Money, Fame and the Allocation of Talent: Brain Drain and the Institution of Science*

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Abstract

The earning structure in science is known to be flat relative to the one in the private sector, which could cause a brain drain toward the private sector. In this paper, we assume that agents value both money and fame and study the role of the institution of science in the allocation of talent between the science sector and the private sector. Following works on the Sociology of Science, we model the institution of science as a mechanism distributing fame (i.e. peer recognition of priority). We show that since the intrinsic performance is less noisy signal of talent in science than in the private sector, a good institution of science can mitigate the brain drain. We also find that the availability of extra monetary incentives through the market might undermine the incentives provided by the institution and thereby worsen the brain drain. Finally, we study the optimal balance between monetary and non-monetary incentives in science and show the optimality of the flat earning structure in science.

Keywords: Fame, Science, Brain Drain, Incentives, Asymmetric Information

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"The purest treasure mortal times afford is spotless reputation; that away, men are but gilded loam or painted clay." - William Shakespeare in Richard II

1 Introduction

Inducing talented people to become scientists is a national priority for all countries since a nation's economic future is closely linked to its scientific capacity in today's knowledge-based economy. However, the private incentive for a talented agent to choose a scientific career may not be well aligned with the social incentive because she has many other attractive alternatives. For instance, in the U.S., bright young people with college degrees can pursue graduate study in one of the major professional fields such as medicine, law and business. Compared to advanced study in science, these fields promise a much shorter period in school and substantially more lucrative job prospects.[1] This might generate a brain drain from the science sector to the private sector. Currently, both in the U.S. and in Europe, there are concerns about a shortage of scientists and engineers[2].

This paper studies the allocation of talent between the science sector and the private sector in an economy in which each agent makes an occupational choice between becoming a scientist and becoming a professional. We make a departure from the conventional assumption that only monetary payoffs matter and assume that each agent values fame as well. We use a rather narrow definition of fame as the amount of recognition that an agent receives from her peers as a function of her performance and study the allocation of talent by focusing on the difference between the two sectors in terms of the mapping from talent to performance.

A fundamental difference between the two sectors is that agents in the private sector can more or less appropriate their contribution to the society through profits while scientists (in pure science) cannot because of the public good nature of science. This difference in turn generates another important difference in terms of allocation of fame; the market provides an objective measure of each agent's performance (i.e. her profit)

[1] Butz et als. (2003) compare an estimate of annualized earnings for Ph.D.s with earnings of professional degree holders in U.S. such as MDs, DDSs, DVMs, JDs, and MBAs and find that professional degree holders earn more at nearly every age and considerably more over an entire life career.

[2] For instance, New York Times (May 5, 2004) says "The Unites States faces a major shortage of scientists because too few Americans are entering technical fields and because international competition is heating up for bright foreigners who once filled the gap" referring to the report of National Science Board (2004). Concerning Europe, see the recent report of the European Commission (2003).

and accordingly distributes fame while, the science sector, in order to have an objective measure of each scientist's performance, needs an institution which certifies the scientific contribution of each work. According to the sociologists of science such as Merton (1957, 1973), science is a social institution which defines originality as a supreme value and allocates fame and recognition according to priority so that the augmenting of knowledge and the augmenting of personal fame go hand in hand.[3] This incentive role of peer recognition for scientists is also recognized by Paul Samuelson who said

"In the long run, the economic scholar works for the only coin worth having - our own applause" (Merton, 1968, p. 341).

We build a simple model in which each agent has private information about her level of talent and her intrinsic preference between the two occupations (scientist and professional) and the government builds a public science sector. An agent can be either talented or not while her occupational preference has a support wide enough that there is a positive fraction of both talented and not-talented agents in each sector in equilibrium. We focus on the refereeing and publication process of the institution of science and define the quality of the institution as the quality of the mapping from intrinsic outcomes of scientific work to perceived outcomes. The perceived outcome of each scientist is observed by the government and her peers: the former provides monetary rewards and the latter provides non-monetary rewards (i.e. peer recognition) depending on the perceived outcome. In contrast, in the case of professionals, we do not make any distinction between intrinsic and perceived outcomes since we assume that each professional's profit is observable.

We investigate three related issues in this setting. First, we study the brain drain generated by lower monetary returns to talent in science and how it is affected by peer recognition and the quality of the institution of science. Second, we study how the availability of additional monetary incentives through the market (for instance, from licensing patents) affects the brain drain. Last, we consider a more general framework in which the government uses two instruments, salaries and research grants, without any constraint in order to investigate whether a relatively flat earning structure in science can arise as an optimal feature compatible with the absence of brain drain.

[3] According to Merton (1957), the institution of science has developed a priority-based system for allocating (honorific) rewards. Heading the list of recognition is eponymy, the practice of affixing the names of the scientist to all or part of what she has found, as with the Copernican system, Hooke's law etc. Other rewards include prizes, medals, memberships in honorary academies. Last, publication and citation constitute rewards available to most scientists.

In the absence of fame, a brain drain toward the private sector arises in our basic model because we assume that the monetary reward to talent is higher in the private sector than in the science sector. This assumption is true in (Continental) Europe in which most institutions of higher education follow a system based on seniority and performance has virtually no impact on salary.[4] It also holds in U.S. since the profile of earnings in science is known to be rather flat[5] while the returns to talent in the private sector are large.[6] We could find only weak evidence of the brain drain that the number of US citizens with very high GRE-score (>750) headed for science and engineering graduate studies declined by more than 8 % between 1992 and 2000 (Zumeta and Ravelingals, 2002)[7]. However, predictions of a shortage of scientists both in Europe and U.S. on the one hand and increasing rewards to talent in the private sector[8] on the other hand well justify our concerns about the brain drain.

Central in our model is the assumption that the intrinsic outcome of a scientist is a less noisy signal of talent than that of a profession in the private sector. This makes peer recognition have a potential role in attracting talent to science. We have three main justifications for this assumption. First, research is individual work while business is team work: the average number of authors per research paper is four (Adams et als. 2004) while production and marketing processes of a firm involve a much larger number of people. Second, originality has a supreme value in science while, in other professions without much team work such as lawyers and medical doctors, tasks are often routine and repetitive: a path-breaking discovery is a clear sign of genius while one does not need to be a genius in order to perform routine tasks well. Last, openness (i.e. making one's discovery public) is the norm in science because of priority recognition while secrecy

[4] See for instance Aghion and Cohen (2004) and the Wall Street Journal Europe (September 3, 2004).

[5] The average full professor earns only about 38 to 109 percent more than the average new assistant professor depending on the discipline (Ehrenberg, 1992). Even the best-paid professor in the fifty leading universities seldom receives three times as much salary as the worst-paid professor (Stigler, 1988).

[6] Although Butz et als. (2003) show that professionals make more money than Ph.D.s, there is no empirical work comparing the monetary rewards to talent in both sectors. However, top money managers, for instance, can earn more than $250 million a year (NY Times, "Doesn't Anyone Want to Manage Harvard's Money?", August 5, 2005) and it is needless to say that no professor's salary can be that high.

[7] They also find that among US citizens and long-term residents, the share of the science and engineering majors from leading colleges or universities planning immediate advanced study in a science or engineering discipline fell from 17% in 1984 to 12% in 1998.

[8] See the literature on superstars (Rosen, 1981), complementarity and positive sorting (Kremer, 1993), skill-biased technological changes (Caselli, 1999) and the finance literature on CEO compensation (Murphy, 1999).

is the norm in the private sector because of profit seeking, which makes filtering out of noise in performance more difficult in the private sector. As a consequence of the assumption, the expected non-pecuniary reward to talent in terms of peer recognition is higher in science than in the private sector when the institution of science is perfect.

As a benchmark, we study the first-best allocation of talent when the government can observe the level of talent and make the wage of each scientist depend on her level of talent. It is widely believed that real innovation in science depends less on the many "worker bees" than on the presence of a small number of great minds. This, together with the huge positive externality of a great scientific discovery on society, would make talent more productive in science than in the private sector. Then, in the first-best outcome, the fraction of scientists is higher among talented agents than among not-talented agents.

Under incomplete information about talent, the government can make the wage of a scientist depend only on her perceived outcome. We assume an upper bound on the wage differential within the science sector that makes the monetary reward to talent lower in science than in the private sector. In the absence of utility from fame, this leads to a brain drain toward the private sector. However, when agents derive utility from fame, a good institution of science can mitigate the brain drain (and may even achieve the first-best allocation) by providing a non-monetary reward to talent higher than in the private sector while a bad institution of science exacerbates it. We also find that progressive profit taxes help to mitigate the brain drain.[9]

In Section 4, we study how making extra monetary incentives through the market available to scientists affects the brain drain. For instance, measures such as Bayh-Dole Act (1980) in the U.S. enable universities to claim ownership of the intellectual property rights generated from federally funded research and provide scientists with opportunities to earn money and most OECD countries emulated the American experience. However, Florida (1999) argues that high involvement of universities in commercialization can restrict universities' ability to attract and produce top talent. In our analysis, we depart from a simple linear relationship between basic and applied science and introduce what we call the Pasteur's Quadrant (PQ)[10] coefficient to capture the degree to which basic

[9] The question of how progressiveness of tax rates affects the allocation of talent has not been studied in the optimal taxation literature (Mirrlees 1971, Saez 2002, etc). Furthermore, most papers in the literature find the negative effect of progressive taxes in terms of incentive to work.

[10] Pasteur's Quadrant is the title of the book written by Stokes (1997) who mainly argues against the standard distinction between basic and applied science as two distinct categories by pointing out that Patseur made pioneering discovery although he was motivated to find solutions to practical problems.

research can generate patentable scientific knowledge. We find that licensing opportunity reduces the brain drain when the PQ coefficient is high while it can worsen it when the coefficient is low. We also identify a trade-off between the direct effect on the reward to talent from extra income generated by licensing and the indirect effect from the change in the mapping from talent to intrinsic outcome which occurs when licensing opportunity causes a shift from basic to applied research.

In Section 5, we study the optimal balance between monetary and non-monetary incentives in science in a general setting in which the government uses two instruments, salaries and research grants. We assume away any restriction on the instrument and show that a relatively flat earning structure in science is optimal when the institution of science is good and scientists highly value priority recognition and that this is compatible with no brain drain. In our setting, the government observes an individual signal correlated with each scientist's talent and awards research grants as a function of the signals. The characterization of the optimal research grants is done in terms of what we call *the benefit-adjusted social marginal cost of providing grants*, which decreases with the quality of the institution of science. This implies that as the quality of the institution increases, one should increase the relative weight of the non-monetary incentive over the monetary one.

Although there are papers on economics of science which refer to the sociology of science (Dasgupta and Paul, 1987, 1994 and Stephan 1996), they have not built any formal model to address the allocation of talent between the private sector and the science sector. Furthermore, the existing literature on the brain drain under asymmetric information initiated by Kwok and Leland (1982) studies only the migration from one country to another but does not study the brain drain from the science sector to the private one in a closed economy.

In terms of modeling incentives from non-monetary rewards, our paper is related to Benabou and Tirole (2003) and Besley and Ghatak (2005). The former builds a signaling model in which reputation from social groups provides incentives to engage in pro-social behavior such as blood donation. The latter studies the incentive issues in mission-oriented organizations such as schools and find a potential benefit of the market in inducing a good match among the principals and the agents with different mission preferences. Both papers focus on how non-monetary rewards can help to solve moral hazard while we focus on how non-monetary rewards can help to screen agents with

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Rosenberg (2004) also argues in a similar spirit that causation between science and technology runs both ways.

different levels of talent.

With respect to the principal-agent theory, our paper is related to the literature on non-responsiveness (Guesnerie and Laffont, 1984), which focuses on a strong conflict between the allocation preferred by the principal and the allocations implementable under incentive constraints. In our paper, the conflict arises since the principal (the government) wants the fraction of scientists among talented agents to be larger than the one among not-talented agents while the incentive constraints may constrain the principal to implement only those allocations in which the latter is larger than the former. Our problem is symmetric to the one analyzed by Jeon and Laffont (1998) who study the optimal mechanism for downsizing the public sector when workers have private information on their productivity although they consider neither science nor fame.

