# A multiregional endogenous growth model with forward looking agents

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comments welcome

**Abstract**

The paper presents a multiregional endogenous growth model designed for calibration with real world data and for numerical policy evaluation. It integrates four strands of research: (1) the celebrated RAMSEY model of consumer behavior, (2) TOBIN's q-theory of investment, (3) ROMER's theory of endogenous growth through horizontal product innovation, and (4) the DIXIT-STIGLITZ-ETHIER theory of intra-industry trade. Integrating the latter into a multiregional model is also an essential ingredient of the New Economic Geography. Thus, the paper is related to this literature as well, but lacks another essential feature of this tradition; the model to be presented does not exhibit catastrophic agglomeration. A symmetric first nature will always generate a "flat earth" steady state equilibrium. The model has an arbitrary (possibly large) number of regions with a representative household and a production sector in each of them. There are three types of goods, non-tradable local goods, horizontally diversified tradables, and designs of tradable products that are the exclusive property of their producers (called "blueprints", for short). Goods are produced combining — in identical proportions for all three of them — four inputs, labor, capital local and tradable goods. Blueprint production benefits from a technological positive externality. Technologies and preferences are uniform across space. Beyond goods markets and real factor markets there is a frictionless global bond market. All markets are perfectly competitive except the tradables market, which is monopolistic in the familiar DIXIT-STIGLITZ-ETHIER style. The engine of sustainable long run growth is the accumulation of blueprints. Agents act under perfect foresight. The paper explains the formal structure, the solution and calibration techniques and illustrates the application by a small example.

JEL classification: C15, H54, H71, R11, R13, R53, R58

## 1 Introduction

The European Union spends roughly one third of its budget for cohesion policies, applying a diverse bundle of instruments like investment subsidies, financial support for transport

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and communication infrastructure as well as innovation policies. There is a practical need for evaluating the effectiveness of these market interventions in terms of the stated objectives to promote a more equitable spatial development and economic growth. Applied dynamic equilibrium analysis is a suitable instrument to that end. It is desirable that the models applied in this context take up the theoretical advances of the last two decades by incorporating agglomeration forces as well as endogenous growth. Setting up such a framework is the aim of this paper.

`To do: what others have achieved, what is missing, which gap do I close.`

It presents a multiregional endogenous growth model designed for calibration with real world data and for numerical policy evaluation. It integrates four strands of research: (1) the celebrated RAMSEY model of consumer behavior, (2) TOBIN's q-theory of investment, (3) ROMER's theory of endogenous growth through horizontal product innovation, and (4) the DIXIT-STIGLITZ-ETHIER theory of intra-industry trade. Integrating the latter into a multiregional model is also an essential ingredient of the New Economic Geography. Thus, the paper is related to this literature as well, but lacks another essential feature of this tradition; the model to be presented does not exhibit catastrophic agglomeration. A symmetric first nature will always generate a "flat earth" steady state equilibrium. The model has an arbitrary (possibly large) number of regions with a representative household and a production sector in each of them. There are three types of goods, non-tradable local goods, horizontally diversified tradables, and designs of tradable products that are the exclusive property of their producers (called "blueprints", for short). Goods are produced combining — in identical proportions for all three of them — four inputs, labor, capital local and tradable goods. Blueprint production benefits from a technological positive externality. Technologies and preferences are uniform across space. Beyond goods markets and real factor markets there is a frictionless global bond market. All markets are perfectly competitive except the tradables market, which is monopolistic in the familiar DIXIT-STIGLITZ-ETHIER style. The engine of sustainable long run growth is the accumulation of blueprints. Agents act under perfect foresight. The paper explains the formal structure, the solution and calibration techniques and illustrates the application with a small example.

The following Section 2 explains the basic model without policy intervention. Section 3 shows how to solve it, and Section 4 briefly discusses calibration. Section 5 shows a stylised policy application. Section 6 concludes.

# 2 The model

I begin with a non-formal description. Consider a closed economy with $n$ regions, $n$ possibly being large, one thousand, say. In each region $r$ resides one immobile infinitely lived household supplying inelastically and constantly $\mathcal{L}_r$ units of labor and owning assets worth $A_r(t)$ units of account. $t$ denotes continuous time. Total assets, summed across regions, equal the total market value of firms at any time. Households consume local goods produced in their own region and a CES index good composed of tradables from all regions. They maximize discounted utility over their infinite lifetime subject to a budget constraint requiring the present value of consumption expenditures not to exceed the present value of wage incomes plus the initial value of assets owned.

Each region is also the home of a set of firms producing three kinds of goods, non-tradable local goods, horizontally diversified tradable goods, and designs for tradable products, called "blueprints", for short. Production can be thought of as being a two-stage process. In the first stage the local good is produced by combining four inputs in a constant returns to scale CD production function, namely labor, physical capital, local goods and a CES composite of tradable goods. In the second stage certain shares of these local goods are transformed into tradables or blueprints under constant returns to scale. Tradables are horizontally diversified. For being able to offer goods of a certain variety a producer has to own a blueprint, that has to be produced in the same region where it is used. Blueprints are thus also non-tradable. I briefly discuss an alternative design in the conclusion.

I assume firms owning a blueprint also to own the exclusive right to use it; nobody is allowed to copy it. It should be mentioned that in a non-spatial world it is sufficient to assume copying to be costly to exclude the option of copying, even if costs of copying are small. If, in a non-spatial world, a firm decided to copy an existing blueprint, it would blunder into BERTRAND competition with the other firm already using the blueprint. This would drive prices down to marginal cost, and nothing would be left to finance even small copying costs. This is obviously different in a spatial world. An imitator at a distance from the other blueprint owner would still have some market power, due to a certain degree of protection from the competitor. If this market power allowed for an operating profit sufficient to finance the capital cost of copying, there would still be an incentive to copy. Though empirically relevant, I have to exclude this possibility for the sake of tractability. I implicitly assume that coping costs always exceed the present value of attainable operating profits of an imitator.

The productivity of blueprint production is endogenous; it is the higher, the more knowledge is acquired by looking at existing blueprints in the own as well as in other regions. Knowledge acquisition (called "learning", for short) is a costly activity. In order to learn from a certain blueprint one must visit the place where it is applied and pay learning plus travel cost, both in terms of local goods used up in this activity. The blueprint owner, however, is not remunerated for the right to learn from his or her blueprint. Hence, in addition to make the production of a certain product variety possible, a blueprint exerts a positive technological externality in the innovation process. This turns out to be key for sustainable growth. Blueprint producers (called "innovators", for short) choose destinations and levels of learning activity such that blueprint production costs are minimised. The expansion of product variety in the course of time lets the price index of tradables' composites decline. This is the ultimate engine of growth. Following the literature I shall assume that the benefit from learning is just big enough to guarantee steady state growth at a constant rate. Both kinds of goods, non-tradables and tradables, are used as inputs, as consumption goods and for physical capital investment. Regarding the latter, I assume quadratic adjustment cost to avoid unrealistic corner solutions. Cost per unit of investment is assumed to increase in the rate of capital growth, yielding Tobin's q-theory of investment.

There are $6n + 1$ markets in this economy that have to be in equilibrium at each moment, six markets per region and one global asset market. Per region there is (respective prices in parentheses)

1. a labor market $(w_r(t))$,

2. a market for the service of capital installed in production $(r_r(t))$,

3. a market for the local good ($p_r^m(t)$),

4. a market for tradables stemming from the respective region ($p_r^m(t)$),

5. a market for the stock of physical capital installed ($q_r(t)$), and

6. a market for the stock of blueprints applied in the region ($v_r(t)$).

Regarding (4), there is strictly speaking not just one such market, but as many as there are varieties produced in the respective region. But by a symmetry assumption they are all alike, such that we only need one price and one equilibrium condition for all of them. Choosing dimensions suitably, this price turns out to be the same as for local goods. All markets are perfectly competitive, except the market for tradables, which is monopolistically competitive in the familiar Dixit-Stiglitz style with iceberg transportation cost.

Finally there is one frictionless perfectly competitive world asset market with a single uniform nominal world interest rate. As one is free to choose a time path of the unit of account, I choose it such that the uniform interest rate is the time preference parameter $\rho$, and thus constant over time. This is standard and convenient, but it is important to keep in mind that any other choice will do, without affecting results in real terms. Households hold shares in firms that themselves hold two types of assets, physical capital and blueprints. Both kinds of assets earn the same nominal interest all over the economy, and there is no uncertainty. Households thus do not care about their respective portfolio composition. If the economy faces an unexpected shock, however, portfolio compositions do matter, because different assets are in general affected by revaluations as a response to the shock in a different way. I shall come back to this important point.

