HOUSING RANKING: A MODEL OF EQUILIBRIUM BETWEEN BUYERS AND SELLERS EXPECTATIONS

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Abstract

The equilibrium set of housing units (alternatives) can be characterized from the standpoint of both the demander and the supplier. The current work describes an application of the multicriteria single price model to the ranking of alternatives. By a generalization of the single price model and from both viewpoints an efficiency index can be calculated. We demonstrate how, in equilibrium, the two viewpoints result inevitably in inverse orders of ranking. The model is illustrated by a sample of housing units in the city of Valencia, Spain.

1. Introduction

Whatever the economic and financial situation at the time, the decision to buy or sell a home should be rational, based on clearly defined aims and taking account of all the available market information. From the sellers' viewpoint, his/her aim must be to maximise the ratio between the sale price and the features and attributes of the property. That means obtaining the highest possible price in line with the market, considering the property's area, age, location, etc. On the opposing side, buyers will try to obtain the best combination of those variables – subject to their personal preferences – at the lowest price possible.

This being the context, it becomes necessary to identify the features that are relevant to price formation and to quantify their respective importance. In the literature, this has usually been done by means of hedonic price models (Rosen, 1974). The hedonic approach views a residential property as a homogeneous possession, but conceptualises it as made up of a basket of individual attributes such that each of them contributes to providing one or more of the home's services. Hedonic prices are defined as the implicit prices of those attributes of the possession.

Sellers have an interest in knowing whether the price they are asking is or is not above the market value of the property (obtained from a set of recent transactions). Conversely, buyers have an interest in knowing whether the property on offer is being overvalued or whether its price is a good market fit. Sometimes there are buyers who may be willing to pay a higher price based on subjective factors. Under this circumstance the seller can get a price which is higher than the "objective" market price of the property. Furthermore, the role played by investors in search of a real estate portfolio should be considered. These are interested in buying and selling, but not at any cost: if and only if the transaction cost is reasonable. All sellers, buyers and investors seek to know the "objective" market price of the properties, which depends on the features of the properties. This information is of great interest for housing sellers and buyers in the dealing process and can help investors to identify the best investment opportunities in the housing market.

To compare and rank dwellings, it is fundamental to establish the weight (valuation) of the different attributes that define a property. Considering the most general form of a utility function, Ballestero and Romero (1991, 1993) make use of Compromise Programming (Yu, 1973; Zeleny, 1973, 1974) to establish a weighting system in which the weight of each attribute is inversely proportional to the difference between its ideal value and anti-ideal. The weights are conceptualized as shadow prices and are directly applicable on different economic scenarios posed by the same authors (Ballestero and Romero, 1994). Among these, noteworthy is the full ranking of organizational units in the efficiency models (Ballestero, 1999). A more detailed economic interpretation can be found in Ballestero (2002, pages 90-94). The following section also provides a brief interpretation of this choice of weights.

The single price model (SPM) of Ballestero (1999) makes it possible to perform a hierarchy of the efficient alternatives, giving rise to what is known as an *efficient alternatives ranking*. SPM computes a cardinal ranking of the units in a simple way, and is connected with an economic scenario where the only hypothesis assumed is a moderate pessimistic attitude towards the decision maker's risk (buyer or seller in our context).

It thus offers a possibility that is especially attractive in the field of selling and buying residential properties. Suppose an owner decides to put his or her home up for sale, and sets a price for it. The seller will not only want to know whether that price undervalues the property in comparison with other similar sold properties; the seller will also want to know what position his or her offer occupies in relation to these properties. In addition, SPM makes it possible to perform a sensitivity analysis of the results and reply to questions like, “By how many positions will the ranking of a property change if the price is modified?” And a similar analysis can be performed from the buyer’s viewpoint.

The full ranking of alternatives is by no means a new question for researchers, especially in the multicriteria area. The well-known DEA (Charnes *et al.*, 1978) attempts to distinguish between efficient and non-efficient alternatives, called Decision Making Units (DMU), and also to provide useful benchmarks (target projection on the efficiency frontier, set of efficient peers). The efficient alternatives are all assigned the same efficiency index (EI), namely 1, so that they all have the same priority. Only the inefficient alternatives can be differentiated by the EI, which is less than 1 for all of them. So DEA is primarily intended to differentiate between inefficient alternatives, but not to differentiate between those that are efficient (Ballestero and Maldonado, 2004).

Most of the proposals based on DEA to perform a full ranking are reasoned on graphic illustration of the DMU's on attributes axes (Sexton *et al.*, 1986; Andersen and Petersen, 1993; Sinuany-Stern *et al.*, 1994; Ertay and Ruan, 2005).

However, making a comparison between SPM and DEA is not an objective of this study, since the methodologies were conceived under different hypotheses and also for different purposes. SPM and other well known multicriteria ranking methods such as ELECTRE (Roy, 1968), AHP (Saaty, 1980), TOPSIS (Hwang and Yoon, 1981) or PROMETHEE (Brans et al., 1986) are not comparable either, since the originality of the SPM model arises from the relation established between the compromise programming and the utility function(Ballestero y Romero, 1991).

