The Polar Projection – an Alternative Approach for Spatial Analysis

Tadeusz Zipser
Department of Spatial Planning
Wrocław University of Technology, *tadeusz.zipser@pwr.wroc.pl*
Phone/fax numbers: +48-71-3206-354/+48-71-3212-448

Wawrzyniec Zipser
Department of Spatial Planning
Wrocław University of Technology
wawrzyniec.zipser@pwr.wroc.pl

Keywords: simulation; intervening opportunities; distance; travel time; density; spherical projection

Abstract: The polar projection presented here results from the need to have a uniform yet fairly simple method for recording the various phenomena and relations pertaining to land development.

The contact infrastructure anisotropy and variable population and development density lead to socioeconomic space discontinuities.

In the "spherical" model all subareas are characterized by coordinates on some "basic" spherical surface. The basic "air" distance between them can thus be identified directly. However, from the points corresponding to the subareas we can draw the radii of the sphere to represent the concrete connection between two subareas available in this moment by means of a segment connecting the corresponding radii at different distances from the centre of the sphere. Thus connections longer or shorter than the basic distance are to be found on an other sphere than the basic sphere. Accordingly, the degree of connectedness is characterized by the corresponding length of radius to express several alternative connections to employ a spatial or time measure and to state whether the lengthening of the distance is due to a "roundabout" connection or to the low speed parameters etc.

It is thus possible to dodge the difficulties which in such cases emerge because of the necessity to employ both types of models – the gravity and the "opportunity" models.

In order to simplify things instead of concentric spheres we can use parallel planes, intersected by a bundle of lines.

A projection of the developed area can consist of selecting the base surface where the element distribution density matches the "standard" density. Now any deviation from this standard density can be presented as moving the surface towards or further away from the pole.

Using this approach to register the socioeconomic space, we can bring all information down to the length of the "projection ray".

Many of the tasks pertaining to modeling events and processes require certain arithmetic operations or use of mathematical functions to measure results. This is performed "on the side" without visual "contact" with the area. A twin system, which can serve as the geometrical basis for registering and building certain relations may be involved.

Each land area development project can advocate a special way of viewing space and the phenomena occurring within it.

These phenomena are mainly contact infrastructure anisotropy and highly variable population and development density. This leads to a variety of socio-economic space discontinuities, even up to the point of total lack of communication infrastructure or lack of investments in certain areas.

Both of the main characteristics of these discontinuities, i.e. anisotropy and variable saturation, are difficult to model. This pertains to either of them separately, not to mention if we had to uniformly deal with them in their entirety.

The basis for modelling such a system could be derived from the concept of the "spherical contact space". The starting point for the herein proposed notation of quantitative characteristics of territorial development should be established as a predefined point, which will serve as the pole of central projection. The rays coming out from this point form pyramids, which intersect with parallel planes or spherical surfaces at various distances, forming different size yet similar (in the sense of Euclidean geometry) figures. Let's assume that one of these surfaces is ‘special’, serving as a reference. All cross-sections closer to the pole from this surface will have a smaller area, whilst the ones further away, will have a larger area. The same will obviously apply to the lengths of sections defined by pairs of rays originating from the pole, as well as the volumes of related 3D objects. If we will treat the two-dimensional figures or sections on the reference plane, (as well as possibly 3D objects supported on that plane), as clearly defined elements, then all objects of such type, defined by the given pyramid at any position, can be viewed as a certain quantitative characteristic of the given element. Applying this general scheme to area development phenomena consists of assuming that the reference element is the spherical surface of earth, a real and tangible territory, divided into different regions, which identify actual places.

Of course this is an idealized surface, treated as if having only two dimensions. The measurement units selected for each given projection have been chosen, such as to relate closely to the real and natural dimensions, if the measurements are made at the base surface.

