The Credit Market and the Determinants of Credit Crunches:

An Agent Based Modeling Approach

Ulf Holmberg
Department of Economics
Umeå School of Business and Economics
Umeå University
SE-901 87 Umeå

Abstract

This paper presents a credit market model and finds, using an agent based modeling approach, that credit crunches have a tendency to occur; even when credit markets are almost entirely transparent in the absence of external shocks. We find evidence supporting the asset deterioration hypothesis and results that emphasize the importance of accurate firm quality estimates. In addition, we find that an increase in the debt's time to maturity, homogenous expected default rates and a conservative lending approach, reduces the probability of a credit crunch. Thus, our results suggest some up till now partially overlooked components contributing to the financial stability of an economy.

1 Introduction

During a time span of over twenty years, from the early nineties to present date, nearly all developed countries have experienced some form of supply side credit crunch in parts of their economies. Following Udell (2009), economists generally define a credit crunch as a "significant contraction in the supply of credit reflected in a tightening of credit conditions". Thus, during a crunch seemingly eligible borrowers find it hard to get credit under reasonable terms, forcing firms that rely on external capital to a halt.

Why do providers of credit suddenly mobilise their lending strategies in such a way? Existing theories provide a useful platform when building an understanding of the determinants of crunches. According to the Risk-Based Capital hypothesis (RBC), the implementation of new risk-based regulatory rules governing lenders' allocation of resources, may have a significant negative impact on the supply of credit. Berger and Udell (1994) tested the RBC hypothesis on the perceived crunch in the United States after the implementation of the first Basel Accord in the late eighties/early nineties. They found some support in favor of the RBC hypothesis but refrained from ruling out competing theories. Sharpe (1995), on the other hand, claimed that banks reduce credit supply due to unpredicted losses in bank capital. In analogy with the RBC hypothesis, he concluded that the reduction in credit coincides with banks having difficulties in meeting the minimum regulatory capital requirements. Pazarbasioglu (1996) found evidence in line with the asset deterioration hypothesis, suggesting that banks become less willing to supply credit during periods associated with a deterioration in asset quality. In addition, according to the financial instability hypothesis, first discussed by Minsky (1977, 1992), economies have a tendency to naturally evolve into a "Ponzi phase" in which firms are forced to borrow to meet their obligations on existing liabilities. Since lenders judge liability structures subjectively, sudden drops in the supply of credit may occur when corporate debt reaches some unforeseen threshold.

Turning to the existing literature on banking and credit, it is well understood that lenders are forced to deal with excessive information asymmetry problems since borrowers have reason to withhold information in order to gain credit. Lenders seek to resolve this problem by practicing screening (Allen, 1990) and monitoring (Winton, 1995) thus reducing their exposure to counter party risk. If the estimates used in these procedures are based on subjective judgments of acceptable liability structures or fail to incorporate risks driven by exogenous shocks, such shocks may lead to a reduction in credit supply due to unforeseen losses. In addition, Kiyotaki and Moore (1997) developed a real business cycle model of a credit market in which lending only occurs if the debt is collateralized. Thus, a recession is amplified with the decrease in the value of collateral during an economic

downturn. Further, the burdens of asymmetric information may in itself lead to cyclical credit and unexpected downturns in lending, a point that is emphasized by Suárez and Sussman (1997, 2007). They developed a dynamic rational expectations model in which business cycles are created endogenously. Their findings indicate that cyclical contractions of credit are driven by a moral hazard problem between firms and financial intermediaries. This implies that a credit crunch may manifest itself solely due to the inherent imperfections of the credit market.

Previous literature suggests that credit crunches are either driven by exogenous shocks (e.g., new risk based regulatory rules) or caused endogenously by problems springing from asymmetric information between the borrower and the lender. A natural conclusion is thus that credit markets grow "safer" with transparency, suggesting that policy makers concerned with financial stability should concentrate their efforts to transparency increasing measures. However, in the light of the money market meltdown of 2008, one cannot avoid wondering if some other mechanism, inherent in the credit market, is equally to blame. In this paper, we derive a simplified banking model in which banks screen applicants in order to pick suitable clients with an acceptable level of default risk. We use this model as a base on which we build an Agent Based Model (ABM) of a credit market. Using this model, we find that supply side driven credit crunches occur even if credit markets are almost entirely transparent. This result originates from the banks' expectations about future credit risk. If banks have adaptive expectations about the risk in their credit portfolios, the credit market may evolve into periods in which banks acquire riskier debt than what is specified by its profit maximizing condition. Such periods are swiftly followed by periods in which banks try to cut back on risky debt, making credit difficult to obtain. If such cutbacks are coordinated across banks, the market may experience the eruption of a credit crunch. These crunches are seemingly spontaneous but highly dependent on the level of conservatism practiced when banks pursue their internal credit risk goals. If banks tend to react slowly to new credit risk goals, i.e. have a conservative approach to new risky ventures or more risky debt, the probability of a credit crunch is reduced.

We also find that an increase in the debt's average time to maturity, reduces the probability of a credit crunch. This result is related to the arguments made in the work of Andreasen et al. (2011) but differs in terms of the nature of the result. In Andreasen et al. (2011), the authors argue that banks, by offering long-term credit to firms, attenuate firms' output responses to technological shocks. In this paper, we find that the mechanism causing a decrease in the probability of a credit crunch due to an increase in the maturity time of debt is far less complicated. An increase in the debts' time to maturity simply reduces the probability of sequential bad lending, i.e. it reduces the probability that banks lend

2

to a sequence of firms associated with a profit reducing level of credit risk. In addition, we are able to find evidence in line with the asset deterioration hypothesis as well as to confirm the importance of accurate estimates in the banks' screening procedures. Thus, by adopting the ABM approach, we are able to find the determinants of credit crunches through a simple mechanism linked to the banks' credit risk valuation procedure while embracing the possible affects on lending caused by random spillover effects of counter party risk.

The outline of the paper is as follows. The next section discusses the theoretical underpinnings of the model. This is followed by a description of the artificial economy, its agents and the conditions driving the behavior of the agents. In the final sections we present and discuss the results derived from the simulations and conclude.

2 Theoretical underpinnings

We define a credit crunch as in Udell (2009), i.e. as a significant contraction in the supply of credit reflected in a tightening of credit conditions. Viewing banks as financial intermediators and providers of investment capital, this definition suggests that the onset of credit crunches are related to the banks' screening and monitoring procedures. The information production in imperfect screening and its effects have been previously studied by Broecker (1990), Chiesa (1998) and Gehrig (1998) among others. However, in this section, we seek a simple mechanism that can be linked to the onset of credit crunches. As such, we initially consider a perfectly transparent credit market such that banks practice costless and perfect screening in order to reduce their exposure to credit risk. In contrast to previous studies, we consider a continuum of firm qualities and view screening as a method of choosing suitable clients by truncating the distribution function defining firm quality.

Consider a two-period economy under the supervision of a financial authority. The economy is made up of a *finite* number of risk-neutral firms, $k = 1, ..., M$, and banks, $i = 1, 2, ..., N$, providing unsecured credit to firms. Firms are assumed to be heterogeneous in terms of quality summarised by $\theta_k \in [0, 1]$. At the initial date, firms are given the choice of carrying out a risky project lasting one time period. To undertake the project, firms need to raise external capital equivalent to l_k on the credit market. The gross return of the investment, $R(\theta_k) \in [0, \infty]$, is realized after one time period and retrieved with probability $1 - \theta_k$. Firm returns are increasing in θ_k such that firm quality also represents the riskiness of firm actions. A high quality firm is thus characterised by a low value of θ_k. The distribution of firm returns are binary and the success rate of the investment is firm size independent. For simplicity, it is assumed that in case of failure the firm de-

faults without liquidation value, allowing us to interpret θ_k as the firm's probability of default. Thus, firms are protected by limited liability such that they only care about the payoffs when the project succeeds. As such, the firms always implement their projects when granted a loan.

