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Revisiting the Case for a Populist Central Banker

by Francesco Lippi

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REVISITING THE CASE FOR A POPULIST CENTRAL BANKER

by Francesco Lippi*

Abstract

It is known that discretionary policy may give rise to an inflationary bias if wages are negotiated in nominal terms. It has recently been argued that this bias can be eliminated, and welfare maximized, by the appointment of a central banker who does not care at all about inflation (a “populist” central banker). A conceptual flaw of the latter result is identified here. It is shown that when wages are negotiated in nominal terms the result is true only in the special case of a single, all-encompassing, union. In the more general case of multiple unions, however, inflation increases linearly with their number and a populist central bank may turn out to decrease welfare.

JEL classification: E5, J5.

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1. Introduction

Recent contributions have shown that the macroeconomic effects of monetary institutions may depend on the labor market structure. Among the main variables that characterize the latter are the number of unions that bargain wages in an independent manner, the degree of labor substitutability and the unions’ aversion to inflation. Cukierman and Lippi (1999) and Guzzo and Velasco (1999) provide models where the effects of central bank independence on inflation and employment depend on these labor market features.

But the models of Cukierman and Lippi and Guzzo and Velasco (CL and GV henceforth) produce rather different results. Perhaps the most striking difference is that in GV both inflation and employment are at their pareto-optimal level when the central banker does not care at all about inflation (what they label a “populist” central banker), while this is not true in general in CL. Given the priority currently attributed to the inflation goal by most central banks, the robustness of such a proposition seems relevant.

This paper shows that the source of several differences in the results of the two papers, among which the proposition concerning the (unconditional) optimality of a populist central banker, is not in the different underlying models of the economy that are used in these papers. Rather, it lies in the different assumptions that CL and GV make concerning the wage bargaining process. Both CL and GV claim that the unions’ strategic choice variable is the nominal wage, i.e. that each union in the bargaining process sets its nominal wage taking the nominal wages of the other unions as given, what will be called “nominal wage bargaining”. Despite their claim, however, GV solve their model by implicitly assuming that each union chooses its nominal wage taking the real (not the nominal) wages of other unions as given.

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1 I have benefited from the comments of Ken Rogoff, an anonymous referee and Harald Uhlig. The views are personal and do not involve the responsibility of the institutions with which the author is affiliated. E-mail: lippi@dada.it.

2 See Cukierman and Lippi (1999) for a survey of the literature on labor market structure and monetary institutions.

3 CL label this type of central banker as “ultra liberal”. Here the more parsimonious terminology of GV is used.

4 Nominal wage bargaining is an essential ingredient of the credibility problem. If wages were negotiated in real terms, the inflation bias would not arise.
It will be shown that the approach of GV is inconsistent with the assumption of nominal wage bargaining. Intuitively, the inconsistency occurs because under nominal wage bargaining a non-atomistic union understands that an increase in its nominal wages reduces the other unions’ real wages (since inflation increases and the other unions’ nominal wages are constant). GV implicit assumption of constant other unions’ real wages implies that GV’s “equilibrium” is not, in general, a Nash equilibrium. As a consequence, most of their “results” are not warranted by their formal analysis.

We reformulate the problem of GV under nominal wage bargaining (Section 2) and demonstrate that in such a case GV’s results are not an equilibrium (Section 3). The correct Nash equilibrium of the game is derived in Section 4. It will be shown that under nominal wage bargaining the GV model produces exactly the same result obtained by CL (Section 5): a populist central banker maximizes welfare only if there is a single union. As the number of unions that take part in the wage bargaining increases, inflation rises linearly and welfare is not necessarily maximized. A final section draws conclusions.

