Growth: With or Without Scale Effects?

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Abstract

The property that ideas are nonrivalrous leads to a tight link between idea-based growth models and increasing returns to scale. In particular, changes in the size of an economy's population generally affect either the long-run growth rate or the long-run level of income in such models. This paper provides a partial review of the expanding literature on idea-based models and scale effects. It presents simple versions of various recent idea-based growth models and analyzes their implications for the relationship between scale and growth.

JEL Classification: O40, E10

1 Introduction

The discovery of new ideas is the engine of growth in many recent growth models. As emphasized by Romer (1986, 1990), ideas are different from most goods analyzed in economics in that they are nonrivalrous: the use of an idea by one person does not preclude, at a technological level, the simultaneous use of the idea by another person, or even by many people. This leads to a tight link between idea-based growth models and increasing returns to scale.

To take a simple example, consider the production of the latest best-selling novel, the hottest-selling computer game, or the new Volkswagon Beetle. To produce the first unit of any of these items requires a large amount of effort: the novel must be written, the computer game must be created and the Beetle must be (re)designed. But clearly these are one-time costs. The "idea" underlying each product only needs to be created once. Afterwards, subsequent units might plausibly be described as being produced with a constant returns to scale production function, following the standard replication argument. The idea is nonrivalrous in the sense that it can be used for each unit simultaneously. Total production of novels, computer games, and automobiles is then characterized by increasing returns once the fixed cost of creating the idea is taken into account. It is this fundamental link between ideas and returns to scale that gives rise to a basic scale effect in idea-based growth models.

In the first wave of such models in the recent growth literature — the models of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) — this scale effect shows up in a particularly troublesome way. The growth rate of the economy is proportional to the total amount of research undertaken in the economy. An increase in the size of the population, other things equal, raises the number of researchers and therefore leads to an increase in the growth rate of per capita income. Taken at

face value, this prediction is problematic because it means that population growth should lead to accelerating per capita income growth. As pointed out by Jones (1995b), this prediction is strongly at odds with 20th century empirical evidence.

Subsequent idea-based growth models have attempted to eliminate this prediction. Jones (1995a) and several recent papers including Kortum (1997) and Segerstrom (1998) follow a strategy that leads to a model in which long-run per capita growth is proportional to the rate of population growth. That is, the scale effect shows up in the level of per capita income instead of its growth rate. An implication of this line of research is that subsidies to research may affect the level of income, but not its long-run growth rate.[1]

The latest line of research on scale and growth, including Young (1998), Peretto (1998), Aghion and Howitt (1998, Chapter 12), and Dinopoulos and Thompson (1998b), proposes a novel method for eliminating the growth effect of scale. These papers add a second dimension to the Romer/Grossman-Helpman/Aghion-Howitt (R/GH/AH) models. Research can increase productivity within a product line, or it can increase the total number of available products. As in R/GH/AH, growth depends on the amount of research effort in each product line. These papers propose that an increase in scale increases the number of products available in direct proportion, leaving the amount of research effort per sector — and therefore growth — unchanged. This class of models is important for a number of reasons. First, it reintroduces the result that changes in policy can have effects on the long-run rate of growth. Second, in the Jones/Kortum/Segerstrom (J/K/S) models, exponential growth cannot be sustained in the absence of population growth. The Young/Peretto/Aghion-Howitt/Dinopoulos-Thompson (Y/P/AH/DT)

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[1]One must be careful about the policy invariance result and the exogeneity of long-run growth suggested in these models. These conclusions are modified in models with endogenous fertility (Jones 1998).

models overturn this prediction.[2]

This paper presents a simple framework for analyzing the three classes of models which explains some of the key differences among the results and provides some direction for future research.

2   The Romer/Grossman-Helpman/Aghion-Howitt Models

The R/GH/AH models contain a number of important insights concerning the microfoundations of growth and the distortions associated with the research process which potentially affect the allocation of resources. Nevertheless, these models share a feature — the effect of scale on growth — that is worth reconsidering. To present this feature in the clearest fashion, consider the following toy model which abstracts from many of the important insights in these papers.