Banks act as information producers about the firms' investment projects and we let the banks observe the distribution of firm quality, $f(\theta)$, from which they make a noisy firm quality estimate $\theta_{k,i}^b$. We let the interest rate on external capital, r, and the deposit rate, ρ, be exogenous to the model and assume that lending is the banks' only source of profit. Given the above, the representative bank's *unconditional* expected profit function is:

$$\pi_b^e = \sum_k^{m \in M} \left[(1+r)(1-\theta_k^b) - 1 \right] l_k - \rho D, \tag{1}$$

where m is the subgroup of firms facing their demand towards the representative bank and D is the bank's deposits. To purely study the affects of lending while ignoring the bank's exposure to deposit risks, it is assumed that the bank finances lending using a stock of own capital, i.e. equity. The bank's equity is given by E such that $\sum_k^{\hat{m} \in M} l_k \leq E$ and $E - \sum_k^{\hat{m} \in M} l_k \geq \hat{E}$ where $\hat{E}$ is the minimum capital requirement as decided by the financial authorities and $\hat{m}$ is the number of firms granted credit. As such, the deposit costs in (1) can be ignored. Since banks observe the distribution of firm quality, the banks' beliefs about θ_k are taken on M. Using this, we rewrite the representative bank's unconditional expected profit function in (1) as:

$$\pi_b^e = [(1+r)(1-\theta^e) - 1] \sum_k^{m \in M} l_k, \tag{2}$$

where θ^e is the expected default rate (quality). From the bank's expected profit function in (2), it is fairly obvious that above some value of θ^e, expected bank-profit turns negative. More specifically, in the unconditional case the bank only participates on the credit market if:

$$\theta^e \leq r/(1+r). \tag{3}$$

However, as discussed by Gehrig (1998), when a contract is negotiated, banks may prefer to screen applicants in order to assess their credit risks. As such, it is assumed that the bank resolves the possibility of negative profits by screening applicants to identify risky firms which are removed from the bank's credit portfolio.

Since we seek a simple mechanism that can be linked to the banks' lending decisions, we assume for the remainder of this section that the credit market is perfectly transparent such that a bank has the ability to practice perfect and costless screening, i.e. $\theta_k^b = \theta_k$.[1]

---

[1]We will relax this assumption when we move over to the artificial economy in Section 3.
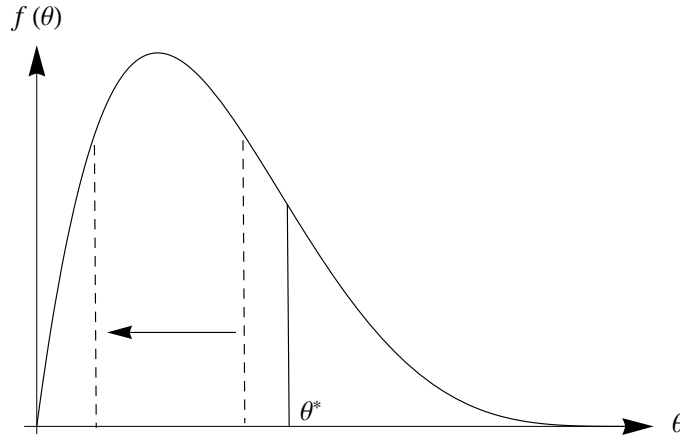
Figure 1: Screening reduces the expected default rate ($E[\theta|\theta_k \leq \theta^*]$).

Recalling the participation constraint in (3), it may be tempting to argue that each bank lends to firms with $\theta_k \leq r/(1+r)$ up to the point when the bank runs out of equity, adjusted for the minimum capital requirements. However, the expected profit from a loan issued to a firm with high θ and a firm with a low θ is fundamentally different since a firm with a high θ is less likely to repay the debt. Recalling that the economy consists of a finite number of firms, the heterogeneity of firm quality leads to a trade-of between quality and quantity of credit. To see this, we acknowledge that the process of screening loan applicants ultimately aims to discriminate between firms and only picking applicants that live up to some minimum requirements for credit (given exogenous interests rates). Since the bank observes the distribution of firm quality, $f(\theta)$, the screening procedure can be thought of as choosing a suitable value of a truncating function λ, constructed to be the function that solves:

$$E[\theta|\theta_k \leq \theta^*(\lambda)] = \lambda\theta^e, \quad 0 \leq \lambda \leq 1, \tag{4}$$

where $\theta^*(\lambda)$ is the truncation point on $f(\theta)$, monotonically increasing in λ. Thus, the criterion needed for credit is represented by $\theta^*(\lambda)$ and the expression in (4) states the expected default rate (quality) in the subpopulation of firms below the truncation point, i.e. the *conditional* expected default rate. The distribution of firm quality for some general distribution is displayed in Figure 1 in which we see that the bank, by screening applicants and truncating the distribution of firm quality, reduces its exposure to default risk.

To understand the trade-of between quality and quantity of credit, we move over to the supply of credit and acknowledge that a bank's *expected* credit supply function can be written as the product of the *m* firms' demand for credit and the probability that a firm

meets the requirements of the bank:

$$L = \sum_k^{m \in M} l_k \int_0^{\theta^*} f(\theta) \, d\theta = \sum_k^{\hat{m}(\theta) \in m} l_k, \tag{5}$$

where $\hat{m}(\theta)$ is the number of firm's eligible for credit. For the analysis below, it is essential to know how screening affects the bank's expected credit supply. Thus, we consider a tightening in the criterion needed for credit, i.e. a reduction in θ^*. It follows that, for any probability density function of firm quality for $\theta_k \in [0, 1]$ and a finite sample of firms (M), a decrease in θ^* will shrink the sample size of eligible firms. This in turn will reduce the bank's expected credit supply. Formally, since $\partial \hat{m} / \partial \theta^* > 0$ and since $\sum_k^{\hat{m}(\theta^*) \in m} l_k \leq \sum_k^{m \in M} l_k$, it follows that $\partial L / \partial \theta^* > 0$. This is summarised in the following Proposition;

Proposition 1: *A bank facing a finite number of applicants (firms) that tightens the criterion needed for credit will reduce the amount of supplied credit.*

A key result from Proposition 1 is that an there exists some profit maximizing value of θ^* implying some profit maximizing value of λ. Thus, the bank's optimization problem boils down to a decision between quality and quantity of credit. Hence, if the bank tightens the criterion needed for credit, i.e. it reduces θ^*, fewer firms will default on their loans but the supply of credit will drop, reducing the bank's *potential* profits. This crucial link between the bank's credit supply and the screening procedure of loan applicants provides a useful platform when forming a understanding of the determinants of credit crunches.

For tractability, let the expected credit supply function be based on the profit maximizing value of θ^*. This allows us to define a weight, ω, that scales the now constant probability in (5). Since θ^* is monotonically increasing in λ and since $E[\theta | \theta_k \leq \theta^*(\lambda)]$ is linear in λ, we solve the bank's expected credit supply function by scaling ω with λ, restricting the weight to positive values. This allows us to rewrite (5) as:

$$L(\lambda) = \lambda \omega \sum_k^{m \in M} l_k. \tag{6}$$

Combining (6) with the definition of the conditional expected default rate in (4) and the bank's expected profit function in (2) gives us the bank's *conditional* expected profit function:

$$E[\pi_b | \theta_k \leq \theta^*(\lambda)] = [(1 + r)(1 - \theta^e \lambda) - 1] \lambda \omega \sum_k^{m \in M} l_k. \tag{7}$$

Maximizing (7) with respect to λ and simplifying, results in the bank's first order condition[2]:

$$\partial E[\pi_b|\theta_k \leq \theta^*]/\partial \lambda = \omega \left[r - 2(1+r)\theta^e \lambda\right] \sum_{k}^{m \in M} l_k = 0, \tag{8}$$

such that $\lambda^* = \lambda^*(\theta^e, r)$ conditioned on the profit maximizing value of θ^*. More specifically, we use the first order condition in (8) and solve for the profit maximizing value of the truncating function:

$$\lambda^* = \frac{r}{2\theta^e(1+r)}. \tag{9}$$

Since $\partial \lambda^*/\partial \theta^e < 0$ and since θ^* is monotonically increasing in λ, it follows that $\partial \theta^*/\partial \theta^e < 0$, i.e. an increase in the unconditional expected default rate, reduces the bank's chosen truncation point. In addition, since $\partial \lambda^*/\partial r > 0$, it also follows that $\partial \theta^*/\partial r > 0$ such that the bank tends to accept a higher level of default risk when interest rates are increased. These results are summarised in Proposition 2;

Proposition 2: *A bank that screens applicants in order to maximize profits, tightens the criterion needed for credit if the unconditional default rate is increased or if the interest rate is decreased.*

Since a credit crunch is intrinsically related to the criterion needed for credit, Proposition 2 give some clues regarding the determinants of credit crunches.

