



BANCA D'ITALIA
EUROSISTEMA

Temi di discussione

(Working papers)

A beta based framework for (lower) bond risk premia

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September 2008

Number

689

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A BETA BASED FRAMEWORK FOR (LOWER) BOND RISK PREMIA

by Stefano Nobili* and Gerardo Palazzo*

Abstract

We use a no-arbitrage essentially affine three-factor model to estimate term premia in US and German ten-year government bond yields. In line with the existing literature, we find that estimated premia have followed a downward trend since the 1980s: from 4.9 per cent in 1981 to 0.7 per cent in 2006 for the US bond and from 3.3 to 0.5 per cent for the German one. Subsequently, using an Error Correction Model (ECM) we prove that the decline is explained by a decrease in global output variability and an increase in the power of ten-year government bonds to diversify the investors' portfolios. In addition, the ECM also forecasts both the US and the German term premia converging to around one percentage point over a five year horizon. Long-term return expectations for ten-year government bonds will have to incorporate bond risk premia that - while in line with average excess returns during the twentieth century - are significantly lower than average excess returns over the last two decades.

JEL Classification: C13, C14, E43, E47, G12, G15.

Keywords: term structure model, bond risk premium, modern portfolio theory.

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1. Introduction¹

The dynamics of bond risk *premia* has been investigated in a large number of recent papers. Even though there is no consensus on this hotly debated topic, the prevailing view is that *premia* have trended downwards since the mid-1980s. The main arguments put forward to explain this reduction are the *great moderation* of economic systems, monetary policy credibility established over an extended period, and the widespread recourse to efficient tools in the evaluation and transfer of financial risks. In recent years and before the onset of the crisis that started in the US mortgage market, both an increase in investors' risk appetite and an excess demand for government bonds by long-term institutional investors and central banks may also have played a role in moderating risk *premia* (Kim and Wright, 2005; OECD, 2006).

Using a term structure model recently developed in the literature and the basic framework of Modern Portfolio Theory, in this paper we add evidence of the declining trend of estimated risk *premia* for US and German ten-year government bonds. We then show that this reduction is associated with a decrease in global output variability and an increase in the power of government bonds to diversify global investment portfolios. The main contribution of our research is to look at the question in an international perspective, as prescribed by Modern Portfolio Theory, while – at least to our knowledge – recent papers restrict their analysis to the domestic macroeconomic and financial context.

The work is organized in two parts. Firstly, we estimate risk *premia* for ten-year government bonds using a no-arbitrage affine term structure model with a flexible specification for the market price of risk (*essentially* affine Gaussian model). Duffee (2002), Dai and Singleton (2002), Duarte (2004) and Kim and Orphanides (2005), who have estimated this type of model with relatively long samples of US Treasury term structure data, are our main references.

Secondly, given that the model used by this strand of literature does not allow for an easy economic interpretation of the results, our ambition is to fill this gap. In order to do so, we study the relationship between government bond risk *premia* and the risk associated with such securities in the context of fully integrated financial markets. Our basic conjecture is fairly simple and is derived from Modern Portfolio Theory: holders of a financial asset can expect a premium over the risk-free

¹ Any views expressed in this paper are the authors' and do not necessarily represent those of the Bank of Italy. We thank Marco Fanari, Gioia Guarini, two anonymous referees and seminar participants at the Bank of Italy for helpful discussions. Special thanks go to Michele Manna who suggested we work on this topic. Without his support, this paper would never have been written.

asset only if they bear systematic risk. In turn, this risk is measured by the covariance between the asset returns and returns of a global market portfolio.

We estimate an Error Correction Model (ECM) to analyze the co-movements between bond *premia* – as inferred using the affine model – and two variables: i) the standard deviation of the world GDP growth rate and ii) the correlation between government bond returns and the returns of a portfolio diversified by asset class, geographic region and currency. The use of variables that may be regarded as proxy for the systematic risk allow us to interpret the ECM fitted *premia* as the fair values that investors require to remunerate the risk of long-term government bonds.

This two-stage approach is not an absolute novelty and Backus and Wright (2007) is only the most recent example: they first estimate risk *premia* using an affine model similar to the one we present in this paper, then they show that its dynamics is associated with the US unemployment rate and the dispersion of US long-term inflation expectations.

We show that the decrease in government bond *premia* since the mid-1980s is mainly due to the reduction in the systematic component of risk. Against this result, the low *premia* at the end of 2006 – 0.7 per cent in the US and 0.5 per cent in Germany – were broadly consistent with the then prevailing level of risk. This finding confirms the conclusion for US risk *premia* reached by Campbell, Sunderam and Viceira (2007), who also offer a macroeconomic insight into the reduction of the systematic risk. In a nutshell, their idea is that – over the last two decades – the US macroeconomic environment has not been affected by supply shocks or changing inflation expectations, a lull that has brought about an increase in the correlation between the inflation rate and the real GDP growth rate. Under such macroeconomic conditions, nominal bond returns have proved to be countercyclical, making nominal bonds a desirable hedge against business cycle risk.

In this regard, it is interesting to observe that yields on long-term government bonds were also low in the 1950s and 1960s. With long-term inflation expectations apparently anchored at low levels and with the prospect of continued economic stability, market participants may currently believe that it is appropriate to price bonds more or less in the same way as four or five decades ago.

We believe these results are important for long-term investors. The lower the risk of government bonds, the lower the premium investors require (*ex ante*) to hold such securities, and the higher the price they are willing to pay. In a period marked by declining required bond *premia*, as 1980-2005 seems to have been, (*ex-post*) excess returns on government bonds were highly significant, especially when compared with the average values recorded over the last century. Dimson, Marsh and Staunton (2006) show that from 1980 to 2005 an investment in government

bonds returned an annual excess return of 4.3 per cent in the US and 2.3 per cent in Germany, much higher than the average excess returns in the 1900-2005 period, equal to 1 per cent for the United States and -2.4 per cent for Germany (Table 1 and Figure 1).

Of course, in order to answer the question about future bond risk *premia* one has to explicitly express a view about the most plausible future evolution of the current macroeconomic environment. To the extent that the *great moderation* and the well-anchored inflation expectations we have experienced in the last two decades may be traced to structural changes (e.g. market deregulation, improved inventory control methods, better risk sharing in financial markets, and improved macroeconomic policies), investors may be confident that long-run bond risk *premia* will remain low. Conversely, if the reduction in macroeconomic uncertainty has been the lucky upshot of fewer and smaller shocks hitting the economy, the outlook for long-term government bonds is gloomy.

The main conclusion of this paper is that investors' expectations about future returns should incorporate a much lower premium than that actually achieved in recent decades. Investors will have to reconsider values more in line with the average over a really long-term horizon, such as the last century.

The remainder of the paper is organized as follows. The next section presents a brief survey of the most recent literature on the causes of the downward trend in government bond risk *premia*. Sections 3, 4 and 5 describe the affine model, the estimation technique based on the maximum likelihood criteria and the final *premia* estimates. Sections 6, 7 and 8 introduce the fair value model employed to determine the equilibrium values for *premia*, present the analysis of government bonds' systematic risk and discuss the estimated results. The main conclusions are summarised in the last section.

2. Review of the possible causes of the decline in term premia

A broad consensus has emerged as regards the reduction in the US long-term government bond risk *premia* over the last two decades (Bernanke, Reinhart and Sack, 2004; Cochrane and Piazzesi, 2005; Kim and Wright, 2005; Rudebusch and Wu, 2006). This result appears to be robust to different methodologies and data sample length.²

² A recent paper (Taboga, 2007) contends the prevailing view using a small-scale macro-econometric model that takes into account changing expectations about the real natural interest rate and the long-term inflation rate. He shows that although risk *premia* did diminish in the most recent years, their current level is not unusual if considered from the perspective of the last two decades.

The recent literature discusses the hypothesis of a reduction in bond *premia* mainly as a result of the decrease in the quantity of the risk and, for the most recent years, an increase in investors' risk appetite and excess demand for long-term government bonds by institutional investors and central banks.

The quantity of risk and investors' risk appetite

The significant reduction in the variability of a wide range of economic indicators such as the rate of output growth and the rate of employment in the G7 countries since the mid-1980s is a well documented fact which has found a successful summary in the expression *great moderation* (Stock and Watson, 2002; Bernanke, 2004; Summers, 2005). This background may explain the reduction in the so-called macroeconomic risks, and hence the risk premium investors demand to hold risky assets in their portfolios.

Another plausible contributing factor is the progress achieved by central banks in steering inflation expectations around low and fairly stable levels. This is described mainly as a result of a successful mix of central banks independence, transparency and effective communication (Bernanke, Reinhart and Sack, 2004).

Recent papers that explicitly connect US macroeconomic uncertainty and US bond risk *premia* are Backus and Wright (2007), Kim and Orphanides (2007) and Campbell, Sunderam and Viceira (2007).

Backus and Wright (2007) use the unemployment rate and the dispersion of consensus expectations about long-term inflation as explanatory variables for the ten-year US government bond risk premium in a simple linear regression model. The two independent variables may explain respectively the countercyclical behaviour of the premium and its downward trend over the last two decades. In a similar way, Kim and Orphanides (2007) find evidence of a positive correlation between the premium and the dispersion of consensus expectations about future long-term inflation and the three-month nominal interest rate.

Campbell, Sunderam and Viceira (2007) show that the premium is associated with the covariance between inflation and the real side of the economy. As this covariance switches sign, so does the risk premium. Their argument is that when inflation is cyclical – as is the case if the economy is not affected by supply shocks or changing inflation expectations so that the Phillips Curve remains stable – bonds tend to display countercyclical returns, making them desirable hedges against business cycle risk. By contrast, when inflation is countercyclical – as will be the case if the

macroeconomic system moves along a Phillips Curve that shifts in or out – bond returns are cyclical and investors demand a positive risk premium to hold them.

Further explanations for lower risk *premia* insist on the development of financial markets and financial innovation in general. As traded volumes in the stock markets grow, investors are content with lower liquidity *premia*. Other factors that may have contributed are the increasing availability of tools for the efficient valuation and transfer of risk (Ferguson, 2005) and the growing amount of assets managed by entities that are better equipped, in terms of capital and skills, to deliver portfolio diversification policies (Avramov, Chordia and Goyal, 2006).

The explanation of the reduction of bond *premia* (and of *premia* for almost all other asset classes) based on a general increase in the investors' risk appetite focuses on the most recent period, particularly 2003-2004. Over these years, the sustained phase of expansionary monetary policies in major economic areas brought down short-term interest rates and encouraged a *search for yield* process with investors increasing their exposure in financial assets with higher expected returns such as stocks and long-term bonds.³

Higher demand for long-term government bonds

The increase in the global resources available for financial investments and portfolio reallocation policies of institutional investors such as central banks, pension funds and insurance companies constitute a further hypothesis put forward to explain the risk *premia* reduction.⁴ Instead of analysing investors' preferences, in this case the interest rate level is analysed and explained using the traditional framework of supply and demand curves.

The best-known explanation in this field is perhaps the *global saving glut* (Bernanke, 2005). This may be summarised as a global excess supply of savings, mainly as a result of a decrease in investment in Asian economies following the financial crises at the end of the 1990s.

Strong demand for long-term government bonds by pension funds and insurance companies, and the massive purchases of dollar-denominated bonds by Asian central banks and oil exporting countries may have contributed to lower long-term bond risk *premia* (Bernanke, Reinhart and Sack, 2004). Recent empirical works suggest that at the end of 2004, all other things being equal, ten-year US government bond yield to maturity would have been between 100 and 150 basis points higher

³ ECB, FSR December 2006 and ECB, FSR June 2007.

⁴ "With the economic outlook held constant, changes in the net demand for long-term securities have their largest effect on the term premium. In particular, if the demand for long-dated securities rises relative to the supply, then investors will generally accept less compensation to hold longer-term instruments – that is, the term premium will decline." Bernanke (2006).

had there been no foreign official flows into US government bonds from 2002 onward (Frey and Moëc, 2005).

3. The affine model

Affine models constitute a special case of no-arbitrage term structure models. The simplifying assumption here is that zero coupon bond yields, their physical dynamics and their equivalent risk neutral dynamics may all be written as linear (affine) functions of an underlying state vector. Because of their great tractability and richness, the affine class has become popular among the finance profession. Early works by Vasicek (1977) and Cox, Ingersoll and Ross (1985), in which the risk-free interest rate is the only state variable, are the first examples of such models. The generalization of these simple one-factor models in the affine class has been formalized by Duffie and Kan (1996).

In this paper we employ the *essentially* affine three-factor model already estimated by Duffee (2002), Dai and Singleton (2002), Duarte (2004) and Kim and Orphanides (2005). In the affine class of models, the essentially affine subclass identifies models in which the market price of risk varies over time and eventually takes negative values. This result is an improvement on *completely* affine models, in which the price of risk can only take positive values. The *EA0(3)* model, in Duffee's (2002) terminology, adequately fits the interest rates in cross section at a fixed date and produces a more accurate yield forecast (both in- and out-of-sample) than other more sophisticated affine models. Yet the *EA0(3)* model still implies time-invariant volatility of yields (Gaussian model).

The three state variables included in the model are latent factors, that is they have a merely statistical meaning. This preserves the robustness of the results against changes of the model specification (Kim and Wright, 2005) but, at the same time, makes their economic interpretation anything but straightforward. By adding two macroeconomic factors related to US inflation and economic growth, Ang and Piazzesi (2003) improve the accuracy in explaining the changes of the short-term rates (up to 12 months). Nevertheless, they do not achieve the same result for long-term rates, whose dynamics are still mostly explained by the latent factors. Mönch (2006) uses only macroeconomic factors and his results strictly improve those of Ang and Piazzesi (2003); still the *EA0(3)* model emerges as the one with the best forecasting (out-of-sample) accuracy for the ten-year government bond yield.

The essentially affine three-factor Gaussian model

The *EA0(3)* model consists of three main equations. The first is the multivariate Ornstein-Uhlenbeck process that describes the continuous time dynamics of the vector of the three state variables z_t :

$$dz_t = K(\mu - z_t)dt + \Sigma d\omega_t \quad (1)$$

where ω_t is a three dimension standard Brownian motion [$\omega_t \sim N(0, I_3)$], μ is a 3×1 vector, K and Σ are 3×3 matrices. Σ is the (diagonal) matrix of the Ornstein-Uhlenbeck process volatilities.

The second equation describes the short-term interest rate r_t as an affine function of the state variables:

$$r_t = \rho_0 + \rho^T z_t \quad (2)$$

where ρ_0 is a scalar and ρ is a 3×1 vector.

The third equation describes the dynamics of the market price of risk. The sufficient condition for the absence of arbitrage in the zero coupon bond market is the existence of a risk neutral measure of probability Q , equivalent to the physical one P ,⁵ under which:

i) the state variables evolve according to the process:

$$dz_t = K^*(\mu^* - z_t)dt + \Sigma d\omega_t^*$$

where $\omega_t^* \sim N(0, I_3)$, under the measure Q ;

ii) the price at time t of a bond whose pay-off is a monetary unit at the time $t + \tau$ is:

$$p_t^\tau = E_t^Q \left[e^{(-r_t)\tau} p_{t+\tau}^{\tau-1} \right]$$

where E_t^Q is the expectation operator under the measure of probability Q , conditioned to the information available at time t .

In general, the vector μ^* and the matrix K^* are different from μ and K , while the hypothesis of equivalence between P and Q ensures that Σ remains unchanged. The physical probability P and the risk neutral probability Q are connected by the time varying dynamics of the market price of risk λ_t :

$$\lambda_t = \lambda_a + \Lambda_b z_t \quad (3)$$

⁵ The two probability measures Q and P are defined to be equivalent if they assign zero probability to the same events.

where λ_b and λ_a are implicitly defined as: $K^* = K - \Sigma\lambda_b$ and $\mu^* = K^{*-1}(K\mu - \Sigma\lambda_a)$. According to Cameron, Martin and Girsanov's theorem (Kallenberg, 1997), the relationship between P and Q is formally written as:

$$E_t^P \left[\frac{dQ}{dP} \right] = \prod_{j=1}^{\infty} \exp \left[-\frac{1}{2} \lambda_{t+j-1}^T \lambda_{t+j-1} - \lambda_{t+j-1}^T \omega_{t+j} \right]$$

so that the stochastic discount factor under the measure of probability P is:

$$m_{t+1} = \exp \left(-r_t - \frac{1}{2} \lambda_t^T \lambda_t - \lambda_t^T \omega_{t+1} \right) \quad (4)$$

and can be recursively used for the pricing of the bond with maturity $t+\tau$:

$$p_t^\tau = E_t^P [m_{t+1} p_{t+1}^{\tau-1}] \quad (5)$$

Zero coupon bond yields turn out to be an affine function of the state variables z_t :

$$y_t^\tau = -\frac{1}{\tau} \ln(p_t^\tau) = A(\tau) + B(\tau)^T z_t \quad (6)$$

where y_t^τ is the *yield* at time t of a bond whose pay-off is one monetary unit at the time $t+\tau$ and $A(\tau) = -a(\tau)/\tau$ and $B(\tau) = -b(\tau)/\tau$ (Riccati equations) are coefficients which solve the following differential equation system:

$$a'(\tau) = -\rho_o + b(\tau)^T K^* \mu^* + \frac{1}{2} \sum_{i=1}^N [b(\tau)^T \Sigma_i]^2 \quad (7)$$

$$b'(\tau) = -\rho - K^{*T} b(\tau)$$

with initial conditions $a(0)=0$ and $b(0)=0$.