2. The elements of the GV model

The fundamental equations of the GV model are reported below (GV numbering):

$$Y = \left( \int_0^1 L_i \frac{\bar{z}_{i+}}{\bar{z}_{i-}} \, di \right)^{\frac{\bar{z}_{i+}}{\bar{z}_{i-}}} , \quad 0 < \alpha < 1, \sigma > 1 \quad \text{(GV 2.1)}$$

$$U_i = \log C_i - \frac{\gamma}{2} \left( \log L_i \right)^2 - \frac{\beta_p}{2} \bar{\pi}^2 , \quad \beta_p \geq 0, \quad \gamma > \alpha \quad \text{(GV 2.8)}$$

$$V_j = n \int_{j-n-1}^j U_i \, di \quad \text{(GV 2.10)}$$

$$J = \int_0^1 \left[ \log C_i - \frac{\gamma}{2} \left( \log L_i \right)^2 \right] \, di - \frac{\beta_g}{2} \bar{\pi}^2 , \quad \beta_g \geq 0 \quad \text{(GV 2.15)}$$

$$L_i = \alpha^{\frac{1}{1-\sigma}} \left( \frac{W_i}{W} \right)^{-\sigma} W^{-\frac{1}{1-\sigma}} \quad \text{(GV B.2)}$$

$$C_i = W_i L_i + D_i = \alpha^{\frac{1}{1-\sigma}} \left( \frac{W_i}{W} \right)^{1-\sigma} W^{-\frac{1}{1-\sigma}} + D_i \quad \text{(GV 2.9)}$$

$$C_i = \left[ \alpha^{\frac{1}{1-\sigma}} \left( \frac{W_i}{W} \right)^{1-\sigma} + (1-\alpha) \alpha^{\frac{1}{\sigma}} \right] W^{-\frac{1}{\sigma}} \quad \text{(GV B.5)}$$

$$W \equiv \left( \int_0^1 W_i^{1-\sigma} \, di \right)^{\frac{1}{1-\sigma}} , \quad W_i \equiv \frac{1 + \omega_i}{1 + \pi} \quad \text{(GV 2.4, GV 2.11)}$$
\[ \pi = \frac{\alpha - \gamma \int_0^1 \log L_i \, di}{(1 - \alpha) \beta_g}. \] (GV 3.1)

A profit-maximizing representative firm produces a consumption good \((Y)\) with technology (GV 2.1) where \(L_i\) is worker \(i\)'s labor input (distributed over the unit interval), \(\sigma\) is the labor substitution elasticity and \(\alpha\) is a return to scale parameter. Workers are organized in \(n \geq 1\) unions (indexed by \(j\)), each of which has a set of members of measure \(n^{-1}\) on whose behalf it sets nominal wages.\(^5\) Worker \(i\)'s utility \((U_i)\) is (GV 2.8) where \(\gamma\) and \(\beta_p\) are preference parameters and \(C_i\) and \(\pi\) are, respectively, \(i\)'s consumption and the inflation rate. The representative union maximizes the utility of its members \(V_j\) (GV 2.10). The government objectives (GV 2.15) differ from the individual unions' objectives because the government accounts for all workers and \(\beta_g\) may differ from \(\beta_p\).

The demand for labor type \(i\) is (GV B.2) where \(W\) and \(W_i\) are the aggregate and individual real wages, respectively, (GV 2.4) and (GV 2.11) \((\omega_i\) is the percent increase in the nominal wage of the union to which worker \(i\) belongs). The representative worker budget constraint is (GV 2.9) (dividends \(D_i\) are taken as given). The government, instead, does not take \(D_i\) as given (GV B.5).

Let the strategic choice variable of union \(j\) be the nominal wage growth, \(\omega_j\), identical across all of its workers (i.e. \(\omega_i = \omega_j; \text{ all } i \in j\)). From (GV 2.4) and (GV 2.11) we derive aggregate nominal wage growth \((\omega)\)

\[ W = \frac{1 + \omega}{1 + \pi}, \quad \text{where} \quad \omega \equiv \left[ \int_0^1 (1 + \omega_i)^{1-\sigma} \, di \right]^{\frac{1}{1-\sigma}} - 1. \] (1)

Equation (1) implies that, in a symmetric equilibrium, union \(j\) perceives that its nominal wage growth increases aggregate nominal wage growth by a factor of \(1/n\), in direct proportion to its size \((\frac{d\omega}{d\omega_j} = \frac{1}{n})\).

