The Impact of Carbon Emission Considerations on Manufacturing Value Chain Relocation

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Abstract

This paper investigates how increasing carbon emission costs affect the operations strategy and relocation decisions for a manufacturer across the world's regions. Specifically, we develop a theoretical model which explicitly accounts for carbon emission costs and compares the manufacturer's decisions under all possible cost and emission structures in two regions. The model is generalisable to more regions. The results show that the carbon emission costs may have significant influence on the relocation decisions under certain circumstances. For example, as carbon emission costs increase, manufacturers may first off-shore and then near-shore. High demand rate could favor production in a high cost region as emission costs become significant.

Keywords: Operations Strategy, Relocation, Emissions, Cost structures, EOQ

1. Introduction

As a stark contrast to the diminishing media profile of the UN climate change talks, the global manufacturers appear to have become more carbon aware than ever before. Carbon audits have been carried out within many corporations to assess the carbon intensity of production processes (Piecyk, 2010). This is to address cost issues both of the present (such as the recent rise in fossil fuel prices) and those anticipated of the future (e.g.

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new carbon related taxes and trade tariffs). Moreover, the adoption of low carbon, clean manufacturing processes has become an increasingly prominent part of branding for many products, which could affect market share and business performance in ways that go beyond questions of cost competitiveness.

Against the background of globalization, the manufacturing facilities of many products have spread all around the world. Meanwhile, the locations of these manufacturing facilities are not static. According to a recent report from Accenture[2], 61% managers in their survey of 287 manufacturers are currently considering to relocate the manufacturing facilities in this rapidly changing world economy, creating different trends such as off-shoring, on-shoring and near-shoring. In the medium to long term, the manufacturing facilities migrate in various patterns under different motivations of which cost reduction is regarded as a critical drive (Lewin and Couto, 2007). Since carbon emissions have become an emerging factor which influences cost both directly and indirectly, the distribution of manufacturing value chain may be reshaped by the carbon emissions further into the future.

We are interested in how the carbon emission costs affect the operations strategy and relocation decision for a manufacturer, as well as the consequent carbon emission variations. Although there are various means to control the carbon emission, such as emissions cap and trade, increasing carbon emission costs is often considered as a direct way to achieve this. Since carbon emissions are not easy to measure precisely, increasing carbon emission cost could be approximated by increasing the fossil fuel cost. It is still unclear whether increasing carbon cost would cause manufacturers to agglomerate in certain regions and what kind of cost and emission structure are more attractive to manufacturers under higher carbon emission cost. We build a theoretical model in which the carbon emission costs are accounted for explicitly, carbon efficiency in different regions is characterized and traditional manufacturing concerns such as scale economies are considered. Through this model, we attempt to predict the dynamics of how manufacturers relocate under different carbon emission costs and market conditions.

The impact of carbon emissions on economics and management has drawn more and more attention in academia. Holtz-Eakin and Selden (1992) study the relationship between carbon emission and economy development. Jorgenson and Wilcoxen (1993) analyze how to reduce carbon emission through methods such as carbon tax. McKinnon (2010) discusses the feasibility and necessity of product level carbon audit. Gong and Zhou (2010) derive the dynamic optimal inventory planning for a manufacturer with emission trading. These studies provide a thorough understanding about carbon emissions and its impact on economy, industry and enterprise. However, location decisions are rarely covered.

Meanwhile, much research work has been contributed to the location problem without consideration of carbon emissions. For example, Grossman and Rossi-Hansberg

[2] http://www.accenture.com/us-en/Pages/insight-manufacturings-secret-shift-gaining-competitive-advantage-summary.aspx

(2008) and Markusen (2004) study the production unbundling problem among different countries. These papers analyze the problem from a macro-economic perspective, focusing on trade, transaction cost and so on. Owen and Daskin (1998) provide an extensive review on how to choose the optimal location under deterministic and stochastic circumstances. Shen et al. (2003) develop a model to determine which node of the supply network should be set as a hub in order to reduce the cost. These models are micro-scope studies, addressing many details of operations management, such as lead time, demand uncertainty etc.

Section 2 below introduces assumptions and setup the model. In Section 3 we discuss cost minimization by choosing appropriate batch size and location; the effect of rising carbon emission costs on the optimal production location distribution is analyzed through numerical experiments. Section 4 concludes the paper.

