

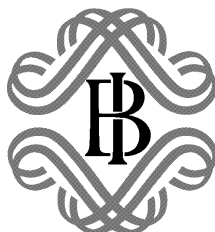
BANCA D'ITALIA

# **Temi di discussione**

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**Bootstrap bias-correction procedure in estimating long-run  
relationships from dynamic panels, with an application  
to money demand in the euro area**

by Dario Focarelli



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# BOOTSTRAP BIAS-CORRECTION PROCEDURE IN ESTIMATING LONG-RUN RELATIONSHIPS FROM DYNAMIC PANELS, WITH AN APPLICATION TO MONEY DEMAND IN THE EURO AREA

by Dario Focarelli \*

## Abstract

In dynamic panel data models, which are particularly well-suited to cross-country analysis, the Mean Group estimator (Pesaran and Smith, 1995) is under certain quite strong conditions consistent, but theoretical and empirical evidence indicates that it can be biased when the number of time observations is small. Possible explanations are sample-size bias and omitted variables or measurement errors that are correlated with the regressors. I find support for both hypotheses using a Monte Carlo experiment which analyzes cointegrated systems. A possible solution for the MG estimator bias is a bootstrap bias-correction procedure, but Pesaran and Zhao (1999) show that it performs well only when the true coefficient of the lagged dependent variable is small. In this paper, I test three different bootstrap procedures and obtain an appreciable reduction in the MG estimator bias, especially when the suggestions of Li and Maddala (1997) are applied. Finally, I use bootstrap bias-corrected estimators to investigate the long-run properties of money demand in the euro area.

JEL classification: C13 ,C15, C23, E41.

Keywords: dynamic panels, bias-corrected estimator, long-run coefficients, money demand.

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## 1. Introduction<sup>1</sup>

In recent years there has been increasing interest in dynamic panel data models where the number of time series observations  $T$  is comparable with  $N$ , the number of groups (Pesaran, Shin and Smith, 1999). In most applications of this type, the parameters of interest are the long-run effects and the speed of adjustment to the long run. Such panels can be very useful in cross-country analysis.

Four procedures are commonly used to compute long-run relationships from such panels: (i) applying aggregate time-series regression (TS estimator); (ii) estimating equations for each group and then averaging the coefficients over groups (the Mean Group estimator, MG, proposed by Pesaran and Smith, 1995); (iii) pooling the data, imposing the same slope allowing for fixed or random common intercepts, and estimating pooled regressions (DFE or DRE estimator); (iv) running a cross-section estimate with long-period averages for each country's variable (CS estimator).

Pesaran and Smith (1995) show that while in the static case all four methods give consistent estimates of the average coefficients, in dynamic models this does not hold. In particular, they show that under certain quite strong conditions (namely, the group-specific parameters are distributed independently of the regressors and the regressors are strictly exogenous) the MG and the CS estimators give consistent (unbiased) estimates of the average group parameters. In contrast, the estimates obtained from ATS and DFE estimators can produce inconsistent and potentially highly misleading estimates. The problem arises when the regressors are serially correlated, so that neglecting coefficient heterogeneity induces serial correlation in the disturbances, which generates inconsistent estimates. Their conclusion is that “individual micro-relations should be estimated separately and the averages of the estimated micro-parameters and their standard errors calculated explicitly”.

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However, as noted in Pesaran, Smith and Im (1996) and Pesaran and Zhao (1999), theoretical considerations and Monte Carlo evidence indicate that the MG estimator can be affected by small-sample bias. Further, empirical applications estimating separate relationships for a number of groups sometimes find differences in coefficients that are significant but economically implausible. The MG estimator tends to be sensitive to these abnormal coefficients. Pesaran, Shin and Smith (1999) argue that “one possible explanation is that the group-specific estimates are biased because of sample-specific omitted variables or measurement errors that are correlated with regressors”. This may become a big problem when dealing with a large number of groups, since it is very difficult to use additional data or a more appropriate specification for each group.

One possible way of tackling this problem is bootstrap bias-correction. To reduce the small-sample bias of the Mean Group estimator in dynamic heterogeneous panels with  $T=20$  and  $N=20$ , Pesaran and Zhao (1999) test such a procedure (together with three alternatives) using standard bootstrap techniques. Their results show that the procedure performs well when the true coefficient of the lagged dependent variable is small but poorly when it is large.

The aim of this paper is twofold. First, via Monte Carlo simulations I further explore the characteristics of the MG estimator bias by considering cointegrated systems, which are in fact the most common case in empirical applications. Second, I further investigate the possibility of using bootstrap techniques to correct the bias of the MG estimator, moving from the standard bootstrap to a more sophisticated design according to the suggestions of Li and Maddala (1997), i.e. using the moving block bootstrap and considering that if  $x_{it}$  is an  $I(1)$  process, bootstrapping the two innovations that drive the cointegrated system simultaneously is convenient.

The Monte Carlo analysis confirms the existence of a downward bias in MG estimates of the long-run coefficient in a cointegrated system. In particular, the bias diminishes as the number of time-observations increases, and increasing the number of groups reduces the variance of the bias. These results are consistent with the hypothesis that small-sample bias plays a major role. Further, in a cointegrated system when strict exogeneity of the regressor is ruled out and the two innovations driving the cointegrated system are allowed to be correlated, the MG estimator has a very pronounced downward bias if the correlation is

negative and an upward bias if it is positive. However, I also tested a different procedure (MG-FMOLS, obtained by averaging FMOLS estimates computed applying the suggestions of Hansen, 1992): this estimator shows a much smaller bias when a non-zero correlation between the regressor and the error is allowed.

My simulations also show that the bootstrap bias-corrected estimators, based on the suggestions of Li and Maddala (1997), can produce an appreciable correction of the MG estimator bias.

Finally, I present an application of this procedure to investigate the long-run properties of money demand in the euro area. The monetary policy debate in the euro area has shown the necessity for a reliable estimate of money demand, as is attested by any number of econometric papers (among these, Monticelli and Papi, 1996; Fagan and Henry, 1998; Coenen and Vega, 1999; Dedola, Gaiotti and Silipo, 2001; Golinelli and Pastorello, 2000; Brand and Cassola, 2000).

In particular, as noted by Dedola, Gaiotti and Silipo (2001), the magnitude of income elasticity determines whether or not there is a trend in the velocity of circulation, which in turn helps determine the reference value for money growth used by the ECB as the “first pillar” of its strategy (European Central Bank, 1999a). The ECB sets this value assuming that the growth rate of real output lies in the range between 2 and 2.5 per cent and that M3 income velocity (the ratio of nominal GDP to M3 money) declines at a trend rate of 0.5 to 1 per cent a year (European Central Bank, 1999b). Assuming that the other variables included in the money demand equation are stationary in the long term, the ECB implicitly assumes that the income elasticity ranges between 1.2 and 1.5 per cent, with a central value of 1.35. This is consistent with previous studies, which use different methodologies and definitions of money: the estimated income elasticity ranges between 1.14 per cent (Coenen and Vega, 1999) and 1.55 per cent (Fagan and Henry, 1998).

The paper is organized as follows. The next section briefly describes the MG estimator in a heterogeneous dynamic model and investigates its bias for the long-run coefficients by means of a Monte Carlo experiment. Section 3 presents three bootstrap procedures and tests their ability to reduce the bias with various data generating processes. In Section 4 an

application to euro-area money demand is presented. Conclusions are set out in the final section.

