The effect of private road supply on the volume/capacity ratio when firms compete Stackelberg in Road Capacity

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Abstract

We study road supply by competing firms between a single origin and destination. In previous studies, firms simultaneously set their tolls and capacities while taking the actions of the others as given in a Nash fashion. Then, under some widely used technical assumptions, firms set the same volume/capacity ratio as a public operator would and thus have the same amount of congestion and travel time. We find that this result does not hold if capacity and toll setting are separate stages—as then firms want to limit the competition in the toll stage by setting lower capacities—or when firms set capacities one after the other in a Stackelberg fashion—as then firms want to limit their competitors' capacities by setting higher capacities. In our Stackelberg competition, the firms that act last have few if any capacity decisions to influence. Hence, they are more concerned with the toll competition substage, and set a higher volume/capacity ratio than the public operator. The firms that act first care more about their competitors' capacities they can influence: they set a lower ratio and have the largest capacities. The average volume/capacity ratio is below the public ratio, and hence the average private travel time is too short. Still, in our numerical model, for three or more firms, welfare is higher under Stackelberg competition than under Nash competition, because of the larger Stackelberg capacity expansion and lower tolls.

1. Introduction

As government budgets become increasingly tight, it becomes ever harder for them to finance road expansion in the face of ever growing congestion. This has sparked a rising interest in private firms supplying road capacity instead of the government. Furthermore, there is a widespread view that private firms operate more efficiently because of their profit motive, thereby lowering the cost of capacity. Finally, private roads offer a way to introduce congestion-externality tolling in the face of strong public opposition to tolling of existing public roads. In Western Europe, about a third of the highway network is currently under concession (Verhoef, 2007). In the USA, private roads and express-lanes are becoming increasingly common; and also in developing countries private roads are increasingly popular.

Yet, there are also disadvantages to private road supply. The private road invariably has marketpower: it is after all impossible to have an infinite number of private roads in parallel to ensure a perfectly competitive outcome. Hence, firms can set tolls and capacities that might be profit maximising but that are not socially optimal. An important question is how harmful for welfare is this and how does this change with the number of firms.

The early literature looked at toll setting by a monopolist on a given road. Unless demand is perfectly elastic, the monopolist generally sets a higher toll than the public operator and consequently has less congestion and shorter travel times (see, e.g., Buchanan (1956), Mohring (1985), Verhoef, Nijkamp and Rietveld (1996), and de Palma and Lindsey (2000)).

But this argument ignores capacity setting. Xiao, Yang and Han (2007) study private firms

building and operating parallel roads between a single origin and destination. Firms simultaneously determine capacity and tolls while taking the actions of the other as given: i.e. there is a single Nash-competition stage (also called an open-loop game in the literature). Now, firms set the same volume/capacity ratio—i.e. a lower service quality—as a public operator, and hence have the same amount of congestion and travel time. Moreover, the ratio is independent of the actions of other firms and the number of firms. Thus, private supply does not lead to a distorted choice of the amount of congestion. Still, firms do set a higher tolls and lower capacities, and have fewer users. Wu, Yin and Yang (2011) find that the constant ratio result also holds in a general network. These results are conditional on there being neutral scale economies in road building and travel time being homogeneous to the degree zero in volume and capacity: i.e. if both the number of cars and capacity double, travel time remains the same, and travel time only depends on the volume/capacity ratio.

The crucial assumption behind the above results is that capacities and tolls are set simultaneously. De Borger and Van Dender (2006) have separate Nash-competition substages for capacity and toll (i.e. two-substages Nash or a closed-loop game). Now, firms set a lower capacity and higher ratio, as this lessens the toll competition and increases equilibrium tolls. Hence, private roads have more congestion and longer travel times than the first-best network. This is opposite to the result in the older literature that only looked at toll setting.

Separate stages for capacity and toll seem more realistic as it takes a long time to build a road whereas the toll could be changed at any moment. However, this still assumes that all firms build their roads at the same time. This also seems unrealistic. Certainly, historically not all toll roads were build at the same moment. And if firms play a sequential capacity game, they will not take the actions of others as given and will not compete Nash in capacities.

We introduce firms first setting their capacities sequentially in a Stackelberg fashion and then simultaneously setting their tolls in a Nash fashion. By setting a larger capacity, a firm can induce firms that follow to set lower capacities, which increases its marketpower and profit. Accordingly, the first few firms to enter set a higher capacity than they would without these strategic considerations, and this means that their volume/capacity ratios are lower than the public operator's. The effect from De Borger and Van Dender (2006) that they want to set a lower ratio to lessen the toll competition still occurs, but for these first firms this is dominated by the capacity effect. The last few firms to enter have few if any competitors' capacities to hinder. Hence, they care more about the toll substage and set a higher ratio. In our numerical model, with two firms, the first firm has a lower ratio than the public operator; the second has a higher ratio. With five firms, the first three firms have a higher ratio and the last two a lower. The net result is that the average volume/capacity ratio on the private roads is lower than in the public case, and hence average travel time is too short.

Our analyses could also be applied on rail transport, airports, and seaports, but also non-transport infrastructure such as waste disposal and telecommunication. For instance, there is an extensive literature on airlines competing in different market structures (see, e.g., Daniel (1995), Breuckner (2002), and Breuckner and Verhoef (2010)). Zhang and Zhang (2006) study a monopolist airport's choice of capacity and landing fare while its cariers have market power, and find that a profit maximiser sets a lower volume/capacity ratio than socially optimal. Basso and Zhang (2007) extend this by having two airports competing single- or two-substages-Nash. Also in this setting our Stackelberg-capacity game could be used.

2. Analitics

2.1. Basic policies

There are multiple roads connecting the single origin and destination that all have the same congestion technology and free-flow travel time. Generalised travel cost, c^i, of link i (henceforth referred to as cost for brevity) is homogeneous to the degree zero in the number

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of users, q^i, and capacity, s^i. Hence, cost and travel time only depend on the volume/capacity ratio q^i/s^i. The derivative of cost to the number of users is always positive, and to capacity it is always negative. These properties are, for example, met by the commonly used Bureau of Public Roads (BPR) formulation. Capacity costs are proportional to capacity and follow $C^{s,i} = k \cdot s^i$. Throughout the paper, we assume that demand is price sensitive and all roads congestible. We indicate total capacity by S, and total number of users by Q.

We first discuss some basic policies that act as benchmarks for the oligopolistic cases. Since, most of these are basic, we generally keep the discussion short, for a more detailed discussions please see, for example, Verhoef and Small (2007). In the **first-best** (FB) case, the toll equals the congestion externality and capacity is set so that marginal cost of capacity expansion, k, equals the reduction in travel costs it causes:

$$\tau^{FB} = c_Q \cdot Q \,, \tag{1}$$
$$k = -c_S \cdot Q \,. \tag{2}$$

Here, subscripts indicate derivatives; superscripts indicate the situation or road. So, c_Q is the derivative of travel cost to the number of users Q.