2. The model

Consider a manufacturer who may produce goods in two different locations, N and S. The locations N and S may represent the northern and southern countries in the world, or regions in a country with different development levels where the cost and emission structures are different. Here we assume that the products from N and from S are perfectly substitutable as far as the consumer is concerned. The manufacturer pursues to minimize the total operational cost. Its decision is of two levels. The first level is to choose the location of the production. We use L to denote this decision, $L \in \{N, S\}$ This is a meso-level decision, which affects the spatial distribution of manufacturing. The second level decision is the production batch size, which is denoted by Q in this paper. Here we employ the EOQ (Economic Order Quantity) model, which is widely used in industrial production management software, to capture the economies of scale and trade-off among various costs. In reality manufacturers do not make relocation decisions as frequent as production batch size decisions. However, in the medium term, the difference in the operational cost is an important drive which points to the direction of future relocation. Hence we put the two decisions together and investigate the relocation trend of the manufactures. Technology improvement usually happens in the long term, thus we do not consider technology improvements in this model and assume the costs and emission structures are static.

The parameters of the model are summarized as follows.

$\lambda =$ the demand rate for the final product.

$K^L =$ the fixed production cost in L per batch.

$k^L =$ the variable production cost in L.

$E^L =$ the fixed emission for production in L per batch.

$e^L =$ the variable emission for production in L.

$c =$ the carbon cost per unit of emission.

$h =$ the unit warehouse cost for the final product.

$e_h =$ the unit emission of warehousing the final product.

The fixed production cost K^L and the fixed emission E^L are independent of the order size. The fixed production cost usually consists of negotiation cost, administrative order processing cost, transport and inventory facility cost, etc. The fixed emission is usually caused by setting up the facilities and equipment, and the business operations that are commonly considered as ‘overheads’. The variable cost per unit k^L and the variable emission e^L represents the rate at which the cost and emission increases with the batch size. The variable cost k^L usually reflects the value of raw materials, energy, labour, transport and so on that are directly used in the production process. The variable emission e^L usually captures the emissions for making, processing and transporting a product. Here we ignore the cost of shipping after the product leaves the factory gate and do not distinguish h, e_h and λ geographically as the conditions for warehouse are identical. Further, we assume that E^L and e^L are already optimized for location L given the prevailing technologies adopted, and a further increase of the unit carbon cost at the location L will not alter E^L and e^L. This is a reasonable assumption given that in many industries, the energy and carbon efficiency improvements that can be achieved through adopting any known technologies have already become very limited. In the rest of this paper, we name $k^L + ce^L$ as total variable cost, and $K^L + cE^L$ as total fixed cost.

Denote the final average cost as $TC(Q, L)$. The optimization problem is formulated as follows

$$\min_{\substack{Q>0 \\ L\in\{N,S\}}} TC(Q,L) = \frac{K^L + cE^L + (k^L + ce^L)Q + \frac{1}{2}(h + ce_h)Q\frac{Q}{\lambda}}{\frac{Q}{\lambda}}. \quad (1)$$

The denominator of equation (1) is the time of one production cycle. The first two items in the nominator is the total fixed cost. Following that is the variable cost including emission costs. The last item in the nominator is the average warehousing cost. Compared with the classical EOQ model, this cost function incorporates the emission costs explicitly and provides a basic model for introducing location decisions of manufacturers.

We solve problem (1) in two steps. The first step is, given L, find the optimal production quantity Q_L^*. The second step is to plug Q_L^* into the cost function and find the optimal location decision, denoted by L^*.

Given L, equation (1) is a standard EOQ model. By taking derivatives with respect to Q, we can easily get the closed form expression of the optimal production quantity

$$Q_L^* = \sqrt{\frac{2\lambda(K^L + cE^L)}{h + ce_h}}. \quad (2)$$

Plug in Q_L^* into the cost function, we have the following optimization problem

$$\min_{L\in\{N,S\}} TC(Q_L^*, L) = \sqrt{\lambda}\left[(ce^L + k^L)\sqrt{\lambda} + \sqrt{2(h+ce_h)(K^L + cE^L)}\right]. \tag{3}$$

The location decision can be then made by comparing $TC(Q_N^*, N)$ and $TC(Q_S^*, S)$. We are particularly interested in how the influence on location choice of the unit carbon cost c and market demand rate λ, which are two key inputs of this model and can be significantly affected, respectively, by government policy and market conditions. However, different relationships between cost and emission parameters could lead to various results. Table 1 cover all possible scenarios.

