A Simple Access Pricing Rule to Achieve the Ramsey Outcome for Interconnected Networks*

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Abstract

This paper considers a general and informationally efficient approach to determine the optimal access pricing rule for interconnected networks. It shows that there exists a simple rule that achieves the Ramsey outcome as the unique equilibrium when networks compete in linear prices without network-based price discrimination. The approach is informationally efficient in the sense that the regulator is required to know only the marginal cost structure, i.e. the marginal cost of making and terminating a call. The approach is general in that access prices can depend not only on the marginal costs but also on the retail prices, which can be observed by consumers and therefore by the regulator as well. In particular, I consider the set of linear access pricing rules which includes any fixed access price, the Efficient Component Pricing Rule (ECPR) and the Modified ECPR as special cases. I show that in this set, there is a unique rule that implements the Ramsey outcome as the unique equilibrium independently of the underlying demand conditions.

Keywords: Networks, Access Pricing, Interconnection, Competition Policy, Telecommunications

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1 Introduction

Access pricing rules constitute the core of the policy issues regarding interconnected net-
works. More precisely, studying how access prices affect competition between networks
and determining the optimal access prices form the central questions of the seminal papers
on two-way network interconnection in Telecommunication Industry (Armstrong 1998,
Laffont-Rey-Tirole (LRT, hereafter), 1998a,b) and the papers that followed.[1] Although
the papers vary in terms of the retail prices they consider (linear versus non-linear prices,
with or without network based price discrimination), the degree of customer heterogeneity
and whether or not they explicitly consider receivers etc., all the papers have a common
trait in that they consider a fixed access price. This approach may not be restrictive
when networks compete in two-part tariffs as long as we neglect call externality[2] since
LRT (1998a,b) show that in this case, by choosing an access price equal to the termination
cost, one can achieve the Ramsey outcome. In contrast, when networks compete in linear
prices, the results obtained with this approach are not satisfactory. More precisely, LRT
(1998a) find that when networks compete in linear prices without network-based price
discrimination[3] (i) the Ramsey access price must be lower than the termination cost but
the equilibrium does not exist if the access price is different from the termination cost and
the services provided by different networks are substitutable enough (ii) if access prices
are determined through private negotiations, networks can achieve the monopoly outcome
by coordinating on a certain level of access price. Furthermore, their Ramsey access price
is informationally demanding since it requires the regulator to possess precise information
regarding both the cost and the demand structure.

In this paper, I consider a general and informationally efficient approach to determine
the optimal access pricing rule and show that there exists a simple rule that achieves
the Ramsey outcome as the unique equilibrium when networks compete in linear prices
without network-based price discrimination. My approach is informationally efficient in
the sense that the regulator is required to know only the marginal cost structure, i.e. the
marginal cost of realizing and terminating a call. The approach is general in that access

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[1]See, for instances, Carter and Wright (1999, 2003), Dessein (2003), Gans and King (2000, 2001),
Hahn (2004), Hermalin and Katz (2001, 2004), Laffont-Marcus-Rey-Tirole, (2003), Jeon-Laffont-Tirole
(2004), Valletti and Cambini (forthcoming), Wright (2002).

[2]See the conclusion for the discussion of the case of competition with two-part tariffs in the presence
of call externality.

[3]The results under linear pricing with network-based price discrimination have a similar flavor: see
LRT (1998b).

prices can depend not only on the marginal costs but also on the retail prices which can be observed by consumers and therefore by the regulator as well. In particular, I consider the set of linear access pricing rules in which the access price mark-up with respect to the termination cost is a linear function of the retail prices and the marginal cost of a call. The set includes any fixed access price and the Efficient Component Pricing Rule (ECPR) as special cases. I show that in this set, there is a unique access rule that implements the Ramsey outcome as the unique equilibrium, *independently of the underlying demand conditions*, as long as there exists at least a mild degree of substitutability between networks' services.

Making access prices depend on retail prices is an old idea in the case of one-way access. The well-known ECPR[4] achieves the efficient entry by equalizing the access price that an entrant should pay to the incumbent with the sum of the cost of providing the access and the latter's opportunity cost (i.e. the incumbent's retail price mark-up) when the incumbent's retail price is regulated. However, the ECPR is not good at promoting competition in retail prices when the retail prices are not regulated since the access price that the incumbent receives increases with its retail price. This motivated Sibley et al. (2004) to consider the Generalized Efficient Component Pricing Rule (GECPR) in which the access price that an entrant pays is, roughly speaking, equal to the sum of the cost of providing the access and the entrant's opportunity cost (i.e. the entrant's retail price mark-up). Sibley et al. (2004) find that since the entrant can reduce its access charge payment by lowering its retail price, the GECPR is good at intensifying retail competition.

In the case of two-way access, LRT (1998a) examine various interpretations of the ECPR and show that when networks can privately negotiate on a fixed level of access price, the ECPR allows them to achieve the monopoly outcome. More importantly, Mialon (2004) studies the Modified Efficient Component Pricing Rule (MECPR) in LRT's framework.[5] Her MECPR is essentially the same as the GECPR considered by Sibley et al. (2004) and therefore the mark-up of the access price that network i pays to other network(s) is equal to the former's retail price mark-up. She finds that there always exists

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[4]See Baumol (1983), Baumol and Sidak (1994) and Willig (1979). For an introduction to the ECPR, see Armstrong (2002) and Laffont and Tirole (2000).

[5]Doganoglu and Tauman (2002) also consider a linear access pricing rule which depends on retail price. More precisely, in their paper, the access price that network i receives from network j is a (positive and) constant fraction of the linear retail price that network i charges. This rule is included as a special case in the set of the access pricing rules that I consider. As is explained in section 3.4, this kind of rule cannot be optimal since network i has an incentive to increase (instead of reducing) its retail price in order to receive a higher access payment.

a unique equilibrium and that the equilibrium retail price is lower than any equilibrium price obtained with a fixed access price larger than the termination cost. Although the MECPR has some desirable properties, its conceptual foundation is weak as long as retail competition is the main issue: there is no rationale for making the mark-up of the access price that network i pays to other network(s) exactly equal to the former's retail price mark-up except for the intuition that this reduces each network's incentive to choose a high retail price. Since this intuition does not involve the opportunity cost reasoning that underlies the ECPR and since, a priori, any access pricing rule that makes the access price that network i pays to other network(s) increase with the network i's retail price has the same effect of promoting retail competition, there is no particular reason to choose the MECPR. In fact, I show that there exists a unique rule achieving the Ramsey outcome in the set of linear access pricing rules which includes the MECPR as a special case, and the optimal rule is different from the MECPR, which implies that the MECPR does not achieve the Ramsey outcome.[6] A remarkable feature of the unique optimal access pricing rule is that it does not depend on the demand structure under the full coverage assumption, which is assumed by LRT; the regulator does not need to know anything about the demand side. Furthermore, both LRT and Mialon consider duopoly in the Hotelling model with constant elasticity demand function while I consider a general model of horizontal differentiation in which both the Hotelling model and the circular city model are special cases and show that the key insight of the main result obtained in the duopoly case extends to the case of n-network competition.

