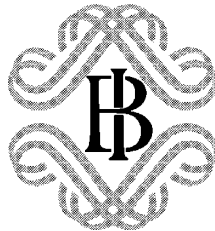# BANCA D'ITALIA

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**Is There an Equity Premium Puzzle in Italy? A Look
at Asset Returns, Consumption and Financial Structure Data
over the Last Century**

by Fabio Panetta and Roberto Violi

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IS THERE AN EQUITY PREMIUM PUZZLE IN ITALY?
A LOOK AT ASSET RETURNS, CONSUMPTION AND FINANCIAL STRUCTURE DATA
OVER THE LAST CENTURY

by Fabio Panetta and Roberto Violi (∗)

Abstract

This paper reconstructs the series of the real returns on Italian equities, bank and PO deposits and long-term government bonds from 1860 to today. In the long-run the return on shares was much higher than that on government securities and also that on bank and PO deposits. However, this summary assessment is considerably influenced by the exceptional falls in the real value of government securities and bank deposits caused by the hyperinflation that occurred in conjunction with the two world wars. Within the period, there were alternate phases, paralleling the economic cycle and the main institutional changes, in which the return on shares was higher than those on the other two instruments and vice versa.

Overall, the Italian equity market provided long-run returns to investors comparable to those of other major countries, although a large fraction of the risk premium for the whole period can be accounted for by the performance following of the hyperinflation episodes of the wars. However, the risk-return trade-off, owing to much larger volatility, compared unfavourably with other markets. Moreover, the Italian equity market in the last 30 years (up to 1994), when equity prices barely kept up with inflation, looks very different.

The econometric analysis suggests the presence of an equity premium puzzle in Italy during the estimation period, 1892-1993. In contrast, for government securities the observed returns were approximately in line with the theoretical values. The estimates show that both the returns on government securities and those on shares include an inflation risk premium. For government securities, this was estimated at around 0.8 percentage points. The inflation risk premium was smaller for shares.

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Contents

1. Introduction[1]

Modern finance theory explains the expected excess return on any risky asset over the riskless interest rate (e.g. risk premia) as the "quantity" of risk times the price of risk. There is a vast literature, starting with the seminal Mehra and Prescott (1985) paper, attempting to account for a relatively widespread and persistent empirical phenomenon of market economies: the return on stocks tends to be far greater than that on lower risk assets, such as bonds and Treasury bills.[2]

In the leading international financial markets the long-run profit rate, approximated by the return on shares, has exceeded the interest rate by much more than can be explained by aggregate risk and the values normally attributed to savers' risk aversion. For example, over the last one hundred years the average real return on stocks in the United States has been about 6 percent per year higher than that on Treasury bills. At the same time, the average real return on Treasury bills has been about 1 percent per year. The source of the equity premium puzzle identified by Mehra and Prescott lies in the difficulty in reconciling this empirical evidence with predictions based on a standard consumption-based asset pricing model, where the excess returns over a riskless asset are attributed to the extent to which a security's return covaries with consumption growth. However, the smoothness of consumption, another well established stylised fact of modern developed economies, makes the covariance of stock

[2] See the survey reporteded in Kocherlakota (1996). Adding to the Mehra and Prescott (1985) findings, Hansen and Singleton (1983) and Aiyagari (1993) document that similar phenomena characterize post Second World-War monthly data in the United States; Roy (1994) confirms the existence of the two puzzles in quarterly data, during the same period, in Germany and Japan. Siegel (1992) reports empirical evidence on annual returns in the United Kingdom across almost 2 centuries, broadly confirming the pattern found in the US.

returns with consumption low; hence the equity premium can only be explained by a very high price of risk, i.e. by a large coefficient of risk aversion.

The purpose of this paper is to measure the magnitude of the risk premia embodied in Italian financial assets across a century of data. The conceptual framework provided by the literature on asset price determination is used to guide our analisys of the data for the Italian financial markets. Our work is organized our work around the question of whether the Italian equity market provided long-run returns to investors comparable to those of other major countries. Answering this question may also help to shed some light on the peculiarity of the Italian equity market during the last 30 years (up to 1994), when stock prices barely kept up with inflation. We also investigate the risk return trade-off of equity vs. bond returns, comparing it with that estimated for the US, as well as the extent to which Italian assets have helped domestic investors hedge their financial wealth against inflation.

The paper is organised as follows: Section 2 reconstructs the series of the ex post real returns on the main Italian financial assets from 1860 to 1994 and examines the underlying trends in relation to the principal economic and institutional changes that occurred during that period. Section 3 contains a description of the theoretical reference model, based on the simultaneous choice of consumption/saving and portfolio selection. Section 4 estimates the risk premia implicit in the real returns on shares, government securities and bank deposits, separating the component of the premia linked to the economic cycle from that associated with inflation; conclusions are summarised in Section 5. The Appendix contains some technical features of the estimated model and a description of the sources and methods used to construct the data.

2. Returns in the Italian Financial Markets

This Section is devoted to describing the features and underlying trends of the returns series covering the period from 1860 to today. They also evaluated the main economic and institutional developments that occurred during the time period. The length of the period considered naturally makes a detailed analysis impossible. Consideration is given to the ex post holding period gross returns on listed shares, medium and long-term government securities, and bank and PO deposits. The returns on shares and government securities

include both capital gains and losses and dividends or coupons paid. The evaluation was carried out at constant prices, by deflating the returns series using the consumer price index. For a detailed description of the sources and methods used for the calculation of the indices and the nominal and real returns, see Appendix II.

Table 1 shows some descriptive statistics regarding the series. For the period as a whole, the average annual real return[3] on shares (6.7 per cent) was much higher than that on government securities (1 per cent) or on bank and PO deposits (-1.1 and -2.1 per cent, respectively). As was to be expected, shares showed a much higher volatility than the other instruments. The foregoing results are also confirmed by Figure 1, which shows the price indices of the individual financial assets[4] for the whole period: at the end of 1994 the real value of 100 lire invested in shares in 1862 was 6,680 lire,[5] against 80.4 lire for government securities and 5.6 lire for bank deposits. However, this summary assessment is considerably affected by the impact on government securities and bank deposits of the hyperinflation that occurred in conjunction with the two world wars (Figure 2). Within the period, there were alternate phases, paralleling the economic cycle, in which the return on shares was higher than those on the other two instruments and vice versa (Figure 3). Overall, the Italian equity market provided long-run returns to investors comparable to those of other major countries, although a large fraction of the risk premium for the whole period can be accounted for by the performance following of the hyperinflation episodes of the wars. However, the risk return trade-off, owed to much larger volatility, compared unfavourably with other markets. Moreover, the Italian equity market in the last 30 years, when equity prices barely kept up with inflation, looks very different. In the following pages the main trends in the identified sub-periods will be outlined.

Shares are the only instrument whose value rose in line with income and consumption, albeit with wide fluctuations. Figure 4 compares the real values of the prices of shares and

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[3] In this Section reference is made to the arithmetic mean of annual returns.

[4] The return on PO deposits is ignored in the analysis since it followed a path closely linked to that of bank deposits.

[5] If the investment in shares had been made in 1860, the first year for which data are available, its real value at the end of 1994 would have been 8,417 lire.

government securities with those of real per capita consumption and GDP from 1890 to today, i.e. from the time when statistics on the macroeconomic aggregates are available. From the very beginning of this period the value of investment in shares was almost always in line with the indices of consumption and GDP and in two sub-periods — from the end of the Great Depression to the beginning of the Second World War and during the post-war reconstruction — it rose much faster. The sharp decline of the Sixties and Seventies, which was partially offset in the Eighties, left in the share index slightly below that of GDP, but still basically in line with that of consumption. By contrast, the rise in the index of government securities was much smaller.

2.1 From the unification of Italy to the First World War (1860-1914)

The government securities index rose sharply in this period. During the 1860s and 1870s the rise was fuelled by stable high real interest rates due — in a phase of political and administrative reorganisation of the State — to a high level of public borrowing and a very low saving rate. In the decades that followed, the rise in the index was the result of the large capital gains that stemmed from the fall in nominal and real interest rates; these were made possible not only by the reduction of the budget deficit — achieved through substantial increases in taxation as well as the confiscation of large amounts of church property — but also by the positive effects of the solution of the Roman question. The decline in the rate of inflation compared to the high levels of the 1860s and 1870s was also a contributing factor.

In the early part of this period, share prices closely followed those of government securities. A short-lived exception occurred when the stock market[6] rose rapidly at the beginning of the 1870s, in response to the impulse imparted by the construction of the first railways. However, the rise in share prices and in the number of listed companies soon subsided and the market remained small, a result of its having only been established a short time coupled with the limited number of joint stock companies. From the last decade of the nineteenth century onwards, the share index rose very rapidly, following of the fast growth in

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[6] In the second half of the nineteenth century the most important exchange was that of Genoa. It was not until the end of the century that the Milan stock exchange drew level.

GDP attributable to the country's industrialisation and the expansionary phase of the economic cycle abroad.

2.2 From 1915 to the end of the Second World War

During the First World War, the increase in inflation and the public debt caused a large rise in nominal and real interest rates. The indices of government securities and shares fell sharply, declining between 1914 and 1920 by respectively 70.2 and 47.7 per cent in real terms.

In the early years of the following decade, a policy designed to curb the budget deficit was launched and the Government sought to reduce the enormous mass of short-term government securities in circulation, without much success, however. After the failure in 1924 of the voluntary conversion of such paper into long-term securities, in 1926 the government issued the Littorio loan and imposed the conversion of outstanding Treasury bills and Treasury bonds maturing by the end of 1930. This operation resulted in a further fall in security prices. The attempts to reduce the burden of the public debt in the years leading up to the Second World War were no more than moderately successful, both due to the effects of the 1934 conversion[7] and also to the large increase in the debt itself associated with Italy's war activities. The hyperinflation that occurred during the war and in its immediate aftermath caused the real value of government securities to slide.

After the share index had fluctuated for most of the 1920s, the Great Depression caused it to fall by 37 per cent in real terms between 1929 and 1931, with turnover contracting even more. In the years that followed the government adopted numerous legislative and fiscal measures, in some cases of ambiguous effect. However, the expansion of production associated with rearmament fuelled the rapid recovery of the share index in the

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[7] In an attempt to benefit from the reduction in interest rates that had occurred in 1933, the government decided in 1934 to convert its 5% consolidated stock into 3.5% redeemable bonds amortizable over forty years. "Even though it was voluntary, the conversion was a success, partly owing to the premiums assigned, the tax exemptions granted ... and the restrictions the law introduced on applications for reimbursement" (See Bianchi, 1979).

years leading up to the Second World War. The exceptional rise in inflation during the war caused the share index to plunge by 84.5 per cent in real terms between 1943 and 1947.

2.3 From 1946 to today

This period can be divided into three main phases (Figure 5). The first, which lasted from the end of the war to the beginning of the 1960s, was characterised by rapid income growth and low and stable positive real interest rates. The growth of Italian industry benefited not only from the mobilisation of resources for post-war reconstruction but also from calm labour relations and the gradual liberalisation of trade. Between 1948 and 1962 the share index rose in real terms by a factor of nearly eight, while that of government securities rose by no more than 47.5 per cent.

In the second phase, which lasted until the early of the 1980s, the growth of Italian industry was much slower. The first signs of instability emerged at the beginning of the 1960s, when wages accelerated and the share of profits in income consequently declined. Numerous factors subsequently held back the growth of industry, the most important of which were the explosion of labour unrest regarding wages and other employment conditions from 1969 onwards and the oil price hikes from 1973 onwards. The rate of inflation rose sharply during the Seventies. The prices of government securities registered a large fall in this period as a result of capital losses — caused by the rise in nominal interest rates — and the persistence of substantially negative real interest rates. Their returns nonetheless remained above that of shares throughout the period. In the early part of the period, in fact, the share market was depressed not only by the slowdown in economic growth but also by a series of other factors, such as the nationalisation of the electricity companies and the introduction of a withholding tax on dividends. More generally, the political changes that occurred at the beginning of the 1960s led to changing attitudes towards financial capital. In the 1970s the difficulties of the stock market were aggravated by the crisis in large firms, which had pursued a strategy of broadening their capital without modernising it. After rising rapidly to a new record high in 1961, real share prices had fallen by nearly half by the end of 1964. After a temporary lull, prices began to fall again in the beginning of the Seventies and

the downward trend was accentuated by the oil crisis. By the end of 1977 the real value of the share price index had fallen to about one quarter of its December 1970 value.

The 1980s and the early 1990s were marked by a recovery in bond prices and fluctuating share prices. As inflation fell, the decline in nominal interest rates reinforced the positive effect produced on the index of government securities, at a time in which the public debt was expanding rapidly, due to the large rise in real interest rates, which became positive again and settled at historically high levels. In the first part of this period, up to 1987-88, the rise in share prices was due both to the expansionary phase of the cycle and to the improved profitability of large firms, which benefited not only from the slowdown in wage increases compared to the previous decade but also from the far-reaching reorganisation that they had implemented at the beginning of the decade; their profitability rose to the highest level since the 1950s. In addition, there was the effect of the changes in the structure of the stock market made during the 1980s. The most important of these was probably the introduction of investment funds, which contributed to the bull market of 1985-86. The recession that overtook the world economy at the end of the 1980s mainly affected the stock market, while its impact on the prices of government securities was less pronounced.