Table 1 Different cost structure scenarios

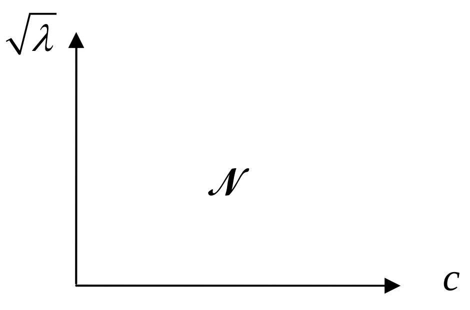
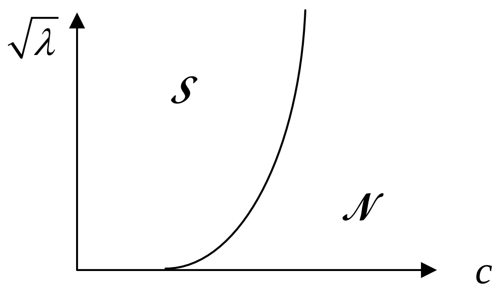
	$K^N \le K^S, k^N \le k^S$	$K^N \le K^S, k^N > k^S$	$K^N > K^S, k^N \le k^S$	$K^N > K^S, k^N > k^S$
$E^N \le E^S, e^N \le e^S$	Scenario1	Scenario 2	Scenario 3	Scenario 4
$E^N \le E^S, e^N > e^S$	Scenario 5	Scenario 6	Scenario 7	Scenario 8
$E^N > E^S, e^N \le e^S$	Scenario 9	Scenario 10	Scenario 11	Scenario 12
$E^N > E^S, e^N > e^S$	Scenario 13	Scenario 14	Scenario 15	Scenario 16

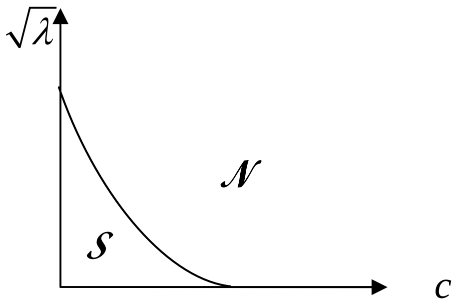
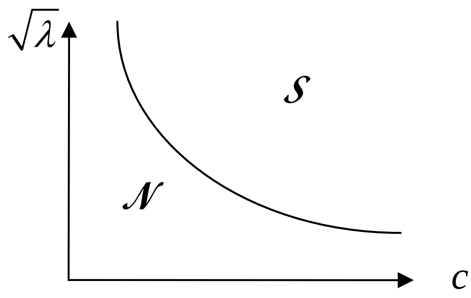
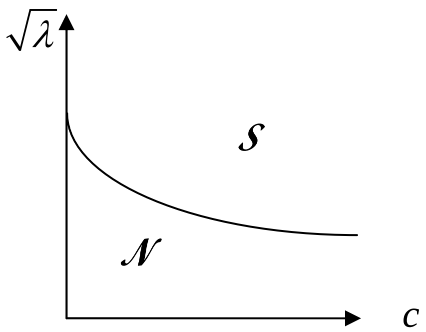
It is obvious that exchanging N and S will not affect the structure of the decision problem. For example, Scenario 8 is equivalent to Scenario 9 if we swap N and S. Hence, we can explore all the possible structures of the optimal location decision by studying the first 8 scenarios. Before discussing the optimal location, we state the difference of operational cost between N and S after optimizing Q as follows

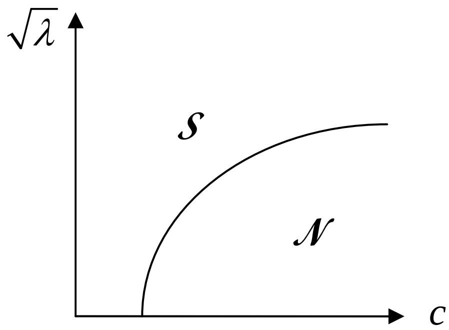
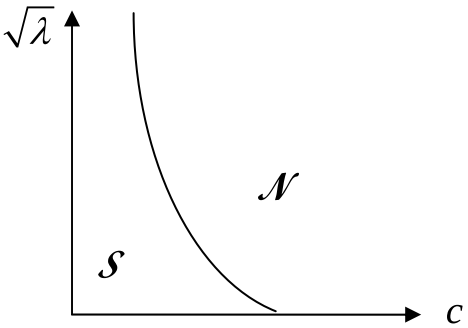
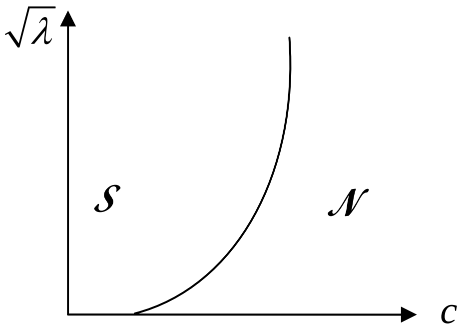
$$\begin{aligned}&(ce^N + k^N)\sqrt{\lambda} + \sqrt{2(h+ce_h)(K^N + cE^N)} - (ce^S + k^S)\sqrt{\lambda} + \sqrt{2(h+ce_h)(K^S + cE^S)} \\ &= [c(e^N - e^S) + k^N - k^S]\sqrt{\lambda} + \sqrt{2(h+ce_h)}[\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S}]\end{aligned}. \tag{4}$$

The comparison of cost and derivation of optimal location decision are in the appendix. The optimized location decisions under the 8 scenarios are summarized in Table 2. The area where location N is optimal is marked by $\mathcal{N}$ and the area where location S is optimal is marked by $\mathcal{S}$.