In practice, there are cases in which access prices (or termination charges) depend on retail tariffs. In the context of termination charges for mobile phone service, the Australian competition and consumer commission (2001) adopted what they call "retail benchmarking approach" which means that "access prices for GSM termination will fall at the same rate as retail prices for mobile services provided by a mobile carrier (p.89)." This paper provides a theoretical support for the retail benchmarking approach although it does not apply the framework to mobile phone communications.[7]

Section 2 presents the general model, defines the set of linear access pricing rules and characterizes the Ramsey outcome. Section 3 establishes the main result and compares different access pricing rules. Section 4 discusses the robustness of the result to introducing

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[6]In fact, the equilibrium price under the MECPR is higher than the Ramsey price.

[7]Another example of pegging access price to retail tariffs can be found in the international postal service. For instance, access prices (i.e. what they call "termination dues") among European countries should be set at 80% of domestic tariffs (Ghosal, 2002).

receivers' surplus, to relaxing the full coverage assumption and to introducing network-based price discrimination. Section 5 concludes.

2 Framework

2.1 The model

I present the general model of n-network competition which includes the duopoly model of LRT (1998a) as a special case. There is a mass one of consumers. There are $n \geq 2$ number of networks having the same cost structure that is specified below. They compete in linear prices and each network can cover all the consumers. Let $\mathbf{p} \equiv (p_1, ..., p_n) \in \Re^n_+$ represent the vector of retail prices and let $\mathbf{p}_{-i} \equiv (p_1, .., p_{i-1}, p_{i+1}, ..., p_n)$.

- *Firm's demand (or market share):*

The networks (i.e. firms) provide horizontally differentiated services and we assume that the measure of consumers subscribing to network i, denoted by $\alpha_i(p_i; \mathbf{p}_{-i})$, satisfies the following properties:

Property 1 (symmetry): for any $i, j = 1, ..., n$ and $i \neq j$ and for any p and p_i, $\alpha_i(p_i; p, ..., p) = \alpha_j(p_j; p, ..., p)$ if $p_i = p_j$.

Property 2 (monotonicity): for any $i, j = 1, ..., n$ and $i \neq j$, $\alpha_i(p_i; \mathbf{p}_{-i})$ is differentiable with respect to each retail price and decreases with p_i and increases with p_j; it strictly decreases with p_i and strictly increases with p_j for $\alpha_i \in (0, 1)$.[8]

Property 3 (full coverage): $\sum_{i=1}^{n} \alpha_i(p_i; \mathbf{p}_{-i}) = 1$ for all relevant $\mathbf{p} \in \Re^n_+$.

The three properties are satisfied by the Hotelling model of LRT (1998a) and the circular city model with $n = 2$ or 3 (Salop, 1979).[9] The symmetry and the full coverage imply

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[8]Property 2 can be more rigorously defined as follows. Given $\mathbf{p}_{-i}$, let $\underline{p}_i$ be the maximum $p_i \in \Re_+$ making $\alpha_i(p_i; \mathbf{p}_{-i}) = 1$ and let $\overline{p}_i$ be the minimum $p_i \in \Re_+$ making $\alpha_i(p_i; \mathbf{p}_{-i}) = 0$. If the maximum does not exist, we take $\underline{p}_i = 0$ and if the minimum does not exist, we take $\overline{p}_i = \infty$. Then, α_i strictly decreases with p_i for $p_i \in \left[\underline{p}_i, \overline{p}_i\right]$. Similarly, given $\mathbf{p}_{-j}$ with $j \neq i$, let $\underline{p}_j$ be the maximum $p_j \in \Re_+$ making $\alpha_i(p_i; \mathbf{p}_{-i}) = 0$ and let $\overline{p}_i$ be the minimum $p_j \in \Re_+$ making $\alpha_i(p_i; \mathbf{p}_{-i}) = 1$. If the maximum does not exist, we take $\underline{p}_j = 0$ and if the minimum does not exist, we take $\overline{p}_j = \infty$. Then, α_i strictly increases with p_j for $p_j \in \left[\underline{p}_j, \overline{p}_j\right]$.

[9]For $n > 3$, our model is more natural than the circular city model since in the latter, a minor price change of network i affects only the demands of its direct neighbors (network $i - 1$ and network $i + 1$) but does not affect the demands of other neighbors.

$\alpha_i = \frac{1}{n}$ for all $i = 1, ..., n$ if $p_i = p$ for all $i = 1, ..., n$. Regarding the full coverage property, LRT (1998a) assume that each consumer derives a constant utility from subscribing to one of the networks, which is large enough to ensure that all consumers always choose to join one of the networks. Since the total mass of consumers is equal to one, under the full coverage, the mass of consumers subscribing to network i (i.e. α_i) is equal to network i's market share.

- *Cost*:

Concerning the cost side, I use the same technology that is used in LRT (1998a). Serving a customer involves a fixed cost $f > 0$, say of connecting the customer's home to the network and of billing and serving her. A network also incurs a marginal cost c_0 per call at the originating and terminating ends of the call and marginal cost c_1 in between. Therefore, the total marginal cost of a call is

$$c \equiv 2c_0 + c_1.$$

- *Individual demand*:

Let $u(q)$ be the utility that a consumer derives from placing q volume of calls. The utility function $u(\cdot)$ is twice continuously differentiable, with $u' > 0, u'' < 0$, which implies that demand function is differentiable. Let $q(.)$ denote the demand function, given by $u'(q(p)) = p$: the volume of calls placed by a customer of network i is given by $q(p_i)$. Let $v(p)$ be the indirect utility function, i.e.,

$$v(p) = \max_q \{u(q) - pq\}.$$

Let $R(p) \equiv (p - c)q(p)$ represent the revenue per consumer. We assume that $R(p)$ is strictly concave with $R(\infty) = 0$ and has a unique maximum at $p = p^m$ with $c < p^m < \infty$; therefore, p^m denotes the monopoly price. Let R^m denote the monopoly revenue per consumer (i.e. $R^m = R(p^m)$). We assume $R^m > f$.

2.2 Access pricing rules

I consider simple access pricing rules which are not informationally demanding. More precisely, the informational constraint that the regulator faces is defined as follows.
- *The regulator's informational constraint*:

On the one hand, I assume that the regulator (or the competition authority) has limited information about the market such that she is not informed about the individual demand $q(p)$, each firm's demand (hence $\alpha_i(\mathbf{p})$) and the value of the fixed cost f. On the other hand, she knows the marginal costs (c, c_0). Furthermore, she and consumers observe retail prices (p_1, p_2).

The firms are assumed to know all the relevant information regarding both the demand and the cost sides.

- *The linear access pricing rules*:

Let a_{ij} with $i \neq j$ denote the access charge that network i receives from network j. In order to consider simple rules, I limit my attention to the following linear access pricing rule:

$$a_{ij} - c_0 = h(p_i, p_j, c) = h_1 p_i + h_2 p_j + h_3 c + h_4 \text{ for any } i, j = 1, ..., n \text{ and } i \neq j, \quad (1)$$

where $(h_1, h_2, h_3, h_4) \in \Re^4$ is a vector of constants. Note that I consider a reciprocal access pricing rule since the coefficients (h_1, h_2, h_3, h_4) do not depend on firms' identities. This is without loss of generality given that I consider symmetric networks.[10] Let Λ_n^L be the set of linear access pricing rules satisfying the above form (1). Some special cases of linear access pricing rules are:

- Marginal cost pricing rule: $a_{ij} = c_0$.