If the initial capacity, s^0, remains untolled while the new public capacity, s^1, can be tolled, we are in the **second-best** (SB) situation. Now, the toll has a term that equals the externality on the tolled road and a negative term to attract users away from the untolled road:[1]

$$\tau^{SB} = c_{q^1} \cdot q^1 - c_{q^0}^0 \cdot q^0 \cdot \left(\frac{-D_Q}{c_{q^0}^0 - D_Q} \right), \tag{3}$$

where D_Q is the derivative of (inverse) demand to the total number of users. Conversely, the capacity setting basically follows the same rule as before—the cost of a marginal capacity expansion equals the travel cost reduction on the priced link this achieves:

$$k = -c_{s^1}^1 \cdot q^1. \tag{4}$$

This means that the tolled road has the same volume/capacity ratio as the first-best network. But since there is also the untolled road, the ratio for the entire second-best network is higher.

Also a **single firm** (SF) offering capacity in parallel to an untolled road uses capacity rule (4) and hence has the same volume/capacity ratio on its road. But its toll is higher as, following (5), the firm adds a positive mark-up to the congestion-externality charge, $c_{q^1} \cdot q^1$, as long as demand is not perfectly elastic (i.e. $-D_Q = 0$) or the untolled road uncongestible (i.e. $c_{q^0}^0 = 0$).[2] The firm internalises the congestion externality as any reduction in congestion costs can be met by a toll increase. The capacities in the single-firm and second-best cases are generally different: the higher private toll means that there are fewer users, which given that the ratio q^1/s^1 is same means that the capacity of the single firm is lower. If capacity costs were non-linear, the firm would generally set a different ratio than the public opperator. But given the toll, the ratio is optimal; only the choice of toll is distorted by market power.

$$\tau^{SF} = c_{q^1} \cdot q^1 + q^0 \cdot c_{q^0}^0 \cdot \left(\frac{-D_Q}{c_{q^0}^0 - D_Q} \right), \tag{5}$$

We also look at an untolled road with in parallel private firms in perfect competition. This

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[1] The second term is negative since D_Q is negative while all other variables are positive.
[2] Conversely, in Knight (1924) and Edelson (1971), the firm does not add a mark-up because these authors assume $c_{q^0}^0 = 0$.

perfect competition (PC) case is equivalent to welfare maximisation under a zero-profit constraint (see Verhoef (2008)). The corresponding lagrangian is

$$\Lambda^{PC} = \int_0^Q D[n]dn - q^0 \cdot c^0 - q^1 \cdot c^1 - k \cdot s^1 + \lambda^0 \left(D[Q] - c^0 \right) + \lambda^1 \left(D[Q] - c^1 - \tau^1 \right) + \lambda^P \left(\tau^1 \cdot q^1 - k \cdot s^1 \right).$$

The first order conditions for toll and capacity are

$$\partial \Lambda^{PC} / \partial \tau^1 = 0 = -\lambda^1 + q^1 \lambda^P, \tag{6a}$$
$$\partial \Lambda^{PC} / \partial s^1 = 0 = -q^1 \cdot c_{s^1}^1 - k - \lambda^1 \cdot c_{s^1}^1 - k \cdot \lambda^P. \tag{6b}$$

Eq. (6a) implies $\lambda^1 = q \cdot \lambda^p$, and inserting this into (6b) results in the capacity rule

$$k = -c_{s^1}^1 \cdot q^1, \tag{7}$$

Thus, the volume/capacity ratio is again the same as with public pricing. Interestingly, the toll follows the same formula as the first-best toll:

$$\tau^{PC} = c_{q^1} \cdot q^1. \tag{8}$$

The intuition why follows the self-financing (i.e. zero-profit) result of Mohring and Harwitz (1962) with first-best pricing. Since, the volume/capacity ratio is the same as with first-best pricing, the first-best toll rule also leads to zero profit with perfect competition; and zero profit is what we need for perfect competition. Note also that with (8) the private toll equals the capacity cost per user. Moreover, eq. (8) also implies that the level of the perfectly-competitive toll equals the first-best toll. The congestion externality is only a function of the volume/capacity ratio. Thus, if the first-best Q/S equals the perfectly competitive q^1/s^1, the tolls—which equal the externalities—will be the same.

If demand is price sensitive and the untolled capacity congestible, the perfectly competitive outcome has a higher toll than the second-best case, and thus there are fewer users on the private capacity and capacity is lower. Conversely, the competition means that the perfectly-competitive toll is lower than the toll of the single firm.

Finally, we look at **only capacity expansion** (OCE) where the capacity is set welfare maximising but the road is untolled. This is good for the consumer as the expansion lowers travel costs while there is no toll to correct demand being too high as travel costs are partly external (i.e. the market failure of the congestion externality leads to demand being too high). But it is bad for the government as there is no toll revenue to pay for the capacity. The capacity setting rule has a second-best correction that compensates for the remaining market failure. Still, the expansion does lower the deadweight loss from the externality by reducing marginal social and external costs for a given demand. The lagrangian is

$$\Lambda^{OCE} = \int_0^Q D[n]dn - Q \cdot c - k \cdot S + \lambda \left(D[Q] - c \right); \tag{9}$$

and the first-order conditions are

$$\partial \Lambda^{OCE} / \partial Q^1 = 0 = D - c - c_Q \cdot Q + \lambda \left(D_Q - c_Q \right), \tag{10a}$$
$$\partial \Lambda^{OCE} / \partial s^1 = 0 = -Q \cdot c_S - k - \lambda \cdot c_S, \tag{10b}$$
$$\partial \Lambda^{OCE} / \partial \lambda^1 = 0 = D - c. \tag{10c}$$

These conditions imply that the volume/capacity ratio following (11) is above the first-best

ratio following (2), since the term in brackets is larger than 1 as $c_Q>0$ and $D_Q<0$. There will be more users relative to capacity to compensate for the externality that remains untolled. Still, total capacity might be higher than the first-best capacity (and is so in our numerical model), because the absence of a toll means that there are more users.

$$k\left(1+\frac{c_Q}{-D_Q}\right)=-c_{s^1}^1 \cdot q^1 \tag{11}$$

2.2. Single-stage Nash

In all oligopolistic regimes, the initial capacity remains toll free and limits the marketpower of the firms. Our first oligopolistic market structure is the **single-stage Nash** game from Xiao et al. (2007). Firms set their capacities and tolls simultaneously. If firms take the tolls and capacities of the others as given, they set the same volume/capacity ratio as the public operator. At this ratio, the capacity cost of a marginal capacity expansion equals the travel cost reduction on this link it causes. Any reduction in the travel costs can be met by an equal toll increase, so setting capacity at this level maximises profits. If the firm offered a higher capacity this would reduce travel costs and it could ask a higher toll, but then the extra revenue would be smaller than the extra capacity costs. Conversely, a lower capacity results in a capacity cost reduction that is below the revenue loss. Tolls are higher than with perfect competition as firms have marketpower, and this also means that total number of users and capacity are lower. Yet, as DeVany and Saving (1980) and Engel, Fisher and Galetovic (2004) show for a given capacity, as the number of firms increases the Nash-equilibrium toll decreases and approaches the perfectly competitive outcome. Moreover, as the number of firms increases, capacity also approaches the perfectly-competitive outcome.

2.3. Two-substages Nash

In the **two-substages Nash** set-up of De Borger and Van Dender (2006) the capacity setting precedes the toll setting, while in each substage firms take the actions of the others *in that substage* as given. Now firms have an incentive to set a lower capacity, as this lessens toll competition and increases Nash-equilibrium tolls: the lower your competitors' capacities are, the higher the toll one can set due to the higher congestion on your competitors' roads. This alteration of the capacity setting rule means that firms generally set a higher volume/capacity ratio than the public operator.