## 2. Mean Group Estimator and its Bias

### 2.1 Mean Group Estimator

Consider the following heterogeneous dynamic model, extensively analyzed by Pesaran and Smith (1995) and Pesaran and Zhao (1999):

$$y_{i,t} = \mathbf{a}_i + \mathbf{l}_i y_{i,t-1} + \mathbf{b}_i x_{i,t} + \mathbf{e}_{i,t}, \quad i = 1, 2, \dots, N, t = 1, 2, \dots, T \quad (1.1)$$

where  $i$  denotes groups and  $t$  is the time index,  $\mathbf{e}_{i,t}$  is assumed to be independently and identically distributed with mean zero and variance  $\mathbf{s}_i^2$ , and  $\mathbf{e}_{i,t}$  is independent of  $\mathbf{a}_i$ ,  $\mathbf{b}_i$ ,  $\mathbf{l}_i$  and  $x_{i,t}$ .

The mean group estimator is based on individual group estimates. For the  $i$ -th group, the estimate of the long-run coefficient is given by:

$$\hat{\mathbf{q}}_i = \hat{\mathbf{b}}_i / (1 - \hat{\mathbf{l}}_i), \quad i = 1, 2, \dots, N \quad (1.2)$$

and the speed of adjustment is given by:

$$\hat{\mathbf{f}}_i = (\hat{\mathbf{l}}_i - 1) \quad i = 1, 2, \dots, N \quad (1.2')$$

where  $\hat{\mathbf{l}}_i$  and  $\hat{\mathbf{b}}_i$  are the OLS estimates of  $\mathbf{l}_i$  and  $\mathbf{b}_i$  respectively.

The mean group estimator of  $\mathbf{q} = E(\mathbf{q}_i) = E(\mathbf{b}_i / (1 - \mathbf{l}_i))$  is given by:

$$\hat{\mathbf{q}}_{MG} = \frac{1}{N} \sum_{i=1}^N (\hat{\mathbf{b}}_i / (1 - \hat{\mathbf{l}}_i)), \quad (1.3)$$

with its variance consistently estimated by:



$$\text{Var}(\hat{\mathbf{q}}_{MG}) = \frac{1}{N(N-1)} \sum_{i=1}^N (\hat{\mathbf{q}}_i - \hat{\mathbf{q}}_{MG})^2 \quad (1.4)$$

The Mean Group estimator and its variance for the speed of adjustment can be computed analogously.

Theoretically  $\hat{\mathbf{q}}_{MG}$  converges on the true  $\mathbf{q}$  as both T and N go to infinity. However, the estimator can be biased, for three reasons:

- when T is small, the presence of the lagged dependent variable, which biases the OLS estimator of the coefficients of  $\hat{\mathbf{I}}_i$  and  $\hat{\mathbf{b}}_i$  (Pesaran and Zhao, 1999);
- the fact that  $\hat{\mathbf{q}}_i$  is a nonlinear combination of  $\hat{\mathbf{I}}_i$  and  $\hat{\mathbf{b}}_i$  (Pesaran and Zhao, 1999);
- the group-specific estimates of  $\hat{\mathbf{I}}_i$  and  $\hat{\mathbf{b}}_i$  may be biased because of sample-specific omitted variables or measurement errors that are correlated with the regressors (Pesaran, Shin and Smith, 1999).

## 2.2 The Data Generating Process and Mean Group Estimator

In order to evaluate the characteristics of the MG estimator bias in a cointegrating system, I adapt the data generating process used by Pesaran and Zhao (1999) to the nonstationary case. This process (DGP1) allows for parameter heterogeneity across the different groups:

$$y_{i,t} = \mathbf{a}_i + \mathbf{I} y_{i,t-1} + (\mathbf{1} - \mathbf{I}) \mathbf{q}_i x_{i,t} + \mathbf{e}_{i,t} \quad i = 1, 2, \dots, N, t = 1, 2, \dots, T \quad (2.1)$$

where

$$x_{i,t} = x_{i,t-1} + \mathbf{u}_{i,t} \quad i = 1, 2, \dots, N, t = 1, 2, \dots, T \quad (2.2)$$

In each experiment, the disturbances and the parameters are generated according to

$$\begin{aligned}
\mathbf{u}_{i,t} &\sim N(0, \mathbf{t}_i^2), \quad \mathbf{e}_{i,t} \sim N(0,1) & i = 1, 2, \dots, N, \quad t = 1, 2, \dots, T \\
\mathbf{a}_i &\sim N(1,1), \mathbf{q}_i \sim N(1,1), \mathbf{m}_i \sim N(1,1) & i = 1, 2, \dots, N.
\end{aligned} \tag{2.3}$$

As in Kiviet (1995) and Pesaran and Zhao (1999), the values of  $\mathbf{t}_i^2$  across  $i$  are generated imposing the value of the signal-to-noise ratio  $\frac{\mathbf{s}_s^2}{\mathbf{s}_e^2} = \frac{R^2}{1-R^2} = \frac{\text{Var}(\mathbf{y}_t) - \mathbf{s}_e^2}{\mathbf{s}_e^2}$  where  $R^2$  is the population value of the squared multiple coefficient of (2.1). Without loss of generality I set  $\mathbf{s}_e^2 = 1$ , so it is easily seen that  $\mathbf{s}_s^2$  is equal to:

$$\mathbf{s}_s^2 = \left( \frac{\mathbf{I}^2}{1 - \mathbf{I}^2} \right) + \mathbf{q}_i^2 \text{Var}(x_{it}). \tag{2.4}$$

Since  $\text{Var}(x_{it})$  grows as  $\mathbf{t}_i^2 T$ , by inverting (2.4) we can compute

$$\mathbf{t}_i^2 = \frac{\left( \mathbf{s}_s^2 - \frac{\mathbf{I}^2}{1 - \mathbf{I}^2} \right)}{\mathbf{q}_i^2 T}.$$

In the simulations, I set  $\mathbf{I} = 0.8$  and  $\mathbf{s}_s^2 = 2$ , which Pesaran and Zhao (1999) show to be the case with the largest bias for the MG estimator when the  $x_{it}$  are stationary and for  $T=N=20$ . In fact, they show that the choice of  $\mathbf{s}_s^2 = 8$  reduces the bias appreciably. It is worth noting that  $\mathbf{s}_s^2 = 2$  is equivalent to an  $R^2$  of 0.67, while  $\mathbf{s}_s^2 = 8$  is equivalent to an  $R^2$  of 0.89.

It is important to note that in DGP1 I retain the assumption that the  $x_{it}$  are strictly exogenous. In the standard case discussed in the time-series literature on cointegrated systems, however, the dependence of  $x_{it}$  on  $\varepsilon_{it}$  is not ruled out. I will remove this assumption later, in sub-section 3.3.

I experimented with  $T = (10,20,50,100)$ ;  $N=(10,20,50,100)$ . I first generated  $T+50$  observations for  $x_i$  and  $y_i$  (with  $x_{i,0} = 0$  and  $y_{i,0} = 0$ ) and then dropped the first 50 observations for each  $i$ . Only replications yielding a stable estimate of  $\lambda$  (namely those with  $|\lambda| < .99$ ) are included in the experiments.