3. Modelling consumption-saving decision and portfolio selection

This Section provides a description of the main features of the theoretical model used to obtain the empirical measures of the risk premia implicit in the returns on financial instruments. In the first paragraph, we outline the main theoretical issue raised as well the empirical evidence confronted in modelling financial market returns vis-à-vis macroeconomic fundamentals as identified in the literature. In the second paragraph we set out the model applied to the Italian data.

3.1 Risk premia and the equity premium puzzle: a review of the literature

There is a vast literature, starting with the seminal Mehra and Prescott (1985) paper, attempting to explain a relatively widespread and persistent empirical phenomenon of market

economies: returns on stocks tends to be far greater than those on lower risk assets, such as bonds and Treasury bills.[8] For example, over the last one hundred years the average real return on stocks in the United States has been about 6 percent per year higher than that on Treasury bills. At the same time, the average real return on Treasury bills has been about 1 percent per year.

The source of the equity premium puzzle identified by Mehra and Prescott lies in the difficulty in reconciling this empirical evidence with predictions based on a standard general equilibrium, representative agent, model typically used by financial economists, where these differences in average returns are attributed to the extent to which a security's return covaries with consumption stream. Equity is not the only example of asset receiving lower return than that implied by standard Arrow-Debreu general equilibrium theory. Currency, for example, is dominated by Treasury bills with positive nominal yields, yet sizeable amount of currency are held. As pointed out by Mehra and Prescott, the equity premium puzzle can be regarded as analogous to the so called "rate of return dominance" puzzle that motivates much of modern monetary theory.

Empirical evidence suggests that there is little comovement between stock returns and per capita consumption. Hence a very large risk aversion parameter seems to be necessary to generate sufficiently high real stock returns (equity premium puzzle). However, if a representative investor strongly dislikes bearing risk, she wants consumption to be smooth over different states of the world, and she also desires smoothness of consumption over time. Yet, empirical evidence seems to suggest otherwise, since individuals defer consumption (that is, save) at a sufficiently fast rate to generate per capita consumption growth at around 2 percent per year, despite very low risk-free rates, oftentimes close to zero; this is what Weil (1989) calls the risk-free rate puzzle. These puzzles have implications not only in explaining portfolio selection, but, more broadly, they are indicative of the large gaps in our

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[8] See the survey reporteded in Kocherlakota (1990). Adding to the Mehra and Prescott (1985) findings, Hansen and Singleton (1983) and Aiyagari (1993) document that similar phenomena characterize post Second World-War monthly data in the United States; Roy (1994) confirms the existence of equity and risk-free rate puzzles in quarterly data, during the same period, in Germany and Japan. Siegel (1992) reports empirical evidence on annual returns in the United Kingdom across almost two centuries, broadly confirming the pattern found in the US.

understanding of the macroeconomy. The equity premium puzzles indicates that we do not know why people keep on saving even when returns are low; models of aggregate savings behaviour may be omitting some crucial variable. The risk-free rate puzzle demonstrates our poor understanding as to why individuals are so averse to the highly procyclical risk associated with stock returns. Without this knowledge, we cannot hope to give a meaningful answer to the relevant question, raised by Lucas (1987), about how costly individuals find business cycle fluctuations in consumption growth and how far portfolio diversification can go to hedge the macro-risk brought about by output fluctuations.

Attempts have been made along several lines to resolve the two puzzles. Plausible explanations for the low value of the risk free rate can be found by abandoning the standard preference orderings imposed by the expected utility assumption. For the large equity premium, ot is more open to debate as to which improvements to the basic model would be required to deal adequately with the empirical findings. Recently suggested approaches rely on two key features: generalised expected utility and habit persistence preferences on the demand side; multisector technology with limited intersectoral mobility of factors of production on the supply side.[9] On the demand side, habit formation can decouple risk aversion and marginal elasticity of substitution, as generally achieved by getting away from separable utility function. On the supply side, limited factor mobility frustrates consumption smoothing by moving the price of capital adversely with respect to the pattern of the marginal utility of consumption; as a result, the price of capital is not as low as it should be during a recession and vice-versa, inducing larger risk on equity. However, these modifications do not go far enough in accounting for the low consumption growth-equity return correlation and the countercyclical behaviour of investment goods price in the data.

Trading frictions, determined by incomplete market and high transaction costs for equity, have been introduced to explain asset returns data. It is important to recall that under complete markets, Constantinides (1982) showed that there always exists a "representative" agent model which duplicates asset prices for any heterogeneous economy facing the same uncertainty. Therefore, the effect of heterogeneity of preferences or endowment can still be

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[9] See Boldrin, Christiano and Fisher (1996).

treated within the context of a traditional single agent model.[10] A recent branch of literature explores the joint effect of market incompleteness and wealth inequality on asset pricing. Constantinides and Duffie (1996) show that the absence of labour income insurance markets, combined with the permanence of labour income shocks, has the potential to generate a risk free rate that may be much lower than the complete markets risk free rate. Similarly, if a sizeable fraction of individuals faces borrowing and short sale constraints, the risk-free rate may drop substantially below the predictions of the representative agent model.[11] However, the assumption of permanent (perfectly autocorrelated labour) income shocks is critical. If shocks are less than fully persistent, Heaton and Lucas (1996) show that the risk-free rate and the equity premium are largely unaffected by the absence of markets, because investors can effectively use the accumulated assets to self-insure against idiosyncratic risk (i.e. shocks to individual income).

Investors who try to engage in asset trade face all sorts of transaction costs, e.g. the bid-ask spread, brokerage fees, load fees, taxes and informational costs. However, transaction costs can have a significant impact on equity premium only if one can assert that there are significant differences in trading costs across the stock and bond markets; under this assumption, the premium on stocks represents compensation not for risk, but rather for bearing additional transaction costs; as a result, bonds are held despite their low rate of return because they are less costly to trade. The transaction costs explanation resembles the solution adopted in modelling the demand for money when facing the "rate of return dominance puzzle". Empirically, it is unclear whether the sizes and sources of trading costs are sufficiently documented to support this explanation.[12] Theoretically, an infinitely lived investor is able to smooth consumption substantially by simply buying and selling the asset that is cheapest to trade. She can consequently contemplate making arbitrage profits by buying and holding stock for a long period of time and therefore amortise the initial trading cost. The suggested possibility of a "market segmentation effect", resulting from limited

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[10] See Gollier (1998) for some results on the equilibrium price of assets under complete markets stemming from the interaction between wealth inequality and social risk preference.

[11] See Telmer (1993) and Lucas (1994).

[12] See however Fisher (1994) for a different assessment.

market participation by investors, has also been considered as a potential explanation. In principle, it might be possible that equity returns would be more correlated with consumption growth of stockholders than the consumption growth of non-stockholders. However, Mankiw and Zeldes (1990 provide direct evidence for the US market that the stockholders covariance is still not large enough to support the market segmentation hypothesis as an explanation of the equity premium puzzle. Limited market participation can be a reason for lacking liquidity. Recently, the risk of stocks suddenly becoming illiquid has been offered by Brown, Goetzmann and Ross (1995), under the label "survivorship bias hypothesis", as explanation for the inexplicable part of the equity premium. Historically, asset trading broke down on several occasions, thus drastically reducing stock market liquidity. They estimate how much extra-return investors would require from equities to compensate them for the risk that the market might not survive. Assuming an 80 percent survival rate for markets, a figure well above the historical average, would be consistent with about half of the premium observed on US and UK stocks. The "survivorship bias" explanation is a refinement of the peso's problem-type of argument proposed by Rietz (1988), suggesting that infrequently occurring stock-market "crashes" can explain higher ex-ante equity premiums with standard risk aversion parameters. In a related paper Goetzmann and Jorion (1997) try to test the "survivorship bias" by estimating the long-term returns to investing in a broad cross-Section of international markets over the twentieth century. Their main conclusion is that the real return on US stocks over the period 1921-1995 (almost 5 percent per annum) appears rather exceptional, as other markets typically returned 3 percent less than US equities.

The recent concept of Rational Beliefs Equilibrium provides a fresh perspective for the equity premium debate.[13] This new theory accounts for observed variations in asset prices in excess of market fundamentals by introducing the notion of endogenous uncertainty into the standard model. Beliefs heterogeneity across market participants propagates price uncertainty endogenously for which investors need to compensated. The size of the premium increases by the extent to which endogenous uncertainty affects assets volatility in excess of

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[13] See Kurz and Beltratti (1996).

the variability of fundamentals. Similar results can be obtained under Knightian uncertainty, where probability function can only be vaguely defined. Aversion to such uncertainty by a representative investor may be sufficient to generate changes in asset prices without movements in fundamentals.[14]

3.2 Generalised expected utility preferences

The main benefit of modelling preferences according to the generalised expected utility model is that the investor's attitudes toward investment risk and consumption growth are separeted. This allows intertemporal substitution and risk aversion to be high simultaneously, therefore it can explain the combination of a high equity return and a low risk-free rate, while being consistent with the stylised facts about per capita consumption growth (very smooth pattern and low correlation with stock returns). In addition, the generalised expected utility model has the desirable attribute of being suitable to econometric implementation, thus allowing the estimation of risk premia. This is a clear advantage over most of the other suggested resolutions to the equity premium puzzle reviewed in Section 3.1, which have not yet faced the challenge of econometric implementation and testing, partly because of the lack of relevant data over a long sample period.

The generalised expected utility model is based on the simultaneous determination of the demand for consumer goods and financial assets. The latter are held for the purpose of guaranteeing the desired pattern of consumption over time. Accordingly, the risk that consumers assign to each security depends on the correlation between its returns and the desired consumption.

Consider an economy in which there are N assets with gross real return factors (e.g. 1+ real return) between time t and $t+1$ equal to $R_{t+1} = (R_{t+1}^1, R_{t+1}^2, ..., R_{t+1}^N)$. The assets in question can include non-tradable forms of wealth, such as human capital or pension entitlements. The shares of the individual assets owned by a typical consumer at the end of

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[14] See Epstein and Wang (1995).

each period are defined as $\omega_{t+1} = (\omega_{t+1}^1, \omega_{t+1}^2, ..., \omega_{t+1}^N)$. The decisions to consume and allocate wealth between the various assets are made on the basis of an intertemporal objective function, subject to the budget constraint that defines the growth in total net wealth:[15]

(1)
$$Max \; W[C_1, C_2, ..., C_T, ..., C_\infty]$$

$$s.t. \quad A_{t+1} = (A_t - C_t)\omega_t R_{t+1} \qquad \forall t = 0,1,..,T,..,\infty$$

where C_t and A_t represent the consumption in period t and the real wealth at the end of the period, respectively. Since the returns are stochastic, the specification of preferences must identify both the choice between present and future consumption and the degree of risk aversion. Working with the preferences of a representative agent can be defended on three grounds: (i) by the assumption of identical agents in the economy; (ii) by defining the social welfare function, of the economy; (iii) by identifying the marginal investor who consumes a basket equal to average consumption and holds the market portfolio of assets. In this paper the two models most commonly used in the literature have been adopted. The first provides for the maximisation of expected utility, assuming the intertemporal separability of utility and constant relative risk aversion:

(2)
$$W = E_t \sum_{s=1}^{\infty} \beta^s U(C_{t+s}) \qquad \text{where} \quad U(C) = C^{1-\gamma} / (1-\gamma)$$

where γ represents the relative risk aversion,[16] β the subjective rate of intertemporal preference and E_t the expectation the basis of information available at time t. This is the standard expected utility model, which constrains the coefficient of relative risk aversion, γ, to be equal to the reciprocal of the elasticity of intertemporal substitution; it is a highly restrictive assumption, which can be relaxed by generalising the preferences structure.

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[15] The shares ω can be interpreted as net positions and can take on a negative sign, i.e. become liabilities. This makes it possible for net income flows (including taxes) to be included in the budget constraint.

[16] In this formulation the relative risk aversion coincides with the elasticity of marginal utility with respect to changes in consumption, i.e. with the curvature of the utility function.