We continue with some additional properties of the theoretical model, later to be used in the artificial credit market as defined in the next section. By combining (9) with the definition of the bank's conditional expected default rate in (4), we get the bank's profit maximizing conditional expected default rate; expressed only as a function of the interest rate:

$$E[\theta|\theta_k \leq \theta^*(\lambda^*)] = \frac{r}{2(1+r)}. \tag{10}$$

The expression in (10) highlights the importance of interest rates on the criterion needed for credit. Remembering the participation constraint in (3), we conclude that the optimal *conditional* expected default rate is simply half the *unconditional* expected default rate. In addition, since $\partial E[\theta|\theta_k \leq \theta^*(\lambda^*)]/\partial r > 0$, an increase in interest rates increases the amount of credit risk undertaken by banks, as previously implied in Proposition 2. By substituting for (9) and (10) in (7), we express the bank's profit maximizing conditional expected profit function in terms of the models exogenous variables:

$$E[\pi_b|\theta_k \leq \theta^*(\lambda^*)] = \omega \sum_{k}^{m \in M} l_k \frac{r^2}{4\theta^e(1+r)} > 0, \tag{11}$$

---

[2]For illustrative reasons the regulatory bodies restriction is ignored.

with $\partial E[\pi_b | \theta_k \leq \theta^*(\lambda^*)] / \partial r > 0$ and $\partial E[\pi_b | \theta_k \leq \theta^*(\lambda^*)] / \partial \theta^e < 0$. Studying the implications of (11), we see that the bank expects positive profits by screening out unwanted firms, conditioned on perfect estimates of firm quality; a result that follows from the bank's "monopoly" power in the screening procedure. However, by viewing the bank's expected profits as expected revenue, (11) also corresponds to the bank's expected deposit costs in a perfectly competitive economic environment.

Summing up our findings so far, in this section, we have derived a simple theoretical banking model in which banks maximize profits by removing risky firms from their credit portfolios. Despite its simplicity, the model is able to highlight the importance of firm quality and interest rates on the criterion needed for credit. Since a credit crunch relates to a period in time in which credit and investment capital are hard to obtain, we argue that a tightening of the criterion needed for credit, and its determinants, is intrinsically related to the onset of a credit crunch. Despite this, however, the theoretical model fails to capture the distinctive nature of credit crunches. Credit crunches are by definition dynamic phenomena since the tightening of the criterion needs to be coordinated across banks throughout a period of time. In addition, in a credit market with a finite number of participants, the decision made by a single bank may affect the pool of potential borrowers of its competitors and it is unlikely that banks face the full set of firms at every instant. To cope with these issues, we view the credit market as a complex adaptive system and proceed with constructing an artificial credit market based on the insights from the theoretical model.

3   An artificial credit market

Through the theoretical two-period model, we found variables that influence the representative bank's decision regarding the criterion needed for credit. However, the model fails to capture the dynamics of a credit market. In addition, in a credit market with a finite number of participants, the decision made by one bank may affect the pool of potential borrowers of its competitors. To cope with these issues, we view the credit market as a complex adaptive system as defined in Tesfatsion (2006). Thus, we construct an Agent Based Model (ABM) of a credit market based on repeated debt contracts. The theoretical model in the previous section is used as the base on which we build the ABM. This allows us to study the credit market in a dynamic framework, without imposing additional restrictive assumptions on the agents behavior. In addition, the ABM allows for random spillover affects of counter party risk. We begin by defining the details of the artificial economy and proceed by deriving the decision rules governing the agents' behavior.

3.1 The model

We first consider the matching process of firms and banks. In reality, this process is likely to be affected by some randomness making the initial match stochastic. In addition, as discussed in the large literature on relationship banking (see Sharpe (1990), Rajan (1992), Petersen and Rajan (1994), Petersen and Rajan (1995) among others), the initial lending may create some information advantage for the initial lender which then leads to some ex-post monopoly situation. However, since we view the interest rate on external capital as exogenous, we can safely ignore the potential ex-post monopoly effect since the lending standards of credit will remain unaffected in either case. Thus, we focus on the initial stochastic matching process and situate the ABM on a finite spaced torus populated with an initial number of firms ($k = 1, ..., M_0$) and banks ($i = 1, ..., N_0$) spread out on a grid at random. Time is discrete and represents new possible debt contracts and/or maturity dates. Following the arguments made in the previous section, firms need external capital in order to undertake a risky project. Firms search the torus for external capital through a 360° random walk where the torus is of size b^2 and where $b \in \mathbb{Z}$ is divisible with remainder. Thus, by situating the agents on a finite spaced torus, the probability of a firm-bank encounter is partly given by the "density" of the credit market, $\mathbb{D}(M_t, N_t, b)$.

Banks are governed by a financial authority stipulating a regulatory rule requiring banks to hold own capital based on the Capital Adequacy Ratio (CAR) such that for any given bank and time:

$$\text{CAR}_{i,t} \geq K, \quad 0 \leq K \leq 1, \tag{12}$$

where K represents the minimum capital requirements. All debt owned by the bank is unweighted and the sum of a bank's Tier-capital is equivalent to the bank's equity capital, henceforth referred to as the banks equity.

When a firm encounters a bank, the firm states its demand for credit which the bank evaluates according to the regulatory rule in (12). If the bank lives up to the requirement, it makes a noisy estimate of the firm's probability of default, $\theta_{k,i}^b = \theta_k + \phi_{k,i}$ with support $[0, \mathcal{T}]$ where $\phi_{k,i}$ is a random draw from a normal distribution with zero mean and standard deviation σ^f. Estimates outside of the support region are re-estimated. If the bank's estimate of firm quality is below the truncation point, i.e. if $\theta_{k,i}^b \leq \theta_{i,t}^*$, a debt contract is formed. If the bank rejects the firm's demand for credit, the firm continues its search for a debt contract. The debt lasts for a minimum of κ time periods and is only repaid upon a firm-bank encounter, making the maturity date of the contract stochastic. Thus, we allow for different maturity dates without specifying the details in the debt contract. Here, it is important to note that a large value of κ increases the *average* time to maturity. In addition, we limit the effect on credit crunches caused by a single firm's performance

by prohibiting firms in debt from additional borrowing until the debt is repaid.

Given the above, the probability of a firm-bank encounter depends on the debt's minimum time to maturity (κ) as well as on the "density" of the credit market ($\mathbb{D}$). Since a firm in debt is restricted from signing a new debt contract until the previously acquired debt is repaid and since debt contracts are only formed when a firm encounters a bank; these variables implicitly define the agents' abilities to sign new debt contracts as well as the "flow of funds". Relating this to the market liquidity literature, in which market liquidity is defined as the ability to trade an assets at short notice (Nikolaou, 2009); we acknowledge that the debt's minimum time to maturity (κ) and the density of the credit market ($\mathbb{D}$), jointly determine something we may call "credit market liquidity" ($\psi(\kappa, \mathbb{D})$). When exploring the properties of credit market liquidity within the model context, we acknowledge that a sparsely populated credit market (relative to the size of the torus) may experience random demand-side drops in credit, reducing the overall indebtedness of firms. However, if $\mathbb{D}$ is large, sudden drops in the aggregate debt level only reflects the decisions made by the suppliers of credit. Thus, by keeping the density of the market high, we are able to study the effects on credit crunches caused by variations in credit market liquidity originating from variations in the minimum time to maturity (κ).

Following this line of reasoning, we state the probability of a debt contract being formed by bank i at any given date as:

$$\Pr(\text{Contract}_{i,t}) = h\left(\psi\left(\kappa, \mathbb{D}\right), \Pr(\theta_k \leq \theta_{i,t}^*), \Pr(\text{CAR}_{i,t} \geq K)\right). \tag{13}$$

The first term in (13) determines how the frequency (from the simulations) in the debt contract formation is affected by credit market liquidity. The second two terms determine how the probability of a debt contract is affected by the supply side of credit.