Using the formulas of De Borger and Van Dender (2006), we can write the capacity rule for a duopoly as

$$k + Stategic\ effect = k - q^i \cdot \frac{\partial \tau^{i,*}}{\partial s^i} = -c_{s^i}^i \cdot q^i, \tag{12}$$

where superscript * indicates that this toll is determined by the Nash toll-setting substage. All this assumes that the outcome is symmetric. De Borger and Van Dender (2006) find that, for their linear congestion technology and with very low marginal costs of capacity (0.25 or lower), an asymmetric equilibrium would result, which has slightly different characteristics. Still, even then, the volume/capacity ratio and tolls seems to be higher. With "Bureau of Public Roads" power-of-four congestion, we have only encountered the symmetric outcome, even for a marginal cost as low as 0.05.[3] Therefore, we focus on the symmetric outcome.

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[3] An extra advantage of the symmetric outcome is that for De Borger and Van Dender (2006), it ensures that the response function of firm *i*'s capacity to firms *j*'s is always negative; with asymmetry this need not be so in a extreme and unlikely case. We assume that capacity costs are high enough to ensure downward-sloping capacity-response functions. In our numerical analyses we have only encountered such function, but our solution technique does not assume this.

2.4. Stackelberg

In our **Stackelberg** game, firms set their capacities one after the other. Then the toll setting substage follows in which tolls are set in a Nash fashion. The first firm to act is the leader and has the best position. With its capacity it can influence the capacity setting of all other firms and the results of the toll substage. By setting a higher capacity, the leader limits the capacities of the other firms, thereby raising its marketpower and profit. Hence, again the capacity setting rule is different than in the fully competitive case, and the leader's volume/capacity ratio is below the public operator's. Still, this extra capacity also has a profit lowering effect: *given the actions of the others* it would be profit maximising to set same ratio as the public operator. The optimal capacity is found when, for a marginal capacity increase, the profit increasing effect of the lower capacities of the competitors equals the profit lowering effects from the too low volume/capacity ratio and lower Nash-equilibrium tolls.

If there are many firms, the second firm to act also has an incentive to set a lower volume/capacity ratio than is socially optimal. But its ratio will be above that of the first firm, as it has fewer firms to influence. The last firm to act has no capacities to influence. So it is only concerned with the toll setting substage. Just as in De Borger and Van Dender (2006), it sets a lower ratio to lessen the toll competition and raise the Nash-equilibrium tolls.

Since the firms set different volume/capacity ratios, their travel times are different. The first firm has the shortest travel time. But since the sum of travel cost and toll (i.e. the generalised price, or price for brevity) must be the same on all roads, this means that the first firm can ask the highest toll. The last firm has the longest travel time and lowest toll.

2.5. Sequential entry

Our last oligopolistic market structure follows Verhoef (2008) and is **sequential entry**. This set-up is in between the two-substages-Nash and Stackelberg set-ups. There are again separate substages for capacity and toll. When the first firm enters, it first sets its capacity and then its toll given that it is the only firm. Since there are no other players to influence, it is profit maximising to have the same volume/capacity ratio as the public operator.

Then, a second firm enters, and optimises its capacity given that there are two firms. The capacity of the first firm is fixed. In the following toll substage, firms set their tolls in a Nash fashion. So the first firm changes its toll, but not its capacity.[4] Each time there is a further entry, it follows the same pattern as for two firms. This set-up might seem a inconsistent in that firms are forward-looking to the toll substage, but are continuously surprised when a further entry occurs (i.e. they are myopic to the next capacity stage). Yet, it also seems plausible that firms do not perfectly know what the future will bring and hence optimise given the current situation. Then a remark could be made against the Nash and Stackelberg games wherein firms have to known how many competitors there will be.

3. Numerical model set-up

We focus on results of the oligopolistic market structures in our numerical model, the calibration of which follows Verhoef (2007; 2008). The model is very simple, but is calibrated to represent a congested peak-hour highway. User cost follows the BPR (Bureau of Public Roads) function, just as in most of the recent literature on private roads:

$$c^i[q^i / s^i] = \alpha \cdot t_f \left(1 + 0.15 \left(\frac{q^i}{s^i} \right)^4 \right). \tag{13}$$

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[4] This is not as restrictive as it seems. The first firm would like decrease its capacity, but this is not directly possible and would certainly not result in its recuperating all capacity costs. Hence, the best it can do is to keep its current capacity.

Free-flow travel time, t^f, is an half an hour. Using a free-flow speed of 120 km/hour, this implies a trip length of 60 kilometres. The value of time, α, is 7.5. Units of capacity are set so that a traffic lane corresponds to a capacity of s^i=1500. Capacity costs follow $C^{s,i} = k \cdot s^i$, where k equals 7. Since our unit of time is an hour, k is the hourly capacity cost. See Verhoef (2008, pp. 476-477) for the derivation from the average yearly capital cost of €5 million per lane-km or $8 million per lane-mile for freeways in the Netherlands. This cost seems in line with the estimates for the USA. Washington State Department of Tranport (2005) reports 15 project from outside the state: median cost is about $8 million per lane-mile while a third is above $10 million. For the 21 project in the state, median cost is around $5 million per lane-mile while a quarter is above $10 million.

All roads have the same technology and free-flow travel time. Thus, a higher ratio q^i/s^i automatically implies a higher travel time. The initial capacity in the base equilibrium is s^0=1500. Inverse demand follows

$$D[Q] = A - B \cdot Q. \tag{14}$$

The A equals 61.27 and B 0.01167. This calibration results in a very congested road with 3500 users and a travel time that is 5.4 times the free-flow travel time. If the initial situation was less congested, then the gain of private road supply as well as of public policies would be lower. In the base equilibrium, the price elasticity is −0.5; whereas in the perfect competition case, it is −0.21. In all other private settings it is in between these two extremes.

4. Numerical results

4.1. Basic policies and Nash capacity competition

Table 1 describes the benchmark equilibria. It gives such things as consumer surplus and welfare (the sum of consumer surplus and system profit). It also gives the volume/capacity ratio on the entire network, on the untolled part, and tolled part. In the **base equilibrium** there is no tolling and capacity is 1500. In the **first-best** case, capacity is more than doubled and the toll equals the congestion externality. In the table, relative efficiency is the welfare gain of a policy from the initial base equilibrium relative to the first-best gain.

In the **second-best** case, the initial capacity s^0 remains untolled, but the extra capacity has a welfare-maximising toll. Optimal capacity is higher than in the first-best case, but the volume/capacity ratio on the tolled part is the same. Due to the low initial capacity, the welfare gain of the second-best option is very close to the first-best gain. With more initial capacity the relative efficiency would be lower, since the capacity expansion would be less important while the detrimental effect of the larger untolled capacity would be larger.