Motivated by the insight that the nonrivalry of ideas leads to increasing returns, suppose that output Y is produced using labor L_Y and the stock of ideas A according to

$$Y = A^\sigma L_Y. \tag{1}$$

There are constant returns to the rivalrous inputs (here, just labor) and increasing returns to labor and ideas together, where the degree of increasing returns is measured by the parameter $\sigma > 0$.

New ideas, $\dot{A}$, are also produced using labor and the existing stock of knowledge:

$$\frac{\dot{A}}{A} = \delta L_A. \tag{2}$$

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[2]In an effort to sort through a number of recent growth papers, I'm coarsely grouping the papers into three categories. While this is useful for the purpose at hand, papers within a category are often very different and contain far more insight and subtlety than is presented in this brief format. Other, more general surveys of this literature can be found in Aghion and Howitt (1998) and Dinopoulos and Thompson (1998a).

In the R/GH/AH model, each unit of research effort can produce a proportionate increase in the stock of knowledge.

Finally, to close this simple model, we assume that a constant fraction s of the total labor force L works in research, so that $L_A = sL$ and $L_Y = (1-s)L$, with $0 < s < 1$.

With these assumptions, it is easy to see that the growth rate of output per worker, defined as g_y, is given by

$$g_y \equiv \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \delta s L. \tag{3}$$

Permanent changes in research intensity s then lead to permanent changes in growth in this model. However, the growth effect of scale is also apparent: with exponential population growth, the growth rate of per capita income in this simple model is itself growing exponentially.

3   The Jones/Kortum/Segerstrom Model

The prediction of the R/GH/AH models that growth rates should themselves be growing exponentially seems to be contradicted by twentieth century experience.[3] J/K/S address this problem by reconsidering the microfoundations of the production function for new ideas. In particular, these papers replace equation (2) by

$$\dot{A} = \delta L_A A^{\phi}, \tag{4}$$

where $\phi < 1$ is imposed. With $\phi > 0$, this formulation allows for increasing returns to scale in the production of new ideas, corresponding to the case in which previous discoveries raise the productivity of current research effort. Alternatively, with $\phi < 0$, the formulation also allows for diminishing returns

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[3]Kremer (1993) shows that this prediction is consistent with evidence prior to the twentieth century, dating back as far as 1 million B.C. However, Kremer also shows that this same evidence is consistent with the Jones (1995a) model, a version of which is described in this section.

in the production of new ideas, for example if past discoveries make it more difficult to find new ideas. The R/GH/AH production function imposes $\phi = 1$, requiring that past discoveries affect the current productivity of research in a very specific fashion.

Using this formulation, together with the assumption that the labor force L grows at an exogenous, constant rate $n > 0$, it is easy to show that there exists a stable balanced growth path for the model where

$$g_A = \frac{n}{1 - \phi}$$

and

$$g_y = \sigma g_A = \frac{\sigma n}{1 - \phi}.$$

This result makes it clear why $\phi = 1$ is a problem. As indicated earlier, the presence of population growth in this case produces explosive growth.

Finally, along the balanced growth path with $\phi < 1$, the level of output per worker $y \equiv Y/L$ is given by

$$y^*(t) = (1 - s) \left(\frac{\delta(1 - \phi)}{n} \cdot s \cdot L(t) \right)^{\frac{\sigma}{1 - \phi}}. \tag{5}$$

Thus, once we relax the assumption of $\phi = 1$ in favor of $\phi < 1$, we see that the model leads to some different results. Changes in research intensity no longer affect the long-run growth rate, but rather affect the long-run level of income along the balanced growth path (through transitory effects on growth). Similarly, changes in the size of the population affect the level of income but not its long-run growth rate. Finally, the long-run growth rate itself is proportional to the population growth rate. In the absence of population growth, exponential growth in per capita output cannot be sustained in this model. These results reflect the increasing returns to scale that results directly from the nonrivalry of ideas (e.g. notice the dependence on $\sigma > 0$).[4]

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[4]Similar models and similar results are found in a number of earlier papers in the growth literature, including Phelps (1966), Nordhaus (1969), and Judd (1985).

The R/GH/AH results that a steady-state growth path can occur in the absence of population growth and that this growth rate depends on research intensity are sensitive to the assumption of $\phi = 1$. More generally, the predictions of those models are likely to be reasonably consistent with data to the extent that $\phi \approx 1$.