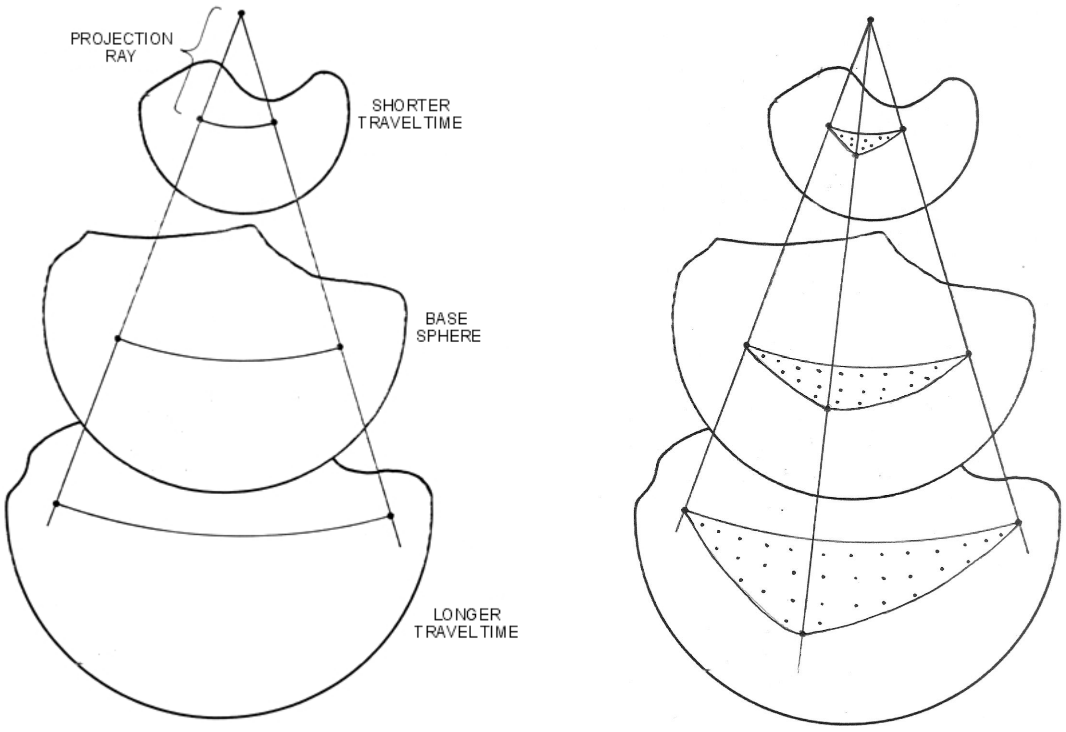
In order to better explain this notation it will be easiest to present an example, which pertains to lengths of sections. This example will illustrate one of the most important theoretical as well as practical aspects. The idea is, that physical distance (measured in meters, kilometers, miles etc.) can be travelled through with different times, depending on the place and conditions – mainly depending on the used mode of transportation. Nevertheless, using the so-called time distances (i.e. distance defined in time units) should not obscure the geodesic distance, which is constant. First of all – because that distance identifies objects in the geographical space, and second – because only knowledge of geodesic distance allows us to define the changes in time distances, in view of network shape, topographic barriers, travel speed and costs.

Two rays coming out from the centre of the sphere (pole) intersect all of these surfaces, defining two points on each of them, separated at various distances. If in all cases we measure this distance using the same unit, which is time, it means that at each of these spherical surfaces these times will be different because of different distance sections.. Let's now assume that these two points on the base sphere define two places within some territory. The geodesic distance is recognizable the whole time, on the other hand, because of the various possible ways of travel, which depends on the communication infrastructure, this will be reflected differently in the individual spheres.

There is also the aspect of variable density of geographical elements, especially the social and economical objects. In this view, the given element becomes part of the space.

Following the earlier approach, a projection of the developed area can consist of selecting the base surface in such a way, that the element distribution density matches the real “standard” density. Now any deviation from this standard density can be presented as moving the surface towards or further away from the pole. When we move this reference surface towards or away from the pole the elements located on it must become more or less densely packed, as in real life (fig. 1).

Fig. 1. The scheme of distance and area projection



Using this approach to register the socio-economic space, we can bring all information down to the information about the projection sphere distance to the centre pole or in other words the length of the “projection ray”.

