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**Scenario based principal component value-at-risk:
An application to Italian banks' interest rate risk exposure**

by Roberta Fiori and Simonetta Iannotti



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SCENARIO BASED PRINCIPAL COMPONENT VALUE-AT-RISK: AN APPLICATION TO ITALIAN BANKS' INTEREST RATE RISK EXPOSURE.

by Roberta Fiori (*) and Simonetta Iannotti (*)

Abstract

The paper develops a Value-at-Risk methodology to assess Italian banks' interest rate risk exposure. By using five years of daily data, the exposure is evaluated through a principal component VaR based on Monte Carlo simulation according to two different approaches (parametric and non-parametric). The main contribution of the paper is a methodology for modelling interest rate changes when underlying risk factors are skewed and heavy-tailed. The methodology is then implemented on a one-year holding period in order to compare the results from those resulting from the Basel II standardized approach. We find that the risk measure proposed by Basel II gives an adequate description of risk, provided that duration parameters are changed to reflect market conditions. Finally, the methodology is used to perform a stress testing analysis.

JEL classification: C14, C19, G21

Keywords: Interest rate risk, VaR, PCA, Non-normality, Non-parametric methods

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(*) Bank of Italy, Competition, Regulation and General Affairs.

1. Introduction*

The aim of this paper is to develop a new Value-at-Risk (VaR) methodology for measuring and monitoring banks' interest rate risk exposure. The main contribution of our work is a new approach to modelling interest rate changes that takes into account the fact that financial data exhibit skewness and fat tails. It is widely known that returns on market variables (such as exchange rates, equity prices and interest rates) systematically depart from normality: financial returns show higher peaks and fatter tails than the normal distribution, especially over short horizons. This implies that extreme events (very large or very small changes in market variables) occur more frequently than predicted under normality assumptions. Failure to account for non-normality may lead to an underestimation of risk.

Additionally, unlike the most common interest rate models in which any relationship between interest rate levels and their correlations and volatilities is dominated by one factor (usually identified with the short rate reflecting the monetary policy stance), we model interest rate changes as a function of three underlying risk factors: shift, tilt and twist, as derived from the Principal Component decomposition of the EU yield curve.

The restatement of observed interest rates in terms of a combination of these underlying risk factors is applied in a Monte Carlo simulation to generate a large number of possible shocks of the yield curve. The profit and loss distribution is then derived from the simulated risk factor distributions by using a delta-gamma approximation function. Finally, the interest rate risk exposure is obtained by selecting the first percentile of the profit and loss distribution according to the VaR definition.

We face the non-normality issue by an appropriate choice of the principal component distribution function. We compare two approaches: the parametric approach, based on the normal distribution hypothesis of the underlying risk factors, and the non-parametric approach, based on kernel densities of the principal component distributions.

The two different approaches are then applied to the balance sheet maturity structure of the major, large and medium-sized Italian banks¹ in a way that strictly reflects the

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¹ The balance sheets are on an individual basis, as consolidated data of assets and liabilities are not available with the desired detail of residual term to maturity. Unfortunately, since in our sample most of the banks belong to banking groups, the distribution of balance sheet items is influenced by the managerial choices

prevailing market conditions both in terms of interest levels and volatility; interest rate risk is computed first on a daily basis and then on a one-year holding period (240 working days).

A bank is exposed to interest rate risk if there is a maturity mismatch between fixed rate assets and liabilities, or between the re-pricing schedules of variable rate positions. Interest rate risk is usually measured by two main methods: the *maturity gap model* and the *duration gap model*. The first approach calculates the effect of interest rate movements on the interest rate margin within a limited time span, generally the one-year period of the income statement. The second approach computes the effect of interest rate movements on the present value of all positions, according to the discounted value of their cash flows. The difference between the duration² of the assets and that of the liabilities gives a measure of the economic capital exposure to interest rate changes: with a positive duration gap (long-term assets are financed with short-term liabilities) the intermediary is exposed to an increase in interest rates, and the value of its economic capital will diminish when rates increase.

Consistently with the *duration gap model*, the standardized method for interest rate risk measurement proposed by the Basel Committee (2003) requires that all assets, liabilities and off-balance sheet items be allocated in 13 maturity buckets according to their remaining time to maturity or, in the case of variable rate items, according to their re-pricing schedule. The net positions for each maturity bucket are then weighted to take into account their sensitivity to interest rate changes: the weighting coefficient results from the product of a) the modified duration of a par bond maturing in the mid-point of the respective bucket and b) a measure of interest rate volatility.³ For each bank, the interest rate risk index is computed as the ratio between the sum of the net weighted positions and supervisory capital.

Most Italian banks use their internal asset liability management (ALM) model to assess the exposure to interest rate risk. Deriving information from the front office system, banks map all cash flows into a specific number of time buckets and calculate the impact of

of each group. For some of the banks, for example, we observe *negative* duration gaps (short-term assets are financed with relatively long-term liabilities), which is counter-intuitive in light of the traditional intermediation activity performed by banks; this can also be due to the particular allocation of assets and liabilities across business units within each banking group.

² Duration is defined as the weighted average maturity of a bond's payment, or the average time of the cash flows, where the weights are the present values of the cash flows.

³ In the first Basel proposal of 1993 (Basel I), on which the current Italian regulation is based, the modified duration is computed for an 8 per cent par bond, and interest rate volatility ranges from 100 basis point for short-term maturities to 60 b.p. for long-term maturities, reflecting the fact that long-term yields are usually less volatile. In the more recent proposal (Basel II, 2001), the duration is computed on a 5 per cent par bond and the volatility is assumed fixed at 200 b.p. for all maturities.

hypothetical interest rate (IR) scenarios on the present value of these cash flows in a baseline scenario. Since the shocks are predetermined, no assumption is made on the type of process driving the IR risk factor.

Our methodology represents an evolution of the standardized approach proposed by the Basel Committee, from which it departs in three respects: 1) first and most importantly, instead of a single scenario we generate a large number of random scenarios in order to derive the banks' profit and loss distributions; 2) we use new duration parameters derived from the interest rate levels prevailing at the time of risk evaluation; 3) we introduce a second order term (convexity) in the approximation function to take into account non-linearity in the relation between interest rate changes and position value changes.

We find that the results from our methodology are consistent with those from Basel II when the duration parameters proposed by the regulation are changed to reflect market conditions. The average risk index for the 18 large Italian banks in the sample, on a one-year risk horizon, is 8.9 per cent of supervisory capital against 8.3 per cent of Basel II with adjusted duration parameters.

Back-testing analysis shows that the parametric approach is well-suited to capture volatility when interest rates are decreasing, but it has some limitations when large positive interest rate changes come into play. The non-parametric approach performs better for banks that are exposed to an increase in interest rates.

To our knowledge this is the first paper that evaluates the Basel regulatory approach for the estimation of the interest rate risk on banks' banking book positions and that applies in a risk management framework a non-parametric estimation procedure to account for the non-normality of the interest rate risk factors. While in the context of credit risk models there is an extensive literature on VaR measures based on the hypothesis of non-normality, we have not been able to find an application of non-normal VaR measures to interest rate risk exposure.

The paper is organized as follows. Section 2 describes the scenario simulation procedure applying PCA. Section 3 provides some evidence on interest rate term structure in the euro area and gives some descriptive statistics of the financial time series used in the analysis. In Section 4 the methodology is applied to a sample of Italian banks, and in Section 5 the performance of different VaR measures is compared through a back-testing analyses. In Section 6, the methodology is extended to a one-year holding period and the risk measure is compared with the regulatory measure proposed by Basel II. In Section 7 we show that scenario simulation based on Principal Component Analysis can be applied to stress testing analysis. The last section summarizes the main results.

2. Scenario simulation of interest rate changes applying PCA

The main obstacle to estimating the VaR of a portfolio by using Monte Carlo simulation is the computational burden of portfolio revaluation due to the high number of risk factors and the large number of positions, which need to be fully revalued under many different scenarios.

Principal Component Analysis is a widely used technique in portfolio risk management that allows to reduce the number of risk factors driving portfolio value changes and therefore the computational burden of portfolio re-evaluation. This technique is especially useful in Monte Carlo simulation, which requires fully re-valuing the portfolio under many different scenarios (Press et al., 1996).