With **only capacity expansion**, the government optimises capacity but there is no toll. The welfare gain of is again close to the first-best gain. Still, this set-up makes a large loss, and the government has to finance this from other sources which might be difficult. The volume/capacity ratio is higher than in the first-best case, since the capacity rule now has a second-best correction. Still, total capacity is higher than in the FB case, as the absence of a toll means that there are more users. This lowers welfare since the marginal benefit for these users is below marginal social cost. Interestingly, in our numerical model, the ratio with **only capacity expansion** equals the one on the entire network in the second-best case, although there the ratio on the tolled part is lower and on the untolled part higher.

A **single firm** building and tolling an extra road is also welfare improving. Private road supply is in our setting always welfare improving: the firm makes a profit; whereas the consumer is never worse off, since if users choose to use the private road it cannot have a higher price than the untolled road. Again, the private road has the same volume/capacity ratio as the first-best network. Still, the price and toll are higher, and capacity is lower.

Table 1. Basic policies

	Base equilibrium	First-best	Second-best	Only capacity expansion	Single Firm	Perfect competition
Relative efficiency	0	1	0.974	0.973	0.453	0.783
Welfare	60984	109468	108206	108163	82946	98968
Consumer surplus	71484	109468	133472	134390	90029	109468
System profit	-10500	0	-25266	-26228	-7083	-10500
Profit tolled part	-	0	-14766	-	3417.32	0
Total capacity (S)	1500	3451.8	3734.0	3746.8	2078.5	2708.7
Total demand (Q)	3500	4331.3	4782.7	4799.1	4331.3	3927.9
Overall Q/S	2.333	1.255	1.281	1.281	1.890	1.599
q^0/s^0 on untolled part	2.333	-	1.320		2.135	1.876
q^1/s^1 on tolled part	-	1.255	1.255	-	1.255	1.255
Price	20.42	11.16	5.46	5.26	15.43	10.72
c^0 on untolled part	20.42	-	5.46	5.26	15.43	10.72
c^1 on tolled part	-	5.14	5.14	-	5.14	5.14
Toll	-	5.58	0.31	-	10.29	5.58

Table 2. Single-stage Nash competition

Number of firms	1	2	3	4	5
Total capacity (S)	2078.5	2411.4	2521.4	2572.7	2602.0
Total demand (Q)	3928.0	4150.5	4219.6	4250.9	4268.6
Average toll	10.29	7.69	6.88	6.52	6.31
Overall Q/S	1.890	1.721	1.673	1.652	1.640
q^0/s^0 on the untolled part	2.135	2.005	1.959	1.937	1.924
q^i/s^i on each private road	1.255	1.255	1.255	1.255	1.255
Price	15.43	12.83	12.03	11.66	11.46
Profit firm i	3417.3	1206.7	557.2	315.9	202.6
Consumer surplus	90029	100518	103892	105440	106319
Welfare	82946	92432	95064	96204	96832
Relative efficiency	0.453	0.649	0.703	0.726	0.739
Welfare gain relative to perfect competition	0.578	0.828	0.897	0.927	0.944

The final case in Table 1 is **perfect competition**, which describes what happens when an infinite number of firms build capacity in parallel to the untolled capacity. This outcome is a useful benchmark for the oligopolistic regimes where firms have marketpower. Yet again the private operators have the same volume/capacity ratio as the public operator.

In the **Single-stage Nash** competition of Table 2 firms set their tolls and capacities at the same time. Since, the equilibrium is symmetric in that all firms have the same tolls and capacities, we give the result for any firm i. Firms take the actions of the others a given. Hence, the best they can do is set the same volume/capacity ratio as the public operator (see Section 2.2). Note that in all oligopolistic setting the outcome with a single firm is the same. We only include it in the tables for comparison sake.

As the number of firms increases, the single-stage Nash outcome approaches the perfectly competitive outcome. With a single firm, the welfare gain is 58% of with perfect competition; with two firms, it is already 83%; and with 5 firms, it is 94% percent.

4.2 Two-substages Nash Competition

Now we come to the first set-up where firms set their capacities strategically to influence the actions the others. These strategic considerations change the capacity setting rule, which means that firms generally have a different volume/capacity ratio than the public operator. As

Table 3 shows, with **two-substages Nash** competition, firms have an incentive to set a lower capacity and higher ratio, because this increases the Nash-equilibrium tolls in the following toll-setting substage. However, this higher ratio comes at a cost: it raises travel time and this lowers the toll users are willing to pay. The optimal capacity is found where, for a marginal capacity increase, the profit enhancing effect of the higher willingness to pay a toll equals the detrimental effect of the lower tolls of the competitors.

With two firms, the welfare gain of two-substages Nash competition is much lower than with a single stage, since capacity is much lower and tolls higher. Yet, as the number of firms increases the advantage for the firms of the separate setting of capacity and toll decreases, and the outcome approaches the single-stage-Nash and perfectly-competitive outcomes.

Table 3. Two-substages Nash competition

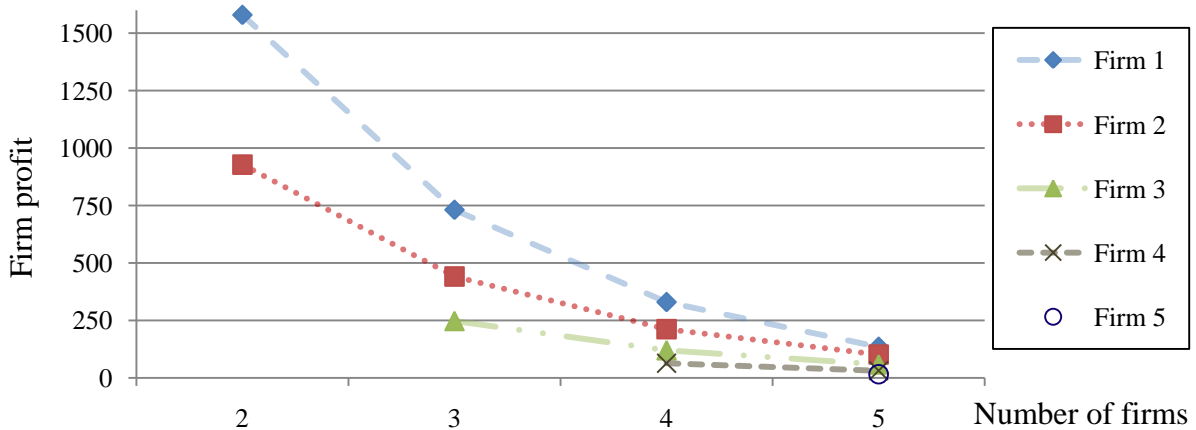| Number of firms	1	2	3	4	5
Total capacity (S)	2078.5	2292.3	2404.4	2470.2	2513.1
Total demand (Q)	3928.0	4087.1	4159.9	4200.0	4225.2
Average toll	10.29	8.27	7.44	6.99	6.72
Overall Q/S	1.890	1.966	1.815	1.747	1.710
q^0/s^0	2.135	2.044	1.999	1.972	1.955
q^i/s^i	1.255	1.288	1.285	1.280	1.277
Price	15.43	13.57	12.72	12.26	11.96
Profit firm i	3417.3	1449.6	772.1	474.1	319.2
Consumer surplus	90029	97472	100971	102931	104170
Welfare	82946	89871	92787	94328	95266
Relative efficiency	0.453	0.596	0.656	0.688	0.707
Welfare gain relative to perfect competition	0.578	0.760	0.837	0.878	0.903

4.3. Sequential entry set-up

The third oligopolistic market structure follows Verhoef (2008). Firms again have separate capacity and toll decisions. The difference is that now firms enter sequentially. First, firm 1 enters, and sets its capacity and then its toll given that it is the only firm. Thereafter, a second firm enters and sets its profit maximising capacity given that there are two firms while taking into account how this affects the toll-setting substage. So the first firm's capacity is fixed, as this is a long term decision; while its toll can be changed, as this is a short run decision. The entry pattern is the same for the third, fourth and so on entrants.