Using the definition of a credit crunch as a period in time in which credit and investment capital is hard to obtain, sudden reductions in the supply of credit can be tracked back to the speed by which new debt contracts are formed. Since the probability in (13) depends on the capital adequacy ratio as well as the acceptable level of credit risk, the model has the ability to capture effects on credit crunches caused by the implementation of new regulatory rules as well as the effects caused by a deterioration in firm quality. In addition, since reductions in credit supply needs to be coordinated across banks in order for a credit crunch to erupt, we state the probability of a debt contract being formed by any bank at time t as:

$$Pr\left(\text{Contract}_t\right) = Pr\left(\cup_{i=1}^{N_t} \text{Contract}_i\right). \tag{14}$$

Hence, the complement of (14) defines the probability that no debt contract will be signed at time t, arguably an important component determining the probability of a credit crunch.

10

Since the probability that a contract will be signed at time t depends on $Pr(\text{CAR}_{i,t} \leq K)$, the probability in (14) relates to the bank's ability to build up capital; which in turn is effected by the bank's expected profit function and the criterion needed for credit. In addition, since the bank's choice of θ^* will depend on its previous encounters and since the market hosts a finite number of firms, the bank's debt portfolio is indirectly dependent on the debt portfolios of its competitors. This since lending reduces the pool of eligible firms. Returning to (13) and acknowledging that the probabilities by this reasoning are dependent, we see that the model allows for random spillover affects of counter party risk.

3.2 The Firms

We seek to keep the firms as simple as possible in order to make the simulations tractable. Thus, we assume that firms are "born" debt free with a pre-specified initial value of equity, E_0^f, identically distributed across firms. In addition, we let firms be defined by the balance sheet identity, allowing us to write the asset value of a representative firm as:

$$A_t^f = E_t^f + L_t^f, \quad t \geq 1,$$

where A_t^f is the firm's asset value, E_t^f is the firm's equity value and L_t^f is the value of firm liabilities at time t. As in the previous section, we let firms be protected by limited liability and assume a need for external capital to fund some risky project. Hence, by the same arguments as in Section 2, they always implement their projects when granted credit. We let the demand for credit vary between time periods to capture the randomness associated with investment opportunities. However, we limit the demand for credit to finite values and let the representative firm's demand for credit be given by a random draw from the firm's equity value:

$$l_t = \eta_t E_t^f,$$

where $\eta_t \sim U(0,1)$ resulting in $0 \leq l_t \leq E_t^f$.

Turning to the granting of credit, as previously mentioned, lending may only occur upon a firm-bank encounter at which the bank estimates the quality of the firm. If the bank's estimate of firm quality lies below the truncation point while the bank meets the requirements made by the model's regulatory body, the firm is granted credit from the bank to fund a risky project. The project lasts until the loan's maturity date on which the firm generates a gross return of R_{T^m} if the project succeeds, where $T^m = t + \tau(\kappa, \mathbb{D})$ denotes the loan's maturity date with $\partial \tau / \partial \kappa > 0$. Here, market density ($\mathbb{D}$) affects the debt't time to maturity since it determines the probability of a firm-bank encounter. Thus, as previously argued, the debt's maturity date is stochastic.

When determining the equation of motion defining the evolution of the firms' asset values, we seek a mechanism that links the performance of firms to their quality. We use the results in the work of Black and Scholes (1973) and Merton (1974) such that the probability that a firm defaults on its loan can be derived from its asset value. Assuming that the firm fails to meet its obligations to the bank if $A_t^f < L_t^f$, we write the equation of motion defining the representative firm's asset value as:[3]

$$A_{T^m}^f = A_{T^m-1} + \frac{E_{T^m-1}}{\Phi^{-1}(\theta)} \Delta W_{T^m}, \tag{15}$$

where $\Phi^{-1}(\theta)$ is the inverse of the standard normal distribution taken at firm quality and where $\Delta W_{T^m} \sim N(0,1)$. Firm quality is drawn from a truncated two parameter beta distribution, $\theta \sim \text{Beta}\,(\alpha,\beta)_{|\theta_k<\mathcal{T}}$, where the beta distribution is chosen for its ability to replicate bounded distributions of firm quality. Note that (15) requires $\mathcal{T} \leq 0.5$ such that $\theta \in [0,0.5]$ due to the symmetry of the standard normal distribution.

Given (15), the asset value of the firm remains constant between maturity dates and the firm defaults with probability θ when the project's profit is realized. If the asset value of the firm drops below zero, the firm files for bankruptcy and fails to meet its obligations to the bank. Thus, we have a steady flow of firms exiting the credit market through bankruptcy forcing the need of a firm-entry process. The firm-entry process is defined by assuming a saturated market. Thus, we let the firm-entry process be governed by a simple rule requiring the number of firms active in the credit market at time t to be approximately equal to the constant and pre-specified finite number of firms, $M_t \approx M$. Hence, in every time period the model gives birth to d_{t-1} new firms, where d_{t-1} is the number of firm defaults in the previous time period. Such an entry process will in the long run affect the distribution of firm quality due to the resampling of θ from $f(\theta)$. This effect is summarised in the following Proposition;

Proposition 3: *Consider an increase in t. If the market is saturated such that it supports a maximum of M firms at each time period, the resampling of firm quality from $f(\theta)$ reduces the unconditional expected default rate, θ_t^e.*

Proposition 3 states that since firms with a high value of θ_k have a high probability of default and since θ_k is drawn from the truncated beta distribution; a consequence of the firm-entry process is that the economic environment grows "safer" with time. Less risky firms will simply crowd-out the riskier ones. To see this, order firm quality such that $\theta_{1,t} < \theta_{2,t} < \cdots \theta_{M_t,t}$ and let S_l denote the state of firm $l < M_t$. Let the state of firm default

---

[3]See Appendix A for details.

be denoted Ω such that $\Pr(S_l = \Omega) < \Pr(S_{M_t} = \Omega)$. Since the sample estimate of the first central moment of θ is given by $\sum_{k=1}^{M_t} \theta_k / M_t$ it follows that $\Pr\left(\theta_t^e < \theta_{t-1}^e\right) > \Pr\left(\theta_t^e > \theta_{t-1}^e\right)$ such that $\partial \theta_t^e / \partial t < 0$.

3.3 The Banks

In analogy with the theoretical model in the previous section, banks use equity to provide firms with loans. The equity value of the banks at the initial date, E_0^b, is pre-specified and identically distributed across banks. At the end of each time period, banks will have accumulated profits from matured loans, funded new projects using its equity and suffered from defaulted loans. Using this, we construct the equation of motion defining the representative bank's asset value from the balance sheet identity:

$$
A_t^b = \left(E_{t-1}^b + \pi_t^b\right) + \left(L_{t-1}^b + \sum_k^{m_t^n \in m_t} l_k - \sum_k^{m_t^d \in \hat{m}_t} l_k\right), \quad t \geq 1,
$$

where A_t^b is the bank's asset value, E_t^b is the bank's equity, L_t^b is the value of the bank's outstanding debt, $m_t \in M_t$ is the number of firms facing their demand towards the representative bank, $\hat{m}_t^n$ is the number of firms granted credit at time t, $\hat{m}_t$ is the number of firms in the bank's debt portfolio at time t and where m_t^d is the number of firms repaying their debt at time t.

As in the previous section, banks maximize profits by screening applicants in order to pick suitable clients with an acceptable level of default risk. This is done by truncating the distribution function defining firm quality. The functional form of the truncating function can be specified in various ways reflecting the decision making process within the bank (e.g., if the decisions are taken at the central or decentralized level). This makes the model flexible for variations in corporate structure. Here, we assume that the bank's management has absolute control over the truncating function allowing us to treat λ as the bank's decision variable. As such, the solution to the bank's optimization problem in the artificial economy bears obvious resemblance to the results derived in the previous section. To see this, define the value of the truncating function at time t as $\lambda_{i,t}$. Using the results in the previous section while acknowledging that the banks now rely on noisy estimates of firm quality, we rewrite the representative bank's objective function as:[4]

$$
E[\pi_{b,t} | \theta_{k,t}^b \leq \theta_t^*] = [(1+r)(1 - \theta_t^e \lambda_t) - 1]\, \lambda_t \omega_t \sum_k^{m_t \in M_t} l_k. \tag{16}
$$

---

[4]Since $E[\theta^b] = E[\theta] + E[\phi] = \theta^e$.