The first to apply Principal Component Analysis to fixed income portfolios were Jamshidian and Zhu (1997) in order to derive a discrete approximation of the portfolio value distribution, while Loretan (1997) and Frye (1997) apply Principal Component Analysis in the context of a Var methodology. In particular, they compute the VaR of a fixed income portfolio by defining principal component based scenarios, where they specify separate “shocks” in each of the directions represented by the PCs and “combined” shocks as linear combinations of the PCs. They use a small set of large prefixed shocks, such as 2.33 times the PCA standard deviation for a ninety-ninth percentile VaR.

When PCA is applied to the term structure of interest rates, a fairly standard result is that three principal components explain a large part of the total variation of the entire yield curve. Moreover, the three-factor structure is consistent across different time periods. Generally, the first PC is interpreted as a “shift” of the yield curve, the second as a “tilt” or “rotation” of the yield curve (change in the steepness) and the third as a “twist” or change in the curvature.

The principal component decomposition can be used to formulate various types of scenarios, along each PC’s direction or through a combination of them (such as an upward parallel shift combined with a flattening of the curve).

Once PCA has been performed, the new risk factors can be “simulated” in order to produce different possible scenarios. Since each PC is a linear combination of the original variables, it is possible to pick tail-event quantiles of their simulated distribution and

generate corresponding tail events of the original risk factors. Several methods can be used in the simulation process: the most commonly used are historical and Monte Carlo simulation.

In historical simulation, the empirical distribution of the risk factors in the past is assumed to be constant and therefore representative of outcomes in the future. It is a non-parametric method that does not depend on any assumption about the probability distribution of the underlying risk factor. However, there is no consensus on the length of the simulation period: in the case of a short period of time the results will be very sensitive to outcomes (possibly accidental) of the recent past; a long period may include information that is no longer relevant to the current situation. Moreover, since historical scenarios include only events that have actually occurred, they may not be representative of all events that could happen in the future.

The Monte Carlo simulation is more flexible than other approaches as the distribution of risk factors shows the full range of all possible realizations and their probabilities. Historical data, while not used to produce scenarios directly, are still needed for calibration. The simulation consists of two steps: a large number of random samples is taken from the assumed risk factor distribution and then portfolio value change is computed for each sample. The Monte Carlo simulation based on Principal Component Analysis is performed by drawing independent random shocks from the distribution of the three PCs underlying the movements of the yield curve, and then inverting the PCA representation to reproduce the correlation structure of the interest rate changes along the various points of the yield curve (see Appendix 1).

Usually, scenarios based on PCs are simulated by assuming that the statistical distributions of risk factors are standard normal. Kreinen et al. (1998) perform a Monte Carlo simulation of the movements along the yield curve by using the PCA results to obtain correlated changes and assuming that the principal components follow a normal distribution. This hypothesis, which allows computational tractability, is far from realistic: empirical returns across different markets show higher peaks and heavier tails than would be predicted by a normal distribution, especially over short horizons. Various studies on market risk factors consistently find higher skewness and heavier tails than implied by the normal distribution.

As in Kreinen et al, we apply Principal Component Analysis to Monte Carlo simulation but we modify their approach to take into account the non-normality of historical observations.

Generally, there are two different approaches in the literature to modelling the non-normality of financial time series. One approach is to use a stochastic volatility model, where conditional return distributions are normal but their variance changes over time. The other approach, used in this paper, is to model directly the unconditional distribution by using a non-normal density function. Various possible distributions have been proposed in the literature, which, since they have fatter tails, allow for larger movements in the extremes of the distribution (for example Student's t distribution, the generalized lambda distribution or the normal mixture approach).

The main contribution of our work is a new method of modelling interest rate changes when the underlying risk factors are skewed and heavy-tailed. Since the PCs retain the statistical properties of the original risk factors, it is possible to account for the non-normality observed in interest rates by an appropriate choice of the principal component distribution functions. In particular, we perform a non-parametric estimation of the PC distribution functions.

We derive the probability densities of the PCs by using a “local smoothing” technique that assumes that the value of the density at each point is mostly influenced by the observations close to that point.⁴ In particular, given the empirical distribution of the principal components, we apply a Gaussian kernel estimator with optimal bandwidth⁵ $\lambda = \sigma N^{-0.2}$. The PC distribution functions are then derived by simply integrating each kernel density.

In Monte Carlo simulation the cumulative distributions need to be inverted in order to calculate the percentiles corresponding to random sets of probabilities. To this end, an analytical expression for the cumulative distributions is derived by fitting each time series of probabilities with an appropriate quasi-likelihood method for fractional logit models,

⁴ Technical references to non- parametric density estimation can be found in Silverman (1986).

⁵ In all kernel estimators, the bandwidth is a crucial parameter determining the size of the region (around the point of interest) that is used to perform the smoothing operation. For that reason we also check the robustness of results by using different estimators and different time bandwidths.

typically used when the dependent variable takes any real value between zero and one (Appendix 2).

The method of scenario simulation using PCA can be summarized as follow:

- 1) Find the principal component decomposition of the yield curve and analyze the statistical properties of the new risk factors (PCs).
- 2) Given the skewness and kurtosis of PC empirical distributions, derive the kernel densities and obtain the corresponding non-parametric probability functions by integration.
- 3) Find an analytical expression for each PCs' distribution function by fitting each time series of PCs' cumulative densities with a fractional logit model. Given the estimated coefficients of the model in terms of log-odds ratios, invert the logit transformation of each PCs' cumulative function (which is linear) to derive the percentile values corresponding to random levels of probability, as drawn from the uniform distribution⁶.
- 4) Once random shocks are generated from each PC non-parametric distribution, apply the PC decomposition of the interest rate term structure in order to reproduce the correlation structure of the original risk factors.

3. The term structure of interest rates: some evidence for the euro area

In our exercise on Italian banks' balance sheets the simulation procedure of the interest rate term structure movements is calibrated over the period from January 4, 1999 to September 30, 2003. The data consists of 1,173 daily observations of government bond par yields in the euro area at tenors of 3 and 6 months and 1, 2, 3, 4, 5, 7, 10, 15, 20, 25 and 30 years.

Table 1 gives a summary description of the yield curve in the EMU from the establishment of the euro. Some of the statistical properties observed for the interest rate term structure in the euro area are in line with the stylized facts known for other markets, in

particular the US market.⁷ The dynamic observed for interest rates indicates that interest rates are persistent in that they spend long, consecutive periods above and below the estimate of the unconditional (or long run) mean and that this behaviour is similar across different maturities (Figures 1a, 1b and 1c show the daily interest rates and their volatility⁸ for the short (3 months), the intermediate (5 years) and the long-term (15 years) maturity). The Dickey-Fuller unit root test confirms the presence of a stochastic trend in the data. Because of this non-stationarity, in analyzing yield curve movements it is necessary to refer to interest rate changes⁹.

Table 2a reports a number of summary statistics for daily interest rate changes, including the Dickey-Fuller unit root test, the ARCH LM test for autoregressive conditional heteroschedasticity and the Shapiro-Wilk test for normality. According to the DF test the daily interest rate changes are stationary. The average volatility of term structure is approximately constant up to one year and then decreasing as maturity becomes longer. The evolution of volatility over time does not show volatility clustering and GARCH effects.¹⁰ The Shapiro-Wilks test fails to accept the null hypothesis of a normal distribution for each rate on the maturity spectrum. The non-normality of data can be related to the asymmetric volatility pattern of interest rate changes, as emerges from a comparison of the volatility of positive and negative changes for several maturities (Figures 2a, 2b and 2c). The charts show that since December 2001 the volatility of positive changes has been larger than the volatility

⁶ Given the estimated coefficients of the model in term of log-odds ratio: $\ln(P/1-P) = a + bX$, it is possible to calculate the percentile X corresponding to a fixed value of P by simply inverting the function : $X = [\ln(P/1-P) - a] / b$.

⁷ The fact that the yield curve follows specific patterns is also used to find specific functional forms matching the curve: for a recent work on the euro yield curve see Brousseau (2002).

⁸ Volatility is measured as the standard deviation of daily rate changes within the month (monthly moving average).

⁹ (Weak) stationarity implies that the first two moments of the distribution are finite and constant, and that the auto-covariance (at various lags) depends only on the lag and is independent of time. If time series are non-stationary, standard test diagnostics are usually biased and one can obtain spurious results. Most financial time series generate prices or yield data that are non-stationary because of a stochastic rather than a deterministic trend; stationarity may be achieved by first differencing the series. In the Dickey Fuller test, the null hypothesis is non-stationarity and the alternative hypothesis is stationarity. For a result on stationarity of interest rate changes, see also Niffiker et al. (2000) and Lardic et al. (2001).