The present study proposes the use of SPM for the objective analysis of efficiency and the cardinal ranking in decisions governing the buying and selling of goods. With the market price of a set of goods and their relevant features as givens, the intent is to arrive at the EI of each and build a full ranking of them. SPM has recently been applied successfully to the purchase of capital goods (Talluri, 2002), to hospital efficiency (Ballestero and Maldonado, 2004), and to selecting textile products (Ballestero, 2004). The novelty of our proposal lies in its field of application, namely the ranking of residential properties, and the double perspective adopted: seller and buyer. Our aim, which is to find a model of equilibrium between the expectations of buyers and sellers, requires some modification of Ballestero's original approach. It will be shown how, in a situation of equilibrium, the differing perspectives of buyer and seller lead inevitably to opposite orders of priority, and that these orders are independent of the decision maker's attitude, whether optimistic or pessimistic. In addition, the weights assigned to each criterion are arrived at even more simply than in the original SPM formulation.

The remainder of this paper is organised as follows. Section 2 briefly summarises the working of SPM and the connection with a well-known multiple criteria technique: Compromise Programming. Section 3 describes the adaptation of the model to a situation of equilibrium between suppliers and demanders in a general context. Section 4 illustrates the foregoing by applying it to a sample of residential properties in the city of Valencia, Spain. Finally, there is a section giving our main conclusions.

2. The single price model

This section intends to provide a summary of the general aspects of the SPM model and its relation to compromise programming, and serves as a basis for the subsequent sections.

SPM treats a set of s benefits and compares them to m costs. In order to draw up a ranking based on the N initial alternatives, aggregation (1) is proposed:

$$Y_j = \sum_{i=1}^{s} u_i y_{ij} \qquad X_j = \sum_{h=1}^{m} v_h x_{hj} \qquad j = 1..N \tag{1}$$

together with its subsequent quotient for calculating the EI (2):

$$EI_j = {Y_j}/{X_j} \tag{2}$$

where Y_j is the aggregate benefit of the j^{th} alternative, X_j is the aggregate cost of the j^{th} alternative, y_{ij} is the i^{th} benefit of the j^{th} alternative, x_{hj} is the h^{th} cost of the j^{th} alternative, with $u_i \geq 0$ and $v_h \geq 0$ being the weights of the i^{th} benefit and h^{th} cost respectively. The problem can now be expressed in terms of how to obtain objectively the values of u_i and v_h, and for this a two-stage solution is offered.

<u>First Step</u>. Classifying the alternatives into inefficient and non-inefficient

In line with the classic DEA model, an alternative is inefficient if and only if it is dominated by a convex combination of other alternatives. Unlike in DEA, the non-dominated alternatives are treated as non-inefficient instead of as efficient.

<u>Second step</u>: Calculating the EI

In this step, the model constructs the EI (2) from the set of alternatives classified in the preceding step as non-inefficient. Building the index requires quantifying weights u_i and v_h in (1). Two assumptions are made for this purpose: 1) the benefits from the non-inefficient alternatives must cover their costs, and 2) in constructing the EI, it is important that the model does not overestimate the difference between benefits and costs in a way that favours any particular alternative. Therefore, the assumption is that the behaviour of those estimating the benefits from the alternatives will be moderate,

since overestimating the benefits of one of them will necessarily entail underestimating the others.
In the context of utilitarianism, the benefits, unlike the costs, follow the rule "more is better". Transforming the latter so that "more is better", and assigning variables to each of the s+m through z_i, the resulting optimization model is (3):

$$\begin{aligned} & \mathrm{Min} \sum_{\lambda=1}^{s+m} w_{\lambda} z_{\lambda q} \\ & s.t. \sum_{\lambda=1}^{s+m} w_{\lambda} z_{\lambda j} \geq 1 \quad j = 1 \ldots n \end{aligned} \tag{3}$$

Where the following transformations were carried out:

$$z_{\lambda j} = y_{ij} \quad \text{for} \lambda, i = 1..s \tag{4}$$

$$z_{\lambda j} = x_{h\max} - x_{hj} \quad \text{for} \lambda = s+1..s+m,\ h = 1..m \tag{5}$$

$$w_{\lambda} = \frac{u_i}{\sum_{h=1}^{m} v_h x_{h\max}} \quad \text{for} \lambda = 1..s, i = 1..s \tag{6}$$

$$w_{\lambda} = \frac{v_h}{\sum_{h=1}^{m} v_h x_{h\max}} \quad \text{for} \lambda = s+1..s+m, h = 1..m \tag{7}$$

Although the difference to best is used in the SPM in order to invert scales, alternative approaches to this end can be found in efficiency analysis, that have different impact on the dataset (Seiford and Zhu, 2002).
The efficient frontier is marked by points (8):

$$E_{\lambda} = (z_{1*}, z_{2*}, \ldots, z_{\lambda-1*}, z_{\lambda}^{*}, z_{\lambda+1*}, \ldots, z_{s+m*}) \tag{8}$$

Where $z_{\lambda *} = \min(z_{\lambda j})$ denotes the anti-ideal or nadir value and $z_{\lambda *} = \max(z_{\lambda j})$ denotes the ideal or anchor value in the λ^{th} criterion, as usually referred to in Compromise Programming. We must remark that anti-ideal and ideal values are obtained from the non-inefficient set of alternatives.
Points (8) are brought into model (3) in the form of constraints:

$$w_{\lambda} z_{\lambda}^{*} + \sum_{\mu} w_{\mu} z_{\mu *} = 1 \qquad \lambda = 1, 2, \ldots, s+m \tag{9}$$

with $\mu = 1,2,..., \lambda - 1, \lambda + 1,..., s + m$. In this way, a linear system of $(s + m)$ equations is obtained. The practical justification for including these constraints will be explained in the next section.