The main output of the model is the inference on the risk premium for a ten-year zero coupon bond from its yield to maturity. This may be thought of as the sum of three components: i) the premium, ii) the expectation of the future average short-term interest rate that will prevail until the maturity of the bond, iii) the Jensen inequality. Formally, the premium is obtained as the observed yield minus the yield \tilde{y}_t^τ that the bond would display under the physical measure P in a world populated by risk neutral investors (for whom bond prices are set as the present value of future cash flows discounted using a risk-free interest rate, instead of an interest rate that takes the risk premium into account).

To work out the yield thus defined we impose that the market price of risk is equal to zero [$\lambda_t=0$ in (4) and (5)], obtaining:

$$\tilde{P}_t^r = E_t^P [e^{(-r_t)} \tilde{P}_{t+1}^{r-1}]. \quad (8)$$

where \tilde{y}_t^r is the yield to maturity that corresponds to such a price and is a function - in a way all but similar to (6) - of coefficients that satisfy an identical differential equation system to (7) with μ and K (drift parameters of the state variables process under the physical measure P) replacing μ^* and K^* (drift parameters of the state variables process under the risk neutral measure Q).⁶

To sum up, setting the market price of risk equal to zero is the same as imposing that the physical and risk neutral measures coincide, and that long-term government bond *premia* are nil.

An alternative measure frequently described in the literature is the *forward risk premium*, calculated as the difference between the instantaneous forward rate at the maturity of the bond and the expected short-term rate at the end of the same period.⁷ Unlike the risk premium we use in this paper, which represents an average premium along the maturity of the bond (yield risk premium), the forward risk premium refers to the premium at a specific date, for instance ten years from now. Where not otherwise specified, in the analysis that follows we will refer to the yield risk premium.

The main advantage gained by using the affine class of model is related to its linearity that allows us to trace back the pricing of bonds as stated in (5) to the solution of the system of ordinary differential equations (ODE) in (7), for which closed formula exist. This turns out to be a much easier task than solving a partial differential equation (PDE).⁸

The representation of the model in state-space form and the estimation technique

There is no unique solution to the model previously specified: a single set of yields to maturity may be obtained under different sets of parameters, reflecting rotations of the factors. Thus, the implementation of the model requires a normalization to rule out invariant transformations (Dai and Singleton, 2000). To that end, we choose the following normalization: i) K is a lower triangular matrix, ii) Σ is diagonal, iii) μ is a vector of zeros, iv) ρ is a vector of ones.⁹ In order to ensure stationarity of the state variables, we also impose that all the K eigenvalues are inside the unit circle.

⁶ Pericoli and Taboga (2006).

⁷ Calculations in Kim and Orphanides (2005) and Kim and Wright (2005) refer to the *forward risk premium*.

⁸ Kim and Orphanides (2005) give closed solutions to the system of equations. In this paper we use Matlab routines packed in the ODE suite. Following Huang e Yu (2006) we use the `ode15s` function.

⁹ The normalization we adopt here is proposed by Kim and Orphanides (2005). It differs slightly from Duffee (2002). See also Kim and Wright (2005).

We assume that the yields to maturity are observed with measurement errors and we estimate the model parameters with the Kalman-filter-based maximum-likelihood method (de Jong, 1999; Piazzesi, 2003; Duffee and Stanton, 2004).¹⁰ Expressing the model in space-state form, which is typically used to apply the Kalman filter methodology, the observation (or measurement) equation is:

$$o_t = \begin{bmatrix} A(\tau_1) \\ \vdots \\ A(\tau_M) \end{bmatrix} + \begin{bmatrix} B(\tau_1)^T \\ \vdots \\ B(\tau_M)^T \end{bmatrix} z_t + \begin{bmatrix} \eta_{t,\tau 1} \\ \vdots \\ \eta_{t,\tau M} \end{bmatrix} \quad (9)$$

where

$$o_t = [y_{t,\tau 1}, \dots, y_{t,\tau M}]^T \quad (10)$$

and τ_1, \dots, τ_M represent a set of M yield used in the estimation and $\eta_{t,\tau 1}, \dots, \eta_{t,\tau M}$ denote yield measurement errors with distribution $N(0, R)$. The state (or transition) equation for the latent state variables is the same as (1) in discrete form:

$$z_t = e^{-Kh} z_{t-h} + (I - e^{-Kh})\mu + \varepsilon_t \quad (11)$$

where $\varepsilon_t \sim N(0, \Omega_h)$ and $\Omega_h = \int_0^h e^{-Ks} \Sigma \Sigma^T e^{-K^T s} ds$; h is the data frequency expressed as a fraction of year (in this work we use monthly data, so $h = 1/12$).¹¹

Model parameters are calculated maximising the likelihood function:

$$\log L(Y_N) = \sum_{t=1}^N \log f(y_t | I_{t-1}) \quad (12)$$

where N is the number of observations and

$$f(y_t | I_{t-1}) = \left\{ (2\pi)^{-1/2} |BP_{t|t-1} B^T + R|^{-1/2} \exp \left[-\frac{1}{2} (y_t - A - B\hat{S}_{t|t-1})^T (BP_{t|t-1} B^T + R)^{-1} (y_t - A - B\hat{S}_{t|t-1}) \right] \right\}.$$

¹⁰ The assumption that all yields are measured with error seems plausible. Data-entry mistakes and interpolation methods for constructing zero-coupon yields are among the obvious sources for such errors. When all yields have errors, it is not possible to invert the yield coefficients in (6) to compute the state vector and Kalman filtering is a useful option, especially when the state vector is normally distributed.

¹¹ The integral solution is: $\text{vec}(\Omega_h) = -((K \otimes I) + (I \otimes K))^{-1} \text{vec}(e^{-Kh} \Sigma \Sigma^T e^{-K^T h} - \Sigma \Sigma)$. The term e^X , where X is a square matrix, denotes the matrix exponential operator $e^X = \sum_{k=0}^{\infty} \frac{X^k}{k!}$. For this calculation we use the `expm` function in Matlab.

After removing the term $(2\pi)^{-1/2}$ (as in Duffie, 2002), picking up the common factor and solving for the logarithm, each of the N addends of (12) becomes:

$$\log f(y_t | I_{t-1}) = -\frac{1}{2} \left\{ BP_{t|t-1} B^T + R \right\} \left(y_t - A - B \hat{S}_{t|t-1} \right)^T \left(BP_{t|t-1} B^T + R \right)^{-1} \left(y_t - A - B \hat{S}_{t|t-1} \right)$$

where the S and P matrices are, respectively, the estimates (worked out at time t-1 for time t) of the expected value and of the variance of the state variables, using the information about the parameters of the measurement and transition equations (available as at time t-1). S and P are calculated and updated at every step t (from 1 to N) using the Kalman filter algorithm described in de Jong (1999).

Long-horizon short-term interest rate expectations and the risk premium

The expected short-term interest rate in w years implicit in the model is:

$$E_t(r_{t+w}) = \rho_0 + \rho^T \left(e^{-Kw} z_t + (I - e^{-Kw}) \mu \right) \quad (13)$$

To ensure that this expectation converges to a finite value as the forecast horizon increases, we constrain the eigenvalues of K to be positive.

As the forecasting horizon w increases, the limit of (13) for the expected short-term rate collapses to the constant ρ_0 and, therefore, the estimated model attributes almost all of the variation in the long-term yield to the variation in term *premia*, with no effect on the future short-term rates expectation. On ten-year horizons, however, the expected short-term rate turns out to be sufficiently flexible.

Premia are calculated subtracting from the observed yields the risk neutral yield \tilde{y}_t^r we obtain by solving the system of differential equations (7) with μ and K replacing μ^* e K^* .

4. The sample data and the estimation strategy

As a general tenet, the length of the data sample is reckoned to be a key factor for robust results in term structure models estimation. Because of the highly persistent nature of interest rates, a data sample spanning 5 to 15 years is not likely to be sufficient to provide a reliable characterization of the dynamics of the interest rate process: in such a short period one may simply not observe a sufficient number of mean-reversions. As a result, model parameters related to the drift of the underlying state variables may not be estimated accurately (Kim and Orphanides, 2005). With reference to US data, there are only two recession periods during the last twenty years ending in 2006 (1990-91, 2001) and five points where short-term rates change directions (Figure 1).

Technically there are two main reasons why this information is not sufficient to adequately estimate the dynamics of the term structure in various phases of the business cycle.

First, it is well known that the estimation of the transition equation (11) in a short sample results in parameter values that tend to underestimate the persistence in the short-term interest rate (so that the long-horizon short-rate expectation is too stable). Kim and Orphanides (2005) report that the long-horizon short-rate expectation they obtain using a limited sample as 1990-2003 turns out to be substantially constant at around 4 per cent. This seems highly implausible given that the US long-horizon inflation expectations, as shown in the Federal Reserve Bank of Philadelphia Survey of Professional Forecasters, decreased from 4 to 2.5 per cent over the same period.

Second, in samples that are *too short*, the likelihood function may well turn out to be substantially flat along many directions in the parameter space. The function would thus have multiple local maxima with similar likelihood values but substantially different implications for the economic quantities of interest, and the standard errors of the parameters would be too large to be useful. A common approach is to restrict several parameters with large standard errors to zero and re-estimate the model (Duffee, 2002; Dai and Singleton, 2002; Duarte, 2004). This narrows confidence intervals for the parameters and for the model-implied interest rate forecasts, however the economic implications of the point estimates, e.g. regarding the evolution of long-horizon short-rate expectations, are substantially similar (Duffee, 2002).¹²

The first problem, the bias of the parameters, can be solved with Monte Carlo simulations that measure the magnitude of the bias. The second one, the multiple local maxima of the likelihood function and the lack of robustness of the estimated solution, can only be reduced by extending the sample information. Kim and Orphanides (2005) analyse the difference in long-horizon short-rate expectations that arise by increasing the depth of the time series (from 1990 to 1965 for the US) or, alternatively, by adding exogenous information such as consensus expectations from the Survey of Professional Forecasters.

We have opted for not using data on consensus expectations considering that the availability of such information for the German market is limited. For reasons of uniformity we have preferred to extend the sample depth. For the US in particular, the choice of the 1964-2006 period is

¹² As Kim and Orphanides (2005) clearly put it: "Should the implications differ, it is unclear which is "better." The issue is that setting some parameters to zero and reestimating is a rather arbitrary procedure, since the individual parameters in a flexibly specified model often do not have a simple economic meaning. Indeed, the procedure is sensitive to the normalisation employed in estimation: a model with a smaller number of free parameters (obtained by setting some parameters to zero) in one normalisation would not necessarily have the same number of free parameters in a different normalisation. More generally, it is unclear how one should determine how small the associated standard errors have to be in order to set a parameter to zero. Setting parameters to zero simply because they are estimated imprecisely also risks introducing significant biases in the resulting estimated model."

supported by the work of Kim and Orphanides (2005). They show that the estimation results on data starting from 1964 are substantially similar to those obtained adding consensus expectations about short-term rates at various horizons (6 months, 12 months and a longer period between 6 and 11 years) to shorter time series (1990-2003).¹³

Obviously, augmenting the length of the time series increases the probability of including structural breaks in the sample. For instance, it is widely accepted that: i) the cyclical fluctuations are less frequent and more attenuated today with respect to the past; ii) the relative importance of the volatility of the inflation expectations and of the real rate has significantly varied (the first has decreased compared to the second); (iii) the volatility of short-term interest rates has decreased. Numerous empirical studies show that such facts may have induced some structural modifications in the dynamics of the term structure.¹⁴ However, as far as the US economy is concerned, disagreement still exists about the nature and the significance of such structural changes.¹⁵

Taking the above discussion into account, for the US model we use time series of zero coupon rates from 12.31.1964 to 12.31.2006. For the period from 12.31.1971 to 12.31.2006 we use the dataset provided by the Fed and described in Gürkaynak, Sack and Wright (2006); for the previous period we turn to the dataset described in McCulloch and Kwon (1993), because the ten-year government bond is only available from 1971 in the Fed dataset. For the German model we use data from 12.31.1971 to 12.31.2006 provided by the Deutsche Bundesbank. In both cases, for the short-term maturities we use 3-month and 6-month money market rates.¹⁶

Following Kim and Orphanides (2005), we consider bond maturity of 1 year, 2 years, 4 years, 7 years and 10 years; the data frequency is monthly. The main difference with Kim and Orphanides (2005) is due to the use of the Fed dataset for the period starting from 12.31.1971 (they use McCulloch and Kwon data, updated by Duffee, 2002). We cannot rule out that this data difference has an impact on the estimate results. The main information for each time series used is summarized in Table 2–(I).

¹³ Expectations are reported by Blue Chip Financial Forecasts.

¹⁴ Campbell and Viceira (2001), Rudebusch and Wu (2004).

¹⁵ With regard to monetary policy, for example, evidence of structural changes highlighted in Clarida, Gali and Gertler (2000) and Cogley and Sargent (2001) contrast with the results in Orphanides (2004) and Sims and Zha (2004) which show that these changes are less important.

¹⁶ Zero coupon rates are not directly observable in the market and are generally inferred from the price of coupon bonds using quantitative techniques. For the most recent period we relied on Fed and Bundesbank data mainly because both use the Svensson (1995) method to estimate zero coupon rates and are therefore homogeneous. Unlike the bootstrapping method, both the McCulloch and Svensson approaches estimate zero coupon rates for different maturities in a simultaneous way with the aim of providing more or less smoothed interest rate curves. The McCulloch approach is based on the cubic splines technique; Svensson is an extension of the Nelson-Siegel (1987) approach and use a polynomial function with five parameters to interpolate rates. The two approaches provide very similar estimates of zero coupon rates; the second is sometimes preferred mainly because - while easier to apply - it provides robust results about long-term forward rates implied in the zero coupon term structure.

In the period under examination, the US and German term structures are both positively sloped on average. In US, the average values of the 3-month and ten-year interest rates are 5.84 per cent and 7.33 per cent respectively; in Germany, 5.32 per cent and 6.68 per cent (Table 3).

The strategy for the likelihood function maximization

The likelihood function (12) displays a high number of local maxima and, around these, is substantially flat for various dimensions of parameter space. Considering that the maximum search takes place with numerical algorithms of local optimization, a four-step maximization strategy is employed (Duffee, 2002):

1. An initial set of parameters is randomly drawn from a multivariate normal distribution with a diagonal variance-covariance matrix. The mean and variances of such multivariate distribution are fixed on plausible values;
2. If the initial parameters do not satisfy the eigenvalue conditions on the K feedback matrix, go back to the previous step, otherwise proceed;
3. A line search algorithm is employed to maximise the likelihood function (12);¹⁷
4. The parameters obtained as a result of the previous step are used as the starting values of a derivative-based algorithm to improve the accuracy of the estimates further.¹⁸

This procedure is repeated 500 times. In most repetitions a negligible increase in the maximum value of (12) is registered after a few hundred iterations. All the solutions represent a maximum value for the likelihood function (12), each reached using a different initial starting point randomly drawn from a multivariate probability distribution.

The different solutions basically imply the same dynamics for long-horizon short-rate expectation and yield risk premium. They mainly differ for a constant term. However the solution associated to the highest likelihood level has an implausible value for the ρ_0 parameter, and therefore for the infinite-horizon short-rate expectation (13).

The problem is well known in the literature. The approach we opt for to stabilize the model implications about long-horizon expectations is to constrain the value of the parameter ρ_0 to a pre-defined level. Kim and Orphanides (2005) state that this can also serve as a substitute for using the

¹⁷ We use the `fminsearch` function in Matlab that implements the simplex method. The maximum number of iteration is set to 1000; the maximum number of the valuation function is set to 10 million; the tolerance for the optimum solution and for the value function are both set to $1e-6$. The optimization algorithm is Levenberg-Marquardt. All the other options are set to standard Matlab values.

¹⁸ We use the `fmincon` function in Matlab. The options for the optimization are set to the same values as for `fminsearch`.

survey data in the estimation.¹⁹ For the US model, ρ_0 is set equal to 4.5 per cent, a plausible value considering the most recent estimates about the “natural rate”, equal to 2 per cent,²⁰ and the consensus long-term inflation expectations, equal to 2.5 per cent. For the German model, considering a “natural rate” between 1.5 and the 2 per cent and consensus long-term inflation expectations equal to 2 per cent, we set ρ_0 to 4 per cent.

The choice to constrain ρ_0 in the final solution may affect the level of the premium but that does not affect its trend, whose analysis is the main object of this paper.

The standard deviation of the parameters

The standard deviation of the parameters is calculated by bootstrapping the residuals implied by the optimal solution of the model.²¹ The following procedure is repeated 500 times:²²

1. The yields to maturity fitted by the model are subtracted from the observed yields to obtain the fitting errors implied by the optimal solution of the model;
2. A random vector of errors is drawn (bootstrapping with repetition) from the vector of fitted errors;²³
3. The vector is added to the fitted yields to obtain a new initial dataset of yields to maturity;
4. The model is estimated on the new dataset applying the search and derivative-based line algorithms in sequence. The initial set of parameters is that of the optimal solution.

The standard deviation of the 500 optimal solutions is the standard deviation of the parameters.

5. The estimate results

The parameters and fitting of the model

The optimal solutions and the standard deviations of the parameters for the US and German model are reported in Table 4.