¹⁰ In a GARCH model, interest rate changes are assumed to be generated by a stochastic process with time-varying volatility: the conditional distributions change over time in an autocorrelated way. The ARCH LM test for autoregressive conditional heteroschedasticity shows insignificant autocorrelation (see Table 2). This could

of negative changes, probably reflecting an asymmetric behaviour of interest rate changes when interest rate levels are low and close to their minimum boundary.

Another aspect worth noting is that there is a strong correlation between daily interest rate changes at different maturities, which confirms that movements of term structure are determined by a limited number of common factors. Since correlation is not equal to one, however, yield curve movements should be determined by more than one factor.

During the period from January 1999 to September 2003, Principal Component Analysis revealed that three components were sufficient to explain 95 per cent of total variation (Table 3). Table 4 shows the factor loadings¹¹ of the first three principal components. The first principal component is highly and positively correlated with all rate changes and can be interpreted as a shift of the yield curve, which means that all interest rates move in the same direction and by the same amount. In our analysis, 69 per cent of the total variation in the yield curve over the sample period can be attributed to parallel shifts. The second component represents the tilt of the yield curve: the factor loadings are monotonically decreasing from 0.82 on the three-month rate to -0.38 on the long rate. Thus, an upward movement in the second principal component induces a change in the slope of the yield curve: short maturities move up and long maturities move down. In our analysis, 21 per cent of the total variation can be attributed to a tilt of the yield curve. The factor loadings on the third component are positive for very short rates, but decreasing and becoming negative for the medium-term rates, and then increasing and becoming positive again for the longer maturities. Therefore, the third component induces the convexity of the yield curve and it represents a “twist” component that causes 5 per cent of the total variation (see Figure 3). As regards the impact on various points of the yield curve, the maturities ranging from 2 to 15 years are more correlated with the first shift factor, whereas short maturities are significantly affected by the second tilt factor. The third factor has a significant impact on the short-term and on the long-term segment.

We replicate the Principal Component Analysis on different sets of daily observations and we obtain that the three-factor specification is consistent across different

also be due to an asymmetric behaviour of positive and negative interest rate changes (see *infra*). The presence of asymmetric Garch effects is beyond the scope of this research and has not been explored here.

¹¹Factor loadings measure the correlation of all 13 points on the yield curve with respect to each PC.

time periods. These findings are fairly standard and consistent with several empirical studies for the US and the EU interest rate markets. Since the work of Litterman and Sheinkman (1991) for the US, various empirical studies have shown that around 99 per cent of the variation in yield changes is explained by three common factors, and that the first factor alone explains around 90 per cent of the variation. This result has been confirmed for other markets as well (Alexander, 2000 and 2001; for Italian interest rates before the euro, see D'Ecclesia and Zenios, 1994).¹²

In Table 5 we present some statistics of the distribution of the principal components, which show that they are skewed and heavy-tailed.¹³ If we compare each principal component relative frequency distribution with a normal density of the same mean and standard deviation, we see that the first distribution is slightly leptokurtic with an extra weight in the right-hand tail of the distribution (positive skewness), the second and the third PC have higher peaks (that is, more weight around the mean) as well as more weight in the tails with a negative skewness (Figure 4a). For these two distributions, the mid-range values on either side of the mean have less weight than the normal distribution; this means that the rotation and the twist of the yield curve are likely to be very small or very large, but are less likely to take values between these two extremes. Figure 4b shows the PC cumulative distribution functions.

4. The principal component Value-At-Risk: some evidence for Italian banks

Our research assesses the interest rate risk exposure of a sample of Italian banks. We choose the 18 largest Italian banks in terms of total assets, as they represent a large fraction of the Italian banking system in terms of total assets and at the same time their balance sheet composition is varied enough to reflect various possible situations in terms of their exposure to interest rate risk.

¹² Prevailing interest rate models interpret these findings in the sense that any relationship between the level of interest rates and their expected changes and volatilities is dominated by one factor (one-factor models). The same finding justifies in some way the use of hedging methods that rely on the assumption of parallel risk movements. This single underlying random factor is usually identified with the instantaneous or short rate of interest, which is interpreted as the change in the stance of monetary policy. For a recent review of interest rate models, see Rebonato (2003).

¹³ Positive excess kurtosis indicates that the probability of extreme movements is higher than implied by the normal distribution.

The interest rate risk assessment refers to individual banks' balance sheet positions and covers both the banking and the trading book. Only euro positions are considered, representing around 90 per cent of total assets.

The first 13 banks in the sample have positive duration gaps (*asset sensitive banks*), while the remaining 5 banks show negative duration gaps (*liability sensitive banks*). Asset sensitive banks tend to finance medium and long-term assets with short-term liabilities, being exposed to interest rate risk when interest rates go up.¹⁴ Conversely, the liability sensitive banks tend to go "short" up to 5 years, with relatively small "long" positions in the highest maturities, being exposed to decreasing interest rates. With respect to portfolio composition, no systematic differences emerge between the two categories of banks.

As in the Basel Committee's standardized approach, banks' on and off-balance sheet positions are distributed along 13 different buckets according to their remaining time to maturity, or residual time to re-appreciation. The net positions in each bucket are then weighted to reflect their sensitivity to interest rate changes.

In order to obtain an interest rate risk measure that is more responsive to the evolution of market conditions, we derive the sensitivity parameter, i.e the modified duration, from the interest rate levels prevailing on the market at the time of risk evaluation. Moreover, a second order sensitivity factor, convexity, is introduced to take into account the non-linearity of the relation between interest rate changes and position value changes.

Formally, net position value changes are approximated by the (non-linear) delta-gamma approximation function, which takes into account the first and second order sensitivity factors to interest rate movements:

$$\frac{dP}{P} = -D * \Delta r + \frac{1}{2} C \Delta r^2$$

where D^* is the modified duration, C is the convexity of the net positions in each maturity bucket and Δr represents the simulated change in interest rates. Duration and convexity are

¹⁴Generally speaking, banks performing the traditional activity of maturity transformation between (short term) deposit liabilities and (long term) loans tend to be asset sensitive and exposed to an increase in interest rates.

calculated on the basis of euro par yields prevailing on the market at the end of September 2003 (Table 6).¹⁵

The interest rate shock for each maturity bucket is derived from the scenarios simulation procedure based on the PC representation of the yield curve. Scenarios are generated by calibrating the simulation procedure on the historical observations of the interest rate changes from January 1999 to September 2003 at the 13 different maturities: 3 and 6 months and 1, 2, 3, 4, 5, 7, 10, 15, 20, 25 and 30 years, respectively.

The total net position value changes for each bank are computed over a large number of scenarios (around 30,000) through the delta-gamma approximation function. The daily VaR is then obtained by choosing the 1st percentile of the profit and loss distribution.¹⁶

The one-day VaR is evaluated according to two different approaches: the parametric approach, based on the normal distribution of the underlying risk factors, and the non-parametric approach, which takes into account the skewness and fat tails observed for both interest rate changes and principal components. The estimates are then compared with the forecast obtained by historical simulation, which is based on the empirical distribution of the interest rate changes, assumed to be constant and representative for the coming day.

Table 7 lists the results for the historical VaR and the principal component VaR (parametric and non-parametric). Panel 7.1 shows the results for the 13 banks with positive duration gaps (*asset sensitive banks*); panel 7.2 shows the results for the remaining 5 banks with negative duration gaps (*liability sensitive banks*).

The historical VaR is computed using the last 250 and 500 historical one-day investment results, respectively. The historical VaR estimate is lower when calculated over a shorter time period. This result is probably due to the gradual decrease of European interest rate volatility in the last two years.¹⁷

As regards principal component VaR, two important findings are worth noting. For banks exposed to an interest rate increase, losses from historical simulation tend to exceed

¹⁵ They are the duration and the convexity of a representative par bond maturing in the mid point of each bucket.

¹⁶ Profit and losses for a given day are computed under the assumption that the balance sheet positions are unchanged and that any gain or loss is due to the (simulated / observed) movement of the term structure.

both PC VaR measures, especially the one computed under the normal distribution hypothesis (Table 7, panel 7.1). On the contrary, the PCA VaR measures, under hypotheses of both normality and non-normality, are systematically higher than the historical simulation VaR for those banks that are exposed to negative interest rate changes (see Table 7, panel 7.2). These results show that - given the higher volatility of positive interest rate changes - the principal component VaR measures may underestimate risk when rates are increasing. This is especially true for the normal PC VaR, probably because of the limits of the parametric approach in modelling interest rate changes when their distribution is skewed and heavy-tailed. On the other hand, the historical simulation VaR probably reflects the higher interest rate changes prevailing in the past and it could be argued that the risk measure computed by this method represents an over-conservative risk estimate, requiring an amount of capital for interest rate risk that is too high given current volatility conditions.