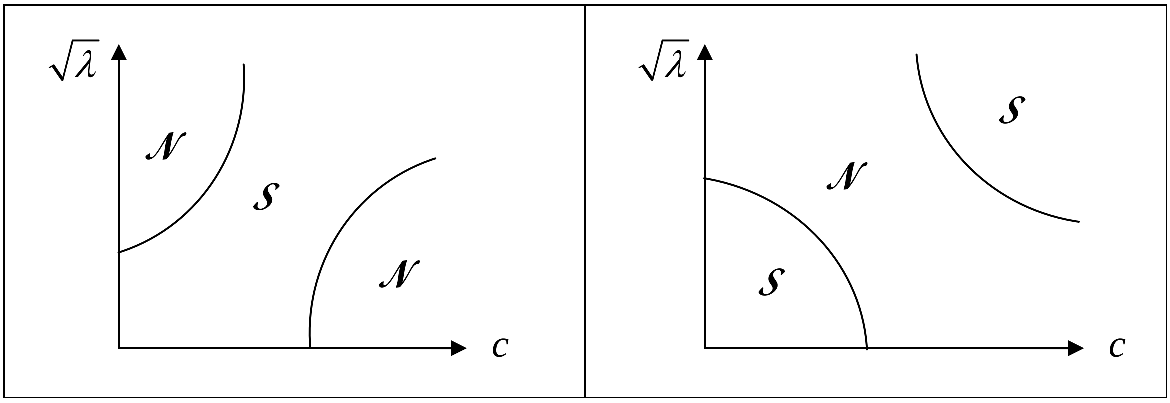
Table 2 Location choice as demand rate and emission cost changes

Scenario 1: $K^N \le K^S, k^N \le k^S, E^N \le E^S, e^N \le e^S$	Scenario 2: $K^N \le K^S, k^N > k^S, E^N \le E^S, e^N \le e^S$
	

<table>
<tr><td>Scenario 3: $K^N > K^S, k^N \le k^S, E^N \le E^S, e^N \le e^S$</td><td>Scenario 5: $K^N \le K^S, k^N \le k^S, E^N \le E^S, e^N > e^S$</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Scenario 6: $K^N \le K^S, k^N > k^S, E^N \le E^S, e^N > e^S$</td><td>Scenario 8: $K^N > K^S, k^N > k^S, E^N \le E^S, e^N > e^S$</td></tr>
<tr><td>

[3]</td><td>
</td></tr>
<tr><td colspan="2">Scenario 4: $K^N > K^S, k^N > k^S, E^N \le E^S, e^N \le e^S$</td></tr>
<tr><td>Sub-case A: $\frac{k^N - k^S}{e^S - e^N} < \frac{K^N - K^S}{E^S - E^N}$</td><td>Sub-case B: $\frac{k^N - k^S}{e^S - e^N} \ge \frac{K^N - K^S}{E^S - E^N}$</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td colspan="2">Scenario 7: $K^N > K^S, k^N \le k^S, E^N \le E^S, e^N > e^S$.</td></tr>
<tr><td>Sub-case A: $\frac{k^N - k^S}{e^S - e^N} < \frac{K^N - K^S}{E^S - E^N}$</td><td>Sub-case B: $\frac{k^N - k^S}{e^S - e^N} \ge \frac{K^N - K^S}{E^S - E^N}$</td></tr>
</table>

[3] The line could also be increasing, depending on the value of the parameters.



3. Impact of carbon emission costs

Based on the above model, we have the following observations which shed light on the impact of carbon emission cost on manufacturers' location decisions.

Observation 1. As we have already assumed above that the fixed and variable emissions are already well optimized through adopting the prevailing technologies in each region, a rise in carbon emission costs has limited effect on reduction in the volume of carbon emissions without relocation. Thus a further rise in carbon emissions cost would have only a minor effect.