A recursive specification of preferences allows the second model to distinguish between pure risk aversion and the intertemporal substitution elasticity of consumption.[17] In particular, the following recursive structure is adopted for the intertemporal profile of preferences:

$$V_t = W[C_{t+1}, m(V_{t+1})] \qquad \forall t = 0, 1, .. T, .. \infty$$

(3)

$$where: \quad W = [(1-b)C^{(1-\frac{1}{s})} + bm^{(1-\frac{1}{s})}]^{\frac{s}{s-1}}$$

$$m(V) = \left[E_t V_{t+1}^{(1-g)}\right]^{\frac{1}{1-g}}$$

where σ indicates the substitution elasticity of consumption. Equation (3) reduces to equation (2) when $\gamma = 1/\sigma$, i.e. when the coefficient of relative risk aversion is equal to the reciprocal of the intertemporal substitution elasticity. Furthermore, if $\gamma = \sigma = 1$ (substitution elasticity and risk aversion both equal to one), we have the special case of logarithmic utility. Under hypothesis (2), the first-order conditions (Euler equations) which must be satisfied by the optimal choice of consumption and portfolio selection are as follows:

(4)
$$E_t\left[b\left(\frac{C_{t+1}}{C_t}\right)^{-g} R_{t+1}^i\right] = 1 \qquad \forall i = 1, .. N; \quad \forall t = 0, 1, .., T, .., \infty$$

under hypothesis (3), alternatively, the Euler equations are represented by (see Epstein, 1992, sect.4.2):

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[17] See Epstein (1992) for a detailed description of the approach adopted here.

$$E_t\left(\frac{Z_{t+1}^{1-\gamma}-1}{1-\gamma}\right)=0 \quad \forall i=1,..N \quad \forall t=0,1,..,T,..\infty$$

$$E_t\left(\frac{Z_{t+1}^{1-\gamma}}{R_{t+1}^m}R_{t+1}^i\right)=1$$

(5)

$$\text{where} \quad Z_{t+1} \equiv \beta^{\frac{\sigma}{\sigma-1}}\left(\frac{C_{t+1}}{C_t}\right)^{\frac{1}{1-\sigma}}(R_{t+1}^m)^{\frac{\sigma}{\sigma-1}}$$

$$R_{t+1}^m \equiv \sum_{i=1}^{N}\omega_i^* R_{t+1}^i$$

where ω_i^* represent the optimal shares of each asset in the consumer's portfolio, and R_{t+1}^m is the average real return of the optimal portfolio, generally known in the literature as the "market portfolio". In order to define the concepts of risk premium and risk-free return, the second equation of (5) can be rewritten as:[18]

$$Cov(Q_{t+1},R_{t+1}^i)+E_tQ_{t+1}E_tR_{t+1}^i=1 \quad \forall i=1,..,N \quad \forall t=0,1,..,T,..\infty$$

(6)

$$\text{where} \quad Q_{t+1} \equiv \left(\frac{Z_{t+1}^{1-\gamma}}{R_{t+1}^m}\right)$$

by definition, the risk-free real return, denoted by R_{t+1}^f, is not correlated with any of the variables contained in the consumer's information set and thus is not correlated even with Q_{t+1} (the intertemporal marginal rate of substitution, hereinafter IMRS). Hence, setting the covariance in (6) equal to zero, we have:

(7)
$$E_t(Q_{t+1})E_t(R_{t+1}^f)=1$$

and, in view of the deterministic nature of the risk-free return, we can write:

(8)
$$R_{t+1}^f = \frac{1}{E_tQ_{t+1}}.$$

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[18] Recalling that $cov\,(x,y)\,=\,Exy-ExEy$.

The risk-free return at time $t+1$ is thus equal to the reciprocal of the IMRS expected at time t. The risk premium, equal to the difference between the expected return on the security and the risk-free return, is obtained by substituting (8) into (6):

(9)
$$\frac{E_t R_{t+1}^i - R_{t+1}^f}{R_{t+1}^f} = -Cov(Q_{t+1}, R_{t+1}^i) \qquad \forall i = 1, N; \forall t = 0,1,..,T,..,\infty$$

from (9) it can be seen that the risk premium on security i is positive — or in other words that the expected return is greater than the risk-free return — only if the covariance between the return on the security and the IMRS is negative, since R^f (equal to 1 + the risk-free rate) is positive. In economic terms, the consumer requires a compensation with respect to the risk-free rate of return if the return on the security is low when consumption is low or, in other words, when it is more "costly", in terms of welfare, to make the inter-temporal substitution of consumption. Conversely, the return required is lower if the wealth asset helps to attenuate the risks associated with the inter-temporal substitution of consumption. In other words, in the model analysed the popularity of each security is linked to its "insurance" function: the most desirable securities, for which the required return is lower, are those with a higher return in adverse phases, i.e. when consumption is low.[19]

The risk premium given by (9) can be reinterpreted in terms of the spread of a security with a return that is negatively correlated with IMRS:

(10)
$$E_t R_{t+1}^i - R_{t+1}^f = \beta_Q^i \left(E_t R_{t+1}^Q - R_{t+1}^f \right) \quad \forall i = 1,..N \quad \forall t = 0,1,..,T,..\infty$$
$$\beta_Q^i \equiv \frac{Cov(R_{t+1}^Q, R_{t+1}^i)}{Var(R_{t+1}^Q)}$$
$$R_{t+1}^Q = -\theta Q_{t+1} \qquad \theta \geq 0$$

in the financial literature, R_{t+1}^Q is generally known as the "market" return (Constantinides, 1982), while β_Q^i, which is comparable to a regression coefficient, is known as the "beta" coefficient and represents the sensitivity of the risk premium to the spread of R_{t+1}^Q. In the

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[19] This principle closely resembles that underlying the models normally used to analyse stock market returns, such as the CAPM.

present context, however, the latter does not normally coincide with R_{t+1}^m.[20] More generally, Q_{t+1} can be interpreted as a stochastic discount factor, which represents the prices of payoffs in different parts of the world for any financial asset (see Campbell, Lo and MacKinlay, 1997, ch. 8). Under complete markets, the absence of arbitrage is equivalent to the existence of a stochastic discount factor (also called equivalent martingale measure; see Duffie, 1992), implying the linear capital asset pricing relation displayed in equation (10).

Since it depends on information available in period t, the (9)-(10) risk premium is affected by all sources of uncertainty present in the economy. Consequently, it includes a premium for the risk of inflation, which is a function of the covariance between real returns and consumer prices. Imagine that it is necessary to completely immunise the real return of a security from the risk of inflation; for this purpose an indexation mechanism can be defined whereby the adjusted real return ($R_{t+1}^{i,\pi}$) both has zero covariance with inflation and satisfies the equilibrium condition defined by the Euler equation of (5).

Defining the inflation risk premium as the difference between the expected return of the security and the expected return of a security fully immunised against inflation, it can be shown that (see Appendix I):

$$\frac{E_t R_{t+1}^i - R_{t+1}^f}{E_t R_{t+1}^{i,\pi} - R_{t+1}^f} = \frac{\beta_Q^i (E_t R_{t+1}^Q - R_{t+1}^f)}{\beta_Q^{i,\pi} (E_t R_{t+1}^Q - R_{t+1}^f)} = \frac{\beta_Q^i}{\beta_Q^i + \beta_Q^\pi \beta_\pi^i} \qquad \forall i = 1,..N \qquad \forall t = 0,1,..T,..\infty$$

$$(11) \qquad \begin{aligned} \beta_Q^{i,\pi} &\equiv \frac{Cov_t(R_{t+1}^{i,\pi}, R_{t+1}^Q)}{\mathrm{Var}_t(R_{t+1}^Q)} \\[2mm] \beta_\pi^i &\equiv \frac{Cov_t(\Pi_{t+1}, R_{t+1}^i)}{\mathrm{Var}_t(\Pi_{t+1})} \\[4mm] \beta_Q^\pi &\equiv \frac{Cov_t(\Pi_{t+1}, R_{t+1}^Q)}{\mathrm{Var}_t(R_{t+1}^Q)} \end{aligned}$$

[20] In fact, it is a non-linear transformation (see equation (7)). However, see Merton (1973) and Cox, Ingersoll and Ross (1985) for a survey of the conditions in which R_{t+1}^Q and R_{t+1}^m coincide, with temporally separable preferences, constant returns to scale and maximisation of expected utility.

the inflation risk premium, which is given by (see Appendix I)

$$(11') \qquad \begin{aligned} & E_t(R_{t+1}^i - R_{t+1}^{i,\pi}) = -\beta_\pi^i \beta_Q^\pi R_{t+1}^f Var_t Q_{t+1} \\ & \forall i = 1, N; \forall t = 0,1,2,...,T,...,\infty \end{aligned}$$

is positive if the correlation between inflation (denoted as Π_{t+1}) and asset returns and that between inflation and the IMRS (or stochastic discount factor) is negative, that is to say if the product of the corresponding beta coefficients

$$(12) \qquad \beta_\pi^i \beta_Q^\pi \leq 0 \quad \forall i = 1, N \,.$$

Furthermore, no inflation risk premium is incorporated in real asset returns if at least one of the correlations is equal to zero.[21] It follows from (12) that a positive correlation between the real return of a security and inflation is not sufficient *per se* to justify the existence of an inflation risk premium or to determine its sign.[22] For this purpose, it is in fact necessary to establish the sign of the correlation between inflation and the IMRS. The sign of such correlation is not obvious a priori, since it depends not only on consumption preferences and the growth of consumption but also on the return of the market portfolio and hence on the correlation between inflation and all the existing assets. In the case of a negative correlation between the rate of inflation and the IMRS, a positive correlation between real returns and inflation would not be sufficient to protect against the risk of inflation and the return on the security would still include an inflation premium. This "perverse" effect could also occur in the event of a negative correlation between the return on the security and inflation — the so-called "Mundell effect". Ultimately, the evaluation of the inflation risk must take account of the link between inflation and the IMRS, i.e. of the interaction between inflation, consumption and the market portfolio. The method used here to evaluate the inflation risk premium is of a general nature and, in principle, could be

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[21] This, however, does not exclude the possibility of an inflation risk premium being incorporated in expected nominal returns. Such a premium, in the absence of correlation between inflation and the real variables, would be attributable exclusively to the variance of inflation (or, more exactly, to the covariance between expected inflation and actual inflation, as shown by Cesari ,1992, for an equilibrium model of the Cox, Ingersoll and Ross, 1985, type).

[22] See, for example, Fama and Schwert (1977) for an evaluation of the inflation risk premium based on the correlation between real returns and inflation (both anticipated and unanticipated).

applied, *mutatis mutandis*, to evaluate the impact on the expected return of all the sources of uncertainty present in the economy.

3.3 Consumption, inflation and real returns in a log-normal model

The evaluation of risk premia — including that associated with inflation — was implemented by estimating equations (4) and (5), using data on the ex post real returns on shares, government securities and bank deposits. The estimate of the premia should help to explain the differences described in Section 2 between the returns observed for the three types of instrument. The literature has underscored the difficulties encountered when attempting to explain the behaviour of returns using portfolio selection models such as the Consumption Capital Asset Pricing Model (CCAPM), developed by Merton (1973), extended by Breeden (1979) and made popular in macroeconomics by Lucas (1978) and Hall (1978).[23] The main problem, known as the equity premium puzzle (see Mehra and Prescott, 1985), is how to explain the fact that actual returns on equity are much higher than those predicted by the standard CCAPM. This latter requires that two highly implausible conditions be satisfied: exceptionally high levels of risk aversion and excessively low (and ultimately negative) values of the subjective intertemporal discount rate,[24] at a time of real growth in consumption. It is not clear how to render a negative risk-free rate of return compatible with a production function characterised by a positive intertemporal rate of transformation.[25] This obviously makes it more difficult to "explain" the behaviour of returns in terms of that of consumption.

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[23] See Roy (1994) for a multicountry comparison extending to Japan and Germany the CCAPM rejection found for the US.

[24] These two conditions give rise to the so-called risk-free rate puzzle, i.e. to a theoretical risk-free rate below the observed one (see Weil, 1989. Nonetheless, Kocherlakota, 1990, has shown that a negative subjective intertemporal discount rate is not incompatible with the hypothesis of intertemporal maximization for an economy which is growing and in which there is uncertainty.

[25] Recently, Cochrane and Hansen (1992) have shown that the statistical properties of the time series of consumption and real returns are very different from those that theory would suggest (correlation and conditional moment puzzles). Observed consumption and returns are only weakly correlated. In particular, the changes in the rate of growth of consumption are much less pronounced than those in real returns.

Whereas the empirical testing of equation (4) is not affected by problems concerning the observability of the variables, the estimation of (5) requires the return of the "market portfolio" to be known, and this cannot be observed directly. Recourse is commonly made in the literature to variables whose returns are deemed, a priori, to proxy the real return of the wealth of the economy (including human capital).[26] This solution, which in any case is not without its own problems, cannot be adopted here owing to the lack of a reliable indicator of the behaviour of such wealth in the period considered. Accordingly, for the purpose of identifying the model and estimating the parameters of (5), recourse has been made here to the following simplifying assumptions: (i) joint lognormality of real returns, the growth rate of consumption and inflation; (ii) no change over time in the matrix of the covariance of real returns, the growth rate of consumption and inflation; and (iii) constraints on the stochastic process followed by the return on wealth.