5. Validation and back-testing of principal component VaR

A shortcoming of the VaR methodology as a risk management tool is that it conveys nothing about the size of violations when they occur (e.g. Basak and Shapiro, 2001 and Berkowitz and O'Brian, 2002). In other words, the VaR measure reflects only the probability that a certain threshold is overcome, but is not informative on the amount of the losses exceeding the threshold. It is therefore of interest to examine the empirical evidence on the magnitude of excesses.

A usual procedure to evaluate the accuracy of VaR models (based on scenario simulation) is "back-testing analysis". The essence of back-testing is to compare model-generated results with actual results: the principle is that the model is deemed to be acceptable if it approximates quite well subsequent historical performance.¹⁸

In this section we provide an evaluation of the accuracy of the different methods outlined in the previous paragraph by comparing the potential losses calculated over a calibration period with the actual losses observed over an out-of-sample period. The scenario

¹⁷ A similar result has been found by Vlaar (2000) on Dutch government bond portfolios.

¹⁸ The 1996 Amendment to the Basel accord describes the form of backtests that must be undertaken by banks wishing to use a VaR model for the calculation of market risks. Regulators recommend using the last 250 days of P&L data to back-test the 1 per cent 1-day VaR predicted by their internal model.

simulation procedure is calibrated on historical observations for euro par yields, from January 1, 1999 to December 31, 2001. The calibrated simulation procedure is used to generate scenarios over the out-of-sample testing period from January 1, 2002 to February 5, 2004.

As a first step, to verify whether our procedure produces realistic scenarios, we construct an “inclusion envelope” measure by linking the forecasts for the 1st and 99th percentiles of the simulated distribution of interest rate changes at each maturity, for both normal and non-parametric distributions (Figures 5a and 5b). For each key rate, the percentiles of the historical distribution of interest rates in the out-of-sample period are then compared with the inclusion envelope. The distance between the simulated worst scenarios and the observed percentiles gives a measure of the realism of the simulation.

From a comparison of the actual outcomes with the simulated distributions it appears that the normal distribution is not able to capture the right-hand tail of the realized distributions, corresponding to positive interest rate changes. In this respect, the non-parametric distribution gives a better fit of reality. Therefore, whereas simulations from normal distributions produce realistic scenarios when interest rates are decreasing, they lead to an underestimation of risk when interest rates are increasing (Figure 5a). On the contrary, simulations from non-parametric probability distributions produce realistic scenarios for positive interest rate variations (Figure 5b).

In a second step, to evaluate the accuracy of VaR models, the scenarios generated by the calibrated procedure are used to compute worst-case potential exposures for the banks’ balance sheets at the 99 per cent confidence level. At each point in time (daily), the simulated worst-case exposures result in an envelope of potential exposures, which is compared with the exposures realized during the historical testing period on the basis of the observed interest rate changes.

The vast majority of back-testing techniques are based on hypothesis testing: when the null hypothesis is rejected, the VaR model does not conform with the characteristics required by the back-testing model and it is therefore deemed to be inaccurate. One type of test is based on the frequency of exceptions and compares the number of days in which the loss exceed the VAR measure and the relative coherence with the VAR confidence level.

A 1 per cent daily VaR gives the level of loss that in normal market conditions is expected to be exceeded one day in every 100 in normal market conditions, under the assumption that the positions in the portfolio are unchanged. So, if the VaR model is accurate, when it is tested over a period of 1000 days one would expect 10 losses exceeding the VaR level. However, if the model underestimates the interest rate risk, more than 10 exceptional losses will be observed. The total number of exceptional losses may be regarded as a random variable from a binomial distribution. For a 1 per cent VaR the probability of an exceptional loss is $p=0.01$ and the number of trials is the number of days in back-testing analysis (in our case $n=526$). Then the expected number of exceptional losses is $np=5.26$ and the standard deviation of the expected value is $\sqrt{np(1-p)}=2.28$. Therefore, using the fact that a binomial distribution is approximately normal when n is large and p is small, a 90 per cent confidence interval for the number of exceptional losses, assuming that the VaR model is accurate, is approximately:

$$(5.26-1.645*2.28, 5.26+1.645*2.28) = (1.5, 9)$$

that is, one can deem the VaR model to be accurate (that it does not underestimate risk) if no more than 9 exceptions are observed.

To verify whether the number of exceptions observed empirically is significantly different from the theoretical one implied by the confidence level chosen for the VaR model we run the likelihood ratio test of unconditional coverage, a *proportion of failure test* (Kupiec, 1995).¹⁹

In Table 8 we report for four banks in our sample:²⁰ the average VaR for the different methods (historical simulation VaR, delta-gamma normal principal component VaR, delta-gamma non-parametric principal component VaR); the percentage of coverage of actual

¹⁹ The likelihood ratio is given by: $LR_{uc}(\alpha) = -2 * \ln \left[\frac{\alpha^x (1-\alpha)^{n-x}}{\pi^x (1-\pi)^{n-x}} \right]$, where $L_0(\alpha) = \alpha^x (1-\alpha)^{n-x}$ is the

likelihood function under the null hypothesis and $L_0(\pi) = \pi^x (1-\pi)^{n-x}$ is the function under the alternative hypothesis. The asymptotic value of the test is distributed as a χ^2 with one degree of freedom. An important aspect in back-testing is the level of statistical significance of the test: for higher levels, the test is more “selective” in the sense that the probability of type II errors, of accepting as good a bad model, is reduced. For risk management purposes, a level of statistical significance of 10 per cent is usually chosen.

²⁰ To test the accuracy of our VaR methodology we have chosen the two banks with the largest negative duration gaps and the two banks with the largest positive duration gaps. Very similar results are obtained for the other banks in the sample.

losses, the number of days in which the losses realized are higher than the potential losses; the percentage of exceptional losses. In the same table we report the value of the unconditional LR test, which has to be compared with the 10 per cent critical value.

The back-testing shows that the number of exceptional losses as well as the magnitude of violations are not relevant for banks with a negative duration gap: the percentage of coverage of the actual VaR is higher than 100 per cent (bank 14 and bank 16). On the contrary, for asset sensitive banks (bank 5 and bank 7) the number of exceptional losses is higher and their size in our sample are quite far beyond the VaR. Moreover, the normal approach is not adequate to forecast potential losses: the percentage of exceptional losses is higher than 1 per cent. This result is due to the non-normality of the PC risk factors, which are skewed and heavy-tailed. These features are well-captured by the non-parametric VaR, which shows a percentage of exceptional losses close to the expected 1 per cent: the LR unconditional test shows that in this case the null hypothesis of accuracy of the VAR model can be accepted.

In Figures 6a-6d we display the time series of the four banks' daily profit and loss from January 2001 to February 2004 (dotted line) and the corresponding one-day ahead 1st percentile VaR forecast. It can be seen that the 30,000 scenarios²¹ generated from the PC non-parametric distributions produce potential exposures that are systematically above the corresponding estimates from the normal distribution.

The plots confirm the acceptable performance of normal VaR for banks vulnerable to negative interest rate changes (Figures 6c and 6d, bank 12 and bank 16). As a result, delta-gamma VaR models based on normal distribution give an accurate description of risk for liability-sensitive banks (negative duration gap); vice versa, non-parametric delta-gamma VaR models are more effective for asset sensitive banks exposed to an increase in interest rates. This is the case for most banks, as well as for the majority of the banks in our sample, and it can be linked to the traditional intermediation activity and the maturity transformation of short-term liabilities in long-term assets.

²¹ In standard Monte Carlo methods, estimates are based on around 10,000 random samplings from the assumed distribution. In our simulation procedure, since there are three independent risk factors for each portfolio, and given the one-day prediction horizon, 30,000 random samplings per day are drawn.

Summarizing, the non-parametric VaR measure is more able to capture the fat-tailedness of the empirical distribution of interest rates and is therefore more apt to capture large interest rate movements in periods of high volatility, which - given the low level of rates in recent years – are observed especially with increasing interest rates. This feature has a cost in terms of excessive conservatism of the VAR in periods of decreasing interest rates and low volatility: for banks exposed to negative shocks the percentage of violations tends to be lower than implied by the confidence interval of the VAR measure.