Regarding papers on the allocation of talent (Acemoglu and Verdier 1998, 2000, Murphy, Shleifer and Vishny 1991, Grossman and Maggi 2000, and Grossman 2004), none of them model fame or study allocation of talent between the science sector and the private sector.

The paper is organized as follows. Section 2 describes the basic model. Section 3 analyzes the model and focuses on the brain drain. Section 4 analyzes how the availability of extra monetary incentives through the market affects the brain drain. Section 5 analyzes the optimal balance between monetary and non-monetary incentives in science. Section 6 discusses our results and Section 7 concludes. All the proofs are in Appendix.

2 The basic model

2.1 Adverse selection and outcomes

There is a mass one of risk-neutral agents in the economy. Let I be the set of all the agents. Each agent should make an occupational choice between becoming a professional in the private sector and becoming a scientist. Although a lot of scientific research is carried out by the private sector in reality, in our model, "becoming a professional" is equivalent to "going to the private sector". Agent i has private information about her level of talent (or intelligence), denoted by θ_i, and her intrinsic preference between the two professions, denoted by γ_i. For simplicity, θ_i can take two values: $\theta_i \in \Theta \equiv \{T, N\}$; $\theta_i = T$ is called a talented type and $\theta_i = N$ is called a not–talented type. θ_i is identically and independently distributed. Since we focus on the choice between professional and scientist, we do not lose much generality by considering a uni-dimensional talent space.[11] Let $\nu \in (0,1)$ denote the probability that $\theta_i = T$; hence $1-\nu = \Pr\{\theta_i = N\}$. Let $I_T \equiv \{i \in I \mid \theta_i = T\}$ and $I_N \equiv \{i \in I \mid \theta_i = N\}$. When we do not refer to a specific agent, we drop the subscript i; for instance, we use θ instead of θ_i.

γ_i represents *the difference* between the intrinsic (non-monetary) pleasure that agent i derives from being professional and the intrinsic pleasure from being scientist such that $\gamma_i < 0$ means that agent i has a relative preference for scientist over professional. For instance, the intrinsic pleasure from becoming scientist can include love of science or satisfaction from solving puzzles (Levin and Stephan, 1991). Since what matters for social welfare is each agent's choice between the two professions and intrinsic pleasure affects agent i's choice only through the relative pleasure γ_i, we normalize, without loss of generality, each agent's absolute pleasure from becoming scientist at zero. For simplicity, we assume that γ_i is identically and independently distributed over i according to a uniform distribution with support $[-\gamma, \gamma]$ and that there is no correlation between θ_i and γ_i. We discuss the case of correlation in section 6.

Let $O_i \in \{R, S\}$ represent agent i's occupational choice: $O_i = R$ ($O_i = S$) when she becomes professional (scientist). We assume for simplicity that the outcome that an agent realizes after choosing an occupation has a binary support: it can be high or low. More precisely, a type θ scientist realizes a high outcome (i.e. a path-breaking discovery) with probability p_θ^S and a low outcome (i.e. an ordinary discovery) with probability $1-p_\theta^S$. We focus on pure scientific research which does not produce any direct monetary gain to the scientist but increases the productive potential of the future economy. We assume that the social monetary value of a path-breaking discovery is $s^H > 0$ and that of an ordinary discovery is $s^L \in (0, s^H)$. A type θ professional produces a high profit (before tax) $\pi^H > 0$ with probability p_θ^R and a low profit $\pi^L \in (0, \pi^H)$ with probability $1 - p_\theta^R$. Obviously $\Delta p^O \equiv p_T^O - p_N^O > 0$ for $O \in \{R, S\}$. Let $\Pi_\theta \equiv p_\theta^R \pi^H + (1-p_\theta^R)\pi^L$ and $S_\theta \equiv p_\theta^S s^H + (1-p_\theta^S)s^L$. The profit realized by an professional is verifiable such that her tax payment depends on it.

2.2 Institution of science and fame

There are many factors affecting the quality of the institution of science. In this paper, we take a narrow angle and focus on refereeing and publication process. We define the

[11]By contrast, if we study a choice between entrepreneur and researcher, we need to consider a multi-dimensional type space since to be a good entrepreneur, one needs mutiple skills (Lazear, 2002).

quality of the institution of science as the quality of the mapping from the intrinsic outcomes of scientists to the perceived outcomes. The intrinsic outcome refers to the original value of a scientific work and the perceived outcome refers to the certification label that the work receives through refereeing and publication process. The intrinsic outcome can be high or low as was defined in section 2.1. We assume that the perceived outcome is either high or low as well. Let $q_r \in \left[\frac{1}{2}, 1\right]$ denote the probability that a high intrinsic outcome is perceived as high, which is assumed to be equal to the probability that a low intrinsic outcome is perceived as low for simplicity. Therefore, q_r[12] is a measure of the quality of the institution of science.

In our definition of fame, we focus on peer recognition and define an individual's fame as the total recognition she gets from her peers. The amount of recognition that agent i receives is assumed to increase with the level of her outcome perceived by the peers and with the number of the peers. In order to focus on the information structure mapping talent to perceived outcome in each sector, we assume that the measure of the peers is the same in each profession.[13] Let η denote the measure of peers. For simplicity, we assume that if agent i's perceived outcome is low, she gets zero recognition while if it is high, she gets a unit of recognition from each peer such that the total amount of peer recognition is η.[14] Therefore, the expected fame of a type θ professional is $p_\theta^R \eta$ and that of a type θ scientist is $\left[p_\theta^S q_r + (1 - p_\theta^S)(1 - q_r)\right] \eta$.

We make the following assumption:

Assumption 1: The intrinsic outcome is a less noisy signal of talent in science than in the private sector (i.e. $\Delta p^S > \Delta p^R$).

We gave in the introduction three reasons for why assumption 1 is likely to hold. This assumption implies that when the quality of the institution of science is perfect (i.e. $q_r = 1$), the difference between a talented agent's expected fame and that of a not-talented agent is larger in the science sector than in the private sector; in other words, the non-pecuniary reward to talent in terms of fame is higher in the former than in the latter.

Agent i's payoff U_i is given as follows:

$$U_i = m_i + \alpha f_i + \gamma_i \mathbf{1}_{[O_i = R]}$$

[12] q_r means quality of refereeing.

[13] Our results are not affected even though we assume that the measure of peers depends on the profession as long as we do not endogenize it, which is beyond the scope of this paper.

[14] The quality of the institution of science can affect the amount of recognition that one obtains from a high perceived outcome. Including this aspect into our model does not affect our results qualitatively.

where m_i is her monetary income, $\alpha(\geq 0)$ is the weight parameter for fame and f_i is her fame.

2.3 Government

In our model, the main role of the government consists in building a (public) science sector to produce knowledge. The government can pay wages to induce agents to become scientists and can make a scientist's wage contingent on her perceived outcome. Let w be the basic salary that every scientist receives. Let $b \geq 0$ be the bonus that a scientist receives if her perceived outcome is high. This bonus can be interpreted as the increase in salary following a promotion.[15] We assume that there is an upper bound on b, denoted by $\bar{b} > 0$.

The government levies taxes on the profits of professionals.[16] Let t^H (t^L) denote the tax when the profit is high (low) and let $\mathbf{t} \equiv (t^H, t^L)$. Define $T_\theta \equiv p_\theta^R t^H + (1 - p_\theta^R) t^L$: it represents the expected tax payment of a type θ professional. $\tau \equiv \frac{t^H - t^L}{e^H - e^L}$ is a parameter representing the progressiveness of taxes; we assume $1 > \tau > 0$. Given that the profit taxes are levied on many other occupations and we focus on the choice between professional and scientist, we consider $\mathbf{t} \equiv (t^H, t^L)$ given. We make the following assumption:

Assumption 2: The monetary reward to talent is higher in the private sector than in science: $\Delta p^R \left[\left(\pi^H - t^H \right) \; - \left(\pi^L - t^L \right) \right] > \Delta p^S \bar{b}$.

The inequality says that the difference between a talented professional's expected net profit and that of a not-talented one is higher than the difference between a talented scientist's expected monetary income and that of a not-talented one even when $q_r = 1$. This implies that the monetary reward to talent is larger in the private sector than in the science sector for any value of $q_r \in \left[\frac{1}{2}, 1\right]$. This assumption captures the stylized fact that monetary incentives are lower-powered in academia than in the private sector. We provided the detailed justifications of the assumption in the introduction.

[15] This description is far from the reality at least in some European countries such as Italy. According to Perotti (2002)'s study of the promotion to full professorship in economics in Italy, (i) an outsider needs 13 more refereed publications than an insider in order to compensate for the advantage the latter has; (ii) even in the competition among outsiders, the effect of a publication in a high-quality journal is not statistically different from the effect of a publication in a low-quality journal.

[16] We assume that (b, w) are monetary payments from the government net of taxes.

In the absence of fame, the expected payoff that a type θ agent with intrinsic occupational preference γ_i expects to obtain upon becoming professional is given by $\Pi_\theta - T_\theta + \gamma_i$. The agent chooses to become scientist if the following inequality holds:

$$w + p_\theta^S b \geq \Pi_\theta - T_\theta + \gamma_i.$$

An allocation of talent between the two occupations is characterized by $\boldsymbol{\phi} \equiv (\phi_T, \phi_N) \in [0,1]^2$ where ϕ_T (ϕ_N) denotes the fraction of the talented (not-talented) agents becoming scientists. In the absence of fame ($\alpha = 0$), social welfare, denoted by SW, is given as follows:

$$SW(\phi_T, \phi_N) \equiv \nu(1-\phi_T)\Pi_T + (1-\nu)(1-\phi_N)\Pi_N + \nu\phi_T S_T + (1-\nu)\phi_N S_N + \int_{I_R} \gamma_i di,$$

where $I_R \equiv \{i \mid O_i = R\}$ is the set of professionals. We assume that the government maximizes the above objective regardless of whether $\alpha > 0$ or $\alpha = 0$. In other words, we assume that the government does not care about recognition per se but cares about it since it affects monetary social welfare through the allocation of talent. In reality, it is hard to measure the aggregate level of fame or recognition in an economy and to make the government accountable for it.[17]

In Section 5 where we study the optimal monetary and non-monetary rewards to scientists in terms of salary and research grant, we introduce a positive shadow cost of public funds $\lambda\,(> 0)$, meaning that each dollar spent by the government is raised through distortionary taxes (labor, capital and commodity taxes) and costs society $1 + \lambda$ dollars (Laffont and Tirole, 1993). In all other sections, we assume $\lambda = 0$ for simplicity: our results hold for $\lambda > 0$ as well but considering $\lambda > 0$ makes the exposition lengthy without adding any new insight.

2.4 Timing

We consider a game with the following timing:

1. For each $i \in I$, nature draws θ_i and γ_i and they become agent i's private information.
2. The government announces $\{w, b\}$.
3. Each agent makes her occupational choice.
4. Each agent's outcome is realized.
5. Each professional pays tax and each scientist receives wage (and bonus).

[17] Furthermore, what people care about is often relative recognition rather than absolute recognition and when we aggregate relative recognition, its sum is zero by definition.

3 Allocation of talent and brain drain

3.1 Benchmarks without fame

We first study two benchmarks for an economy without fame ($\alpha = 0$). In the first benchmark, we derive the first-best allocation of talent when the government has complete information about each agent's talent. In the second benchmark, we study the second-best allocation in a more realistic setting in which each agent has private information about her talent and there is an upper bound on b.