Table 1

**Simulation Results for the Bias and the RMSE of Mean Group Estimator**  
**DGP2 (2.1')-(2.4'): 1000 Monte Carlo replications**

Groups		10		20		50		100	
		$\lambda$	$\theta$	$\lambda$	$\theta$	$\lambda$	$\theta$	$\lambda$	$\theta$
Observations	Bias	-0.451	-0.142	-0.454	-0.252	-0.451	-0.121	-0.451	-0.185
	St. Dev.	0.109	7.759	0.074	6.159	0.049	3.634	0.034	2.434
	RMSE	0.464	7.756	0.460	6.161	0.454	3.634	0.452	2.439
	M	269		597		1433		3029	
10	Bias	-0.247	0.201	-0.245	-0.218	-0.244	-0.09	-0.246	-0.126
	St. Dev.	0.067	5.447	0.048	2.908	0.031	2.144	0.021	1.466
	RMSE	0.256	5.447	0.249	2.915	0.246	2.144	0.247	1.470
	M	53		115		299		589	
20	Bias	-0.105	-0.112	-0.103	-0.132	-0.104	-0.069	-0.103	-0.067
	St. Dev.	0.037	2.174	0.026	1.600	0.016	0.988	0.012	0.697
	RMSE	0.111	2.175	0.106	1.605	0.105	0.990	0.104	0.700
	M	1		4		3		3	
50	Bias	-0.052	-0.120	-0.052	-0.044	-0.052	-0.002	-0.052	0.014
	St. Dev.	0.023	1.558	0.016	1.103	0.010	0.669	0.007	0.458
	RMSE	0.057	1.562	0.055	1.103	0.053	0.668	0.053	0.458
	M	0		0		0		0	
100	Bias	-0.052	-0.120	-0.052	-0.044	-0.052	-0.002	-0.052	0.014
	St. Dev.	0.023	1.558	0.016	1.103	0.010	0.669	0.007	0.458
	RMSE	0.057	1.562	0.055	1.103	0.053	0.668	0.053	0.458
	M	0		0		0		0	

The results, summarized in Table 1, are based on 1000 replications and were computed using GAUSS. The following general conclusions may be drawn:

- The average bias for  $\lambda$  depends solely on the number of observations; it decreases from -.45 for  $T=10$  to -.05 for  $T=100$ . The standard deviation of the bias for  $\lambda$  tends to diminish with the increase in the number of groups; when  $N=100$  the standard deviation is about one fifth as large as when  $N=10$ .
- The bias for  $\theta$  is substantially smaller than that for  $\lambda$  when  $T=(10,20)$ , whereas the two biases are comparable when  $T=(50,10)$ . The standard deviation of the bias for  $\theta$  is much higher than that for  $\lambda$ ; it tends to diminish, as the number of groups increases, faster than in the case of  $\lambda$ .

- The number of cases where the absolute value of the estimates of  $\lambda$  was greater than 0.99 (denoted as M in the tables) is not negligible for T=10 (approximately 0.3 per cent of the cases) or for T=20 (approximately 0.03 per cent of the cases). For T=50,100 it is practically nil.
- Finally, in unreported simulations, I compared these results with those obtained when x is I(0).<sup>2</sup> The magnitude of the bias turned out to be very similar for the two sets of simulations. However, the standard deviation of the bias for  $\theta$  is substantially lower when x is I(0) than when it is I(1), while those for  $\lambda$  are similar for the two sets of simulations.

In conclusion, the results show that the MG estimate of the long-run parameter is downward biased, especially when T<100, in cointegrated systems where the signal-to-noise ratio is kept constant and low, the regressor is strictly exogenous, and the true coefficient of the lagged dependent variable is small. The increase in the number of groups has a limited effect on the bias, but it does reduce its standard deviation. These results are consistent with the hypothesis that the major source of bias is small sample size.

### 3. Bootstrap Bias-corrected Estimators

Bootstrap methods can be used to make the bias correction, in particular for pivotal statistics (Li and Maddala, 1996a). As reported in Pesaran and Zhao (1999), Kiviet noted that  $\left(\hat{\mathbf{q}}_i - \mathbf{q}\right)$  is asymptotically pivotal; it can then be shown that the bootstrap bias correction will also lead to an estimator which is unbiased to order  $O(T^{-1})$ .

In the Monte Carlo experiments, I use the DGP1 presented in sub-section 2.2 and focus on two cases: (T=20,N=20), which was examined by Pesaran and Zhao (1999); and (T=65, N=11), which is consistent with the information in the euro-area money demand data-set. For each case, I test three different bootstrap procedures. Below, I illustrate the 3 procedures.

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<sup>2</sup> In particular I used the DGP described in Pesaran and Zhao (1999, pp. 312-313), which is different from DGP1 here because the regressor is stationary:  $x_{i,t} = \mathbf{m}_i(1 - \mathbf{r}) + \mathbf{r}x_{i,t-1} + u_{i,t}$  with  $\mathbf{r} = 0.95$ . I experimented with T = (10,20,50,100); N=(10,20,50,100), while Pesaran and Zhao (1999) simulations were focussed on T=N=20.

Pesaran and Zhao (1999) proposed a standard bootstrap bias-corrected (BSBC1) estimator designed in the following manner:

**Procedure 1 (BSBC1)**

- 1) compute the OLS estimates  $\hat{\mathbf{a}}_i, \hat{\mathbf{I}}_i$  and  $\hat{\mathbf{b}}_i$  from equation (2.1), as well as the long-run coefficients estimates  $\hat{\mathbf{q}}_i$ . The Mean Group estimate for the parameter  $\mathbf{q}$  is given by  $\hat{\mathbf{q}}_{MG} = \frac{1}{N-M} \sum_{i=1}^{N-M} \left( \hat{\mathbf{b}}_i / (1 - \hat{\mathbf{I}}_i) \right)$ , by excluding the M cases where the absolute value of  $\hat{\mathbf{I}}_i$  is greater than .99;
- 2) for the j-th bootstrap replication, generate bootstrap samples  $\hat{\mathbf{e}}_{i,t}^j, i = 1, 2, \dots, N; t = 1, 2, \dots, T$  by drawing randomly with replacement from the OLS residuals  $\hat{\mathbf{e}}_{i,t}$  of equation (2.1);
- 3) for the j-th bootstrap replication, generate bootstrap samples  $\mathbf{y}_{i,t}^j$  using  $\mathbf{y}_{i,t}^j = \hat{\mathbf{a}}_i + \hat{\mathbf{I}}_i \mathbf{y}_{i,t-1}^j + \hat{\mathbf{b}}_i \mathbf{x}_{i,t} + \hat{\mathbf{e}}_{i,t}^j, i=1, i = 1, 2, \dots, N; t = 1, 2, \dots, T; j = 1, 2, \dots, B$  where  $\mathbf{y}_{i,0}^j = \mathbf{y}_{i,0}, i=1, 2, \dots, N$ ;
- 4) for the j-th bootstrap replication, use  $\mathbf{y}_{i,t}^j$  and the original observations  $\mathbf{x}_{i,t}$  to compute the OLS estimates  $\hat{\mathbf{a}}_i^j, \hat{\mathbf{I}}_i^j$  and  $\hat{\mathbf{b}}_i^j$ , as well as  $\hat{\mathbf{q}}_i^j$ ;
- 5) repeat steps 2) through 4) B times;
- 6) compute the bootstrap estimates  $\hat{\mathbf{q}}_{B1} = \frac{1}{(N - \bar{M}_i)B} \sum_{j=1}^B \sum_{i=1}^{N-\bar{M}_i} \hat{\mathbf{q}}_i^j$ , by excluding the  $\bar{M}_i$  cases where  $\hat{\mathbf{I}}_i^j$  is greater than .99 in absolute value<sup>3</sup>. The bias-corrected estimator is then given by:  $\hat{\mathbf{q}}_{BSBC1} = 2 * \hat{\mathbf{q}}_{MG} - \hat{\mathbf{q}}_{B1}$ .

