0.11	0.46	0.01
Uncond. flex-price output gap	0.65	0.72	0.93	0.49	0.40	0.24
Cond. flex-price output gap	0.58	0.65	0.84	0.38	0.44	0.14
Real marginal cost	0.38	0.43	0.55	0.01	0.38	0.00
AR	0.69	0.77	1.00	-0.20	0.65	0.04
RW	0.90	1.00	1.29	-0.03	0.90	0.00
Horizon 8q						
Trend output gap	0.71	1.09	1.35	0.10	0.70	0.01
HP output gap	0.33	0.51	0.64	-0.11	0.32	0.01
Uncond. flex-price output gap	0.53	0.81	1.01	0.51	0.27	0.26
Cond. flex-price output gap	0.46	0.71	0.87	0.40	0.30	0.16
Real marginal cost	0.23	0.36	0.44	0.01	0.23	0.00
AR	0.53	0.81	1.00	-0.18	0.49	0.03
RW	0.65	1.00	1.24	-0.08	0.64	0.01