13

We condition on the profit maximizing value of θ^* and maximize (16) with respect to λ_t, including the regulatory bodies constraint (12). This gives us the optimal value of the truncating function for the representative bank in the artificial economy:

$$\lambda_t^* = \begin{cases} \dfrac{r}{2\theta_t^e(1+r)} & \text{if } \text{CAR}_t > K \\ 0 & \text{if } \text{CAR}_t \leq K, \end{cases}$$

indicating that in an economic environment with fixed interest rates, the criterion needed for credit only varies with the estimate of θ_t^e. Recalling that $\partial\theta_t^e/\partial t < 0$ and that $\partial\lambda_t^*/\partial\theta_t^e < 0$, it follows that $\partial\theta_t^*/\partial t > 0$, using that θ_t^* is monotonically increasing in λ_t. In other words, a bank tends to decrease the criterion needed for credit with the passage of time.

Proposition 4: *Consider an increase in t. If the market is saturated such that it supports a maximum of M firms at each time period, a bank that screens applicants to maximize profits will tend to reduce the criterion needed for credit with time.*

From Proposition 4 it follows that banks tend to take on more risky debt as the economy evolves. However, the economy will suffer from short term fluctuations around the time path of the criterion needed for credit due to noisy estimates of firm quality. To see this, we acknowledge that $E[\theta|\theta_k \leq \theta_t^*(\lambda_t^*)] \neq E[\theta|\theta_k^b \leq \theta_t^*(\lambda_t^*)]$ where the inequality is due to imperfect estimates of firm quality.[5] It is reasonable to assume that banks learn about the quality of firms by interim information production, Besanko and Kanatas (1993) and Holmström and Tirole (1997). Thus, we assume that the bank observes the true quality of firms for the subpopulation of firms currently in its debt portfolio. Using this, we let the bank have adaptive expectations of (4) such that $E[\theta|\theta_k^b \leq \theta_t^*(\lambda_t^*)] = \sum_k^{\hat{m}_{t-1}} \theta_{k,t-1}/\hat{m}_{t-1}$. Relating this to the profit maximizing conditional default rate in (10), we let the bank solve for the point of truncation by an iterative procedure stated as:

$$\theta_t^* = \begin{cases} \theta_{t-1}^* - c, & \text{if } E[\theta|\theta_k^b \leq \theta_t^*(\lambda_t^*)] > \dfrac{r}{2(1+r)} \\ \theta_{t-1}^*, & \text{if } E[\theta|\theta_k^b \leq \theta_t^*(\lambda_t^*)] = \dfrac{r}{2(1+r)} \\ \theta_{t-1}^* + c, & \text{if } E[\theta|\theta_k^b \leq \theta_t^*(\lambda_t^*)] < \dfrac{r}{2(1+r)} \end{cases}$$

where $0 \leq c \leq r/(2(1+r))$ is a parameter representing the speed by which banks move towards the optimal truncation point. In addition, the bank is refrained from lending if $\text{CAR}_t \leq K$, honouring the regulatory rule in (12).

---

[5]See Appendix B for details.

Examining the iterative procedure defining θ_t^* above, we acknowledge four things. First, since the optimal truncation point, θ_t^*, represents the criterion needed for credit and since the truncation point determines the riskiness of the bank's credit portfolio; movements towards the optimal truncation point can be thought of as movements towards the bank's internal credit risk goal. Thus, c represents the speed of adjustment to the bank's internal credit risk goal. Second, given the parameter space of c, the bank may "overshoot" its own credit risk goal and acquire a debt portfolio characterized by more risky debt than in (10). This opens up for periods characterised by "over-lending" in which over-lending banks try to reduce their exposure to credit risk by tightening the criterion needed for credit. Third, if such a tightening occurs simultaneously across banks, the economy may move into a time period in which credit and investment capital is hard to obtain. Fourth, the bank's initial debt contracts may influence the bank's future decision regarding θ^*. To reduce this effect, we set $\theta_0^* = 0$ allowing the bank to steadily build up the riskiness of its credit portfolio using the iterative procedure as stated above.

Banks with a low value of c take small steps towards the optimal level of credit risk. Hence, the bank's speed of adjustment to its internal credit risk goal reflects the level of conservatism within the bank's organisational structure where conservative banks have a relatively low value of c. Relating c to the real world, the speed of adjustment to the bank's internal credit risk goal can be thought of as a parameter reflecting the bank's willingness to engage in new risky ventures or as its willingness to use new and unexplored debt instruments characterised by more unexplored risk. Since we are interested in the determinants of credit crunches, we study the case in which all banks are equally conservative. This allows us interpret c as a parameter reflecting the general level of conservatism in the economy.

4   Simulations

In order to find the determinants of credit crunches, we simulate the artificial economy in different economic states, implementing the framework discussed in the previous sections.[6] We first define a restrictive measure of a credit crunch within the context of the model and then explore the properties of the artificial economy through a selected simulation. The selected simulation is chosen as to illustrate the features of a progressive economy populated with many creditworthy firms.

When defining a restrictive and measurable variable of a credit crunch, we first recall the definition in Udell (2009), suggesting that a credit crunch is reflected in a tightening

---

[6]The NetLogo environment is used for the simulations. The code is available on request.

of credit conditions, here represented by a decrease in $\theta_{i,t}^*$. Thus, if the average truncation point drops below some threshold, investment capital becomes hard to obtain since only a small sample of firms are eligible for credit. Using this, in the absence of a stringent formal definition, we define an indicator variable of a credit crunch as:

$$\delta = \begin{cases} 1, & \text{if } \theta_{i,t}^* = 0, \ \forall i, \quad t > 0 \\ 0, & \text{if else} \end{cases}, \tag{17}$$

such that $\delta = 1$ in the case of a credit crunch which by all means of measurement, represents an increase in the criterion needed for credit. Arguably, the indicator variable in (17) relates to the probability in (14) since $\sum_i^N \theta_{i,t}^* \to 0 \Rightarrow \text{Pr}(\text{Contract}_t) \to 0$. However, the definition above neglects the potential effects on the supply of credit caused by (i) the banks' potential inability to live up to the capital requirements, and (ii) the potential effects on crunches caused by risk based regulatory changes affecting (14) through $Pr(\text{CAR}_{i,t} \geq K)$. Remembering that all debt is unweighted in this version of the model, we neglect these issues.

4.1 Selected simulation

The properties of the model are illustrated through a selected simulation of a credit market in which banks have close to perfect firm quality estimates ($\sigma^f = 0.0001$) and where the unconditional expected default rate (θ_t^e) is lower than the banks' optimal expected default rate as stipulated in (10). Thus, since $\mathcal{T} = 0.5$, *ex-ante* we may expect banks with almost perfect firm quality estimates to set $\theta_{i,t}^* = 0.5$. However, since banks may oversample from the pool of risky firms, occasional decreases in the average truncation point is expected. The parameters of the beta distribution are chosen to be $\alpha = 2.6$ and $\beta = 150$ such that firm quality is distributed with a heavy tail to the right. Given this, the unconditional expected default rate at the initial time period is $\theta_0^e \approx 1.7$ percent. The interest rate on external capital is set to $r = 4$ percent such that the *optimal* conditional expected default rate is 1.92 percent, i.e. 22 basis points higher than the unconditional expected default rate. We set the minimum capital requirements at $K = 8$ percent, replicating the capital requirements enforced by the bank for international settlements in Basel, assuming that banks are refrained from holding capital to mitigate future risks. Figure 2 illustrates the evolution of debt and the average truncation point in an artificial economy lasting 5000 time periods where the first 500 observations have been removed in order to get rid of transients. The model is simulated with $M_0 = 2000$ firms, $N_0 = 5$ banks and the torus is constructed from $b = 11$. The initial equity of the banks is set to $E_0^b = 2$ and firms are born with $E_0^f = 1$. The level of conservatism in the economy, i.e. speed of adjustment
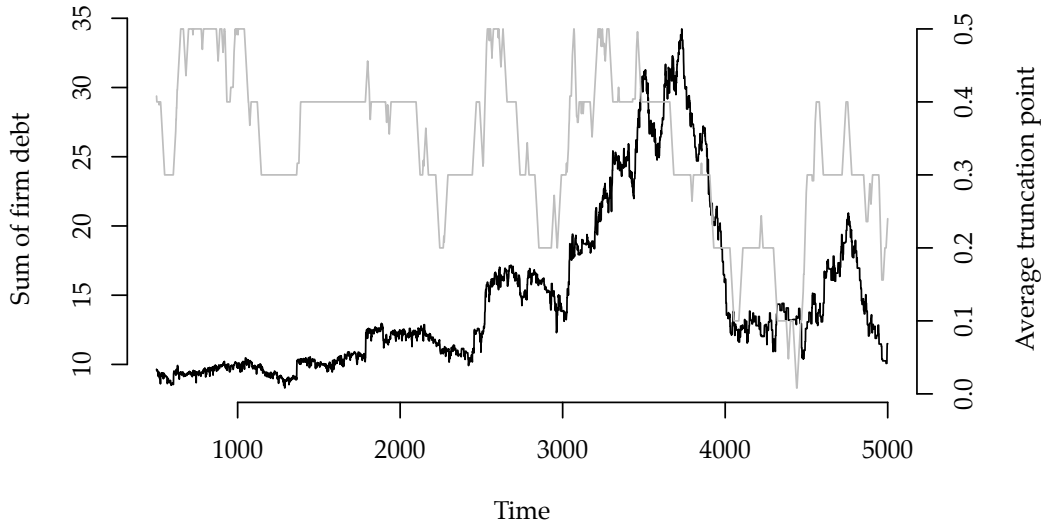
Figure 2: Sum of firm debt (L_t^f) (solid line) and the average truncation point (θ_t^*) (gray line).