The difference observed when comparing the actual “network” level, where we don't deal with theoretically shortest geometrical connection but with the actual length of given section, to the reference ('geodesic') level, results from the track elongation owing to network's shape on the given area. In this manner we receive a numerical compact characterisic of the network and its critical parts.

Of course the significant changes in the projection rays can be attributed to access to higher level “contact infrastructure” as for example airplane connections.

The proposed notation assumes that there is a certain “base sphere”, which relates to the assumed 'standard' travel speed. All distances on the base surface can therefore be stated in time units, assuming that defined travel speed. Besides the base sphere, we can imagine an unlimited multitude of other spherical surfaces, having a common curve centre with the base sphere. I.e. concentric surface at different ray length distances from the centre (fig.2).

In the “spherical” model all subareas are characterized by coordinates of their situation on some “basic” spherical surface. The basic “air” distance between them can thus be identified directly. However, from the points corresponding to the subareas we can draw the radii of the sphere to represent the concrete connection between two subareas available in this moment by means of a segment connecting the corresponding radii at different distances from the centre of the sphere. Thus connections somewhat longer than the basic distance are to be found on a more external sphere than the basic sphere. Accordingly, the degree of connectedness is characterized by the curvature of the sphere used or simply by the corresponding length of radius. In this way it is possible to express several alternative connections between the subareas, to employ a spatial or time measure, to note the frequency of connections in the form of two segments and the areas contained between them etc. An adequate "spherical analysis" of the communication network makes it possible to clearly state whether the lengthening of the distance between two subareas is due to a "roundabout" connection or to the low speed parameters on the way etc.

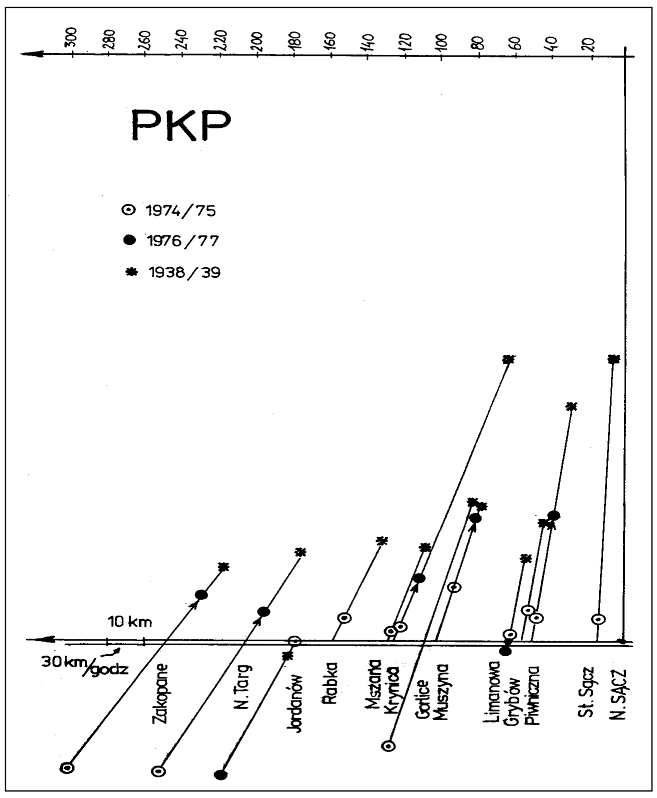
It is therefore possible to notice that for two points (for example two cities) we can have alternative ways of travelling the distance - bus, passenger car, express train, plane. We can apply this also to information flow post, telephone, radiocommunication, hotlines, e-mail etc.

In order to simplify things, instead of the polar projection we can use an alternative presentation form. Instead of concentric spheres we can use parallel planes, intersected by a bundle of straight lines. The Tales theorem guarantees us that we maintain the same relation between the lengths of the sections between the points on the surface and the distance of the surface from the "central" point of the straight line bundle. However, instead of talking about shortening the ray length, here we talk about the surface being closer to the central point.