We use the optimized emissions as a benchmark. If the manufacturer sets its goal as minimization of carbon emissions and ignore all other costs, then its optimal production batch will be

$$\hat{Q}_L^* = \arg\min \frac{E^L + e^L Q + \frac{1}{2} e_h Q \frac{Q}{\lambda}}{\frac{Q}{\lambda}} = \sqrt{\frac{2\lambda E^L}{e_h}} \tag{5}$$

The resulting optimal emission level is $TE_L^* = e^L\lambda + \sqrt{2\lambda E^L e_h}$. While taking operations cost and carbon emission cost into consideration, the actual average emission is also a function of c, denoted by $TE_L(c)$.

$$TE_L(c) = e^L\lambda + \frac{E^L\lambda}{Q_L^*} + \frac{1}{2} e_h Q_L^* \tag{6}$$

In Equation (6) Q_L^* varies as c increases. However, $TE_L(c)$ is not very sensitive to c. A numeric example is provided as follows. Let $K^N = 60$, $k^N = 2$, $E^N = 10$, $e^N = 8$, $e_h = 1$, $h = 1$, $\lambda = 100$. We vary the carbon cost c from 0 to 10. The ratio $[TE_N(0) - TE_N(c)]/TE_N(0)$ reflects how the carbon emission reduces as c increases. The ratio $TE_N(c)/TE_N^*$ reflects how much the carbon emissions exceed the theoretical optimal value.

Table 3 Batch size and emission changes as emission cost varies

c	Q_N^*	$\hat{Q}_N^*$	$TE_N(c)$	$TE_N(c)/TE_N^*$	$[TE_N(0) - TE_N(c)]/TE_N(0)$

0	109.54	44.72	863.90	102.27%	0.00%
0.1	105.31	44.72	862.15	102.06%	0.20%
0.2	101.65	44.72	860.66	101.89%	0.37%
0.3	98.45	44.72	859.38	101.74%	0.52%
0.4	95.62	44.72	858.26	101.60%	0.65%
0.5	93.09	44.72	857.29	101.49%	0.77%
0.6	90.83	44.72	856.42	101.39%	0.87%
0.7	88.78	44.72	855.65	101.29%	0.95%
0.8	86.92	44.72	854.97	101.21%	1.03%
0.9	85.22	44.72	854.35	101.14%	1.11%
1	83.67	44.72	853.79	101.07%	1.17%
10	53.94	44.72	845.51	100.09%	2.13%

From the column of $TE_N(c)/TE_N^*$ we find that even when no carbon emission cost is charged, the emission level is only 2.27% higher than its optimal level. From the column $[TE_N(0)-TE_N(c)]/TE_N(0)$, we know that even when the carbon emission costs are much higher than the production cost, the reduction in emission is almost negligible (about 2%). This is to say that carbon emission costs have very limited impact on carbon emissions if adjusting the batch size is the only option. In order to reduce emissions significantly, the manufacturer will need to switch to a more carbon-efficient location.

Observation 2. Carbon emission cost could cause relocation of production, hence may have a significant impact on carbon emission level.

Under certain conditions, the location decision is quite sensitive to the carbon emission cost variations. A small rise in carbon emission cost may lead to a switch of production location. The consequent carbon emission reduction can be much more obvious than what can be achieved in the original location. We demonstrate this observation again by numerical results. The cost and emission in N is the same as last numerical experiment. The holding cost, holding emission, and the demand remain the same. Let $K^S=50,\quad k^S=3,\quad E^S=14,\quad e^S=2$. We summarize the results in Table 4.

Table 4 Cost, emission and location changes as emission cost varies

c	$TC(Q_N^*,N)$	$TC(Q_S^*,S)$	Location	$TE(c)$	$\frac{TE(c)}{TE^*}$	$\frac{TE(0)-TE(c)}{TE(0)}$
0	210.95	310.00	N	863.90	341.58%	0.00%
0.1	291.58	330.63	N	862.15	340.89%	0.20%
0.11	299.65	332.70	N	861.99	340.82%	0.22%
0.12	307.71	334.76	N	861.84	340.76%	0.24%
0.13	315.77	336.82	N	861.68	340.70%	0.26%
0.14	323.83	338.88	N	861.53	340.64%	0.27%

0.15	331.89	340.95	N	861.38	340.58%	0.29%
0.16	339.95	343.01	N	861.23	340.52%	0.31%
0.17	348.02	345.07	S	262.11	103.63%	69.66%
0.18	356.08	347.13	S	262.01	103.60%	69.67%
0.19	364.14	349.20	S	261.92	103.56%	69.68%
0.2	372.20	351.26	S	261.83	103.52%	69.69%

The results in Table 4 shows that as c increase from 0 to 0.16, which is not very high, the production location switches from N to S. Consequently, the carbon emission drops dramatically, at a percentage of about 70%. At the same time, the carbon emission level is only about 3% higher than its theoretical minimal level.