- Efficient component pricing rule (ECPR): $a_{ij} - c_0 = p_i - c$.

- Modified efficient component pricing rule (MECPR): $a_{ij} - c_0 = p_j - c$.

In the case of the ECPR, the access price that network j pays to network i is the sum of the termination cost and network i's opportunity cost (i.e. its retail price mark-up). In contrast, in the case of the MECPR, the access price that network j pays to network i is the sum of the termination cost and network j's opportunity cost (Sibley et al. 2000, Mialon 2004).

2.3   Ramsey benchmark

For future reference, we derive the social optimum in the ideal case in which the regulator knows all the relevant information and can dictate the prices under the constraint that

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[10]In the case of asymmetric networks, we need to consider non-reciprocal rules such that the coefficients depend on the firms' identities. See Carter and Wright (2003) for the study of asymmetric networks.

the industry breaks-even. Consumer variable welfare is

$$W(\mathbf{p}) = \sum_{i=1}^{n} \alpha_i(\mathbf{p})v(p_i) - T\left[\alpha_1(\mathbf{p}), ..., \alpha_n(\mathbf{p})\right] \tag{2}$$

where $T(\alpha_1, ..., \alpha_n)$ denotes the average consumer's utility from not being able to consume her preferred service. We assume that $T(\alpha)$ is minimized at equal market share $\alpha_i = \frac{1}{n}$. The industry budget constraint is

$$\sum_{i=1}^{n} \alpha_i(\mathbf{p})R(p_i) = f. \tag{3}$$

Maximizing (2) subject to (3) yields a symmetric solution, $p_i = p^R$ for all $i = 1, ..., n$, where the Ramsey price p^R is the lowest price that satisfies the budget constraint:

$$R(p^R) = f.$$

Since we assume $R^m > f$, we have $p^R < p^m$. Let $q(p^R) \equiv q^R$.

2.4   The main assumption and the timing

Since $R(p)$ is strictly concave and continuous with $R(\infty) = 0$, there exists a $\overline{p}$ such that $R(\overline{p}) = f$ with $\overline{p} > p^m$. In what follows, we make the following assumption on the degree of substitutability among the networks:

Assumption 1: The services provided by the networks are at least mildly substitutable in the sense that $\alpha_i(\overline{p}; \mathbf{p}_{-i}) = 0$ at $\mathbf{p}_{-i} = (p^R, ..., p^R)$.

Assumption 1 says that there is a minimum degree of substitutability among the networks such that if a network charges $\overline{p}(> p^m)$ while all the other networks charge the Ramsey price, then the former gets zero market share. Since $\overline{p}$ is much larger than p^m, the assumption implies that the services provided by the networks are at least mildly substitutable.

The timing of the game I consider is the following:
1. The regulator (or the competition authority) chooses a linear access pricing rule in Λ_n^L.
2. Each network simultaneously chooses its retail price.
3. Consumers make subscription and consumption decisions.

3 The main result

I first state the main result.

Proposition 1 *Under assumption 1, for any demand structure satisfying Properties 1-3, there is a unique linear access pricing rule in Λ_n^L defined by $a_{ij} - c_0 = \frac{n}{n-1}(p_j - c)$ that implements, independently of the underlying demand conditions,*

(i) the Ramsey outcome ($p_i = p^R$ for all $i = 1, ..., n$) as the unique symmetric equilibrium for $n \geq 2$

(ii) the Ramsey outcome as the unique equilibrium for $n = 2$.

Note first the remarkable result that the optimal rule implementing the Ramsey outcome does not depend on the demand structure as long as it satisfies Properties 1-3. Proposition 1(ii) is stronger than Proposition 1(i) since in the former, I prove that no asymmetric equilibrium exists: Although I did not prove that no asymmetric equilibrium exists for $n \geq 3$, I conjecture that it would hold.[11] In what follows, I prove Proposition 1(i) step by step in the main texts and provide the intuition. The proof that no asymmetric equilibrium exists for $n = 2$ is done in Appendix. At the end of the section, I compare different access pricing rules in an intuitive way.

Given a linear access pricing rule belonging to Λ_n^L and under the assumption of balanced calling patterns[12], the profit of network i is given by:

$$\pi_i(p_i : \mathbf{p}_{-i}) = \alpha_i \left\{ (p_i - c)q(p_i) - f \right\} + \sum_{j \neq i} \alpha_i \alpha_j \left\{ h(p_i, p_j, c)q(p_j) - h(p_j, p_i, c)q(p_i) \right\}, \quad (4)$$

where the first term represents the retail profit and the second term represents the net access revenue (or deficit).

3.1 Uniqueness of the candidate rule to achieve the Ramsey outcome

I show that among all the access pricing rules belonging to Λ_n^L, there is a unique candidate rule that satisfies a necessary condition to implement the Ramsey outcome ($p_i = p^R$ for

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[11]To prove this, we need to define Property 1 (Symmetry) with respect to any permutation of prices.

[12]The assumption is from LRT (1998a, b). It means that a consumer has an equal chance of calling a given consumer belonging to his network and another given consumer belonging to any other rival network.

$i = 1, ..., n$). From (4), the first-order derivative of π_i with respect to p_i is given by:

$$\frac{\partial \pi_i}{\partial p_i} = \frac{\partial \alpha_i}{\partial p_i} \{(p_i - c)q(p_i) - f\} + \alpha_i \left\{q(p_i) + (p_i - c)\frac{dq(p_i)}{dp_i}\right\} \tag{5}$$

$$+ \sum_{j \neq i} \left[\frac{\partial \alpha_i}{\partial p_i}\alpha_j + \frac{\partial \alpha_j}{\partial p_i}\alpha_i\right] \{h(p_i, p_j, c)q(p_j) - h(p_j, p_i, c)q(p_i)\}$$

$$+ \alpha_i \sum_{j \neq i} \alpha_j \left\{h_1 q(p_j) - h_2 q(p_i) - h(p_j, p_i, c)\frac{dq_i}{dp_i}\right\}.$$