The dynamics of the state variables (1) estimated by the model is substantially in line with that of the principal components identified by Litterman and Scheinkman (1991): i) level, ii) slope

¹⁹ “Furthermore, we have tried imposing the condition that ρ_0 be determined by the condition that the unconditional mean $E(r_t)=\rho_0+\rho^T\mu$ equals a predetermined value, e.g., $E(r_t)=0.045$. We find that this helps stabilize the model’s implications about the long-horizon expectations, and can serve as a substitute for using the long-horizon survey data in the estimation, to some extent.”, Kim and Orphanides (2005).

²⁰ Laubach and Williams (2003).

²¹ Chernick (1999).

²² The literature does not prescribe a precise number of bootstrap iterations; in general the number has increased as available computing power has increased. We limited ourselves to run 500 iterations because it took nearly 40 hours to complete the simulations on our PC (Intel Pentium 1.0 GHz).

²³ We used the `bootstrap` function in Matlab.

and iii) curvature of the term structure²⁴ (Figure 3). The solution of the system of ordinary differential equations (7) identifies the factor loadings for the state variables, whose economic interpretation is similar to that of a principal component analysis.

The dynamics of the instantaneous interest rate implicit in the model (2) is in line with that of the 3-month money market rate used for the parameter estimates (Figure 4).

The fitting of the observed yields to maturity (6) is extremely accurate, especially for long-term maturities where the average value of the Absolute Fitting Error (AFE) does not exceed 5 basis points (Figure 5 and Table 5). The analysis of the residuals shows however a significant degree of autocorrelation. This result, already described in Piazzesi (2003), may be due to the omission of an explicative variable and/or to the presence of a non linear relation that is not taken into account by the model.

The risk premium

Starting from the beginning of the 1980s, ten-year government bond risk *premia* show a general tendency to decrease in both the areas (Figure 6). The average values are 2.1 per cent in the US and 2.2 per cent in Germany; standard deviation of the *premia* is equal to 1 and 0.80 per cent respectively. At the end of 2006 *premia* are equal to 0.7 per cent in the US and to 0.5 per cent in Germany.

As far as the US premium is concerned, similar results are described in various recent papers employing different methodologies and this may be seen as a sign of the robustness of the findings. In the period 1984-2005, an approach based on a vector autoregressive model (VAR) estimates a reduction of the premium from 5 to 0.5 per cent,²⁵ with a similar model, Bernanke, Reinhart and Sack (2004) estimate a reduction from 6 to 0.5 per cent,²⁶ Cochrane and Piazzesi (2005), from 4 to -2.5 per cent,²⁷ Kim and Wright (2005) from 4 per cent to almost zero,²⁸ Rudebusch and Wu (2006) from 3 to 1.5 per cent.²⁹ Such data compares with a reduction from about 5 to 0.5 per cent obtained in this work.

Between 30 June 2004 and 30 July 2005, the Fed raised the monetary policy rate from 1.25 per cent to 3.25 per cent. Over the same period the ten-year instantaneous forward rate (as

²⁴ The level is defined as the ten-year rate, the slope as the difference between the ten-year and 3-month rates; the curvature as the difference between two times the two-year rate and the sum of the 3-month and ten-year rates.

²⁵ Result cited in Rudebusch, Sack and Swanson (2006). The variables included in the VAR model are unemployment, inflation and the 3-month interest rate.

²⁶ The VAR model forecasts were improved imposing no-arbitrage conditions.

²⁷ The estimated premium is based on a linear combination of forward rates.

²⁸ Their model is very similar to that of the present work.

²⁹ They estimate a macroeconomic structural model with no-arbitrage conditions.

calculated by the Fed) fell by about 150 basis points, from 6.4 to 4.9 per cent. Our model assigns three quarters of the decline in the forward rate to a decrease in the forward risk premium and about one quarter to a reduction in the long-horizon short-term interest rate. The result is in line with Kim and Wright (2005). If measured in terms of yield risk premium, our preferred measure, the reduction has been equal to 130 basis points (from 1.4 to 0.6 per cent).

During the first six months of 2007, ten-year government bond yields increased to 5.0 per cent (+30 bp) compared to the 3-month money market rate and the monetary policy rate substantially stable at 5.4 and 5.25 per cent respectively. Over the same period, the risk premium increased from 0.7 to about 1 per cent.

As regards the risk premium of German government bonds, the decline we obtain is larger than the one described by Pericoli and Taboga (2006) who estimate a drop from about 2 per cent at the beginning of 1984 to about 1 per cent at the end of September 2005 (respectively 2.9 and 0.5 per cent in our study).³⁰ Hördhal and Tristani (2007) estimate a reduction from 2.5 per cent at the end of 1999 to 1 per cent at the end of 2005 (from 1.6 to 0.3 per cent in our study).³¹

Obviously much care has to be taken in the comparison of the results previously described for the US and the German rates as they are obtained by solving different models with different parameter uncertainty. Moreover, although our model accurately fits the data, the standard deviation of the parameters may imply substantially different levels for risk *premia*.

6. Modern Portfolio Theory as a framework for premia determinants

The essentially affine model described in the previous sections uses a set of latent factors to estimate *premia* in long-term government bonds. Compared to alternative models that include macroeconomic variables, purely statistical ones show a better fitting performance. The price they pay for accuracy, however, is the loss of a direct economic interpretation of the *premia* determinants.

Following Backus and Wright (2007), Kim and Orphanides (2007) and Campbell, Sunderam and Viceira (2007), in this paper we concentrate on the study of the government bonds risk *premia* as a function of the risks investors are exposed to when holding these securities. We believe, however, that limiting the analysis to the domestic macroeconomic and financial context, as all these three papers do, is a severe restriction, especially if the approach has to be extended to other

³⁰ The approach is an extension of the affine model augmented with macroeconomic factors presented in Ang and Piazzesi (2003).

³¹ This is a macroeconomic structural model with no-arbitrage conditions. The authors estimate the model on a data sample with 88 monthly observations from January 1999 to April 2006.

countries apart from the US. For this reason we choose to model the systematic risk of government bonds in a context of fully integrated financial markets.

Modern Portfolio Theory (MPT) offers a simple and rigorous framework to identify the main determinants of the *premia* dynamics. The Capital Asset Pricing Model (CAPM) and its more recent extension, the International CAPM (ICAPM), state that - in equilibrium - *premia* for holding financial assets are the product between the market price of risk and the systematic risk, that is the risk that may not be diversified away by all investors contemporaneously (Pericoli and Sbracia, 2006).³²

This risk is measured by the covariance between bond returns and returns of the global market portfolio. To get a better understanding of the various dimensions of systematic risk, covariance may be further decomposed as the product of the standard deviations of bond and portfolio returns and their correlation: while the standard deviations capture the volatility of returns considered separately from each other, the correlation isolates the information regarding their co-movement and is independent of volatility.

Referring to this framework, the variables we identify to explain the dynamics of bond risk *premia* are the variability of the global economy growth rate, as measured by the standard deviation of the quarterly growth rates (with a 5-year rolling window), and the correlation between the excess returns of government bonds and those of a market portfolio diversified per asset class, geographical area and currency.

The inclusion of the variability of the global economy growth rate in a framework related to MPT may seem too subjective yet it has a strong economic intuition: a lower (higher) variability of the economic activity should be reflected, *ceteris paribus*, in a lower (higher) uncertainty of expected cash flows for all the financial assets in the market portfolio, and so for government bonds too. In addition, the global economy growth rate can be thought of as the most direct and easily available *proxy* for what is normally referred to as macroeconomic uncertainty and this allows us to

³² The assumptions of the CAPM are: i) all investors share the same expectations about the expected return and expected risk of the securities; ii) investors are risk adverse and maximize a quadratic utility function of expected wealth over a one period horizon; iii) investors perceive only the risk resulting from the portfolio of financial assets (there are no risks from other assets that constitute their wealth such as the present value of labour income); iv) investors can borrow or lend at a risk-free interest rate; v) there are no limits to investing activities (eg. no short-sale constraints; limits that prevent the increase of an exposure over a certain amount; limits on the borrowing or lending activities); vi) there are no taxes; vii) markets are perfect (individual investors cannot influence prices); viii) there are no transaction costs nor costs to acquire information. (Reilly and Brown, 2000).

check if the results of the important strand of literature that looks at *great moderation* as one of the main determinants of lower *premia* are still valid in our framework.³³

The quarterly growth rates of national economies are considered in local currency and, in order to ensure a better diversification of the aggregate, are equally weighted. The volatility of these rates shows a clear downward trend since the mid-1980s (Figure 7). We use data from national statistical agencies for GDP and from the IMF for inflation (Table 2 - (II)). Similar downward trends are obtained if, alternatively, we use the variability of: 1) growth rates of economies converted into dollars and GDP-weighted (the source of this data is the IMF; Figure 7); 2) financial markets returns on an equally-weighted portfolio consisting of all stocks and ten-year bond markets covered by Datastream³⁴ (5-year rolling window; Figure 7); 3) individual growth or inflation rates of the two economies that are the main topic of this paper: the US and Germany (Figure 8). This supports the robustness of our results.

As far as the correlation between ten-year zero coupon returns and the market portfolio is concerned, it is the most intuitive *proxy* of the capability of bonds to diversify investment portfolios. The idea underlying the CAPM is that the higher the return correlation of an asset with the returns of the market portfolio (and ultimately with the economic cycle), the higher the risk that investors holding that asset have to bear: assets that show positive returns in recession periods diversify investors' wealth and are considered less risky.

In order to keep our model parsimonious, we do not include the volatility of bond returns in the final version of the fair value model. In our data sample the dynamics of this variable are strongly related to the correlation indicator and add no further information.

Nor do we take the market price of risk explicitly into account. In general, the identification of a proxy for this variable is complex and highly uncertain. Illing and Meyer (2005) present an overview of the various risk aversion indicators proposed so far in the literature and show that many of them exhibit a divergent pattern. Tarashev and Tsatsaronis (2006) estimate the market price of risk based on the comparison between statistical and risk-neutral probability distributions of returns on several asset classes; for each of them the outcome is highly correlated with the implied volatility inferred from the option prices written on the same asset.

³³ Alternative proxies for the macroeconomic variability such as the dispersion of consensus expectations of economic growth, inflation and long-term interest rates - as used in Backus and Wright (2007) - are not available with a proper historical depth for the German case.

³⁴ BIS (2006) contains an analysis of the long-term volatility of equities and bonds in major international financial markets. Results show that volatility, while remaining above the average level over the last 150 years, has been subdued since the 1980s, especially for government bonds.

Bearing this in mind, we use the VIX index (a measure of the implied volatility in the prices of the S&P500 options elaborated by the Chicago Board Options Exchange; CBOE) as a proxy for the market price of risk. We do accept that this variable may have played a key role in the reduction of *premia* occurred during the most recent years, but a simple regression exercise shows that it cannot be considered as a significant factor to explain the downward trend of *premia* in the last two decades. During the period 2004-2006, a decrease in the US ten-year government bond premium is actually associated with the downward trend of the VIX.³⁵ However, when we add the VIX index as a third explanatory variable in our fair value model for the US premium and re-estimate it using data beginning from 1990 (when the CBOE began calculating the VIX) the relationship between the premium and VIX turns out to be not statistically significant. This is mainly due to the fact that the VIX shows no obvious trend over the 1990-2006 period.³⁶

7. Systematic risk of government bonds

The CAPM requires that the market portfolio to be used as a benchmark must include all categories of existing assets, securities and real estate, each with a weight equal to the ratio of its capitalization to the total. This section describes in detail how we identify the market portfolio and then calculate the systematic risk of government bonds.

The market portfolio

The theoretical framework we use for the choice of the market portfolio is a simplified version of the International Capital Asset Pricing Model (ICAPM). This choice is warranted by the fact that the hypothesis of the CAPM describes an excessively simplified environment (Solnik and McLeavey, 2005): investors consume the same basket of goods and services in each country and the real prices of these baskets are identical; bilateral exchange rates movement between two currencies simply reflect the changes in the interest rate differential (Purchasing Power Parity – PPP – holds at any time); the ex-ante (nominal) foreign currency risk coincides with the inflation uncertainty, which is limited on average. In this extremely simplified world, the real exchange rate is supposed to be constant.

Actually, the empirical evidence shows that the deviations of the exchange rate from the level which ensures PPP and the difference between the consumption preferences of investors in different countries are considerable, especially in the short period. The real exchange rate does not

³⁵ At the end of 2003 the implied volatility was 18 per cent and the bond risk premium was 1.5 per cent while at the end of 2006 these values were 11 and 0.70 per cent respectively.

³⁶ We cannot exclude that risk aversion is correlated with systematic risk and the impact of the former on *premia* is partly absorbed by the latter.

remain constant and then the risk that the same basket of goods and services has different (real) prices in different countries may not be negligible. According to the ICAPM, in a system of $k+1$ countries (and currencies), the expected excess return for each financial asset is expressed as:

$$E(R_i) - R_0 = \beta_{iw} RP_w + \gamma_{i1} FRP_1 + \gamma_{i2} FRP_2 + \dots + \gamma_{ik} FRP_k \quad (14)$$

where $E(R_i)$ is the expected return for asset i , R_0 is the domestic risk-free short-term interest rate, RP_w is the world market risk premium, β_{iw} is the sensitivity of asset i domestic currency returns to market movements (market exposure), FRP_1 to FRP_k are the foreign currency risk *premia* on currencies 1 to k , and γ_{i1} to γ_{ik} are the currency exposures, that is the sensitivities of asset i domestic currency returns to the exchange rate on currencies 1 to k . All returns in (14) are measured in domestic currency. The ICAPM differs from the CAPM in two fundamental aspects: 1) the relevant market risk is extracted from a globally diversified portfolio, not a domestic one; 2) further risk factors (and risk *premia*) arise because of the existence of a co-movement between asset class returns and currencies returns.³⁷ The two models coincide if, and only if, the returns on an asset (expressed in domestic currency) are not correlated with the returns on the foreign currency (also expressed in domestic currency).

Noticeably, the mean expected values of these currency risk *premia* are estimated to be around a few basis points³⁸ and these estimates are characterized by a high degree of uncertainty (Solnik and McLeavey, 2005). Hence, in the analysis that follows we assume that the currency risk *premia* are nil and maintain the result that the relevant market portfolio for the determination of the degree of systematic risk is to be considered at an international level:

$$E(R_i) - R_0 = \beta_{iw} RP_w \quad (15)$$

Theoretically, the choice of a poorly diversified market portfolio produces an inaccurate estimate of systematic risk. Although we acknowledge that with reference to US assets the choice of

³⁷ As for the CAPM, the intuition behind the ICAPM is once again that investors demand a premium for holding an asset whose risk can not be eliminated by a naïve diversification of the portfolio. The currency risk, of course, cannot be hedged in aggregate, although hedging by single investors is possible by using forward contracts for example. From an economic point of view the currencies risk *premia* arise from the fact that investors in different countries may show different degrees of risk aversion and have different net foreign investment positions. In equilibrium, this will result in positive or negative currency risk *premia*. For example, if US investors have positive net foreign investments and are more risk adverse than foreigners, their hedging demand (implemented by selling foreign currency forward) will be greater than that of foreign investors (by selling the US dollar forward). This imbalance will create a downward pressure on the foreign currency forward rate and it will be lower than the expected value of the foreign currency spot rate for the delivery date (in other words the forward premium is no longer an unbiased estimator of the expected change in the foreign currency spot rate). In equilibrium, the US investors will be willing to pay a risk premium on their hedging transactions, while foreign investors demand this premium to hold dollar assets.

³⁸ Litterman (2003) shows that US investors pay a premium of about 0.22 per cent on euro assets and requires a premium of 0.40 per cent on yen assets. In the calculation, a constant risk aversion parameter is assumed.

the S&P 500 is not overly restrictive – given the relative importance of the US economy and financial market –, the extension of this approach to the German case could be less cautious.

Moreover, while characterized by a higher degree of diversification, we cannot rule out that a global index in which individual markets are weighted according to their capitalization does not pass a formal test on portfolio efficiency. To further complicate the matter, such a test – based on the mean-variance criterion and on the exclusive use of historical data – may not provide conclusive results as investor expectations about returns to be used to determine efficient portfolios may be different from the historical estimates (Gibbons, Ross and Shanken, 1989; Grinold, 1992).

The approach we opt for to minimize this hindsight bias is the use of equally weighted portfolios, which can be thought of as efficient portfolios for investors with diffuse priors about the expected returns (means, variances and covariances are identical for all asset classes; Amenc, Goltz and Le Sourd, 2006). So we calculate the performance of a global portfolio in which all domestic stock markets and bond indices covered by Datastream are equally weighted.

At the beginning of 2007, this portfolio included 52 stock markets and 26 bond markets. The weight attributed to each of them was about 1 per cent, while the weight of the US and German markets were 36 and 4 percent respectively in the market-weighted global stock market (Table 6).³⁹ Price data used to compute global market portfolio returns are available from 1.1.1973 for equity markets and from 1.1.1980 for bond markets⁴⁰ (Table 2 – (III) and Table 2 – (IV)).

Betas, covariances and correlations

The degree of systematic risk in government bonds is measured by the covariance between the excess returns of a zero coupon bond with a maturity of 10 years and the excess returns of the market portfolio.