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\(^5\) GV assume \(n \geq 2\). However, as will become clear later, only when \(n = 1\) the populist central banker is unconditionally optimal. Therefore \(n \geq 1\) is assumed here in order to consider the special case of \(n = 1\). They also assume \(\alpha \in [0, 1]\) while \(\alpha \in (0, 1)\) is assumed here to remove the degenerate corner solutions which arise when \(\alpha\) takes the extreme values of either 0 or 1.
2.1 The reaction function of policy to nominal wages

A two-stage game is considered. Nominal wages are set in the first stage in a Nash game between the \( n \) unions. In the second stage inflation is chosen by the government to maximize (GV 2.15) with respect to \( \pi \) subject to (GV B.2), (GV B.5), (1) taking nominal wages as given. The first order condition of this problem yields the reaction function (GV 3.1). Since the unions’ strategic choice variable is the nominal wage growth \( (\omega_j) \), we express the reaction function in terms of nominal wages, using equations (1) and (GV B.2) into (GV 3.1). This yields \(^6\)

\[
\pi = \frac{\alpha (1 - \alpha) - \gamma \log \alpha + \gamma \left[ (1 - \alpha) \sigma \int_0^1 (\omega_i - \omega) di + \omega \right]}{(1 - \alpha)^2 \beta_y + \gamma}.
\]

An important implication of (2) is that a non-atomistic union perceives that the growth of its nominal wages raises inflation. The perceived impact effect of \( \omega_j \) on the inflation rate, when the other unions’ nominal wages (label those \( \omega_{-j} \)) are taken as given, is

\[
\frac{d\pi}{d\omega_j} \bigg|_{\omega_{-j}} = \frac{\gamma}{n \left[ (1 - \alpha)^2 \beta_y + \gamma \right]} \equiv s(\beta_y, n) \in (0, 1)
\]

which we label \( s \).\(^7\) It appears that the impact effect depends on the central bank inflation aversion \( \beta_y \) and on the union’s size. This shows the relevance of labor market structure: atomistic unions \( (n \to \infty) \) perceive their impact on inflation is zero. A non-atomistic union, however, perceives that raising its nominal wages increases the inflation rate \( (s > 0) \) and that this increase is smaller the more the central bank is inflation averse.

3. Nominal wage bargaining versus GV

GV (on p.1324) claim to solve the unions’ problem under the assumption of nominal wage bargaining (NWB henceforth): “the union sets the rate of increase of the nominal wages of its members. [..] In doing so, it takes the nominal wages set by other unions as given”. Let us verify what the NWB assumption implies for the real wage elasticity of labor demand, a

\(^6\) As done in GV the approximations \( \log W \approx \omega_i - \pi \) and \( \log W \equiv \omega - \pi \) are used throughout.

\(^7\) Equation (3) gives the impact effect of \( \omega_j \) on inflation evaluated at a symmetric equilibrium, where all wages are identical, which implies that the term \( \frac{d\pi}{d\omega_j} \left[ \int_0^1 (\omega_i - \omega) di \right] \) is zero.
variable that is key in the determination of equilibrium outcomes (see equations 3.4 and 3.6 in GV). Under NWB the real wage elasticity of labor demand, which we label \( \tilde{\psi} \), is (Appendix A)

\[
\tilde{\psi} \equiv - \frac{d \log L_j}{d \log W_j} \bigg| \omega_{-j} = \left[ \psi + \frac{\sigma(1 - \alpha) - 1}{1 - \alpha} \frac{(n - 1)s}{n(1-s)} \right] \in (1, \infty).
\]