to the banks' internal credit risk goals, is set to $c = 0.02$ and the debts minimum time to maturity is set to $\kappa = 10$.

From Figure 2 we see that the aggregated debt level has a positive trend, exhibiting cyclical tendencies. In addition, we acknowledge that firm debt is closely related to variations in the average truncation point (the criterion needed for credit). The average truncation point occasionally deviates from the profit maximizing solution and at $t = 4440$ the economy evolved into a two period credit crunch. The crunch, and the preceding decrease in the average truncation point, caused a 58.37 percent drop in debt compared to the aggregate debt's local maximum at $t = 3640$. Since all parameters are held constant during the simulation period, this indicates that crunches have a natural tendency to occur; this even if banks have near to perfect estimates of firm quality in the absence of new regulatory rules or sudden variations in firm quality.

During the time period preceding the credit crunch, the aggregate debt level experienced growth, despite occasional decreases in the average truncation point. The sudden downturn in debt due to the spontaneously coordinated tightening of the criterion needed for credit (reduced $\theta_{i,t}^*$), forced the onset of a credit crunch. Recalling the lending mechanism discussed in the previous section, this indicates that banks tend to engage in pe-
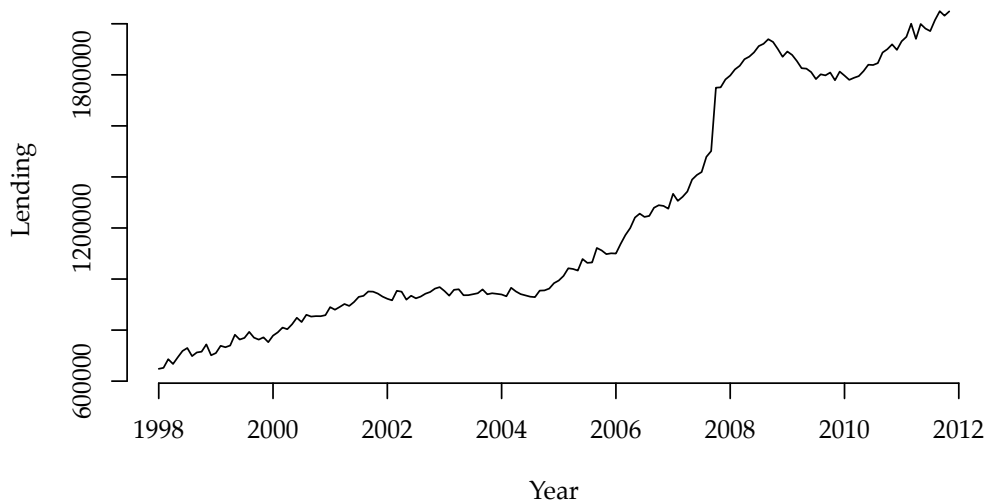
17

Figure 3: Value of lending to non-financial firms by Swedish banks in billion SEK. Source: Statistics Sweden.

riodic over-lending, acquiring a debt portfolio characterised by more risk than the profit maximizing level of credit risk. When realized, the banks seek to "wash-out" previously acquired bad debt by tightening the criterion needed for credit. For comparison, Figure 3 exhibits the evolution of lending made by Swedish banks to Swedish non-financial firms from January 1998 to November 2011. The series shows a reduced growth in lending after the internet bubble of 2001 and a sharp drop in lending during the aftermath of the financial crisis of 2008. By comparing the evolution of lending during the financial crisis and the evolution of debt in the artificial economy, we see an obvious resemblance.

The evolution of the average of firms' assets, on the other hand, is characterised by a positive trend as illustrated in Figure 4. On average, the firms' asset values grew with 5 basis points per time period.[7] The positive trend is frequently broken by sequential downturns due to reduced lending and sequential firm defaults. Such "busts" are highly dependent on the criterion needed for credit since the equation of motion defining the evolution of firms' asset values is defined by firm quality. Time periods characterised by little or no lending reduces the supply of investment capital. As such, firms have no means of funding potentially fruitful projects, reducing the aggregate growth level of firm

---

[7]We only measure firms active on the credit market, i.e. firms granted credit at least once, since non-participants have a constant asset value defined only by E_0^f.
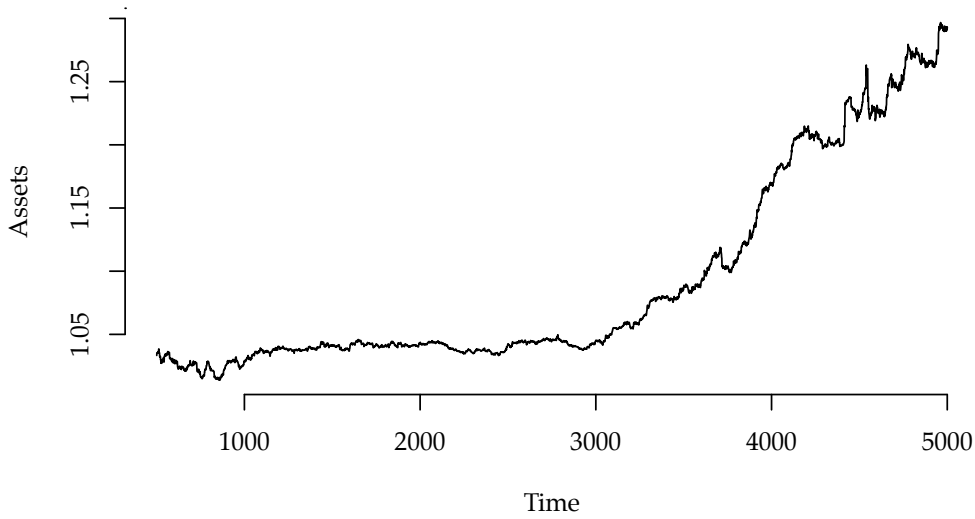
Figure 4: Average of firms' assets (A_t^f).

assets. In addition, the series is characterised by seemingly random "booms" caused by an increase in project funding and numerous successful projects. Furthermore, we acknowledge that the series shows signs of increased volatility after the onset of the credit crunch at $t = 4440$ due to the small number of new debt contracts.

4.2 The determinants of credit crunches

From the selected series, we acknowledge that the artificial credit market has a natural tendency to spontaneously evolve into a credit crunch. However, the determinants of crunches remain undetermined. In order to find the parameters of the model that can be held accountable for sudden supply side drops in credit, data is collected from simulations of the artificial credit market, limited to sequences of 5000 time periods. The experimental plan used in the study is presented in Table 1.