This simplification can also prove convenient for other applications, especially when treating the "base surface" in a more abstract way or when having to form some geometrical constructions using it.

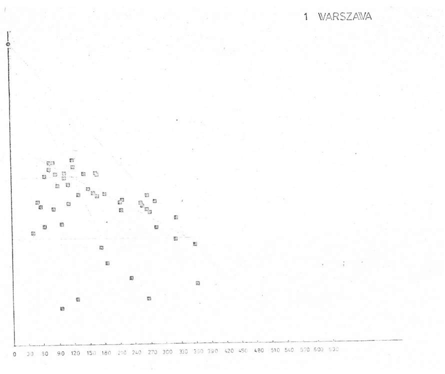
A good example illustrating the changes occurring in time is the polar projections of Nowy *Sącz* connections to other cities in years 1938, 1974 and 1976 (fig.2).

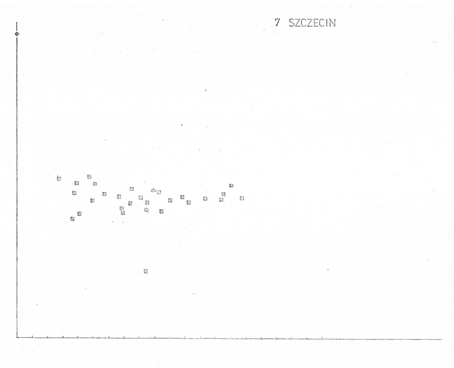
Fig. 2. Changing time of railway connections in 3 periods.

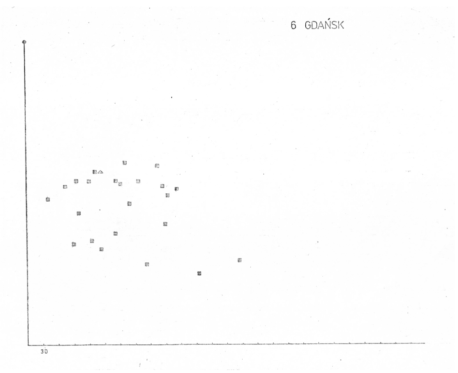


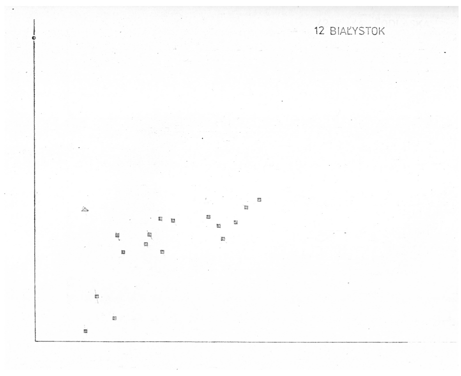
Using this technique, we collected information about the Polish railway connections in selected periods as well as several transeuropean links. This kind of visualisation allowed for a comparison of the patterns of direct railways connections being at disposal of about fifty main urban centers of the country. By erasing the radial and time distance lines we obtained a “cloud” of points. The shape, density and alternatively stratified desintegration of such cloud make a basis for classification and interpretation of a status in accessability system. Assuming a reference travel speed at for example 50 km/h, we were able to plot straight-line sections, visualizing the travel times between certain stations. Superimposed sections of different lengths, mean different travel times i.e. travelling the same route with different speeds. These sections allow for easy comparison to each other, either comparing the same connections during different time periods or comparing totally different travel routes to each other.

Fig. 3. Comparison of railway connections of 4 cities.









Now we have to extent the possible application of spherical or better to say polar-layer projection in order to grasp some aspects of the process of economic and social development. For, it is only with these two systems: that of information flow and the transport network that we can safeguard a full field of penetration which is a precondition of relations between settlement units.

Somewhat connected with migration is the need to define the admissible reach of everyday commuting as still acceptable. Neither the gravity model nor the “intervening opportunities” model use such a distinct threshold distance (the latter model knows no absolute distance at all). Thus in spite of the fact that the adoption of distance intervals in the “general shift” variant of the Bernoulli model (the mathematical basis of “intervening opportunities” in M. Schneiders formulation) yields a similar effect as the differentiation of admissible distances in different categories of motions creates difficulties here. At any rate we still believe the intervening opportunities approach is capable of generating good reconstruction of real processes in settlement systems.