Note that this is not the elasticity used by GV (equation 2.13 of the GV paper). Instead, when the other unions’ real wages (\( W_{-j} \)) are assumed to be invariant to changes in union \( j \)’s nominal wages (\( \omega_j \)), the real wage elasticity is (Appendix A)

\[
\psi \equiv - \frac{d \log L_j}{d \log W_j} \bigg| \omega_{-j} = \sigma - \frac{\sigma(1 - \alpha) - 1}{1 - \alpha} \frac{n}{n}
\]

which is equal to equation (2.13) of the GV paper. Note that \( \tilde{\psi} = \psi \) only in the special cases of a single union or an atomistic labor market (respectively, \( n = 1 \) or \( n \rightarrow \infty \)), because in neither case unions perceive to affect the other unions’ real wages under NWB. The above demonstrates that GV actually solve the unions’ problem by making each union choose the nominal wage taking the real (not the nominal) wages of other unions as given. This assumption is inconsistent with NWB because the increase in the nominal wages of a non-atomistic union reduces the other unions’ real wages (as inflation rises and the other unions’ nominal wages are constant).

Since GV assume the unions’ strategic choice variable is the nominal wage (i.e. they aim at modelling NWB), the implicit assumption of constant other unions’ real wages implies that the GV “equilibrium” is not a Nash equilibrium, i.e. the unions’ nominal-wage strategies they consider are not mutual best responses. Indeed, as shown in the next section, equilibrium employment and inflation under NWB are not the ones identified by GV. As a consequence their results 1b, 2, 3, 4, 5, 6 are not warranted by their model (result 1a is the only one that is correct as it is).

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8 The real wage elasticity \( \tilde{\psi} \) under NWB is obtained mapping nominal wage growth (the unions’ strategic choice variable) into real wage growth, according to: \( \frac{d \log W_j}{d \omega_j} = 1 - s \), yielding \( \tilde{\psi} \equiv - \frac{d \log L_j}{d \omega_j} \bigg| \omega_{-j} = \frac{1}{1 - s} \). This is convenient because it makes our results directly comparable to those of GV.

9 Technically, this can be seen from the fact that the correct real wage elasticity under NWB is not the one used by GV (Appendix A), which implies that their first order condition (3.2) is wrong under NWB in all cases except when \( n = 1 \) or \( n \rightarrow \infty \).
It may appear at first that GV results might be resurrected by assuming they are derived under the assumption of “real wage bargaining”, i.e. a situation in which the unions’ strategic choice variable is the real wage. In this case, in fact, $\psi$ is the correct real wage elasticity to be used. Unfortunately, however, the “real wage bargaining” assumption is only useful to resurrect the real outcomes of the model, not the inflation bias result. The reason is that the bias disappears from the model if unions bargain real wages, as the central bank cannot affect employment in such a case. Indeed, under “real wage bargaining”, the equilibrium inflation rate is always zero in this model. This makes the GV “results” vacuous for the analysis of the optimal degree of central bank conservatism.

4. Equilibrium under nominal wage bargaining (NWB)

The problem faced by the typical union $j$ under NWB yields the first order condition (Appendix B)

\begin{equation}
-\beta_p \pi \frac{s}{1-s} - \alpha \left[\psi - 1\right] + \gamma \psi \log L = 0
\end{equation}

which indicates that an increase in the wages of union $j$ has two opposing effects on the utility of workers: on one hand, it decreases utility since it increases inflation and reduces consumption (the first and second terms in (6), respectively). On the other hand, it increases utility since it raises leisure. Equation (6) shows that union $j$ trades off these marginal benefits and costs according to its preferences about inflation, consumption and leisure ($\beta_p$ and $\gamma$).