Since π_i is a continuous function of p_i, a necessary condition to implement the Ramsey outcome is that the first-order derivative is zero at $p_i = p^R$ when all the other networks charge $p_j = p^R$ for $j \neq i$ and $j = 1, ..., n$. Since $R(p^R) = f$ and $h(p_i, p_j, c)q(p_j) = h(p_j, p_i, c)q(p_i)$ at the symmetric equilibrium candidate with the Ramsey price, the first and the third terms are zero in the above first-order derivative. Since $q(p_i) = q^R$ and $\alpha_i = \frac{1}{n}$ for $i = 1, ..., n$ at the symmetric equilibrium candidate, the necessary condition holds only if the following conditions are satisfied by $h(p_i, p_j, c)$:

$$1 + \frac{n-1}{n}(h_1 - h_2) = 0$$

$$p^R - c - \frac{n-1}{n}\left[(h_1 + h_2)p^R + h_3 c + h_4\right] = 0.$$

Given the regulator's informational constraint introduced in section 2.2, we find from the two conditions that $h_1 = 0, h_2 = \frac{n}{n-1}, h_3 c + h_4 = -\frac{n}{n-1}c$. Therefore, we obtain the unique candidate in the set of linear access pricing rules as follows:

$$a_{ij} - c_0 = \frac{n}{n-1}(p_j - c).$$

3.2 Existence of the symmetric equilibrium

I now show that under the access pricing rule $a_{ij} - c_0 = \frac{n}{n-1}(p_j - c)$ and under assumption 1, the symmetric equilibrium with $p_i = p^R$ for $i = 1, ..., n$ always exists. Given the access pricing rule $a_{ij} - c_0 = \frac{n}{n-1}(p_j - c)$, network i's profit is given by:

$$\pi_i(p_i : \mathbf{p}_{-i}) = \alpha_i \left[R(p_i) - f\right] + \frac{n}{n-1}\alpha_i \sum_{j \neq i} \alpha_j \left[R(p_j) - R(p_i)\right] \tag{6}$$

Suppose that all the other networks except network 1 charge p^R. Then, because of the symmetry and the full coverage, we have $\alpha_2 = ... = \alpha_n = \frac{1-\alpha_1}{n-1}$ and network 1's profit is

given by;

$$\pi_1(p_1 \quad : \quad p^R, ..., p^R) = \alpha_1 \left[R(p_1) - f\right] + \frac{n}{n-1}\alpha_1(1-\alpha_1)\left[f - R(p_i)\right]$$

$$= \quad \alpha_1 n(n-1)\left(\alpha_1 - \frac{1}{n}\right)\left[R(p_1) - f\right].$$

Note first that $\pi_1 = 0$ when $p_1 = p^R$ and $\pi_1 = 0$ for $p_1 \geq \overline{p}$ under assumption 1. Consider any p_1 with $p_1 < p^R$. Then, we have $\alpha_1 > \frac{1}{n}$ and $R(p_1) < f$, implying $\pi_1 < 0$. Consider now $p_1 \in \left(p^R, \overline{p}\right)$. Then, we have $\alpha_1 < \frac{1}{n}$ and $R(p_1) > f$, implying $\pi_1 < 0$ if $\alpha_1 > 0$. Therefore, the symmetric equilibrium always exists.

To give the intuition, I consider the case of $n = 2$ and examine network 1's price choice given $p_2 = p^R$. Consider first $p_1 \in \left(p^R, \overline{p}\right)$. In this case, network 1's retail profit per customer is $R(p_1) - f > 0$. Its access revenue per customer is $2(1-\alpha_1)R(p_2) = 2(1-\alpha_1)f$ while its access payment per customer is $2(1-\alpha_1)R(p_1)$, implying that it has a net access deficit per customer equal to $2(1-\alpha_1)\left[f - R(p_1)\right]$. Since $\alpha_1 < \frac{1}{2}$ for $p_1 \in \left(p^R, \overline{p}\right)$, the access deficit is larger than the retail profit and therefore the firm makes a loss. In contrast, in the case of $p_1 < p^R$, the firm has a retail deficit per customer equal to $R(p_1) - f < 0$ while it has a net access profit per customer equal to $2(1-\alpha_1)\left[f - R(p_1)\right]$. Since $\alpha_1 > \frac{1}{2}$, the access profit is not large enough to cover the retail deficit and the firm's profit is still negative. In other words, the coefficient in the optimal linear access pricing rule (2 when $n = 2$) is such that (i) when $p_1 = p^R$, network 1's profit is zero, (ii) when $p_1 \in \left(p^R, \overline{p}\right)$, its retail profit per communication is equal to its net access deficit per communication but, since its market share is smaller than a half, the total amount of on-net communications is smaller than the total amount of off-net communications, implying that it makes a loss (iii) when $p_1 < p^R$, its retail deficit per communication is equal to its net access revenue per communication but, since its market share is larger than a half, the total amount of on-net communications is larger than the total amount of off-net communications, implying that it makes a loss.

Note that in LRT (1998a), the non-existence of equilibrium occurs since a network can have an incentive to corner the market by deviating to a price lower than the price in the equilibrium candidate. In our equilibrium achieving the Ramsey outcome, the cornering strategy does not make any sense since it requires the deviating network to charge a price lower than p^R, implying that the firm makes a loss after cornering the market.

3.3 Non-existence of other symmetric equilibrium

I now show that under the access pricing rule $a_{ij} - c_0 = \frac{n}{n-1}(p_j - c)$ and under assumption 1, no other symmetric equilibrium exists except $p_i = p^R$ for $i = 1, ..., n$. Let p be a symmetric equilibrium candidate. First, it is obvious that neither $p < p^R$ nor $p > \overline{p}$ can be an equilibrium since then each firm makes a negative profit. Therefore, I consider only $p \in \left(p^R, \overline{p}\right]$. Consider first $p = \overline{p}$. Then, each firm gets zero profit without deviation. Suppose now that network 1 deviates to $p_1 = p^m$ while all the other networks continue to charge $\overline{p}$. Then, network 1's profit is given by:

$$\pi_1(p^m; \overline{p}, ..., \overline{p}) = \alpha_1 n(n-1)\left(\alpha_1 - \frac{1}{n}\right)[R^m - f] > 0,$$

where $\alpha_1 = \alpha_1(p^m; \overline{p}, ..., \overline{p}) > \frac{1}{n}$. Therefore, no symmetric equilibrium with $p = \overline{p}$ exists.

Let us consider now $p \in \left(p^R, \overline{p}\right)$. Then, from (6), the first-order derivative of π_i with respect to p_i is given by:

$$
\begin{aligned}
\frac{\partial \pi_i(p_i : \mathbf{p}_{-i})}{\partial p_i} &= [R(p_i) - f]\frac{\partial \alpha_i}{\partial p_i} + \alpha_i \frac{dR(p_i)}{dp_i} - \frac{n}{n-1}\alpha_i \sum_{j \neq i}\alpha_j \frac{dR(p_i)}{dp_i} \qquad (7)\\
&\quad + \frac{n}{n-1}\sum_{j \neq i}\left[\alpha_j \frac{\partial \alpha_i}{\partial p_i} + \alpha_i \frac{\partial \alpha_j}{\partial p_i}\right][R(p_j) - R(p_i)].
\end{aligned}
$$

At $p_i = p$ for $i = 1, ..., n$, since $\sum_{j \neq i}\alpha_j = \frac{n-1}{n}$, the first-order derivative is given by:

$$\frac{\partial \pi_i(p : p, ..., p)}{\partial p_i} = [R(p) - f]\frac{\partial \alpha_i}{\partial p_i} < 0 \text{ for } p \in \left(p^R, \overline{p}\right). \qquad (8)$$

Therefore, each firm has an incentive to undercut and no other symmetric equilibrium exists.