As Table 4 shows, even though all firms have the same costs structures and congestion technologies, they are now ex-post asymmetric. This is due to the sequential decision making. The first firm sets a much higher capacity than it would under Nash competition, and this limits the capacities the others can set. Yet, this sequential decision making need not be good for the firms as becomes clear from comparing the profits in Fig. 1 with those in Table 3. For 4 or more firms, firm 1's profit is lower with sequential entry (Fig. 1) than with two-stage Nash (Table 3). Firm 2 always has a lower profit with sequential entry. Sequential entry leads to a much higher total capacity, and thus to stronger toll competition. For the up to 5 firms we study, sequential entry market gives a higher welfare gain than Nash competition.

As the number of firms increases, capacities increase and tolls decrease (see Fig 2), which leads to lower profits. Still, Firm 1 always attains the largest profit due to its largest size. The later a firm entered, the lower its profit will be.

Table 4. Results under sequential entry

| Number of firms	1	2	3	4	5
Total capacity (S)	2078.5	2399.1	2576.4	2670.3	2718.6
Total demand (Q)	3928.0	4138.4	4237.3	4285.0	4308.4
Average toll	10.29	7.86	6.80	6.30	6.05
Overall Q/S	1.890	1.725	1.645	1.605	1.585
q^0/s^0	2.135	2.012	1.946	1.912	1.894
Capacity firm 1 (s^1)	578.5	578.5	578.5	578.5	578.5
Capacity firm 2 (s^2)	-	320.5	320.5	320.5	320.5
Capacity firm 3 (s^3)	-	-	177.4	177.4	177.4
Capacity firm 4 (s^4)	-	-	-	93.8	93.8
Capacity firm 5 (s^5)	-	-	-	-	48.3
Price	15.43	12.98	11.82	11.26	10.99
Consumer surplus	90029	99931	104767	107138	108308
Welfare	82946	91939	95687	97365	98153
Relative efficiency	0.453	0.638	0.716	0.750	0.767
Welfare gain relative to perfect competition	0.578	0.814	0.910	0.953	0.973

Fig. 1. Firm profit by the number of firms under sequential entry

Fig. 2. The toll of each firm by the number of firms under sequential entry
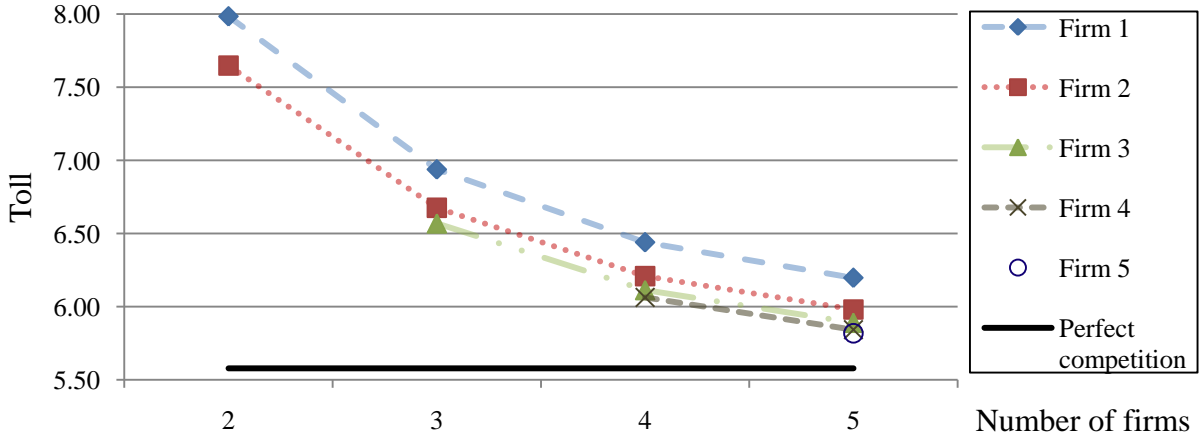
The sequential-entry market structure never reaches perfect competition. With 5 firms, total capacity is already above the perfectly-competitive level (see also Fig. 8 in Section 4.5), and the entries of the sixth and seventh firms only increases capacity (although these cases are not shown). It is surprising that an above perfectly-competitive capacity level can be

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profitable. The reason is that, with five firms, a firm still has marketpower and adds a mark-up to the toll (see Fig. 2); whereas with perfect competition, the toll equals the congestion externality and the mark-up is zero. At some point in the game there will be no further entry, as this would decrease marketpower so much that the entering firm would make a loss. Still, the welfare loss from this game never reaching perfect competition is limited. Two firms entering sequentially gives a consumer surplus that is 9% lower and welfare gain that is 19% lower than with prefect competition; for five firms these figures are respectively 1% and 3%.

The most intriguing result from the sequential-entry market structure is the development of the volume/capacity ratios in Fig 3. When firm 1 enters, it sets the same ratio as the public operator since there are no other players to influence. Therefore, the best it can do is set its capacity so that the decrease in travel costs due to a marginal capacity expansion—which can be converted into toll payments—equals marginal capacity cost.

When firm 2 enters, it sets a higher ratio, because this increases the Nash-equilibrium tolls in the toll-setting substage, thereby increasing its profit. So the second firm has a second-mover advantage. This is the opposite from standard Stackelberg competition, where the firm that moves first has the advantage.[5] Because the first capacity is fixed but the new entry attracts users away, the first firm's volume/capacity ratio decreases and is now below the public ratio. The average ratio on the private roads also decreases, as firm 1 is larger. For later entries a similar pattern holds: the entrant sets a ratio above the public ratio to limit the toll competition, and the ratios of the incumbents and the overall ratio decrease.

Fig. 3. Volume capacity ratio under sequential entry
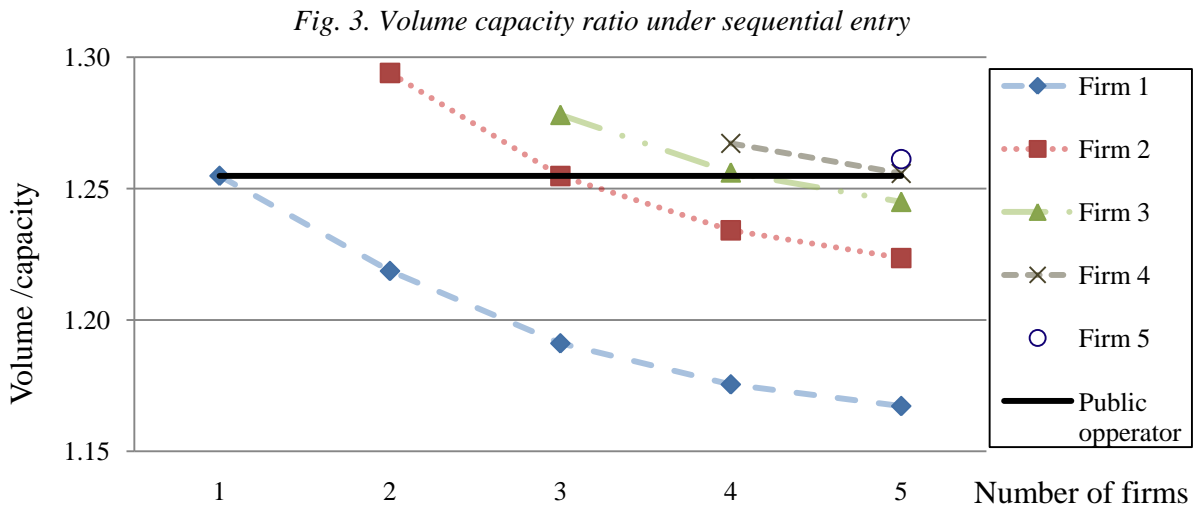
4.4. Stackelberg capacity competition