To avoid confusion it is important to distinguish between the world asset market (with uniform nominal interest rate $\rho$), the markets for regional physical capital stocks (with stock prices $q_r(t)$) and the markets for the the services of regional physical capital (with rental rates $r_r(t)$). $\rho$ is the payment per annum per unit of an asset that is also measured in units of account. It thus has dimension "per annum" (p.a.). $q_r(t)$ is the payment per unit of installed physical capital. Its dimension is "unit of account per unit of physical capital". Finally, $r_r(t)$ is the per annum payment for using the service of one unit of capital. It is thus measured in terms of "units of account per unit of physical capital per annum". Unlike $\rho$, the latter two prices vary across time and space.

I now display the model structure formally. I use calligraphic letters like $\mathcal{L}$ for real variables, upper case Latins like $A$ for nominal variables, lower case latins like $p^c$ for prices and lower case Greeks like $\rho$ for parameters. Lower case Greek parameters are constant across regions and over time. All variables refer to one region and are functions of continuous time. All equations also refer to one region. If not needed for understanding, the regional subscript and the time argument are omitted to avoid notational clutter. I first deal in turn with households, production, investment and innovation, and then derive the equilibrium conditions.

## 2.1 Households

Households in a region are regarded to represent the present as well as all future generations. They maximize CIES utility over an infinite time horizon,

$$\max_{\mathcal{C}} \int_0^\infty \frac{\mathcal{C}^{1-1/\theta} - 1}{1 - 1/\theta} \exp(-\rho t) \, \mathrm{d}t$$

subject to the flow budget constraint

$$\dot{A} = w\mathcal{L} + \rho A - C, \tag{1}$$

with wage rate $w$, real consumption $\mathcal{C}$ and nominal consumption $C = \mathcal{C}p^c$. As usual, dots denote time derivatives. $p^c$ is the consumers' price index, $\theta > 0$ is the intertemporal elasticity of substitution, $\rho > 0$ is the rate of time preference. $A$ is the nominal asset value owned by the household. As mentioned, I choose the time path of the unit of account such that $\rho$ is also the nominal interest rate. The budget constraint says that nominal saving, which is the increase of the nominal asset value per unit of time, equals wage income $w\mathcal{L}$ plus interest income $\rho A$ minus nominal consumption.

The household is supposed to start at $t = 0$ with an asset $A_0$,

$$A(0) = A_0. \tag{2}$$

There is also a "final" boundary condition, a limiting condition for $t$ tending to infinity. No household would want to hold assets that in present values tend to a strictly positive amount, if time goes to infinity. Thus, no household is willing to lend amounts tending to a positive present value, and therefore no household can end up with a debt tending to a positive amount in present values. From these considerations follows the familiar limiting condition

$$\lim_{t\to\infty} A(t) \exp(-\rho t) = 0. \tag{3}$$

Equations (1), (2) and (3) jointly say that the present value of wage income plus the initial asset value must equal the present value of consumption expenditure.

The first order condition for optimality (which is necessary and due to concavity also sufficient) requires the discounted marginal utility to be equal to the discounted price index, up to an unknown constant $c$. Rearranging yields consumption expenditures

$$C = c(p^c)^{1-\theta}. \tag{4}$$

A special more familiar case is $\theta = 1$, implying constant consumption expenditures. The constant $c$ must be such that the limiting condition (3) is going to hold.

Real consumption is a CD composite of local goods and the tradables' composite, with respective expenditure share $\epsilon$ and $1 - \epsilon$. Thus the corresponding consumer's price index is

$$p^c = (p^m)^\epsilon (p^d)^{1-\epsilon}. \tag{5}$$

$p^d$ denotes the price index of the tradables' composite, which is supposed to be a CES composite of varieties from all over the world. The mill price of a variety from region $s$ is $p_s^m$, its destination price in region $r$, including iceberg transportation cost, is $p_s^m \Theta_{sr}$, with cost mark up factor $\Theta_{sr} \geq 1$ for flows from $s$ to $r$. The price index for the composite is thus

$$p_r^d = \left( \sum_s \mathcal{N}_s (p_s^m \Theta_{sr})^{1-\sigma} \right)^{1/(1-\sigma)} . \tag{6}$$

$\mathcal{N}$ is the measure of varieties, $\sigma$ is the elasticity of substitution between varieties.

## 2.2 Production

Local goods are produced by a CD technology with input shares $\alpha$, $\beta$, $\gamma$ and $\eta$ for labour, physical capital, non-tradables and tradables, respectively. The local goods price is thus

$$p^m = \phi w^\alpha r^\beta (p^m)^\gamma (p^d)^\eta. \tag{7}$$

$\phi$ is an arbitrary scaling parameter allowing to choose units of local goods in a convenient way. The price for tradables' composites is $p^d$, the same as for consumers. I assume firms to buy the same composite as consumers, and later I assume this composite also to be used for investment.

As mentioned, local goods can be transformed into tradables under constant returns to scale. I choose units of tradables such that $(\sigma - 1)/\sigma$ input units are required for one unit of tradables. Producers of tradables are well known to face a price elasticity equal to $-\sigma$, due to the CES form of tradables demand. The monopolistic price mark up factor is therefore $\sigma/(\sigma - 1)$, and the mill price of tradables just equals the price for local goods $p^m$.

## 2.3 Investment

Local goods producers decide at any moment about both, production and investment into physical capital. They face adjustment costs of investment, in addition to the cost for the investment itself; these costs — represented by the second term in brackets in equation (8) — start at zero for zero gross investment and increase as the rate of capital growth goes up. Following the literature (see e.g. [2]) I assume quadratic investment costs (see Figure 1),

$$J = p^c \mathcal{I} \left( 1 + \frac{\zeta}{2} \frac{\mathcal{I}}{\mathcal{K}} \right) . \tag{8}$$

$\mathcal{I}$ is real gross investment, $\mathcal{K}$ is the real stock of capital, $J$ is nominal investment cost, and $p^c$ is the price of the final demand commodity. It is a CD composite of local and tradable goods. The same composite is used for consumption and for investment. $\zeta$, measured in years, is the adjustment cost parameter. The higher $\zeta$, the more adjustment costs increase with increasing speed of capital accumulation, and the more sluggish investment is going to respond to changes in capital returns.

By introducing adjustment costs I rule out an implausible outcome of the basic open-economy version of the RAMSEY model, where the adjustment of capital stocks is done through an instantaneous jump to other locations as a response to an external shock affecting returns to capital. The existence of adjustment costs implies that the stock of capital in a region has a stock price $q$ that in general differs from the price of the investment good, by which the stock is built up. Taking the stock price at any point in

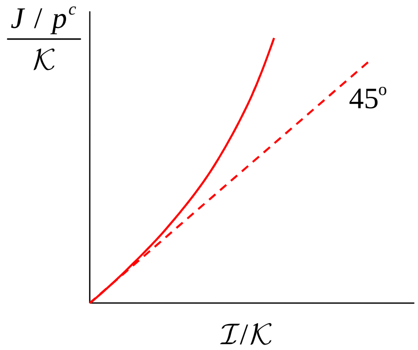


Figure 1: Investment cost

time as given, firms invest until the marginal cost of investment equals its marginal return $q$, leading to the investment function

$$\mathcal{I} = \mathcal{K}\frac{q/p^c - 1}{\zeta}. \tag{9}$$

$q/p^c$ is called "TOBIN's q" in the literature. Capital depreciates at a rate $\delta$ p.a., hence

$$\dot{\mathcal{K}} = \mathcal{I} - \delta\mathcal{K}. \tag{10}$$

With investment according to (9) we thus obtain

$$\hat{\mathcal{K}} = \frac{q/p^c - 1}{\zeta} - \delta \tag{11}$$

with initial boundary condition

$$\mathcal{K}(0) = \mathcal{K}_0 \tag{12}$$

to describe the dynamics of the capital stock. Hats indicate growth rates, e.g. $\hat{\mathcal{K}} = \dot{\mathcal{K}}/\mathcal{K}$.

## 2.4 Innovation

Innovations consist in new blueprints for tradables. The measure of blueprints in a region is denoted $\mathcal{N}$, a non-negative real variable. Varieties, and thus blueprints, are understood as a continuum. One unit of the measure is simply called "one blueprint". $\dot{\mathcal{N}}$, the increase in the measure of blueprints per unit of time, is called "innovation". Certain firms, called "innovators", produce innovations with local goods as the only input under perfect competition. An innovator needs $\imath$ units of the local good per blueprint produced. The innovator can decide to spend more or less for learning from others to reduce $\imath$, but other determinants of $\imath$ are out of his or her control. $\imath$ depends on the regional rate of innovation $\hat{\mathcal{N}}$ as well as on the blueprint stocks in the own as well as in all other regions according to

$$\imath = \psi \exp(\lambda\hat{\mathcal{N}})\mathcal{P}^{-\chi}, \tag{13}$$

with "learning potential"

$$\mathcal{P}_r = \sum_s \mathcal{N}_s \exp(-\mu\Phi_{rs}).$$

The term $\exp(\lambda\hat{\mathcal{N}})$ is an adjustment cost term, similar to the adjustment cost specification for physical capital investment. Cost per blueprint goes up with an increasing rate of innovation. This term prevents unrealistic corner solutions, where innovation is entirely concentrated to just a single region offering the highest operating profit per blueprint. Note that, as $\lambda\hat{\mathcal{N}}$ is a small number (typically in the order of 0.1), $\exp(\lambda\hat{\mathcal{N}}) \approx 1 + \lambda\hat{\mathcal{N}}$. Hence, the difference between this specification and the one in (8) is negligible. The choice is made here for mathematical convenience. Individual innovators regard $\hat{\mathcal{N}}$ as an externality.