To complete this survey of the imperfections of the model in its present form it is to be indicated that the simulation model of settlements development ought to represent not only individual inhabitants but also social groups, especially families as well as certain social organizations and institutions. This is in accordance with the above-discussed requirements concerning the stratified nature of the system of accessibility and, probably, of information and of modelling the endogenous forces.

After this critical review of the “intervening opportunities” simulation model let us briefly point out a few regularities which we suppose govern the growth of the settlement network and as such must be considered in the model.

The communication system understood as comprising the system of transportation and the system of information transmission is built so that the links between two centres are built usually by the stronger one (stronger in virtue of functional rank and of employment).

The subsystems of communication are different on different levels of contact (as a rule, one centre is a node in several subsystems).

Centres of higher rank have a broader range of functions (usually without any detrimental effect on the basic functions) and thus a more comprehensive range of contact levels.

As the links of the subsystems of communication are built by the stronger centre this latter must perforce be interested in putting up such links (the given link favours the maintenance, or attainment, of equilibrium in the stronger system).

There is probably a certain threshold of additional benefit which conditions a change in location of a physical unit or institution.

Different subjects exhibit different times of reaction.

The above list should be completed with the regularities which underlie the existing simulation models of urban development, especially the principle of least effort which is involved both in “opportunity” — type and gravity models, or the balance principle which demands an agreement of the number of destinations and the number of acceptances in separate settlement units, what generates the concentration phenomena.

In addition to the above observations concerning actual phenomena it is also necessary to take into account the operations aspect of the prospective new model, above all the concomitant constraints.

Still worse, this applied occasionally to procedures which though subsidiary are indispensable and hence that laboriousness may not be noticed immediately as it is not rooted in the complexity of the model itself. A good example of this are the tables of distances between the subareas of the model which involve the creation and, subsequently, the handling of excessively big matrices. More analogous examples could be given. If the model is to analyze many variants and if, moreover, it should easily admit new extensions in the form of new detailed procedures then the purely operative aspect of the model may ultimately prove to be of decisive importance. Therefore in the endeavour to introduce a new model we bring up these problems to the fore with a view to finding a convenient operative procedure. It is advisable to obtain a physical analogue for the model as this may make it easier both to devise a calculation apparatus together with its further sophistication as well as an actual construction of the analogue as a concrete physical object.

The shape of such an analogue in the model is largely determined by the difficulties inherent in the description of the mutual spatial dependencies, which usually boil down to the table of distances. As this touches upon the “spherical” model of the communication network let us say but a few words to this problem.

A similar abstract structure constitutes the analogue framework of the extended model. Subareas — usually coinciding with settlement units — are determined on the basic sphere (which, e.g., may correspond to the speed of a walker or of the slowest means of transportation). The radii running out of the subareas converge in the centre of the sphere (or, in more general applications, in several “centres” of the curvature). The radii coincide with the regions’ “axis of development” on which are marked the individual spheres of activity, creating spatial relations. Therefore on the “bottom” of the axis its point or segment represent an individual inhabitant of the settlement as an individual object moving to work or to other destinations. Another segment represents the family as an object of the daily relations of more differentiated and more numerous relations. Different places at the axis mark workplaces of different specificity, administrative organs, as well as individuals of some degree of “institutionalization”, including outstanding experts, social leaders etc.

The particular zones of the radius correspond to different modes of representation of the geographical space. The closer the centre of the curvature the closer the points of intersection of the given sphere by the radii pertaining to other subareas. But no all subareas need be represented by corresponding objects or subjects, in some parts the course of their axes may be “empty”. In this manner, for instance, the sphere of contacts of some institution or some specialist will not include minor localities with but a poor range, of functions although physically they may be quite close. By considering the contacts upon a definite curvature it is thus possible to “jump over” those units and to dodge the difficulties which in such cases are bound to emerge because of the necessity to employ both types of models — the gravity and the “opportunity” models.