3.1.1 First-best: complete information on talent

Suppose that the government has complete information on θ_i, although γ_i is agent i's private information. Suppose also that it is possible to make a scientist's wage depend on her talent. Let w_N and w_T denote wages for talented and not-talented scientists, respectively. Given w_T, agent i with $\theta_i = T$ becomes scientist if and only if $\Pi_T - T_T + \gamma_i < w_T$. Hence, if $w_T - (\Pi_T - T_T) \in [-\gamma, \gamma]$, the agent with $\gamma_i = w_T - (\Pi_T - T_T)$ becomes the cut-off type among talented agents in that all talented agents with γ_i lower than $w_T - (\Pi_T - T_T)$ become scientists and the fraction of scientists among talented agents is $\phi_T = \frac{w_T - (\Pi_T - T_T) + \gamma}{2\gamma}$. Similarly, given w_N, the cut-off type for not-talented agents is given by $\gamma_i = w_N - (\Pi_N - T_N)$ and the fraction of scientists among not-talented agents is $\phi_N = \frac{w_N - (\Pi_N - T_N) + \gamma}{2\gamma}$. Writing w_T (w_N) as a function of ϕ_T (ϕ_N) shows that a given interior allocation of talent $(\phi_T, \phi_N) \in (0,1)^2$ can be achieved by the government if wages are chosen as follows:

$$\Pi_T - T_T + 2\gamma\phi_T - \gamma = w_T \tag{1}$$

$$\Pi_N - T_N + 2\gamma\phi_N - \gamma = w_N. \tag{2}$$

Given an allocation of talent (ϕ_T, ϕ_N), the cut-off types are determined by $\gamma_i = \gamma(2\phi_\theta - 1)$ for $\theta \in \{T, N\}$ and we can compute the sum of the agents' intrinsic pleasure from their occupations as follows:

$$\int_{I_R} \gamma_i di = \nu \int_{\gamma(2\phi_T - 1)}^{\gamma} \frac{z}{2\gamma} dz + (1-\nu) \int_{\gamma(2\phi_N - 1)}^{\gamma} \frac{z}{2\gamma} dz = \gamma \left[\nu\phi_T (1 - \phi_T) + (1-\nu)\phi_N (1 - \phi_N)\right].$$

Hence, social welfare is given as follows:

$$\begin{aligned} SW(\phi_T, \phi_N) \equiv\ & \nu(1 - \phi_T)\Pi_T + (1-\nu)(1-\phi_N)\Pi_N + \nu\phi_T S_T + (1-\nu)\phi_N S_N \\ & + \gamma\left[\nu\phi_T (1-\phi_T) + (1-\nu)\phi_N (1-\phi_N)\right]. \end{aligned} \tag{3}$$

The government maximizes SW with respect to (ϕ_T, ϕ_N) in $[0,1]^2$. The first order conditions (for an interior allocation) are given as follows:[18]

$$\Pi_T + \gamma(2\phi_T - 1) = S_T \tag{4}$$
$$\Pi_N + \gamma(2\phi_N - 1) = S_N \tag{5}$$

From the first order conditions, we find that for each $\theta \in \{T, N\}$, the social marginal value that the cut-off type produces as a professional is equal to the one she produces as a scientist where the social marginal values take into account the intrinsic preferences for occupations. Next proposition characterizes (ϕ_T^*, ϕ_N^*), the first-best allocation of talent. Notice that the same allocation would arise if the government could observe (θ_i, γ_i) for every i and dictate each agent's occupational choice.

Proposition 1 *(The first-best allocation of talent) Suppose that the government has complete information on θ_i such that it can make each agent's wage depend on her level of talent.*
(i) The first-best allocation of talent is given by:

$$\phi_T^* = \frac{\gamma - \Pi_T + S_T}{2\gamma}, \qquad \phi_N^* = \frac{\gamma - \Pi_N + S_N}{2\gamma}. \tag{6}$$

The optimal wages are obtained from (1) and (2).
(ii) In (ϕ_T^*, ϕ_N^*), the fraction of scientists is larger among talented agents than among not-talented agents if and only if talent is more productive in the science sector than in the private sector: $\phi_T^* > \phi_N^*$ if and only if $S_T - S_N > \Pi_T - \Pi_N$.

In the rest of the paper we make the following assumption, which implies $\phi_T^* > \phi_N^*$:

Assumption 3: Talent is more productive in the science sector than in the private sector: $S_T - S_N > \Pi_T - \Pi_N$.

We have $S_T - S_N = \Delta p^S(s^H - s^L)$. It is widely believed that real innovation in science depends less on the many "worker bees" than on the presence of a small number of great minds (i.e. Δp^S high). This fact, together with the huge positive externality of a great scientific discovery on society (i.e. $s^H - s^L$ high) makes assumption 3 quite plausible.

[18] Throughout the paper we assume that the optimal allocations are interior; in the proofs in the appendix we describe the conditions under which this is the case. Allowing for corner allocations is straightforward but complicates the exposition without yielding any additional insight.

Note that the first-best allocation of talent is given by (6) even when agents derive some utility from fame since social welfare depends only on the allocation of talent and not on fame regardless of the value of α.

3.1.2 Incomplete information outcome without fame

In this subsection we study the government's optimal choice of (w, b) under the assumption that agent i has private information on θ_i and γ_i, but we still keep $\alpha = 0$. We focus on how the incomplete information together with assumption 2 restricts the set of implementable allocations of talent. In order to achieve an interior allocation of talent $(\phi_T, \phi_N) \in (0,1)^2$, it is necessary that (w, b) satisfy the following incentive constraints:

$$
\begin{aligned}
(IC_T) \quad \Pi_T - T_T + 2\gamma\phi_T - \gamma &= \beta_T b + w; \qquad (7)\\
(IC_N) \quad \Pi_N - T_N + 2\gamma\phi_N - \gamma &= \beta_N b + w : \qquad (8)
\end{aligned}
$$

where $\beta_\theta \equiv p_\theta^S q_r + (1 - p_\theta^S)(1 - q_r)$ represents the probability that a type-θ scientist will have a high perceived outcome. If (IC_θ) holds, all type-θ agents with intrinsic occupational preference higher (lower) than $2\gamma\phi_\theta - \gamma$ become professionals (scientists) since the type with preference $2\gamma\phi_\theta - \gamma$ is indifferent between the two occupations. Then, the fraction of type-θ agents becoming scientists is just ϕ_θ.

As long as $q_r \in (\frac{1}{2}, 1]$, it is possible to solve (7)-(8) with respect to (w, b) because $q_r > \frac{1}{2}$ implies $\beta_T > \beta_N$. The solution is given by:

$$
w = \frac{\beta_T A_N - \beta_N A_T}{\beta_T - \beta_N}, \qquad b = \frac{A_T - A_N}{\beta_T - \beta_N}, \tag{9}
$$

where A_θ is the left hand side in (IC_θ). Therefore, for any given allocation (ϕ_T, ϕ_N), when $q_r > \frac{1}{2}$ we can find a pair (w, b) which implements (ϕ_T, ϕ_N) if we neglect the constraint that b must belong to $\left[0, \overline{b}\right]$.

After performing a simple manipulation and using $\beta_T - \beta_N = \Delta p^S(2q_r - 1)$, we find that b in (9) satisfies $b \leq \overline{b}$ if and only if

$$
\phi_N - \phi_T \geq \frac{(\Pi_T - T_T) - (\Pi_N - T_N) - \Delta p^S(2q_r - 1)\overline{b}}{2\gamma}. \tag{10}
$$

Recall that $\Delta p^S(2q_r - 1)\overline{b}$ is the maximal monetary reward to talent in the science sector given q_r. As q_r (or $\overline{b}$) increases, the maximal monetary reward to talent in the science sector becomes larger and it is possible to induce a larger fraction of talented agents to become scientist. However, under assumption 2, the first best allocation (ϕ_T^*, ϕ_N^*) cannot be implemented for any $q_r \in [\frac{1}{2}, 1]$; since the monetary reward to talent in the private sector $(\Pi_T - T_T - (\Pi_N - T_N))$ is larger than $\Delta p^S(2q_r - 1)\bar{b}$ under the assumption and $\phi_T^* > \phi_N^*$ holds, (10) is violated for any q_r at $(\phi_T, \phi_N) = (\phi_T^*, \phi_N^*)$.

Therefore, we find the second-best allocation of talent in the absence of fame, denoted by $(\phi_T^{**}, \phi_N^{**})$, by solving the following program[19]

$$\max_{(\phi_T, \phi_N) \in [0,1]^2} SW \text{ subject to (10)} \tag{11}$$

Next proposition characterizes it:

Proposition 2 *(The second-best in the absence of fame) Suppose that (θ_i, γ_i) is agent i's private information and $\alpha = 0$. Under assumptions 2 and 3,*
(i) The second-best allocation of talent $(\phi_T^{**}, \phi_N^{**})$ is given by:

$$\begin{array}{c} \phi_T^{**} = \phi_T^* - \frac{\mu^{**}}{2\nu\gamma}; \\ \phi_N^{**} = \phi_N^* + \frac{\mu^{**}}{2(1-\nu)\gamma} : \end{array} \tag{12}$$

where $\mu^{**}(> 0)$ is the multiplier associated with the constraint (10) and is given by

$$\mu^{**} = 2\nu(1-\nu)\gamma(\frac{B}{2\gamma} + \phi_T^* - \phi_N^*), \tag{13}$$

where $B \equiv (\Pi_T - T_T) - (\Pi_N - T_N) - \Delta p^S(2q_r - 1)\bar{b} > 0$.
(ii) The second-best allocation of talent $(\phi_T^{**}, \phi_N^{**})$ is such that
a. There is a brain drain from the science sector to the private sector ($\phi_T^* > \phi_T^{**}$)
b. The fraction of not-talented agents becoming scientists is larger than that of talented agents: $\phi_N^{**} > \phi_T^{**}$
c. The total number of scientists is the same as in the first-best: $\nu\phi_T^* + (1-\nu)\phi_N^* = \nu\phi_T^{**} + (1-\nu)\phi_N^{**}$.

Proposition 2 establishes that there is a brain drain from the science sector to the private sector in that the number of talented scientists is smaller in the second best than in the first-best outcome: $\phi_T^{**} < \phi_T^*$. It also establishes that a larger number of not-talented agents are now in the science sector: $\phi_N^{**} > \phi_N^*$. Figure 1 describes the first-best and the second-best allocations of talent in the absence of fame. As we have mentioned above, the brain drain is generated by assumption 2, according to which the cap on

[19] Since in the first best the inequality $b \leq \bar{b}$ is violated, we will find $b = \bar{b}$ in the second best; hence $b \geq 0$ is satisfied.

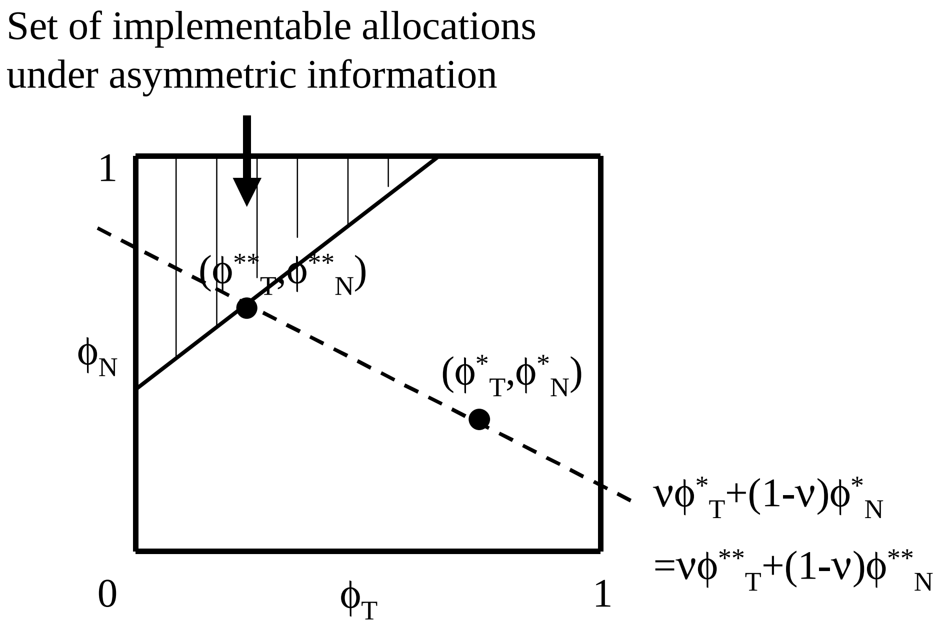

Figure 1: The first-best and the second-best allocations of talent in the absence of fame

the bonus in the science sector $\bar{b}$ makes the monetary reward to talent in the science sector smaller than the one in the private sector for any q_r. This makes talented agents have larger incentives to become professionals than not-talented agents such that the fraction of scientists is larger among not-talented agents than among talented agents (i.e. $\phi_N^{**} > \phi_T^{**}$). As $\bar{b}$ increases, there is less distortion in the allocation of talent from (12) and (13) and social welfare increases. However, because of the specific linear-quadratic structure of (11), the total mass of scientists in the second best $\nu\phi_T^{**} + (1-\nu)\phi_N^{**}$ is equal to the first-best level $\nu\phi_T^{*} + (1-\nu)\phi_N^{*}$.