Using a theorem from Ballestero and Romero (1993), it can be demonstrated that when the set of constraints (9) is added to model (3) the solution for w is unique and is given by expression (10) independently of the alternative that is under consideration in the objective function:

$$w_\lambda = \frac{1}{(z_\lambda^* - z_{\lambda*})[1 + \sum_{\mu=1}^{s+m} z_{\mu*} \big/ (z_\mu^* - z_{\mu*})]} \qquad \lambda = 1,2,..., s+m \tag{10}$$

In this way, the EI of the j^{th} non-inefficient alternative can be calculated by ratio (11):

$$EI_j = \frac{\sum_{i=1}^{s} w_i y_{ij}}{\sum_{h=1}^{m} w_{s+h} x_{hj}} \tag{11}$$

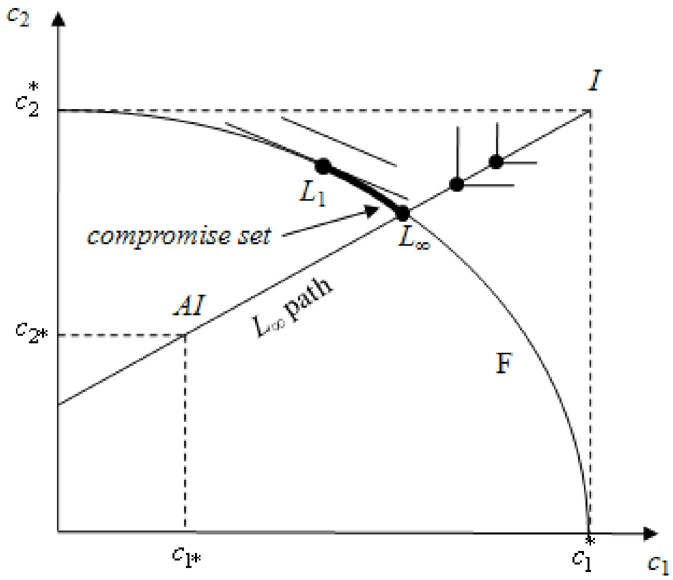
and from that the ranking of altematives can be arrived at directly.

As stated in the introduction, the weights w_λ are inversely proportional to the difference between the ideal value and the anti-ideal in the criterion λ^{th}. Figure 1 represents the problem in a bicriteria space. Suppose that the criteria follow the rule "more is better", and that locus F (convex) is defined by the set of non-dominated alternatives. The criteria c_1 (c_2) has the ideal value $c_1^*(c_2^*)$ and the anti-ideal $c_{1*}(c_{2*})$. Consequently, the ideal point I of coordinates (c_1^*, c_2^*) is located in the non-feasible region. Following Zeleny's axiom of choice, the F alternatives closest to I will be preferable.

Among the different norms that can be used to quantify the distance to I is the infinite norm, which is the norm used to represent the L_∞ path. The weights, which must hold with the equality $w_1(c_1^* - c_1) = w_2(c_2^* - c_2)$ are derived specifically from this path. The cross point between the boundary F and the L_∞ path identifies the feasible alternative closest to the ideal I in infinite norm. Point L_1 corresponds to the alternative closest to the ideal point in norm one. In a bicriteria problem, the application of other norms would give rise to other solutions within the segment delimited by L_1 and L_∞, the so-called compromise set (Yu, 1973).

Ballestero and Romero (1991) demonstrate how under the hypothesis of the marginal rate of substitution law, any utility function defined on the criteria c_1 and c_2 reach a solution within the compromise set.

Figure 1. Compromise set in a bicriteria space



3. Full ranking of goods by means of an adapted single price model

As stated in the Introduction, this study proposes SPM be used for the objective analysis of efficiency in decisions concerning sale and purchase of goods (alternatives). Our proposal should be understood to be a generalization of the SPM model, in which the viewpoints of both the buyer and the seller, rather than just one of their viewpoints, are considered in the full ranking of goods. In our proposal it is assumed that all the decision makers have the same objective preferences so as to exclude the subjectivity of the analysis. The exclusion of subjectivity, understood as the individual decision-maker preferences, ensures to get a one and only ranking of alternatives. If the perception of each criterion is different depending on the particular decision-maker, or the weight of the criteria is different for each decision-maker, there will not be an only ranking. In this case, the relative position of the alternatives could be modified depending on who is the decision-maker. When applying the proposed model, the decision maker must be aware of and test the moderate attitude which is assumed to be basic in the model, as well as the features of the equilibrium set obtained in each particular application.

The proposal depends on modifying the original model, and for that we must first give some definitions.

Definition 3.1: Good non-inefficient for the buyer

A good is to be considered non-inefficient from the buyer's viewpoint if there is no convex combination of goods that would have a lower or equal price with a higher or equal level of features.

Definition 3.2: Good non-inefficient for the seller

A good is to be considered non-inefficient from the seller's viewpoint if there is no convex combination of goods that would have a higher or equal price with a lower or equal level of features.

Definition 3.3: Equilibrium set

Given a set of goods whose sale/purchase price is known and a vector of features that are relevant to the valuation of the goods, then the equilibrium set of goods is composed of those that are non-inefficient from the viewpoint of both the buyer and the seller.