On the basis of the aforementioned assumption (i) applied to the system of equations (5), the following equilibrium conditions for the expected returns and the risk-free return (and hence on the risk premia) in relation to the rate of growth in consumption hold (see Epstein, 1992, sect. 4.4):

(13)
$$E_t r_{t+1}^i = r_{t+1}^f - \frac{1}{2}\sigma_i^2 + \left[\frac{(1-\gamma)}{(\sigma-1)}\right]\sigma_{c,i} + \left[\frac{(\sigma\gamma-1)}{(\sigma-1)}\right]\sigma_{m,i} \quad \forall i = 1,.N \quad \forall t = 0,1,.T,.\infty$$

$$r_{t+1}^f = E_t r_{t+1}^m - \left[\frac{(1-\gamma)}{(\sigma-1)}\right]\sigma_{c,m} - \left\{\frac{[\gamma\sigma - \frac{1}{2}(\sigma+1)]}{(\sigma-1)}\right\}\sigma_m^2$$

$$E_t r_{t+1}^m = \kappa_m + \left(\frac{1}{\sigma}\right)E_t\Delta c_{t+1}$$

$$\kappa_m \equiv -\left\{\log\beta + \frac{1}{2}\left(\frac{1-\gamma}{\sigma-1}\right)\left[\left(\frac{1}{\sigma}\right)\sigma_c^2 + \sigma\sigma_m^2 - 2\sigma_{c,m}\right]\right\}$$

where $\sigma c,i$ and $\sigma m,i$ indicate the covariance between the real return on the security and, respectively, the growth rate of consumption and the return on total wealth,[27] while σ_m^2 and

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[26] Epstein and Zin (1991a) use a mean of the returns of certain sectors of the stock market.

[27] In what follows, lower case letters are used to indicate the log of the previously defined variable.

$\sigma_{c,m}$ indicate, respectively, the variance of the return on wealth and the covariance between the latter and the growth rate of consumption. The following conclusions can be drawn from (13):

a) the risk premium of each asset is a function of the covariance of its return with consumption and with the return on wealth, the degree of risk aversion and the elasticity of intertemporal substitution of consumption;[28]

b) the risk-free return depends on the expected rate of growth in consumption, the variance of the return on wealth, the variance of the growth in consumption and the covariance between the two latter variables. It is also proportional to the subjective intertemporal discount rate and is influenced by the degree of risk aversion and the elasticity of intertemporal substitution of consumption.

The sign of the premium may be positive or negative, depending on the impact of the two factors of systematic risk: consumption and the real return on wealth (the latter in view of the recursive nature of preferences). In order to clarify the effect of the two factors, it is worth obtaining the risk premium with the standard assumption of risk aversion and the elasticity of intertemporal substitution being equal (see equation (4)):

(14)
$$E_t r_{t+1}^i = r_{t+1}^f - \frac{1}{2}\sigma_i^2 + \gamma\sigma_{c,i} \quad \forall i = 1,..N \quad \forall t = 0,1..,T,.\infty$$
$$r_{t+1}^f = E_t \Delta c_{t+1} - (\log\beta + \frac{1}{2}\gamma^2\sigma_c^2) \ .$$

It can be seen from (14) that a security with a return characterised both by a large premium with respect to the risk-free rate and by a low correlation with consumption implies a high value of γ (the degree of risk aversion) in the first equation. If the variability of the growth rate of consumption is limited — so that the term $\gamma^2\sigma_c^2$ is low, despite the high degree of risk aversion — and the expected growth in consumption is large, the values of ß required to obtain a theoretical risk-free rate in line with the observed value may be greater

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[28] The variance of the return on the security only reflects a Jensen inequality type effect.

than one or, in other words, the subjective intertemporal discount rate may become negative. As shown by Epstein and Zin (1991a), the inclusion of the market portfolio in (13) tends to ease these difficulties.[29] Alternatively, adding a slow-moving "habit", or time-varying subsistence level, to the basic power utility consumption-based model, as in Campbell and Cochrane (1994), can also help to match excess returns on equity with reasonable risk-aversion parameters.

The inflation risk premia, determined on the basis of (11), are given by (see Appendix I):

$$E_t r_{t+1}^i - E_t r_{t+1}^{i,\pi} = -\frac{\sigma_{\pi,i}}{\sigma_\pi^2}\left[\frac{1}{2}\sigma_{\pi,i} + \frac{\sigma(1-\gamma)}{(\sigma-1)}\sigma_{\pi,m} + \frac{1-\gamma}{(\sigma-1)}\sigma_{\pi,c}\right]$$

(15)
$$E_t r_{t+1}^m - E_t r_{t+1}^{m,\pi} = -\frac{\sigma_{\pi,m}}{\sigma_\pi^2}\left\{\left[\frac{1}{2} + \frac{\sigma(1-\gamma)}{(\sigma-1)}\right]\sigma_{\pi,m} + \frac{1-\gamma}{(\sigma-1)}\sigma_{\pi,c}\right\}$$

$$\forall i = 1,..N \qquad \forall t = 0,1,..T,..,\infty$$

the second assumption — of no change over time in the matrix of the covariance of real returns, the growth rate of consumption and inflation — implies no change over time in the risk premia of (13) and (14). Substituting the second and third equations of (13) in the first gives:

$$E_t\left(r_{t+1}^i - \left(\frac{1}{\sigma}\right)\Delta c_{t+1} - A_i\right) = 0 \qquad\qquad \forall i = 1,..N \quad \forall t = 0,1,..,T,..,\infty$$

(16)
$$A_i \equiv -\left[\frac{(1-\gamma)}{(\sigma-1)}\right]\sigma_{c,m} - \left\{\frac{[\sigma\gamma - \frac{1}{2}(\sigma+1)]}{(\sigma-1)}\right\}\sigma_m^2 -$$

$$\left\{\log\beta + \frac{1}{2}\left(\frac{1-\gamma}{\sigma-1}\right)\left[\frac{\sigma_c^2}{\sigma} + \sigma\sigma_m^2 - 2\sigma_{c,m}\right]\right\} - \frac{1}{2}\sigma_i^2 + \left[\frac{(1-\gamma)}{(\sigma-1)}\right]\sigma_{c,i} + \left[\frac{(\sigma\gamma-1)}{(\sigma-1)}\right]\sigma_{m,i}$$

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[29] Nonetheless, Weil (1989) and Epstein and Zin (1991b) explain that specification (14) is still affected by one of the fundamental limitations associated with the principle of the maximization of the expected utility as formulated by von Neumann and Morgenstern, insofar as the risk premium is proportional to the variance of the return exposed to risk. This indicator of dispersion is of the second order, (in a Taylor expansion around a fixed wealth level) and therefore it leads to risk premia that tend to be small when the return dispersion is limited (see Bekaert, Hodrick and Marshall, 1994, for details).

the evaluation of the covariance of the returns and of consumption requires the specification of an equation of consumption and inflation. Analogously, the determination of the inflation risk premia in (14) makes it necessary to specify an equation for inflation. The functional form adopted for the two equations is the following:

$$E_t \Delta c_{t+1} = \phi_0 + \phi_1 \Delta c_t + \sum_{i=1}^{N} \delta_i r_t^i$$

(17)
$$E_t \pi_{t+1} = \varphi_0 + \varphi_1 \pi_t + \varphi_2 \Delta y_t$$

$$\forall t = 0,1,..,T,..,\infty$$

in the first equation expected consumption growth at time t is a linear function of current realised consumption growth and the level of realised real asset returns. In the second equation the expected inflation at time t is a linear function of current inflation and GDP growth (defined as Δyt). This latter equation can be interpreted as a supply curve representing a standard NAIRU relationship (or Phillips curve), in which the expected change in inflation depends upon current inflation and an excess demand term (an indicator of the output gap) given by the deviation of current output growth from its (constant) trend. Such an equation can be derived from the aggregation of optimal price-setting decision by monopolistically competitive firms adjusting their prices with a constant probability in any given period.[30]

The impossibility of observing the return on total wealth requires, for the purpose of estimating the covariance between this return and the returns on securities, consumption and inflation, the addition of the third assumption indicated above. Specifically, it is shown that the following approximation holds (see Appendix I):[31]

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[30] Such price-setting structure, which can also arise under alternative assumptions of intertemporal optimisation (e.g. quadratic adjustment costs or deterministic time-dependent rules with staggered pricing), was first introduced by Calvo, 1983, and has been frequently adopted in macroeconomic applications; (see Yun, 1996, and Woodford, 1996, for a formal derivation).

[31] See also Campbell (1993) and Campbell and Viceira (1996) for more details.

$$\sigma_{m,i} = \frac{\sigma_{c,i}}{[1+(1-\sigma)\chi(\lambda)]}$$

(18)
$$\sigma_{m,c} = \frac{\sigma_c^2}{[1+(1-\sigma)\chi(\lambda)]}$$

$$\sigma_{m,\pi} = \frac{\sigma_{c,\pi}}{[1+(1-\sigma)\chi(\lambda)]}$$

where χ is a function of the mean of the consumption/wealth ratio, λ. Using (18), it is possible to express the covariances with the return on wealth as a function of the observable variables: consumption, inflation and observed returns. However, the full identification requires further assumptions about the function $\chi(\lambda)$, since the mean of the consumption/wealth ratio is not observable, because total wealth (including human capital) is unobservable. Both χ and β (the utility discount factor) are parameters entering the system of equations (16) only through the set of constant terms, *Ai*; as a result, they cannot be estimated separately.

4. Econometric estimation: methodology and application

After substituting (18) in (16), the parameters of equations (16) and (17), i.e. $\{\gamma, \sigma, \varphi_0, \varphi_1, \varphi_2, \phi_0, \phi_1, \sigma_{c,,i}, \sigma_{\pi,,i}, \sigma_{c,\pi}, \sigma_c^2, \sigma_\pi^2\}$, can be estimated using the generalised moments method (GMM),[32] taking account of the constraints imposed on the parameters by the covariance between returns, consumption and inflation. In total, the model involves the estimation of 17 equations: 3 regarding the real returns of deposits, government securities and shares; 2 for consumption and inflation; and 12 for the constraints represented by the aforementioned covariances (5 variances for the observed variables plus the 7 respective covariances). The instruments used in the estimation are the 5 observed variables with a lag of one period and a constant. The restrictions on the moments required by the estimation

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[32] See Hansen and Singleton (1982). For an introductory presentation, see Hamilton (1994), Chapter 14.

imply 102 conditions of orthogonality.[33] Since there are 25 parameters to be estimated, there remain 77 over-identification restrictions, equal to the degrees of freedom of the Hansen test,[34] used to assess whether the sample moments fulfil the orthogonality conditions imposed by the model. This statistical indicator also serves as a criterion for the estimation of the two unidentifiable parameters, β and χ, which are set by means of a grid-search procedure designed to test the congruency of a restricted version of the model (e.g. by fixing β and χ) to the observed data. This procedure, similar to a Lagrange-multiplier type of diagnostic test for model specification,[35] led to the two parameters in question being set equal to zero.[36]

The parameters' estimation is carried out by using TSP version 4.3, option GMM, which initialises the weighting matrix for the orthogonality restrictions by a three-stage least-squares procedure; such matrix is then kept constant during the iteration process leading to the coefficients estimate; a Newey-West type of correction is introduced in estimating the autocovariance matrix and a Bartlett-type of spectral density kernel is used to adjust the weighting scheme, so that positive semi-definiteness for the errors' covariance matrix is preserved under residuals' correlation.

4.1 Consumption, inflation and real returns in Italy: some descriptive statistics

Before analysing the results of the estimation, it is worth recalling some descriptive statistics and properties of the time series of the observable variables. All the variables included in the estimated equations — the real returns, the growth rates of consumption and GDP, and the inflation rate (all measured in log, e.g. continuously compounded) — were found to be stationary series in the estimation sample (annual series over the period 1892-1993), at least at the 5 per cent level, measured using the Augmented Dickey Fuller test.

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[33] The number of restrictions is equal to the product of the number of equations (17) and the number of instruments (6).

[34] See Hansen and Singleton (1982).

[35] See Hamilton (1994), p. 430, for an illustration of the procedure, which requires the equivalence between GMM and maximum likelihood estimators. This condition is fulfilled in our case.

[36] The Hansen statistics proved not to be very sensitive to changes in the two parameters in question.

The sample periodogram of the series,[37] under the assumption of covariance-stationarity, can provide useful information about typical periodicity displayed by asset returns, inflation and consumption growth; it also gives some clue about the magnitude of the contribution made by different frequency in determining the total sample variability.[38] For all the series considered here it has a relatively flat profile, with only one fairly pronounced peak, primarily attributable to the two world wars and characterised by high inflation and the collapse of real returns, consumption and economic activity. There is evidence of a ten-year cycle (slightly shorter for shares) in which real returns, consumption, economic activity and inflation are pro-cyclical. There is also evidence, albeit less clear, of a shorter cycle of just under six years for economic activity and the return on shares. The cycle for the returns on government securities and bank deposits is shorter still, on the order of 3-4 years, and even less pronounced.