The second term represents innovation spillovers. $\Phi_{rs}$ is the business travel distance. Innovators are supposed to learn from blueprints in use elsewhere, and they benefit more from those that are close by than those that are further way. The distance decay is parameterised by $\mu > 0$. A higher learning potential reduces innovation cost with an elasticity $-\chi < 0$. The term generalises the typical specification of knowledge spillovers in ROMER models. One of the textbook versions (see e.g. [1, Ch. 13.2]) assumes blueprints to be produced only by labour. In that version, to allow for permanent growth, one has to assume the input needed per blueprint to be inverse proportional to the stock of blueprints in the economy. This specification emerges with $\mu = 0$ and $\chi = 1$. A positive $\mu$ lets geography enter the scene. Allowing $\chi$ to differ from one is necessary to allow for sustained steady state growth in our setting, differing from the mentioned textbook model by assuming innovation not to be produced by labour alone, but by the local good. This is tantamount assuming it to be made by labour, physical capital and tradable and non-tradable inputs in just the same proportions as they apply to local goods production. It resembles the specification that ACEMOGLU [1, Ch. 13.1] calls the "lab equipment" model.

Equation (13) can be derived from an optimal learning approach. Let the input requirement per blueprint be

$$\imath_r = \tilde{\psi} \exp(\tilde{\lambda}\hat{\mathcal{N}}_r) \left( \sum_s \mathcal{N}_s^{1-\nu} \ell_{rs}^{\nu} \right)^{-\tilde{\chi}} .$$

$\ell$ is the learning activity of innovators from $r$ in $s$. $\tilde{\psi}$, $\tilde{\lambda}$, $\tilde{\chi}$ and $\nu$ are positive parameters. $\nu$ is less than one. Learning and existing knowledge, which is given from the point of view of the innovator, are combined according to a CD function to generate a learning output (call it a "lesson") $\mathcal{N}_s^{1-\nu} \ell_{rs}^{\nu}$ in destination region $s$. Summing over all destinations yields the total lessons learned, which reduce the cost with elasticity $-\tilde{\chi}$. Learning is costly; units of learning are chosen such that one unit of learning in $s$ entails costs of $\exp(\tau\Phi_{rs})$ in terms of the local good, with a cost parameter $\tau > 0$. Learning costs per unit are one, if distance is zero. As shown in the appendix, $\imath_r$ in (13) is obtained by optimizing the learning activity, with

$$\psi = \tilde{\psi}^{\frac{1}{1+\tilde{\chi}\nu}} (1 + \tilde{\chi}\nu)(\tilde{\chi}\nu)^{-\frac{\tilde{\chi}\nu}{1+\tilde{\chi}\nu}},$$

$$\lambda = \frac{\tilde{\lambda}}{1 + \tilde{\chi}\nu},$$

$$\mu = \frac{\tau\nu}{1 - \nu},$$

and

$$\chi = \frac{\tilde{\chi}(1 - \nu)}{1 + \tilde{\chi}\nu}.$$

Due to constant returns to scale and perfect competition, the equilibrium price $v$ of a blueprint must be just equal to the cost of a blueprint, $v = p^m \imath$. Inserting (13) for $\imath$ and solving for $\hat{\mathcal{N}}$ gives the innovation equation

$$\hat{\mathcal{N}} = \frac{1}{\lambda} \log \frac{v \mathcal{P} \chi}{p^m \psi}, \tag{14}$$

with initial boundary condition

$$\mathcal{N}(0) = \mathcal{N}_0. \tag{15}$$

The innovation rate is the higher, the higher the real blueprint price in terms of the local good, the higher the learning potential and the lower the adjustment cost.

## 2.5 Equilibrium

There must be six equilibrium conditions per region for the six markets enumerated above, plus one for the global asset market. I deal with them in the same order as they are enumerated above.

1. Labor market:

$$\alpha M = w \mathcal{L}. \tag{16}$$

   $M$ is output value.

2. Market for the service of capital installed:

$$\beta M = r \mathcal{K}. \tag{17}$$

3. Market for tradables stemming from the respective region:

$$S_r \frac{\sigma}{\sigma - 1} = \mathcal{N}_r (p_r^m)^{1-\sigma} \sum_s \left( (\Theta_{rs}/p_s^d)^{1-\sigma} D_s \right), \tag{18}$$

   with tradables demand $D_s$ in region $s$,

$$D = (1 - \epsilon)(C + J) + \eta M. \tag{19}$$

   $S$ is the input value of tradables. As $\sigma/(\sigma - 1)$ is the monopolistic mark-up factor, $S\sigma/(\sigma - 1)$ is the output value. It must be equal to the value of demand from all regions. Demand from one region $s$ for tradables from $r$ is $D_s$ times the expenditure share going to $r$, which is the log derivative of $p_s^d$ with respect to $p_r^m$ by Hotelling's lemma.

4. Market for local goods: The value of supply must be equal to the value of demand,

$$M = \epsilon(C + J) + \gamma M + H + S. \tag{20}$$

   $H$ is the input value for innovation,

$$H = v \dot{\mathcal{N}}. \tag{21}$$

5. Market for the physical capital stock: For households to be willing to hold the existing stock of physical capital nailed to the ground somewhere, $q$ units of account paid per unit of capital must earn the uniform world interest rate $\rho$,

$$\rho q = \dot{q} - \delta q + r - J_{\mathcal{K}}. \tag{22}$$

This is a non-arbitrage condition. The last term is minus the partial derivative of $J$ with respect to $\mathcal{K}$, i.e. the marginal reduction of investment cost per unit of capital. The explicit formula is

$$J_{\mathcal{K}} = -p^c \frac{\zeta}{2} \left( \frac{\mathcal{I}}{\mathcal{K}} \right)^2 .$$

There is no initial boundary condition on $q$; the stock price is a "jumper". But optimizing behavior of firms implies a transversality condition [2, p. 120] requiring the present value of the capital stock to tend to zero as time goes to infinity,

$$\lim_{t\to\infty} q(t)\mathcal{K}(t)\exp(-\rho t) = 0. \tag{23}$$

6. Market for the stock of blueprints: A similar no-arbitrage condition as for physical capital has to hold for the stock of blueprints,

$$\rho v = \dot{v} + \frac{S}{\mathcal{N}(\sigma - 1)}. \tag{24}$$

Note that $S\sigma/(\sigma-1)$ is the output value in the tradables industry. Subtracting the input value $S$ gives the operating profit $S/(\sigma-1)$. The last term in (24) is thus the operating profit per blueprint. A similar boundary condition as for physical capital has to hold for the value of the blueprint stock,

$$\lim_{t\to\infty} v(t)\mathcal{N}(t)\exp(-\rho t) = 0. \tag{25}$$

Finally, there is the world asset market. For the entire world, the increase in the value of stocks per unit of time must be equal to total net saving,

$$\sum_r \dot{A}_r = \sum_r (\dot{\mathcal{K}}_r q_r + \mathcal{K}_r \dot{q}_r + \dot{\mathcal{N}}_r v_r + \mathcal{N}_r \dot{v}_r).$$

By Walras' law this condition automatically holds, if all the others are fulfilled. It thus needs not be explicitly taken account of.

Taking stock, we have 5 dynamic variables per region, $A$, $\mathcal{K}$, $q$, $\mathcal{N}$ and $v$. I gather their respective logs[1] in the vector $X \in \mathbb{R}^{5n}$ for later reference.[2] There are 5 corresponding differential equations, (1), (11), (22), (14) and (24), with 5 boundary conditions, (2), (12), (23), (15) and (25). Furthermore, we have 12 time-varying variables per region, $C$, $p^c$, $p^d$, $p^m$, $J$, $\mathcal{I}$, $w$, $r$, $S$, $D$, $M$ and $H$. I gather their logs in the vector $Y \in \mathbb{R}^{12n}$. There

[1] For the sake of simplicity I assume here $A$ never to become negative, though solutions with negative $A$ can be feasible; households could be indebted, even forever. It is possible to solve the system along similar lines treating $A$ instead of $\log A$ as a variable and admitting it to be negative, but equations then look less tidy.