Since credit market liquidity, ψ, is jointly determined by the minimum time to maturity (κ) and the density of the market, $\mathbb{D}(M_t, N_t, b)$, we choose to hold the size of the torus (b^2) constant throughout the simulation periods since variations in this parameter only varies the density of the market. In addition, since all debt is unweighted in this version of the model, we deem it unlikely that regulatory changes between states will affect the criterion needed for credit. Thus, we keep the minimum capital requirements (K) constant at 8 percent in all simulations. Since the parameters of the beta distribution defines the

Table 1: Experimental plan used for the simulations.

Variables	Treatments	
Conservatism (c)	0.0001	0.01
Interest rate (r)	2%	4%
Initial number of firms (M_0)	1000	2000
Initial number of banks (N_0)	3	5
Minimum time to maturity (κ)	1	10
α, β	1.67, 100	2.5, 150
σ^f	0.0001	0.01

Constants	Value
Minimum capital requirements (K)	8%
Initial truncation point ($\theta_{i,0}^*$)	0
Initial bank capital ($E_{i,0}^b$)	2
Initial firm equity ($E_{j,0}^f$)	1
Torus size parameter (b)	11

evolution of firm assets as well the probability of firm default, these parameters represent the state of the economy. The parameters of the beta distribution are varied in two states such that θ_0^e takes on the same value for different values of α and β in a subset of the simulations.

Given the experimental plan in Table 1, we simulate the artificial economy in 256 different states with 100 replications resulting in a total of 25 600 observations. If the economy experiences a crunch during a simulation period, the result is documented and a new simulation is initiated. Thus, the onset of a credit crunch is defined as a dichotomous variable with one observation per simulation run. We acknowledge that the variable of interest is dependent on the vector of observables such that the probability of a crunch can be estimated using a standard logit model. To determine how the parameters of the beta distribution affect the probability of a credit crunch we estimate two models. The estimates from the logit models are displayed in Table 2 from which we only seek to interpret the signs of the estimates due to the theoretical nature of the model.

Examining Table 2, we conclude that an increase in the speed of adjustment to the banks' internal credit risk goals (c) has a positive effect on the probability of a credit crunch. This implies that a more conservative approach to lending reduces the probability of sudden supply side drops in credit, even in the absence of variations in the economic

Table 2: Maximum likelihood estimates from the logit models on credit crunches (δ). All parameter estimates are significant at the 0.001 level, $n = 25600$.

Variables	Model 1	Model 2
Intercept	2.8792	10.726
Conservatism (c)	540.31	527.28
Interest rate (r)	-383.66	-375.20
Initial number of firms (M_0)	-0.0004	-0.0004
Initial number of banks (N_0)	-0.6529	-0.6415
Initial unconditional expected default rate (θ_0^e)	505.65	
Minimum time to maturity (κ)	-0.0264	-0.0259
α		4.0634
β		-0.0643
σ^f	139.80	137.38
Nagelkerke R^2 index	0.8254	0.8213

conditions; this being a partially overlooked component contributing to the financial stability of an economy. In addition, an increase in the debts' minimum time to maturity (κ) decreases the probability of a credit crunch. This result suggests that an increase in the average time to maturity *reduces* the probability of a credit crunch. We also see that an increase in market density, working through an increase in the initial numbers of firms (M_0) and banks (N_0), reduces the probability of a credit crunch.

To fully understand these findings, we need to view them in the light of how the artificial economy is constructed. Due to random movements of a finite number of firms on a torus, banks do not meet the full distribution of eligible firms at every instant. Since banks have adaptive expectations about the credit risk in their debt portfolio, they continue to increase $\theta_{i,t}^*$ until the credit risk in its debt portfolio equals/or overrides their profit maximizing level of credit risk. Acknowledging that firms are allowed to make repayments on matured debt in every time period, the risk associated with a bank's debt portfolio can increase rapidly if the bank grants credit to risky firms at the same instant as less risky firms meet their obligations to the bank. Thus, the faster a bank adjusts to its internal credit risk goal, i.e. the larger the c, the higher the probability of retrieving a debt portfolio defined by a suboptimal expected default rate. Simultaneous reductions in truncation points due to spontaneous wash-outs of bad debt may then lead to an absolute tightening of the criterion needed for credit, forcing the onset of a supply side credit crunch. As such, if the lending capacities of banks are locked in contracts with long maturity dates, the prob-

ability of hastily increasing the bank's credit risk goal above the bank's optimal level is reduced. Thus, an increase in the minimum time to maturity (κ) decreases the probability of issuing credit to numerous risky firms at the same instant as less risky firms repay its debt, reducing the probability of a poorly diversified credit portfolio. This result indicates that an increase in the maturity time of debt may offset some of the negative side effects caused by rapid variations in the banks' truncation points.

The effects on crunches caused by the parameters of the beta distribution are more easily understood if we view them in the light of this new insight. If α is increased, the mode of the distribution defining firm quality is moved to the right, reducing the proportion of firms afflicted with an acceptable default risk. Hence, an increase of α can be thought of reducing the sample size of eligible firms. A rapid increase in the truncation point, conditioned on a relatively large value of α, may result in an oversampling of risky firms from the bank's perspective, forcing a tightening of the criterion needed for credit. This corresponds to an increase in the unconditional expected default rate (θ^e) since an increase in α moves the mode of the truncated beta distribution to the right. Thus, an increase in the unconditional expected default rate at the initial date (θ_0^e) increases the probability of a credit crunch, as previously suggested in the theoretical part of this paper. In contrast, an increase in β reduces the probability of a credit crunch. Such an increase centers the probability density mass around the mode of the distribution, increasing the "distance" to riskier loans. This can be thought of as homogenising firm quality which tends to reduce the probability of a crunch.

Since θ_k is drawn from the truncated beta distribution and since θ_k defines the evolution of firm assets, the results regarding the parameters of the beta distribution are fully in in line with predictions from the asset deterioration hypothesis. In addition, we find that an increase in interest rates (r) has a significant and negative impact on the probability of a crunch. Relating a credit crunch to the criterion needed for credit, this result is fully in line with the findings in the theoretical part of this paper. If the interest rate is lowered, the pool of firms that have the ability to bear a positive contribution to the banks' expected profits is reduced. The banks react to this by only granting credit to a subgroup of firms that add positive value to the banks' expected profits. Rapid variations in the banks' truncation points may then lead to an oversampling from the segment of value reducing firms with reduced lending as a direct consequence.

5   Concluding remarks

This paper analyses the determinants and causes of credit crunches. We start by deriving a simple theoretical banking model in which banks screen applicants in order to pick

firms with an acceptable probability of default. We then use the mechanisms from the theoretical model and construct an Agent Based Model (ABM) of a credit market. Through simulations of the ABM, we show that crunches have a natural tendency to occur if banks have adaptive expectations about the risk in their credit portfolios. We also find that an increase in the speed by which banks adjust to their internal credit risk goals, increases the probability of a credit crunch. We link this parameter to the level of conservatism in the market and conclude that a more conservative approach to lending leads to fewer credit crunches; an up till now partially overlooked component contributing to the financial stability of an economy. In addition, we are able to show that the onset of crunches are affected by variations in the market conditions defining the evolution of firm assets. If the economy is in a state characterised by few creditworthy firms, the probability of a credit crunch is increased, fully in line with the asset deterioration hypothesis. In addition, we find that homogenous markets, in terms of firm quality, tends to be associated with a lower probability of a credit crunch. The simulations also show that an increase in the debts time to maturity reduces the probability of a credit crunch since the lending capacities of banks are locked in credit with long maturities. This, in turn, reduces the probability of a poorly diversified credit portfolio. Thus, this paper adds new insights to current theory as well as provides new perspectives on the nature of sudden reductions of credit. In addition, this paper highlights the importance of time to maturity and a conservative approach to lending if policy makers seek to reduce the probability of a credit crunch.

Acknowledgments

References

ALLEN, F. (1990): "The Market for Information and the Origin of Financial Intermediation," *Journal of Financial Intermediation*, 1, 3–30.

ANDREASEN, M. M., M. FERMAN, AND P. ZABCZYK (2011): "The Business Cycle Implications of Banks' Maturity Transformation," Working Paper.

BERGER, A. N. AND G. F. UDELL (1994): "Did Risk-Based Capital Allocate Bank Credit and Cause a "Credit Crunch" in the U.S.?" *Journal of Money, Credit and Banking*, 26, 585–628.

BESANKO, D. AND G. KANATAS (1993): "Credit Market Equalibrium with Bank Monitoring and Moral Hazard," *Review of Financial Studies*, 6, 213–232.