3.2 Fame and allocation of talent

In this section agent i still has private information on θ_i and γ_i but derives utility from fame ($\alpha > 0$) and we study how fame affects the allocation of talent. Now the incentive constraints need to take into account the utility from recognition and in order to achieve an interior allocation of talent $(\phi_T, \phi_N) \in (0,1)^2$, it is necessary that the following incentive constraints are satisfied:

$$(IC_T) \quad \Pi_T - T_T + 2\gamma\phi_T - \gamma + \alpha p_T^R \eta = \beta_T(b + \alpha\eta) + w; \tag{14}$$

$$(IC_N) \quad \Pi_N - T_N + 2\gamma\phi_N - \gamma + \alpha p_N^R \eta = \beta_N(b + \alpha\eta) + w. \tag{15}$$

The non-pecuniary reward to talent in the private sector is equal to $\alpha\eta\Delta p^R$, while the non-pecuniary reward to talent in science is $\alpha\eta(\beta_T - \beta_N) = \alpha\eta\Delta p^S(2q_r - 1)$. From assumption 1, when $q_r = 1$, the latter is larger than the former. In contrast, when $q_r = \frac{1}{2}$, the latter is zero and thus smaller than the former. Therefore, there exists a threshold $\overline{q}_r \in \left(\frac{1}{2}, 1\right)$ such that the non-pecuniary reward to talent is larger in the science sector than in the private sector if and only if the quality of the institution of science is higher than $\overline{q}_r$.

For any given (ϕ_T, ϕ_N), we can find a pair (w, b) which satisfies (14) and (15) if $q_r > \frac{1}{2}$. However, this pair may violate $b \leq \overline{b}$ and this constraint imposes a restriction on the set of implementable allocations as follows:

$$\phi_N \geq \phi_T + \frac{(\Pi_T - T_T) - (\Pi_N - T_N) - \Delta p^S(2q_r - 1)\overline{b} + \alpha\eta\left[\Delta p^R - \Delta p^S(2q_r - 1)\right]}{2\gamma} \quad (16)$$

which is an analog of the constraint (10). Our previous discussion implies that the presence of fame (i.e. $\alpha > 0$) relaxes (16) if and only if the quality of the institution of science is higher than $\overline{q}_r$. In particular, if $q_r > \overline{q}_r$ and α is large such that the non-pecuniary reward to talent in the science sector is much larger than the one in the private sector, the first-best allocation can be achieved: (16) is satisfied at $(\phi_T, \phi_N) = (\phi_T^*, \phi_N^*)$.

When the first-best allocation cannot be achieved, we solve the following program:

$$\max_{(\phi_T, \phi_N) \in [0,1]^2} SW \text{ subject to (16).} \quad (17)$$

Let $(\phi_T^{**}(\alpha), \phi_N^{**}(\alpha))$ denote the solution of the above program; it represents the optimal allocation in the presence of fame if (ϕ_T^*, ϕ_N^*) violates (16). We have:

Proposition 3 *(The effects of fame) Suppose that (θ_i, γ_i) is agent i's private information and $\alpha > 0$. Under assumptions 1 to 3, there exists a threshold $\overline{q}_r \in (\frac{1}{2}, 1)$ such that:*

(i) The first-best allocation of talent can be achieved if the institution of science is good enough ($q_r > \overline{q}_r$) and the weight on fame α is large enough.

(ii) Otherwise, the optimal allocation of talent $(\phi_T^{**}(\alpha), \phi_N^{**}(\alpha))$ is such that:

a. There is a brain drain ($\phi_T^* > \phi_T^{**}(\alpha)$) and the total number of scientists is the same as in the first-best: $\nu\phi_T^{**}(\alpha) + (1-\nu)\phi_N^{**}(\alpha) = \nu\phi_T^* + (1-\nu)\phi_N^*$.

b. (comparative statics on the brain drain)

- As the quality of the institution of science increases, the brain drain decreases: $\frac{\partial(\phi_T^* - \phi_T^{**}(\alpha))}{\partial q_r} < 0$.

- As the weight on fame α increases, there is less (more) brain drain if the quality of the institution of science is higher (lower) than $\overline{q}_r$: $\frac{\partial\left(\phi_T^* - \phi_T^{**}(\alpha)\right)}{\partial \alpha} \gtreqless 0$ *if* $q_r \lesseqgtr \overline{q}_r$.

- The brain drain decreases as the profit taxes become more progressive: $\frac{\partial(\phi_T^* - \phi_T^{**}(\alpha))}{\partial \tau} < 0$.

A good institution of science improves the allocation of talent and mitigates the brain drain by increasing both the monetary and non-monetary reward to talent in the science sector: in particular, if the agents put sufficient weight α on fame, a good institution of science can allow the government to achieve the first-best allocation. If the first-best cannot be achieved, there is a brain drain and how α affects the brain drain depends on the quality of the institution of science. In particular, if the quality of the institution of science is bad such that the non-pecuniary reward to talent in terms of fame is larger in the private sector than in the science sector, an increase in α makes choosing professional even more attractive to talented agents and therefore aggravates the brain drain. In other words, the existence of fame can reduce the brain drain only if the quality of the institution is above a certain level. Last, as the profit taxes become more progressive, the monetary reward to talent in the private sector is reduced and therefore, ceteris paribus, the brain drain is mitigated.

Our results suggest that if, in the past, the western countries succeeded in inducing talented people to become scientists without giving large monetary returns to talent, it is mainly because they built a good institution of science to generate large non-pecuniary returns to talent in the science sector. The results also suggest that highly progressive income taxes in Europe have a secondary effect of mitigating the brain drain due to low-powered monetary incentives in academia while, in the U.S., relatively low progressiveness of the income taxes requires relatively high-powered monetary incentives in academia in order to mitigate the brain drain.

4 Extra monetary rewards through the market and the allocation of talent

Salary and bonus are not the only sources of income for scientists since they can generate revenue from consulting fees, patents, prizes etc. Furthermore, specific measures such as the Bayh-Dole Act (1980) in the U.S. enable universities to claim ownership of the intellectual property rights generated from federally funded research and provide scientists in academia with incentives to commercialize innovations. In this section, we make an extension of the previous model to examine how the availability of extra monetary rewards through the market (in particular, those from licensing patents) affects the brain drain. More precisely, we study the conditions under which it enlarges or reduces the set of implementable allocations of talent.

One of the main concerns regarding the Bayh-Dole Act is that it can divert scientists' research from basic science to applied one (Cohen et als. 1998, Florida, 1999, National Science Board, 2004, Thursby and Thursby 2003).[20] We focus on this aspect and consider a simple moral hazard problem; each scientist decides whether or not to divert some effort from basic to applied research. However, we depart from a simple linear relationship between basic and applied science and introduce what we call the Pasteur's Quadrant (PQ) coefficient, denoted by y_b[21], to capture the fact that basic research can to some extent generate patentable scientific knowledge. Therefore, even though a scientist does not divert her effort, she can make extra money from the licensing opportunity. More precisely, if a type-θ scientist does not divert her effort, her probability of making a path-breaking discovery is p_θ^S and generates a social benefit of $p_\theta^S y_b$ (in expected terms) from licensing in addition to $s^H (> 0)$. If there is diversion, her probability of making a path-breaking discovery decreases by Δ_θ (with $p_\theta^S > \Delta_\theta > 0$ and $p_T^S - \Delta_T > p_N^S - \Delta_N$) and the social benefit from licensing is equal to $(p_\theta^S - \Delta_\theta) y_b + \Delta_\theta y_a$ with $y_a \geq 0$ where the subscript a means applied science. Note that we assume that the market is efficient in that even though a path-breaking discovery is recognized as a low outcome, it generates y_b. This makes sense since even though an important discovery is not published in a top journal, it can obtain a patent. We assume that a scientist captures a share $\delta \in (0, 1]$ of the social value generated from licensing and that the government cannot make a scientist's salary depend on whether or not she diverts effort as is the case in reality.

In this setting, given (w, b) chosen by the government, the payoff of a type-θ scientist is $w + \beta_\theta (b + \alpha\eta) + \delta p_\theta^S y_b$ if she fully dedicates herself to basic research and $\delta \Delta_\theta y_a + w + (\beta_\theta - (2q_r - 1)\Delta_\theta)(b + \alpha\eta) + \delta (p_\theta^S - \Delta_\theta) y_b$ otherwise. Therefore, she does not engages in applied research regardless of her type if and only if the PQ coefficient is larger than the threshold $\overline{y}_b(b)$ given by:

$$\overline{y}_b(b) \equiv \frac{\delta y_a - (2q_r - 1)(b + \alpha\eta)}{\delta}. \tag{18}$$

[20] However, the empirical evidence is mixed. For instances, Cohen et als (1998) provide evidence of countervailing effects of industry collaboration on faculty productivity in terms of publications while Thursby and Thursby (2003) find that licensing did not affect the portion of faculty's research that is published in basic journals.

[21] The subscript b means basic science.

When $y_b \geq \overline{y}_b(b)$, basic research itself produces patentable knowledge and therefore scientists do not need to engage in applied research.

In what follows, we suppose that both before and after making licensing opportunity available, the first-best allocation of talent[22] cannot be implemented because of the constraint $b \leq \overline{b}$ (and therefore it binds). Next proposition describes how the availability of licensing opportunity affects the set of implementable allocations of talent.

Proposition 4 *Suppose that the government provides scientists with the opportunity to patent and license their research.*

(i) If the Pasteur's Quadrant (PQ) coefficient y_b is larger than the threshold $\overline{y}_b$, providing the opportunity does not affect scientists' research pattern and reduces the brain drain where

$$\overline{y}_b \equiv \frac{\delta y_a - (2q_r - 1)\left(\overline{b} + \alpha\eta\right)}{\delta}.$$

(ii) If the Pasteur's Quadrant (PQ) coefficient y_b is smaller than the threshold $\overline{y}_b$, providing the opportunity induces scientists to divert part of their attention from basic to applied science. Furthermore;

(a) If talented scientists divert more than not-talented scientists ($\Delta_T \geq \Delta_N$), providing the opportunity reduces the brain drain.

(b) If talented scientists divert less than not-talented scientists ($\Delta_T < \Delta_N$), there is a threshold $\underline{y}_b (< \overline{y}_b)$ such that providing the opportunity reduces (worsens) the brain drain if $y_b \geq \underline{y}_b$ ($y_b \geq \underline{y}_b$) where

$$\underline{y}_b \equiv \overline{y}_b \frac{\Delta_N - \Delta_T}{\Delta p^S + \Delta_N - \Delta_T}.$$

The above proposition first reveals the importance of the Pasteur's Quadrant (PQ) coefficient in determining the impact of the licensing opportunity on the research pattern and the brain drain. If the coefficient is high enough or if the quality of the institution is good enough and the monetary and non-monetary incentives that it provides ($b+\alpha\eta$) are large enough, we expect that providing licensing opportunity has no impact on research pattern since the opportunity cost of diverting attention to applied research increases as q_r (or $b + \alpha\eta$) increases. In this case, the brain drain is reduced since a talented scientist's expected income from licensing is higher than that of a not-talented one by $\Delta p^S y_b$. Even though we do not model different research fields, in reality, the coefficient

[22] It is different from (6) and is defined in the proof of proposition 4.

should depend on the field: for instance, it should be high for life science and engineering and low for physics and astronomy etc.