4 The Young/Peretto/Aghion-Howitt/Dinopoulos-Thompson Models

The results in the J/K/S models that policy typically has no long-run growth effects and that exponential growth depends on population growth are sufficiently at odds with the spirit of the endogenous growth literature that a number of other researchers have sought an alternative way to eliminate the effect of scale on growth in idea-based models. Recently, the Y/P/AH/DT papers have studied an important alternative, to which we now turn.

Suppose that aggregate consumption (or output) is a CES composite of a variety of goods:

$$C = \left(\int_0^B Y_i^{1/\theta} di \right)^{\theta}, \tag{6}$$

where B measures the variety of goods available, Y_i is the consumption of variety i, and $\theta > 1$ is related to the elasticity of substitution between products. Let each variety Y_i be produced according to the R/GH/AH model we set up in equations (1) and (2).

To complete the model, we need to explain how B, the total variety of consumption goods, evolves over time. For simplicity, assume that

$$B = L^{\beta}, \tag{7}$$

where for the moment, we allow β to be any real number. In the Y/P/AH/DT models, $\beta = 1$ is maintained so that the variety of consumption goods is pro-

portional to the population of the economy.[5]

For simplicity, assume that each intermediate good Y_i is used in the same amount, so that $Y_i = Y$ and $C = B^\theta Y$.[6] Per capita output is then given by $c = B^\theta y$, where $c \equiv C/L$, and per capita output growth is

$$
\begin{aligned}
g_c &= \theta g_B + \sigma g_A \\
&= \theta \beta n + \sigma g_A.
\end{aligned}
\tag{8}
$$

With the R/GH/AH production function for new ideas, the growth rate of A now depends on research effort per variety L_A/B:

$$
\begin{aligned}
g_A &= \delta s L/B \\
&= \delta s L^{1-\beta}.
\end{aligned}
\tag{9}
$$

Substituting this result into equation (8) yields the growth rate of per capita output in the model:

$$
g_c = \theta \beta n + \sigma \delta s L^{1-\beta}.
\tag{10}
$$

With $\beta = 1$, that is with $B = L$, we have the key result of the Y/P/AH/DT models. The scale effect on growth is eliminated, changes in research intensity s affect long-run growth, and exponential growth in per capita output occurs even in the absence of population growth. The intuition for these results is that an increase in population results in a proportionate increase in the number of sectors in the economy. This means that the size of each sector — and in particular the number of researchers in each sector — does not change in response to the rise in population. This neutralizes the growth

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[5]The reduced form relationship in equation (7) can be derived from a production function for varieties, at least along a balanced growth path. For example, suppose $\dot{B} = L B^\gamma$. Then, along a balanced growth path, a relationship similar to that in equation (7) holds, with $\beta = 1/(1-\gamma)$.

[6]Such an assumption is not needed, but could be justified with a Leontief technology in equation (6).

effect of scale present in the R/GH/AH models. Notice, however, that population growth still affects per capita output growth, just as in the J/K/S models, through the first term in equation (10).

These features of the model make it quite appealing. However, it is unclear how robust these results are. In particular, the Y/P/AH/DT models assume $\beta = 1$, and the results in those models hinge importantly on this assumption.[7]

First, consider the case of $\beta < 1$. In this case, the number of sectors grows less than proportionally with population. The size of each sector grows over time, and since productivity growth in each sector is proportional to its size, the model once again exhibits scale effects in growth. This is apparent in equation (10).

Alternatively, suppose $\beta > 1$. In this case, the number of sectors in the economy grows more than proportionally with population. The size of each sector is declining over time, and therefore so is productivity growth in each sector. The model exhibits a negative scale effect in growth. Asymptotically, productivity growth in each sector is zero, and the only component of per capita growth that remains is the first term in equation (10), which is proportional to the rate of population growth.

The papers by Y/P/AH/DT emphasize that the growth effect of scale can be eliminated while maintaining the other implications of the R/GH/AH models. What we see from this analysis is that this result relies on the special case of $\beta = 1$. If $\beta < 1$, the model once again exhibits scale effects in growth, so that the problem is not resolved. The model behaves just like those in R/GH/AH. On the other hand, if $\beta > 1$, then the model has a balanced growth path, but growth is once again proportional to the rate of population growth. That is, the model is (asymptotically) returned to the J/K/S class.