The possibility of using the bootstrap for bias correction in cointegrated systems was investigated by Li and Maddala (1997). They applied bootstrap methods to different asymptotic procedures that correct for endogeneity and serial correlation in a cointegrating

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<sup>3</sup> Pesaran and Zhao (1999) explicitly exclude Monte Carlo replications where  $\hat{\mathbf{I}}_i$  is greater than .99 in absolute value from the MG estimator. However, they do not specify whether or not they exclude from the computation of the bootstrap bias-corrected estimates the bootstrap replications where  $\hat{\mathbf{I}}_i^j$  is greater than .99 in absolute value. Their results show an abnormal increase of the RMSE of the bias for the bootstrap bias-corrected estimator; therefore, I infer that they did not exclude such bootstrap replications. If this is the case, the algorithm I use is different from theirs.

regression. For the Philips and Hansen (1990) fully modified OLS (FMOLS) estimates, they found that bootstrap procedures reduce the bias to some degree. In particular, they adopted a bootstrap procedure that differs from standard bootstrap in two ways:

- a) to use the information that  $x_{i,t}$  is I(1), they define  $\hat{n}_{i,t} = \Delta x_{i,t}$  and, after centering, bootstrap the pairs  $(\hat{e}_{i,t}, \hat{n}_{i,t})$ ;
- b) since the errors driving cointegrated systems are typically autocorrelated and of unknown structure, they use the moving block bootstrap (MBB) (Künsch, 1989, and Liu and Singh, 1992). With this method, the T observations are divided into T-k+1 overlapping blocks of length K, and b=T/K of these blocks (with repeats allowed) are selected. Their simulation results show that this method works well, although as they comment “the theoretical justification for the MBB bootstrap is extremely (almost impossibly) complicated”.

Therefore, the second method I use is exactly the Li and Maddala procedure (BSBC2). Namely:

### Procedure 2 (BSBC2)

- A) compute the FMOLS estimates  $\hat{q}_i$  from equation:  $y_{i,t} = g_i + q_i x_{i,t} + e_{i,t}$ . The Mean Group estimator is given by  $\hat{q}_{MGFMOLS} = \frac{1}{N} \sum_{i=1}^N \hat{q}_i$ ;
- B) for the j-th bootstrap replication, calculate the FMOLS residuals  $\hat{e}_{i,t}$  and the set of residuals  $\hat{n}_{i,t} = \Delta x_{i,t}$ . After centering these residuals, form the residual moving block pairs of length k  $\{\hat{e}_{i,t}, \dots, \hat{e}_{i,t+k-1}; \hat{n}_{i,t}, \dots, \hat{n}_{i,t+k-1}\}$ ,  $t = 1, 2, \dots, T-k+1$ . Draw  $b=T/k$  blocks  $\{\hat{e}_{i,t}^{jb}, \dots, \hat{e}_{i,t+k-1}^{jb}; \hat{n}_{i,t}^{jb}, \dots, \hat{n}_{i,t+k-1}^{jb}\}$ ,  $jb=1, 2, \dots, b$  randomly with replacement from the residual moving block pairs to obtain  $\hat{e}_{i,t}^j$  and  $\hat{n}_{i,t}^j$ ,  $i = 1, 2, \dots, N$ ;  $t = 1, 2, \dots, T$ ;  $j = 1, 2, \dots, B$ ;
- C) for the j-th bootstrap replication, generate bootstrap samples  $x_{i,t}^j$  and  $y_{i,t}^j$ , using, respectively,  $x_{i,t}^j = x_{i,t-1}^j + \hat{n}_{i,t}^j$  and  $y_{i,t}^j = g_i + b_i x_{i,t}^j + \hat{e}_{i,t}^j$ ,  $i = 1, 2, \dots, N$ ;  $t = 1, 2, \dots, T$ ;  $j = 1, 2, \dots, B$  where  $x_{i,0}^j = x_{i,0}$ ,  $i=1, 2, \dots, N$ ;
- D) for the j-th bootstrap replication, use  $y_{i,t}^j$  and  $x_{i,t}^j$  to compute the FMOLS estimates  $\hat{q}_i^j$ ;

E) repeat steps B) through D) B times;

F) compute the bootstrap estimates  $\hat{\mathbf{q}}_{B2} = \frac{1}{NB} \sum_{j=1}^B \sum_{i=1}^N \hat{\mathbf{q}}_i^j$ . The bias-corrected estimator is

then given by:  $\hat{\mathbf{q}}_{BSBC2} = 2 * \hat{\mathbf{q}}_{MGFMOLS} - \hat{\mathbf{q}}_{B2}$ .

Finally, I test a third bootstrap method (BSBC3). In this case I apply the Li and Maddala suggestions reported above as points a) and b) to bootstrap the Mean Group estimates of  $\lambda$  and  $\theta$  in a heterogeneous dynamic model. Therefore, the bootstrap scheme is the following:

### Procedure 3 (BSBC3)

i) compute the OLS estimates  $\hat{\mathbf{a}}_i$ ,  $\hat{\mathbf{I}}_i$  and  $\hat{\mathbf{b}}_i$  from equation (2.1), as well as the long-run coefficient estimates  $\hat{\mathbf{q}}_i$ . The Mean Group estimate for the parameter  $\mathbf{q}$  is given by

$\hat{\mathbf{q}}_{MG} = \frac{1}{N-M} \sum_{i=1}^{N-M} \left( \hat{\mathbf{b}}_i / (1 - \hat{\mathbf{I}}_i) \right)$ , by excluding the M cases where  $\hat{\mathbf{I}}_i$  is greater than .99 in absolute value;

ii) for the j-th bootstrap replication, calculate the OLS residuals  $\hat{\mathbf{e}}_{i,t}$  and the set of residuals  $\hat{\mathbf{n}}_{i,t} = \Delta \mathbf{x}_{i,t}$ . After centering these residuals, form the residual moving block pairs of length k  $\{\hat{\mathbf{e}}_{i,t}, \dots, \hat{\mathbf{e}}_{i,t+k-1}; \hat{\mathbf{n}}_{i,t}, \dots, \hat{\mathbf{n}}_{i,t+k-1}\}$ ,  $t = 1, 2, \dots, T-k+1$ . Draw  $b=T/k$  blocks  $\{\hat{\mathbf{e}}_{i,t}^{jb}, \dots, \hat{\mathbf{e}}_{i,t+k-1}^{jb}; \hat{\mathbf{n}}_{i,t}^{jb}, \dots, \hat{\mathbf{n}}_{i,t+k-1}^{jb}\}$ ,  $jb=1, 2, \dots, b$  randomly with replacement from the residual moving block pairs to obtain  $\hat{\mathbf{e}}_{i,t}^j$  and  $\hat{\mathbf{n}}_{i,t}^j$ ,  $i = 1, 2, \dots, N$ ;  $t = 1, 2, \dots, T$ ;  $j = 1, 2, \dots, B$ ;

iii) for the j-th bootstrap replication, generate bootstrap samples  $\mathbf{x}_{i,t}^j$  and  $\mathbf{y}_{i,t}^j$  using, respectively,  $\mathbf{x}_{i,t}^j = \mathbf{x}_{i,t-1}^j + \hat{\mathbf{n}}_{i,t}^j$  and  $\mathbf{y}_{i,t}^j = \hat{\mathbf{a}}_i + \hat{\mathbf{I}}_i \mathbf{y}_{i,t-1}^j + \hat{\mathbf{b}}_i \mathbf{x}_{i,t}^j + \hat{\mathbf{e}}_{i,t}^j$ ,  $i=1, 2, \dots, N$ ;  $t = 1, 2, \dots, T$ ;  $j = 1, 2, \dots, B$  where  $\mathbf{x}_{i,0}^j = \mathbf{x}_{i,0}$  and  $\mathbf{y}_{i,0}^j = \mathbf{y}_{i,0}$   $i=1, 2, \dots, N$ ;

iv) for the j-th bootstrap replication, use  $\mathbf{y}_{i,t}^j$  and  $\mathbf{x}_{i,t}^j$  to compute the OLS estimates