The time series properties of these variables are quite different. Even though there is evidence of heteroskedasticity, (the log of) the index of capitalisation of the real returns on shares basically resembles a random walk with drift. In fact, the persistence index proposed by Campbell and Mankiw (1987) has a value of 1.05, which is not significantly different from 1, which corresponds to the random walk case (see Table 2). In short, the real return on shares is very close to a stochastic process of the white-noise type and hence unpredictable. Similar considerations apply to (the logs of) GDP and consumption. The persistence indices are equal, respectively, to 0.88 and 1.58 and are not significantly different from 1. By contrast, inflation and the real returns on government securities and bank deposits are clearly mean reverting. In all three cases the indices are equal to around 2.8 and significantly different from 1 at the 1 per cent level. For shares and government securities, the characteristics of the real returns are repeated for the differentials with respect to the return on bank deposits.

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[37] Not reported here, but available from the authors.

[38] Acyclical phenomena, which would show at zero frequency, cannot properly be treated in this framework, hence we are ruling out exotic dynamic patterns in the data (e.g., chaotic dynamics).

The structure of auto-correlation shows a broadly similar pattern. For the return on shares (and the related differential with respect to bank deposits), the null hypothesis, corresponding to zero auto-correlation, cannot be rejected (at least up to the tenth order). For government securities and bank deposits, auto-correlation is significant up to the second order (only that of the second order for the corresponding differential), and the same holds for the growth rates of GDP and consumption. For inflation, auto correlation tends to last longer and indeed it is significant to the third order.

4.2 Results of the econometric estimates on Italian data

The joint modelling of variables with different statistical properties undoubtedly raises serious econometric problems. These are aggravated by the radical transformation in the country's economic structure produced by the upheavals that accompanied two world wars and the tumultuous economic growth that followed the Second world war. The specification adopted does not pretend to provide a definitive solution to the econometric problems involved. Accordingly, no attempt has been made to make ad hoc adjustments to take account of the wars.

The results of the estimates appear satisfactory on the whole (Table 3a). The orthogonality conditions imposed on the moments for the estimation of the parameters, evaluated using the Hansen test, cannot be rejected; the significance of the test is very high. The equations for inflation and the return on shares appear satisfactory, while those for consumption and the returns on bank deposits and government securities reveal the existence of serial auto-correlation among the residuals (Table 3b). For the same variables, and partly for the same reason, analogous problems are encountered with regard to the equations of the respective variances and covariances with respect to consumption. The existence of these auto-correlations may partly be a reflection of some degree of misspecification in the model. One possible source of misspecification might be related to the assumption of constant risk premia. Their estimated values are therefore to be interpreted as the permanent (long-term) component possibly time-varying risk-premia.

The estimated parameters appear to be highly significant — the standard errors have been corrected, using the procedure proposed by Newey and West (1987), to take into account the auto-correlation of the residuals.[39]

The estimations of the model with expected utility (Table 4) imply a value of the coefficient of risk aversion of around 4.2, which is in line with the values estimated in other works,[40] and a large negative subjective intertemporal discount rate (-11 per cent). In short, these values of the parameters constitute a risk-free rate puzzle. Hansen's test, however, rejects this model.

Using the model with recursive preferences, the steady-state growth rate of consumption implicit in the estimates is equal to 2.4 per cent (2.1 per cent on average in the sample) and that of inflation is egual to 7.8 per cent (8.3 per cent in the sample), including, it should be noted during the two world wars. The estimated coefficient of risk aversion (equal to 41.4; Table 5), is high and not very different from the values estimated for other countries using similar methods;[41] by contrast, at 0.31 the elasticity of intertemporal substitution, σ, is rather low. The difference between the two parameters is statistically significant, which helps to explain the weakness of the model with expected utility. The value of the parameter σ is of considerable importance, since it measures the reactivity of consumption (and hence of saving) with respect to movements in real interest rates. The relatively low estimate obtained suggests that saving was influenced by financial market returns to a lesser extent in Italy than in the other leading countries.[42]

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[39] In order to ensure the consistency of the estimates of the standard errors, the covariance matrix of the estimated parameters is calculated using a Bartlett window of size 20. Making the correction for the size of the sample proposed by MacKinnon and White (1985) would involve multiplying the standard errors calculated by the coefficient $1.192 (=(102/(102-22))^{0.5})$, where 102 and 22 are, respectively, the size of the estimation sample and the number of parameters estimated. This correction would not alter the significance of the estimations.

[40] See Guiso, Jappelli and Terlizzese (1992).

[41] See Attanasio and Weber (1989), Epstein and Zin (1991a) and Deaton (1992).

[42] See Hu (1993) and, for Italy, Favero (1993).

4.3 Model-based estimates of the risk premia on Italian assets

The risk premia implicit in the estimates of the parameters — obtained using (13) and (15) — are summarised in Table 5; these are all statistically significant. Standard error are obtained by constructing a Wald-type test on the non-linear functions of the model's coefficients entering the risk premia.[43]

The total estimated risk premium is positive for all three of the instruments considered and appears to reflect their riskiness: 0.46 per cent for bank deposits, 2.1 per cent for government securities and 3.9 per cent for shares. The risk premium associated with the implied return on total wealth is estimated to have been 5.5 percentage points, hence larger than that estimated for the observable assets; this might be explained by the relative magnitude of the risk embodied in human capital, which is an important component of total wealth, greater than that of non-human capital. For the whole sample period, the estimated risk-free interest rate was equal to -0.4 percentage points, or around 3.4 points above the average return observed for deposits.[44]

The component of the risk-premia attributable to the inflation risk is statistically significant for all asset returns. It is very similar for bank deposits and government securities (80-85 basis points), while for shares it is equal to 55 basic points; the inflation risk premium for market portfolio (total wealth) is virtually nil (4.1 basis points). The order of magnitude of the estimated inflation risk premia is consistent with the conventional wisdom that stocks are a better hedge against inflation uncertainty than bonds or deposits. Also, the very low inflation premium on total wealth may again be related to the importance of the human capital component in total wealth, since it is very likely that the average wage rate closely tracks the inflation rate Thus the assumption of Fisher neutrality of inflation is clearly rejected for bonds and stocks in the historical period considered, whereas it would be a fair approximation for the return on total wealth.[45]

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[43] Risk premia are a higly non-linear function of the model's parameters; the tests of the restriction that risk premia do not differ from zero is carried out by using the "Analyz" procedure in TSP 4.3.

[44] The returns shown have been calculated as geometric means.

[45] Similar conclusions are drawn by Buraschi (1996) for the US in the period 1964-1992.

Inflation risk premia appear to be larger than those estimated in Labadie (1989) in a monetary version of Mehra and Prescott's model where a cash-in-advance constraint is always binding; the largest inflation premia generated by this model is only 30 basis points for equities. The empirical evidence analysed in Bagliano and Beltratti (1996) regarding the relationship between stock market returns and inflation confirms that the Italian equity market has been an imperfect hedge against inflation, especially during the last 30 years.[46]

All the estimated risk premia should be interpreted as unconditional, e.g. constant, measures of long-run average excess returns. It will be left for future work to explore the issue of time-varying risk premia over such a long period of data. It would be interesting to investigate whether the evidence assembled in Blanchard (1993), pointing towards a relative decline in the equity premium and a rise in real bond excess return in more recent decades, can be accounted for by relaxing the assumption of constant second moments for the underlying economic fundamentals. The presence of heteroskedasticity in the driving forces of the economy is also suggested by the cross-countries evidence assembled in Canova and De Nicolo (1995), who document important cross-country and sub-samples heterogeneity in equity premia and risk free rates. Heterogeneity across countries point to the lack of an integrated world capital markets, however Campbell (1996) has pointed out that (short-run) cross-country correlations of stock returns are higher than the (short-run) cross-country correlations of consumption or dividend growth rate. Hence international stock markets comovement is greater than warranted by the cross-country correlations of fundamentals. Accounting for such heterogeneity poses further challenges which we prefer to leave to future research.

5. Conclusions

In this paper we have estimated the magnitude of the risk premia embodied in Italian financial assets over a century of data using the conceptual framework provided by recent

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[46] See also Banca Commerciale (1995), pp. 17-18, for further evidence.

literature on assets price determination; estimates for italian assets are compare with the findings by several studies for the US financial markets.

In Italy the long-run (from 1860 to 1994) ex post returns on shares (6.7 per cent) was much higher than that of government securities (1 per cent) and those of bank and PO deposits (-1.1 and -2.1 per cent, respectively). Obviously, shares also showed a much higher volatility than the other financial assets. However, this summary assessment is considerably influenced by the exceptional falls in the value of government securities and bank deposits caused by the hyperinflation that occurred in conjunction with the two world wars. Within the period, there were alternate phases, paralleling the economic cycle and the main institutional changes, in which the returns on shares were higher than those on the other two instruments and vice versa.

Overall, the Italian equity market provided long-run returns to investors comparable to those of other major countries, although a large fraction of the risk premium for the sample period can be accounted for by the performance in the wake of the hyperinflation episodes of the wars. However, the risk return trade-off, owing to much larger volatility, compared unfavourably with other markets. Moreover the Italian equity market over the last 30 years (up to 1994), when equity prices barely kept up with inflation, looks very different.

The econometric analysis shows that in the estimation period (1892-1993) the return on shares was much higher than justified by the risk parameters; also the estimated risk-free rate appears to be lower than warranted by the risk parameters. These results, known as the equity premium and risk-free rate puzzles, are approximately in line with those obtained for the other leading countries (for example the US). Specifically, the econometric results indicate that the model-based equilibrium spread (risk premium) between shares and bank deposits (i.e. the difference between the returns on the two financial instruments) should be around 3.3 percentage points, whereas its empirical value in the period considered was around 5.4 points.[47] By contrast, for government securities the observed premium was approximately in line with its theoretical value (1.7 and 1.5 points, respectively).

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[47] In order to make the theoretical and observed returns comparable, geometric means have been used.

These estimates, both the real returns on government securities and those on shares, include an inflation risk premium. For government securities, the model-based, equilibrium, inflation risk premium is estimated at around 0.8 percentage points, a figure which approximates the average annual saving the Treasury could have made by issuing index-linked securities over the sample period. The inflation risk premium was smaller, but not negligible, for shares (0.55 percent), which represent a claim on real assets and thus provide better protection against unforeseen changes in the monetary yardstick. Overall, the average inflation hedge delivered by equity holding has been better than that of bond holdings, yet imperfect over the sample period.

These results are of interest in several respects. First of all, previous work pointed out the inconsistency between the results of empirical research suggesting that the conditions for dynamic efficiency are satisfied in Italy[48] and the fact that for a long time the rate of growth of the Italian economy exceeded the return on financial assets, approximated by the real return on government securities.[49] Since under uncertainty the returns on individual financial assets may differ, this inconsistency could be due to the choice of government securities as a proxy for the rate of return on capital in the Italian economy. The size of the estimated equity premium does not support such a proxy for the real return on capital. Over the long run, real stock returns — both actual and expected — are on average greater than the growth rate of per capita GDP. As a result, the evaluation of the dynamic efficiency hypothesis for the Italian economy based on stock prices confirms previous findings in the literature based on the comparison of profits and investments growth.

Secondly, the econometric evidence allows us to reject the hypothesis that the Italian stock exchange's failure to grow was due to the insufficient profitability of investments in listed shares. The empirical evidence gathered shows that in the long run the value of shares follows, albeit with fluctuations, the growth of GDP and consumption more closely than ther

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[48] See Abel *et al.* (1989), who show the equivalence between the comparison of profits and investment growth rates and the comparison of real interest rate and growth rate in assessing dynamic efficiency (see Diamond, 1965, for the theoretical underpinning of the latter condition). The main OECD countries surveyed in the Abel *et al.* paper passed the dynamic efficiency test (based on the profit-investment relationship), including Italy.

[49] See the arguments suggested in Blanchard and Weil (1990) and, for Italy, in Galli and Giavazzi (1992).

financial assets. This feature is especially important at a time when pension funds are being developed.