[2] I drop here the convention to reserve upper case Latins for nominal variables.

| $X$ | $\xi$ | $X$ | $\xi$ |
|---|---|---|---|
| $A$ | $1-1/\theta$ | $\mathcal{N}$ | $(\sigma-1)\Omega$ |
| $\mathcal{K}$ | $1$ | $v$ | $1-1/\theta-(\sigma-1)\Omega$ |
| $q$ | $-1/\theta$ | | |

| $Y$ | $\varphi$ | $Y$ | $\varphi$ |
|---|---|---|---|
| $C$ | $1-1/\theta$ | $w$ | $1-1/\theta$ |
| $p^c$ | $-1/\theta$ | $r$ | $-1/\theta$ |
| $p^d$ | $-\epsilon\Omega-1/\theta$ | $S$ | $1-1/\theta$ |
| $p^m$ | $(1-\epsilon)\Omega-1/\theta$ | $D$ | $1-1/\theta$ |
| $J$ | $1-1/\theta$ | $M$ | $1-1/\theta$ |
| $\mathcal{I}$ | $1$ | $H$ | $1-1/\theta$ |

Table 1: Steady state growth rates, up to a common multiplyer $x$

are 12 corresponding algebraic equations (4), (5), (6), (7), (8), (9), (16), (17), (18), (19), (20) and (21). Finally, there is one endogenous constant $c_r$ per region with corresponding limiting condition (3). One may count the latter also as a dynamic variable with the trivial differential equation $\dot{c}=0$ and limiting condition (3). But in the literature on solution techniques of boundary value problems that I rely on [5], these variable are treated as an extra class, called "parameters". It is numerically easier to work with logs. Hence I introduce the vector $d=(\log c_1,\ldots,\log c_n)\in\mathbb{R}^n$.

The system is then compactly written as

$$\dot{X} = f(X,Y), \tag{26}$$

$$0 = h(X,Y,d), \tag{27}$$

with $3n$ initial ((2), (12), (15)) and $3n$ limiting ((3), (23), (25)) boundary conditions. $f$ is a $\mathbb{R}^{17n}$ to $\mathbb{R}^{5n}$ and $h$ is a $\mathbb{R}^{18n}$ to $\mathbb{R}^{12n}$ mapping. This is a differential algebraic equation system with the extra complication that some boundary conditions are given at the initial point in time and others at infinity. I am able to solve it using a solver for two-point boundary value problems. To make this possible, I first develop an approximate solution, a trajectory based on a linear approximation around the steady state. Then I replace the limiting conditions by boundary conditions at a finite horizon. These conditions require the system to attain the stable manifold of the approximation. Choosing the finite horizon far enough in the future allows to approach the true trajectory as close as one likes.

# 3 Solution

The following proposition shows that, under a suitable parameter restriction, growth rates and equilibrium conditions remain unchanged, if the endogenous variables grow for an arbitrary length of time at rates proportional to the numbers displayed in Table 1, with

$$\Omega=\frac{\alpha}{1-\gamma-\epsilon(\alpha+\beta)}=\frac{\alpha}{\eta+(1-\epsilon)(\alpha+\beta)}. \tag{28}$$

**Proposition 1** *If $\xi$ and $\varphi$ are defined as in table 1, and if*

$$\chi = 1 - \frac{\eta + \beta(1-\epsilon)}{\alpha(\sigma - 1)}, \tag{29}$$

*then, for any $X$, $Y$, and $d$ such that $h(X, Y, d) = 0$, and any real number $g$,*

$$f(X + g\xi, Y + g\varphi) = f(X, Y) \tag{30}$$

*and*

$$h(X + g\xi, Y + g\varphi, d) = 0. \tag{31}$$

A steady state is a trajectory where all variables grow at constant, though possibly different rates. It s thus a real number $x$ (the growth rate of real consumption) and a pair of trajectories $X(t) = X^* + x\xi t$ and $Y(t) = Y^* + x\varphi t$ such that

$$x\xi = f(X^*, Y^*), \tag{32}$$

$$0 = h(X^*, Y^*, d). \tag{33}$$

These trajectories obviously fulfill equations (26) and (27). One can pick one point on the trajectory, denoted $(x, \bar{X}, \bar{Y})$, by adding a restriction, e.g. $\sum_r \log \mathcal{N}_r = 0$.

The parameter $\chi$ measures the strength of spillovers just required for endogenous permanent growth. Larger spillovers lead to explosion, smaller to stagnation in the long run. The more the innovation producers benefit from product diversity in the economy, the smaller are the externalities just necessary for long run steady growth. Innovators' benefits from diversity are the bigger, the smaller is $\sigma$ and the more capital intensive and tradables intensive the economy is, that is the smaller the labour share $\alpha$, the larger the share of capital and tradables in production, and the larger the share of tradables in final demand. It may seem strange that parameter restriction (29) is needed to allow for sustained growth. The fact that sustained growth only works with such a kind of knife-edge parametrization is however an inevitable property of any endogenous growth model. In one version of textbook models, called "lab equipment model" by Acemoglu [1, Ch. 13.1], blueprints are assumed to be produced by the final good, which itself is made by labor and the composite of diversified goods. In such a specification sustained growth works without an externality (i.e. with $\chi = 0$), if $\sigma$ is small enough, namely $\sigma = 1/\alpha$ (see [1, p. 434]). The formula in (29) renders this as a special case: for $\sigma = 1/\alpha$, $\eta = \epsilon = 0$ and $\beta = 1 - \alpha$ we get $\chi = 0$. Another version assumes labor to be the only input in blueprint production [1, Ch. 13.2]. In this case productivity of innovation does not benefit from product diversity, and one must assume $\chi = 1$. Our specification resembles the lab equipment model, but choosing $\sigma$ low enough to make $\chi = 0$ is no suitable choice. Given realistic numbers for the other parameters, the elasticity of substitution would have to be less than two (but bigger than one, as it should be), which is clearly too low. $\sigma = 1.8$, say, would imply a price markup of 125 % (i.e. a price more than double the marginal cost), which is obviously out of range. In the trade and Economic Geography literature one finds elasticities of at least five, which implies a $\chi$-parameter between zero and one, a good compromise between the extreme cases of $\chi = 0$ and $\chi = 1$ in the textbook models.

To derive the linear approximation, I Taylor-approximate to the first order equations (26) and (27) around $(x, \bar{X}, \bar{Y})$ and some guess $\bar{d}$ for $d$. This yields the inhomogeneous linear constant coefficients system

$$\dot{X} = x\xi + \Xi(X - \bar{X}) + \Gamma(d - \bar{d}), \tag{34}$$

with $\Xi = f_X - f_Y h_Y^{-1} h_X$ and $\Gamma = -f_Y h_Y^{-1} h_d$. $f_X$ and so forth denote Jacobians, evaluated at $(\bar{X}, \bar{Y}, \bar{d})$. With

$$Z = X - \bar{X} - x\xi t + \Xi^{-1}\Gamma(d - \bar{d})$$

this becomes a homogeneous system in $Z$,

$$\begin{aligned} \dot{Z} &= \dot{X} - x\xi \\ &= \Xi(X - \bar{X}) + \Gamma(d - \bar{d}) \\ &= \Xi(X - \bar{X} - x\xi t) + \Gamma(d - \bar{d}) \\ &= \Xi Z. \end{aligned}$$

From the second to the third line I used $\Xi\xi = 0$, which follows from derivating (30) and (31) with respect to $g$, yielding $f_X\xi + f_Y\varphi = 0$ and $h_X\xi + h_Y\varphi = 0$. Solving the latter for $\varphi$ and inserting into the former yields $(f_X - f_Y h_Y^{-1} h_X)\xi = \Xi\xi = 0$.

The general solution of $\dot{Z} = \Xi Z$ is

$$Z(t) = V \exp(\Lambda t) u,$$

where $V$ denotes the matrix of eigenvectors, $\Lambda$ denotes the diagonal matrix of eigenvalues of $\Xi$ and $u \in \mathbb{R}^{5n}$ denotes a yet undetermined vector of weights. The initial boundary condition can be written as

$$BX(0) = R,$$

with a vector $R \in \mathbb{R}^{3n}$ containing the logs of $A$, $\mathcal{K}$ and $\mathcal{N}$ and a $(3n \times 5n)$-matrix $B$. Inserting $X(0) = Vu + \bar{X} - \Xi^{-1}\Gamma(d - \bar{d})$ into the boundary condition yields a linear system in $u$ and $d$,

$$B(V, -\Xi^{-1}\Gamma) \begin{pmatrix} u \\ d \end{pmatrix} = R - B(\bar{X} + \Xi^{-1}\Gamma\bar{d}). \tag{35}$$

These are $3n$ equations in $6n$ unknowns $(u, d)$. $3n$ degrees of freedom must be closed by the boundary conditions at infinity. Fortunately, we find exactly $3n$ eigenvalues with real parts equal to or bigger than $\rho - x(1 - 1/\theta)$. The respective weights in the vector $u$ must be restricted to zero. Otherwise the assets $A$ and stock values $v\mathcal{N}$ and $q\mathcal{K}$ grow at rates equal to or higher than $\rho$, which violates the boundary conditions at infinity. Thus $3n$ components remain, leaving us with a linear system of $3n$ equations in $3n$ unknowns. One of the remaining eigenvalues is zero; the respective eigenvector is (a multiple of) $\xi$, as already shown. All remaining $2n - 1$ eigenvalues turn out to have strictly negative real parts.