$\hat{\mathbf{a}}_i^j$ ,  $\hat{\mathbf{I}}_i^j$  and  $\hat{\mathbf{b}}_i^j$ , as well as  $\hat{\mathbf{q}}_i^j$ ;

v) repeat steps ii) through iv) B times;

vi) compute the bootstrap estimates  $\hat{\mathbf{q}}_{B3} = \frac{1}{(N - \overline{M}_i)B} \sum_{j=1}^B \sum_{i=1}^{N-\overline{M}_i} \hat{\mathbf{q}}_i^j$ , by excluding the  $\overline{M}_i$  cases where  $\hat{\mathbf{I}}_i^j$  is greater than .99 in absolute value. The bias-corrected estimator is then given by:  $\hat{\mathbf{q}}_{BSBC3} = 2 * \hat{\mathbf{q}}_{MG} - \hat{\mathbf{q}}_{B3}$ .

### 3.1 The simulation results

The simulation results are summarized in Table 2 and were computed using GAUSS. They are based on 1000 Monte Carlo replications, and the number of bootstrap replications B is set equal to 200.

The FMOLS estimates were computed by using a prewhitened kernel estimator (specifically the Quadratic spectral kernel recommended by Andrews; 1991) with the plug-in bandwidth recommended by Andrews and Monahan (1992). According to Hansen (1992), the use of the plug-in bandwidth parameter eliminates the arbitrariness of the choice and can dramatically improve the estimates of cointegrating relationships. I used a Gauss code prepared by Hansen (available at the web page [http://www.ssc.wisc.edu/~bhansen/progs/jbes\\_92.html](http://www.ssc.wisc.edu/~bhansen/progs/jbes_92.html)).

The block length in BSBC2 and BSBC3 is set equal to one fifth of T. As noted by Berkowitz and Kilian (2001), choosing a block length involves a tradeoff. As the block size becomes too small, the moving block bootstrap destroys the time dependency of the data which is the reason why MBB is believed to improve over the standard bootstrap. As the block size becomes too large, pseudo-data will tend to look alike. Several procedures have been proposed to set the block size automatically (see the discussion in Li and Maddala, 1996, and Berkowitz and Kilian, 2001). The block length chosen here is the same as in Li and Maddala (1997) and is consistent with the results in Berkowitz and Kilian (2001).

The following general conclusions may be drawn from these results:

- The MG-FMOLS estimator bias (computed by averaging the FMOLS estimates for the long-run parameter  $\theta$  over the groups) is greater than that of the standard MG estimator (computed by averaging the OLS estimates of the dynamic model over the groups).



- The bootstrap bias-corrected estimators significantly reduce the downward bias of the Mean Group estimators. In particular, the bias reduction is more effective when the Li and Maddala (1997) guidelines are applied. However, the bias reduction is associated with a slight increase in both standard error and RMSE.
- The bootstrap bias-corrected estimator applied in a dynamic model (BSBC3) shows the smallest bias for the long-run coefficient  $\theta$ .

Table 2

<b>Simulation Results for the Bias of Mean Group Estimators And Bootstrap Bias-corrected Estimators</b>						
<b>DGP1 (2.1-2.4); 1000 Monte Carlo replications; 200 Bootstrap replications</b>						
		<b>BSBC1</b>		<b>BSBC2</b>	<b>BSBC3</b>	
		<b>(1)-(6)</b>		<b>(A)-(F)</b>	<b>(i)-(vi)</b>	
		$\lambda$	$\theta$	$\theta$	$\lambda$	$\theta$
<b>Panel A: N=20, T=20</b>						
Mean Group Estimator	Bias	-0.245	-0.218	-0.334	-0.245	-0.218
	St. Dev.	0.048	2.908	4.318	0.048	2.908
	RMSE	0.249	2.915	4.329	0.249	2.915
Bootstrap Bias-corrected Estimator	Bias	-0.084	-0.184	-0.225	-0.106	-0.108
	St. Dev.	0.056	4.040	4.994	0.057	3.937
	RMSE	0.101	4.042	4.996	0.120	3.937
<b>Panel B: N=11, T=65</b>						
Mean Group Estimator	Bias	-0.082	-0.124	-0.201	-0.082	-0.124
	St. Dev.	0.030	1.825	1.895	0.030	1.825
	RMSE	0.087	1.828	1.904	0.087	1.828
Bootstrap Bias-corrected Estimator	Bias	-0.011	-0.092	-0.154	-0.023	-0.067
	St. Dev.	0.033	1.942	2.098	0.034	2.096
	RMSE	0.034	1.943	2.102	0.041	2.096

### 3.2 A further investigation when $x_{it}$ is not strictly exogenous

As was discussed in sub-section 2.2, in DGP1 the  $x_{it}$  are considered as strictly exogenous, thus ruling out the possible dependence of  $x_{it}$  on  $\varepsilon_{it}$  (for some  $t$ ). I now introduce a second Data Generating Process (DGP2) that, as is standard in the time-series literature on cointegrated variables, allows for a non-zero correlation between the two sets of residuals  $u_{it}$  and  $\varepsilon_{it}$ .

To illustrate the cases under investigation, I use the notation of cointegrated systems used by Li and Maddala (1997):

$$y_{i,t} = \mathbf{q}_i x_{i,t} + \mathbf{w}_{i,t} \quad i = 1, 2, \dots, N, t = 1, 2, \dots, T \quad (3.1)$$

where

$$x_{i,t} = x_{i,t-1} + \mathbf{n}_{i,t} \quad i = 1, 2, \dots, N, t = 1, 2, \dots, T \quad (3.2)$$

The DGP2 for the cointegrated system (3.1) and (3.2) posits that  $\eta_{i,t} = (\omega_{i,t}, \nu_{i,t})'$  follows for each group  $i$  a stationary VAR(1) process

$$\begin{pmatrix} \mathbf{w}_{i,t} \\ \mathbf{u}_{i,t} \end{pmatrix} = \mathbf{j}_i \begin{pmatrix} \mathbf{w}_{i,t-1} \\ \mathbf{u}_{i,t-1} \end{pmatrix} + \begin{pmatrix} \mathbf{e}_{i,t} \\ \mathbf{u}_{i,t} \end{pmatrix} \quad (3.3)$$

where

$$\mathbf{j}_i = \begin{pmatrix} \mathbf{f}_{i,11} & \mathbf{f}_{i,12} \\ \mathbf{f}_{i,21} & \mathbf{f}_{i,22} \end{pmatrix} \text{ and } \begin{pmatrix} \mathbf{e}_{i,t} \\ \mathbf{u}_{i,t} \end{pmatrix} \sim IIDN(0, E) \equiv IIDN \left( \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \begin{pmatrix} \mathbf{S}_{i,e}^2 & \mathbf{S}_{i,eu} \\ \mathbf{S}_{i,eu} & \mathbf{S}_{i,u}^2 \end{pmatrix} \right). \quad (3.4)$$

It is assumed that  $E > 0$ , a positive definite matrix. In the simulation, I set:

$$\mathbf{j}_i = \begin{pmatrix} .8 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } E = \begin{pmatrix} 1 & k \\ k & \mathbf{S}_{i,u}^2 \end{pmatrix}.$$

The free parameter  $K$  is set equal to  $(-.5, 0, .5)$ . It is worth noting that when  $u_{it}$  and  $\varepsilon_{it}$  have zero correlation ( $K=0$ ) we are in the same case as DGP1 (namely,  $x_{it}$  is strictly exogenous). In all cases, the true cointegrating parameter is chosen as  $\mathbf{q}_i \sim N(1, 1)$   $i = 1, 2, \dots, N$ . For the sake of brevity I consider only the case where  $N=11$  and  $T=65$ .