As carbon emission cost has significant impact on relocation decisions, we have the following observations about the direction of manufacturing relocations.

Observation 3. There are significant variations across the different scenarios depending on the relative cost and emissions structures. As the carbon emission cost increases, the optimal location could either change from N to S or from S to N. And it is also possible that the location changes back if the emission cost keeps on rising.

Whether the production location is changing from N to S or from S to N depends on which location is more efficient in production cost as well as in emissions per unit of output. Because both fixed cost and variable cost (including the emissions cost) are considered, the cost and emission efficiency may be not monotone to unit carbon emission cost c. In other words, as c increases, the location may change from N to S, and then change back to N as c keeps on increasing. This happens in Sub case A in Scenario 7. In that scenario, when c is small, the production cost K^L and k^L dominates the location decision. N has higher fixed production cost and lower variable production cost. As long as λ is large enough, it is optimal to produce in N as the fixed production cost can be diluted. As c increases, total variable cost changes and S begins to have a lower variable cost because the emissions cost has a larger share in total variable cost than in the total fixed cost in Sub-Case A. Since S already has a lower total fixed cost, the location changes to S. However, if c keeps on increasing, then N will gain advantage in total fixed cost because $E^N \leq E^S$. In this situation, a smaller batch will be more suitable for production in N. As long as e_h is large enough, the high emission in warehousing will keep the batch size small enough. Then it is attractive again to move the production back to N.

In other words, production originally located in N is to take advantage of lower variable costs. As c increases, the advantage disappears hence S becomes a better location. When c is high enough, locating production back to N is to take advantage of lower fixed total costs and lower warehouse emission costs.

Observation 4. In different scenarios, as the demand rate λ increases, it is possible that the production moves from N to S or from S to N.

Take Scenario 4 sub-case A as an example. The cost and emission structure is $K^N > K^S, k^N > k^S$, $E^N \le E^S, e^N \le e^S$. This is commonly observed in the world's regions today: N or the North has a higher production cost but lower emissions because of technology advantages. It is generally believed that when the demand rate is high and the production batch is large, it is better to locate production in low cost region S. However, this may not be true in Scenario 4 when emission cost takes a higher proportion in total variable cost than in total fixed cost: there are values of c that will lead to lower total variable cost and higher total fixed cost in N. Larger demand rate λ will cause production to move to N because of scale economies.

4. Conclusions

In this paper we study the possible impacts of carbon emissions on the manufacturing facility relocation. The carbon emissions and their costs are explicitly characterized and the economies of scale are captured in our model. The results show that the carbon emissions cost has a significant influence on the relocation decisions under certain cost structures and emission structures. As carbon emission cost rises, the optimal location could either change from N to S or from S to N. If the emission cost keeps on rising, it is possible that the optimal location switch again under certain circumstances. The demand rate, which determines the batch size, also plays an important role in the location decisions.

In the current study, both the demand rate and the carbon emission cost are exogenously given. However, the demand rate may depend on the carbon emissions cost. On the one hand, the demand rate may decrease as the cost increases. On the other hand, the demand rate may increase as the emissions level decreases because customers may have a preference for a greener product. In a future study, the relationship between demand rate and carbon emissions cost may be endogenized. The location of the final market may also bring interesting variations to the current model. Furthermore, lead times, transport costs and adaptation to demand change may also be important in relocation decisions of a manufacturer.

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Appendix

The optimal location decisions in 8 scenarios are derived as follows.

Scenario 1: $K^N \le K^S, k^N \le k^S, E^N \le E^S, e^N \le e^S$.

In this scenario, production in N incurs lower cost as well as lower emission. It is always true that expression (4) is non-positive. Therefore, it is optimal to locate the production in N in Scenario 1.

Scenario 2: $K^N \le K^S, k^N > k^S, E^N \le E^S, e^N \le e^S$.

In this scenario, production in N has advantage in fixed cost(including the emission cost). But whether the variable cost(including the emission cost) favors production in N depends on the cost of carbon emission c. Besides, the sign of expression (4) also depends on the demand rate λ.

If $c \ge \dfrac{k^N - k^S}{e^N - e^S}$, then $c(e^N - e^S) + k^N - k^S \le 0$ and expression (4) is non-positive. It is optimal to locate the production in N.