When the PQ coefficient is lower than the threshold $\overline{y}_b$, scientists divert part of their attention from basic to applied science. In this case, the effect on the brain drain depends both on the PQ coefficient and on which type of scientists divert more. The change in research pattern affects the allocation of talent through three different channels. First, it reduces the licensing income originated from basic research by $\delta\Delta_\theta y_b$ for each type of scientist. Second, it increases the licensing income originated from applied research by $\delta\Delta_\theta y_a$ for each type of scientist. When $\Delta_T > \Delta_N$ ($\Delta_T < \Delta_N$), the first two effects increase (reduce) the reward to talent by $\delta\left|\Delta_T - \Delta_N\right|(y_a - y_b)$ since $y_a > y_b$ when $y_b < \overline{y}_b$ holds. Last, it affects the information structure in science and changes the reward to talent from the institution of science: for instance, if talented scientists divert more than not-talented ones ($\Delta_T > \Delta_N$), this makes the intrinsic outcome of science a noisier signal of talent and thereby reduces the monetary and non-monetary return to talent provided by the institution of science by $(2q_r - 1)(\Delta_T - \Delta_N)(b + \alpha\eta)$. The first two are direct effects and the last is an indirect effect. The total effect of the change in research pattern on the reward to talent is given by

$$(\Delta_T - \Delta_N)\left[\delta(y_a - y_b) - (2q_r - 1)(b + \alpha\eta)\right].$$

Since the term in the bracket is positive when $y_b < \overline{y}_b$, the change in research pattern increases the reward to talent in science if and only if $\Delta_T > \Delta_N$. Since the availability of licensing opportunity increases the reward to talent by $\Delta p^S y_b$ in the absence of the change in research pattern, it reduces the brain drain whenever $\Delta_T \geq \Delta_N$. If $\Delta_T < \Delta_N$ holds, we have to compare the positive effect from the licensing opportunity with the negative effect from the change in research pattern. Since the positive effect increases with y_b while the negative effect decreases with y_b, there is a threshold $\underline{y}_b$ such that the availability of licensing opportunity worsens the brain drain if and only if $y_b \leq \underline{y}_b$.

Therefore, we found that the availability of licensing opportunity worsens the brain drain when the basic research does not generate much patentable scientific knowledge and the availability of licensing opportunity induces not-talented scientists to be specialized in applied research. Even though this specialization makes the mapping from the talent to the outcome a less noisy signal of talent in science and therefore has a potential to reduce the brain drain, its is dominated by the direct effect from the licensing revenue which reduces the monetary return to talent in science.

Remark 1: optimality of providing extra monetary incentives through the

market

We did not analyze whether providing licensing opportunity is optimal at the first place from a social welfare point of view. If it does not affect research pattern, obviously it is optimal to provide licensing opportunity. If it does affect research pattern, under complete information (hence when there is no brain drain), it is socially optimal to provide the opportunity regardless of type if $\Delta_\theta y_a + \left(p_\theta^S - \Delta_\theta\right) y_b > \Delta_\theta (s^H - s^L)$ for $\theta = T$ or N. Our analysis suggests that even though this condition holds, the extra incentives can reduce the social welfare under incomplete information if it causes a brain drain.

5 Optimal allocation of public funds between research grants and salaries

The analysis of the previous sections identified two kinds of rewards to scientists: monetary and non-monetary rewards. The government usually has two instruments to control them: salaries affect the monetary rewards while research grants affect the non-monetary rewards when a scientist's probability to make a path-breaking discovery increases with the amount of her research grant. In this section, we assume away any constraint on the instruments (in particular, $b \leq \bar{b}$ and assumption 2) and derive the optimal wages and grants and show the optimality of relatively flat wages in science. For this purpose, we enrich the basic model in three respects.

First, after each agent makes her occupational choice, for each scientist i, the government observes a signal σ_i which is positively correlated with θ_i but is not correlated with θ_j for any $j \neq i$. The signal can be either good or bad: $\sigma_i \in \{G, B\}$. For instance, σ_i represents scientist i's performance in the early stages of her career. Let $q_s \in [\frac{1}{2}, 1]$ represent the quality, or precision, of the signal in the following sense:

$$q_s \equiv \Pr\{\sigma_i = G \mid \theta_i = T\} = \Pr\{\sigma_i = B \mid \theta_i = N\}.$$

For simplicity, however, we assume that recognition depends only on the (final) perceived outcome and not on the early signal.

Second, the government allocates research grants to scientist i on the basis of σ_i; let g_G (g_B) represent the research grant given to scientist i when $\sigma_i = G$ (when $\sigma_i = B$). A scientist's probability of making a path-breaking discovery depends both on her talent and on her research grant. More precisely, let $p_\theta^S(g)$ represent the probability for a type-θ

scientist to make a path-breaking discovery when she receives grant g. Assumption 4 below specifies the properties of the functions $p_T^S(g)$ and $p_N^S(g)$.

Last, there is a positive shadow cost of public funds $\lambda > 0$.

We make the following assumption regarding $p_T^S(g)$ and $p_N^S(g)$:

Assumption 4: (i) $p_T^S(0) \geq p_N^S(0)$ and $\frac{dp_T^S}{dg} \geq \frac{dp_N^S}{dg} \geq 0$ for any $g > 0$; $\frac{dp_N^S(0)}{dg} > \frac{1+\lambda}{s^H - s^L}$;
(ii) $0 > \frac{d^2 p_T^S}{dg^2} > \frac{d^2 p_N^S}{dg^2}$ whenever $\frac{dp_N^S}{dg} > 0$;
(iii) $\frac{d^3 p_\theta^S}{dg^3} \geq 0$ for $\theta \in \{T, N\}$.

The first part of the assumption says that the marginal productivity is positive and a talented scientist's marginal productivity is larger than that of a not-talented scientist; the assumption on $\frac{dp_N^S(0)}{dg}$ implies that the optimal g is strictly positive for both signals. The second part says that the marginal productivity decreases and it does so faster for a not-talented scientist than for a talented scientist. The last part says that marginal productivity decreases in a decreasing way. For instance, $\frac{dp_\theta^S(g)}{dg} = \max\{c_\theta - d_\theta g, 0\}$ with $c_T > c_N > \frac{1+\lambda}{s^H - s^L}$ and $d_N > d_T > 0$ satisfies assumption 4.

In this section we allow for $\alpha \geq 0$ but impose an upper bound on α:

Assumption 5: The social gain from a path-breaking discovery in the science sector is larger than the private non-pecuniary gain in terms of fame: $s^H - s^L > \alpha\eta$.

Even though fame induces a scientist to internalize the social benefit from a path-breaking research, assumption 5 says that this internalization is partial even when $q_r = 1$.

In what follows, we proceed in two steps. First, we fix an allocation of talent (ϕ_T, ϕ_N) that the government wants to achieve and study the optimal allocation of public funds between salaries and research grants. In particular, we examine how this allocation of funds is affected by a change in parameters $\alpha, q_s, q_r, \lambda$. Second, we characterize the optimal allocation of talent.

In this setting, we assume that the government can make the salary of a scientist depend both on her signal and on her perceived outcome. However, because of risk neutrality of the agents and the government, it turns out that there is no need to specify the structure of the salaries and only the expected monetary rewards matter. Let m_T^e (m_N^e) represent the expected monetary payoff to a talented scientist (a not-talented scientist). As in the previous sections, β_T (β_N) is the probability for a talented scientist (a not-talented scientist) to get a high perceived outcome and is now given by:

$$\beta_T \equiv \left[q_s p_T^S(g_G) + (1-q_s) p_T^S(g_B)\right] q_r + \left[1 - (q_s p_T^S(g_G) + (1-q_s) p_T^S(g_B))\right](1-q_r); \quad (19)$$

$$\beta_N \equiv \left[q_s p_N^S(g_B) + (1-q_s)p_N^S(g_G)\right] q_r + \left[1 - (q_s p_N^S(g_B) + (1-q_s)p_N^S(g_G))\right](1-q_r). \quad (20)$$

In order to implement a given allocation (ϕ_T, ϕ_N), it is necessary and sufficient that (m_T^e, m_N^e, g_G, g_B) satisfy the following incentive constraints

$$(IC_T) \quad \Pi_T - T_T + 2\gamma\phi_T - \gamma + \alpha\eta p_T^R = m_T^e + \alpha\eta\beta_T; \quad (21)$$

$$(IC_N) \quad \Pi_N - T_N + 2\gamma\phi_N - \gamma + \alpha\eta p_N^R = m_N^e + \alpha\eta\beta_N. \quad (22)$$

In order to find m_T^e and m_N^e which satisfy (21)-(22), it is necessary that there is some non-zero correlation between the signals the government observes and the talent. Hence, we consider $q_s > \frac{1}{2}$ and/or $q_r > \frac{1}{2}$. Note first that the left hand side of the incentive constraint (IC_θ) represents the reservation utility of a type-θ scientist having the intrinsic preference $\gamma_i = 2\gamma\phi_\theta - \gamma$. Given an allocation of talent, the reservation utility is fixed. Therefore, an increase in g_G or g_B increases the non-pecuniary rewards to both types of scientist through an increase in the probability of making a path-breaking discovery and this in turn decreases their monetary rewards m_T^e and m_N^e through (21)-(22).

Since (ϕ_T, ϕ_N) is given, the contribution to social welfare generated by the private sector is constant and the objective of the government is the social welfare generated by the science sector minus the social cost of salaries and grants. We denote this objective by SW^S and let $S_T(g) \equiv p_T^S(g)s^H + (1-p_T^S(g))s^L = s^L + (s^H - s^L)p_T^S(g)$, $S_N(g) \equiv s^L + (s^H - s^L)p_N^S(g)$. Then, we have:

$$\begin{aligned} SW^S \;=\; & q_s\left\{\nu\phi_T\left[S_T(g_G) - (1+\lambda)g_G\right] + (1-\nu)\phi_N\left[S_N(g_B) - (1+\lambda)g_B\right]\right\} \\ & +(1-q_s)\left\{\nu\phi_T\left[S_T(g_B) - (1+\lambda)g_B\right] + (1-\nu)\phi_N\left[S_N(g_G) - (1+\lambda)g_G\right]\right\} \\ & -\lambda\left[\nu\phi_T m_T^e + (1-\nu)\phi_N m_N^e\right]. \end{aligned}$$

After expressing m_T^e and m_N^e as functions of (g_G, g_B) from (21) and (22) and inserting them into SW^S, we obtain a concave function of (g_G, g_B) which is maximized at the solution of the following first-order conditions:[23]

$$\nu\phi_T q_s\left(\frac{dp_T^S(g_G)}{dg_G} - k\right) + (1-\nu)\phi_N(1-q_s)\left(\frac{dp_N^S(g_G)}{dg_G} - k\right) = 0; \quad (23)$$

$$\nu\phi_T(1-q_s)\left(\frac{dp_T^S(g_B)}{dg_B} - k\right) + (1-\nu)\phi_N q_s\left(\frac{dp_N^S(g_B)}{dg_B} - k\right) = 0; \quad (24)$$

[23] We cannot have $g_G = 0$ and/or $g_B = 0$ in the optimum because of assumption 4(i). Furthermore, a (unique) solution to (23)-(24) exists because p_T^S and p_N^S are bounded above and therefore $\frac{dp_\theta^S(g)}{dg} \to 0$ as $g \to +\infty$.

where

$$k = \frac{1+\lambda}{s^H - s^L + \alpha\lambda\eta(2q_r - 1)}$$

In the special case in which the signal is perfectly correlated with the type (i.e. $q_s = 1$), we have:

$$\frac{dp_T^S(g_G)}{dg_G} = k = \frac{dp_N^S(g_B)}{dg_B}.$$

We give an economic interpretation of k through this special case. Consider a unitary increase in g_G for instance. On the one hand, the social marginal cost of providing a unit of grant is $1 + \lambda$. On the other hand, there are two social marginal benefits. One is the direct benefit from an increased probability of having the path-breaking research, which is equal to $\frac{dp_T^S(g_G)}{dg_G}\left(s^H - s^L\right)$. The other is the indirect benefit related to the reduction in the monetary reward necessary to maintain the given allocation of talent, which is equal to $\frac{dp_T^S(g_G)}{dg_G}\alpha\eta(2q_r - 1)$. Therefore, the total social marginal benefit is $\frac{dp_T^S(g_G)}{dg_G}\left[s^H - s^L + \alpha\lambda\eta(2q_r - 1)\right]$. Observe now that the numerator of k is the social marginal cost of grants while the denominator represents the social marginal benefit from an increase in p_T^S. Therefore, we call k the *benefit-adjusted social marginal cost of providing grants.* Let $(g_G^{**}(\alpha, q_s, q_r, \lambda), g_B^{**}(\alpha, q_s, q_r, \lambda))$ denote the optimal grants and $(m_T^{e**}(\alpha, q_s, q_r, \lambda), m_N^{e**}(\alpha, q_s, q_r, \lambda))$ the optimal expected salaries.