The formula we use to calculate the monthly returns (holding period return) of ten-year zero coupon bonds from their yield to maturity is from Campbell, Lo and MacKinlay (1997):

$$ret_{t+1} = \frac{1}{12} y_{nt} - \frac{1}{12} (n-1)(y_{n-1,t+1} - y_{n,t}) \quad (16)$$

³⁹ In early 2000, the capitalization of global equity and bond markets was approximately equal to 36 and 31 trillion dollars respectively (Dimson, Marsh and Staunton, 2002).

⁴⁰ Although extremely diversified, the market portfolio we refer to in this paper does not include real estate or corporate bonds. To support the robustness of this choice we refer to results in Stambaugh (1982). Having analyzed alternative portfolio compositions, including bonds and real estate, the author concludes that portfolio efficiency does not change significantly if the proportion of shares in the portfolio is at least equal to 10 per cent.

where ret_{t+1} is the monthly performance from t to $t+1$, $y_{n,t}$ is the ten-year zero coupon rate at time t (at the beginning of the period), $y_{n-1,t+1}$ is the zero coupon rate on bonds with a 9 years and 11 months maturity at $t+1$ (end of period), n is the maturity of the zero coupon (equal to 120 if expressed in months).

The covariance of the returns is estimated each month using a rolling window of 60 months. We calculate rolling betas (covariance standardized for the variance of market portfolio returns) and rolling correlation in the same way. All these three indicators show a similar trend within each of the two areas. Some differences emerge, however, between the two areas: while the US downward trend starting in mid-1980s is fairly clear, in Germany, the reduction starting from the mid-1990s was followed by a recovery in mid-2003 (Figure 9).⁴¹

Determining the economic causes of the reduction in the systematic risk of government bonds is beyond the scope of this work. In this regard, the most recent literature has advanced a possible hypothesis. With reference to the US market, Campbell, Sunderam and Viceira (2007) argue that this reduction may be traced back to the increase of the correlation between inflation and real economic growth that has occurred since the mid-1980s. Their underlying idea is that in a macro-economic environment with inflation following a cyclical trend, nominal bond returns display a countercyclical pattern: their risk decreases because investors may use bonds to hedge the business cycle risk.⁴²

8. Fair value model estimation and results

The equilibrium level for the ten-year bond risk premium is the fitted value in a linear regression of the risk premium (calculated using the affine model) on the same-time value of two variables: the standard deviation of the GDP growth rate and the correlation between bond and market portfolio excess returns. The use of *proxies* of systematic risk for government bonds as independent variables in the regressions allows us to consider the fitted value of the premium as the excess return that investors require to remunerate the government bonds risk.

To estimate the relationship between these three variables, a necessary first step is to verify the stationarity of the time series over the period considered. The hypothesis of stationarity has been tested using the Augmented Dickey Fuller Test (ADF). For the three variables, the test does not

⁴¹ Starting from mid-2003, US bond beta is not statistically different from zero. The same result is obtained for German bond beta starting from the end of 2004.

⁴² It is possible to interpret the change in the correlation between inflation and economic growth using the reasoning scheme of the Phillips curve. Inflation is cyclical when high economic growth is associated with high inflation (and vice versa): the macroeconomic system moves along a stable Phillips curve. Inflation is countercyclical when shocks from the supply side and/or revisions of inflation expectations affect the economic system: the Phillips curve shifts up and down.

reject the null hypothesis that they follow a random walk (Table 7); the test was then conducted for first differences, and the null of a random walk was rejected. We conclude that the variables are integrated of order 1, I(1).

The existence was then verified of a cointegration between variables, according to the Johansen methodology. The relationship between the variables can be expressed in the form of an Error Correction Model (ECM):

$$\Delta x_t = \Pi x_{t-1} + \sum_{s=1}^{l-1} \Gamma_s \Delta x_{t-s} + \varepsilon_t \quad (17)$$

where x_t is the vector of the k (three) variables considered, Π and Γ_s are matrices of coefficients ($k \times k$): $\Pi = \sum_{i=1}^l A_i - I$ and $\Gamma_i = -\sum_{j=i+1}^l A_j$, l is the number of lags.

If the matrix Π has reduced rank, i.e. $r < k$, then matrices α and β must exist both with size ($k \times r$) and rank r , so that $\Pi = \alpha\beta'$ and $\beta'x_t$ is integrated of order zero: I(0). r is the number of cointegration relationships and each column of β represents the coefficients of the cointegration vector. The elements of α represent the coefficients of the error corrections (or adjustment coefficients).

The Johansen methodology estimates the Π matrix and tests if it is possible to reject the restrictions implied by a reduced rank. Using standard information criteria (SBC and AIC) we estimate the cointegration relation with one lag. The test rejects the non-existence of a cointegration vector for both the US and the German market and does not reject the existence of at most one cointegration vector (Table 8).

The coefficients of the long-term relationship between the three variables (coefficients of the cointegration vector) have the expected sign and are statistically significant. The bond premium increases with the volatility of GDP and with the correlation between bond and market portfolio. Results show that the reduction in the correlation is not only a possible explanation for the decline in the premium but it is also independent from the reduced variability of economic cycles explanation (Figure 10).

The estimated equilibrium premium at the end of 2006 is equal to 1.1 per cent in the US (compared to an actual value estimated by the affine model equal to 0.7 per cent) and to 1.5 per cent in Germany (actual premium equal to 0.5 percent).

A simple approach to forecast future risk *premia* is using the cointegration relation we have estimated so far. Of course, this is a purely statistical exercise that does not take into account

macroeconomic factors exogenous to the model but simply extrapolates into the future the co-dynamics of the variables observed in the reference sample. The forecasted premium converges to 1 per cent, both in the US and the German case, very quickly (Figure 11).

9. Conclusions

We examined the dynamics of the risk premium required by investors in order to hold long-term US and German government bonds and tried to identify the main financial and economic determinants using Modern Portfolio Theory as a general framework.

Findings confirm the commonly held view that there has been a downward trend in bond *premia* since the mid-1980s. This is due to reduced macroeconomic uncertainty and the increased power of diversification of government bonds.

The low levels reached by *premia* at the end of 2006, 0.7 per cent in the US and 0.5 per cent in Germany, is in line with reduced levels of risk. Results of a forecasting exercise conducted using the ECM show that future *premia* on ten-year government bonds are predicted to be around 1 per cent in both areas.

This result should not be dismissed by investors holding a long-term horizon. On the basis of our analysis, long-term return expectations for ten-year government bonds will have to incorporate bond risk *premia* that, while in line with the average excess return of the whole of the 20th century, are significantly lower than average excess returns over the last two decades.

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Table 1

Long-term US and German government bonds

Real returns (in the lower triangular part of the matrix) and excess returns (in the upper triangular part); percentage values

US

	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000	2005
1900	-	1.9	2.0	1.0	0.3	0.8	0.4	-	0.1	0.3	0.6	1.0
1910	1.7	-	2.1	-0.5	1.1	1.5	0.8	0.3	0.1	0.5	0.9	1.3
1920	-1.4	-4.5	-	1.2	2.7	2.8	1.6	0.8	0.5	0.9	1.3	1.7
1930	1.3	1.1	7	-	4.3	3.6	1.7	0.7	0.4	0.9	1.3	1.7
1940	2.7	3.1	7	7.1	-	2.8	0.4	-0.5	-0.6	0.2	0.8	1.3
1950	1.7	1.8	3.9	2.4	-2.1	-	-1.9	-2.1	-1.7	-0.4	0.4	1.1
1960	1.1	0.9	2.3	0.8	-2.1	-2.2	-	-2.3	-1.5	0.1	1	1.7
1970	0.8	0.6	1.7	0.4	-1.8	-1.6	-1	-	-0.7	1.3	2.1	2.9
1980	0.5	0.3	1.1	-0.1	-1.8	-1.7	-1.4	-1.7	-	3.4	3.6	4.3
1990	1.2	1.1	1.9	1.1	0	0.5	1.4	2.6	7.2	-	3.7	4.9
2000	1.6	1.6	2.4	1.8	0.9	1.5	2.4	3.6	6.4	5.7	-	7
2005	1.9	1.9	2.7	2.2	1.4	2.1	3	4.2	6.6	6.2	7.1	-

Germany

	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000	2005
1900	-	4.0	5.4	3.0	1.5	5.4	4.2	3.2	2.6	2.2	1.8	1.4
1910	-2.3	-	6.8	-2.4	-0.5	-5.7	-4.2	-3.1	-2.4	-2	-1.5	-1.1
1920	-11.3	-19.5	-	3.4	3.1	-5.3	-3.5	-2.3	-1.6	-1.2	-0.8	-0.4
1930	-9.1	-12.7	-3.3	-	2.9	-8.6	-5.3	-3.4	-2.3	-1.8	-1.3	-0.8
1940	-4.5	-5.3	3.6	9.5	-	-18.9	-9.1	-5.4	-3.6	-2.7	-2	-1.3
1950	-8.2	-9.7	-5.9	-6.9	-20.8	-	1.8	2.2	2.1	1.8	1.8	2.2
1960	-6.2	-7	-3.4	-3.5	-9.4	3.8	-	2.5	2.2	1.8	1.8	2.3
1970	-4.9	-5.3	-2.1	-1.9	-5.4	3.5	3.1	-	1.9	1.4	1.6	2.2
1980	-4.1	-4.3	-1.4	-1.1	-3.6	2.9	2.5	1.9	-	0.9	1.5	2.3
1990	-3.1	-3.2	-0.6	-0.2	-2.1	3.3	3.1	3.1	4.4	-	2.1	3.3
2000	-2.3	-2.3	0.2	0.6	-0.8	3.8	3.8	4	5	5.6	-	5.3
2005	-1.8	-1.7	0.6	1.1	-0.1	4.1	4.1	4.4	5.4	6.1	6.8	-

Source: ABN-AMRO Global Investment Returns Yearbook 2006, Elroy Dimson, Paul Marsh and Mike Staunton.
London Business School, February 2006.

Table 2 - (I)

Data and data sources
Money market interest rates and government bond zero coupon rates

Variable name	Source	Provider	Code
BD GERMAN EURO-MARK - 3 MONTH (LONDON, EP)	Main Economic Indicators - OECD	Datastream	BDEURO3.R
BD GERMAN EURO-MARK - 6 MONTH (LONDON, EP)	Main Economic Indicators - OECD	Datastream	BDEURO6.R
BD INTEREST RATE ON NOTIONAL ZERO COUPON BONDS: 1 YEAR TO MAT	Deutsche Bundesbank	Datastream	BDWZ9808
BD INTEREST RATE ON NOTIONAL ZERO COUPON BONDS: 2 YEARS TO MAT	Deutsche Bundesbank	Datastream	BDWZ9810
BD INTEREST RATE ON NOTIONAL ZERO COUPON BONDS: 4 YEARS TO MAT	Deutsche Bundesbank	Datastream	BDWZ9814
BD INTEREST RATE ON NOTIONAL ZERO COUPON BONDS: 7 YEARS TO MAT	Deutsche Bundesbank	Datastream	BDWZ9820
BD INTEREST RATE ON NOTIONAL ZERO COUPON BONDS: 10 YRS TO MAT	Deutsche Bundesbank	Datastream	BDWZ9826
US TREASURY BILL 2ND MARKET 3 MONTH - MIDDLE RATE	FED	Datastream	FRTBS3M
US TREASURY BILL 2ND MARKET 6 MONTH - MIDDLE RATE	FED	Datastream	FRTBS6M
US INTEREST RATE ON NOTIONAL ZERO COUPON BONDS: 1 YEAR TO MAT	FED	www.federalreserve.gov/pubs/feds/2006	SVENY01
US INTEREST RATE ON NOTIONAL ZERO COUPON BONDS: 2 YEARS TO MAT	FED	www.federalreserve.gov/pubs/feds/2006	SVENY02
US INTEREST RATE ON NOTIONAL ZERO COUPON BONDS: 4 YEARS TO MAT	FED	www.federalreserve.gov/pubs/feds/2006	SVENY04
US INTEREST RATE ON NOTIONAL ZERO COUPON BONDS: 7 YEARS TO MAT	FED	www.federalreserve.gov/pubs/feds/2006	SVENY07
US INTEREST RATE ON NOTIONAL ZERO COUPON BONDS: 10 YRS TO MAT	FED	www.federalreserve.gov/pubs/feds/2006	SVENY10
US INTEREST RATE ON NOTIONAL ZERO COUPON BONDS	McCulloch-Know (1993)	www.econ.ohio-state.edu/jhm/ts/mcckwon/mccull.htm	ZEROYLD1

Table 2 – (II)

Data and data sources
US and German CPI; GDP various countries

Variable name	Datastream Code
US CHANGE IN CPI NADJ	USI64..XF
BD CHANGE IN CPI NADJ	BDI64..XF
WD WORLD GDP (CONSTANT, % CHANGE)	WDI99BPX
EA GDP CONA	EAGDP...D
EM GDP CONA	EMGDP...D
BD GDP CONA	BDGDP...D
CB GDP CONA	CBGDP...D
CL GDP CONA	CLGDP...D
CZ GDP CONA	CZGDP...D
FN GDP CONA	FNGDP...D
GR GDP CONA	GRGDP...D
IR GDP CONA	IRGDP...D
IS GDP CONA	ISGDP...D
IT GDP CONA	ITGDP...D
KO GDP CONA	KOGDP...D
NW GDP CONA	NWGDP...D
PH GDP CONA	PHGDP...D
PT GDP CONA	PTGDP...D
SD GDP CONA	SDGDP...D
SW GDP CONA	SWGDP...D
TH GDP CONA	THGDP...D
MX GDP CONA	MXGDP...D
HN GDP CONA	HNGDP...D
OE GDP CONA	OEGDP...D
VE GDP CONA	VEGDP...D
EJ GDP CONA	EJGDP...D
EX GDP CONA	EXGDP...D
FR GDP CONA	FRGDP...D
NZ GDP CONA	NZGDP...D
AG GDP CONA	AGGDP...D
JP GDP CONA	JPGDP...D
BG GDP CONA	BGGDP...D
LX GDP CONA	LXGDP...D
AU GDP CONA	AUGDP...D
DK GDP CONA	DKGDP...D
SP GDP CONA	SPGDP...D
SA GDP CONA	SAGDP...D
CN GDP CONA	CNGDP...D
UK GDP CONA	UKGDP...D
NL GDP CONA	NLGDP...D
LN GDP CONA	LNGDP...D
CP GDP CONA	CPGDP...D
US GDP CONA	USGDP...D

Table 2 – (III)

**Data and data sources
Datastream equity indices**

Variable name	Datastream Code
ARGENTINA-DS Market	S51324(RI)
AUSTRALIA-DS Market	S53424(RI)
GERMANY-DS Market	S43224(RI)
BELGIUM-DS Market	S51924(RI)
BULGARIA-DS Market	L13224(RI)
BRAZIL-DS Market	S59224(RI)
COLOMBIA-DS Market	S58224(RI)
CHINA-DS Market	L09624(RI)
CHILE-DS Market	S58024(RI)
CANADA-DS Market	S52324(RI)
CYPRUS-DS Market	L12424(RI)
SRI LANKA-DS Market	L11424(RI)
CZECH REP.-DS Market	L08224(RI)
DENMARK-DS Market	S50024(RI)
SPAIN-DS Market	S50824(RI)
FINLAND-DS Market	S50224(RI)
FRANCE-DS Market	S42824(RI)
GREECE-DS Market	S53224(RI)
HONG KONG-DS Market	S41224(RI)
HUNGARY-DS Market	L08424(RI)
INDONESIA-DS Market	S53624(RI)
INDIA-DS Market	S41424(RI)
IRELAND-DS Market	S52924(RI)
ISRAEL-DS Market	L09024(RI)
ITALY-DS Market	S47824(RI)
JAPAN-DS Market	S42024(RI)
KOREA-DS Market	S51124(RI)
LUXEMBURG-DS Market	S41024(RI)
MEXICO-DS Market	S50624(RI)
MALAYSIA-DS Market	S53824(RI)
NETHERLAND-DS Market	S42424(RI)
NORWAY-DS Market	S52124(RI)
NEW ZEALAN-DS Market	S54024(RI)
AUSTRIA-DS Market	S51724(RI)
PERU-DS Market	S58624(RI)
PHILIPPINE-DS Market	S54224(RI)
PAKISTAN-DS Market	L11024(RI)
POLAND-DS Market	S59624(RI)
PORTUGAL-DS Market	S55024(RI)
ROMANIA-DS Market	L11224(RI)
RUSSIA-DS Market	L08624(RI)
SOUTH AFRI-DS Market	S51524(RI)
SWEDEN-DS Market	S52724(RI)
SINGAPORE-DS Market	S54424(RI)
SLOVENIA-DS Market	L13424(RI)
SWITZ-DS Market	S91824(RI)
TAIWAN-DS Market	S54624(RI)
THAILAND-DS Market	S54824(RI)
TURKEY-DS Market	S55224(RI)
UK-DS Market	S19824(RI)
US-DS Market	S41624(RI)
VENEZUELA-DS Market	L12624(RI)

Table 2 – (IV)

Data and data sources
Datastream government bond indices (10-year constant maturity)