As $\xi$ is an eigenvector with a generally non-zero weight, the solution is independent of which point on the steady state trajectory one chooses as $\bar{X}$. Any multiple of $\xi$ that we may add to $\bar{X}$ is perfectly collinear to the just mentioned eigenvector representing the nullspace of $\Xi$.

I am not able to show analytically, for a general version of the model, that the counting condition on eigenvalues always holds. The observation in the last paragraph is obtained from numerical experience, where I never encountered a wrong number. It is known that perfect foresight models under perfect competition and without externalities generically exhibit local uniqueness, implying just the right number of eigenvalues that must be equated to zero. But there are also examples of models with non-perfect competition or externalities (or both) that do not meet the counting condition, either in one or the other direction [4]. There may be too many eigenvectors whose weights must be restricted to zero; then there is no stable path. Or there may be too few, then there is a non-singleton continuum of stable paths. Stock prices are fundamentally indeterminate in this case. As in this case agents may rationally respond to fundamentally irrelevant signals these equilibria are called sunspot equilibria.

Once the linear approximation is solved, the nonlinear solution is simple. I solve (27) for $Y$ and insert it into (26) to obtain a differential equation

$$\dot{X} = F(X, d)$$

with "parameter" $d$. I apply the solution algorithm `bvp5c` in MATLAB (see [5]), with the approximate solution as initial guess of the trajectory to be found. As there are $6n$ unknowns, we need $6n$ boundary conditions. The initial boundary condition is $BX(0) = R$, giving $3n$ constraints. The other boundary condition requires $X(T)$ at some horizon $T$ far in the future to be a point on the solution trajectory of the linear approximation. This is to say that there shall be a vector $u^s \in \mathbb{R}^{3n}$ such that

$$V^s u^s = X(T) - \bar{X} + \Xi^{-1}\Gamma(d - \bar{d}). \tag{36}$$

$V^s$ ist the stable basis, i.e. the $(5n \times 3n)$-matrix of those columns of $V$ that correspond to eigenvalues with zero or negative real parts. $u^s$ is the corresponding vector of weights. Note that I dropped the term $-x\xi T$ because it is collinear with one of the columns in $V^s$. If $V^\perp$ denotes a basis of the nullspace of $V^s$, the boundary condition can be written as

$$(V^\perp)'\Big(X(T) - \bar{X} + \Xi^{-1}\Gamma(d - \bar{d})\Big) = 0. \tag{37}$$

This delivers $3n$ linear constraints at finite time $T$. Jointly with the initial boundary condition we thus have $6n$ boundary conditions as required to determine $X \in \mathbb{R}^{5n}$ and $d \in \mathbb{R}^n$.

# 4 Calibration

For a numerical implementation of the model I need numbers for the fundamental parameters, i.e. all parameters denoted in lower case Greek letters. My choices are displayed in Table 5 in the Appendix, and I try to give some motivation. Beyond these parameters, trade cost mark-ups $\Theta_{rs}$ are needed; see [3] for estimates. A difficult issue is calibrating the distance decay of learning. The distance $\Phi_{rs}$ can well be approximated by costs of business travel, but the parameter $\mu$ (or the parameters $\nu$ and $\tau$ that $\mu$ is composed of) is more difficult to quantify. $\tau$ is the ratio of travel cost per unit of distance over the total cost of a learning trip. It is hard to come up with a plausible number. Some reflections on this are in the Appendix, but a better empirical underpinning is called for.

One furthermore needs information on initial boundary values $A(0)$, $\mathcal{K}(0)$ and $\mathcal{N}(0)$. Regarding $\mathcal{K}(0)$ and $\mathcal{N}(0)$, a typical approach is to assume the richest regions to move along the steady state, while the others lag more or less behind in terms of either $\mathcal{K}$, $\mathcal{N}$, or both. One chooses the degree of backwardness in terms of these variables such that the ratio of GDP per employee of the backward region to GDP per employee in the advanced region is reproduced in the equilibrium solution. Unfortunately, this gives only one observation for two initial values to be determined. A second information that one can make use of is the share of real investment expenditure in GDP. For a region lagging behind more in terms of real capital than in terms of its level of technology this share should be comparatively large. Similarly, the share of innovation expenditures in GDP could also be used, but it is much less easily available.

The most difficult initial value is the asset distribution. Very little is known about the level of assets, and even less about the portfolio composition. The latter does not matter as long as the economy is assumed to move along a perfect foresight path, because the nominal returns of all types of assets are identical, and there is no risk. This is however different if the economy is shocked in an unpredicted way. Then assets devaluate or revaluate differently, and thus asset gains or losses vary, depending on portfolio composition. I show in the policy experiment in the next Section that this point really matters. Regarding the asset levels, balance of payment data offer information on the net asset values in the rest of world held by domestic households, but on a sub-national regional level this information is difficult to obtain. My way out is to assume the asset value held by households in a region to be initially just equal to the equilibrium market value of real capital and the stock of blueprints in that region, or formally, $A_r = Q_r\mathcal{K}_r + v_r\mathcal{N}_r$ for each region $r$ at $t = 0$.

# 5 A numerical policy application

I study policy interventions in an artificial example. Poor regions get subsidies either for real investments or for innovation. Subsidies are paid by an income tax raised in all regions at a uniform tax rate. A real world application is still to be done. The geography of my tiny world is depicted in Figure 2. It might be taken as a stylised map of the EU. There are seven regions, the three blue ones in the North-West are the rich ones and are not subsidised. The green ones in the South lag somewhat behind and get a relatively small subsidy, the red ones in the East are lagging behind most and receive the highest subsidy. Distances are the Euclidian distances between the respective dots. The longest distance (between regions 4 and 7) correspond to 28 hours travel time by car (Madrid to Warsaw). The size of transport cost is chosen such that transport cost between regions 4 and 7 amounts to 50% of the trade value. The corresponding car travel times are inserted for the distance $\Phi_{rs}$ in the learning potential. We will see that this implies a strong distance decay of learning. The green and red regions have all the same labour force normalised to one per region, the three blue regions have labour forces of two each.

To study convergence I give the rich regions a head start in terms of both, blueprints per capita and capital per capita.[3] They start at their respective steady state levels with regard to these two endowments, while the green and red regions lag behind their respective steady state endowments by 20 % and 40 %, respectively. Table 2 collects some information on the economies at $t = 0$ in the benchmark scenario without any policy intervention.

As seen in Table 3 and Figure 3, regions converge, but not fully. After 100 years the steady state is virtually attained. The poor regions stay relatively poor forever. The reason is not the initial disadvantage! The steady state distribution is almost independent of the initial endowment. There is only a small effect due to the fact that the regional

[3]Per capita and per employee are taken as synonymous, because one person supplies one unit of labour, see above.

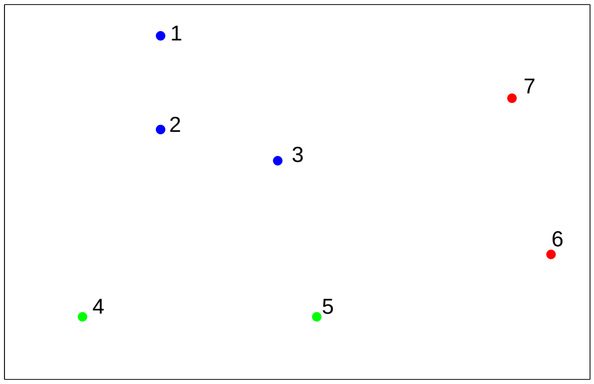


Figure 2: Location of regions

market size is somewhat affected by the initial asset distribution. But this effect is small. The green and red regions lag behind due to their relatively poor market access, both because their respective home markets are small (one unit of labour instead of two) and because they are located in the periphery. It has a double effect: first, as in Economic Geography models, they face relatively unfavourable terms of trade — larger prices for the tradables' composite bought and lower prices for the tradable varieties sold. Second, innovation is more costly because of the smaller learning potential. In the numerical example the latter effect is the more important one. In fact, assuming for example a distance of 28 hours between regions 4 and 7 implies that the weight of region 7's blueprint stock in region 4's learning potential is more than 16 times ($\exp(-\mu \cdot 28) < 1/16$) smaller than that of the blueprint stock in region 4 itself. This is a strong distance decay leading to pronounced spatial variation in learning potentials. It is a matter of future empirical research to disentangle the respective strengths of these two peripherality disadvantages. Note that lagging behind in terms of capital endowment leads to larger investment ratios ($J$/GDP), but the shares of innovation expenditure in GDP ($H$/GDP) are almost uniform across space.

| Region | GDP/$\mathcal{L}^*$ | $\mathcal{K}/\mathcal{L}^*$ | $\mathcal{N}/\mathcal{L}^*$ | $J$/GDP$^{**}$ | $H$/GDP$^{**}$ |
|---|---|---|---|---|---|
| 1 | 13.92 | 17.37 | 27.73 | 22.31 | 11.55 |
| 2 | 24.25 | 25.94 | 52.05 | 22.27 | 11.51 |
| 3 | 21.71 | 24.26 | 41.89 | 22.19 | 11.46 |
| 4 | -26.14 | -26.77 | -54.79 | 25.01 | 11.54 |
| 5 | -19.69 | -21.54 | -47.12 | 24.88 | 11.45 |
| 6 | -40.12 | -45.55 | -74.52 | 27.57 | 11.31 |
| 7 | -33.81 | -41.29 | -66.91 | 27.71 | 11.41 |

$^*$Deviations from weighted average in percent

$^*$Shares in percent

Table 2: Some statistics at $t = 0$

I now study two types of policy interventions, a subsidy for real investments and a subsidy for innovation. At $t = 0$ the introduction of a subsidy comes as a full surprise.