<b>Simulation Results for the Bias of Mean Group Estimators</b>							
<b>And Bootstrap Bias-corrected Estimators</b>							
<b>DGP2 (3.1)-(3.4); N=11,T=65; 1000 Monte Carlo replications; 200 Bootstrap replications</b>							
		<b>BSBC1</b>		<b>BSBC2</b>		<b>BSBC3</b>	
		<b>(1)-(6)</b>		<b>(A)-(F)</b>		<b>(i)-(vi)</b>	
		$\lambda$	$\theta$	$\theta$		$\lambda$	$\theta$
		.....		.....		.....	
<b>Panel A: <math>S_{i,\mu}^2</math> set according to eq. (2.4)</b>							
<b>K=-.5</b>							
Mean Group Estimator	Bias	-0.042	-0.505	-0.164	-0.042	-0.505	
	St. Dev.	0.058	0.202	0.144	0.058	0.202	
	RMSE	0.072	0.543	0.218	0.072	0.543	
Bootstrap Bias-corrected Estimator	Bias	0.020	-0.466	-0.001	0.048	-0.076	
	St. Dev.	0.061	0.219	0.164	0.054	0.238	
	RMSE	0.064	0.515	0.164	0.072	0.249	
<b>K=.5</b>							
Mean Group Estimator	Bias	-0.537	0.303	0.069	-0.537	0.303	
	St. Dev.	0.096	0.107	0.064	0.096	0.107	
	RMSE	0.546	0.321	0.094	0.546	0.321	
Bootstrap Bias-corrected Estimator	Bias	-0.508	0.304	0.002	-0.446	0.123	
	St. Dev.	0.104	0.115	0.073	0.113	0.121	
	RMSE	0.519	0.325	0.073	0.460	0.173	
<b>Panel B: <math>S_{i,\mu}^2 = 1</math></b>							
<b>K=-.5</b>							
Mean Group Estimator	Bias	-0.008	-0.074	-0.129	-0.008	-0.074	
	St. Dev.	0.032	0.100	0.099	0.032	0.100	
	RMSE	0.033	0.124	0.135	0.033	0.124	
Bootstrap Bias-corrected Estimator	Bias	0.038	-0.043	-0.034	0.026	0.011	
	St. Dev.	0.031	0.108	0.109	0.029	0.106	
	RMSE	0.049	0.116	0.089	0.039	0.107	
<b>K = 0</b>							
Mean Group Estimator	Bias	-0.048	-0.023	-0.091	-0.048	-0.023	
	St. Dev.	0.023	0.083	0.096	0.023	0.083	
	RMSE	0.053	0.086	0.105	0.053	0.086	
Bootstrap Bias-corrected Estimator	Bias	-0.005	-0.006	-0.030	-0.011	0.007	
	St. Dev.	0.023	0.086	0.104	0.025	0.093	
	RMSE	0.024	0.087	0.084	0.027	0.093	
<b>K=.5</b>							
Mean Group Estimator	Bias	-0.157	0.023	-0.036	-0.157	0.023	
	St. Dev.	0.034	0.076	0.086	0.034	0.076	
	RMSE	0.160	0.080	0.072	0.160	0.080	
Bootstrap Bias-corrected Estimator	Bias	-0.121	0.028	-0.012	-0.101	0.007	
	St. Dev.	0.037	0.080	0.093	0.037	0.085	
	RMSE	0.126	0.085	0.072	0.107	0.085	

The three bootstrap bias-corrected estimates are computed as illustrated at the beginning of this section. Again, the number of Monte Carlo replications is 1000 and that of bootstrap replications B is 200.

Panel A gives the results when  $\mathbf{S}_{i,u}^2$  is set according to eq. (2.4); they suggest the following:

- In contrast to DGP1, the MG-FMOLS estimator bias (computed by averaging the FMOLS estimates for the long-run parameter  $\theta$ ) is much smaller than that of the MG estimator (computed by averaging the OLS estimates) where  $K = (-.5,.5)$ ; this result is also confirmed in unreported simulations where  $K = (-.9,-.3,.3,.9)$ .
- As in DGP1, the bootstrap bias-corrected estimators significantly reduce the absolute value of the Mean Group estimator bias, irrespective of the value for k. In particular, the bias reduction is greater for the moving block bootstrap computed applying the guidelines of Li and Maddala (1997). The bias reduction is associated with a slight increase in both the standard error and the RMSE.
- The bootstrap bias-corrected estimator applied to the FMOLS estimates (BSBC2) shows the smallest bias for the long-run coefficient  $\theta$ .

Panel B gives the results when  $\mathbf{S}_{i,u}^2 = 1$ , which implies a much better fit of the data and is common in the cointegrated system literature. Consistent with the simulations in Pesaran and Zhao (1999), there is a dramatic bias decrease for both the MG and MG-FMOLS estimators; in particular, the two biases turn out to be very similar in size. Again, the bootstrap bias-corrected estimators significantly reduce the absolute value of the Mean Group estimator bias.

In summary, where there is a non-zero correlation between the two sets of residuals  $u_{it}$  and  $\varepsilon_{it}$ , averaging the FMOLS estimates over the groups leads to a smaller bias in cointegrated systems, but when  $x_{it}$  is strictly exogenous averaging the dynamic OLS estimates is less biased. In both cases, the application of the guidelines of Li and Maddala (1997) for bootstrapping cointegrating regressions leads to an appreciable improvement in the bootstrap's ability to reduce the bias.

#### 4. An application to euro-area money demand

I consider two alternative specifications of the euro area's money demand: from Dedola, Gaiotti and Silipo (DGS, 2001), and from Coenen and Vega (CV, 1999). A comparison of these two models is well beyond the scope of this paper; a detailed discussion is presented in Dedola, Gaiotti and Silipo (2001).

Both models make real money a function of real output in the long run. However, the models differ in their treatment of interest rates; in particular, DGS includes in the long run relationship the own rate of money, which is expressed as a differential with respect to either the short-term or long-term market rate:

$$rm_{i,t} = \mathbf{b}_{1,i} y_{i,t} + \mathbf{b}_{2,i} sl_{i,t} + \mathbf{b}_{3,i} ll_{i,t} \quad (4.1)$$

where  $rm$  is the logarithm of real M3 money,  $y$  is the logarithm of seasonally adjusted real output,  $sl$  is the spread between the short-term money market rate and the own rate of money (short-term opportunity cost),  $ll$  is the spread between the long-term money market rate and the own rate of money (long-term opportunity cost).

The Coenen and Vega model includes in the long run relationship the spread between the long-term and short-term market rates, and the rate of inflation (4.2).

$$rm_{i,t} = \mathbf{b}_{1,i} y_{i,t} + \mathbf{b}_{3,i} sp_{i,t} + \mathbf{b}_{3,i} inf_{i,t} \quad (4.2)$$

where  $sp$  is the spread between the long-term and short-term money market rate (yield curve steepness), and  $inf$  is the annualized quarterly inflation rate.

I have data for 11 countries (N=11) and for the whole area; the time span ranges from 1983q1 to 1999q1 (T=65). A full description of the data is given in Dedola, Gaiotti and Silipo (2001); they also performed unit root tests, where the ADF statistics show that almost all the variables included in the two models are I(1).