The asymmetric feature of the non-parametric VAR measure can be regarded as a direct consequence of the skewness of the empirical, *unconditional*, distribution of the interest rate risk factors. In the context of capital allocation, this means that there is a trade-off between the ability to capture large movements in interest rates and the excessive amount of capital required in periods of low volatility.

6. An application of principal component Value-at-Risk to a one-year holding period: a comparison with the Basel II proposal

In the previous paragraph we presented empirical evidence of the performance of principal component VaR to predict portfolio VaR for the next day. The next exercise compares the PC VaR on a one-year holding period with the risk measures proposed by the Basel Committee on Banking Supervision. We compute the interest rate risk measure on an annual basis in two ways, first by extending the daily measure through the square root of time rule, and then by recalibrating the PC VaR simulation procedure on a one-year holding period.

According to the Basel regulation, within each maturity bucket portfolio value changes can be computed through a linear approximation, which considers only the first order sensitivity to interest rate changes:²²

$$\frac{dP}{P} = -D * \Delta r$$

²² Duration is useful only for small changes and it does not take into account changes in the shape of the yield curve.

where D^* is the modified duration of a government bond issued at par and yielding 8 per cent in each maturity bucket, and Δr are the interest rate shocks computed over a 240-day holding period and ranging from 100 to 60 basis points as maturity increases (Basel I). In the new proposal of the Basel Committee, the modified duration D^* is that of a government bond yielding 5 per cent and the shocks are supposed to be equal to 200 basis points for all maturities (Basel II).²³

The aim of our exercise is to evaluate to what extent the Basel Committee's recommendations reflect the actual interest rate risk, given current market conditions.

The simulation methodology discussed in this paper produces independent and identically distributed scenarios. Using the square root of time rule it is possible to compute the effective time horizon underlying the Basel Committee's proposal with respect to the daily forecasts based on real markets conditions (Table 9A). If we consider the non-parametric PC VaR, the Basel I risk measure covers on average one-month of potential losses while the Basel II risk estimate covers a six-month time horizon. Why does the Basel II risk measure correspond to a six-month time horizon if the standardized shock of 200 basis points is calibrated on a one-year (240 working days) holding period? This cannot be ascribed to the fact that the hypothesized interest rate change is not large enough. In fact, if we compare the Basel II scenarios with the 1st and the 99th percentile of observed interest rate changes using a one-year (240 working days) holding period, we observe that the hypothesis of a parallel shift of 200 basis points is quite prudential. The positive interest rate changes exceed the Basel Committee's scenarios one day every 100 only for maturities ranging from one month up to three years (Table 10).

The explanation is therefore in the duration parameters. The next exercise on the Italian banks compares the Basel II results with those that would be achieved if the duration parameter was changed to reflect the conditions prevailing on the market as of September 2003.²⁴ Looking at Table 9, if one replaces the duration parameters as they result from the

²³ For exposures in G10 currencies, Basel II proposes a standardized interest rate shock that would be based on an upward and downward 200 basis points parallel rate shock or on the 1st and the 99th percentile of observed interest rate changes using a one-year (240 working days) holding period and a minimum five years of observations.

²⁴ In the exercise the prudential scenario of 200 basis points are retained.

current market conditions, the interest rate risk exposure according to *adjusted* Basel II increases on average by almost 25 per cent and the time horizon associated with the one-day PC VaR gets close to one year (210 working days). This is due to the fact that the duration is longer, as in September 2003 interest rates were lower (the lower the interest rate levels, the higher the sensitivity of balance sheet positions to interest rate changes).²⁵ Moreover, if we order the 18 banks according to their different levels of risk exposure according to the different methodologies (Table 9B, where the reference ranking is the one in the first column of the table, corresponding to the one-year non-parametric VaR), we observe a very similar ranking between the PC VaR models (both parametric and non-parametric) and the adjusted Basel II (only 3 inversions between consecutive banks). The ranking becomes significantly different when we compare Basel II with Basel I (see, for example, bank i).

A further comparison with Basel II can be made by computing the PC VaR on a one-year holding period. The PC VaR methodology can be replicated for different holding periods by simply calibrating the PCA representation on the appropriate time series. In particular, in order to evaluate the VaR on a one-year horizon, interest rate changes have to be computed on a one-year holding period (on a rolling basis). Then, the PCA representation can be applied to produce scenarios that reflect the correlation structure of the yield curve.

In order to make an adequate comparison with the Basel regulatory risk measure, we compute the VaR of the 18 Italian banks on a 240-working day holding period according to the parametric approach (PC VaR based on normal distribution).²⁶ In fact, the empirical probability distribution of principal components appears too irregular when evaluated on a one-year holding period and it cannot be well approximated by a Logit function. This can be due to the fact that interest rate changes are computed on one-year rolling windows, which are partially overlapping.

²⁵ There is in fact an inverse relationship between the duration and level of interest rates: lower yield bonds have longer duration. At lower yields, the more distant payments made by the bond have relatively greater present value and account for a greater share of the total bond value. Thus, in the weighted average calculation of duration, the distant payment receives greater weights, which leads to a longer duration measure.

²⁶ During the observed period from December 1999 to September 2003, the Principal Component Analysis reveals that the first two components (PC) are sufficient to explain 98 per cent of the total variation. As in daily data, the first principal component is highly and positively correlated with all rate changes. As a result, it moves all interest rates in the same direction (parallel shift). The second component is positively correlated with short-term maturities and negatively correlated with long maturities and it can be seen as a tilt of the yield curve.

Table 11 compares the results, in percentage of supervisory capital, obtained from: a) Basel regulation in the modified version; b) the VaR on a one-year holding period, simulated under the normality hypothesis; c) the Var on a one-year holding period, obtained by simply multiplying the daily VAR based on the non-parametric approach times the square root of 240; d) the historical simulation²⁷.

With respect to the historical simulation measure, the PC VaR tends to be higher: the risk index frequently exceeds the corresponding index based on historical simulation. This result is not surprising, since the historical simulation on a one year holding period is influenced by the gradual decrease in the volatility of European interest rates in recent years while the PC VaR measure is based on random shocks.

The results from the principal component VaR models are consistent with the Basel II risk measure when the duration parameters are modified to reflect market conditions. The average risk index for the 18 banks is 8.9 per cent of supervisory capital against 8.3 per cent for modified Basel II. Looking at the risk index distribution, the risk index from the distribution simulated under the normality hypothesis is generally lower than that implied by Basel II for the most asset sensitive banks. This evidence confirms the limitations of the parametric approach based on the normal distribution to capture volatility when interest rates are increasing; in this case the non-parametric measure is more effective.

7. An application of principal component based scenarios to stress testing analysis

Scenario simulation based on Principal Component Analysis has a natural application to stress testing analysis. Stress testing is really a part of scenario analysis, but instead of considering expected changes in normal market circumstances, one looks at the portfolio value when risk factors assume extreme positions.

Stress testing results depend crucially on the choice of scenarios, which should reflect exceptional but plausible events: if the available historical data do not adequately reflect the potential risks for the future, it would be useful to artificially generate extreme scenarios of

²⁷ For the historical simulation, the distribution of the observed interest rate changes on 240-day rolling windows from December 1999 through September 2003 is assumed to be representative for the next 240 working days. It has to be noted that the historical simulation could be biased due to the fact that the daily time series of overlapping one-year changes exhibits violations of independency.

the main risk factors. However, standard methodologies give no idea of the probability with which stressed scenarios may occur; often they have no statistical foundation or justification, making the interpretation of results difficult.

The simulation procedure based on PCA limits discretion in the choice of scenarios and gives an idea of the plausibility of the results in terms of confidence levels. In the inverse PC representation $X = PW'$, interest rate changes X are expressed as a function of the new risk factors P , where the weighting coefficients W (the so called “factor loading”) capture the correlation in the system and account for the contribution of each risk factor to the overall variance (see Appendix 1).

Stress testing analysis in the context of PCA can be performed by changing the volatility of each principal component, and hence of each interest rate along the yield curve and/or the correlation structure of the data. One can choose to stress correlation by modifying the matrix of factor weights, while assuming constant volatility. Conversely, one can shock the volatility of interest rate changes while maintaining the matrix of factor loading fixed at historical values.

We perform a stress testing exercise on a one-year risk horizon. The aim of the stress testing exercise is to evaluate what happens under different hypotheses on correlation and volatility. These hypotheses can be compared with those under the Basel II proposal, where the assumed scenario corresponds to a parallel shift of the yield curve of 200 basis points when there is also perfect correlation (all rates shift up or down together).