It can be seen that definition 3.3 makes a good deal of sense economically speaking. If the goods in a set S all possess the same features but different prices, then the dearest of them, A, is non-inefficient for the seller, while the least expensive of them, B, is non-inefficient for the buyer. However, neither of them will likely be chosen for the transaction. In that set, good A will be the choice of the seller but the least attractive to buyers. The same reasoning can be applied to good B, with the result that neither of them will end up being sold. In fact, no other good in set S is likely to change hands if the market is transparent, because both sellers and buyers can find better alternatives within the same set. Consequently, the equilibrium set will contain only those goods that are equally attractive to both buyer and seller, that is to say non-inefficient from both points of view. In other words, the assumption is that a sale is only likely to be transacted when neither buyer nor seller can find a more efficient alternative. If the data set only comprises already sold goods, and not a combination of offered and demanded goods, then the reason why A and B should be excluded from the equilibrium set is also clear: we would have alternatives with similar features but with a different price, which in a transparent market might imply that (i) some relevant criteria have not been considered or that (ii) the perception of some of these criteria is different depending on the buyer/seller which take part on the transaction. This would fail to fulfil the nonsubjectivity assumption previously remarked. In this situation, both A and B should be excluded from the equilibrium set.

First step: Determining the equilibrium set of goods

The buyer seeks to maximise the ratio between the utility of the features in the vector of features of the good and the offering price, while the seller does the opposite. To put it in the terminology of efficiency analysis, for the buyer the price acts as the single cost (what the buyer gives) and the features of the good as the different benefits (what the buyer receives), and vice versa for the seller. Take c_{ij} as the value of the i^{th} feature of the j^{th} good and p_j as the price of the j^{th} good, then the equilibrium set of goods is arrived at by model (12) for $a = 1\ldots N$.

$$
\begin{aligned}
&\text{Min } \frac{1}{2}(\varphi_a^s + \varphi_a^b) \\
&s.t. \sum_{j=1}^{N} \varphi_j^s c_{ij} \le c_{ia} \ \forall i \\
&\quad \sum_{j=1}^{N} \varphi_j^s p_j \ge p_a \\
&\quad \sum_{j=1}^{N} \varphi_j^s = 1 \\
&\quad \sum_{j=1}^{N} \varphi_j^b c_{ij} \ge c_{ia} \forall i \\
&\quad \sum_{j=1}^{N} \varphi_j^b p_j \le p_a \\
&\quad \sum_{j=1}^{N} \varphi_j^b = 1 \\
&\quad \varphi^s, \varphi^b \ge 0
\end{aligned}
\qquad (12)
$$

A good is deemed non-inefficient if the objective function takes value 1, and inefficient otherwise. Essentially, a good will be non-inefficient if it is non-inefficient both for the buyer and the seller. Consequently, model (12) simply includes the buyer and seller models in a single mathematical programming model.

The computing cost entailed in this step is O*(N)*.

Second step: Full ranking of the goods

The second step only treats the goods constituting the equilibrium set from the first step. One of the difficulties in applying SPM in this step is the need to distinguish between costs and benefits. The problem arises because what is a cost for the buyer is a benefit for the seller; and vice versa, what the seller sees as a cost the buyer considers as a benefit. Nevertheless, Proposition 3.1 below demonstrates that the criteria weights are independent of whether the criterion is cost or benefit. This makes it possible to implement the second step by means of a model that is even simpler than the proposal of Ballestero (1999).

Proposition 3.1: The weight of a criterion is independent of whether the criterion is considered a cost or a benefit.

Suppose a set of s benefits corresponding to m costs. In SPM, the constraint corresponding to the fictitious alternatives $w_\lambda z_\lambda^* + \sum_\mu w_\mu z_{\mu*} = 1$ generates the following set of equations:

$$w_1(z_1^* - z_{1*}) = w_2(z_2^* - z_{2*}) = \cdots = w_{s+m}(z_{s+m}^* - z_{s+m*}) \tag{13}$$

Take $v > s$ and $\boldsymbol{h} = v - s$. Applying a trivial transformation on the original criteria results necessarily in:

$$w_v(z_v^* - z_{v*}) = w_v[(x_{h\max} - x_{h\min}) - (x_{h\max} - x_{h\max})] =$$

$$= w_v(x_{h\max} - x_{h\min}) \tag{14}$$

Thus, (12) can be expressed as a function of the $s+m$ original criteria:

$$w_1(y_1^* - y_{1*}) = \cdots = w_s(y_s^* - y_{s*}) = w_{s+1}(x_1^* - x_{1*}) = \cdots = w_{s+m}(x_m^* - x_{m*}) \tag{15}$$

with $y_i^* = max(y_{ij})$, $y_{i*} = min(y_{ij})$, $x_h^* = max(x_{hj})$, and $x_{h*} = min(x_{hj})$.

Expression (15) provides the same solution as (10), if we perform the transformations $z_{\lambda j} = y_{ij}$ for $\lambda, i = 1..s$ and $z_{\lambda j} = x_{h\max} - x_{hj}$ for $\lambda = s+1..s+m, h = 1..m$. Thus it is demonstrated that the weights are independent of whether a specific criterion is a cost or a benefit.

Corollary 3.1: The EI regarded from the buyer's viewpoint is inversely proportional to the EI from the seller's viewpoint.