Our last market structure extends the previous one by making firms forward looking: they recognise that their capacity influences the capacity setting of all following firms as well as the Nash toll-setting substage. The difference with the previous setting is that now firms know how many firms there will be, whereas before they assumed that they were the last entrant.

As Table 5 and Fig. 4 show, with two firms, the leader sets a higher capacity and lower volume/capacity ratio than the follower. Still, the leader's capacity is below the one with sequential entry, as this lower capacity lessens the toll competition, which raises profit even though it also raises the capacity of firm 2. The leader's capacity is well above the single- or

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[5] There other cases where moving later gives an advantage. With incomplete information, actors that move later gain information from the actions that were taken. In common-value auctions the value of the item is the same for all bidders, but this value is unknown and each bidder has different information: e.g. when bidding for an oil field, bidders might have different geological estimates on the amount of oil. By gaining information from another bid, a bidder can update her beliefs and bid (Klemperer (2004)). With complete information, second-mover advantages also occur if firms compete in prices while there are no capacity constraints and *no congestion* (see Gal-Or (1985)).

two-substages-Nash level, as this lowers the follower's capacity and increases the leader's marketpower. For three or more entrants the set-up and the results follow the same lines.

Table 5. Results under Stackelberg capacity competition

Number of firms	1	2	3	4	5
Total capacity (S)	2078.5	2397.6	2564.2	2647.2	2686.6
Total demand (Q)	3928.0	4137.8	4233.6	4279.0	4301.2
Average toll	10.29	7.87	6.82	6.32	6.06
Overall Q/S	1.890	1.726	1.651	1.616	1.601
q^0/s^0	2.135	2.013	1.949	1.916	1.890
Capacity firm 1 (s^1)	578.5	576.2	543.4	489.7	428.3
Capacity firm 2 (s^2)	-	321.5	336.5	350.7	345.8
Capacity firm 3 (s^3)	-	-	184.3	201.0	226.5
Capacity firm 4 (s^4)	-	-	-	105.8	122.
Capacity firm 5 (s^5)	-	-	-	-	63.2
Price	15.43	12.98	11.86	11.33	11.08
Consumer surplus	90029	99902	104580	106837	107949
Welfare	82946	91916	95558	97177	97943
Relative efficiency	0.453	0.638	0.713	0.746	0.762
Welfare gain relative to perfect competition	0.578	0.814	0.910	0.953	0.973

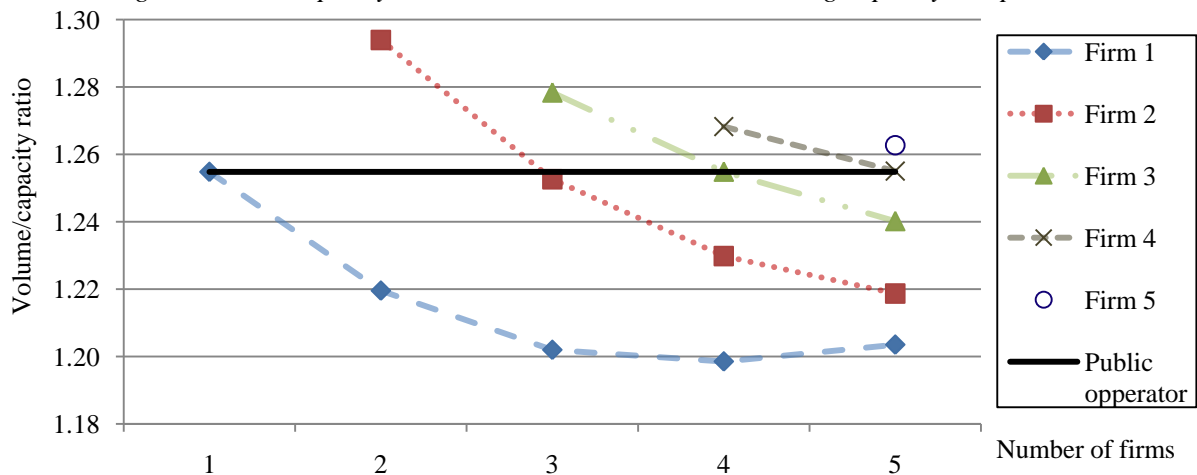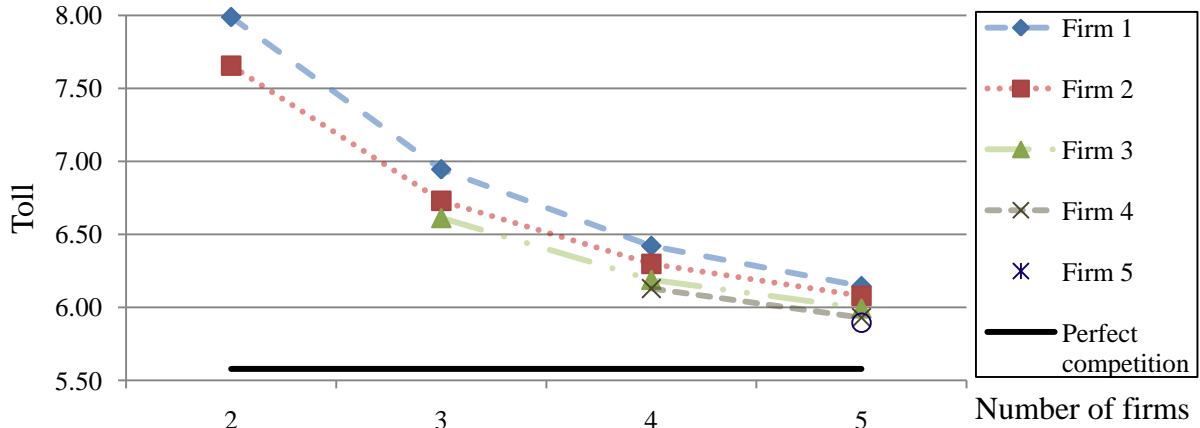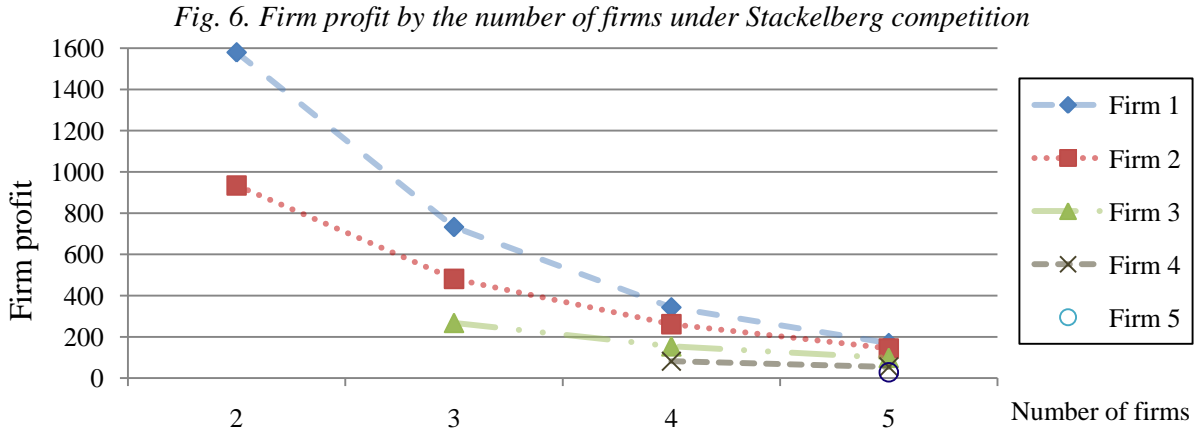*Fig. 4. Volume capacity ratio on each road under Stackelberg capacity competition*