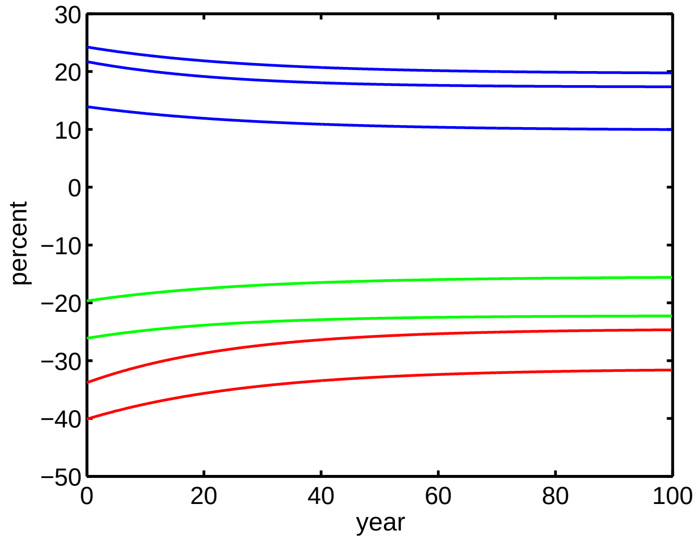


Figure 3: GDP per capita, deviations from weighted average in percent

| Region | GDP/$\mathcal{L}^*$ | $\mathcal{K}/\mathcal{L}^*$ | $\mathcal{N}/\mathcal{L}^*$ | $J$/GDP$^{**}$ | $H$/GDP$^{**}$ |
|---|---|---|---|---|---|
| 1 | 9.96 | 7.65 | 20.26 | 23.03 | 11.38 |
| 2 | 19.76 | 15.50 | 42.29 | 23.03 | 11.38 |
| 3 | 17.36 | 14.01 | 33.22 | 23.04 | 11.39 |
| 4 | -22.27 | -17.80 | -45.07 | 23.05 | 11.62 |
| 5 | -15.61 | -11.90 | -36.12 | 23.06 | 11.63 |
| 6 | -31.62 | -25.19 | -60.62 | 23.09 | 11.86 |
| 7 | -24.67 | -19.43 | -49.72 | 23.08 | 11.84 |

$^*$Deviations from weighted average in percent

$^*$Percent

Table 3: Some statistics at $t = 100$ years

Its announcement leads to a revaluation of the present value of capital installed and blueprints in use, quantified by the stock price $q$ and blueprint price $v$, and thus also to a revaluation of assets held by the households. Thus $A$, $q$ and $v$ jump at the day of announcement. Subsidy rates are 10 % of investment or innovation cost respectively in regions 4 and 5 and 20 % in regions 6 and 7. Subsidies are fully paid for 20 years and then phase out over a decade (see Figure 4). The time schedule of subsidies is common knowledge, once the subsidy plan is announced, and it is taken as 100 % credible by the public. The subsidy starts at the date of announcement.

There is no problem to let the announcement precede the realisation date within the model. In this case the dynamic state variables still jump at the announcement day and move along a continuous path from there on. They may have kinks, but no jumps when the subsidy actually starts. Such a modification will not alter anything important, so I dispense with announcement effects. The reader might wonder why a subsidy should come as a surprise, if everybody knows about subsidies in the EU. This is true, but two worlds, one with and the other without the policy in place, can be compared only under one set of expectations holding in both worlds. Either the policy is expected, then the no-policy world comes as a surprise, or the no-policy world is expected, then the policy scenario comes as a surprise. I work out the latter, because it makes a comparison of different policies against one single reference more easy.

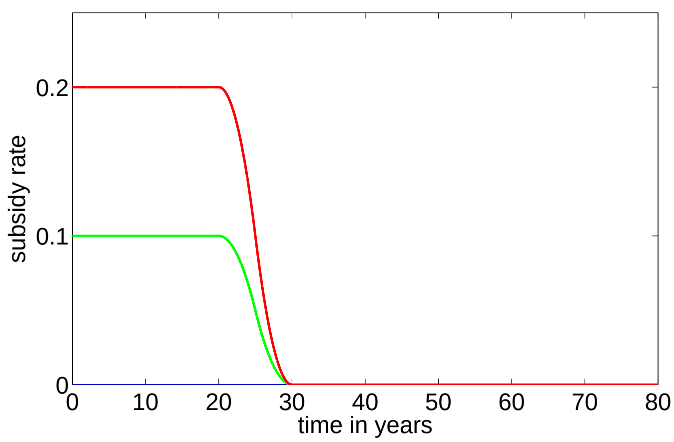


Figure 4: Subsidy rate

Subsidising private investment means that, in lagging regions, the government rebates to the investor a certain share of the investment cost. If the government (and eventually the tax payer) bears the share $\Gamma$ of the investment costs, investors maximise $q\mathcal{I} - (1-\Gamma)J$ rather than $q\mathcal{I} - J$. This leads to a replacement of (9) with

$$\mathcal{I}/\mathcal{K} = \frac{q/\Big((1-\Gamma)p^c\Big) - 1}{\zeta}. \tag{38}$$

and (22) with

$$\rho q = \dot{q} - \delta q + r - (1-\Gamma)J_{\mathcal{K}}. \tag{39}$$

Clearly, investments are the higher, the more they are subsidised, other things equal.

Subsidies are financed by an income tax

$$\sum_r \mathfrak{t}(w_r\mathcal{L}_r + \rho A_r) = \sum_r \Gamma_r J_r, \tag{40}$$

with endogenous tax rate $\mathfrak{t}$. The budget constraint (1) then becomes

$$\dot{A}_r = (1-\mathfrak{t})(w_r\mathcal{L}_r + \rho A_r) - C_r.$$

There is a minor nuisance here. So far we claimed that the nominal interest rate can be chosen arbitrarily, even varying arbitrarily over time. This affects just the time path of the unit of account, no real variable. After specifying the tax system according to equation (40) this is not true anymore, because we tax nominal, not real interest income. Other choices are possible, e.g. taxing real interest income or taxing only labour income, but the impact is small. An interpretation of (40) is that, whatever the nominal interest rate actually is, the state raises a tax $\mathfrak{t}\rho A$ on the asset value held by a household.

To study effectiveness of the subsidy I compare the subsidy scenario with an alternative where the households in the lagging regions obtain a lump-sum transfer matching in real terms the payments that the regions receive in the subsidy scenario. This is achieved by modifying the households' budget constraint (1) to

$$\dot{A}_r = Z_r + (1-\mathfrak{t})(w_r\mathcal{L}_r + \rho A_r) - C_r.$$

$Z_r$ is the nominal transfer received: $Z_r = \Gamma \tilde{J}_r p^c_r / \tilde{p}^c_r$. Tildes indicate the solution of the subsidy scenario, no tilde indicates the lump-sum transfer scenario. I emphasise that "effectiveness" does not mean efficiency. Effectiveness only means that one achieves more for the lagging region by a subsidy than by a lump-sum transfer. Global efficiency would mean that a policy maximises some global welfare criterion, and efficient redistribution would mean that a certain distributional objective is achieved by the smallest loss in terms of a global welfare criterion. Both aspects of efficiency can also be studied by the model at hand, but go beyond the purpose of this paper.

Subsidising innovation investment means that, in lagging regions, the government rebates to the innovator a certain share of the innovation cost. If the government bears the share $\Gamma$ of the innovation costs, the equilibrium price of a blueprint $v$ must be equal to the private cost of producing it, $v = (1-\Gamma)p^m i$. Equation (14) thus becomes

$$\hat{\mathcal{N}} = \frac{1}{\lambda} \log \frac{v\mathcal{P}^\chi}{(1-\Gamma)p^m\psi}. \tag{41}$$

The policy is compared with a lump-sum transfer scenario in the same way as before.