Suppose without loss of generality that price is the first criterion and that the *m* features influencing the price occupy the next following positions. Then the EI on the seller's side can be calculated by (16):

$$\mathrm{EI}_j \text{ seller} = w_1 y_j \Big/ \sum_{h=2}^{m+1} w_h x_{hj} \tag{16}$$

while the buyer's side index requires expression (17):

$$\mathrm{EI}_j \text{ buyer} = \sum_{h=2}^{m+1} w_h x_{hj} \Big/ w_1 y_j \tag{17}$$

Resulting from Proposition 3.1, and given that the equilibrium set is the same for both sides, the weights of each criterion are likewise identical for both buyer and seller. It follows that expression (17) is the exact inverse of (16). This relationship only holds if the second step is applied to the goods in the equilibrium set and not to the two sets of non-efficient goods that would result from taking the viewpoints of buyer and seller separately.

Definition 3.4: Moderate pessimism (Ballestero, 2002)

A moderately pessimistic decision maker is one who assumes conservatively that the most favourable in a set of possibilities is not the one that will ultimately take place (without making conjectures as to the other possibilities).

This is a key definition in the SPM approach, as was indicated previously. Including the set of fictitious alternatives that make up the system of equations (9) –called a marginal set in Ballestero (2002) – is clearly justifiable on practical grounds. It deals with alternatives that have extreme values for their criteria (the highest value for one of the criteria, the lowest value for the rest), which makes them less attractive than other, better-balanced criteria. Ballestero (2002) shows that this constraint makes the non-inefficient alternatives attain values greater than unity; that is, they are preferable to the fictitious alternatives. The fictitious alternatives are all assigned a value of 1, so that they are all equally preferable for a moderately pessimistic decision-maker. The equal ranking for these alternatives is not followed by other MCDA approaches, as swing weights in MAUT models, that explicitly ask the decision maker to compare and rank such alternatives. Nevertheless, since our main objective is to get a one and only ranking of the alternatives, this ranking can not depend on the individual preferences of a single buyer/seller. This would mean, in the most extreme case, to have as many rankings as buyers or sellers.

Let the set of alternatives be the following:

$$a_1 = [z_1^*, z_{2*}, \dots, z_{s*}, \dots, z_{s+m*}]$$

$$a_2 = [z_{1*}, z_2^*, \dots, z_{s*}, \dots, z_{s+m*}]$$

$$\dots$$

$$a_s = [z_{1*}, z_{2*}, \dots, z_s^*, \dots, z_{s+m*}]$$

$$\dots$$

$$a_{s+m} = [z_{1*}, z_{2*}, \dots, z_{s*}, \dots, z_{s+m}^*] \qquad (18)$$

Presented with this set, an extreme pessimist would only consider a single alternative, the one consisting of the worst values for the criteria. A moderately pessimistic decision maker admits the possibility that one criterion may reach the highest possible value while the others take the minimum value. Taking this moderately pessimistic approach, let us compare, without loss of generality, alternatives a_1 and a_2. It follows from definition 3.4 that a decision maker would set aside the first and second criteria, z_1 and z_2, because they are the most favourable to alternatives a_1 and a_2 respectively. In this way, the two alternatives would be composed of the remaining criteria, and they would be (i) indistinguishable from one another, with values $[z_{3*}, \dots, z_{s+m*}]$ for the criteria, for which reason they can all be assigned the same ranking (e.g., a value of 1); and (ii) because they have the worst possible values for their criteria, they would be less preferable than any of the non-fictitious alternatives.

Although the moderately pessimistic attitude was originally introduced by Ballestero in order to deal with the problem of the choice of alternatives under uncertain scenarios (Ballestero, 2002), later the same author applied it in a multicriteria context (Ballestero, 2004). Let us reflect on the existing link between both approaches, since a priori they might seem to be in conflict. As mentioned before, to rank a set of alternatives it is necessary to quantify the weight of each of the criteria which take part in the determination of their EI. Without loss of generality and from the seller's point of view: given an initial set of goods, suppose the seller decides to compare the a_i and a_j alternatives, in such a way that a_i exhibits the greatest value over a_j in the z_i criteria, and a_j exhibits the greatest value over a_i in the z_j criteria. Hence, z_i and z_j are the most favourable criteria for a_i and a_j, respectively. When comparing both alternatives, the moderately pessimistic seller will be sceptical about the relevance of criteria z_i and z_j. In fact, believing that the criteria for which his/her property gets the greatest value are the most relevant in the market is typical of an optimistic seller, not of a moderately pessimistic one. Therefore, the decision-maker fears that alternative a_i (a_j) will not be so lucky as it would be the case if its most favourable criteria were the most relevant to the market (Ballestero, 2004, p. 148).

When Definition 3.4 states that the most favourable in a set of possibilities is not the one that will ultimately take place, it means that this possibility will not be considered by the moderately pessimistic decision-maker when taking his/her decision.

Definition 3.5: Moderate optimism

A moderately optimistic decision maker assumes that the most unfavourable of a set of possibilities is not the one that will ultimately take place (without making conjectures about the other possibilities).

Given this attitude, the decision maker would consider as fictitious alternatives those that have only a single criterion at its lowest value and all the rest at their highest value (19):

$$a_1 = [z_{1*}, z_2^*, \ldots, z_s^*, \ldots, z_{s+m}^*]$$

$$a_2 = [z_1^*, z_{2*}, \ldots, z_s^*, \ldots, z_{s+m}^*]$$

$$\ldots$$

$$a_s = [z_1^*, z_2^*, \ldots, z_{s*}, \ldots, z_{s+m}^*]$$

$$\ldots$$

$$a_{s+m} = [z_1^*, z_2^*, \ldots, z_s^*, \ldots, z_{s+m*}] \qquad (19)$$

Like the moderate pessimists, the moderate optimists would compare any two fictitious alternatives, and because of their attitude they would eliminate the attributes with the lowest value. Let the two alternatives again be a_1 and a_2. When criteria z_1 and z_2 are removed, the alternatives are composed of the same maximum values in the rest of the attributes $[z_3^*, \ldots, z_{s+m}^*]$. Unlike for the moderate pessimist, for the moderate optimist the fictitious alternatives represent better options than the non-fictitious alternatives; from which it follows that if the former are allocated unity as index of efficiency, the latter are bound to take lower values.