The access price rule $a_{ij} - c_0 = \frac{n}{n-1}(p_j - c)$ intensifies retail price competition since by reducing p_j network j can reduce the access price that it should pay to the rival networks. In particular, at any symmetric price p that allows networks to realize a positive retail profit (i.e. $R(p) > f$), each network has an incentive to choose a price lower than p. From (7), when network i reduces its retail price, there are three effects on its profit. First, given its retail price, its retail profit increases through its expansion of market share. Second, given each network's market share, its retail revenue per consumer decreases while its access payment per consumer decreases as well. Third, given each network's retail price, the changes in the market shares affect its net access payment. In any symmetric equilibrium candidate with $p_i = p \in \left(p^R, \overline{p}\right)$ for $i = 1, ..., n$, the second and the third effects are zero and the first is positive. Therefore, each firm has an incentive to deviate in order to increase its market share.

3.4   Comparison with other rules when $n = 2$

Suppose that the regulator should choose an access pricing rule without knowing the demand structure while she only knows the marginal cost structure (c, c_0). Consider duopolistic competition[13] and, for simplicity, let a_i denote the access charge that network i receives from the rival network. Then, from Proposition 1(ii), we have the following corollary.

Corollary 1 *Under assumption 1, the social welfare is strictly higher under the access pricing rule $a_i - c_0 = 2(p_j - c)$ than under any other fixed access price (including $a_i = c_0$), under the ECPR ($a_i - c_0 = p_i - c$) and under the MECPR ($a_i - c_0 = p_j - c$).*

In order to give the intuition, I examine the first order derivative of network i's profit in each access pricing rule assuming that a symmetric equilibrium with $p_1 = p_2 = p < p^m$ exists under each rule.

First, under a fixed and reciprocal access price rule $a_1 = a_2 = a$, network i's profit is given by:
$$\pi_i(p_i; p_j) = \alpha_i \left[R(p_i) - f\right] + \alpha_i(1 - \alpha_i)(a - c_0)\left[q(p_j) - q(p_i)\right].$$

Therefore, the first-order derivative with respect to p_i at $p_i = p_j = p$ is given by:
$$[R(p) - f] \frac{d\alpha_i}{dp_i} + \frac{1}{2} \frac{dR_i}{dp_i} - \frac{(a - c_0)}{4} \frac{dq(p_i)}{dp_i}. \tag{9}$$

Consider first the case of the marginal cost pricing ($a = c_0$). In this case, for any market share, each network has zero net access profit. Since $\frac{d\alpha_i}{dp_i} < 0 < \frac{dR_i}{dp_i}$, the first order condition holds only for $p > p^R$ such that $R(p) > f$. Hence, the marginal cost pricing cannot achieve the Ramsey outcome. From (9), it is clear that as the access price becomes larger than the termination cost, network i has an extra incentive to raise p_i since by reducing the demand of its own customers, it can reduce its access payment. Since an increase in the reciprocal access price results in an increase in the retail price, LRT (1998a) find that networks can achieve the monopoly outcome if they can choose access price through private negotiation. In contrast, as the access price becomes smaller than the termination cost, network i has an extra incentive to reduce p_i in order to increase its access revenue. This is why LRT (1998a) find that the Ramsey access charge requires

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[13]Although I restrict my attention to the case of $n = 2$ since non-existence of asymmetric equilibrium is not proved for $n > 2$, the intuition obtained in this section applies to the case of $n > 2$ as well.

an access charge lower than the termination cost. More precisely, they find that Ramsey access charge, denoted by a^R, is given by:

$$\frac{a^R - c_0}{2} = -(1 - \frac{1}{\eta})(p^m - p^R),$$

where η is the elasticity of demand and is assumed to be constant and larger than 1. Note that in order to be able to compute the Ramsey access price, the regulator should have a precise knowledge about the demand structure such that she should be able to compute η, p^m and p^R. Furthermore, LRT (1998a) show that the equilibrium does not exist for $a \neq c_0$ if the degree of substitutability of the two networks is high enough.

Second, in the case of the ECPR, network i's profit is given by:

$$\pi_i(p_i; p_j) = \alpha_i \left[R(p_i) - f \right] + \alpha_i(1 - \alpha_i) \left[q(p_j)(p_i - c) - q(p_i)(p_j - c) \right].$$

Therefore, the first-order derivative with respect to p_i at $p_i = p_j = p$ is given by:

$$\left[R(p) - f \right] \frac{d\alpha_i}{dp_i} + \frac{1}{2} \frac{dR_i}{dp_i} + \frac{1}{4} \left[q(p) - (p - c) \frac{dq(p_i)}{dp_i} \right] \tag{10}$$

The first two terms in (10) are what we found in the first-order derivative under $a = c_0$ and have to do with the retail profit. The last term in (10) has to do with the access revenue and since $p > c$ and $\frac{dq(p_i)}{dp_i} < 0$, it induces network i to increase its retail price. Since under the ECPR a network can increase its revenue by increasing its retail price, the ECPR induces each network to choose a price higher than the one under $a = c_0$.

Last, consider the following rule $a_i - c_0 = \kappa(p_j - c)$ where $\kappa(\geq 0)$ is a constant. For instances, if $\kappa = 1$, we have the MECPR and if $\kappa = 0$, we have the marginal cost pricing. Then, network i's profit is given by:

$$\pi_i(p_i : p_j) = \alpha_i \left[R(p_i) - f \right] + \kappa \alpha_i(1 - \alpha_i) \left[R(p_j) - R(p_i) \right].$$

Therefore, the first-order derivative with respect to p_i at $p_i = p_j = p$ is given by:

$$\left[R(p) - f \right] \frac{d\alpha_i}{dp_i} + \frac{1}{2} \frac{dR_i}{dp_i} - \frac{\kappa}{4} \frac{dR_i}{dp_i}. \tag{11}$$

The first two terms in (11) are what we found in the first-order derivative under $a = c_0$ and have to do with the retail profit. The last term in (11) has to do with the access revenue and, since $\frac{dR_i}{dp_i} > 0$, an increase in κ induces network i to reduce its retail price. This implies that the retail price under the marginal cost pricing is higher than the retail price under the MECPR, which is higher than the retail price under when $\kappa = 2$ (i.e. the Ramsey price). Note that from (11), when $\kappa = 2$, the only price satisfying the first-order condition is the Ramsey price.

4 Robustness

In this section, we discuss the robustness of our results to relaxing some of our assumptions.

First, introducing receiver's surplus as in Jeon-Laffont-Tirole (2004) does not affect our result as long as the receivers are not charged for the reception. Note first that introducing receiver's surplus does not affect the Ramsey price which is the lowest price allowing networks to recover their fixed cost (i.e. $R(p^R) = f$). Second in the proofs of Proposition 1, I only use the three properties regarding market share α_i introduced in Section 2 and these properties remain intact even though receiver surplus is introduced.

In what follows, I examine the robustness of the result to relaxing the full coverage assumption and introducing the network-based price discrimination.