The specific manner of putting up contacts is characterized by a sphere specific for the given type of subject. For instance, a sphere of a big radius corresponding to slow means of movement, say on foot, by streetcar or shuttle train, will be used for daily commuting to work, whereas a specialist's contacts may take place by means of an express train or by air, that is on another “curvature”. Analogously, for contacts of different institutions between each other a “curvature” for telephone or e–mail connections may be used. In exceptional cases, a “red line” permitting immediate contact may represent a zero curvature radius.

Nevertheless the mode of performing of a contact like a choice of transportation means is now not the most important factor. It has to play a supportive role. The crucial scheme is to reflect the diversity of activities, their selectivity and hierarchical arrangement. So the lenghts and areas represent more abstract features and the flows of information as well as the presence of structural patterns are now essential.

This two-fold process of emergence of relations which in most cases are not emitted “blindfold” but are preceded by a penetration of the area or by the spread of information suggests the idea to create two separate but interrelated spaces in the model: the space of stimulation and the space of realization of the contact. This may be envisaged by means of another sector of the sphere obtained by producing the radii-axes of the subareas beyond the centre of the sphere. In this way we obtain another picture of the area described which is symmetric with respect to the sphere centre.

Of course other pictures of that duality can also be envisaged. What is essential is only that the subarea should find its representation in the two systems, one to be used in the simulation of the processes located in the “information or stimulation space” and the other in the geographical space serviced by concrete communication systems. It must be pointed out that, in this model, means of contact such as telephone or e–mail networks are thought to belong to the space of realizing the contacts, for although they do not serve the transport of people or cargo they nevertheless involve the choice of one concrete addressee (which, incidentally, does not preclude any further consequences resulting from the information conveyed in such a way). In the stimulation space, instead, we have to do with the spreading of information without any concrete address, unlimited information so to say. The circumstance that such information does not reach all potential subjects of the relations is not caused by any strict selection of the receivers but by the fact that one type of information naturally does not concern all. For example it is futile to make public the cooperation requirements of some enterprise to another, or the news of a shop having been opened in a town may be of interest to the people living an its neighborhood but need not be of any direct importance for research institutions.

If we assume that the existence of some functions in the subarea can be associated with some specific means of contact for each of them, then we could considerably facilitate the description of such networks which in turn may permit to make the algorithms more intricate. The existence of a function in the region also implies that there is a system of contacting on the “curvature” corresponding to it or that the system tends to such a system as both necessary and desirable. For instance, it is normally unlikely that a big city should not have a train connection to the capital city or that an enterprise should have not a single telephone number.

But then, if relatively good connections exist already, certain functions are attracted to the region. In such cases we have to do with a sequence of feedback relations and the model must not ignore it. The mechanism of developing a network of connections may be regulated in this model by the gravity algorithm. The computed power of mutual "attraction" of centres pushes them, in a sense, upward along the radii into some larger curvature where distance decreases (simultaneously overcoming the resistance of installation cost or the cost of rationalizing a new pattern of connections defined by a separate submodel).

The tasks to be fulfilled by the algorithm based on the idea of "intervening opportunities" include, among others, assessing the process of realizing the proper contacts and signalizing the existing deviations from equilibrium (i.e. surpluses or deficits, of acceptances with respect to the number of opportunities) along the pertinent radius. Such signals are transformed in the information (stimulation) space into corresponding information emission or absorption. After the simulation of the process, that is after the information has spread, the signals from the stimulated subareas will be conveyed along their radii to the physical space (the realization space) where they may evoke some concrete response, say a migration or a location of a new function.

In that way the physical neighbourhood relation will be replaced by a hierarchical network in wich some separated and distant points may be very close to eachother in the sense of preference and priority.

This last fragment is a description of the basic simulation procedure of the model, which consists in a continual circulation of impulses between the two spaces. But it is known empirically that the time of reaction differs from situation and from subject to subject. For instance, mean reaction time is relatively short when an individual makes a decision to change his job, the place of shopping or even place of residence, but the relocation of an industrial plant or improvements in traffic lines require as a rule much more time. The model makes the following provisions for this variation in reaction time.