Proposition 3.2: The approaches of the moderate pessimist and the moderate optimist generate the same vector of criterion weights.

In the previous section, it was set forth that the solution to the second step in the full ranking process was provided by the system of equations associated with fictitious alternatives $w_\lambda z^*_\lambda + \sum_\mu w_\mu z_{\mu*} = 1$, with $\lambda = 1,2, \dots ,s+m$.

Extrapolating the system to alternatives (19), it is easy to deduce the same solution (10) for the weights.

To sum up, the criteria weights are independent not only of whether the decision makers are sellers or buyers, but also of whether they have an optimistic or a pessimistic attitude. The weights remain constant provided the decision makers maintain a moderate attitude in line with definitions 3.4 and 3.5.

4. Case study

For a practical application of the model expounded in the previous section, a database was built of properties in the city of Valencia, Spain, compiled from data provided by a major Spanish valuation company (TABIMED). The information relates to transactions carried out during the second half of 2007.

The model could also be applied to a data base of offered houses; however, in this case, differences between seller and buyer points of view should be considered as a limitation. While housing price is real for the seller, in the sense that he or she shows the willingness to sell the dwelling at the offered price, the same does not occur for the buyer. Price just will be real for the buyer when he/she comes to a deal with the seller about the transaction. In Spain, for example, the final price is estimated to be an average of 5% lower than the offered one. However, when the data base is only comprised by sold housings, like in our case study, prices have been agreed to by sellers and buyers; hence, they could be considered real prices for both sides.

In this case study the variables can be grouped into three categories:

I. Variables at individual property level: price (in Euros), usable space (in square metres), number of bedrooms, number of bathrooms, area of the balcony or terrace (in square metres), floor on which the property is located, quality of construction (on a scale of 1 to 5).

II. Variables at entire building level: number of storeys, lift (a binary no/yes variable), age (in years).

III. Environmental variables: urban environment quality (scaled from 1 to 4), commercial environment variable (1 to 3), income level (rising from 1 to 3).

The variable 'orientation' was removed from those provided by the valuers because it turned out not to be statistically significant for explaining price. The qualitative variables were determined according to the criterion of 'better if more valuable', and were assessed by the whole team of valuers assigned by the firm to the city of Valencia. For example, to assess the value of the urban environment on a scale of 1 to 4, the valuers took account of a series of factors: local district communications (bus, underground, tram), green spaces and recreation areas, distance from the city centre and other important places in the town, good maintenance of road and pavement surfaces, lighting, cleaning, historic importance, and so on.

Before applying the models, it was necessary to transform some of the original variables. For instance, the variables 'number of bedrooms' and 'number of bathrooms' were replaced by the ratios 'area/number of bedrooms' and 'number of bathrooms/number of bedrooms' respectively. The reason for the change in the first case was that if two properties have exactly the same area, the one with larger bedrooms is valued more highly. The second ratio was introduced for a similar reason: the number of bathrooms cannot be valued in absolute terms but only relative to the number of bedrooms.

In order to limit the number of properties analysed and ensure a minimum of homogeneity throughout the sample, they have been taken only from the areas with postcodes 46010, 46020, 46021, 46022 and 46023. These are areas that are close to one another and, most importantly, they share a similar degree and type of urban development. Table 1 is a compilation of the principal statistics for all the properties in the sample.

Table 1. Basic statistics of the variables measured in the sample

	Minimum	Maximum	Average	Standard deviation
Price (euro)	150,000	590,000	259,730.6	91,438.6
Usable area (sq m)	55	176	100.0	23.2
Ratio area / number of bedrooms	20	77	34.0	8.8

Ratio bathrooms / bedrooms	0	1	0.5	0.2
Balcony or terrace area (sq m)	0	80	0.9	6.3
Ratio floor / number of storeys	0	1	0.6	0.3
Construction quality (1-5)	1	5	2.0	0.9
Lift (0/1)	0	1	0.8	0.4
Age (years)	0	77	17.7	12.2
Urban environment quality (1-4)	1	4	2.2	0.6
Commercial environment quality (1-3)	1	3	2.2	0.4
Income level (1-3)	1	3	1.6	0.7

Applying the first step described above produced a total of 32 non-inefficient properties. Their characteristics are shown in Table 2.

At the second stage, the adapted SPM (see Section 3) was applied to the previously mentioned set of properties. It follows from Proposition 3.1 that calculating these weights does not require transforming the criteria which act as cost, and the result is invariant with respect to the viewpoint adopted (seller or buyer) and to whether the decision maker has an optimistic or pessimistic outlook. All that is required is that the decision maker's attitude be moderate. The last column of Table 2 shows the weights that result from applying expression (10) on the original criteria.