The choice of relevant scenarios typically depends on the type of balance sheet. For example, if the yield curve twists anti-clockwise, with a higher rise in the long rate and a smaller rise in the short rate, the risk exposure of an asset sensitive bank would be greater than one estimated under a parallel shift²⁸.

Given the low level of interest rates in recent years, in our stress testing exercise we explore the magnitude of positive shocks under the stress hypotheses. In particular, we evaluate the impact of a 30 per cent increase in volatility. Additionally, we specify separate shocks in each PC direction and combined shocks of PCs (Table 12) by changing the matrix

of factor loading while maintaining historical volatility fixed (Figure 7). In particular, we compare the hypothesis in which all interest rates move together (perfect correlation) with the situation in which interest rate changes are different for each rate along the yield curve (the second factor is flattening or steepening). In all cases, the stress events are obtained by selecting the 99th percentile of the risk factors' simulated distributions under the different hypotheses²⁹.

Looking at Table 12, we see that all stress scenarios reflect some stylized facts which make them plausible. Specifically, the short and medium rates are characterized by higher volatility than long-term rates: when short rates move up, the long rates tend to increase more gradually.

For banks with a positive duration gap the worst situation occurs when the correlation between rates becomes higher and the yield curve flattens (Table 12, last column). In that situation, the medium-term rates are much more volatile than the short and long rates. Conversely, the extreme hypothesis of an inversion of the yield curve does not seem plausible on the basis of observed volatility and correlation. Finally, in any scenario interest rate changes for the longer maturities are never greater than the 200 basis point of the Basel Committee proposal; in this regard, the hypothesis of a 200 basis points parallel rate shock seems to be quite prudential.

8. Conclusions

This paper develops a Value-at-Risk methodology for measuring interest rate risk on both banking and trading book items of banks' balance sheets that is responsive to market conditions in terms of interest rate levels and volatility. By using 5 years of daily data, the risk is evaluated through a VaR measure based on a principal component Monte Carlo simulation of interest rate changes. The bank profit and loss distributions are then derived from the simulated risk factor distributions through the delta-gamma approximation

²⁸ In this sense, the Basel Committee shock of a parallel shift of 200 basis points may be too simplifying. Scenarios with a higher rise in the long rates and a smaller rise (or even a drop) in the short rates can be plausible in short holding periods (one-day or ten-day).

²⁹ It has to be noted that the probability of worst case scenario at each maturity is 1 per cent but the probability of having the exact combination of them is lower.

function, in which we compute the duration parameter according to the interest rate level observed in the market at the time of the risk evaluation and introduce convexity to take into account the non-linearity of the relation. The interest rate risk measure is obtained by selecting the first percentile of the profit and loss distribution.

The VaR is computed according to two different approaches: the parametric approach, based on the normal distribution of the principal component underlying risk factors, and the non-parametric approach, which represents the main novelty of this paper.

The PC VaR model is applied to the balance sheet maturity structure of the 18 largest Italian banks in terms of total assets, first on a daily basis and then on a one-year holding period (240 working days). The results are consistent with Basel II when the duration parameters proposed by the regulation are changed to reflect market conditions. The average risk index for the 18 largest Italian banks on a one-year risk horizon is 8.9 per cent of supervisory capital against 8.3 per cent for the “modified” Basel II proposal.

Back-testing analysis shows that the parametric approach entails some limitations in capturing volatility when interest rates are increasing. Especially from December 2001, positive interest rates changes in the euro area have shown a higher volatility than negative changes, probably owing to the low levels of interest rates, which are close to their minimum boundary. In the presence of such an asymmetric pattern of volatility, the non-parametric approach performs better for banks that are exposed to an increase in interest rates.

The simulation procedure based on the principal components can be used for stress testing analysis. It limits the discretion in the choice of stress scenarios and gives an idea of plausibility of results in terms of confidence levels. We evaluate the impact of a 30 per cent increase in volatility of all rates in the yield curve; alternatively, we specify separate and combined shocks of PC direction by changing the matrix of factor loading while maintaining historical volatility fixed. The stress events are obtained by selecting the appropriate percentiles of the risk factors’ simulated distributions under the different hypotheses. We find out that for banks with positive duration gaps the worst situation occurs when the correlation between rates becomes higher and the yield curve flattens; conversely, the extreme hypothesis of an inversion of the yield curve does not seem plausible on the basis of observed volatility and correlation.

Appendix

1.1. Monte Carlo Simulation of Interest rate changes applying PCA.

Principal Component Analysis is a statistical technique that is used to determine whether the observed correlation between a given set of variables can be explained by a smaller number of unobserved and unrelated common factors (Press et al. 1996). It is employed to reduce data dimension to a tractable threshold, without committing to any particular strong hypothesis on the data generating process. In the simulation process, the reduction in the number of factors increases computational efficiency.

The original number of variables is compressed into a small set of underlying factors through appropriate transformations of the original data.³⁰ Formally, given the \mathbf{X} matrix of the standardized interest rate changes,³¹ the PCs are orthogonal linear combinations of the original risk factors:

$$\mathbf{P}=\mathbf{XA}$$

where \mathbf{A} is the orthogonal matrix of eigenvectors of the variance and covariance matrix of \mathbf{X} and \mathbf{P} is the matrix of factor scores (the results of the linear combinations). The principal components are orthogonal, and therefore additive and statistically independent.³²

A Monte Carlo simulation³³ based on Principal Component Analysis is performed by drawing independent random shocks from each PC distribution, and then inverting the PCA

³⁰ The number of relevant PCs (risk factors) is determined by the correlation structure of the data: if the data are all highly correlated, a few principal components are sufficient to explain most of the variation in the data .

³¹ Prior to applying PCA to the financial series, it is important to determine whether PCA is in fact a meaningful procedure given the distributional properties of the data. In particular, one needs to check non-stationarity of the data, which would imply the existence of a stochastic trend. Various studies have found that levels of interest rates are non stationary whereas first differencing achieves stationarity (Niffiker et al, 2000; Lardic et al., 2001). Lardic et al. (2001) have also shown that the original variables should be centered and variance-reduced, which amounts to using the correlation matrix of the changes. Moreover, the use of daily data leads to more accurate results.

³² For risk management purposes, additivity is important because it allows evaluation of the impact of say one unit of added parallel shift risk to an existing position. Statistical independence is important because it allows the factors to be managed separately, say to hedge a parallel shift without having to think about its effect on the other factors (Niffiker, 2000).

representation of the observed term structure in order to reproduce the correlation structure of the original risk factors:

$$\mathbf{X}=\mathbf{P}\mathbf{W}'$$

The original risk factors \mathbf{X} are then expressed as a linear combination of the principal component \mathbf{P} where the coefficients \mathbf{W}' , known as *factor loading*, give the sensitivity of interest rate changes along the yield curve with respect to each PC. The restatement of market movements provided by the factor loading is similar in spirit to a linear regression where the principal components play the role of explanatory variables and the factor loading plays the role of regression coefficient. Thus, factor loading restates each day's yield curve movement as a combination of the movements of principal components.

Since the vectors of factor loading can be expressed as the eigenvectors times the square root of corresponding eigenvalues, the shock vector \mathbf{U} can be written as follows:

$$\mathbf{U}=\mathbf{A}'\sqrt{\Lambda}\boldsymbol{\eta}=\eta_1\sqrt{\lambda_1}\mathbf{A}_1+\eta_2\sqrt{\lambda_2}\mathbf{A}_2+\dots+\eta_k\sqrt{\lambda_k}\mathbf{A}_k$$

where \mathbf{A} is the matrix of the orthogonal eigenvectors of the original variance–covariance matrix of \mathbf{X} , Λ is the diagonal matrix of the corresponding eigenvalues, and $\boldsymbol{\eta}=(\eta_1, \eta_2, \dots, \eta_n)$ are the vectors of independent shocks. Thus, the vector \mathbf{U} represents the simulated scenarios where the vector \mathbf{A}_j gives the direction of the j -th principal component (risk factor) and $\sqrt{\lambda_j}$ gives its contribution to the whole variance.³⁴

Usually, scenarios based on PCs are simulated through vectors of standard normal shocks $\eta_j \sim N(0, I_k)$. The co-dependent structure is then derived from the PC decomposition of the original variance–covariance matrix, so the normal random vector \mathbf{u} has the same covariance matrix as the original data.