Although information about a new situation in the physical space is furnished to the information space "along the pertinent radii — axes of the subareas" immediately the response signal in the physical space is switched on with some delay, different for different subjects. This can be envisaged in the form as though of a spherical wave which propagates from the sphere centre along the radii of the subareas in the stimulation space arriving latest at those subjects which are situated at the farthest points of the axes (on the smallest curvature). While running through the various spheres the wave "switches on" their response signals thus starting the reaction.

It is seen immediately that owing to such a mechanism before some subjects have responded to certain impulses other subjects of considerably shorter reaction time will not only be able to react but even receive the impulses generated by the secondary action even several times. In this manner there appear cycles of different periods superimposing upon one another, the structure of such “loops” in the algorithm permitting abrupt change-overs from “quicker” to “slower” cycles whenever definite critical values are overstepped. This occurs when the expected, reaction of the more inert subjects must be suddenly modified whenever the situation has changed essentially. Such abrupt change-overs can be called “critical jumps” or “intervening information jump”.

The state of stimulation of the relation as the correspondence of information emission and absorption can be represented in the analogue in the form of loads of different signs and this convention is carried over to the physical space to the above-mentioned mechanism of gravitation between the, subareas such that can affect the “curvature” (sphere) of real contact. Due to this certain masses can, in spite of their dimensions, be indifferent to one another (have the same sign). This is the case when for instance a starlike pattern of airplane connections emerges between the country's capital city and the capitals of the provinces ignoring the need of connecting the provincial cities directly with one another.

It is possible to incorporate the concept of marginal extent of real contact as the boundary time distance beyond which it becomes unrealistic or unprofitable into the model.

Notwithstanding its apparently complicated structure the model can be easily presented in the form of a complex algorithm. Surprisingly enough, the spherical shape of both “information” and “physical” spaces makes it easier to write down many data.

CONCLUSIONS

The justification presented in favour of the polar projection results from the need to have a uniform yet fairly simple method for recording the various phenomena and relations pertaining to land development. Many of the tasks pertaining to modelling events and processes require certain arithmetic operations or use of mathematical functions to measure results. This is performed, as we say “on the side”, without visual “contact” with the area to which they pertain. This fact encouraged us to supplement the polar projection with a twin system, which can serve as the geometrical basis for registering and building certain relations. In other words, it seems that in terms of the new model which can be called the spherical model of the space of activities it should be possible to derive certain interpretations of real phenomena. For instance, the dilatability of functions along the radius of the subarea, in other words, the subarea's participation in many spheres of activity enjoins considerable costs of “organizing the space of contacts” (among others, by maintaining the network of connections to the given subarea from different distances and by different means). The limited possibilities of some subareas as regards expenditures may explain their dissociation from their natural "influence" areas (leaving the lower spheres of activity). This can be observed in cases of highly specialized towns as well as in some of the biggest cities, such as regional or national capitals which concentrate diverse functions. Certain differences between agglomeration types as well as the process of generating conurbations itself can be interpreted similarly.

The polar-spherical projection seems to be appropriate to reflect the multilevel structure of our urbanized civilization still containing many local and spatially limited systems as well several "globalization" phenomena. It is also easy adaptable implement as it operates with very simple formula of a "lenght of radius" inside of flexible set of differently defined 3D space sectors.

REFERENCES

Catanese A.L, 1972, New perspectives in urban transportation research. Toronto - London.

Pyers C.E., Evaluation of the intervening opportunities trip distribution model. 44th Annual Meeting of Highway Research Board, Washington.

Schneider M., 1961, Annex. In: Chicago Area Transportation Study, v. 2, Chicago.

Zipser T., 1973, A simulation model of urban growth based on the model of the opportunity-selection process. Geographia Polonica, v. 27, 119-132.

Zipser T., 1976, The territorial-production complex as a self-balancing network of spatial relations. Economic models in regional development and planning.,149-164.