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[7]Young (1998) considers relaxing the assumption of $\beta = 1$ and derives some of the results given below, in particular that the model can generate either positive or negative scale effects on growth.

These results can be extended and summarized by relaxing the assumption of $\phi = 1$ in the Y/P/AH/DT models — that is, by allowing the production function for new type-A ideas to be of the J/K/S form instead of the R/GH/AH form.[8] Assuming $\dot{A} = \delta L_A A^\phi$, the growth rate of per capita output in equation (10) becomes

$$g_c = \theta \beta n + \sigma \delta s \frac{L^{1-\beta}}{A^{1-\phi}}. \tag{11}$$

This general model embeds each of the three classes of models we have discussed in this paper as special cases and also allows for more general cases. One can show that, asymptotically, growth either explodes or is characterized by one of the three special cases, depending on the values taken on by β and ϕ. The various cases are summarized in Figure 1. For example, if the correct parameter values are such that $\beta \approx 1$ and $\phi \approx 1$, then the Y/P/AH/DT class of models is likely to be a good description of economic growth. Alternatively, if $\beta < 1$ and $\phi \approx 1$, growth is well-characterized by the R/GH/AH models. For all other parameter values, growth either explodes or is asymptotically proportional to the rate of population growth.

Without empirical work designed to estimate the parameter values, it is impossible to say which class of models provides the best characterization of long-run economic growth. Economically speaking, the R/GH/AH models require past discoveries to increase the productivity of current research in a precise fashion. The Y/P/AH/DT models require this restriction together with a restriction that increasing the scale of the economy does not (asymptotically) change the number of researchers in the sectors in which the R/GH/AH productivity spillovers operate.

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[8] An interesting paper by Li (1998) that I became aware of after writing the first draft of this paper proceeds in this direction.

Figure 1: Characterizing Asymptotic Growth in the General Model
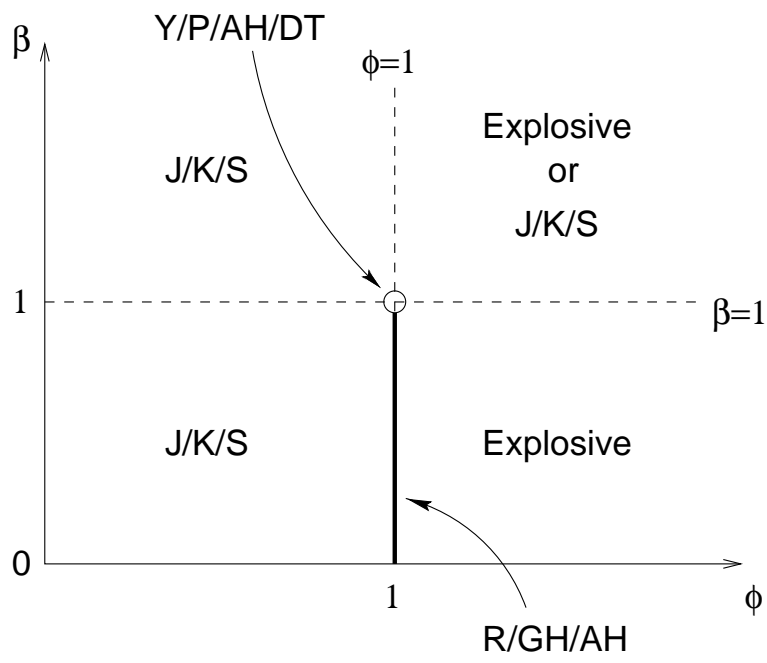
5 Conclusion

That ideas are important to economic growth seems almost a trivial statement. However, the property that ideas are nonrivalrous means that growth and increasing returns to scale are tightly linked. It is this linkage that generally gives rise to the feature that idea-based growth models exhibit some kind of scale effect.

All of the models reviewed in this brief paper exhibit scale effects, notwithstanding some of their titles: the size of the economy affects either the long-run growth rate or the long-run level of per capita income. It is important to keep this in mind when reading many papers on growth and ideas. The phrase "growth without scale effects" is used in the title of three papers reviewed here. Each model in fact does involve scale effects, but on the level of per capita income rather than its growth rate.

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