Table 4 shows welfare effects of the respective subsidy policies and lump-sum scenarios. Figures are relative equivalent variations, that is the percentage increases of real consumption that one would have offer the respective households forever to make them as well off as in the benchmark. I make two opposing extreme assumptions regarding the portfolio composition of assets. "Global portfolio" means that the portfolio compositions are everywhere the same. Each household owns a certain share in total assets of the world. Portfolios are thus perfectly diversified. "Local portfolio" means that households own the respective capital and blueprint stocks in their own regions and nothing elsewhere. Initial assets in the benchmark equilibrium are the same under both assumptions.

| Region | Global portfolio | | | | Local portfolio | | | |
|---|---|---|---|---|---|---|---|---|
| | Real investment subsidy | | Innovation subsidy | | Real investment subsidy | | Innovation subsidy | |
| | Subs. | Lumps.* | Subs. | Lumps.* | Subs. | Lumps.* | Subs. | Lumps.* |
| 1 | -0.59 | -0.45 | -0.03 | -0.20 | -0.56 | -0.45 | -0.04 | -0.20 |
| 2 | -0.58 | -0.45 | -0.01 | -0.20 | -0.54 | -0.45 | -0.02 | -0.20 |
| 3 | -0.55 | -0.46 | 0.04 | -0.21 | -0.49 | -0.46 | 0.04 | -0.21 |
| 4 | 0.69 | 0.65 | 0.61 | 0.32 | 0.59 | 0.65 | 0.62 | 0.32 |
| 5 | 0.72 | 0.63 | 0.68 | 0.31 | 0.64 | 0.64 | 0.68 | 0.31 |
| 6 | 2.32 | 2.28 | 1.47 | 0.98 | 2.15 | 2.30 | 1.51 | 0.98 |
| 7 | 2.28 | 2.31 | 1.38 | 0.99 | 2.09 | 2.32 | 1.41 | 1.00 |

* Lumpsum compensation of households by the same real amount as paid in subsidy scenario.

Table 4: Welfare effects, equivalent variation in percent

The testimonial for real investment subsidies as a means of cohesion policy is devastating. Under the global portfolio the subsidy helps the lagging regions only slightly more than the lump-sum transfer, while the losses in the non-supported regions are considerably larger in the former than in the latter scenario. Things are even worse under the local portfolio assumption. Lump-sum transfers outperform subsidisation; rich regions loose less and poor regions get more! Gains are smaller with a local rather than a global portfolio, because the subsidy devaluates the existing stock. The reason is that it makes capital more redundant and thus depresses capital returns. Reducing the cost of nailing new capital to the ground does no doom to those owning the capital that is already installed, if existing and new capital are perfect substitutes. Figure 5 nicely reveals this mechanism. It shows the percentage deviation of the capital stock price $q$ in the subsidy scenario from the benchmark.

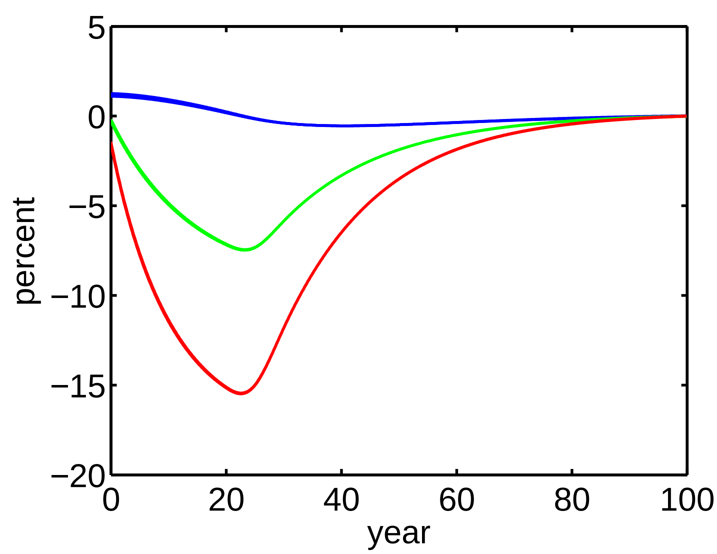


Figure 5: Response of capital stock price to real investment subsidy in percent

Results are much more favourable for innovation subsidies. For the green regions this policy generates twice the welfare gain achieved by a lump-sum transfer, and for the red ones the gain is still one and a half times that of a lump-sum transfer. Furthermore, the losses in the rich regions are much smaller than if they would pay a lump-sum transfer, and the most central region 3, though paying a tax and getting no subsidy, even gains. The reason is the positive externality of innovation. Similar to the response of $q$ to real investment subsidisation, the blueprint price $v$ drops as a response to innovation subsidisation, as shown in Figure 6. But this does not depress the welfare gain under the local portfolio assumption, because at the same time the real capital stock appreciates, as shown in Figure 7. This effect dominates, because more than tree quarters of the assets are real capital.

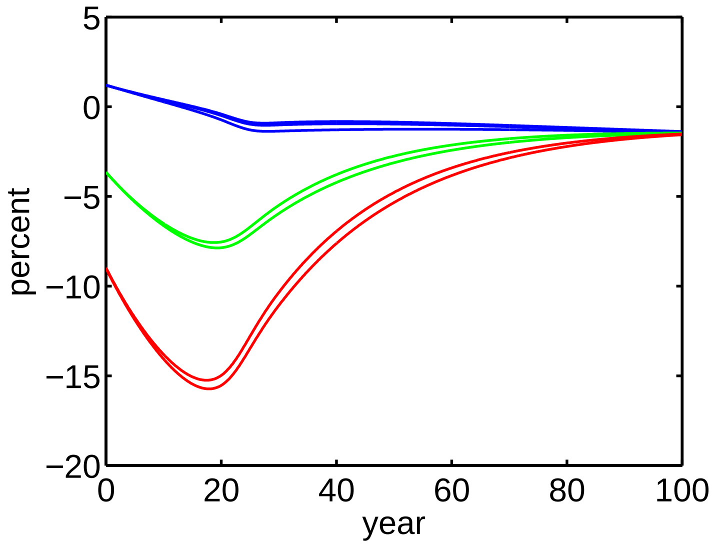


Figure 6: Response of blueprint price to innovation subsidy in percent

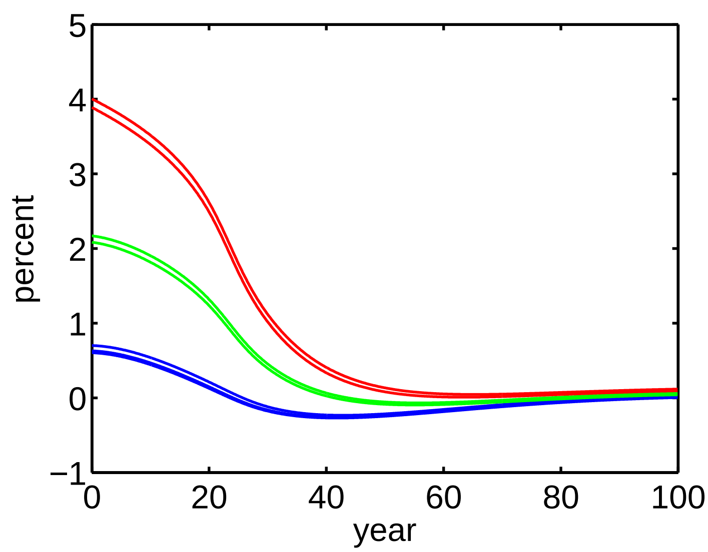


Figure 7: Response of capital stock price to innovation subsidy in percent

# 6 Conclusion

I (hope to) have shown that dynamic general equilibrium models with endogenous growth, Economic Geography ingredients and fully consistent forward looking decisions of consumers, innovators and investors is a viable instrument for policy evaluation in multiregional settings. Though the numerical application is still experimental, the application to the real world with many regions is straightforward, and just a small step to be taken next. The policy conclusion to be drawn from the experiment is that, with a traditional investment subsidy as it is practiced in European cohesion policy, it is hard to beat a simple lump-sum transfer. This is different for innovation subsidisation. One achieves much more than with a lump-sum transfer, and at a considerably lower welfare cost on the side of the non-supported regions.

Beyond real world application, two issues are on top of the agenda for future research. First, one has to make use of the empirical evidence about the role of distance in knowledge diffusion for a better foundation of the parameters in the learning and innovation technology. Second, there are many different model designs to be evaluated with regard to innovation. For example, it is natural to allow some knowledge to be a traded good. One then can assume an innovation to be produced by tradable knowledge components and some local input. The spatial implications might be quite different from what I found in this paper.