Fig. 5. The toll of each firm by the number of firms under Stackelberg competition

Fig. 6. Firm profit by the number of firms under Stackelberg competition

Up to 4 firms, the volume/capacity ratio of firm 1 decreases with the number of firms. Yet, with 5 firms it is higher than with 4 firms. This suggest that as the number of firms increases even further, that the ratios of all firms approach the public ratio. With more firms, it is more difficult to influence the capacity and toll setting of others and this becomes less profit enhancing. Conversely, the profit loss from setting a lower ratio remains, in that the increased capacity costs are larger than the toll revenue gain due to the lower travel time; and for a higher ratio, the capacity cost reduction is offset by the larger loss in revenue. Only when it is possible to influence the actions of the others is it profit increasing to set a different ratio, but with many firms the strategic effect is small. This suggests that our Stackelberg market structure also approaches perfect competition as the number of firms becomes large. So this is just as with standard Stackelberg competition. Certainly, as Figs. 5 and 6 indicate, the tolls and profits approach the perfectly-competitive outcome.

Fig. 7. Weighted average volume/capacity ratio on the private roads
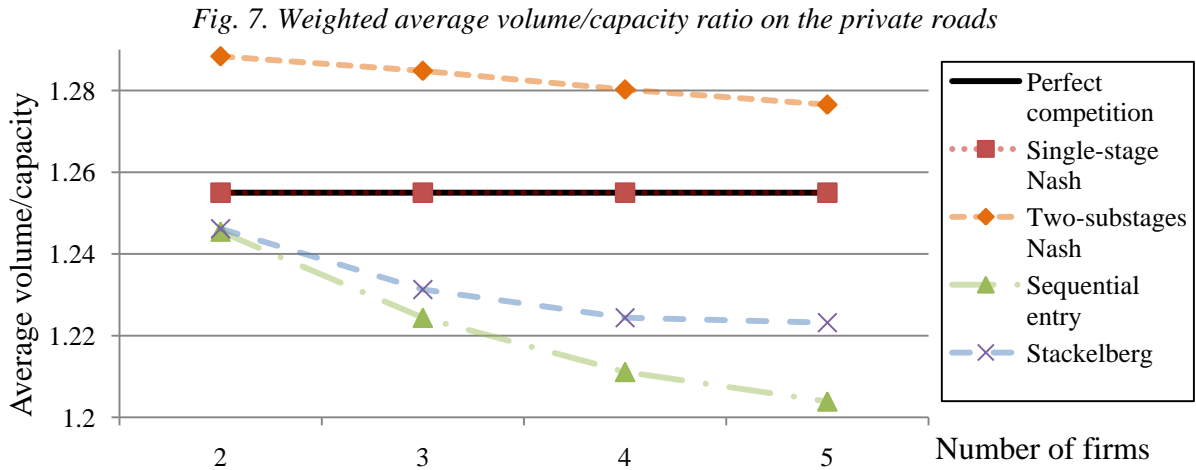
4.5. Comparison of the oligopolistic market structures

Fig. 7 compares the average volume/capacity ratio in the different oligopolistic settings. It also has the ratio with perfect competition as a benchmark, which equals the public one. The single-stage Nash market structure results in constant private volume/capacity ratios that also equal the public one. When the capacity and toll competitions are separate stages, firms set a higher ratio to lessen the later toll competition. Still, this ratio seems to approach the competitive level as the number of firms increase. Conversely, when firms set their capacities one after the other, the average ratio is below perfectly competitive one. For Stackelberg competition, the average seems to level off at five firms. And we expect, when number of firms becomes even larger, that the average will increase again and approach the perfectly-

competitive level. This because with more firms it becomes ever more difficult to influence your competitors' capacities and tolls, and this makes strategic capacity setting less attractive.

Sequential entry never reaches perfect competition because at some point capacity is so high that no further entry is profitable. Moreover, as Fig. 8 also shows, the capacity level with 5 firms is already above with perfect competition, and further entry only increases capacity.

Figs. 8 to 10 compare the capacities, average tolls, and relative efficiencies in the different market structures. All set-ups lead to substantial welfare gains that, even for 2 firms, are relatively close to the perfectly-competitive level. Only with a single firm is the relative efficiency much lower with 0.453. Two-substages Nash attains of all oligopolies the lowest gain, as firms set a lower volume/capacity ratio, build less capacity, and have higher tolls.

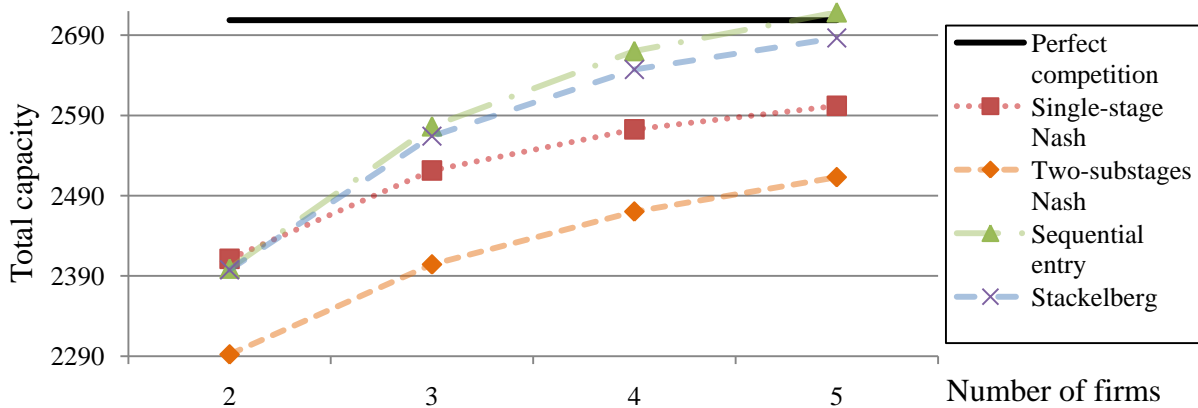*Fig. 8. Total capacity and the number of firms*