Variable name	Datastream Code
AU BENCHMARK 10 YEAR DS GOVT. INDEX	BMAU10Y(RI)
BD BENCHMARK 10 YEAR DS GOVT. INDEX	BMBD10Y(RI)
BG BENCHMARK 10 YEAR DS GOVT. INDEX	BMBG10Y(RI)
CN BENCHMARK 10 YEAR DS GOVT. INDEX	BMCN10Y(RI)
CZ BENCHMARK 10 YEAR DS GOVT. INDEX	BMCZ10Y(RI)
DK BENCHMARK 10 YEAR DS GOVT. INDEX	BMDK10Y(RI)
EMU BENCHMARK 10 YR. DS GOVT. INDEX	BMEM10Y(RI)
ES BENCHMARK 10 YEAR DS GOVT. INDEX	BMES10Y(RI)
FN BENCHMARK 10 YEAR DS GOVT. INDEX	BMFN10Y(RI)
FR BENCHMARK 10 YEAR DS GOVT. INDEX	BMFR10Y(RI)
GR BENCHMARK 10 YEAR DS GOVT. INDEX	BMGR10Y(RI)
HN BENCHMARK 10 YEAR DS GOVT. INDEX	BMHN10Y(RI)
IR BENCHMARK 10 YEAR DS GOVT. INDEX	BMIR10Y(RI)
IT BENCHMARK 10 YEAR DS GOVT. INDEX	BMIT10Y(RI)
JP BENCHMARK 10 YEAR DS GOVT. INDEX	BMJP10Y(RI)
NL BENCHMARK 10 YEAR DS GOVT. INDEX	BMNL10Y(RI)
NW BENCHMARK 10 YEAR DS GOVT. INDEX	BMNW10Y(RI)
NZ BENCHMARK 10 YEAR DS GOVT. INDEX	BMNZ10Y(RI)
OE BENCHMARK 10 YEAR DS GOVT. INDEX	BMOE10Y(RI)
PO BENCHMARK 10 YEAR DS GOVT. INDEX	BMPO10Y(RI)
PT BENCHMARK 10 YEAR DS GOVT. INDEX	BMPT10Y(RI)
SA BENCHMARK 10 YEAR DS GOVT. INDEX	BMSA10Y(RI)
SD BENCHMARK 10 YEAR DS GOVT. INDEX	BMSD10Y(RI)
SW BENCHMARK 10 YEAR DS GOVT. INDEX	BMSW10Y(RI)
UK BENCHMARK 10 YEAR DS GOVT. INDEX	BMUK10Y(RI)
US BENCHMARK 10 YEAR DS GOVT. INDEX	BMUS10Y(RI)

Table 3**Empirical distribution of zero coupon interest rates over time****US – Monthly data from 31.12.1964 to 31.12.2006**

	3 months	6 months	1 year	2 years	4 years	7 years	10 years
Average	5.84%	5.99%	6.41%	6.64%	6.92%	7.17%	7.33%
Standard deviation	2.71%	2.69%	2.78%	2.66%	2.51%	2.38%	2.30%
Asymmetry	0.957	0.863	0.789	0.777	0.838	0.917	0.960
Curtosis	4.568	4.314	4.077	3.903	3.680	3.566	3.533
Minimum	0.89%	0.96%	1.03%	1.33%	1.98%	2.95%	3.67%
1st quartile	4.36%	4.51%	4.83%	5.03%	5.32%	5.55%	5.71%
Median	5.32%	5.50%	5.95%	6.26%	6.59%	6.80%	6.98%
3rd quartile	7.22%	7.34%	7.80%	7.93%	8.02%	8.19%	8.32%
Maximum	15.52%	15.69%	16.11%	15.78%	15.35%	14.99%	14.89%

Germany – Monthly data from 31.12.1971 to 31.12.2006

	3 months	6 months	1 year	2 years	4 years	7 years	10 years
Average	5.32%	5.32%	5.39%	5.65%	6.09%	6.47%	6.68%
Standard deviation	2.54%	2.48%	2.34%	2.19%	2.01%	1.83%	1.69%
Asymmetry	0.968	0.934	0.806	0.551	0.252	0.015	- 0.140
Curtosis	3.236	3.206	3.122	2.683	2.345	2.189	2.160
Minimum	2.01%	1.89%	1.93%	2.04%	2.38%	2.89%	3.21%
1st quartile	3.45%	3.49%	3.65%	3.99%	4.60%	5.12%	5.32%
Median	4.60%	4.63%	4.83%	5.21%	5.85%	6.46%	6.79%
3rd quartile	6.58%	6.69%	6.75%	7.12%	7.68%	8.03%	8.02%
Maximum	13.60%	13.69%	13.17%	12.33%	11.76%	10.90%	10.24%

Table 4

Affine model parameter estimates

$$dz_t = K(\mu - z_t)dt + \Sigma d\omega_t ; r_t = \rho_o + \rho^T z_t ; \lambda_t = \lambda_a + \Lambda_b z_t$$

$$[y_{t,\tau_1}, \dots, y_{t,\tau_M}]^T = \begin{bmatrix} A(\tau_1) \\ \vdots \\ A(\tau_M) \end{bmatrix} + \begin{bmatrix} B(\tau_1)^T \\ \vdots \\ B(\tau_M)^T \end{bmatrix} z_t + \begin{bmatrix} \eta_{t,\tau_1} \\ \vdots \\ \eta_{t,\tau_M} \end{bmatrix}$$

N	variable	US		GERMANY	
		optimum value	standard deviation	optimum value	standard deviation
1	ρ_o	0.045	constrained	0.04	constrained
2	K(1,1)	0.10	0.06	0.64	0.15
3	K(2,1)	-0.02	0.18	-0.90	0.38
4	K(3,1)	0.34	0.30	-0.88	0.77
5	K(1,2)	0.00	constrained	0.00	constrained
6	K(2,2)	1.69	0.30	0.10	0.00
7	K(3,2)	-0.11	0.32	0.50	0.15
8	K(1,3)	0.00	constrained	0.00	constrained
9	K(2,3)	0.00	constrained	0.00	constrained
10	K(3,3)	0.36	0.07	0.75	0.19
11	$\Sigma(1,1)$	0.0486	0.0032	0.0125	0.0015
12	$\Sigma(2,1)$	0.0000	constrained	0.0000	constrained
13	$\Sigma(3,1)$	0.0000	constrained	0.0000	constrained
14	$\Sigma(1,2)$	0.0000	constrained	0.0000	constrained
15	$\Sigma(2,2)$	0.0261	0.0035	0.0176	0.0020
16	$\Sigma(3,2)$	0.0000	constrained	0.0000	constrained
17	$\Sigma(1,3)$	0.0000	constrained	0.0000	constrained
18	$\Sigma(2,3)$	0.0000	constrained	0.0000	constrained
19	$\Sigma(3,3)$	0.0394	0.0031	0.0203	0.0010
20	$\lambda(1,1)$	0.01	0.06	0.13	0.11
21	$\lambda(2,1)$	0.20	0.34	-0.46	0.07
22	$\lambda(3,1)$	-0.10	0.12	0.01	0.02
23	$\Lambda(1,1)$	0.97	0.93	0.82	1.37
24	$\Lambda(2,1)$	-0.01	0.03	0.01	0.02
25	$\Lambda(3,1)$	-2.98	2.53	-0.17	0.21
26	$\Lambda(1,2)$	2.37	3.92	1.45	2.00
27	$\Lambda(2,2)$	0.15	0.28	0.83	1.41
28	$\Lambda(3,2)$	0.72	1.25	1.31	2.38
29	$\Lambda(1,3)$	-0.60	0.43	-1.84	2.63
30	$\Lambda(2,3)$	-0.03	0.07	0.02	0.03
31	$\Lambda(3,3)$	1.08	1.64	0.40	0.61
32	$\eta(1,1)$	0.0012	0.0003	0.0026	0.0002
33	$\eta(2,1)$	0.0014	0.0002	0.0024	0.0004
34	$\eta(3,1)$	0.0021	0.0003	0.0011	0.0001
35	$\eta(4,1)$	0.0008	0.0003	0.0007	0.0001
36	$\eta(5,1)$	0.0008	0.0003	0.0003	0.0000
37	$\eta(6,1)$	0.0006	0.0002	0.0004	0.0001
38	$\eta(7,1)$	0.0000	0.0001	0.0009	0.0001

Table 5

Empirical distribution of errors over time

US							
	3m	6m	1y	2y	4y	7y	10y
Average	0.00%	-0.04%	0.10%	0.01%	-0.01%	0.00%	0.00%
Standard deviation	0.09%	0.10%	0.15%	0.06%	0.06%	0.05%	0.00%
Asymmetry	-1.008	0.143	1.051	0.745	-0.530	-0.584	0.680
Curtosis	6.761	5.672	6.356	8.326	6.922	8.801	8.518
Minimum	-0.51%	-0.56%	-0.30%	-0.22%	-0.40%	-0.04%	0.00%
Maximum	0.32%	0.37%	0.97%	0.38%	0.21%	0.04%	0.00%
Avg Absolute Fitting Errors	0.06%	0.08%	0.13%	0.05%	0.04%	0.02%	0.00%
<i>Correlation matrix</i>							
6m	-0.556						
1y	-0.649	0.190					
2y	-0.613	-0.068	0.705				
4y	0.430	-0.390	-0.772	-0.636			
7y	0.417	-0.232	-0.656	-0.778	0.834		
10y	-0.431	0.296	0.686	0.725	-0.888	-0.958	
<i>Autocorrelation coefficients</i>							
<i>Lags</i>							
1	0.50	0.63	0.73	0.64	0.67	0.72	0.66
2	0.36	0.54	0.60	0.50	0.55	0.62	0.59
3	0.27	0.41	0.51	0.44	0.44	0.56	0.51
4	0.19	0.37	0.42	0.33	0.35	0.48	0.47
5	0.25	0.36	0.43	0.31	0.28	0.44	0.41
6	0.24	0.38	0.40	0.30	0.31	0.42	0.39
7	0.22	0.31	0.37	0.24	0.28	0.38	0.37
8	0.14	0.35	0.34	0.22	0.31	0.35	0.37
9	0.17	0.40	0.35	0.20	0.28	0.36	0.38
10	0.19	0.40	0.32	0.22	0.28	0.34	0.35
GERMANY							
	3m	6m	1y	2y	4y	7y	10y
Average	0.07%	0.03%	-0.02%	-0.01%	0.00%	0.00%	0.00%
Standard deviation	0.23%	0.22%	0.08%	0.06%	0.02%	0.02%	0.06%
Asymmetry	0.034	2.129	0.706	-0.294	0.560	0.407	-0.671
Curtosis	4.914	27.163	11.407	4.377	3.859	4.200	5.373
Minimum	-1.01%	-0.91%	-0.32%	-0.21%	-0.05%	-0.06%	-0.27%
Maximum	0.82%	2.17%	0.55%	0.20%	0.09%	0.09%	0.18%
Avg Absolute Fitting Errors	0.18%	0.14%	0.05%	0.05%	0.02%	0.02%	0.05%
<i>Correlation matrix</i>							
6m	-0.070						
1y	-0.553	-0.413					
2y	-0.356	-0.342	-0.034				
4y	0.290	0.004	-0.389	-0.308			
7y	0.020	0.270	0.255	-0.477	-0.626		
10y	-0.091	-0.013	-0.024	0.468	-0.714	0.157	
<i>Autocorrelation coefficients</i>							
<i>Lags</i>							
1	0.59	0.37	0.61	0.66	0.49	0.37	0.68
2	0.42	0.29	0.50	0.52	0.35	0.17	0.55
3	0.37	0.24	0.42	0.42	0.29	0.16	0.45
4	0.30	0.16	0.34	0.37	0.25	0.14	0.34
5	0.29	0.22	0.33	0.32	0.18	0.11	0.27
6	0.28	0.12	0.28	0.29	0.08	0.03	0.22
7	0.23	0.16	0.28	0.21	0.11	0.09	0.24
8	0.25	0.15	0.25	0.18	0.07	0.06	0.20
9	0.22	0.06	0.23	0.11	0.04	-0.03	0.19
10	0.20	0.15	0.19	0.11	0.04	-0.01	0.18

Table 6

Global market portfolio composition

DATE	German stock market-weight in the market weighted stock market world index	US stock market weight in the market-weighted stock market world index	Countries in the global stock market index	Countries in the global bond market index	Countries in the global market index	Single market weight in the equally-weighted global market portfolio
1973	3.5%	64.9%	17	0	17	5.9%
1974	4.1%	63.8%	17	0	17	5.9%
1975	5.6%	61.1%	17	0	17	5.9%
1976	4.9%	62.0%	17	0	17	5.9%
1977	4.9%	60.8%	17	0	17	5.9%
1978	6.2%	53.3%	17	0	17	5.9%
1979	6.1%	49.0%	17	0	17	5.9%
1980	5.4%	49.5%	17	4	21	4.8%
1981	3.6%	48.5%	18	4	22	4.5%
1982	3.5%	48.5%	18	5	23	4.3%
1983	3.2%	54.9%	19	4	23	4.3%
1984	3.2%	52.1%	19	5	24	4.2%
1985	3.0%	51.9%	19	9	28	3.6%
1986	4.9%	46.0%	19	10	29	3.4%
1987	3.8%	37.1%	20	10	30	3.3%
1988	2.5%	28.8%	25	13	38	2.6%
1989	2.6%	25.5%	29	13	42	2.4%
1990	3.7%	25.2%	31	18	49	2.0%
1991	4.0%	30.6%	36	15	51	2.0%
1992	3.6%	33.7%	37	19	56	1.8%
1993	3.5%	38.4%	41	20	61	1.6%
1994	3.3%	31.7%	45	21	66	1.5%
1995	3.5%	32.1%	49	20	69	1.4%
1996	3.5%	36.3%	49	20	69	1.4%
1997	3.8%	40.4%	50	20	70	1.4%
1998	4.1%	44.9%	50	20	70	1.4%
1999	4.6%	49.3%	51	22	73	1.4%
2000	4.1%	45.0%	51	23	74	1.4%
2001	3.7%	47.9%	52	26	78	1.3%
2002	3.7%	50.2%	52	26	78	1.3%
2003	3.2%	46.8%	52	26	78	1.3%
2004	3.6%	44.3%	52	26	78	1.3%
2005	3.5%	41.7%	52	26	78	1.3%
2006	3.5%	38.1%	52	26	78	1.3%
2007	3.7%	36.2%	52	26	78	1.3%

Table 7

Augmented Dickey – Fuller (ADF) test

$$t\text{-stat of the } \rho \text{ parameter in the regression: } z_t = a + \rho z_{t-1} + \sum_{s=1}^l \rho_s \Delta z_{t-s} + \varepsilon_t$$

US

Lag	1	2	3	4	5	6	7	8
term premium	-1.16	-1.07	-1.31	-1.44	-1.19	-1.06	-0.89	-1.13
correlation	-1.52	-1.51	-1.71	-1.86	-1.64	-1.82	-1.76	-1.24
vol gdp 5y	-1.81	-2.22	-1.85	-1.89	-1.05	-0.99	-0.94	-1.10

Lag	1	2	3	4	5	6	7	8
d(term premium)	-16.57	-13.70	-12.28	-12.07	-11.74	-11.75	-11.13	-11.61
d(correlation)	-16.15	-13.24	-11.97	-11.84	-11.25	-11.16	-11.05	-10.00
d(vol gdp 5y)	-14.68	-12.68	-10.81	-11.47	-9.84	-10.19	-9.74	-9.34

GERMANY

Lag	1	2	3	4	5	6	7	8
term premium	-0.38	-0.66	-0.27	-0.66	-0.70	-0.32	0.54	0.15
correlation	-1.32	-1.52	-1.54	-1.62	-1.64	-1.81	-2.64	-2.62
vol gdp 5y	-1.81	-2.22	-1.85	-1.89	-1.05	-0.99	-0.94	-1.10

Lag	1	2	3	4	5	6	7	8
d(term premium)	-11.97	-11.50	-10.10	-9.64	-9.88	-10.09	-8.66	-10.10
d(correlation)	-14.00	-12.21	-11.29	-11.02	-11.92	-10.24	-10.23	-10.13
d(vol gdp 5y)	-14.68	-12.68	-10.81	-11.47	-9.84	-10.19	-9.74	-9.34

The null hypothesis is that the series have unit roots. The 95° per cent critical value is -2.9152.