Equilibrium outcomes under NWB are obtained combining the reaction function (GV 3.1) and the unions’ first order condition (6). At a symmetric equilibrium this yields

\begin{equation}
\log L = \left(\frac{\alpha}{\gamma}\right) \tilde{\phi} \quad \in (0, 1)
\end{equation}

where

\begin{equation}
0 \leq \tilde{\phi} \equiv 1 - \frac{(1 - \alpha)(1-s)\beta_g}{\beta_p s + (1 - \alpha)(1-s)\beta_g \psi} \leq 1
\end{equation}
which is the equilibrium employment level under NWB. Inflation under NWB is also generally different from that derived in GV (see equation GV 3.6). Equation (7) and (GV 3.1) yield equilibrium inflation under discretionary policy

\[ \pi = \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{1 - \phi}{\beta_g} \right). \]

(9)

5. Revisiting the case for a populist central banker

With a populist central banker (\( \beta_g = 0 \)) each union perceives its impact on inflation is equal to \( s = \frac{1}{n} \) (from equation 3). The equilibrium level for employment thus is (from equation 7)

\[ \log L = \frac{\alpha}{\gamma}. \]

(10)

Equation (10) shows that, as in GV, under a populist CB employment is at its optimal level, i.e. the level where the consumption/leisure marginal rate of substitution (\( \gamma \log L \)) equals the (efficient) technical rate of transformation (\( 1/\alpha \)). The intuitive reason is that such banker would originate an infinite inflation if employment was below the optimal level (\( \frac{\alpha}{\gamma} \)). In order to avoid such a catastrophe, inflation averse unions (\( \beta_p > 0 \)) set real wages consistently with the optimal employment level.

The equilibrium level for inflation is derived from equation (9), yielding

\[ \pi = \frac{\alpha(n - 1)}{\beta_p}. \]

(11)

This result is in stark contrast with the one of GV, where the populist central bank produces zero inflation at all \( n \)’s. Under NWB, this occurs only if there is a single union (\( n = 1 \)). There is an intuitive reason for why this happens. When \( n = 1 \), the single union does not perceive the possibility to increase its real wage above the optimal level (i.e. the level consistent with

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\(^{10}\) A comparison of \( \tilde{\phi} \) with the corresponding GV variable, \( \phi \) (their equation 3.4) is not fully appropriate as the latter is not an equilibrium outcome under NWB (see the previous section). However, since \( \phi \) is the equilibrium outcome under “real wage bargaining”, the comparison of \( \tilde{\phi} \) with \( \phi \) contains information on the employment effects of nominal versus real wage bargaining. Analytical results on this issue are available from the author upon request.
the optimal employment in 10) because a unit increase in \( \omega_j \) is matched by a unit increase in inflation \((s = 1)\). Thus the union has no incentive to increase its nominal wage since that would raise inflation with no beneficial effects in terms of leisure (i.e. real wage).

If there is more than one union in the economy, however, each union perceives that a unit increase in its nominal wages increases its real wages since inflation rises by less than one for one \((s < 1)\). Crucially, inflation does not jump to infinity after a single union’s wage increase, even in the presence of a populist central banker, because the inflation caused by this wage increase reduces the other unions’ real wages leaving the aggregate real wage (hence aggregate employment) unchanged, at the level desired by the CB.\(^{11}\)

Since each individual union has an incentive to raise its real wages above the socially optimal level (a well known coordination failure arising in monopolistic markets), it will do it. Thus, when \( n > 1 \), all unions increase their nominal wages by identical amounts in a symmetric equilibrium, which are transformed \textit{fully} in inflation by the populist CB.

Note from equation (11) that inflation is higher the larger the number of unions in the economy. This occurs because the smaller each union is, the smaller is the perceived impact on inflation (naturally, as each union accounts for a smaller portion of the aggregate nominal wage). This makes the perceived marginal cost of inflation decreasing in the number of unions. Hence the equilibrium nominal wage growth chosen by each union, and therefore equilibrium inflation, increase with \( n \).\(^{12}\)

It should therefore be clear that under NWB the GV result on the optimality of the populist central banker is unconditionally valid (i.e. valid for any \( \beta_p > 0 \)) only in the special case when \( n = 1 \), where both inflation and employment lie at their optimal levels. As \( n \) increases, the inflation rate increases linearly. Therefore, when \( n > 1 \), the optimal level of

\(^{11}\) Equation (14) reveals that an increase in union \( j \)’s wages does not raise the aggregate real wage (employment) under a populist central bank. Notice the difference with the same case under the GV assumption, under which each union perceives that an increase in its own wages raises the aggregate real wage (since the real wages of the other unions are unchanged), lowering employment and hence leading to a hyperinflation.