If $c < \dfrac{k^N - k^S}{e^N - e^S}$, then the sign of expression (4) depends on λ. Economies of scale begin to take effect on the location decision. Since production in S has advantage in variable cost and production in N has advantage in fixed cost, larger production batch will drive the production from N to S. By comparing the cost function, we have the following results. When $\sqrt{\lambda} \ge -\dfrac{\sqrt{2(h+ce_h)}(\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S})}{c(e^N - e^S) + k^N - k^S}$, it is optimal to produce in S. When $\sqrt{\lambda} < -\dfrac{\sqrt{2(h+ce_h)}(\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S})}{c(e^N - e^S) + k^N - k^S}$, it is optimal to produce in N.

Scenario 3: $K^N > K^S, k^N \le k^S, E^N \le E^S, e^N \le e^S$.

In this scenario, $c(e^N - e^S) + k^N - k^S$ is always non-positive, which means that production in N incurs lower variable cost. However, which place has advantage in fixed cost depends on the carbon emission cost c.

If $c \geq \frac{K^N - K^S}{E^S - E^N}$, then $\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S} \leq 0$, i.e. it is advantageous for saving fixed cost in N. Expression (4) is always non-positive thus it is optimal to locate the production in N.

If $c < \frac{K^N - K^S}{E^S - E^N}$, production in S has lower fixed cost while production in N has lower variable cost. Economies of scale take effect and the sign of expression (4) depends on the demand rate λ. Larger production batch will hedge the disadvantage in fixed cost while enhance the savings in variable cost for production in N. When $\sqrt{\lambda} \geq -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S})}{c(e^N - e^S) + k^N - k^S}$, it is optimal to produce in N. When $\sqrt{\lambda} < -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S})}{c(e^N - e^S) + k^N - k^S}$, it is optimal to produce in S.

Scenario 4: $K^N > K^S, k^N > k^S, E^N \leq E^S, e^N \leq e^S$.

In this scenario, both signs of $c(e^N - e^S) + k^N - k^S$ and $\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S}$ depends on the value of c. The carbon emission cost will affect the order of cost efficiency for both fixed cost and variable cost. Furthermore, we can discuss Scenario 4 in two sub-cases.

Sub-case A: $\frac{k^N - k^S}{e^S - e^N} < \frac{K^N - K^S}{E^S - E^N}$

If $c < \frac{k^N - k^S}{e^S - e^N}$, it is obvious that $c(e^N - e^S) + k^N - k^S > 0$ and $\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S} > 0$. Hence it is optimal to locate the production in S.

If $\frac{k^N - k^S}{e^S - e^N} \leq c < \frac{K^N - K^S}{E^S - E^N}$, then $c(e^N - e^S) + k^N - k^S \leq 0$ and $\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S} > 0$. When $\sqrt{\lambda} \geq -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S})}{c(e^N - e^S) + k^N - k^S}$, it is optimal to produce in N. When $\sqrt{\lambda} < -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S})}{c(e^N - e^S) + k^N - k^S}$, it is optimal to produce in S.

If $c \geq \frac{K^N - K^S}{E^S - E^N}$, then $c(e^N - e^S) + k^N - k^S \leq 0$ and $\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S} \leq 0$. As a result, it is preferred to locate the production in N.

Sub-case B: $\frac{k^N - k^S}{e^S - e^N} \geq \frac{K^N - K^S}{E^S - E^N}$

If $c < \frac{K^N - K^S}{E^S - E^N}$, similarly we have $c(e^N - e^S) + k^N - k^S > 0$ and $\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S} > 0$. Hence it is optimal to locate the production in S.

If $\frac{K^N - K^S}{E^S - E^N} \leq c < \frac{k^N - k^S}{e^S - e^N}$, then $c(e^N - e^S) + k^N - k^S > 0$ and $\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S} \leq 0$. When $\sqrt{\lambda} \geq -\frac{\sqrt{2(h + ce_h)}(\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S})}{c(e^N - e^S) + k^N - k^S}$, it is optimal to produce in S. When $\sqrt{\lambda} < -\frac{\sqrt{2(h + ce_h)}(\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S})}{c(e^N - e^S) + k^N - k^S}$, it is optimal to produce in N.

If $c \geq \frac{k^N - k^S}{e^S - e^N}$, then $c(e^N - e^S) + k^N - k^S \leq 0$ and $\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S} \leq 0$. As a result, it is preferred to locate the production in N.

Scenario 5: $K^N \leq K^S, k^N \leq k^S, E^N \leq E^S, e^N > e^S$.

In this scenario, it is always true that $\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S} \leq 0$.

If $c < \frac{k^N - k^S}{e^S - e^N}$, then expression (4) is non-positive and it is optimal to locate the production in N.

If $c \geq \frac{k^N - k^S}{e^S - e^N}$, the sign of expression (4) depends on the value of λ. When $\sqrt{\lambda} \geq -\frac{\sqrt{2(h + ce_h)}(\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S})}{c(e^N - e^S) + k^N - k^S}$, it is optimal to produce in S. When

$\sqrt{\lambda} < -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S})}{c(e^N-e^S)+k^N-k^S}$, it is optimal to produce in N.