Table 8

Cointegration estimates

$$\Delta brp_t = \alpha_{brp} (brp_{t-1} + a * corr_{t-1} + b * vol_{t-1} + c) + d_{brp} \Delta brp_{t-1} + e_{brp} \Delta corr_{t-1} + f_{brp} \Delta vol_{t-1} + \varepsilon_{brp,t}$$

$$\Delta corr_t = \alpha_{corr} (brp_{t-1} + a * corr_{t-1} + b * vol_{t-1} + c) + d_{corr} \Delta brp_{t-1} + e_{corr} \Delta corr_{t-1} + f_{corr} \Delta vol_{t-1} + \varepsilon_{corr,t}$$

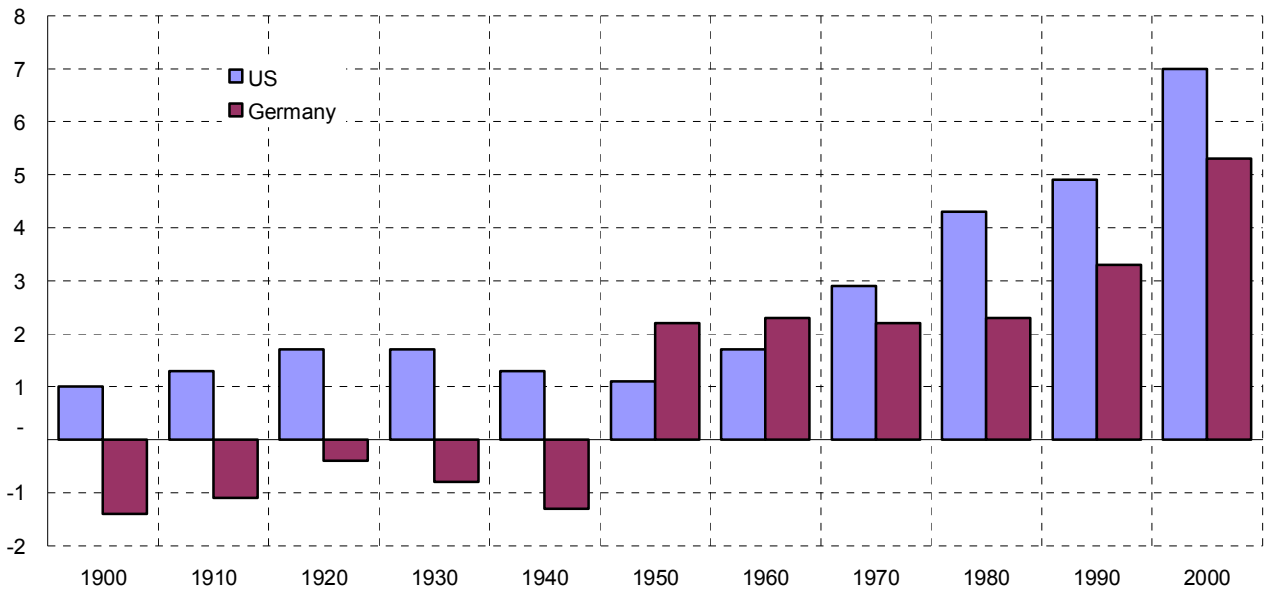
$$\Delta vol_t = \alpha_{vol} (brp_{t-1} + a * corr_{t-1} + b * vol_{t-1} + c) + d_{vol} \Delta brp_{t-1} + e_{vol} \Delta corr_{t-1} + f_{vol} \Delta vol_{t-1} + \varepsilon_{vol,t}$$

	US			Germany		
Cointegration test						
	None	At most 1		None	At most 1	
p-value (Trace stat)	0.02	0.50		0.15	0.90	
p-value (Maximum Eigenvalue)	0.01	0.42		0.04	0.90	
Cointegration vector						
	Coefficient	Standard error	t-stat	Coefficient	Standard error	t-stat
term premium	1			1		
correlation	-0.02	-0.01	3.43	-0.03	-0.01	4.65
vol gdp	-11.60	-1.30	8.89	-7.35	-1.18	6.21
intercept	0.03	-0.01	-4.84	0.01	-0.01	-2.01
Term premium equation						
adjustement	-0.07	-0.04	1.55	-0.09	-0.04	2.19
lag 1 - term premium	-0.02	-0.10	0.23	0.20	-0.10	-2.11
lag 1 - correlation	0.00	0.00	-0.66	0.00	-0.01	0.72
lag 1 - vol GDP	1.74	-1.08	-1.61	-0.05	-0.88	0.06
Correlation equation						
adjustement	0.55	-0.85	-0.65	-0.10	-0.72	0.13
lag 1 - term premium	-1.19	-1.87	0.64	3.80	-1.71	-2.22
lag 1 - correlation	0.00	-0.10	0.00	0.07	-0.10	-0.73
lag 1 - vol GDP	32.31	-20.92	-1.54	4.13	-15.58	-0.27
Vol GDP equation						
adjustement	0.02	0.00	-4.87	0.02	0.00	-4.19
lag 1 - term premium	0.00	-0.01	-0.54	0.00	-0.01	-0.25
lag 1 - correlation	0.00	0.00	-0.69	0.00	0.00	-0.42
lag 1 - vol GDP	0.05	-0.08	-0.64	0.02	-0.09	-0.27

brp is the bond risk premium as estimated by the affine term structure model, $corr$ is the correlation between the bond and the market portfolio excess returns, vol is the standard deviation of the GDP growth rate. α_{brp} , α_{corr} , α_{vol} are the adjustment coefficients (short-term error correction); a , b , and c are the coefficients of the cointegration vector (long-run equation); d , e , and f are the lag coefficients.

Figure 1

Average bond risk premium from year (x) to 2005
 (percentage values; source: Dimson, March and Staunton, 2006)



Average bond risk premium from 1900 to year (x)
 (percentage values; source: Dimson, March and Staunton, 2006)

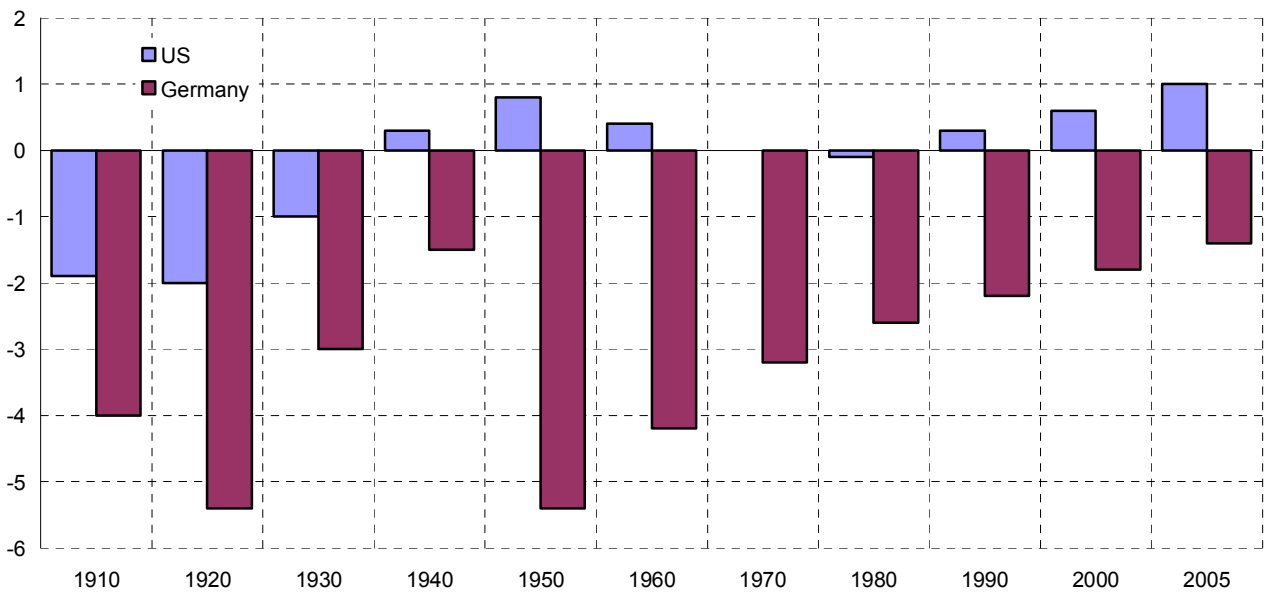
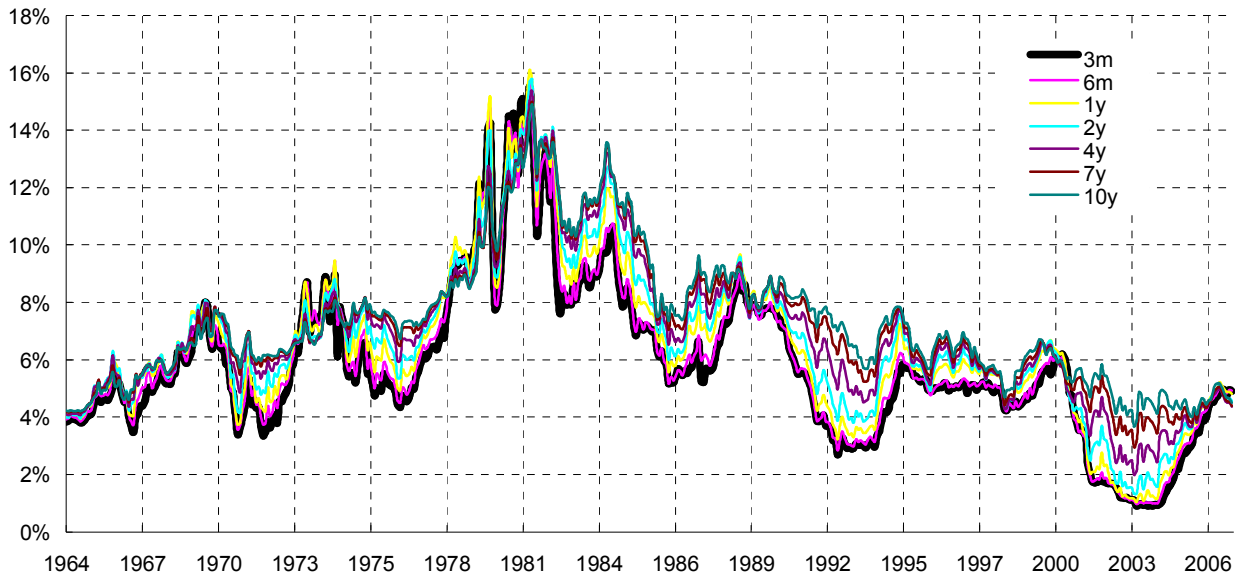


Figure 2

US zero coupon bond rates
(monthly data from 31.12.1964 to 31.12.2006)



German zero coupon bond rates
(monthly data from 31.12.1974 to 31.12.2006)

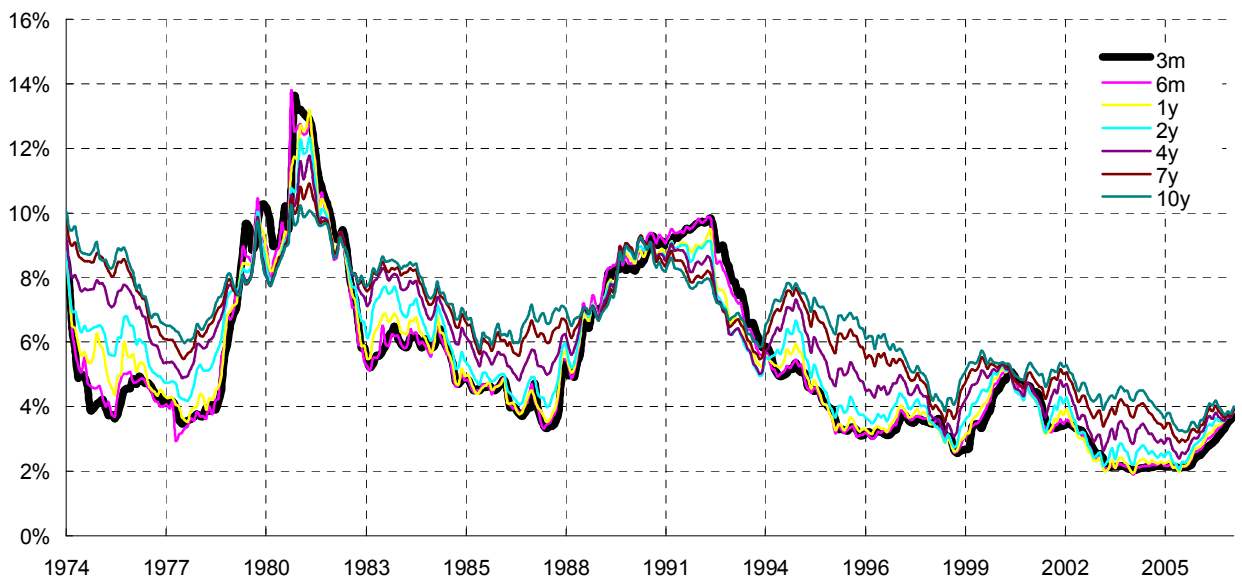


Figure 3

State variables
(monthly data from 31.12.1964 for US and 31.12.1974 for Germany)

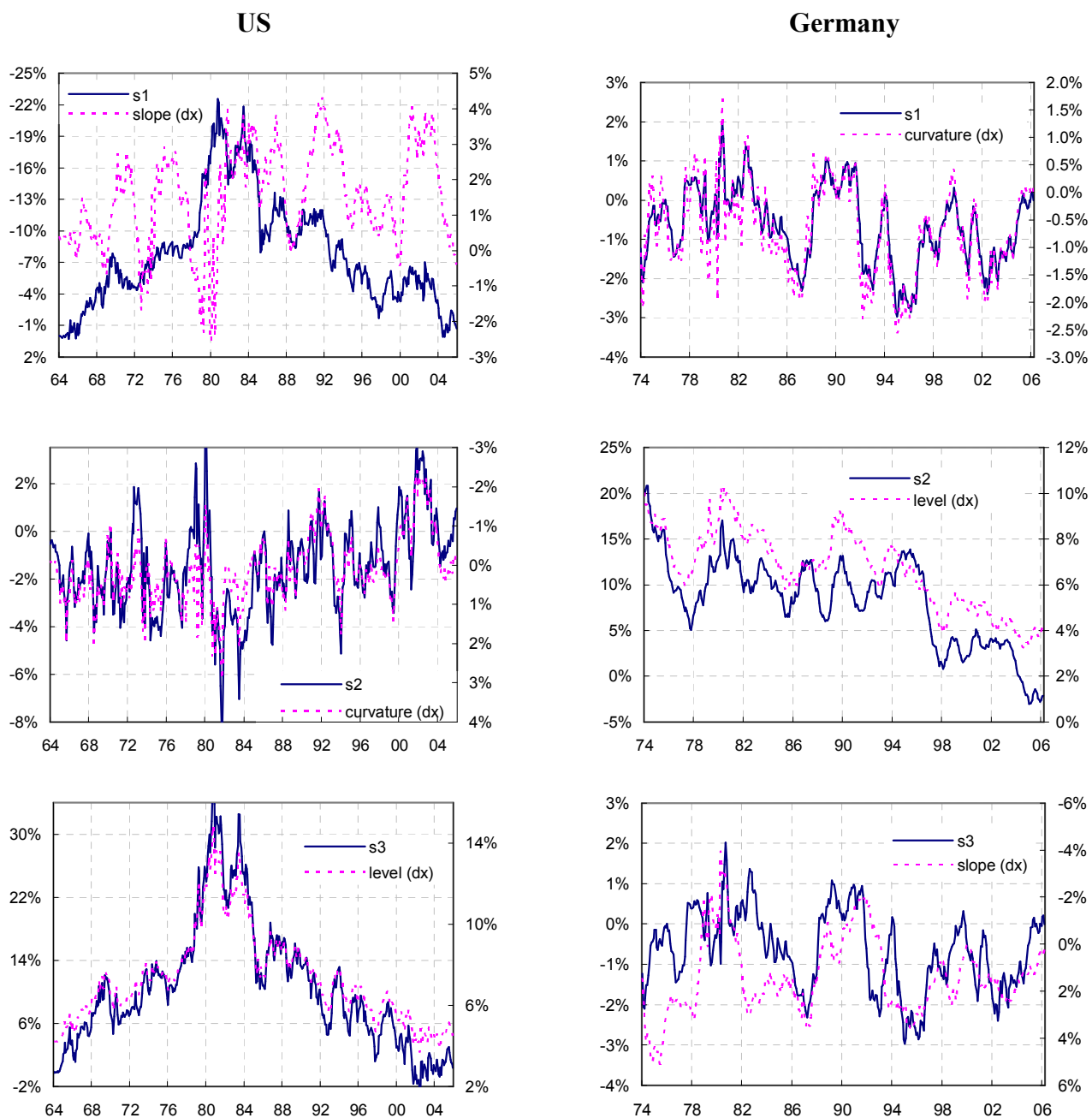
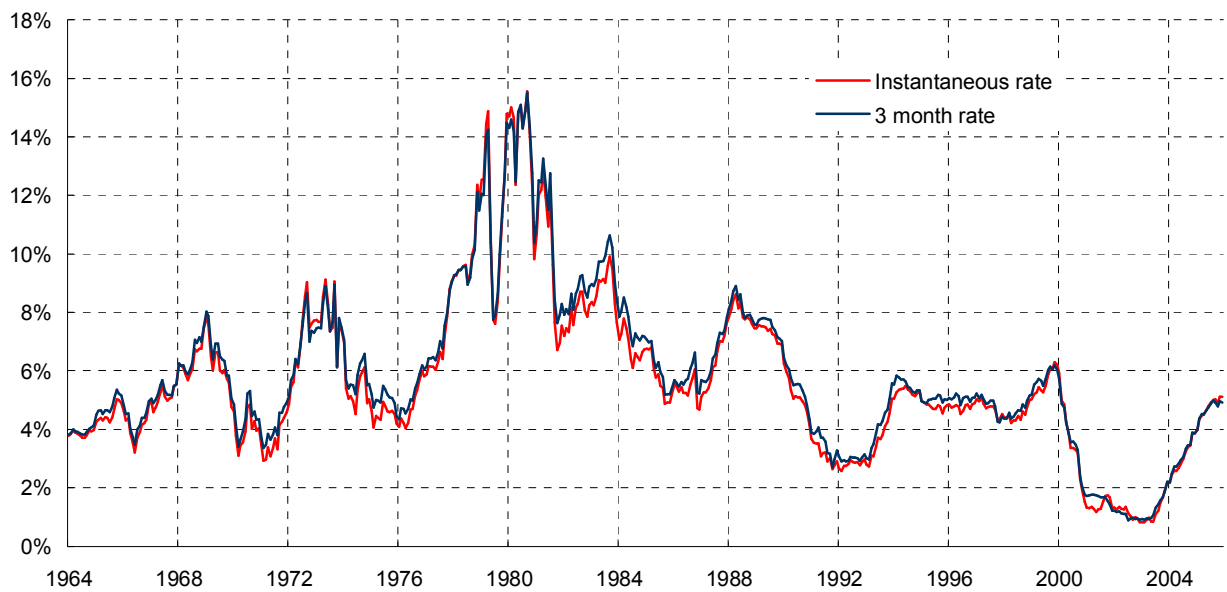