\(^{12}\) The first order condition of the representative union (6), when \( \beta_g = 0 \) (note that 4 implies \( \psi = \sigma \) when \( \beta_g = 0 \)), is

\[-\beta_p \pi \frac{n}{n - 1} - \alpha [\sigma - 1] + \gamma \sigma \log L = 0\]

which reveals that inflation costs are decreasing in \( n \).
CB inflation aversion (the optimal $\beta_g$) depends on workers’ inflation preferences ($\beta_p$). For instance, in a decentralized labor market (high $n$) inflation will be high under the populist central banker, which makes it an improbable social optimum (i.e. given $n$, it is always possible to find a sufficiently large — but finite — $\beta_p$ for which a populist central banker is not optimal).

6. Concluding remarks

Most central banks are concerned with inflation and in many countries this concern has been emphasized and made more explicit in recent years (see Cukierman, 1998). An influential interpretation of these facts relies on Rogoff’s (1985) idea that, in the presence of credibility problems, the government may be better off by delegating monetary policy to a “conservative” central banker. Guzzo and Velasco (1999) have recently challenged this idea. They argued that in a standard setup, where unions negotiate nominal wages, the appointment of a populist central banker (one who does not care at all about inflation) might completely eliminate the inflation bias and increase structural employment.

This paper has shown that the results presented by Guzzo and Velasco are inconsistent with their maintained assumption of nominal wage bargaining. In such a setup, their “equilibrium” results are not a Nash equilibrium. We have shown that this conceptual flaw impairs most of their results.

In particular, when the GV problem is solved correctly under the assumption of nominal wage bargaining (as was also the intention in their paper) the welfare effects of the populist central banker may change radically: with the exception of the special case in which there is a single all-encompassing union, the optimality of a populist central banker is not robust. It is shown that if society (i.e. workers) is sufficiently interested in inflation, a conservative central bank may indeed be welfare improving. This result is almost identical to the one obtained by Cukierman and Lippi (1999). Overall, this casts serious doubts on Guzzo and Velasco’s normative implication that central banks should not be concerned with price stability.
APPENDIX A: The real wage elasticity of labor demand

Using the real wage elasticity definition and equation (GV B.2), straightforward algebra reveals that at a symmetric equilibrium ($W = W_j$):

\begin{equation}
\eta \equiv \left. \frac{d \log L_i}{d \log W_i} \right|_{\omega_{-j}} = \sigma - \left( \sigma - \frac{1}{1 - \alpha} \right) \left. \frac{dW}{dW_j} \right|_{\omega_{-j}}.
\end{equation}

Under the assumption of NWB, let us use the real wage definition (GV 2.4) and (GV 2.11) to calculate:

\begin{equation}
\left. \frac{dW}{dW_j} \right|_{\omega_{-j}} = \frac{W^\sigma}{1 - \sigma} \left[ \int_{i \in j} (1 - \sigma) W_i^{-\sigma} \, di + \int_{i \in -j} (1 - \sigma) W_i^{-\sigma} \left( \frac{d \left( \frac{1 + \omega_{-j}}{1 + \pi_j} \right)}{dW_j} \right) \, di \right].
\end{equation}

Since the wage is the same for all workers of union $j$ (label this $W_j$), and within the group of the workers belonging to “other unions” (i.e. all $W_i$ for which $i \in -j$, label this $W_{-j}$), we can integrate across each of these groups obtaining

\begin{equation}
\left. \frac{dW}{dW_j} \right|_{\omega_{-j}} = W^\sigma \left[ \frac{1}{n} W_j^{-\sigma} + \frac{n - 1}{n} W_{-j}^{-\sigma} \frac{d \left( \frac{1 + \omega_{-j}}{1 + \pi_j} \right)}{dW_j} \right]_{\omega_{-j}}.
\end{equation}