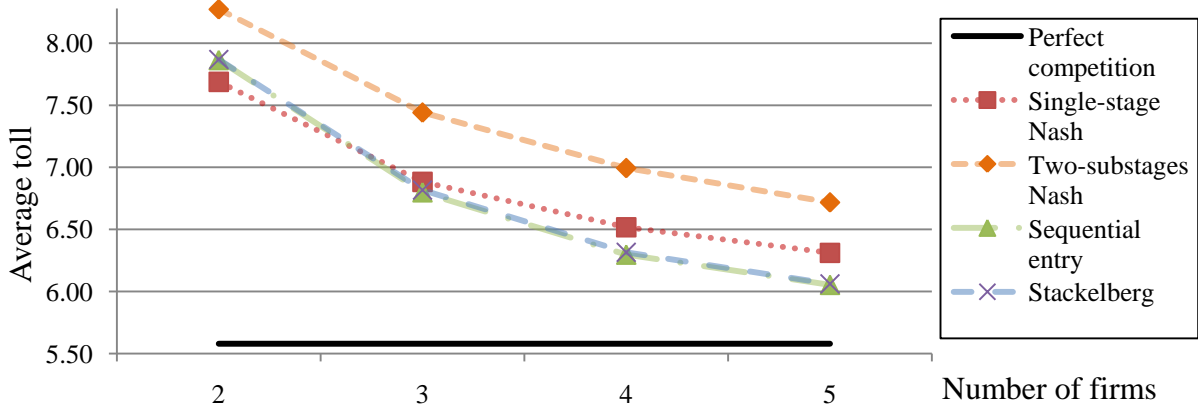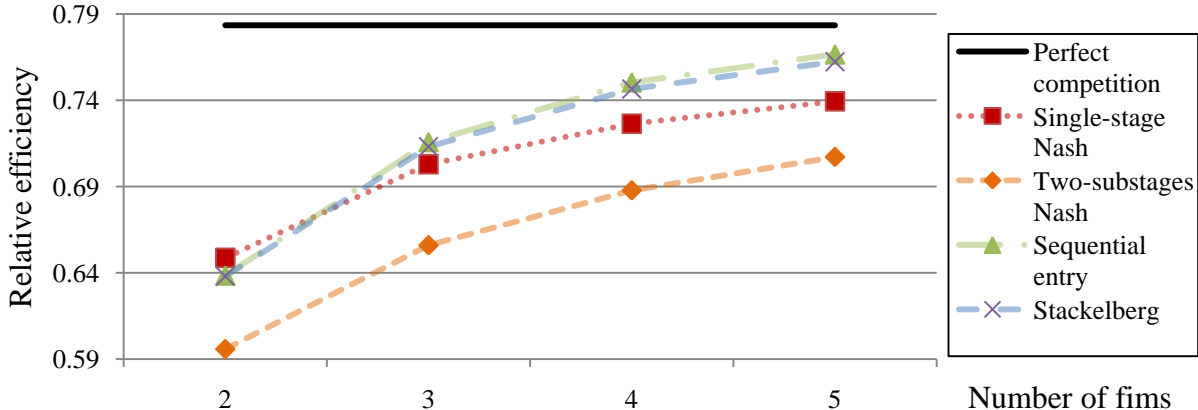*Fig. 9. Weighted average toll and the number of firms*

Fig. 10. Relative efficiency and the number of firms

The Stackelberg and sequential entry games have very similar results. A weakness of the Stackelberg model is that firms have to know how many firms there will be with certainty. Conversely, with sequential entry, firms assume that they are the last entry and are continuously surprised when a further entry occurs. In reality the market structure might be in between these two games: i.e. a firm does not know for certain how many firms there will be, but has a prior belief about the likelihood of each outcome, and optimises given this belief. One would expect that this game would lead to an outcome in between our two set-ups.

It is important to perform sensitivity analyses to important parameters. We focus on the effect of the amount of initial capacity. The effects of other such standard things as the price elasticity and value of time were as one would expect: e.g. more price sensitive demand makes private provision more beneficial as it limits market power. With more initial capacity, the gain of the first-best policy is lower, since there is less to gain from the capacity expansion. For the second-best, single-firm, and perfect-competition cases the gain and relative efficiency are also lower: capacity expansion is less important and there is more initial capacity that remains untolled (see also Verhoef (2007)). The oligopolistic settings also attain lower gains; but compared with the perfect competition they fare better, because the larger untolled capacity limits the oligopolistic market power.

5. Conclusion

This paper re-examined whether a private firm sets the same volume/capacity ratio as a public operator and hence has the same travel time. Previous studies found that, under some technical assumptions, firms competing in parallel set the same ratio if they take the actions of the others as given in a Nash fashion. We find that this single-stage Nash-competition assumption is crucial. If firms can influence the decisions of others with their capacity this changes their capacity-setting rule and they generally set a different ratio than the socially optimal one.

In our Stackelberg market structure, firms first set their capacities one after the other and then set their tolls in a Nash fashion. Firms have two strategic considerations: (1) they want to limit the capacities of firms that follow by setting a higher capacity, and (2) they want to limit the toll competition by setting a lower capacity. The first firms to act have many capacities to influence and hence set a higher capacity and lower volume/capacity ratio than they otherwise would. The last firms have few if any capacities to influence and set a higher ratio.

Strategic setting of a lower capacity to limit toll competition is harmful for welfare as it lowers capacities and increases tolls. Stackelberg setting of higher capacities to limit competitors' capacities can be good or bad for welfare: it increases the marketpower of the leaders, but also tends to lead to higher total capacity and a lower average toll. The Stackelberg oligopoly seems to approach perfect competition as the number of firms increases. In our numerical model, the Stackelberg game attains, with 2 firms, 81% of the perfectly-competitive gain, and with 5 firms this is 97%. A general result is that the effect of private road supply depends on the number of competitors and on the market structure: the outcomes are different in the single-stage Nash, two-substages Nash and Stackelberg set-ups.

The reader might comment that our setting with many firms competing in parallel seems unrealistic, as it is rare to have many toll roads going to the same destination. However, as the number of toll road becomes ever larger—which seems likely—the chance that are multiple roads going roughly in the same direction increases. Moreover, also in different settings strategic considerations alter the capacity setting rule and lead to a too low or high volume/capacity ratios. Consider two firms competing in serial between a single origin and destination, with the first firm having a road on the first leg of the journey and the second on the second leg. A firm then might want to set a higher volume/capacity ratio, as this limits the demand for the first firm which would lower its toll. If a firm is a Stackelberg leader, it might

want a higher ratio to induce the follower to set a higher capacity, which reduces travel cost and enables the leader to set a higher toll. If there are two destinations that are imperfect substitutes, then a firm wants a toll road to the competing destination to be smaller and its tolls higher. How private road supply fares in these settings and in more general, more realistic, networks seems a very interesting topic for further research. Similar structures are possinle for other congestible facilities aswell: two airports could be perfect or imperfect substitutes or compete sequentially (i.e. the first airport is the destination of the second).

Acknowledgements

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