Table 1

REAL HOLDING PERIOD RETURNS ON LIRA DENOMINTED FINANCIAL ASSETS:
(annual data CPI deflated)

| Time period	Stock return		Government bond return		Bank deposit rate		Post-office deposit rate	
	Mean (1)	Standard deviation (2)	Mean (1)	Standard deviation (2)	Mean (1)	Standard deviation (2)	Mean (1)	Standard deviation (2)
1861-1994[3]	6.72 / 3.32	26.50	1.03 / -0.12	13.68	-1.15 / -2.15	11.12	-2.09 / -3.16	11.49
1861-1870	11.15 / 10.34	13.93	5.89 / 4.46	8.25	3.71 / 3.68	2.24	—	—
1871-1880	11.94 / 9.41	26.24	10.09 / 9.44	12.72	2.48 / 2.22	7.86	—	—
1881-1890	4.35 / 3.99	9.18	7.17 / 7.00	6.46	4.29 / 4.25	3.21	4.07 / 4.03	3.26
1891-1900	4.65 / 3.44	17.10	6.46 / 6.37	4.61	3.88 / 3.88	0.97	3.65 / 3.65	0.97
1901-1910	6.66 /6.40	7.95	3.94 / 3.90	3.03	2.02 / 1.99	2.25	1.81 / 1.79	2.25
1911-1920	-6.27 / -7.36	14.24	-10.93 / -11.84	13.15	-8.66 / -9.51	12.87	-8.97 / -9.82	12.82
1921-1930	11.94 / 6.06	37.44	5.02 / 4.19	13.56	1.45 / 1.15	8.21	0.98 / 0.67	8.17
1931-1940	12.69 / 11.05	20.19	3.96 / 3.20	13.32	0.73 / 0.42	8.32	1.49 / 1.22	7.79
1941-1950	2.95 / -10.67	53.64	-22.01 / -26.62	25.92	-23.04 / -28.52	25.53	-22.31 / -28.00	26.25
1951-1960	22.40 / 20.96	20.68	2.24 / 2.19	3.42	-0.86 / -0.89	2.83	-0.04 / -0.07	2.45
1961-1970	-2.88 / -4.10	15.96	0.15 / 0.11	3.17	-0.03 / -0.05	1.99	-0.03 / -0.05	1.88
1971-1980	0.65 / -4.37	35.63	-5.06 / -5.35	7.66	-4.51 / -4.56	3.05	-5.97 / -6.04	3.88
1981-1994	6.93 / 2.59	33.08	6.45 / 5.76	7.30	2.08 / 1.91	2.24	-0.13 / -0.18	3.94

(1) Arithmetic/geometric mean. - (2) Standard deviation of arithmetic mean. - (3) Government bond returns and post-office deposit rates, beginning in 1863 and 1876 respectively.

Table 2

MEASURE OF PERSISTENCE (1)

Variable	Coefficient	Asymptotic standard error
Inflation	2.829	0.762
GDP	0.880	0.403
Consumption	1.578	0.277
Bank deposit return	2.841	0.792
Government bond return	2.807	0.722
Stock return	1.020	0.100

(1) Calculated by means of the formula $[1+\Theta(L)]/[1-\Phi(L)]$ where $\Theta(L)$ and $\Phi(L)$ are the polynormal distributed lags for the moving average and autoregressive component, respectively, for the ARMA process. Maximum likelihood estimation carried out under the assumption of ARMA(2,2); for Government bond return, ARMA(1,5) is assumed.

Table 3

GMM: ESTIMATION:
RECURSIVE-PREFERENCES (1)

Parameters	Coefficient	t-Statistic
ϕ_0	0.023	107.8
δ_D	0.167	242.9
δ_{TS}	-0.139	-131.7
δ_{AZ}	0.020	63.7
ϕ_1	0.098	62.5
φ_0	0.013	27.4
φ_1	0.750	855.4
φ_2	0.967	303.3
σ^2_c	0.134	90.2
$\sigma_{c,D}$	0.621	47.7
σ^2_D	0.040	93.2
σ^2_π	0.025	96.2
$\sigma_{\pi.D}$	-0.025	-100.6
$\sigma_{c.D}$	0.913	66.1
σ^2_{TS}	0.034	82.9
$\sigma_{\pi.TS}$	-0.020	-96.2
$\sigma_{c.TS}$	0.188	52.7
σ^2_{AZ}	0.078	67.7
$\sigma_{\pi.AZ}$	-0.939	-49.2
$\sigma_{c.\pi}$	-0.512	-34.9

Hansen-Singleton test (77 degree of freedom): 51.495;
p-value: 0.989.
(1) Annual data for 1892-1993 period.

Table 4

GMM ESTIMATION:
EXPECTED UTILITY, COSTANT RELATIVE RISK AVERSION

Parameter	Coefficient	t-Statistic
β	1.110	55.57
α	4.228	6.64

Hansen-Singleton test (13 degree freedom): 24.001
p-value: 0.031.

Table 5

GMM: ESTIMATION:
RECURSIVE-PREFERENCES

Parameter	Coefficient	t-Statistic	
α	41.423	61.76	
σ	0.309	891.52	
α-σ	38.187	56.91	
	Risk premium (1) (in percent)		
Return	Total	Inflation	
Bank deposit	0.565 (7.96)	0.809 (15.79)	
Government bond	2.094 (28.908)	0.845 (21.33)	
Shares	3.881 (39.04)	0.595 (33.26)	
Market portfolio	5.494 (138.19)	0.041 (21.05)	

(1) t-statistic in parenthesis.

Figure 1

ITALIAN SECURITIES MARKET: PRICE INDICES

(annual data CPI deflated; 1862 =100)

YEARS

Shares — — Government bonds ····· Bank deposits
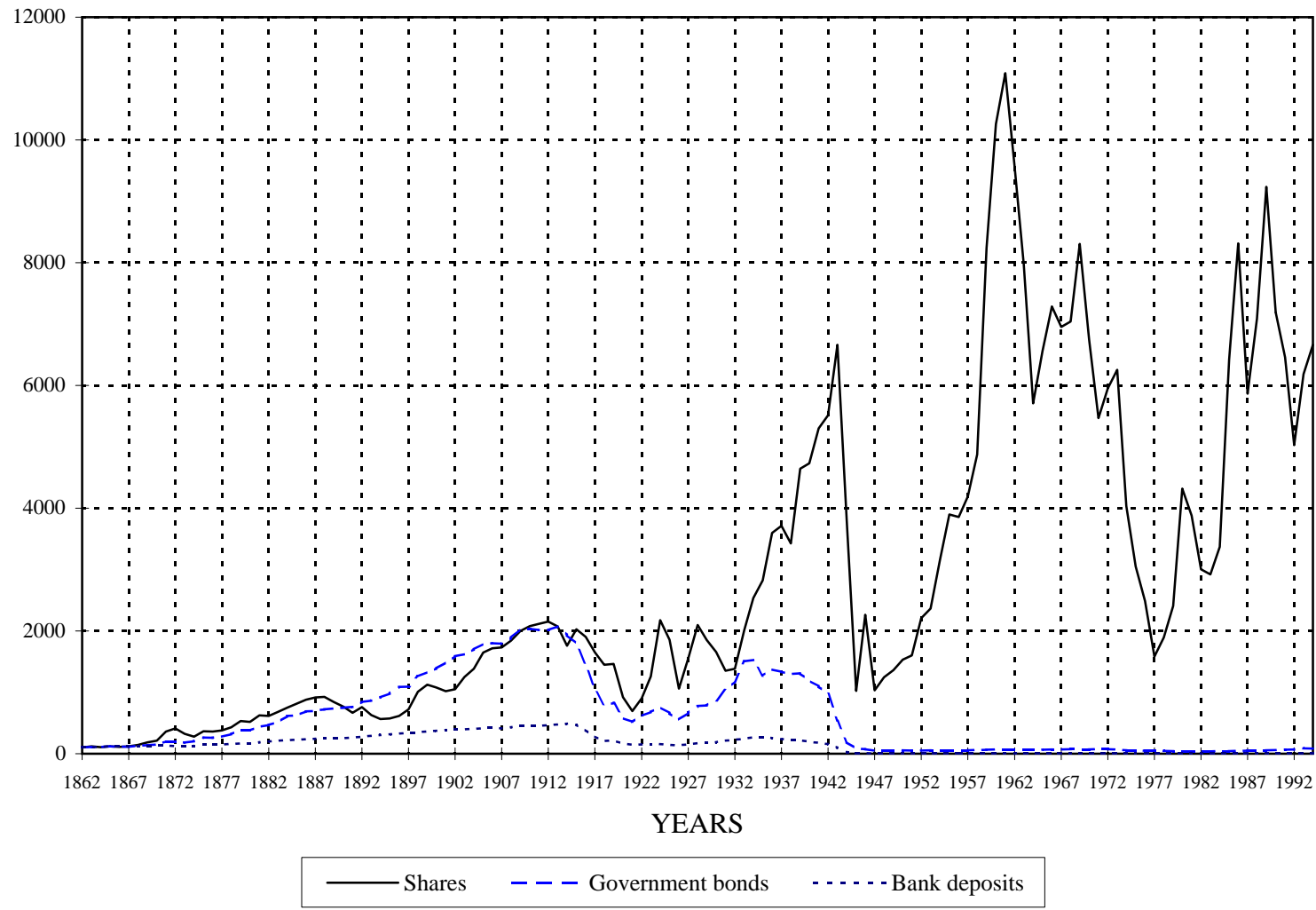
REAL GDP AND CONSUMPTION GROWTH
(percentage change)
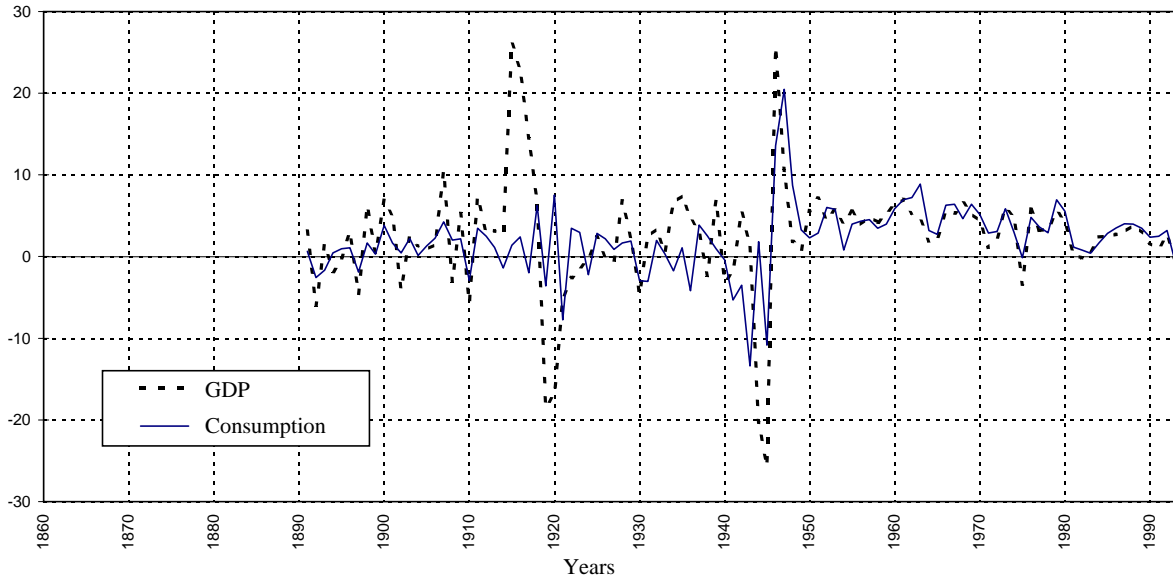
CPI
(percentage change)

Excluding 1944 (349%) and 1945 (97%)
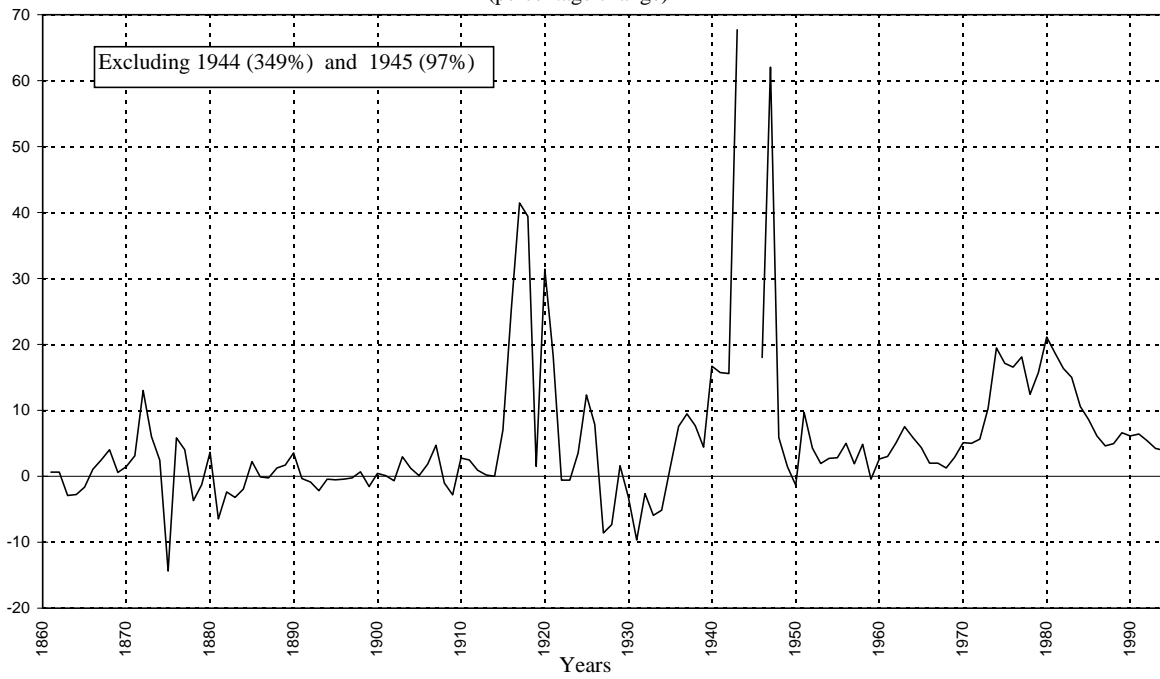
HOLDING PERIOD RETURN ON LIRA DENOMINATED FINANCIAL ASSETS