³³ The Standard Monte Carlo techniques can be performed as follows: a) determine the covariance matrix among the different instruments in the portfolio based on historical data. The original covariance matrix among securities can be substituted by the covariance matrix among risk factors, contemporaneously evaluating the sensitivities of the various instruments to the specific risk factors; b) generate a series of independent random numbers for each of the risk factors. The distribution of the independent shocks should reflect the distribution of original risk factors. Monte Carlo Var standards are based on around 10,000 random sampling from the assumed distribution; c) transform the independent random numbers into random numbers with the covariance structure of the original data by using the Cholesky decomposition; d) revalue the portfolio for each of the simulated scenarios and evaluate the distribution of portfolio returns. The results are subsequently ranked and the Var figure is read off at the required percentile.

³⁴Statistically, $\sqrt{\lambda_j}$ is the standard deviation of j -th principal component. So, all risk factors (PCs) influence the simulated scenarios according to their contribution to the total variance.

1.2. A non-parametric distribution function when risk factors are skewed and heavy-tailed.

To obtain estimates of PC probability density we use the “local smoothing” technique according to which the value of density at each point is influenced mostly by the number of observations close to that point, whereas it is little affected by the data far away from that point. Among the local smoothing estimators, known as “kernel estimators”, we have chosen the Gaussian kernel with a scalar bandwidth given by $\sigma N^{-0.2}$ ³⁵, where σ is the standard deviation of observations. The PC cumulative distribution functions are derived from the non-parametric densities by a simple operation of integration. The probability functions are then fitted by an appropriate quasi-likelihood method for fractional logit models, characterized by continuous dependent variables taking a real value between zero and one. Formally, given an independent sequence of observations $\{x_i, y_i : i = 1, 2, \dots, N\}$, where $0 \leq y_i \leq 1$ and N is the sample size, the assumption is that, for all i ,

$$E(y_i | x_i) = G(x_i \beta) \quad (\text{A.1})$$

where $G(\cdot)$ is the Logit distribution function. Under (A.1), β can be consistently estimated by maximizing the Bernoulli log-likelihood function (see L. Papke and J. Wooldridge, 1996):³⁶

$$l_i(b) = y_i \log[G(x_i b)] + (1 - y_i) \log[1 - G(x_i b)] \quad (\text{A.2})$$

Because (A.2) is a member of the linear exponential family, the quasi-maximum likelihood estimator (QMLE) of β obtained from maximization problem

$$\max_b \sum_i^N l_i(b)$$

is consistent and \sqrt{N} -asymptotically normal regardless of the distribution of y_i conditional on x_i .

³⁵ In all kernel estimators, the bandwidth is a crucial parameter determining the size of the region (around the point of interest) which is used to perform the smoothing operation.

³⁶ Generally, for fractional response variables the method of estimation consists in maximizing the quasi log-likelihood function of a binomial model with a logit link function.

Generally, in statistics and econometrics packages, the coefficients b and the corresponding standard errors and confidence intervals can be expressed in exponential form. For binomial models with logit link function, exponentiation results in odds ratios:³⁷

$$E(\log[y/(1-y)|x]) = x\beta \quad (\text{A.3})$$

where $\log[y/(1-y)]$ can take on any real value as y varies between 0 and 1.

³⁷A logit function $y = \exp(a+bX) / 1 + \exp(a+bX)$ can be made linear by transforming the dependent variable y into the log-odds ratio $\ln(y / 1-y)$.

Tables and Figures

Table 1. Euro area par yield curve - Descriptive statistics (4.1.99-30.9.2003)
Levels (Percentage points)

	3 months	6 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years	15 years	20 years	25 years	30 years
Mean	3.510	3.530	3.520	3.760	3.960	4.140	4.300	4.550	4.810	5.070	5.220	5.330	5.400
St. Dev.	0.847	0.872	0.888	0.859	0.797	0.736	0.682	0.599	0.519	0.443	0.402	0.379	0.365
Min	2.120	2.013	1.811	1.907	2.150	2.422	2.679	3.098	3.540	3.984	4.244	4.415	4.532
Max	5.130	5.207	5.204	5.350	5.358	5.358	5.410	5.495	5.725	5.976	6.127	6.224	6.288
Dickey-Fuller Unit root test*	0.764	0.712	-0.006	-0.407	-0.717	-0.942	-1.127	-1.440	-1.761	-2.011	-2.106	-2.151	-2.177

*5% critical value is equal to -2.86

Table 2A Euro area par yield curve - Descriptive statistics (4.1.99-30.9.2003)
Daily interest rate changes (Percentage points)

	3 months	6 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years	15 years	20 years	25 years	30 years
Mean	-0.001	-0.001	-0.001	-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Std. Dev.	0.025	0.027	0.042	0.049	0.051	0.051	0.050	0.048	0.046	0.044	0.043	0.042	0.042
Min	-0.335	-0.251	-0.215	-0.203	-0.165	-0.166	-0.174	-0.182	-0.181	-0.169	-0.159	-0.176	-0.189
Max	0.203	0.206	0.227	0.219	0.209	0.094	0.206	0.187	0.175	0.170	0.175	0.178	0.180
Skewness	-2.210	-0.620	0.301	0.450	0.475	0.408	0.391	0.355	0.309	0.292	0.281	0.263	0.244
Kurtosis	47.279	16.143	2.795	1.648	1.246	1.018	1.010	1.026	1.010	0.922	0.870	0.888	0.929
Shapiro-Wilk Normality test	13.658	11.560	7.612	6.815	6.548	5.991	5.916	5.870	5.599	5.259	5.055	4.924	4.903
(Pvalue)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ARCH	0.006	0.014	0.068	0.052	0.000	0.094	0.161	0.186	0.158	0.105	0.012	0.064	0.467
(Pvalue)	0.940	0.906	0.795	0.819	0.984	0.759	0.689	0.667	0.691	0.746	0.914	0.801	0.495
Dickey-Fuller Unit root test*	-28.110	-29.150	-35.520	-33.170	-33.680	-33.510	-33.640	-34.130	-34.210	-34.270	-34.190	-34.060	-33.930

Table 3. Principal Component Analysis on daily basis (*): proportion of variance explained by the first three PCs.

Factors	Eigenvalue	Difference	Proportion	Cumulative
1	15.18	10.60	0.69	69.0%
2	4.58	3.51	0.21	89.8%
3	1.07	0.66	0.05	94.7%

(*)The PCA is applied to the interest rate term structure over the period from January 4, 1999 to September 30, 2003. The data consist of 1,173 daily observations of money markets rate and government bond par yields in the euro area at tenors of 3 and 6 months and 1, 2, 3, 4, 5, 7, 10, 15, 20, 25 and 30 years.

Table 4. Principal Component Analysis on daily basis (*): factor loadings.

Maturity	PC1	PC2	PC3
3 months	0.45	0.82	0.18
6 months	0.72	0.64	-0.15
1 year	0.84	0.28	-0.29
2 years	0.93	0.11	-0.29
3 years	0.95	0.01	-0.23
4 years	0.96	-0.06	-0.17
5 years	0.97	-0.12	-0.11
7 years	0.97	-0.20	-0.02
10 years	0.95	-0.28	0.09
15 years	0.92	-0.34	0.19
20 years	0.89	-0.37	0.25
25 years	0.86	-0.38	0.29
30 years	0.83	-0.38	0.30

(*)The PCA is applied to the interest rate term structure over the period from January 4, 1999 to September 30, 2003. The data consist of 1,173 daily observations of money markets rate and government bond par yields in the euro area at tenors of 3 and 6 months and 1, 2, 3, 4, 5, 7, 10, 15, 20, 25 and 30 years.

Table 5. Principal component descriptive statistics.

	PCA1	PCA2	PCA3
Mean	0.000	0.000	0.000
Std.	3.896	2.140	1.035
Min	-13.132	-27.274	-6.933
Max	15.172	15.722	6.889
Skewness	0.382	-2.384	-0.407
Kurtosis	4.140	36.930	10.346
Shapiro-Wilk Normality test	6.11	12.62	9.71
(Pvalue)	0.000	0.000	0.000
ARCHLM	0.594	0.004	0.011
(Pvalue)	0.440	0.951	0.915
Dickey-Fuller Unit root test*	-33.72	-27.18	-32.88

*5% critical value is equal to -2.86

Table 6. Duration and convexity (*)

Maturity	Par yield at the end of September 2003	Sensitivity factors	
		Duration	Covexity
3 months	2.13	0.25	0.00
6 months	2.07	0.49	0.51
1 year	2.03	0.99	1.52
2 years	2.33	1.94	5.00
3 years	2.67	2.86	10.32
4 years	2.98	3.74	17.31
5 years	3.24	4.58	25.81
7 years	3.64	6.13	46.64
10 years	4.05	8.16	85.20
15 years	4.44	10.86	160.92
20 years	4.67	12.91	241.60
25 years	4.82	14.44	320.37

(*) Duration and the convexity parameters of a representative bond issued at par maturing in the mid-point of each time-bucket and yielding the interest rates prevailing in the EU market at the end of September 2003.