We have the following proposition:

Proposition 5 *(optimal allocation of grants and salaries) Suppose that (θ_i, γ_i) is agent i's private information. Under assumptions 4 and 5 and given an allocation of talent $(\phi_T, \phi_N) \in (0,1)^2$,*

(i) The optimal grants and monetary rewards $(g_G^{**}, g_B^{**}, m_T^{e**}, m_N^{e**})$ are characterized by (21)-(24) and are such that $g_G^{**} > g_B^{**}$ for any $q_s > \frac{1}{2}$.

(ii) As the quality of the signal q_s increases, the grant to scientists with a good signal g_G^{**} increases while the grant to scientists with a bad signal g_B^{**} decreases.

(iii) Given $q_s \in (\frac{1}{2}, 1]$, as the weight on fame α increases, or the quality of the institution of science q_r increases, or the shadow cost of public funds λ decreases,

a. both grants g_G^{**} and g_B^{**} increase (thus the monetary rewards to both types m_T^{e**} and m_N^{e**} decrease);

b. $g_G^{**} - g_B^{**}$ increases.

The result of proposition 5(ii) is quite intuitive. In the extreme case of zero precision of the signal (i.e. $q_s = \frac{1}{2}$), it is optimal to give the same level of grant regardless of the signal: $g_G^{**} = g_B^{**}$. As the precision increases from $\frac{1}{2}$, it is optimal to increase the grant to scientists with a good signal and to decrease the grant to those with a bad signal.

An increase in α reduces the benefit-adjusted social marginal cost k of providing grants and therefore increasing the grants for both signals (thus reducing the monetary rewards) is optimal. Furthermore, since the marginal productivity decreases in a decreasing way and that of a talented scientist decreases less quickly than that of a not-talented one, a reduction in k makes it optimal to increase $g_G^{**} - g_B^{**}$ even though the quality q_s of the signal is fixed. Similarly, an increase in the quality of the institution of science q_r increases the non-pecuniary rewards produced by an incremental increase in grants and therefore reduces k.

A decrease in the shadow cost λ of public funds reduces k and increases g_G^{**}, g_B^{**} and $g_G^{**} - g_B^{**}$. This result is not trivial and in order to provide an intuition, we first consider the extreme case of $\lambda = 0$. Since in this case giving salaries is a pure transfer and has no social cost, the optimal grants are determined by equalizing the direct social benefit from increased scientific production to the social cost of providing grants. This can be seen from the fact that k is independent of α and q_r when $\lambda = 0$. As λ increases, the social cost of grants $1 + \lambda$ obviously increases but also the social benefits of grants $s^H - s^L + \alpha\lambda\eta(2q_r - 1)$ increase because the indirect benefit from wage reductions increases with λ. However, a scientist does not fully internalize the social benefit from a path-breaking research from assumption 5 and therefore we have $s^H - s^L > \alpha\eta(2q_r - 1)$. This implies that the increase in the total benefits is relatively smaller than the increase in the cost and therefore k increases as λ increases. Therefore, as λ increases, it is optimal to decrease grants while increasing salaries.

Now we compare the monetary reward to talent in the science sector with the one in the private sector. For this purpose, we modify assumption 1 as follows. Define $\underline{g}_B$ by $\frac{dp_N^S(\underline{g}_B)}{dg} = \frac{1+\lambda}{s^H - s^L}$. Then, we have $g_G^{**}(\alpha, q_s, q_r, \lambda) \geq g_B^{**}(\alpha, q_s, q_r, \lambda) \geq \underline{g}_B > 0$ for all $(\alpha, q_s, q_r, \lambda)$.

Assumption 1': $\Delta p^S(\underline{g}_B) \equiv \left[p_T^S(\underline{g}_B) - p_N^S(\underline{g}_B)\right] > \Delta p^R$.

This assumption is a sufficient condition to make the intrinsic outcome a less noisy signal of talent in science than in the private sector when grants are chosen optimally, for any (α, q_s, λ) (see the proof of proposition 6(ii)). From (21)-(22), the difference between the monetary reward to talent in the private sector $(\Pi_T - T_T - \Pi_N + T_N)$ and the one in the science sector $(m_T^{e**} - m_N^{e**})$ is given by:

$$\alpha\eta\left\{(\beta_T - \beta_N) - \Delta p^R\right\} - 2\gamma\left(\phi_T - \phi_N\right). \tag{25}$$

Therefore, we have:

Proposition 6 *Suppose that (θ_i, γ_i) is agent i's private information. Under assumptions 4 and 5, given an allocation of talent $(\phi_T, \phi_N) \in (0,1)^2$;*
(i) As the quality of the institution of science q_r increases or as the shadow cost of public funds λ decreases, the monetary reward to talent in the science sector decreases with respect to the one in the private sector;
(ii) Under assumption 1', for any $q_r > \widehat{q}_r \equiv \left[\Delta p^S(\underline{g}_B) + \Delta p^R\right] / 2\Delta p^S(\underline{g}_B)$, the monetary reward to talent in the science sector is lower than the one in the private sector if $\phi_T \leq \phi_N + \Phi(q_r)$ where

$$\Phi(q_r) \equiv \frac{\alpha\eta}{2\gamma}\left[(2q_r - 1)\Delta p^S(\underline{g}_B) - \Delta p^R\right] > 0.$$

Proposition 6 is closely related to the results of Proposition 5. An increase in q_r or a decrease in λ increases $g_G^{**} - g_B^{**}$, which increases the non-monetary reward to talent in the science sector. This in turn implies, given an allocation of talent, a decrease in the monetary reward to talent in the science sector relative to the one in the private sector. Proposition 6 (ii) says that when the quality of the institution of science is good enough, it is optimal to have the monetary reward to talent in the science sector lower than the one in the private sector for all allocations satisfying $\phi_T \leq \phi_N + \Phi(q_r)$ where $\Phi(q_r) > 0$. Note that $\Phi(q_r)$ increases with q_r and α. Therefore even though an optimal allocation of talent requires $\phi_T > \phi_N$, if the institution of science is good enough and the weight on fame is large enough, achieving the optimal allocation requires the monetary reward to talent in the science sector to be lower than the one in the private sector. This is because the science sector can provide a high non-monetary reward to talent given that the intrinsic outcome is a less noisy signal of talent in science than in the private sector. Therefore proposition 6(ii) provides a justification for the optimality of relatively flat wages in science.

We now study the optimal allocation of talent given that salaries and grants are chosen optimally. The social welfare is given by:

$$\begin{aligned} SW(\phi_T, \phi_N) &= \nu(1-\phi_T)\Pi_T + (1-\nu)(1-\phi_N)\Pi_N + \gamma\left[\nu\phi_T(1-\phi_T) + (1-\nu)\phi_N(1-\phi_N)\right] \\ &\quad + SW^S(\phi_T, \phi_N, g_G^{**}(\phi_T, \phi_N), g_B^{**}(\phi_T, \phi_N)). \end{aligned}$$

Using the envelope theorem, we find the first order condition with respect to (ϕ_T, ϕ_N) (for an interior maximum) as follows:

$$\Pi_T + \gamma(2\phi_T - 1) = q_s\left[S_T(g_G^{**}) - (1+\lambda)g_G^{**}\right] + (1-q_s)\left[S_T(g_B^{**}) - (1+\lambda)g_B^{**}\right] - \lambda\left(m_T^{e**} + 2\gamma\phi_T\right) \tag{26}$$

$$\Pi_N+\gamma\left(2\phi_N-1\right)=q_s\left[S_N(g_B^{**})-(1+\lambda)g_B^{**}\right]+(1-q_s)\left[S_N(g_G^{**})-(1+\lambda)g_G^{**}\right]-\lambda\left(m_N^{e**}+2\gamma\phi_N\right). \quad (27)$$

The left hand side represents the social gain that the marginal agent who is indifferent between the two professions produces as a professional while the right hand side represents the social gain that she produces as a scientist. The right hand side is composed of the social gain from research minus the social cost of grants and wages: the last term $m_\theta^{e**}+2\gamma\phi_\theta$ is equal to $\frac{\partial\left(\phi_\theta m_\theta^{e**}\right)}{\partial\phi_\theta}$ (i.e. the increase in the wage bill $\phi_\theta m_\theta^{e**}$ induced by a marginal increase in ϕ_θ).

6 Discussions

Our results suggest that the current increase in team size in science[24] might have a negative consequence in terms of the brain drain as scientific production becomes team work similar to production in firms. For instance, it is possible on an experimental article in physics for the author list to be longer than the article and in such a case the role of the individual scientist is almost impossible to evaluate. In fact, Merton (1968) argues that the growth of team work makes the recognition of individual contributions by others problematic. In our model, increase in team size might make the outcome in science a noisier signal of talent.

It would be interesting to study how recognition from non-peers affects the allocation of talent. In general, outsiders would have difficulty to tell whether a professor has good or bad publication records but it would not be difficult for them to know about the institution to which a professor belongs. Since non-peers would give more recognition to professors of prestigious universities than to professors of mediocre universities and becoming professor of prestigious universities would generally require talent, a hierarchical organization of universities as in U.S. could increase the reward to talent in terms of non-peer recognition and hence mitigate the brain drain. In contrast, in (Continental) Europe, most universities are local monopoly and therefore there is not much quality differentiation among them.

Although we focused on the moral hazard in terms of occupational choice in this paper, we can apply the framework of section 5 to study the optimal balance between

[24] Adams et als. (2004) find that team size increases by 50 percent over the period of 1981-1999 in U.S.

the monetary and the non-monetary incentives in the context of designing the optimal incentive schemes to boost research effort in science. Our framework can be also used to study how the balance should depend on the information (signal) available about each researcher's ability from their past research records.

Finally, if most agents highly value autonomy or freedom in academia, this would make wage in academia lower than the one in the private sector as in Aghion, Dewatripont and Stein (2005). Although this can be easily captured in our model with a negative mean value of γ_i, our focus is not about the absolute wage differential in both sectors but about the relatively flat monetary reward to talent in science. If a talented agent appreciates more autonomy than a not-talented one since the former derives more pleasure from puzzle solving than the latter, one can have a relatively flat earning structure in science without brain drain. Although this positive correlation between pleasure from puzzle solving and talent is likely to hold, an explanation entirely based on the positive correlation cannot shed any light on the role of the institution of science as a mechanism distributing priority recognition emphasized by Merton.

7 Conclusion

The earning structure in science is known to be flat relative to the one in the private sector and this raises the concern about the brain drain from the science sector to the private sector. This paper points out that since the performance is less noisy signal of talent in the science sector than in the private sector, if agents care about both money and peer recognition, a good institution of science can mitigate the brain drain by providing a high non-pecuniary reward to talent. Furthermore, when institution of science is good and scientists care a lot about priority recognition, a relative flat earning structure in science is likely to be optimal. Despite the desirability of providing strong monetary and non-monetary incentives to scientists by making their salaries and research grants depend on publications, one should be cautious with introducing extra monetary incentives through the market by encouraging research for commercialization. For instance, the extra incentives can induce a shift from basic to applied research, which might make the performance in science a more noisier signal of talent and thereby undermine the incentives from the institution of science.

Our study offers a useful insight about the primary role of the institution of science in providing monetary and non-monetary rewards to scientists. In this respect, Internet technology creates a unique opportunity to improve the institution of science by making it possible to speed up the refereeing process and to make wide distribution of journals at low (close to zero) marginal cost. However, there exist concerns that the private interests of the commercial publishers having market power might conflict with the realization of the potential gain from the technology.[25]

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[25] For instance, big commercial publishers' bundling practices can force the libraries to spend too much money on their journals, which leaves little money left over for small publishers and builds entry barriers (Edlin and Rubinfeld, 2004, Jeon and Menicucci, 2004).