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`To be completed.`

# Appendix

## Derivation of equation (13)

Innovators solve the convex minimization problem

$$\imath_r = \min_{\ell} \left\{ \tilde{\psi} \exp(\tilde{\lambda} \hat{\mathcal{N}}_r) \left( \sum_s \mathcal{N}_s^{1-\nu} \ell_{rs}^{\nu} \right)^{-\tilde{\chi}} + \sum_s \ell_{rs} \exp(\tau \Phi_{rs}) \right\}.$$

The first order condition is

$$f \tilde{\chi} \nu \Sigma^{-(\tilde{\chi}+1)} (\ell_{rs}/\mathcal{N}_s)^{\nu-1} = \exp(\tau \Phi_{rs}), \tag{42}$$

with

$$\Sigma = \sum_s \mathcal{N}_s^{1-\nu} \ell_{rs}^{\nu}$$

and

$$f = \tilde{\psi} \exp(\tilde{\lambda} \hat{\mathcal{N}}_r).$$

Multiplying (42) by $\ell_{rs}$ and summing over $s$ yields

$$f \tilde{\chi} \nu \Sigma^{-\tilde{\chi}} = \sum_s \ell_{rs} \exp(\tau \Phi_{rs}),$$

and therefore

$$\imath_r = f(1 + \tilde{\chi}\nu) \Sigma^{-\tilde{\chi}}. \tag{43}$$

Raising (42) to the power $\nu/(\nu - 1)$, multiplying both sides by $\mathcal{N}_s$ and summing over $s$ one obtains

$$(f \tilde{\chi} \nu)^{\frac{\nu}{\nu-1}} \Sigma^{1+\frac{\nu(\tilde{\chi}+1)}{1-\nu}} = \sum_s \mathcal{N}_s \exp\left( \tau \frac{\nu}{\nu - 1} \Phi_{rs} \right).$$

Raising to the power of $-\chi$, with $\chi = \tilde{\chi}(1 - \nu)/(1 + \tilde{\chi}\nu)$, leads to

$$(f \tilde{\chi} \nu)^{\frac{\chi\nu}{1-\nu}} \Sigma^{-\tilde{\chi}} = \left( \sum_s \mathcal{N}_s \exp(-\mu \Phi_{rs}) \right)^{-\chi},$$

with $\mu = \tau\nu/(1 - \nu)$. Multiplying by $f(1 + \tilde{\chi}\nu)/(f\tilde{\chi}\nu)^{\frac{\chi\nu}{1-\nu}}$ and using (43) gives the expression in (13) for $\imath_r$.

## Proof of proposition 1

The proof is straightforward. I just go through the equations one by one, showing the growth rates of dynamic state variables and the equalities in the algebraic equations to be unaffected by multiplying the variables listed in table 1 by the respective values for $\exp(g\xi)$, with an arbitrary real number $g$. To ease notation, I write $\xi(A) = 1 - 1/\theta$, $\xi(\mathcal{K}) = 1, \ldots$. I begin with the dynamic equations.

- Equation (1): $\hat{A} = w\mathcal{L}/A + \rho - C/A$. This is unaffected, because $\xi(w) = \xi(A) = \xi(C)$.

- Equation (11): $\xi(q) = \xi(p^c)$.
- Equation (22): $\xi(r) = \xi(q)$, $\xi(p^c) = \xi(q)$ and $\xi(\mathcal{I}) = \xi(\mathcal{K})$.
- Equation (14): $\xi(v) + \chi\xi(\mathcal{N}) = \xi(p^m)$.
- Equation (24): $\xi(S) = \xi(v) + \xi(\mathcal{N})$.

Next, I go through the algebraic equations.

- Equation (4): $\xi(C) = (1-\theta)\xi(p^c)$.
- Equation (5): $\xi(p^c) = \epsilon\xi(p^m) + (1-\epsilon)\xi(p^d)$.
- Equation (6): $\xi(p^d) = -\epsilon\Omega - 1/\theta = \xi(\mathcal{N})/(1-\sigma) + \xi(p^m) = [(\sigma-1)\Omega]/(1-\sigma) + [(1-\epsilon)\Omega - 1/\theta]$.
- Equation (7): $\xi(p^m) = (1-\epsilon)\Omega - 1/\theta = \alpha\xi(w) + \beta\xi(r) + \gamma\xi(p^m) + \eta\xi(p^d) = \alpha[1-1/\theta] + \beta[-1/\theta] + \gamma[(1-\epsilon)\Omega - 1/\theta] + \eta[-\epsilon\Omega - 1/\theta]$.
- Equation (8): $\xi(J) = \xi(p^c) + \xi(\mathcal{I})$ and $\xi(\mathcal{I}) = \xi(\mathcal{K})$.
- Equation (9): $\xi(q) = \xi(p^c)$ and $\xi(\mathcal{I}) = \xi(\mathcal{K})$.
- Equation (16): $\xi(M) = \xi(w)$.
- Equation (17): $\xi(M) = \xi(r) + \xi(\mathcal{K})$.
- Equation (18): $\xi(S) = 1 - 1/\theta = \xi(\mathcal{N}) + (1-\sigma)\xi(p^m) - (1-\sigma)\xi(p^d) + \xi(D) = (\sigma-1)\Omega + (1-\sigma)[(1-\epsilon)\Omega - 1/\theta] - (1-\sigma)[-\epsilon\Omega - 1/\theta] + 1 - 1/\theta$.
- Equation (19): $\xi(D) = \xi(C) = \xi(J) = \xi(M)$.
- Equation (20): $\xi(M) = \xi(C) = \xi(J) = \xi(H) = \xi(S)$.
- Equation (21) can be written as $H/\mathcal{N} = v\hat{\mathcal{N}}$. As shown above, $\hat{\mathcal{N}}$ remains unaffected, and $\xi(H) - \xi(\mathcal{N}) = \xi(v)$.

## Parameter choices

Notes to motivate the choice of parameters:

1. Typical shares from national accounts.
2. Just a guess.
3. The steady state real interest rate is $\rho +$ growth rate$/\theta$. I choose it to be 3.5% p.a., leading to $\rho = 0.035 - 0.02/0.8 = 0.01$, if the steady state growth rate is 2 % p.a., as assumed here (see below).
4. For a given vector of labour stock per region and given transport and communication cost parameters, $\psi$ determines the steady state rate of growth. The smaller $\psi$, the larger is the growth rate. I invert this relation and fix $\psi$ such that the steady state growth rate of real consumption comes out as 2 % p.a. The dimension of $\psi$ is non-intuitive; thus neither the dimension nor the parameter value are reported.

| | Explanation | Dim.* | Value | Note** |
|---|---|---|---|---|
| $\alpha$ | input share of labour in manufacturing | 1 | 0.35 | 1 |
| $\beta$ | input share of capital in manufacturing | 1 | 0.15 | 1 |
| $\gamma$ | input share of tradables in manufacturing | 1 | 0.3 | 1 |
| $\eta$ | input share of locals in manufacturing | 1 | 0.2 | 1 |
| $\epsilon$ | consumers' or investor's expenditure share for local goods | 1 | 0.6 | 2 |
| $\rho$ | rate of consumers' time preference | 1/a | 0.01 | 3 |
| $\delta$ | rate of capital depreciation | 1/a | 0.05 | 5 |
| $\psi$ | scaling parameter for innovation input | | | 4 |
| $\theta$ | elasticity of intertemporal substitution | 1 | 0.8 | 5 |
| $\sigma$ | elasticity of substitution between product varieties | 1 | 6 | 5 |
| $\zeta$ | adjustment cost parameter for real investment | a | 20 | 6 |
| $\lambda$ | adjustment cost parameter for innovation | a | 20 | 6 |
| $\mu$ | distance decay parameter for learning potential | 1/h | 0.1 | 7 |

*Dimension. "1" means dimensionless.

**Reference to following list.

Table 5: Parameter values

5. These are standard choices in the literature.

6. I am experimenting with the adjustment cost parameters $\zeta$ and $\lambda$. Twenty years is a large number, implying that, with 2 % growth p.a., investment costs are almost 50 % larger than they would be with zero growth. Lower adjustment costs lead to an unrealistically high speed of conditional convergence. In a real world application this has to be scrutinised more thoroughly, by looking at the speed of adjustment to unexpected shocks. The lifting of the iron curtain is a good candidate for a natural experiment.

7. If distance $\Phi_{rs}$ is measured in hours one-way travel time, then $\tau$ is the ratio of cost per unit of distance over all other costs related to a business travel trip. 0.1 could be a sensible number meaning that, for a trip to a destination 10 hours away, travel cost is $100(e-1) = 170$ % of total cost. If (by the principle of insufficient reason) I assume $\nu$ to be one half I obtain $\mu = 0.1$ per hour travel time. The implied "half-life" distance of knowledge externalities in the learning potential is 7 hours travel time. By "half-life" distance I mean that a stock of blueprints $\mathcal{N}_s$ in a destination region $s$ that is 7 hours away from a given region $r$ exerts the same externality as $\mathcal{N}_r = \mathcal{N}_s/2$ in region $r$ itself.