BLACK, F. AND M. SCHOLES (1973): "The Pricing of Options and Corporate Liabilities," *The Journal of Political Economy*, 81, 637–654.

BROECKER, T. (1990): "Credit-Worthiness Tests and Interbank Competition," *Econometrica*, 58, 429–452.

CHIESA, G. (1998): "Information Production, Banking Industry Structure and Credit Allocation," *Research in Economics*, 52, 409–430.

GEHRIG, T. (1998): "Screening, Cross-Border Banking, and the Allocation of Credit," *Research in Economics*, 52, 387–407.

HOLMSTRÖM, B. AND J. TIROLE (1997): "Financial Intermediation, Loanable Funds and the Real Sector," *The Quarterly Journal of Economics*, 112, 663–691.

KIYOTAKI, N. AND J. MOORE (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211–248.

MERTON, R. C. (1974): "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29, 449–470.

MINSKY, H. P. (1977): "A Theory of Systemic Fragility," in *Financial Crises: Institutions and Markets in a Fragile Environment*, ed. by E. Altman and A. W. Sametz, New York, NY: John Wiley and Sons, chap. 6, 138–152.

——— (1992): "The Financial Instability Hypothesis," Working Paper Series 74, The Jerome Levy Economics Institute of Bard College.

NIKOLAOU, K. (2009): "Liquidity (Risk) Concepts: Definitions and Interactions," Working Paper Series 1008, European Central Bank.

PAZARBASIOGLU, C. (1996): "A Credit Crunch? A Case Study of Finland in the Aftermath of the Banking Crisis," IMF Working Papers 96/135, International Monetary Fund.

PETERSEN, M. A. AND R. G. RAJAN (1994): "The Benefits of Lending Relationships: Evidence from Small Business Data," *Journal of Finance*, 49, 3–37.

——— (1995): "The Effect of Credit Market Competition on Lending Relationships," *The Quarterly Journal of Economics*, 110, 407–443.

RAJAN, R. G. (1992): "Insiders and Outsiders: The Choice Between Informed and Arm's-Length Debt," *Journal of Finance*, 47, 1367–1400.

SHARPE, S. A. (1990): "Asymmetric Information, Bank Lending and Implicit Contracts: A Stylized Model of Customer Relationships," *The Journal of Finance*, 45, 1069–1087.

——— (1995): "Bank Capitalization, Regulation, and the Credit Crunch: A Critical Review of the Research Findings," Finance and Economics Discussion Series 95-20, Board of Governors of the Federal Reserve System (U.S.).

SUÁREZ, J. AND O. SUSSMAN (1997): "A Stylized Model of Financially-Driven Business Cycles," Papers 9722, Centro de Estudios Monetarios Y Financieros.

——— (2007): "Financial Distress, Bankruptcy Law and the Business Cycle," *Annals of Finance*, 3, 5–35.

TESFATSION, L. (2006): "Agent-Based Computational Economics: A Constructive Approach to Economic Theory," in *Handbook of Computational Economics*, ed. by L. Tesfatsion and K. L. Judd, Elsevier, vol. 2, 831–880.

UDELL, G. F. (2009): "How will a Credit Crunch Affect Small Business Finance?" *FRBSF Economic Letter*.

WINTON, A. (1995): "Delegated Monitoring and Bank Structure in a Finite Economy," *Journal of Financial Intermediation*, 4, 158–187.

Appendix A: The firms' asset values

Assume that the representative firm has a calendar-time counterpart that acts on a credit market where the time horizon is represented by T^m. Fix a probability space $(\Omega, \mathcal{F}, P)$ on which there is a standard Brownian motion W. Let $(\mathcal{F}_t)_{t \in T^m}$ be a filtration on the probability space such that the σ-algebra $\mathcal{F}_t$ represents the collection of observable events up to time t. Given the above, it is assumed that the asset value of the firm's calendar-time counterpart follows a geometric Brownian motion:

$$dA_t^f = \mu A_t^f \, dt + \sigma A_t^f \, dW_t, \tag{A.1}$$

where W is a standard Brownian motion under the probability measure P. Moving over to the agent based model's sequential evolution of time, we rewrite (A.1) as:

$$\Delta A_t^f = A_t^f \left(\mu \Delta t + \sigma \Delta W_t \right). \tag{A.2}$$

Since the evolution is bounded by the endpoint, T^m we let $W_{t-\tau} = W_{t-1}$ such that the firm's asset value remains constant between maturity dates. Given this, we let time evolve in multiples of one such that $\Delta W_{T^m} = W_{T^m} - W_{T^m - 1} \sim N(0, 1)$. By rearranging (A.2) we get:

$$A_{T^m}^f = A_{T^m - 1}^f + \sigma_{T^m}^* \Delta W_{T^m},$$

where $\sigma_{T^m}^* = A_{T^m}^f (\mu / \Delta W_{T^m} + \sigma)$. Since $\Delta W_{T^m} \sim N(0, 1)$ it follows that $A_{T^m}^f \sim N(A_{T^m - 1}^f, \sigma_{T^m}^*)$. Thus, the drift terms enter by asymmetric shocks. Acknowledge that $A_{T^m}^f = A_{T^m - 1}^f + \sigma_{T^m}^* \Delta W_{T^m} = E_{T^m - 1}^f + L_{T^m - 1}^f + \sigma^* \Delta W_{T^m}$. Use that $L_{T^m - 1}^f$ is constant between maturity dates and let the firm default if $A_{T^m}^f < L_{T^m - 1}^f$ with probability θ. It follows that $Pr(A_{T^m}^f < L_{T^m - 1}^f) = Pr(A_{T^m}^f - L_{T^m - 1}^f < 0) = Pr(E_{T^m - 1} + \sigma^* \Delta W_{T^m} < 0) = \theta$. Solve for the $\sigma_{T^m}^*$ that forces the firm to default with probability θ at the maturity date and it follows that $\sigma_{T_m}^* = E_{T^m - 1}^f / \Phi^{-1}(\theta)$ where $\Phi^{-1}(\theta)$ is the inverse of the standard normal distribution taken at firm quality. Thus, we rewrite the representative firm's equation of motion as:

$$A_{T^m}^f = A_{T^m - 1} + \frac{E_{T^m - 1}}{\Phi^{-1}(\theta)} \Delta W_{T^m},$$

where $\theta \in [0, 0.5]$ due to the symmetry of the standard normal distribution.

Appendix B: Expected default rates

Let θ_k represent realizations of $f(\theta)$ and let ϕ_k represent realizations from $f(\phi)$. We seek the conditional expected default rate conditioned on a measurement error in expectations, i.e. $E[\theta|\theta^b \leq \theta^*] = E[\theta|\theta + \phi \leq \theta^*] = E[\theta|\theta \leq \theta^* - \phi] = E[\theta|\theta \leq \hat{\theta}^*]$. As such, we have a random truncation, selected out of a density $f(\hat{\theta}^*)$. Since $\phi \sim N(0, \sigma^f)$ it follows that $\hat{\theta}^* \sim N(\theta^*, \sigma^f)$. However, we truncate the distribution such that $\hat{\theta}^* \in [0, \mathcal{T}]$. Given this, the expected truncation point is:

$$E[\hat{\theta}^*|0 \leq \hat{\theta}^* \leq \mathcal{T}] = \frac{\int_0^{\mathcal{T}} \hat{\theta}^* f(\hat{\theta}^*) d\hat{\theta}^*}{F_{\hat{\theta}^*}(\mathcal{T}) - F_{\hat{\theta}^*}(0)},$$

where $F_{\hat{\theta}^*}(x)$ is the cumulative distribution function of $\hat{\theta}^*$. From this it follows that:

$$E[\theta|\theta_k \leq \theta^*] = \frac{\int_0^{\theta^*} \theta f(\theta) \, d\theta}{F_\theta(\theta^*)} \neq E[\theta|\theta_k^b \leq \theta^*] = \frac{\int_0^{E[\hat{\theta}^*|0 \leq \hat{\theta}^* \leq \mathcal{T}]} \theta f(\theta) \, d\theta}{F_\theta(\theta^*)},$$

where $F_\theta(x)$ is the cumulative distribution function of θ. Hence, the bank fails to find the optimal expected default rate.