(annual data CPI deflated; yearly averages)

1-year holding period return

Returns

Years

Shares — — — Government bonds ······ Bank deposits

Figure 3b

10-years holding period return

Returns

Years

Shares — — — Government bonds ······ Bank deposits
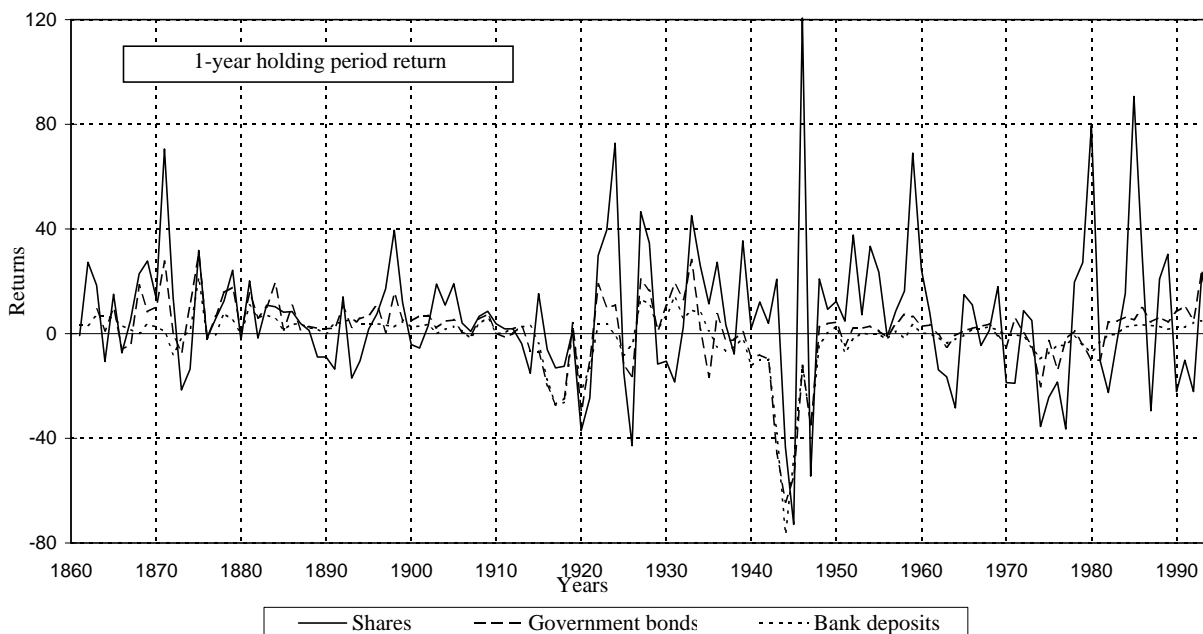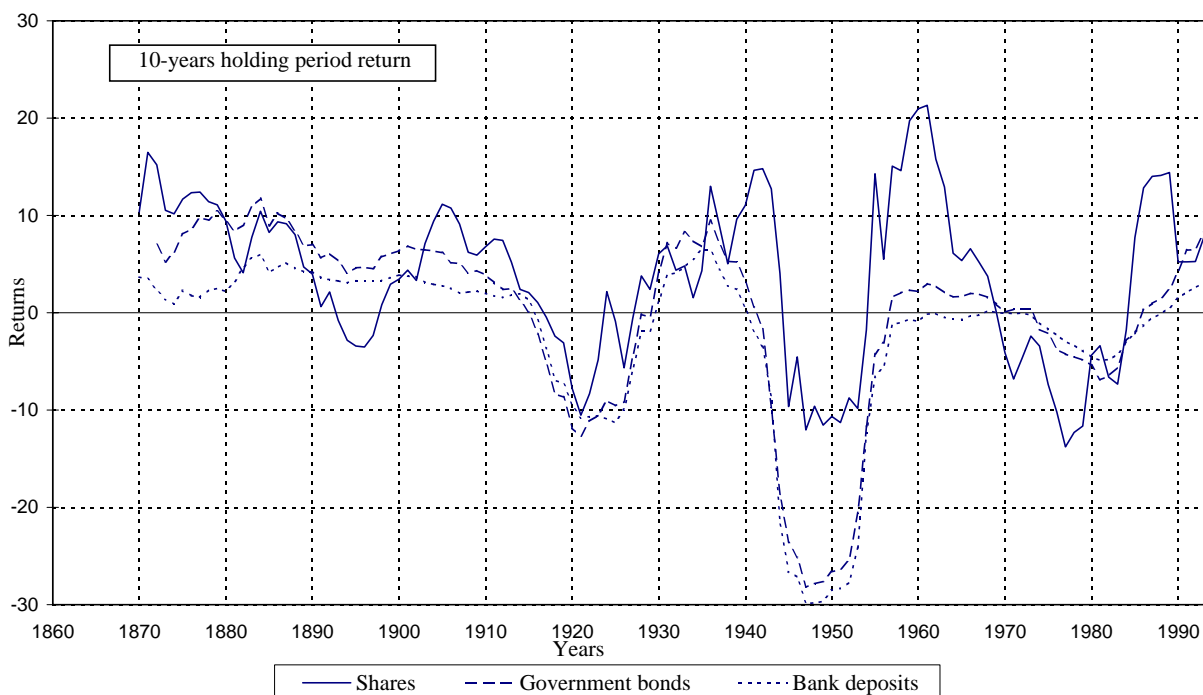
**SHARES AND GOVERNMENT BONDS PRICE INDICES
AND PER CAPITA INCOME AND CONSUMPTION**

(in real terms, 1890=100 - end of year data)

Figure 4

YEARS
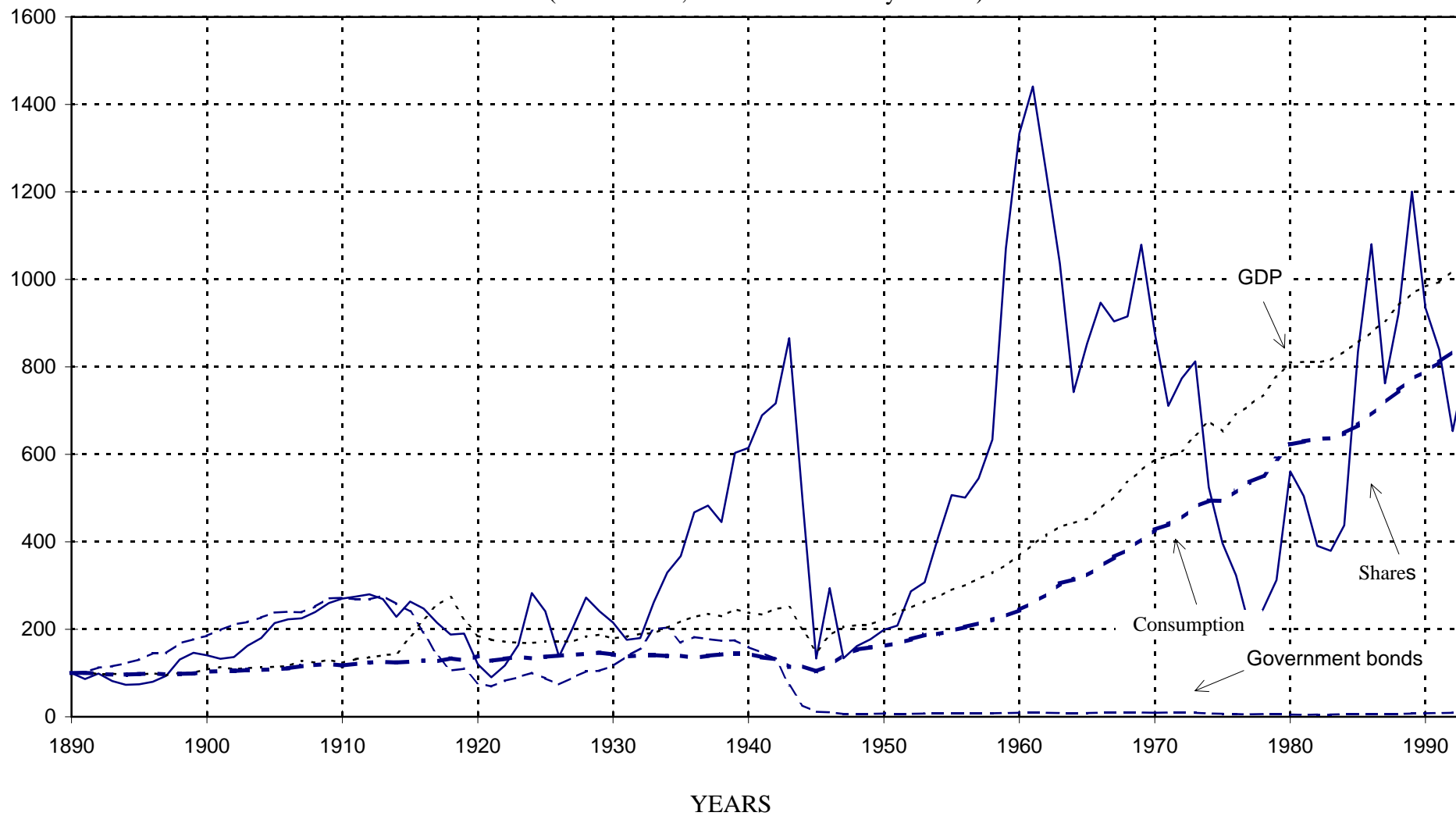
Appendix I

Euler equation for generalised expected utility preferences

Denote by $J(A,I)$ the value of the agent's intertemporal optimization program (1), under recursive preference defined in (4) and beginning with wealth A and current information I, used to predict future rates of return. Following Epstein (1992), J should solve the Bellman equation:

(1a)
$$J(A_t, I_t) = \underset{C_t, \omega_t}{MAX} w \left\{ (1-\beta)C_t^{(1-\frac{1}{\sigma})} + \beta[E_t J[(A_t - C_t)\omega_t R_{t+1}, I_{t+1}]^{(1-\gamma)}]^{\frac{1}{1-\gamma}} \right\}^{\frac{\sigma}{\sigma-1}}$$

$$\forall t = 0,1,2,...,T,...,\infty .$$

By the omogeneity of the planning problem, this optimal value is proportional to wealth; given the structure of the problem, consumption is also proportional to wealth. The maximisation of this right-hand side with respect to consumption implies:

(2a)
$$C_t^{-\frac{1}{\sigma}} = \beta(A_t - C_t)^{-\frac{1}{\sigma}}[E_t(\vartheta_{t+1}\omega_t^* R_t)^{(1-\gamma)}]^{\frac{1}{1-\gamma}}$$

$$J(A_t, I_t) = \vartheta_t A_t$$

$$\vartheta_t \equiv \left\{ \frac{C_t}{A_t} \right\}^{\frac{\sigma}{1-\sigma}}$$

$$\forall t = 0,1,2,...,T,...,\infty .$$

Substituting (2a) into the definition of the value function (1), equation (5) — first equality — can be obtained. Turning to the restrictions implied by the optimal portfolio selection on the right-hand side of (1):

(3a)
$$\underset{\omega_1}{MAX}[E_t(\vartheta_{t+1}\omega_t R_t)^{(1-\gamma)}]^{\frac{1}{1-\gamma}}$$

$$\sum_{i=1}^{N} \omega_t R_t = 1$$

$$\forall t = 0,1,2,...,T,...,\infty .$$

Solving (3a) after substituting (2a), we obtain the necessary (first-order) conditions set out in equation (5).

The evaluation of inflation risk premia

We start by defining the vector of inflation-hedged returns, R_t^π, as the sum of actual returns and an inflation-hedging term (spread), H_t^π:

(4a)
$$R_t^\pi = R_t + H_t^\pi$$
$$\forall t = 0,1,2,...,T,...,\infty ;$$

we assume that the inflation-hedging spread responds in a linear fashion to expected and unexpected inflation:

(5a)
$$H_t^\pi = -\pi_1 \Pi_t + \pi_2 E_{t-1} \Pi_t$$
$$\forall t = 0,1,2,...,T,...,\infty$$

where $-\pi_1$ and π_2 are vectors of coefficients to be determined.