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Appendix
Proof of Proposition 1

(i) The first order conditions (4)-(5) are sufficient for the optimality of an interior allocation since SW is strictly concave in (ϕ_T, ϕ_N). Hence, (6) is optimal if it is interior, which turns out to be the case if and only if $\gamma > S_\theta - \Pi_\theta > -\gamma$ for $\theta \in \{N, T\}$.
(ii) The proof is straightforward. ■

Proof of Proposition 2

(i) Let $B = (\Pi_T - T_T) - (\Pi_N - T_N) - \Delta p^S (2q_r - 1)\bar{b}$ for brevity and define the Lagrangian by $L \equiv SW + \mu(10)$ where μ is the multiplier associated with the (10). Then, the first-order conditions are given by:

$$\frac{\partial L}{\partial \phi_T} = \nu(-\Pi_T + S_T + \gamma(1 - 2\phi_T)) - \mu = 0 \tag{28}$$

$$\frac{\partial L}{\partial \phi_N} = (1-\nu)(-\Pi_N + S_N + \gamma(1 - 2\phi_N)) + \mu = 0. \tag{29}$$

It is straightforward to find $\phi_T^{**} = \frac{\nu(S_T - \Pi_T + \gamma) - \mu}{2\nu\gamma} = \phi_T^* - \frac{\mu^{**}}{2\nu\gamma}$ and $\phi_N^{**} = \frac{(1-\nu)(\gamma - \Pi_N + S_N) + \mu}{2(1-\nu)\gamma} = \phi_N^* + \frac{\mu^{**}}{2(1-\nu)\gamma}$ from (28)-(29). Clearly, we obtain (ϕ_T^*, ϕ_N^*) if $\mu = 0$, thus violating (10); hence, $\mu^{**} > 0$ and (10) binds at $(\phi_T^{**}, \phi_N^{**})$. Plugging these values into (10) yields $\mu^{**} = 2\nu(1-\nu)\gamma(\frac{B}{2\gamma} + \phi_T^* - \phi_N^*) > 0$; hence, (12) is interior if and only if $2\gamma - \nu B > \nu(\gamma - \Pi_T + S_T) + (1-\nu)(\gamma - \Pi_N + S_N) > (1-\nu)B$. Since $\mu^{**} > 0$, we obtain (iia). Result (iib) holds because $B > 0$ by assumption 2. From (12) it is straightforward to verify that (iic) holds. ■

Proof of Proposition 3

(i) If if $q_r > \bar{q}_r$, then $\Delta p^R - \Delta p^S(2q_r - 1) < 0$ and it is obvious that (16) holds at (ϕ_T^*, ϕ_N^*) if α is large.
(iia) The proof is the same as the proof of Proposition 2, except that now B is replaced by $B' \equiv B + \alpha\eta\left[\Delta p^R - \Delta p^S(2q_r - 1)\right]$. In particular, the condition for an interior allocation is now $2\gamma - \nu B' > \nu(\gamma - \Pi_T + S_T) + (1-\nu)(\gamma - \Pi_N + S_N) > (1-\nu)B'$. The first-order conditions with respect to ϕ_θ are still (28) and (29) except for the fact that $\alpha > 0$ affects the multiplier associated with the constraint (16), which is now denoted by $\mu(\alpha)$. Hence, $\phi_T^{**}(\alpha) = \phi_T^* - \frac{\mu^{**}(\alpha)}{2\nu\gamma}$ and $\phi_N^{**}(\alpha) = \phi_N^* + \frac{\mu^{**}(\alpha)}{2(1-\nu)\gamma}$ with $\mu^{**}(\alpha) = 2\nu(1-\nu)\gamma(1+2\lambda)(\frac{B'}{2\gamma} + \phi_T^* - \phi_N^*)$
(iib) When q_r increases, B' decreases and therefore $\phi_T^{**}(\alpha)$ increases. When α increases, B' increases or decreases depending on whether $\Delta p^R > \Delta p^S(2q_r - 1)$ or $\Delta p^R < \Delta p^S(2q_r - 1)$, which is equivalent to saying $q_r < \bar{q}_r$ or $q_r > \bar{q}_r$. When τ increases, B' decreases. ■

Proof of Proposition 4

We only analyze the non-trivial case in which $y_b < \overline{y}_b$. We denote by $(\phi_T^{*ar}, \phi_N^{*ar})$ the first-best allocation conditional on that $y_b < \overline{y}_b$ (i.e. type-θ scientist diverts effort from basic to applied research). We find

$$\phi_\theta^{*ar} = \frac{\gamma - \Pi_\theta + S'_\theta + \left(p_\theta^S - \Delta_\theta\right) y_b + \Delta_\theta y_a}{2\gamma} \tag{30}$$

where $S'_\theta \equiv \left(p_\theta^S - \Delta_\theta\right) s^H + \left(1 - p_\theta^S + \Delta_\theta\right) s^L$.

We can represent the incentive constraints which (w, b) needs to satisfy (in addition to $b \leq \bar{\bar{b}}$) in order to implement a given interior allocation (ϕ_T, ϕ_N) as follows:

$$\begin{aligned} \Pi_T - T_T + 2\gamma\phi_T - \gamma + \alpha p_T^R\eta &= (\beta_T - (2q_r - 1)\Delta_T)(b + \alpha\eta) + \\ & \quad w + \delta\left[\Delta_T y_a + (p_T^S - \Delta_T)y_b\right] && (31) \\ \Pi_N - T_N + 2\gamma\phi_N - \gamma + \alpha p_N^R\eta &= (\beta_N - (2q_r - 1)\Delta_N)(b + \alpha\eta) + \\ & \quad w + \delta\left[\Delta_N y_a + (p_N^S - \Delta_N)y_b\right] && (32) \end{aligned}$$

After solving (31)-(32) with respect to (w, b), we find that $b \leq \bar{b}$ reduces to

$$\begin{gathered} \Pi_T - T_T - (\Pi_N - T_N) + 2\gamma(\phi_T - \phi_N) + \alpha\Delta p^R\eta \leq \\ \delta\left[(\Delta_T - \Delta_N)(y_a - y_b) + \Delta p^S y_b\right] + (2q_r - 1)(\Delta p^S - \Delta_T + \Delta_N)(\bar{b} + \alpha\eta) \end{gathered} \tag{33}$$

As we mentioned before the proposition, (33) is violated by $(\phi_T^{*ar}, \phi_N^{*ar})$. Therefore, (33) is binding in the solution to the second-best problem and $b = \bar{b}$. Providing licensing opportunity relaxes (16) (i.e. (33) is relaxed than (16)) if and only if

$$y_b > \underline{y}_b \equiv \frac{(\Delta_T - \Delta_N)\left[(2q_r - 1)(\overline{b} + \alpha\eta) - \delta y_a\right]}{\delta\left[\Delta p^S - \Delta_T + \Delta_N\right]}. \tag{34}$$

Note first that $\Delta p^S - \Delta_T + \Delta_N = p_T^S - \Delta_T - (p_N^S + \Delta_N) > 0$. $\overline{y}_b > 0$ holds if and only if $\overline{b} + \alpha\eta \leq \delta y_a$ holds or $\overline{b} + \alpha\eta > \delta y_a$ and $q_r < q_r^{**}$ where q_r^{**} is given by

$$(2q_r^{**} - 1)(\overline{b} + \alpha\eta) = \delta y_a.$$

We only need to consider $\overline{y}_b > 0$; otherwise, providing licensing opportunity does not affect research pattern. Consider first $\Delta_T \geq \Delta_N$. If $\overline{b} + \alpha\eta \leq \delta y_a$ or if $\overline{b} + \alpha\eta > \delta y_a$ and $q_r < q_r^{**}$, we have $\underline{y}_b \leq 0 \leq \overline{y}_b$. Since $y_b > 0$, in this case, providing licensing opportunity relaxes (16) and therefore reduces the brain drain. Consider now $\Delta_T < \Delta_N$. Then, whenever $\overline{y}_b > 0$, $\overline{y}_b > \underline{y}_b > 0$. Therefore, providing licensing opportunity reduces the brain drain if and only if $\overline{y}_b \geq y_b > \underline{y}_b$.

■

Proof of Proposition 5

(i) If $g_G^{**} \leq g_B^{**}$, then $\frac{dp_T^S(g_G^{**})}{dg_G} - k \geq \frac{dp_T^S(g_B^{**})}{dg_B} - k > 0 > \frac{dp_N^S(g_G^{**})}{dg_G} - k \geq \frac{dp_N^S(g_B^{**})}{dg_B} - k$. Then, notice that (a) $\frac{dp_T^S(g_G^{**})}{dg_G} - k$ is multiplied by $\nu\phi_T q_s$ in (23) and $\frac{dp_T^S(g_B^{**})}{dg_B} - k$ is multiplied by $\nu\phi_T(1-q_s)\,(<\nu\phi_T q_s)$ in (24); (b) $\frac{dp_N^S(g_G^{**})}{dg_G} - k$ is multiplied by $(1-\nu)\phi_N(1-q_s)$ in (23) and $\frac{dp_N^S(g_B^{**})}{dg_B} - k$ is multiplied by $(1-\nu)\phi_N q_s\,(>(1-\nu)\phi_N(1-q_s))$ in (24). We infer therefore that the left hand side of (23) is positive if (24) is satisfied, which is a contradiction.

(ii) By applying the implicit function theorem to (23) and (24) we obtain

$$\begin{aligned}
\frac{dg_G^{**}}{dq_s} &= \frac{\nu\phi_T\left(\frac{dp_T^S(g_G^{**})}{dg_G} - k\right) - (1-\nu)\phi_N\left(\frac{dp_N^S(g_G^{**})}{dg_G} - k\right)}{-\nu\phi_T q_s\frac{d^2p_T^S(g_G^{**})}{dg_G^2} - (1-\nu)\phi_N(1-q_s)\frac{d^2p_N^S(g_G^{**})}{dg_G^2}}\\
\frac{dg_B^{**}}{dq_s} &= \frac{-\nu\phi_T\left(\frac{dp_T^S(g_B^{**})}{dg_B} - k\right) + (1-\nu)\phi_N\left(\frac{dp_N^S(g_B^{**})}{dg_B} - k\right)}{-\nu\phi_T(1-q_s)\frac{d^2p_T^S(g_B^{**})}{dg_B^2} - (1-\nu)\phi_N q_s\frac{d^2p_N^S(g_B^{**})}{dg_B^2}}
\end{aligned}$$

By assumption 4(ii), the denominators in $\frac{dg_G^{**}}{dq_s}$ and $\frac{dg_B^{**}}{dq_s}$ are positive. The numerator of $\frac{dg_G^{**}}{dq_s}$ is positive because assumption 4(i) and (23) imply $\frac{dp_T^S(g_G^{**})}{dg_G} - k > 0 > \frac{dp_N^S(g_G^{**})}{dg_G} - k$. The numerator of $\frac{dg_B^{**}}{dq_s}$ is negative because $\frac{dp_T^S(g_B^{**})}{dg_B} - k > 0 > \frac{dp_N^S(g_B^{**})}{dg_B} - k$ by assumption 4(i) and (24).

(iiia) We prove that $\frac{\partial g_\theta^{**}}{\partial \alpha} > 0$. First, notice that $\frac{\partial g_\theta^{**}}{\partial \alpha} = \frac{\partial g_\theta^{**}}{\partial k}\frac{\partial k}{\partial \alpha}$ and $\frac{\partial k}{\partial \alpha} < 0$; hence, it suffices that we prove $\frac{\partial g_\theta^{**}}{\partial k} < 0$. By applying the implicit function theorem to (23) and (24) we find

$$\frac{\partial g_G^{**}}{\partial k} = \frac{\nu\phi_T q_s + (1-\nu)\phi_N(1-q_s)}{\nu\phi_T q_s \frac{d^2 p_T^S(g_G^{**})}{dg_G^2} + (1-\nu)\phi_N(1-q_s)\frac{d^2 p_N^S(g_G^{**})}{dg_G^2}} \tag{35}$$

$$\frac{\partial g_B^{**}}{\partial k} = \frac{\nu\phi_T(1-q_s) + (1-\nu)\phi_N q_s}{\nu\phi_T(1-q_s)\frac{d^2 p_T^S(g_B^{**})}{dg_B^2} + (1-\nu)\phi_N q_s\frac{d^2 p_N^S(g_B^{**})}{dg_B^2}} \tag{36}$$

and $\frac{\partial g_G^{**}}{\partial k} < 0$, $\frac{\partial g_B^{**}}{\partial k} < 0$ by assumption 4(ii). The proof that $\frac{\partial g_\theta^{**}}{\partial q_r} > 0$ is very similar to the proof that $\frac{\partial g_\theta^{**}}{\partial \alpha} > 0$.
In order to prove that $\frac{\partial g_\theta^{**}}{\partial \lambda} < 0$, we observe that $\frac{\partial g_\theta^{**}}{\partial \lambda} = \frac{\partial g_\theta^{**}}{\partial k}\frac{\partial k}{\partial \lambda}$ and $\frac{\partial k}{\partial \lambda} = \frac{s^H - s^L - \alpha\eta(2q_r-1)}{(s^H - s^L + \alpha\eta(2q_r-1))^2} > 0$ by assumption 5.