4.1 Relaxing full coverage

I here assume away the full coverage assumption and assume that $\sum_{i=1}^{n} \alpha_i(p : p, ..., p)$ strictly decreases with p. We continue to normalize the mass of potential consumers at one. Since $\sum_{i=1}^{n} \alpha_i(p : p, ..., p)$ represents the total mass of consumers who subscribe to one of the networks, it cannot be larger than one. In this setting, the Ramsey price is still characterized by $R(p^R) = f$. Let $\alpha_i(p^R : p^R, ..., p^R) = \alpha^R > 0$. Then, we have the following result:

Proposition 2 *Suppose that $\sum_{i=1}^{n} \alpha_i(p : p, ..., p)$ strictly decreases with p. Under assumption 1, for any demand structure satisfying Properties 1 and 2,*
(i) there is a unique linear access pricing rule in Λ_n^L defined by $a_{ij} - c_0 = \frac{1}{\alpha^R(n-1)}(p_j - c)$ that satisfies a necessary condition to achieve the Ramsey outcome ($p_i = p^R$ for $i = 1, ..., n$) as an equilibrium
(ii) under the rule, $p_i = p^R$ for $i = 1, ..., n$ is an equilibrium.

Note that the access pricing rule in Proposition 2 generalizes the one in Proposition 1 since under the full coverage, $\alpha^R = \frac{1}{n}$.

Proof. (i) The first-order derivative of π_i with respect to p_i is given by (5). A necessary condition to implement the Ramsey outcome is that the first-order derivative is zero at $p_i = p^R$ for $i = 1, ..., n$. Since $R(p^R) = f$ and $h(p_i, p_j, c)q(p_j) = h(p_j, p_i, c)q(p_i)$ at the symmetric equilibrium candidate, the first and the third terms are zero in (5) at $p_i = p^R$ for $i = 1, ..., n$. Since $q(p_i) = q^R$ and $\alpha_i = \alpha^R$ for $i = 1, ..., n$ at the symmetric equilibrium

candidate, the necessary condition holds only if the following conditions are satisfied by $h(p_i, p_j, c)$:

$$1 + (n-1)\alpha^R (h_1 - h_2) = 0$$
$$p^R - c - (n-1)\alpha^R \left[(h_1 + h_2) p^R + h_3 c + h_4\right] = 0.$$

From the two conditions, we find that $h_1 = 0, h_2 = \frac{1}{\alpha^R(n-1)}, h_3 c + h_4 = -\frac{c}{\alpha^R(n-1)}$. Therefore, we obtain the unique candidate in the set of linear access pricing rules as follows:

$$a_{ij} - c_0 = \frac{1}{\alpha^R(n-1)}(p_j - c).$$

(ii) Given the access pricing rule, network i's profit is given by:

$$\pi_i(p_i : p_j) = \alpha_i \left[R(p_i) - f\right] + \frac{1}{\alpha^R(n-1)}\alpha_i \sum_{j \neq i} \alpha_j \left[R(p_j) - R(p_i)\right]$$

Suppose that all the other networks except network 1 charge p^R. Then, because of the symmetry, we have $\alpha_2 = ... = \alpha_n$ and network 1's profit is given by;

$$\begin{aligned}
\pi_1(p_1; p^R, ..., p^R) &= \alpha_1 \left[R(p_1) - f\right] + \frac{1}{\alpha^R}\alpha_1\alpha_2 \left[f - R(p_1)\right] \\
&= \alpha_1 \frac{\left[\alpha^R - \alpha_2\right]}{\alpha^R} \left[R(p_1) - f\right],
\end{aligned}$$

where $\alpha_2 = \alpha_2(p^R; p_1, p^R, ..., p^R)$. Note first that $\pi_1 = 0$ when $p_1 = p^R$ and $\pi_1 = 0$ for $p_1 \geq \bar{p}$ under assumption 1. Consider any p_1 with $p_1 < p^R$. Then, from the monotonicity, we have $\alpha^R > \alpha_2$ and $R(p_1) < f$, implying $\pi_1 < 0$. Consider now $p_1 \in (p^R, \bar{p})$. Then, we have $\alpha^R < \alpha_2$ and $R(p_1) > f$, implying $\pi_1 < 0$ if $\alpha_1 > 0$. ∎

Remark 1: Even though we relax the full coverage assumption, the rule presented in Proposition 1 implements the Ramsey outcome if the market is mature in that the total mass of consumers choosing to join one among the networks is equal to one at the Ramsey price.[14] Otherwise, the regulator needs to know α^R and in this sense the optimal

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[14]However, there might be multiple symmetric equilibria. Under the access rule described in Proposition 2, the first-order derivative of π_i with respect to p_i when all networks charge p with $p \in (p^R, p^m)$ is given by:

$$[R(p_i) - f]\frac{\partial \alpha_i}{\partial p_i} + \frac{\alpha_i}{\alpha^R}\left[\alpha^R - \alpha_i\right]\frac{dR(p_i)}{dp_i}$$

where $\alpha_i = \alpha_i(p; p, ..., p) < \alpha^R$. The first term is negative and the second term is positive. Therefore, the first-order condition may be satisfied for $p \in (p^R, p^m)$.

access pricing rule is informationally demanding. However, even when it is difficult for the regulator to know α^R, this does not imply that she should adopt one of the alternative access pricing rules presented in Section 2.2. As the comparison of different rules in Section 3.4 has shown, the intuition that one can intensify the retail competition by making the access price that network i pays to other networks increase with its retail price holds generally. More precisely, since $\alpha^R \leq 1/n$ holds, we have $\frac{1}{\alpha^R(n-1)} \geq n/(n-1)$. Therefore, one can use the access pricing rule presented in Proposition 1, $a_{ij} - c_0 = \frac{n}{n-1}(p_j - c)$: although the equilibrium price under the rule is higher than the Ramsey price, it is lower than the equilibrium price under any fixed access price (larger than the termination cost), or under the ECPR or under the MECPR. Furthermore, the previous rule is not informationally demanding.

4.2 Network-based price discrimination

I now introduce network-based price discrimination. Following LRT (1998b), let p_i be network i's on-net price and $\widehat{p}_i$ network i's off-net price. For simplicity, I consider the case of $n = 2$. Note first that the Ramsey outcome is not affected by the network-based price discrimination. Since the Ramsey outcome can be achieved by the access pricing rule $a_i - c_0 = 2(p_j - c)$ without network-based price discrimination, the price discrimination has no social value in our framework. Consider the following linear access price rule:

$$\begin{aligned} a_i - c_0 &= h(p_i, \widehat{p}_i, p_j, \widehat{p}_j, c) \\ &= h_1 p_i + \widehat{h}_1 \widehat{p}_i + h_2 p_j + \widehat{h}_2 \widehat{p}_j + h_3 c + h_4, \end{aligned}$$

where each of $(h_1, \widehat{h}_1, h_2, \widehat{h}_2, h_3, h_4)$ is a constant. Let $\widehat{\Lambda}^L$ denote the set of the linear access pricing rules taking the above form. Since I assume that the regulator chooses $(h_1, \widehat{h}_1, h_2, \widehat{h}_2, h_3, h_4)$ without knowing the demand structure, none of $(h_1, \widehat{h}_1, h_2, \widehat{h}_2, h_3, k)$ can depend on the demand-side information. I have a negative result:

Proposition 3 *In the presence of network-based price discrimination, there is no rule to achieve the Ramsey outcome as an equilibrium among the linear access pricing rules in* $\widehat{\Lambda}^L$,

Proof. Network i's profit is given by:

$$\begin{aligned} \pi_i &= \alpha_i \left[\alpha_i R(p_i) + (1 - \alpha_i) R(\widehat{p}_i) - f \right] \\ &\quad + \alpha_i (1 - \alpha_i) \left[q(\widehat{p}_j) h(p_i, \widehat{p}_i, p_j, \widehat{p}_j, c) - q(\widehat{p}_i) h(p_j, \widehat{p}_j, p_i, \widehat{p}_i, c) \right]. \end{aligned}$$

A necessary condition to implement the Ramsey outcome is that the first-order derivative is zero at $p_1 = p_2 = \widehat{p}_1 = \widehat{p}_2 = p^R$. The first-order condition with respect to p_i at $p_1 = p_2 = \widehat{p}_1 = \widehat{p}_2 = p^R$ is given by:

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial p_i} &= \frac{1}{4} \left. \frac{dR(p_i)}{dp_i} \right|_{p_1 = p^R} + \frac{1}{4} q^R (h_1 - h_3) \\
&= \frac{1}{4} q^R (1 + h_1 - h_3) + \frac{1}{4} (p^R - c) \left. \frac{dq(p_i)}{dp_i} \right|_{p_1 = p^R}
\end{aligned}
$$

Therefore, (h_1, h_3) must depend on the information such as q^R, p^R and $\left. \frac{dR(p_i)}{dp_i} \right|_{p_1 = p^R}$ that are not available to the regulator. This proves the result. ∎

In the presence of network-based price discrimination, as long as access prices are not related to on-net prices, each network has an incentive to increase its on-net price above the Ramsey price because of its market power. However, in order to induce each network to charge its on-net price equal to the Ramsey price by making the access prices depend on the on-net prices, the government must possess precise information about the demand function as a necessary condition. Therefore, there is no linear access pricing rule that achieves the Ramsey outcome under the informational constraint described in Section 2.2.

5 Conclusion

I showed that when networks compete in linear prices without network-based price discrimination, there is a simple access pricing rule that achieves the Ramsey outcome as the unique equilibrium independently of the underlying demand conditions. The rule is simple and is not informationally demanding: the regulator only needs to know the marginal costs. The rule intensifies retail competition by making the access price that network i pays to other networks increase with its retail price. Our result implies that although networks sell differentiated products, they end up having an equilibrium with zero profit.[15]

I showed that the key insight is valid in a general framework of competition among n-networks. Although the optimal rule achieving the Ramsey outcome can be informationally demanding if the full coverage assumption does not hold, I explained intuitively that the optimal access pricing rule conditional on full coverage, which is not informationally

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[15]This can reduce each network's incentive to invest in infrastructure. If this incentive is a central issue, one can study the optimal access pricing rule focusing on the trade-off between investment incentive and retail competition. See Valletti and Cambini (forthcoming) for the study of investment incentive.

demanding, performs better in terms of retail price competition than any fixed access price (larger than the termination cost) or the ECPR or the MECPR. I also found that there is no simple rule achieving the Ramsey outcome when networks can use network-based price discrimination since preventing each network from exercising its market power in terms of its on-net price requires the regulator to have very precise information about the demand structure as a necessary condition. Therefore, banning network price discrimination increases social welfare in our case. The result that network-based price discrimination can reduce social welfare is reminiscent of the finding in Jeon-Laffont-Tirole (2004) in which they show that network-based price discrimination can generate connectivity breakdown.

Finally, it would be interesting to apply my approach to the case of network competition with two-part tariffs. The question is to know whether pegging access prices to retail prices can help internalize call externalities even in the absence of reception charges since reception charges are not used in many countries.[16]

Appendix

In the appendix, I prove four lemmas and this establishes that there is no asymmetric equilibrium when $n = 2$.

Lemma 1 *There exists no "cornered-market" equilibrium.*

Proof. Suppose that network 1 corners the market with $\pi_1 > 0$. Then, network 2 can charge $p_2 = p_1$ and make a profit $\pi_1/2 > 0$ and therefore we get a contradiction. Suppose that network 1 corners the market with $\pi_1 = 0$. This implies that $p_1 = p^R$ or $p_1 = \overline{p}$. The proof of Proposition 1 in Section 3.3 has shown that if $p_1 = \overline{p}$, network 2 can realize a strictly positive profit by charging $p_2 = p^m$. Suppose $p_1 = p^R$. From the proof of Proposition 1 in Section 3.2, we know that any price p_2 different from p^R and $\overline{p}$ and satisfying $\alpha_2(p_2; p^R) > 0$ makes $\pi_2 < 0$. However, $(p_1 = p^R, p_2 = \overline{p})$ cannot be an equilibrium since p_1 has an incentive to deviate to p^m. The only remaining possibility is $p_1 = p_2 = p^R$ but then $\alpha_1 = \alpha_2 = 1/2$, which is not a cornered-market equilibrium. ■

Lemma 2 *If (p_1, p_2) is an asymmetric equilibrium with $p_1 < p_2$, then $p_2 > p^m$.*

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[16] Jeon-Laffont-Tirole (2004) show that when receivers derive surplus from communications and networks can charge call receptions, there is a unique access price implementing the Ramsey outcome.

Proof. Given an asymmetric equilibrium (p_1, p_2), network i's equilibrium profit is

$$\pi_i(p_i : p_j) = \alpha_i \left[R(p_i) - f \right] + 2\alpha_i(1 - \alpha_i) \left[R(p_j) - R(p_i) \right].$$

This must be higher than the profit that network i obtains by deviating to p_j: the following inequalities must hold

$$\pi_1(p_1 : p_2) \geq \frac{1}{2} \left[R(p_2) - f \right]; \tag{12}$$

$$\pi_2(p_2 : p_1) \geq \frac{1}{2} \left[R(p_1) - f \right]. \tag{13}$$

Adding (12) and (13) yields,

$$(\alpha_i - \frac{1}{2}) \left[R(p_1) - R(p_2) \right] \geq 0.$$

Since $\alpha_i > \frac{1}{2}$, a necessary condition to have $R(p_1) \geq R(p_2)$ and $p_1 < p_2$ is $p_2 > p^m$. ∎

Lemma 3 *If (p_1, p_2) is an equilibrium, then p_i cannot be lower than the Ramsey price and cannot be higher than $\overline{p}$: $p^R \leq p_i \leq \overline{p}$ for $i = 1$ and 2.*

Proof. Step 1: If (p_1, p_2) is an equilibrium, then p_i cannot be lower than p^R.