Scenario 6: $K^N \le K^S, k^N > k^S, E^N \le E^S, e^N > e^S$

In this scenario, $c(e^N-e^S)+k^N-k^S$ is always non-negative and $\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S}$ is always non-positive. The demand rate λ plays an important role in determining the optimal production location. When $\sqrt{\lambda} \ge -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S})}{c(e^N-e^S)+k^N-k^S}$, it is optimal to produce in S. When $\sqrt{\lambda} < -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S})}{c(e^N-e^S)+k^N-k^S}$, it is optimal to produce in N.

Scenario 7: $K^N > K^S, k^N \le k^S, E^N \le E^S, e^N > e^S$.

In this scenario, both signs of $c(e^N-e^S)+k^N-k^S$ and $\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S}$ depends on the value of c. Similar to Scenario 4, we can discuss Scenario 7 in two sub-cases.

Sub-case A: $\frac{k^N-k^S}{e^S-e^N} < \frac{K^N-K^S}{E^S-E^N}$

If $c < \frac{k^N-k^S}{e^S-e^N}$, it is obvious that $c(e^N-e^S)+k^N-k^S < 0$ and $\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S} > 0$. When $\sqrt{\lambda} \ge -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S})}{c(e^N-e^S)+k^N-k^S}$, it is optimal to produce in N. When $\sqrt{\lambda} < -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S})}{c(e^N-e^S)+k^N-k^S}$, it is optimal to produce in S.

If $\frac{k^N-k^S}{e^S-e^N} \le c < \frac{K^N-K^S}{E^S-E^N}$, then $c(e^N-e^S)+k^N-k^S \ge 0$ and $\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S} > 0$. Locating the production in S is optimal.

If $c \ge \frac{K^N-K^S}{E^S-E^N}$, then $c(e^N-e^S)+k^N-k^S \ge 0$ and $\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S} \le 0$.

As a result, When $\sqrt{\lambda} \geq -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S})}{c(e^N-e^S)+k^N-k^S}$, it is optimal to produce in S. When $\sqrt{\lambda} < -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S})}{c(e^N-e^S)+k^N-k^S}$, it is optimal to produce in N.

Sub-case B: $\frac{k^N-k^S}{e^S-e^N} \geq \frac{K^N-K^S}{E^S-E^N}$

If $c < \frac{K^N-K^S}{E^S-E^N}$, similarly we have $c(e^N-e^S)+k^N-k^S < 0$ and $\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S} > 0$. When $\sqrt{\lambda} \geq -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S})}{c(e^N-e^S)+k^N-k^S}$, it is optimal to produce in N. When $\sqrt{\lambda} < -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S})}{c(e^N-e^S)+k^N-k^S}$, it is optimal to produce in S.

If $\frac{K^N-K^S}{E^S-E^N} \leq c < \frac{k^N-k^S}{e^S-e^N}$, then $c(e^N-e^S)+k^N-k^S < 0$ and $\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S} \leq 0$. It is preferred to locate the production in N

If $c \geq \frac{k^N-k^S}{e^S-e^N}$, then $c(e^N-e^S)+k^N-k^S \geq 0$ and $\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S} \leq 0$. When $\sqrt{\lambda} \geq -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S})}{c(e^N-e^S)+k^N-k^S}$, it is optimal to produce in S. When $\sqrt{\lambda} < -\frac{\sqrt{2(h+ce_h)}(\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S})}{c(e^N-e^S)+k^N-k^S}$, it is optimal to produce in N.

Scenario 8: $K^N > K^S, k^N > k^S, E^N \leq E^S, e^N > e^S$.

In this scenario, it is always true that $c(e^N-e^S)+k^N-k^S > 0$.

If $c < \frac{K^N-K^S}{E^S-E^N}$, then $\sqrt{K^N+cE^N}-\sqrt{K^S+cE^S} > 0$. It is optimal to locate the production in S.

If $c \geq \frac{K^N - K^S}{E^S - E^N}$, then the sign of expression (4) depends on the value of λ. When $\sqrt{\lambda} \geq -\frac{\sqrt{2(h + ce_h)}(\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S})}{c(e^N - e^S) + k^N - k^S}$, it is optimal to produce in S. When $\sqrt{\lambda} < -\frac{\sqrt{2(h + ce_h)}(\sqrt{K^N + cE^N} - \sqrt{K^S + cE^S})}{c(e^N - e^S) + k^N - k^S}$, it is optimal to produce in N.