Tables 4 and 5 report in Panel A the long-run elasticities for the individual country and the area-wide equation for both models obtained using FMOLS estimates. These estimates were computed applying the suggestions of Hansen (1992) cited in subsection 3.1. Preliminarily, I removed the effects of outliers and seasonal factors from the dependent

variable by using a two-stage procedure: in the first stage, I ran an OLS estimate with the 3 variables and quarterly seasonal dummies as regressors and identified as outliers the observations estimated with an error greater in absolute value than 3 times the standard error of the regression; in the second stage I included in the OLS regression the additional dummy variables for the outliers identified.<sup>4</sup>

The last column of Panel A reports the  $L_c$  test proposed by Hansen (1992) in order to test parameter stability in the context of fully modified estimation of cointegrated regression models when the likelihood of parameter variation is relatively constant throughout the sample. The  $L_c$  test indicates that for the DGS model there is evidence of parameter instability for four equations (Austria, Germany, Ireland and Portugal), and five for the CV model (Austria, Germany, Luxembourg, Netherlands and Portugal).

The individual country equations allow us to draw two general conclusions. First, the estimates of income elasticity are relatively robust and not too different across countries. Second, the effect of interest rates is quite difficult to capture with precision and there is a high variability across countries.

In the DGS model (Table 4) the long-run income elasticity ranges from .49 in Finland to 1.87 in Belgium with an average of 1.33 (considering only the 7 equations where parameter stability is accepted, the average is equal to 1.35). Using aggregate time-series regression the long-run income elasticity is identical (1.35).

In the CV model (Table 5) the long-run income elasticity ranges from .76 in Italy to 2.05 in Belgium with an average of 1.47 (considering only the 6 equations where parameter stability is accepted, the average is equal to 1.57). Using aggregate time-series regression, the long-run income elasticity is slightly smaller (1.38).

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<sup>4</sup> In unreported estimates, I checked that the results obtained without performing the preliminary estimates of the effects of outliers and seasonal factors are in fact similar to what is presented in Tables 4-5.

Table 4

**EURO AREA MONEY DEMAND LONG-RUN EFFECTS (DGS MODEL)**

Standard errors are reported in italics. The symbol \*\*\* indicates a significance level of 1 per cent or less; \*\* between 1 and 5 per cent; \* between 5 and 10 per cent. In panel A, the Mean Group Estimator is obtained by averaging the FMOLS coefficients across countries. The Bootstrap Bias corrected Estimator is calculated according to the procedure BSBC2 described in Section 3. The block length is equal to 13 observations. Outliers and seasonal factors are eliminated from the dependent variables by using a two-stage procedure: in the first stage, an OLS estimate is run with the 3 variables and quarterly seasonal dummies and outliers are identified as the observations estimated with an error greater in absolute value than 3 times the standard error of the regression; in the second stage, the additional dummy variables for the outliers identified are included in the OLS regressions. The last column of Panel A reports the  $L_c$  test proposed by Hansen (1992) in order to test parameter stability in the context of fully modified estimation of cointegrated regression models. In panel B, the Mean Group Estimator is obtained by averaging the coefficients across countries where the equation is stable (namely those where the speed of adjustment is  $< -0.01$ ). Again, the effects of outliers are eliminated by using the same procedure outlined above. The Bootstrap Bias corrected Estimator is calculated according to the procedure BSBC3 described in Section 3. The block length is equal to 13 observations.

<b>Panel A: FMOLS ESTIMATES</b>									
Country	Real GDP		Short-term opportunity cost		Long-term opportunity cost		Test for parameter instability		
<b>Individual Country Equation</b>									
Austria	1.29	(0.05) ***	-0.94	(0.41) **	2.80	(1.03) ***	0.87 **		
Belgium	1.87	(0.07) ***	0.97	(0.47) **	-3.15	(0.58) ***	0.33		
Finland	0.49	(0.49)	-7.60	(1.86) ***	-3.97	(2.88)	0.61		
France	1.55	(0.10) ***	2.46	(0.42) ***	-1.57	(0.74) **	0.41		
Germany	1.32	(0.04) ***	-3.26	(0.43) ***	-0.98	(0.63)	1.75 ***		
Ireland	1.36	(0.18) ***	-15.57	(4.94) ***	0.21	(2.95)	1.26 ***		
Italy	1.01	(0.20) ***	-3.00	(1.71) *	4.42	(1.55) ***	0.13		
Luxembourg	1.20	(0.10) ***	1.45	(1.61)	-3.13	(2.51)	0.46		
Netherlands	1.65	(0.04) ***	2.13	(0.39) ***	-1.48	(0.73) **	0.29		
Portugal	1.23	(0.18) ***	-4.62	(1.50) ***	-1.09	(1.59)	6.88 ***		
Spain	1.67	(0.09) ***	-1.19	(0.42) ***	1.62	(0.70) **	0.54		
Area-wide equation	1.35	(0.03) ***	0.01	(0.28)	-0.33	(0.39)	0.31		
<b>MG Estimator and Bootstrap Bias corrected Estimator</b>									
MG Estimator	1.33		-2.65		-0.58				
t-percentile confidence intervals									
1 per cent	1.22	1.82	-9.92	-2.00	-1.99	0.54			
5 per cent	1.31	1.71	-8.60	-2.38	-1.77	0.29			
10 per cent	1.34	1.68	-7.76	-2.62	-1.62	0.16			
BSBC Estimator	1.70		-3.76		-0.65				
<b>Panel B: OLS ESTIMATES</b>									
	Real GDP		Short-term opportunity cost		Long-term opportunity cost		Speed of adjustment		
Area-wide equation	1.29	(2.05) ***	0.72	(0.46)	-2.64	(0.87) ***	-0.15 (0.04) ***		
<b>MG Estimator and Bootstrap Bias corrected Estimator (ARDL 1,1,1)</b>									
MG Estimator	1.31		3.07		-5.06		-0.09		
t-percentile confidence intervals									
1 per cent	1.04	1.93	1.11	9.98	-9.81	-2.06	-0.12	-0.03	
5 per cent	1.09	1.79	1.53	7.94	-9.31	-2.81	-0.10	-0.04	
10 per cent	1.13	1.73	1.80	7.12	-8.61	-3.29	-0.10	-0.05	
BSBC Estimator	1.42		3.67		-6.03		-0.06		

Table 5

**EURO AREA MONEY DEMAND LONG-RUN EFFECTS (CV MODEL)**

Standard errors are reported in italics. The symbol \*\*\* indicates a significance level of 1 per cent or less; \*\* between 1 and 5 per cent; \* between 5 and 10 per cent. In panel A, the Mean Group Estimator is obtained by averaging the FMOLS coefficients across countries. The Bootstrap Bias corrected Estimator is calculated according to the procedure BSBC2 described in Section 3. The block length is equal to 13 observations. Outliers and seasonal factors are eliminated from the dependent variables by using a two-stage procedure: in the first stage, an OLS estimate is run with the 3 variables and quarterly seasonal dummies and outliers are identified as the observations estimated with an error greater in absolute value than 3 times the standard error of the regression; in the second stage, the additional dummy variables for the outliers identified are included in the OLS regressions. The last column of Panel A reports the  $L_c$  test proposed by Hansen (1992) in order to test parameter stability in the context of fully modified estimation of cointegrated regression models. In panel B, the Mean Group Estimator is obtained by averaging the coefficients across countries where the equation is stable (namely those where the speed of adjustment is  $< -0.01$ ). Again, the effects of outliers are eliminated by using the same procedure outlined above. The Bootstrap Bias corrected Estimator is calculated according to the procedure BSBC3 described in Section 3. The block length is equal to 13 observations.