The beta coefficient for the inflation-hedged returns can be obtained by applying the asset pricing equation (10) to the inflation-hedged set of returns (4a):

(6a)
$$E_t R_{t+1}^{i,\pi} - R_{t+1}^f = \beta_Q^{i,\pi} \left(E_t R_{t+1}^Q - R_{t+1}^f \right)$$
$$\forall i = 1,..N \quad \forall t = 0,1,..,T,..\infty .$$

Substituting (4a) into the beta-coefficient of equation (6a), we get the following decomposition:

(7a)
$$\beta_Q^{i,\pi} = \beta_Q^i + \beta_Q^{h_i}$$
$$\beta_Q^{i,\pi} \equiv \frac{Cov_t \left(R_{t+1}^{i,\pi}, R_{t+1}^Q \right)}{Var_t \left(R_{t+1}^Q \right)}$$
$$\beta_Q^{h_i} \equiv \frac{Cov_t \left(H_{t+1}^{i,\pi}, R_{t+1}^Q \right)}{Var_t \left(R_{t+1}^Q \right)}$$
$$\forall i = 1, N; \forall t = 0,1,2,...,T,...,\infty$$

to be a valid asset price representation, equation (6a) should fulfil the no arbitrage-restriction implied by the Euler equation (6) of the main text:

$$
\begin{aligned}
&E_t Q_{t+1} R_{t+1}^{i,\pi} = 1 \\
&\forall i = 1, N; \forall t = 0,1,2,...,T,...,\infty
\end{aligned}
$$

(8a)

to provide the required hedge to inflation, the inflation-hedged asset returns, $R_{t+1}^{i,\pi}$, should be uncorrelated with the inflation rate:

$$
\begin{aligned}
&Cov_t(R_{t+1}^{i,\pi}, \Pi_{t+1}) = 0 \\
&\forall i = 1, N; \forall t = 0,1,2,...,T,...,\infty .
\end{aligned}
$$

(9a)

Equations (8a)-(9a), together with the asset price equation for actual returns:

$$
\begin{aligned}
&E_t Q_{t+1} R_{t+1}^{i} = 1 \\
&\forall i = 1, N; \forall t = 0,1,2,...,T,...,\infty ,
\end{aligned}
$$

imply:

$$
\begin{aligned}
&E_t Q_{t+1} H_{t+1}^{i,\pi} = 0 \\
&Cov_t(H_{t+1}^{i,\pi}, \Pi_{t+1}) = -Cov_t(R_{t+1}^{i}, \Pi_{t+1}) \\
&\forall i = 1, N; \forall t = 0,1,2,...,T,...,\infty .
\end{aligned}
$$

(10a)

The vector of inflation hedging coefficients, $[\pi_1, \pi_2]$, is determined by plugging equation (5a) into the set of restrictions obtained in (10a) and solving the corresponding two equation system:

$$
\begin{aligned}
&\pi_1^i = \frac{Cov_t(R_{t+1}^{i}, \Pi_{t+1})}{Var_t(\Pi_{t+1})} = \beta_\pi^i; \\
&\pi_2^i = \frac{Cov_t(R_{t+1}^{i}, \Pi_{t+1})}{Var_t(\Pi_{t+1})} \frac{E_t(\Pi_{t+1}, Q_{t+1})}{E_t(\Pi_{t+1})E_t(Q_{t+1})} = \beta_\pi^i \frac{E_t(\Pi_{t+1}, Q_{t+1})}{E_t(\Pi_{t+1})E_t(Q_{t+1})} \\
&\forall i = 1, N; \forall t = 0,1,2,...,T,...,\infty
\end{aligned}
$$

(11a)

inflation risk premia can then be obtained by taking the (negative) expected value of the inflation-hedge in (5a):

$$(12a) \quad E_t(R_{t+1} - R_{t+1}^{\pi}) = -E_t H_{t+1}^{\pi} = (\pi_1 - \pi_2) E_t \Pi_{t+1}$$
$$\forall t = 0,1,2,...,T,...,\infty$$

and substituting into (12a) the coefficients determined in (11a) as well as the definitions drawn from equations (8) and (10) of the main text:

$$(13a) \quad R_{t+1}^Q = -\theta Q_{t+1}$$
$$R_{t+1}^f = \frac{1}{E_t Q_{t+1}}$$

we end up with the following expression:

$$(14a) \quad E_t(R_{t+1}^i - R_{t+1}^{i,\pi}) = \beta_\pi^i \left[1 - \frac{E_t\left(\Pi_{t+1}, R_{t+1}^Q\right)}{E_t \Pi_{t+1} E_t R_{t+1}^Q} \right] E_t \Pi_{t+1} = -\beta_\pi^i \beta_Q^\pi R_{t+1}^f Var_t Q_{t+1}$$
$$\forall i = 1, N; \forall t = 0,1,2,...,T,...,\infty$$

representing the inflation risk premium.

We turn now to the completion of the proof for equation (11) in the main text, where the beta coefficients for inflation-hedged returns have been set to be equal to:

$$\beta_Q^{i,\pi} = \beta_Q^i + \beta_Q^\pi \beta_\pi^i$$
$$\forall i = 1, N.$$

In light of equation (7a), we just need to prove that:

$$(15a) \quad \beta_Q^{h_i} \equiv \beta_Q^\pi \beta_\pi^i$$
$$\forall i = 1, N.$$

We start by plugging into (7a) the functional form of the inflation hedge (5a):

$$(16a) \quad \beta_Q^{h_i} \equiv \frac{Cov_t\left(H_{t+1}^{i,\pi}, R_{t+1}^Q\right)}{Var_t\left(R_{t+1}^Q\right)} = -\pi_1^i \frac{Cov_t\left(\Pi_{t+1}, R_{t+1}^Q\right)}{Var_t\left(R_{t+1}^Q\right)}$$
$$\forall i = 1, N; \forall t = 0,1,2,...,T,...,\infty;$$

by replacing in (16a) the expression for the first hedging coefficient computed in equation (11a) and then plugging equation (13a) into (16a), we conclude that:

(17a)
$$\beta_Q^{h_i} = -\pi_1^i \frac{Cov_t\left(\Pi_{t+1}, R_{t+1}^Q\right)}{Var_t\left(R_{t+1}^Q\right)} = -\beta_\pi^i \frac{Cov_t\left(\Pi_{t+1}, -\theta Q_{t+1}\right)}{Var_t\left(-\theta Q_{t+1}\right)} = \frac{1}{\theta}\beta_\pi^i \beta_Q^\pi$$
$$\forall i = 1, N; \forall t = 0,1,2,...,T,...,\infty$$

without loss of generality, θ is set equal to 1 in (17a), so that (15a) holds.

Approximating the covariance matrix of returns extended to total wealth

Since, expected return on total wealth and consumption growth are linearly related - see (13):

(12a)
$$E_t r_{t+1}^m = \kappa_m + (1/\sigma)E_t \Delta c_{t+1}$$
$$\forall t = 0,1,2,...,T,...,\infty$$

their innovations are also linked, as a result of the constant parameter (1/σ); assuming that both variables are stationary, we can use an ARMA representation for consumption — with coefficients {χ_i} — which maps the covariance matrix for consumption into the covariance matrix for wealth return (see Campbell and Viceira, 1996, for details). Normalizing such coefficients so that each wealth return innovation is a constant proportion of consumption innovation — hence offsetting the drift term in (12a) — equations (18) are obtained.

Appendix II

Data-set description: sources and methods

This appendix contains a description of the sources and methods adopted to construct the data used in this paper. The ex post returns were calculated for each period as follows: $R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}}$, where P_t represents the price of the securities at the end of year t and D_t the dividend (for shares) or the coupon (for government securities) paid in year t. The real returns are equal to $\frac{1 + R_t}{1 + \pi_t}$, where πt represents the inflation rate.[50]

Government securities

For the period from 1862 to 1942 the data refer to the return of the consolidated debt of the Treasury. The main source was Bianchi (1979), where, on page 166, there is a detailed description of the origin of the data (for the most part the "Listini ufficiali" of the various Italian stock exchanges). The data on ex coupon prices and coupons reported in Bianchi (1979) were used to obtain indices of total return including both the coupons paid and capital gains.

From 1862 to 1916 the indices refer to the 5% consolidated Rendita before tax. In calculating the after-tax returns, account was taken of the 8 per cent withholding tax on coupons introduced in 1869; the rate was raised to 13.2 per cent in 1871 and to 20 per cent in 1895, where it remained until 1906.[51] From 1906 to 1911 the 5% Rendita was converted into

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[50] Analogously the total return indices at constant prices were obtained using the formula $\frac{1 + P_t}{1 + C_t}$, where P represents the total return index of the security and C the index of consumer prices.

[51] Before 1868 income from securities was not subject to a specific tax, but it was supposed to be included in recipients' total income. Nonetheless, nearly 90 per cent of the tax on income from Rendita bonds was evaded.

the 3.75% net Rendita and from 1912 onwards into the 3.50% net Rendita. The size of this loan was relatively high in the period considered: at the end of the nineteenth century, it accounted for about 80 per cent of the total debt in issue and 50 per cent of total (public and private) financial assets.

Following Bianchi (1979), when calculating the indices for the period from 1917 to 1946, we have also included the 4th, 5th and 6th national loans (all consolidated), the Littorio loan and the 5% Rendita issue of 1935. Each loan was weighted according to the face value of the securities in issue.

After the Second World War, the huge issues of Treasury bonds (BTPs) reduced the significance of the consolidated loans as indicators of the cost of the public debt. Accordingly, for the period from 1945 onwards we constructed the index of total return on the BTPs listed on the stock exchange, including both coupons and capital gains. The data on monthly ex coupon prices, together with the amortisation plans and monthly amounts in issue of each security, were obtained from the Bank of Italy's "Bollettino Statistico". The overall index was calculated by weighting the index of each security on the basis of the face value in circulation. Over the whole period a total of 44 issues were analysed. On average, the number of securities considered in each month was seven. The coverage of the data was generally satisfactory: the BTPs included in the calculation were 52 per cent of the total value of the medium and long-term government securities in circulation in December 1950, 89 per cent in December 1960 and 72 per cent in December 1970. From December 1983 to May 1988, reference was made to the index of the BTPs listed on the Milan Stock Exchange, published by the Bank of Italy in "Supplemento al Bollettino Statistico del Mercato Finanziario". From June 1988 onwards, reference was made to the index of total return of the BTPs listed on the screen-based secondary market for government securities (MTS), which is also published by the Bank of Italy in "Supplemento al Bollettino Statistico del Mercato Finanziario". The calculation of the before tax returns (and indices) took account of the introduction of the withholding tax on government securities coupons.

Shares

The total return index for shares, inclusive of dividends and capital gains, was obtained from various sources. From 1860 to 1895, we drew on the statistics published by Da Pozzo and Felloni (1964), which refer to the December values of the general index of the Genoa Stock Exchange,[52] adjusted to take account of the distribution of dividends.[53]

From 1896 to 1907, the index was calculated with reference to all the shares listed on the Milan Stock Exchange at the end of December. Prices were obtained from the "Listino Ufficiale", while the information on dividends and operations involving firms' capital was obtained from Credito Italiano (1925)[54] and from the "Bollettino Ufficiale delle società per azioni". From 1907 to 1911 the index was obtained from Aleotti (1990).[55]

From 1912 to 1977, reference was made to the indices of prices and dividends prepared by the Bank of Italy for a sample of 40 companies.[56] Up to 1938, the figures are reconstructed in Rosania (1954); subsequently, they have been published regularly in the Bank of Italy's "Bollettino Statistico".[57] The price and dividend indices have been combined to give the overall return index of capitalisation, which includes both dividends and capital gains.

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[52] See Da Pozzo and Felloni (1964), pp. 499-508.

[53] Appendix XVII of Da Pozzo and Felloni (1964), pp. 465-68, appears to indicate that the index was adjusted for operations involving firms' capital but not for the distribution of dividends.

[54] The edition of *Notizie Statistiche sulle Società per Azioni* published by Credito Italiano in 1925 contains a description of all the capital operations effected by Italian firms from their formation. We are grateful to Baia-Curioni of Bocconi University for this information.

[55] It is not clear whether the indices published in Aleotti (1990) take account of dividends. Since comparison with other available information suggests that they do not, when calculating the returns, a dividend of 5 per cent was included, in line with the average dividend paid in the preceding years and with the figure published by Rosania for the following years.

[56] The representativeness of the 40 shares covered by the Bank of Italy index is generally very high, since they nearly always accounted for at least three quarters of the total market capitalization.

[57] For an analysis of the characteristics of the Bank of Italy index, see also Rosania (1983).

From 1978 to 1994, the index was calculated using the Bank of Italy Research Department's data base on the share market (Sistema Informativo sul Mercato Azionario — SIMA). This course was adopted, in preference to using other indices (such as MIB), in order to permit the adjustment of share prices for the payment of dividends and for operations involving firms' capital. The index refers to all the shares listed in each month considered.

Bank deposits

From 1861 to 1940, the rates on bank savings deposits are those of savings banks reported in Biscaini Cotula and Ciocca (1982), to which the reader is referred for a detailed description of the methods and sources used. From 1941 to 1945, the rates were estimated using an OLS regression of bank deposits on the PO deposits of the previous period. From 1947 to 1961, the figures are those published in Bianchi, Nardi and Veccia (1972). Subsequently, recourse was made to the data published in the Bank of Italy's "Bollettino Statistico".

Inflation and national accounts variables

Inflation refers to the index of consumer prices calculated by Istat. The figures for consumption, investment and GDP, at both current and constant prices, are those published by Rossi, Sorgato and Toniolo (1992).

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