Observe how if the criteria would have been standardized so that, $z'_{\lambda q} = {(z_{\lambda q} - z_{\lambda *})}/{(z^*_{\lambda} - z_{\lambda *})}$ all the criteria would have the same unit weight, which simplifies the mathematical expressions maintaining the same results as in the initial focus in which the weights are calculated based on the original criteria.

The possibility that the introduction of a new alternative might change the relative position of the rest of the alternatives should be pointed out. For example, if the price of the new alternative is lower than the minimum price in the current set of alternatives, the relative ranking of the other alternatives may be modified. Nevertheless, this is a problem shared with other methodologies for the ranking of alternatives.

The EI for each property has been calculated from either expression (16) or (17) according to whether it is being done from the seller's or buyer's viewpoint, and it is listed in columns 14 and 15 of Table 2.

Because the weight that results for the price (5.1176E-07) is relatively low compared to the rest of the criteria, it might be thought that the model is undervaluing this variable despite the fact that it can be considered the most important for practical purposes and sums up all the information in the other criteria. To test this hypothesis, the linear correlation coefficient was calculated between the EI for the seller and each of the criteria, and it was observed that the highest value is precisely that of price (92.7%), followed by area (68.2%), lift (51.7%) and age (-46.8%). Similar values have been obtained from the buyer's viewpoint but with the opposite sign, as was to be expected from what was stated in Corollary 3.1. This constitutes confirmation of the hypothesis that price is the most pertinent variable for calculating the efficiency index of properties. With the aim of comparing and contrasting differences with other known full ranking methods, the EI of the housings which comprise the equilibrium set has been calculated by means of the cross-efficiency analysis (Sexton *et al.*, 1986), both in its aggressive and benevolent versions. The aggressive (benevolent) version seeks to minimize (maximize) the efficiency of the population of the DMUs while maintaining the efficiency of the DMU under consideration fixed (Ertay and Ruan, 2005). One important difference between the SPM and the cross-efficiency analysis is the different treatment for the criteria weights: in the SPM this weights are invariable with respect to the analyzed DMU, while with the cross-efficiency analysis the weights can differ from one DMU to other.

Results from the cross-efficiency analysis application appear in the last columns of the table2. Although in the SPM model the EI from both the buyer and the seller point of view are directly related, the same thing does not occur in the cross-efficiency analysis, due to the different weights obtained for the criteria in each DMU. In the aggressive version, the correlation coefficient between both EI is of -50.9%, while in the benevolent version the correlation is of -74.2%. In our opinion, the use of the same weights for the criteria, independently of the decision-maker is the buyer or the seller, and independently of the analyzed housing, is a SPM model advantage. This makes possible the EI for the buyer to be the inverse of the EI for the seller. In other words, to consider that what is good for the seller is no good for the buyer, and vice versa. This hypothesis is not supported when the correlation coefficient between the EI of the buyer and the seller one distances from the -100%, like in the case of the cross-efficiency analysis applied to the two studied versions.

Table 2. Information relating to non-inefficient properties and their EIs

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
1	590,000	130.0	5	3	43.33	0.67	1.00	1	0	4	3	10	0.122	8.224	0.546	0.383	0.913	0.469
2	583,000	167.0	2	3	41.76	0.50	0.44	1	0	3	3	26	0.137	7.303	0.581	0.334	0.941	0.408
4	535,000	120.0	5	3	60.00	1.00	0.31	1	0	4	3	5	0.112	8.903	0.544	0.385	0.878	0.484
5	527,748	143.0	3	3	47.67	0.67	0.83	1	0	2	3	5	0.123	8.151	0.573	0.357	0.863	0.441
6	526,054	138.2	3	1	46.08	0.67	0.77	1	0	4	3	0	0.130	7.684	0.685	0.352	0.892	0.454
7	525,000	93.0	2	1	31.00	0.67	1.00	1	20	2	1	3	0.154	6.476	0.183	0.275	0.840	0.335
8	520,000	135.0	5	3	45.00	0.67	0.17	1	0	4	3	10	0.117	8.555	0.531	0.401	0.841	0.504
10	500,000	170.0	2	2	42.50	0.50	0.75	1	0	2	2	28	0.129	7.739	0.504	0.375	0.811	0.457
15	475,000	132.0	5	3	33.00	0.75	0.33	1	0	2	3	10	0.114	8.761	0.506	0.407	0.788	0.499
52	350,000	113.1	1	3	37.70	0.67	0.88	1	0	3	3	50	0.079	12.652	0.401	0.519	0.673	0.556
65	339,500	80.0	2	3	40.00	0.50	1.00	1	0	2	2	16	0.091	10.990	0.426	0.455	0.719	0.514
66	336,567	78.6	4	2	39.30	0.50	0.71	1	0	3	2	0	0.094	10.617	0.539	0.483	0.693	0.594
78	320,640	80.0	1	3	40.00	0.50	1.00	1	0	2	2	20	0.088	11.403	0.444	0.453	0.723	0.501
125	271,066	70.8	2	2	23.59	0.33	1.00	1	0	3	3	9	0.077	12.983	0.388	0.546	0.621	0.626
131	264,445	60.7	4	2	60.68	1.00	0.71	1	0	3	2	0	0.066	15.151	0.427	0.604	0.573	0.732
157	243,636	77.5	2	1	25.83	0.33	0.33	1	0	4	3	9	0.078	12.900	0.365	0.562	0.578	0.683
165	240,000	113.0	2	3	56.50	0.50	0.33	0	0	3	3	32	0.064	15.590	0.484	0.660	0.737	0.786
180	230,400	126.0	1	1	42.00	0.33	0.50	0	0	2	2	20	0.088	11.364	0.576	0.541	0.796	0.690
188	226,000	81.0	4	2	27.00	0.33	0.75	0	0	3	2	35	0.070	14.383	0.473	0.718	0.754	0.828
210	213,000	103.0	1	3	51.50	0.50	0.22	0	0	3	3	32	0.061	16.354	0.505	0.653	0.759	0.768
222	205,000	142.7	1	2	28.55	0.40	0.56	1	0	2	2	45	0.059	17.076	0.253	0.806	0.390	0.915
284	180,000	64.0	4	1	64.00	1.00	0.25	1	23	2	3	0	0.044	22.782	0.074	0.834	0.295	1.000
308	167,516	61.0	1	1	20.32	0.33	1.00	0	0	1	1	9	0.086	11.606	0.611	0.522	0.853	0.621
314	165,000	62.1	2	3	31.05	0.50	0.67	0	0	3	3	44	0.046	21.624	0.388	0.876	0.655	0.924
321	161,900	67.0	1	2	22.33	0.33	1.00	1	0	2	3	30	0.047	21.122	0.247	0.851	0.405	0.878
324	161,178	64.0	2	1	32.00	0.50	0.22	1	0	2	2	9	0.060	16.600	0.266	0.691	0.423	0.820
333	157,000	78.0	1	1	19.50	0.25	1.00	0	0	3	2	35	0.058	17.181	0.428	0.800	0.664	0.899
335	156,000	60.0	2	1	30.00	0.50	0.60	0	0	2	3	30	0.056	17.975	0.426	0.739	0.723	0.808
341	151,050	90.0	1	1	22.50	0.25	0.33	0	0	2	2	20	0.070	14.197	0.486	0.667	0.696	0.820
342	151,000	70.0	2	1	23.33	0.33	0.40	0	0	2	2	8	0.070	14.313	0.521	0.657	0.723	0.826