(iiib) Now we prove that $\frac{\partial\left(g_G^{**}-g_B^{**}\right)}{\partial \alpha} > 0$; the same arguments can be used to show that $\frac{\partial\left(g_G^{**}-g_B^{**}\right)}{\partial q_r} > 0$ and $\frac{\partial\left(g_G^{**}-g_B^{**}\right)}{\partial \lambda} < 0$. We find $\frac{\partial\left(g_G^{**}-g_B^{**}\right)}{\partial \alpha} = \frac{\partial\left(g_G^{**}-g_B^{**}\right)}{\partial k}\frac{\partial k}{\partial \alpha}$; therefore we need to prove that $\frac{\partial\left(g_G^{**}-g_B^{**}\right)}{\partial k} < 0$. Using (35)-(36) we see that this condition is equivalent to $\frac{\nu\phi_T(1-q_s)+(1-\nu)\phi_N q_s}{\nu\phi_T(1-q_s)\frac{d^2 p_T^S(g_B^{**})}{dg_B^2}+(1-\nu)\phi_N q_s\frac{d^2 p_N^S(g_B^{**})}{dg_B^2}} > \frac{\nu\phi_T q_s+(1-\nu)\phi_N(1-q_s)}{\nu\phi_T q_s\frac{d^2 p_T^S(g_G^{**})}{dg_G^2}+(1-\nu)\phi_N(1-q_s)\frac{d^2 p_N^S(g_G^{**})}{dg_G^2}}$, which reduces to

$$\begin{aligned}
&(\nu\phi_T(1-q_s) + (1-\nu)\phi_N q_s)\left[\nu\phi_T q_s\frac{d^2 p_T^S(g_G^{**})}{dg_G^2} + (1-\nu)\phi_N(1-q_s)\frac{d^2 p_N^S(g_G^{**})}{dg_G^2}\right] \\
> \;& (\nu\phi_T q_s + (1-\nu)\phi_N(1-q_s))\left[\nu\phi_T(1-q_s)\frac{d^2 p_T^S(g_B^{**})}{dg_B^2} + (1-\nu)\phi_N q_s\frac{d^2 p_N^S(g_B^{**})}{dg_B^2}\right]
\end{aligned}$$

We exploit assumption 4(iii) and $g_G^{**} > g_B^{**}$ to obtain $\frac{d^2 p_T^S(g_G^{**})}{dg_G^2} > \frac{d^2 p_T^S(g_B^{**})}{dg_B^2}$ and $\frac{d^2 p_N^S(g_G^{**})}{dg_G^2} > \frac{d^2 p_N^S(g_B^{**})}{dg_B^2}$. Hence, since $\nu\phi_T q_s(\nu\phi_T(1-q_s) + (1-\nu)\phi_N q_s) - \nu\phi_T(1-q_s)(\nu\phi_T q_s + (1-\nu)\phi_N(1-q_s)) = \nu\phi_T\phi_N(2q_s-1)(1-\nu)$ and $(1-\nu)\phi_N(1-q_s)(\nu\phi_T(1-q_s)+(1-\nu)\phi_N q_s) - (1-\nu)\phi_N q_s(\nu\phi_T q_s + (1-\nu)\phi_N(1-q_s)) = -\nu\phi_T\phi_N(2q_s-1)(1-\nu)$, it is sufficient that we prove the inequality

$$\nu\phi_T\phi_N(2q_s-1)(1-\nu)\frac{d^2 p_T^S(g_G^{**})}{dg_G^2} - \nu\phi_T\phi_N(2q_s-1)(1-\nu)\frac{d^2 p_N^S(g_G^{**})}{dg_G^2} > 0$$

This inequality holds since $\frac{d^2 p_T^S(g_G^{**})}{dg_G^2} > \frac{d^2 p_N^S(g_G^{**})}{dg_G^2}$ by assumption 4(ii).
(iv) The proof is straightforward from (23) and (24), $\frac{dp_T^S(g_B^{**})}{dg_B} \geq \frac{dp_T^S(g_G^{**})}{dg_G} > k > \frac{dp_N^S(g_B^{**})}{dg_B} \geq \frac{dp_N^S(g_G^{**})}{dg_G}$ and assumption 4. ■

Proof of Proposition 6

(i) We have

$$\frac{\partial\left(\beta_T-\beta_N\right)}{\partial g_G}=q_r\left(q_s\frac{\partial p_T^S}{\partial g_G}-(1-q_s)\frac{\partial p_N^S}{\partial g_G}\right)>0;$$

$$\frac{\partial\left(\beta_T-\beta_N\right)}{\partial g_B}=q_r\left(-q_s\frac{\partial p_N^S}{\partial g_B}+(1-q_s)\frac{\partial p_T^S}{\partial g_B}\right)\gtrless 0.$$

In order to study the impact of q_r or λ, we prove first $\frac{\partial(\beta_T-\beta_N)}{\partial g_G}+\frac{\partial(\beta_T-\beta_N)}{\partial g_B}>0$. We have:

$$\frac{\partial\left(\beta_T-\beta_N\right)}{\partial g_G}+\frac{\partial\left(\beta_T-\beta_N\right)}{\partial g_B}=(2q_r-1)\left(q_s\left(\frac{\partial p_T^S}{\partial g_G}-\frac{\partial p_N^S}{\partial g_B}\right)+(1-q_s)\left(\frac{\partial p_T^S}{\partial g_B}-\frac{\partial p_N^S}{\partial g_G}\right)\right). \quad (37)$$

We can express $\frac{\partial p_N^S}{\partial g_G}$ (respectively, $\frac{\partial p_T^S}{\partial g_B}$) as a function of $\frac{\partial p_T^S}{\partial g_G}$ (respectively $\frac{\partial p_N^S}{\partial g_B}$) from (23) and (24), so that $\frac{\partial p_T^S}{\partial g_B}=k-\frac{(1-v)\phi_N q_s(\frac{\partial p_N^S}{\partial g_B}-k)}{v\phi_T(1-q_s)}$ and $\frac{\partial p_N^S}{\partial g_G}=k-\frac{v\phi_T q_s(\frac{\partial p_T^S}{\partial g_G}-k)}{(1-v)\phi_N(1-q_s)}$. Plugging these expression into (37) we obtain:

$$\begin{aligned}&\left[\frac{\partial\left(\beta_T-\beta_N\right)}{\partial g_G}+\frac{\partial\left(\beta_T-\beta_N\right)}{\partial g_B}\right]/(2q_r-1)\\ =\;& q_s\left[\frac{\partial p_T^S}{\partial g_G}-\frac{\partial p_N^S}{\partial g_B}\right]+q_s\frac{(k-\frac{\partial p_N^S}{\partial g_B})\left[(1-\nu)\phi_N\right]^2-(k-\frac{\partial p_T^S}{\partial g_G})\left[\nu\phi_T\right]^2}{\nu(1-\nu)\phi_T\phi_N}.\end{aligned}$$

Therefore, $\frac{\partial(\beta_T-\beta_N)}{\partial g_G}+\frac{\partial(\beta_T-\beta_N)}{\partial g_B}>0$ if the following inequality holds:

$$\begin{aligned}&\left[\nu(1-\nu)\phi_T\phi_N+\left[\nu\phi_T\right]^2\right]\frac{\partial p_T^S}{\partial g_G}+\left[(1-\nu)\phi_N\right]^2k\\ >\;&\left[\nu(1-\nu)\phi_T\phi_N+\left[(1-\nu)\phi_N\right]^2\right]\frac{\partial p_N^S}{\partial g_B}+\left[\nu\phi_T\right]^2k\end{aligned}$$

which holds because $\frac{\partial p_T^S(g_G)}{\partial g_G}>k>\frac{\partial p_N^S(g_N)}{\partial g_N}$ at (g_G^{**},g_B^{**}) for any $q_s\in\left[\frac{1}{2},1\right)$.

When we examine $\frac{\partial(\beta_T-\beta_N)}{\partial q_r}$, we see that $\frac{\partial(\beta_T-\beta_N)}{\partial q_r}=\frac{\partial(\beta_T-\beta_N)}{\partial g_G}\frac{\partial g_G^{**}}{\partial q_r}+\frac{\partial(\beta_T-\beta_N)}{\partial g_B}\frac{\partial g_B^{**}}{\partial q_r}+2\left[q_s p_T^S(g_G^{**})+(1-q_s)p_T^S(g_B^{**})-q_s p_N^S(g_B^{**})-(1-q_s)p_N^S(g_G^{**})\right]$ where the last term is positive and is due to the direct effect of q_r on $\beta_T-\beta_N$ given (g_G^{**},g_B^{**}). Since $\frac{\partial g_G^{**}}{\partial q_r}>\frac{\partial g_B^{**}}{\partial q_r}>0$ holds from Proposition 5, we have $\frac{\partial(\beta_T-\beta_N)}{\partial g_G}\frac{\partial g_G^{**}}{\partial q_r}+\frac{\partial(\beta_T-\beta_N)}{\partial g_B}\frac{\partial g_B^{**}}{\partial q_r}>\left[\frac{\partial(\beta_T-\beta_N)}{\partial g_G}+\frac{\partial(\beta_T-\beta_N)}{\partial g_B}\right]\frac{\partial g_B^{**}}{\partial q_r}>0$. Therefore, an increases in q_r reduces the monetary reward to talent in the science sector.

Last, we have $\frac{\partial(\beta_T-\beta_N)}{\partial\lambda}=\frac{\partial(\beta_T-\beta_N)}{\partial g_G}\frac{\partial g_G^{**}}{\partial\lambda}+\frac{\partial(\beta_T-\beta_N)}{\partial g_B}\frac{\partial g_B^{**}}{\partial\lambda}<\left[\frac{\partial(\beta_T-\beta_N)}{\partial g_G}+\frac{\partial(\beta_T-\beta_N)}{\partial g_B}\right]\frac{\partial g_B^{**}}{\partial\lambda}<0$. Therefore, a decrease in λ reduces the monetary reward to talent in the science sector.

(ii) We have

$$\begin{aligned}
\beta_T - \beta_N &= (2q_r - 1)\left[q_s p_T^S(g_G^{**}) + (1-q_s)p_T^S(g_B^{**}) - q_s p_N^S(g_B^{**}) - (1-q_s)p_N^S(g_G^{**})\right] \\
&= (2q_r - 1)\left\{\left[p_T^S(g_B^{**}) - p_N^S(g_B^{**})\right] + \int_{g_B^{**}}^{g_G^{**}} \left(q_s \frac{dp_T^S(g)}{dg} - (1-q_s)\frac{dp_N^S(g)}{dg}\right) dg\right\} \\
&\geq (2q_r - 1)\left[p_T^S(g_B^{**}) - p_N^S(g_B^{**})\right] \\
&\geq (2q_r - 1)\left[p_T^S(\underline{g}_B) - p_N^S(\underline{g}_B)\right].
\end{aligned}$$

Therefore, when $q_r = 1$, from Assumption 1', $(\beta_T - \beta_N) > \Delta p^R$. When $q_r = \frac{1}{2}$, $\beta_T - \beta_N = 0 < \Delta p^R$. Therefore, the result is obtained from (25). ■