We cannot have an equilibrium in which $p_i < p^R$ for $i = 1$ and 2 since then at least one firm has a strictly negative profit. Suppose $p_1 < p^R \leq p_2$. First, $\alpha_2 > 0$ requires $p_2 < \overline{p}$, which implies $R(p_2) \geq f$. If (p_1, p_2) is an equilibrium, network 1 must not have an incentive to deviate to p_2. In other words, the following inequality should hold;

$$\alpha_1 \left[R(p_1) - f \right] + 2\alpha_1(1 - \alpha_1) \left[R(p_2) - R(p_1) \right] \geq \frac{1}{2} \left[R(p_2) - f \right],$$

which is equivalent to

$$2\alpha_1(\alpha_1 - \frac{1}{2}) \left[R(p_1) - f \right] + 2 \left[\alpha_1(1 - \alpha_1) - \frac{1}{4} \right] \left[R(p_2) - f \right] \geq 0.$$

Since we have $\alpha_1 > \frac{1}{2}, R(p_1) < f \leq R(p_2)$, the left hand side is strictly negative and therefore we have a contradiction.

Step 2: If (p_1, p_2) is an equilibrium, then p_i cannot be higher than $\overline{p}$.

We cannot have an equilibrium in which $p_i > \overline{p}$ for $i = 1$ and 2 since then at least one firm has a strictly negative profit. Suppose $p_1 \leq \overline{p} < p_2$. We distinguish two cases: $p_1 \in (p^m, \overline{p}]$ and $p_1 \in \left(p^R, p^m \right]$. Note that $\left(p_1 = p^R, p_2 > \overline{p} \right)$ cannot be an equilibrium since then network 1 corners the market but no cornered-equilibrium exists from Lemma 1.

Case 1: $p_1 \in (p^m, \overline{p}]$ and $\overline{p} < p_2$.

Given $p_1 \in (p^m, \overline{p}]$, there must be a $p_1' \in \left[p^R, p^m \right)$ such that $R(p_1) = R(p_1') \geq f$. In order to have (p_1, p_2) as an equilibrium, network 1 should not have an incentive to deviate to p_1'. Therefore, the following inequality should hold:

$$\alpha_1 \left[R(p_1) - f \right] + 2\alpha_1 (1 - \alpha_1) \left[R(p_2) - R(p_1) \right] \geq \alpha_1' \left[R(p_1') - f \right] + 2\alpha_1' (1 - \alpha_1') \left[R(p_2) - R(p_1') \right],$$

where $\alpha_1' = \alpha_1(p_1'; p_2) > \alpha_1 = \alpha_1(p_1; p_2) > \frac{1}{2}$. Since $R(p_1) \geq f > R(p_2)$, the left hand side is strictly smaller than the right hand side and therefore we have a contradiction.

Case 2: $p_1 \in \left(p^R, p^m \right]$ and $\overline{p} < p_2$.

We have $R(p_1) > f > R(p_2)$. Network 1's profit is given by:

$$
\begin{aligned}
\pi_1(p_1 \;:\; p_2) &= \alpha_1 \left[R(p_1) - f \right] + 2\alpha_1 (1 - \alpha_1) \left[R(p_2) - R(p_1) \right] \\
&= \alpha_1 \left\{ (2\alpha_1 - 1) \left[R(p_1) - f \right] + 2(1 - \alpha_1) \left[R(p_2) - f \right] \right\}.
\end{aligned}
$$

$\pi_1(p_1 : p_2) \geq 0$ is equivalent to

$$\frac{R(p_1) - f}{f - R(p_2)} \geq \frac{2(1 - \alpha_1)}{2\alpha_1 - 1}.$$

Network 2 should have no incentive to deviate to p_1:

$$\pi_2(p_2 : p_1) = \alpha_2 \left[R(p_2) - f \right] + 2\alpha_2 (1 - \alpha_2) \left[R(p_1) - R(p_2) \right] \geq \frac{1}{2} \left[R(p_1) - f \right]$$

which is equivalent to

$$\frac{R(p_1) - f}{f - R(p_2)} \leq \frac{(1 - \alpha_1)(2\alpha_1 - 1)}{\frac{1}{2} - 2\alpha_1(1 - \alpha_1)} \left(= \frac{2(1 - \alpha_1)}{2\alpha_1 - 1} \right).$$

Therefore, we must have $\pi_1(p_1 : p_2) = 0$ and $\pi_2(p_2 : p_1) = \frac{1}{2} \left[R(p_1) - f \right]$. Then, there is a profitable deviation for network 1: since $\overline{p} < p_2$, from assumption 1, there must be a $p_1' \in \left(p^R, p^m \right]$ that allows it to corner the market and to realize the profit of $R(p_1') - f > 0$. Therefore, we have a contradiction. ∎

Lemma 4 *There is no asymmetric equilibrium with $p_i \in (p^m, \overline{p}]$ for $i = 1$ or 2.*

Proof. **Case 1**: $p_i \in (p^m, \overline{p}]$ for $i = 1$ and 2

Suppose $p_1 < p_2$. Therefore, we have $R(p_1) > R(p_2) \geq f$. There must be a $p_1' \in \left(p^R, p^m \right)$ such that $R(p_1) = R(p_1') > f$. Network 1 should not have any incentive to deviate to p_1':

$$\alpha_1 \left[R(p_1) - f \right] + 2\alpha_1 (1 - \alpha_1) \left[R(p_2) - R(p_1) \right] \geq \alpha_1' \left[R(p_1') - f \right] + 2\alpha_1' (1 - \alpha_1') \left[R(p_2) - R(p_1') \right]$$

where $\alpha'_1 = \alpha_1(p'_1; p_2) > \alpha_1 = \alpha_1(p_1; p_2) > \frac{1}{2}$. Since $R(p_1) > R(p_2) \geq f$, the left hand side is strictly smaller than the right hand side and therefore we have a contradiction.

Case 2: $p_1 \in \left[p^R, p^m \right]$ and $p_2 \in (p^m, \overline{p}]$.

If $\alpha_2 = 0$, network 1 corners the market and we have a contradiction from Lemma 1. Therefore, we consider $\alpha_2 > 0$, which implies that one cannot have $\left(p_1 = p^R, p_2 = \overline{p} \right)$ as an equilibrium. This in turn implies that one of the two following inequalities $R(p_1) \geq f$ or $R(p_2) \geq f$ must hold strictly. Network 2 should have no incentive to deviate to p_1:

$$\alpha_2 \left[R(p_2) - f \right] + 2\alpha_2(1 - \alpha_2) \left[R(p_1) - R(p_2) \right] \geq \frac{1}{2} \left[R(p_1) - f \right],$$

which is equivalent to

$$2\alpha_2 \left(\alpha_2 - \frac{1}{2} \right) \left[R(p_2) - f \right] + 2 \left[\alpha_2(1 - \alpha_2) - \frac{1}{4} \right] \left[R(p_1) - f \right] \geq 0.$$

Since $\alpha_2 < \frac{1}{2}, \alpha_2(1-\alpha_2) < \frac{1}{4}$ and either $(R(p_1) \geq f$ and $R(p_2) > f)$ or $(R(p_1) > f$ and $R(p_2) \geq f)$, the left hand side is strictly negative. Therefore, we have a contradiction. ∎

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