Figure 4

US 3-month interest rate and instantaneous rate estimated with the affine model
(monthly data from 31.12.1964 to 31.12.2006)



German 3-month interest rate and instantaneous rate estimated with the affine model
(monthly data from 31.12.1974 to 31.12.2006)

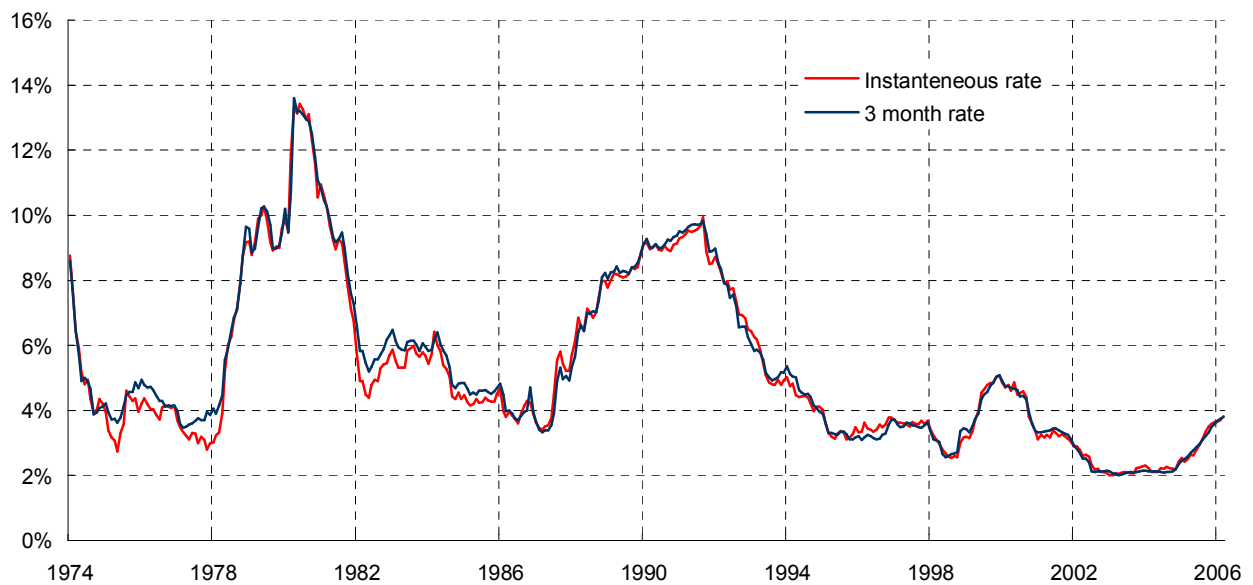
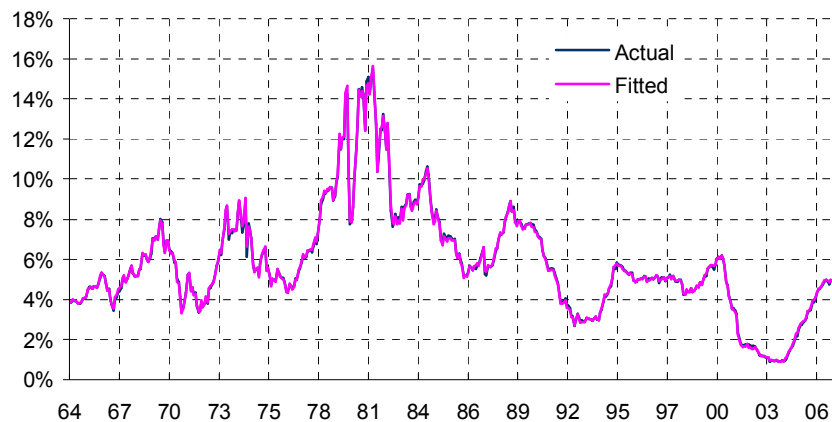


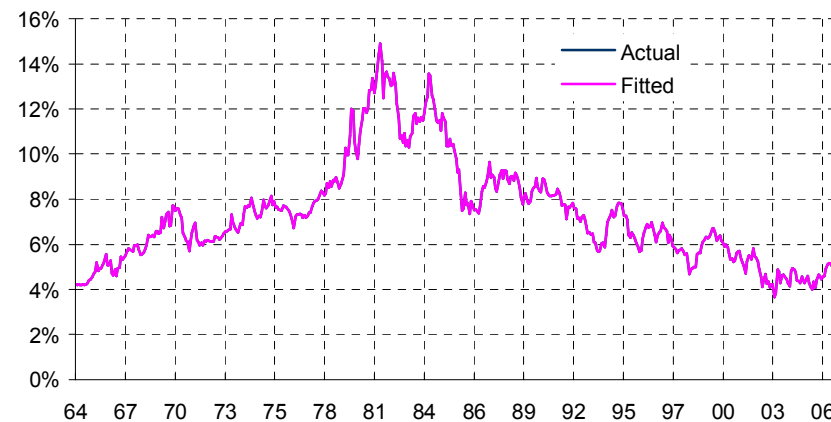
Figure 5

Actual and fitted zero coupon rates
(monthly data from 31.12.1964 for US and 31.12.1974 for Germany)

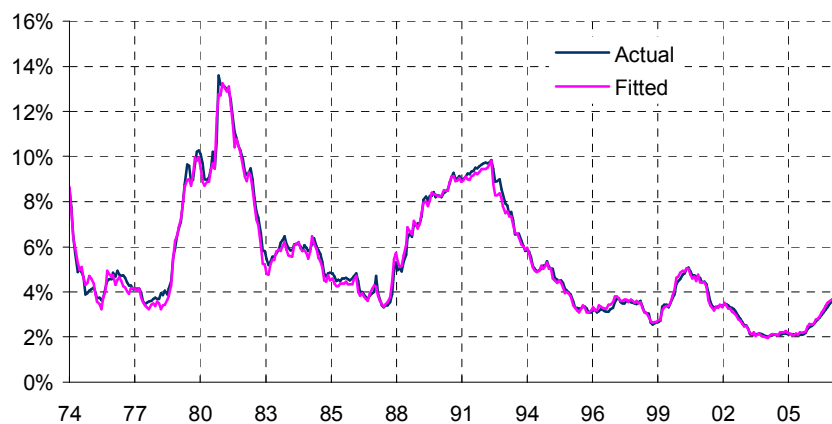
US – 3-month rate



US – 10-year rate



Germany – 3-month rate



Germany – 10-year rate

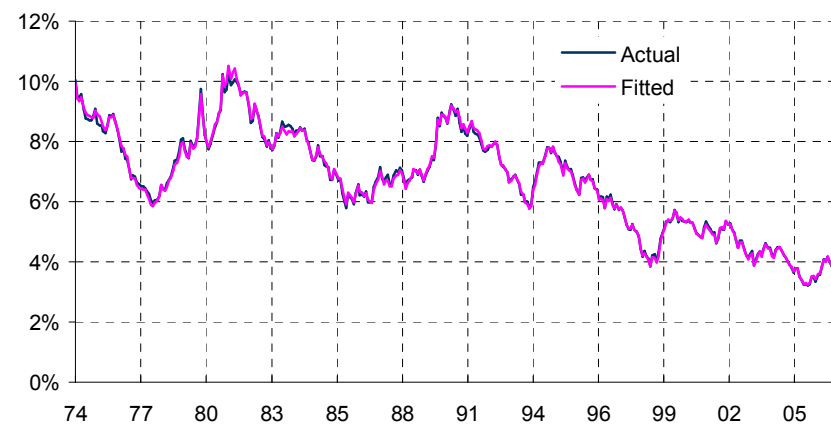
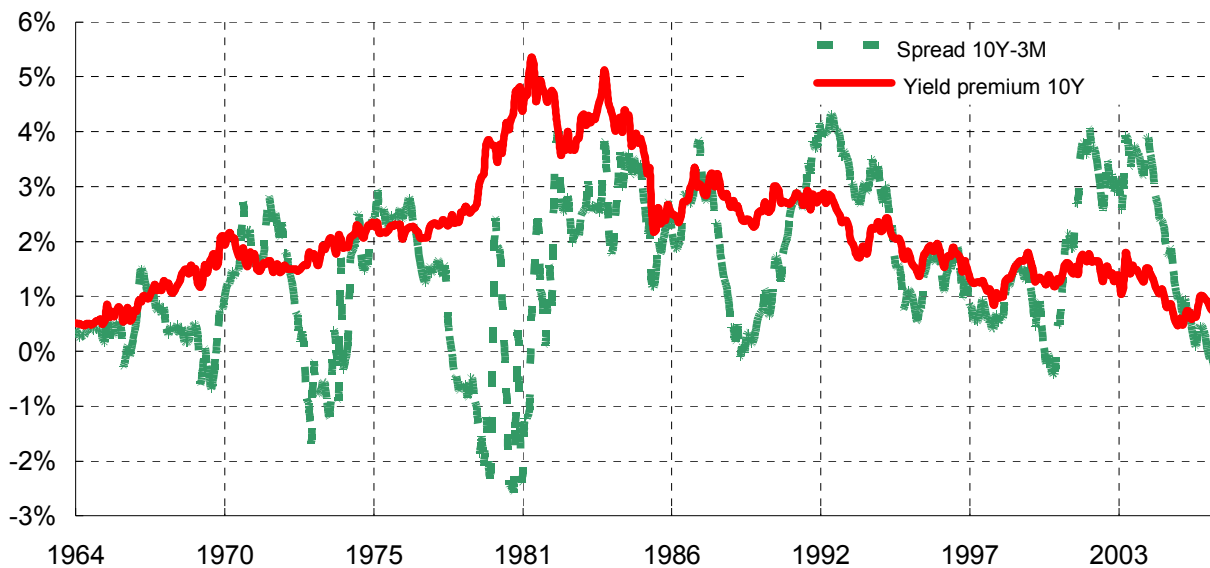


Figure 6

US – Bond risk premium and 10-year – 3-month interest rate spread
(monthly data from 31.12.1964 to 31.12.2006)



Germany – Bond risk premium and 10-year – 3-month interest rate spread
(monthly data from 31.12.1974 to 31.12.2006)

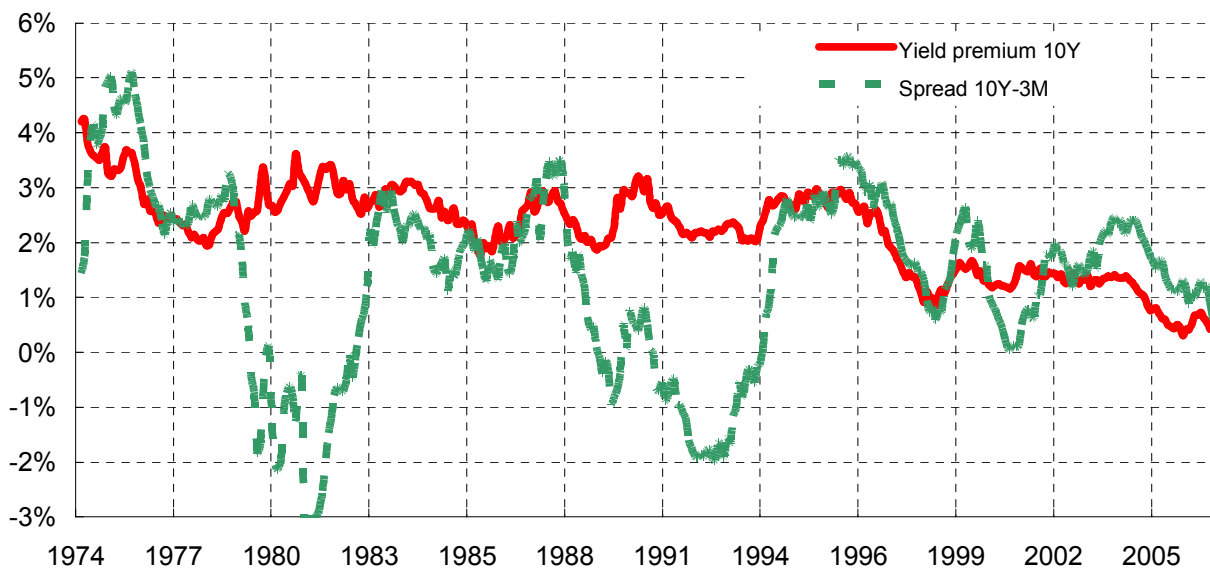
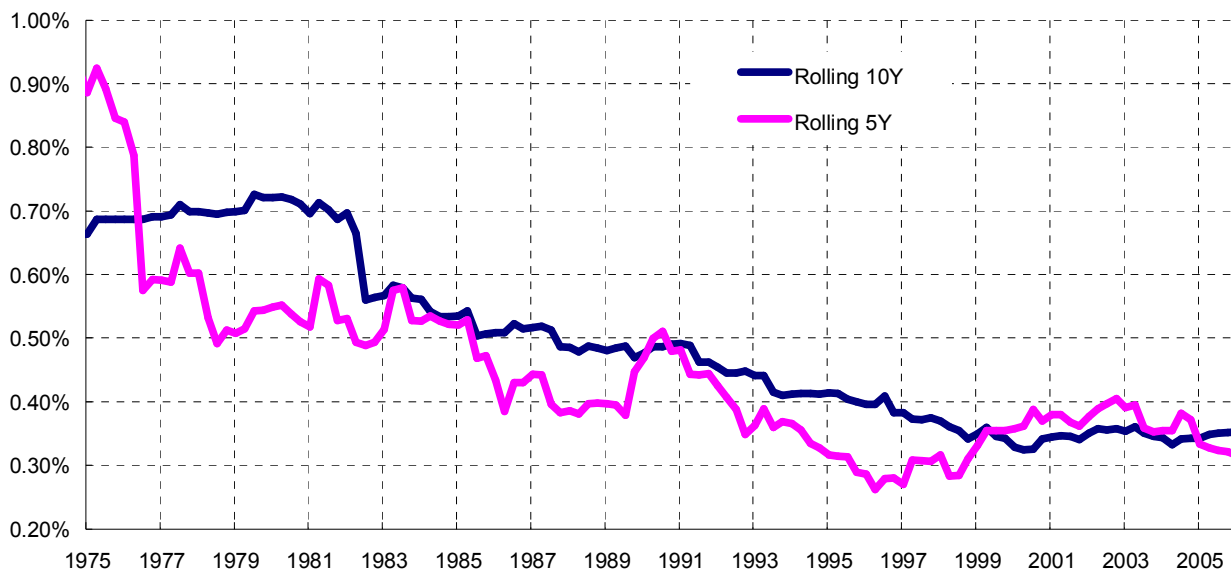


Figure 7

Standard deviation of GDP growth rates
(equally weighted economies; quarterly values from 1975Q4 to 2006Q4)



Rolling standard deviation of GDP growth rates (IMF index) and of financial market rate of returns
(annual data for GDP, monthly data for financial market; from 1975 to 2006)