<b>Panel A: FMOLS ESTIMATES</b>								
Country	Real GDP		Yield curve steepness		Inflation		Test for parameter instability	
<b>Individual Country Equation</b>								
Austria	1.29	(0.06) ***	0.96	(0.49) *	0.17 (0.22)		0.858 **	
Belgium	2.05	(0.08) ***	-1.74	(0.54) ***	-0.24 (0.35)		0.666	
Finland	2.00	(0.57) ***	2.54	(3.12)	-2.62 (1.94)		0.208	
France	1.40	(0.09) ***	-2.11	(0.43) ***	-0.13 (0.38)		0.244	
Germany	1.45	(0.06) ***	2.66	(0.57) ***	-0.43 (0.31)		1.786 ***	
Ireland	1.53	(0.14) ***	3.26	(1.44) **	3.31 (1.31) **		0.381	
Italy	0.76	(0.34) **	4.71	(1.67) ***	-0.78 (1.03)		0.335	
Luxembourg	1.23	(0.05) ***	-2.20	(0.68) ***	-0.24 (0.42)		0.864 **	
Netherlands	1.60	(0.05) ***	-2.63	(0.52) ***	-0.03 (0.38)		1.637 ***	
Portugal	1.17	(0.19) ***	1.96	(1.23)	-0.89 (0.36) **		7.122 ***	
Spain	1.66	(0.07) ***	1.02	(0.41) **	0.19 (0.29)		0.572	
Area-wide equation	1.38	(0.03) ***	0.22	(0.25)	0.21 (0.18)		0.208	
<b>MG-FMOLS Estimator and Bootstrap Bias corrected Estimator</b>								
MG Estimator	1.47		0.77		-0.15			
	t-percentile confidence intervals							
1 per cent	1.42	1.97	0.30	2.72	-1.15	0.44		
5 per cent	1.47	1.88	0.50	2.39	-0.91	0.26		
10 per cent	1.51	1.83	0.60	2.18	-0.81	0.17		
BSBC Estimator	1.85		1.21		-0.24			
<b>Panel B: OLS ESTIMATES</b>								
Country	Real GDP		Yield curve steepness		Inflation		Speed of adjustment	
Area-wide equation	1.15	(0.13) ***	-2.57	(1.31) *	-2.25 (1.18) *		-0.09 (0.04) **	
<b>MG Estimator an Bootstrap Bias corrected Estimator (ARDL 1,1,1)</b>								
MG Estimator	1.03		-3.72		-2.61		-0.09	
	t-percentile confidence intervals							
1 per cent	0.63	1.61	-7.52	-2.43	-4.03	-1.71	-0.12	-0.03
5 per cent	0.78	1.49	-6.18	-2.78	-3.87	-1.96	-0.11	-0.04
10 per cent	0.85	1.44	-5.79	-2.93	-3.65	-2.07	-0.10	-0.05
BSBC Estimator	1.56		-4.10		-2.87		-0.07	



The Bootstrap Bias-corrected estimates based on 1000 replications of the moving-block bootstrap scheme (BSBC2) outlined in section 3 (with block length =13) are reported in the last row of Panel A in Tables 4 and 5. Consistent with the Monte Carlo results presented in section 3, there is greater long-run income elasticity than with the MG estimator: 1.7 in the DGS model and 1.85 in the CV model.

In order to test the robustness of these estimates I also estimate the MG estimator using dynamic OLS estimates. In both models I chose to work with an ARDL model with maximum lag equal to one. Again, I first eliminated the effects of outliers. The individual country estimates (unreported) show much greater variability than those obtained with FMOLS estimates. As far as the long-run income elasticity is concerned, MG estimates<sup>5</sup> are quite similar to those obtained with FMOLS (1.31 in the DGS model and 1.03 in the CV model). Using Bootstrap Bias-corrected estimates (BBSC3), again there is greater long-run income elasticity than with the MG estimator (the long-run income elasticity is 1.42 in the DGS model and 1.56 in the CV model). Finally, I also estimated for both models an ARDL with maximum lag equal to 2. For the long-run income elasticity, the unreported results are very similar to those obtained when the maximum lag was 1.

## 5. Conclusion

In recent years there has been increasing interest in dynamic panel data models where the number of time series observations is comparable to the number of groups. The Mean Group estimator (estimating equations for each group and then averaging the coefficients over groups) is a consistent estimator. However, in a Monte Carlo experiment conducted by Pesaran and Zhao (1999) with 20 groups and 20 time observations, the long-run coefficient of the I(0) regressor was found to be downward biased, in particular when the lagged dependent variable is large (0.8), possibly as an effect of small-sample bias.

In this paper, Monte Carlo simulations are used to further explore the characteristics of the MG estimator bias by considering cointegrated systems, which are in fact the most

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<sup>5</sup> Only stable equations (namely those with  $|\lambda| < .99$ ) are included in the computation of the Mean Group estimator. The number of such cases is 3 in the DGS model (Austria, Ireland and Luxembourg) and 4 in the CV model (those three plus Germany). The frequency of unstable equations is relatively high, possible evidence of model misspecification.

common case in empirical applications; I analyze a wider range of cases defined as a function of the number of groups and time observations.

In cointegrated systems where the signal-to-noise ratio is kept at relatively low, the regressor is strictly exogenous, and the true coefficient of the lagged dependent variable is small, my results confirm the existence of a downward bias for the long-run coefficient of the regressor, in particular when  $T < 100$ . Increasing the number of groups has a limited effect on the bias, but does reduce its variance. These results are consistent with the hypothesis that small-sample bias is a major factor.

When the hypothesis of strict exogeneity is ruled out and the two innovations driving the cointegrated system are allowed to be correlated, the MG Estimator has a more pronounced downward bias if the correlation is negative, but is upward biased if the correlation is positive. The MG-FMOLS estimator (computed averaging FMOLS estimates) displays the same pattern, but with a smaller bias. These results are consistent with the intuition of Pesaran, Shin and Smith (1999), who cite sample-specific omitted variables or measurement errors that are correlated with regressors as another possible explanation of MG estimator bias.

In order to perform bias correction, I apply the guidelines of Li and Maddala (1997) for bootstrapping cointegrating regressions (namely, use the moving block bootstrap and consider that if  $x_{it}$  is an I(1) process it is convenient to bootstrap the two innovations that drive the cointegrated system simultaneously). My simulations show an appreciable improvement in the bootstrap's ability to reduce the bias, in particular when these suggestions are applied to the MG estimator in a heterogeneous dynamic model if  $x_{it}$  is strictly exogenous and to the MG-FMOLS if non-zero correlation between the two sets of residuals  $u_{it}$  and  $\varepsilon_{it}$  is allowed. However, I also find a slight increase in both the standard error and the RMSE.

Finally, I apply these procedures to estimate the long-run coefficients for euro-area money demand in two different models. The Mean Group estimates computed averaging either the FMOLS or the dynamic OLS estimates are quite similar to those obtained by estimating the area-wide equation, consistent with the hypothesis that aggregation bias is not relevant in this framework. With respect to the MG estimators, the Bootstrap bias-correction

estimators produce an increase in the long-run coefficients, consistent with the results of the Monte Carlo experiment. The estimate of the long-run income elasticities is roughly 1.5, which is the upper bound of the ECB's implicit assumption for determining the reference value for money growth. This result is robust to different model specifications and to different treatments of number of lags used in the analysis.

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