Let us use (3) to calculate:

\begin{equation}
\frac{d \left( \frac{1 + \omega_{-j}}{1 + \pi_j} \right)}{dW_j} \bigg|_{\omega_{-j}} = \frac{W_{-j}}{W_j} \left( \frac{\partial \log W_{-j}}{\partial \omega_{-j}} \right) \bigg|_{\omega_{-j}} \left( \frac{\partial \omega_{-j}}{\partial \log W_j} \right) \\
= \frac{W_{-j}}{W_j} \left( \frac{\partial (\omega_{-j} - \pi_j)}{\partial \omega_{-j}} \right) \bigg|_{\omega_{-j}} \frac{1}{1 + \pi_j} = \frac{W_{-j}}{W_j} \left( \frac{1}{1 - s} \right)
\end{equation}

which plugged into (13) yields at a symmetric equilibrium ($W = W_j = W_{-j}$):

\begin{equation}
\left. \frac{dW}{dW_j} \right|_{\omega_{-j}} = \frac{1}{n} - \frac{(n - 1)s}{n (1 - s)} \geq 0.
\end{equation}

Substituting (14) into (12) yields (4) in the main text, which in terms of the basic model parameters is equal to:

\[ \tilde{\psi} = \frac{1}{1 - \alpha} + \left( \sigma - \frac{1}{1 - \alpha} \right) \frac{(1 - \alpha^2 \beta \gamma + \gamma)}{\n - (1 - \alpha^2 \beta \gamma + \gamma)}. \]
The real wage elasticity of GV (their equation 2.13) is obtained from (13) when the rightmost term in the square bracket, which represents the impact of union \( j \) nominal wages on the other unions’ real wages, is set to zero.

**APPENDIX B: A typical union’s first order condition**

The typical union \( j \) maximizes

\[
n \int_{i \in j} \left[ \log C_i - \frac{\gamma}{2} (\log L_i)^2 - \frac{\beta_p \pi^2}{2} \right] di
\]

with respect to \( \omega_j \) subject to (GV 2.9), (GV B.2), (2) and taking \( \omega_{-j} \) and \( D_i \) as given. The partial derivative of (15) with respect to \( \omega_j \) (i.e. \( \omega_i \) for \( i \in j \)) yields

\[
n \int_{i \in j} \left[ \frac{1}{C_i} \frac{dC_i}{d\omega_i} \bigg|_{\omega_{-j}} - \gamma \log L_i \frac{d \log L_i}{d\omega_i} \bigg|_{\omega_{-j}} - \beta_p \pi s \right] di = 0.
\]

Since the wages of union \( j \)'s members are identical we can integrate across them to get

\[
\alpha \left[ 1 - s + \frac{d \log L_j}{d\omega_j} \bigg|_{\omega_{-j}} \right] - \gamma \log L_j \frac{d \log L_j}{d\omega_j} \bigg|_{\omega_{-j}} - \beta_p \pi s = 0
\]

where we used \( \frac{dC_j}{d\omega_j} \bigg|_{\omega_{-j}} = \frac{W_j L_j}{\epsilon_j} \left[ \frac{d \log W_j}{d\omega_j} + \left( \frac{d \log L_j}{d\omega_j} \bigg|_{\omega_{-j}} \right) \right] \), \( \frac{W_j L_j}{\epsilon_j} = \alpha \), and \( \log W_j \simeq \omega_j - \pi \).

The real wage elasticity is: \( \tilde{\psi} \equiv -\frac{d \log L_j}{d \log W_j} \bigg|_{\omega_{-j}} = -\frac{d \log L_j}{d\omega_j} \bigg|_{\omega_{-j}} \frac{1}{1-s} \). Dividing expression (16) by \( 1 - s \) yields equation (6) in the main text (the first order condition for the special case in which \( s = 1 \) is derived in footnote 10).
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