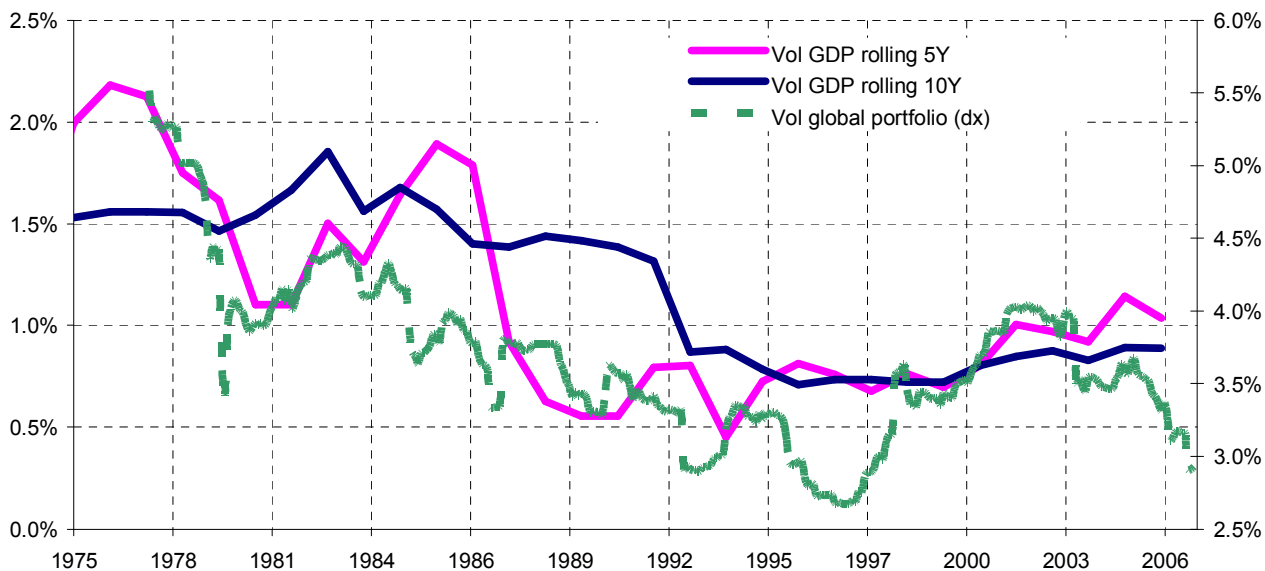
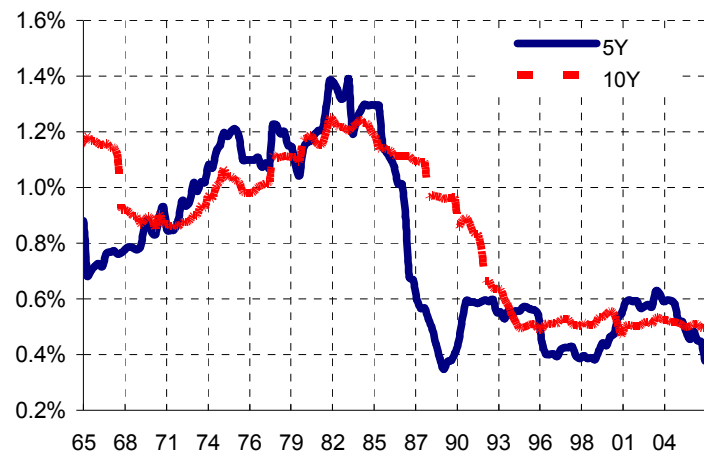
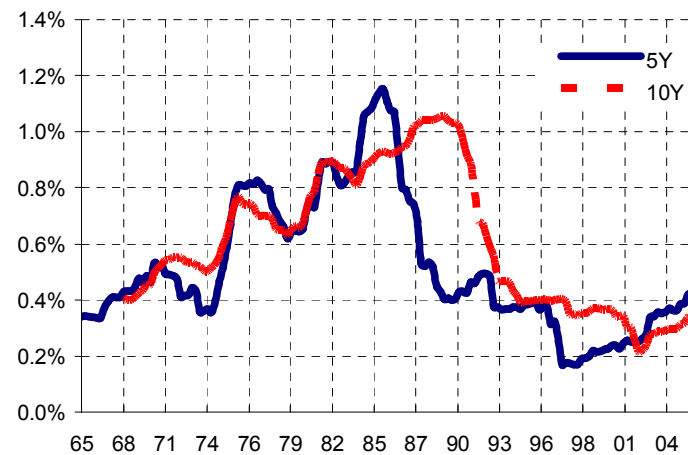


Figure 8

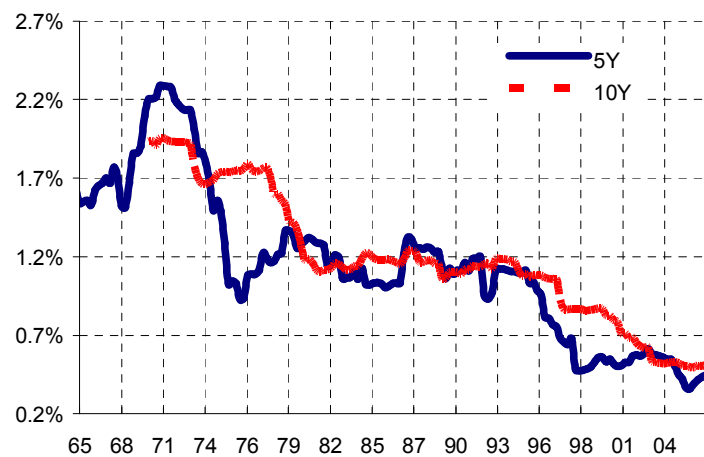
US – GDP standard deviation
(quarterly data, rolling 5 and 10 years, from 1965Q1 to 2006Q4)



US – Inflation standard deviation
(quarterly data, rolling 5 and 10 years, from 1965Q1 to 2006Q4)



Germany – GDP standard deviation
(quarterly data, rolling 5 and 10 years, from 1965Q1 to 2006Q4)



Germany – Inflation standard deviation
(quarterly data, rolling 5 and 10 years, from 1965Q1 to 2006Q4)

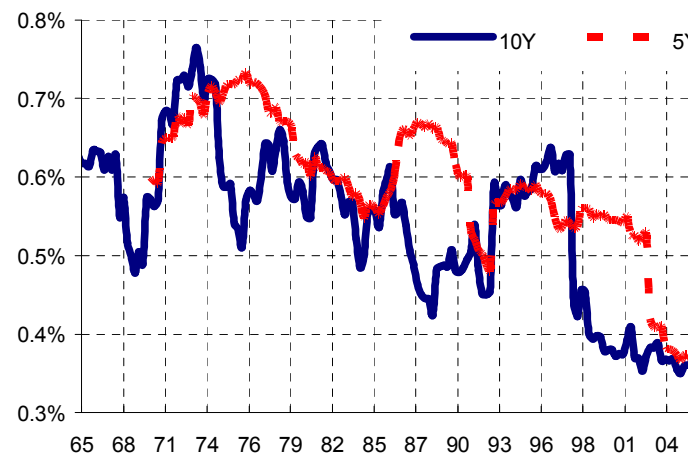
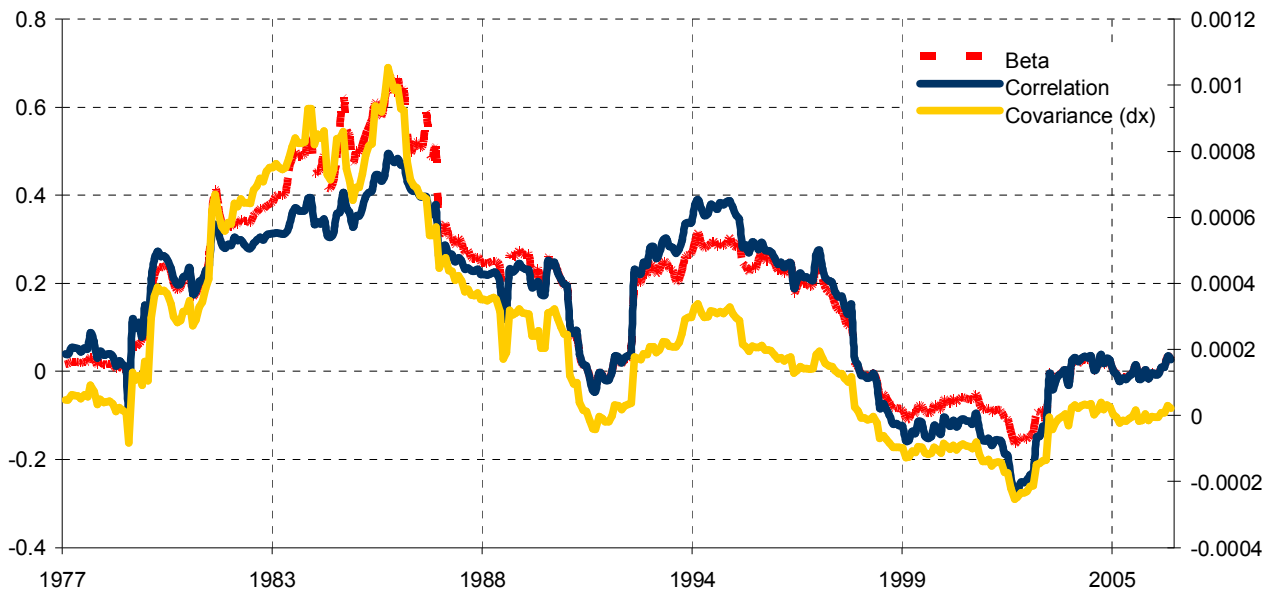


Figure 9

US - Betas, correlations and covariances (rolling 60 months)
between govt. bond and global market index
(monthly data from 31.3.1978 to 31.12.2006)



Germany - Betas, correlations and covariances (rolling 60 months)
between govt. bond and global market index
(monthly data from 31.3.1978 to 31.12.2006)

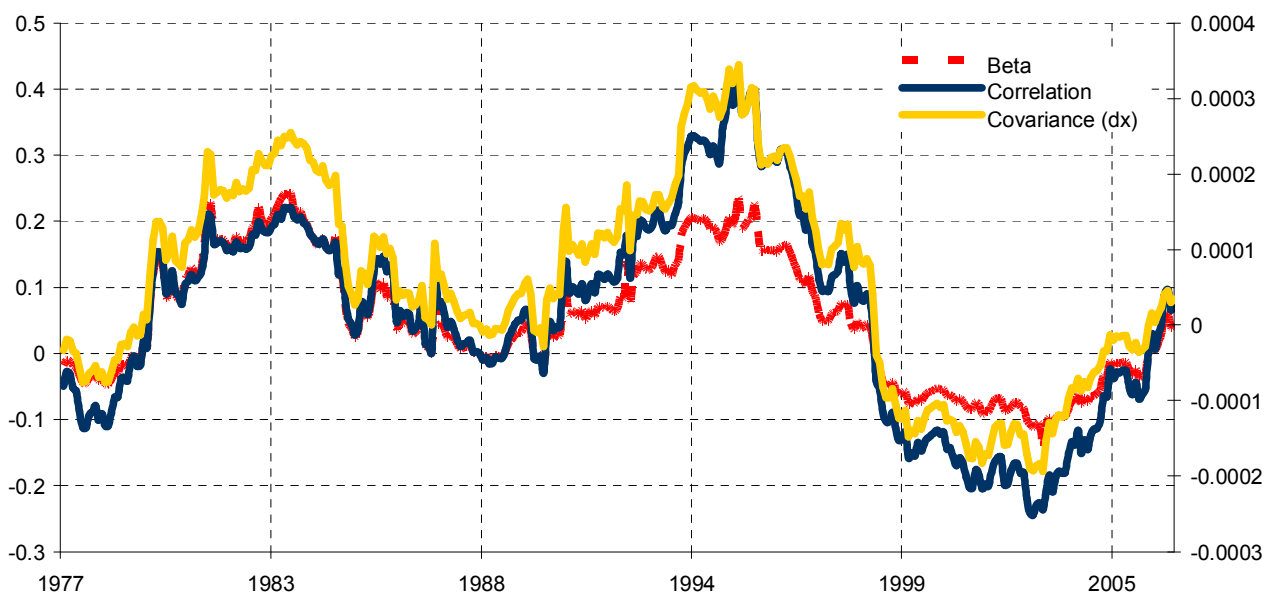
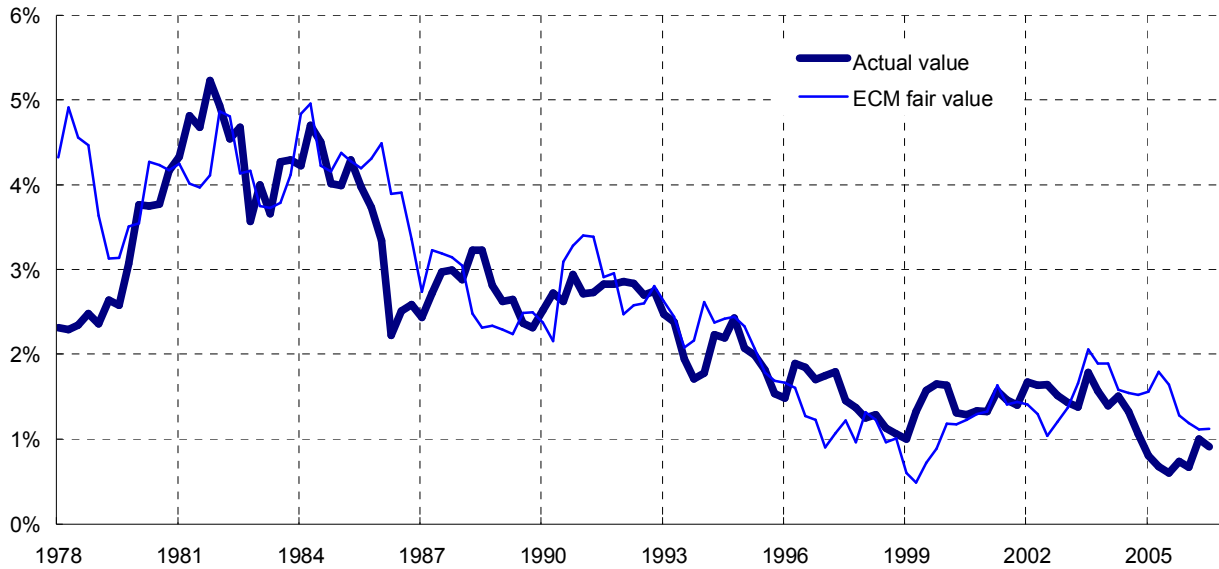


Figure 10

US – Actual premium and ECM fair value premium
(quarterly data from 31.3.1978 to 31.12.2006)



Germany – Actual premium and ECM fair value premium
(quarterly data from 31.3.1978 to 31.12.2006)

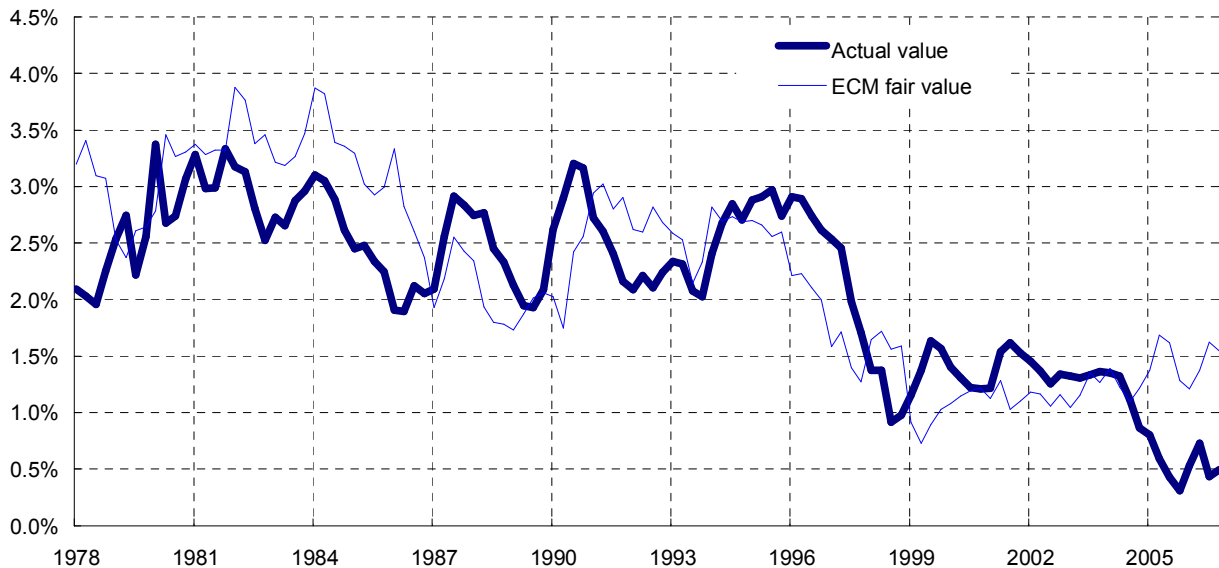
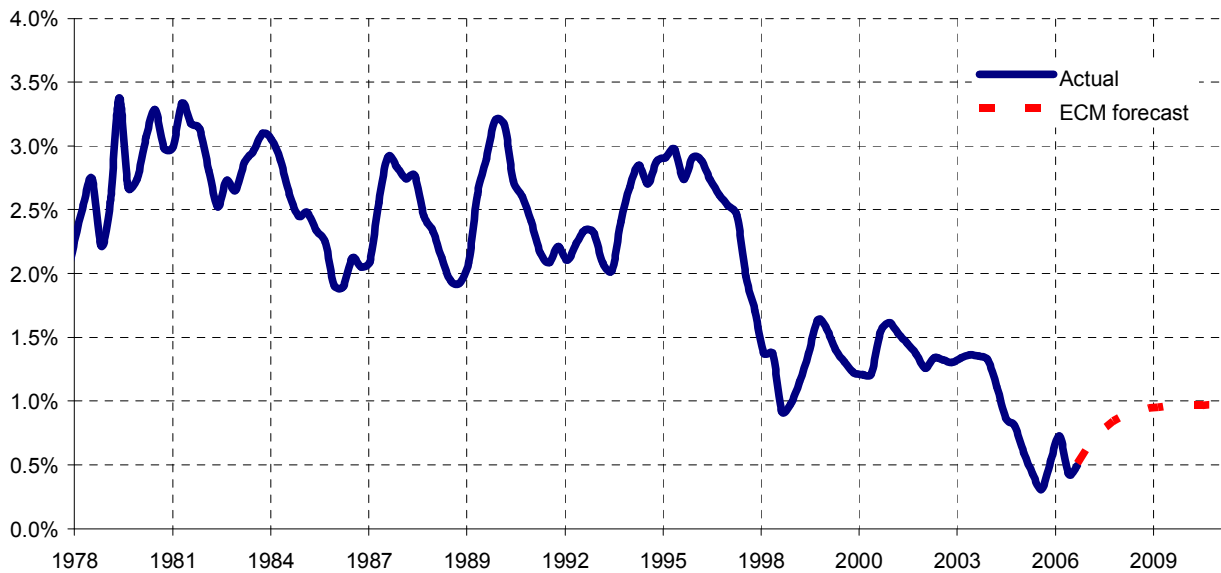


Figure 11

US – ECM Premium forecast
(quarterly data from 31.3.1978 to 31.12.2010)



Germany – ECM Premium forecast
(quarterly data from 31.3.1978 to 31.12.2010)



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