344	150,000	67.0	1	3	33.50	0.50	0.80	0	0	3	3	40	0.042	23.580	0.371	0.915	0.610	0.971
345	150,000	90.0	1	2	22.50	0.50	1.00	0	0	2	2	35	0.050	19.968	0.375	0.868	0.562	0.964
w_j	5.1176E-07	0.00205	0.05629	0.11258	0.00506	0.30022	0.27020	0.22516	0.00979	0.07505	0.11258	0.00450						

Legend:

(1) Property identification number. (2) y_1 - Sale transaction price. (3) x_1 - Usable area. (4) x_2 - Construction quality on a scale of 1-5. (5) x_3 - Income level on a scale of 1-3. (6) x_4 - Ratio usable space / number of bedrooms. (7) x_5 - Ratio number of bathrooms / number of bedrooms. (8) x_6 - Ratio floor where the property is situated / number of storeys in the building. (9) x_7 - Lift. (10) x_8 - Balcony or terrace area. (11) x_9 - Urban environment quality on a scale of 1-4. (12) x_{10} - Commercial environment quality on a scale of 1-3. (13) x_{11} - Age. (14) EI from seller's viewpoint in SPM. (15) EI from buyer's viewpoint in SPM. (16) EI from seller's viewpoint with the aggressive version of cross-efficiency. (17) EI from buyer's viewpoint with the aggressive version of cross-efficiency. (18) EI from seller's viewpoint with the benevolent version of cross-efficiency. (19) EI from buyer's viewpoint with the benevolent version of cross-efficiency.

N.B. The classification of variables as cost (x) or benefit (y) has been done from the seller's viewpoint. To change to the buyer's viewpoint, it is only necessary to invert the notation.

5. Conclusions

This study reports an application of the single price model to the ranking of alternatives or goods in a scenario where multiple sellers and buyers are considered, and an application to the residential market is presented. By making a slight adaptation of the original model from Ballestero (1999), the equilibrium set of goods is characterised for seller and buyer, and from that the EI is calculated.

The model used has a number of advantages over other methods for making a full ranking of a set of efficient alternatives. It is a model based on Compromise Programming, has a robust axiomatic basis; and when it calculates the weights of each attribute, it assumes that the decision maker has a moderate attitude. The study demonstrates that in the model put forward (i) the weights assigned to each of the criteria are independent of whether the decision maker is the seller or the buyer, and this simplifies calculating the EI; (ii) the EI for the seller is inversely proportional to that for the buyer –something which makes good economic sense–; (iii) the calculation of cost and/or benefit weights coincides no matter whether the decision makers are optimistic or pessimistic, provided that in either case they maintain a moderate attitude. Furthermore, the weights of each criterion are independent of the good valued, and determining them does not carry a high computing cost. Indeed the model's implementation in two steps has a cost that increases only linearly with the number of goods analysed. The EI obtained by using this model not only makes it possible to rank the goods in an ordinal way, it also evaluates differences by cardinality.

Finally, the proposed model has been illustrated by taking a broad sample of residential properties in the city of Valencia and observing that price is by far the most significant variable for calculating the EI.

Acknowledgements

The authors are sincerely grateful to the referees for their helpful comments that lead to important improvements in the paper. The authors also gratefully acknowledge the help of the TABIMED in making available the data used for the case study. This paper has been partially funded by Spanish Ministry of Education and Science research project SEJ2007-67937. We would like to thank the R&D&I Linguistic Assistance Office, Universidad Politécnica de Valencia (Spain).

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