Table 7. Banks' losses according to different methods (million of euros and as percentage of capital, end of September 2003).

7.1 Asset sensitive banks*.

Banks	Balance sheets	One-day historical simulation VaR (million euros)		One-day principal component VaR (million euros)		Percent of supervisory capital	
	Assets denominated in euro as a percentage of total assets.	Empirical distribution (**)		Normal distribution	Non-parametric distribution	Normal distribution	Non-parametric distribution
		500 days	250 days				
Bank 1	86.9	-34.9	-34.6	-28.5	-32.5	-0.4	-0.4
Bank 2	96.2	-8.2	-8.1	-6.7	-7.8	-0.8	-0.9
Bank 3	98.3	-8.5	-8.2	-6.1	-7.1	-0.6	-0.8
Bank 4	99.2	-32.9	-31.5	-25.1	-29.0	-1.5	-1.8
Bank 5	74.4	-116.0	-117.1	-91.2	-104.0	-0.4	-0.5
Bank 6	76.8	-23.0	-22.5	-18.4	-22.3	-0.1	-0.1
Bank 7	82.5	-40.8	-41.1	-32.1	-38.3	-0.4	-0.4
Bank 8	89.2	-26.2	-25.4	-19.1	-24.6	-0.3	-0.4
Bank 9	94.6	-14.1	-14.3	-10.6	-12.0	-0.3	-0.3
Bank 10	85.0	-9.1	-9.3	-6.9	-8.1	-0.2	-0.2
Bank 11	77.5	-6.8	-7.0	-5.0	-5.7	-0.2	-0.2
Bank 12	98.0	-16.0	-15.7	-12.0	-14.4	-1.4	-1.7
Bank 13	95.1	-30.0	-29.7	-23.0	-29.2	-0.5	-0.6
Mean	88.7	-28.2	-28.0	-21.9	-25.8	-0.55	-0.65

(*) Liability sensitive banks are those having a negative duration gap. Asset sensitive banks are those having a positive duration gap.

(**) The empirical distribution is represented by the last 250 and 500 historical one-day investment results, respectively.

7.2 Liability sensitive banks*.

Banks	Balance sheets	One-day historical simulation VaR (million euros)		One-day principal component VaR (million euros)		Percent of supervisory capital	
	Assets denominated in euro as a percentage of total assets.	Empirical distribution (**)		Normal distribution	Non-parametric distribution	Normal distribution	Non-parametric distribution
		500 days	250 days				
Bank 14	80.2	-26.3	-25.8	-29.1	-37.4	-0.5	-0.7
Bank 15	82.3	-8.1	-7.7	-9.2	-8.8	-0.1	-0.1
Bank 16	98.9	-13.2	-12.0	-14.1	-16.3	-0.4	-0.4
Bank 17	94.1	-1.0	-0.9	-1.2	-1.3	-0.5	-0.5
Bank 18	88.7	-3.3	-3.1	-4.1	-3.2	-0.3	-0.2
Mean	88.8	-10.4	-9.9	-11.5	-13.4	-0.35	-0.38

(*) Liability sensitive banks are those having a negative duration gap. Asset sensitive banks are those having a positive duration gap.

(**) The empirical distribution is represented by the last 250 and 500 historical one-day investment results, respectively.

Table 8. Back-testing analysis results (million of euros, end of September 2003).

Bank 5: actual and potential losses.

<i>Actual VaR</i>	-135.00
<i>Average exceptional losses</i>	-144.52

VaR models	Potential losses	Percentage of coverage of actual VaR	Number of exceptional losses	Percentage of exceptional losses	LR Test of unconditional coverage (critical value 10%: 2.70)
<i>Historical simulation</i>	-116.72	86.5%	12	2.3%	6.40
<i>Delta-gamma normal VaR</i>	-113.61	84.2%	11	2.1%	4.81
<i>Delta-gamma non-parametric VaR</i>	-128.30	95.0%	6	1.1%	0.10

Bank 7: actual and potential losses.

<i>Actual VaR</i>	-44.50
<i>Average exceptional losses</i>	-51.80

VaR models	Potential losses	Percentage of coverage of actual VaR	Number of exceptional losses	Percentage of exceptional losses	LR Test of unconditional coverage (critical value 10%: 2.70)
<i>Historical simulation</i>	-41.57	93.4%	8	1.5%	1.24
<i>Delta-gamma normal VaR</i>	-40.17	90.3%	10	1.9%	3.41
<i>Delta-gamma non-parametric VaR</i>	-44.78	100.6%	4	0.8%	0.33

Bank 14: actual and potential losses.

<i>Actual VaR</i>	-31.77
<i>Average exceptional losses</i>	-34.97

VaR models	Potential losses	Percentage of coverage of actual VaR	Number of exceptional losses	Percentage of exceptional losses	LR Test of unconditional coverage (critical value 10%: 2.70)
<i>Historical simulation</i>	-34.67	109.1%	1	0.2%	5.23
<i>Delta-gamma normal VaR</i>	-34.99	110.2%	1	0.2%	5.23
<i>Delta-gamma non-parametric VaR</i>	-44.17	139.0%	1	0.2%	5.23

Bank 16: actual and potential losses.

<i>Actual VaR</i>	-12.12
<i>Average exceptional losses</i>	-14.10

VaR models	Potential losses	Percentage of coverage of actual VaR	Number of exceptional losses	Percentage of exceptional losses	LR Test of unconditional coverage (critical value 10%: 2.70)
<i>Historical simulation</i>	-18.38	151.6%	1	0.2%	5.23
<i>Delta-gamma normal VaR</i>	-14.92	123.0%	2	0.4%	2.67
<i>Delta-gamma non-parametric VaR</i>	-15.75	129.9%	1	0.2%	5.23

Table 9A. Comparison of different risk measures in terms of losses and time horizon: Basel versus PC Value at Risk^(*).

(Million of euros; September 2003)

Banks	Losses in million of euros						Time horizon underlying the Basel Committee's measures with respect to the daily forecast from non parametric distribution hypothesis (days)		
	One-day principal component VaR		Basel Accord risk measure (240 working days)						
	Normal distribution	Non-parametric distribution	Basel I	Basel II	Adjusted Basel II	Percent Variation (Adjusted Basel II versus Basel II)	Basel I	Basel II	Adjusted Basel II
Bank 1	-28.5	-32.5	-89.5	-397.3	-463.1	16.6	8	149	203
Bank 2	-6.7	-7.8	-25.0	-72.4	-88.5	22.1	10	87	130
Bank 3	-6.1	-7.1	-19.4	-88.6	-102.0	15.1	7	156	206
Bank 4	-25.1	-29.0	-103.9	-373.6	-446.3	19.4	13	166	237
Bank 5	-91.2	-104.0	-343.4	-1418.0	-1617.7	14.1	11	186	242
Bank 6	-18.4	-22.3	-63.3	-196.7	-242.8	23.4	8	78	119
Bank 7	-32.1	-38.3	-121.3	-508.6	-567.9	11.6	10	176	220
Bank 8	-19.1	-24.6	-95.6	-296.0	-356.5	20.5	15	145	211
Bank 9	-10.6	-12.0	-27.4	-144.7	-165.0	14.1	5	145	189
Bank 10	-6.9	-8.1	-23.5	-96.9	-109.9	13.5	8	142	183
Bank 11	-5.0	-5.7	-4.3	-54.8	-55.4	1.0	1	93	94
Bank 12	-12.0	-14.4	-40.8	-164.6	-190.4	15.7	8	130	175
Bank 13	-23.0	-29.2	-87.0	-328.8	-384.8	17.0	9	127	174
Bank 14	-29.1	-37.4	-184.5	-443.4	-576.2	29.9	24	140	237
Bank 15	-9.2	-8.8	-65.6	-72.5	-106.1	46.4	56	68	146
Bank 16	-14.1	-16.3	-133.7	-237.8	-329.6	38.6	67	213	409
Bank 17	-1.2	-1.3	-10.9	-19.3	-27.8	43.8	70	221	457
Bank 18	-4.1	-3.2	-27.3	-19.9	-36.9	85.7	71	38	131
Mean	-19.0	-22.3	-81.5	-274.1	-325.9	24.9	22	137	209

(*)The table compares the one-day PC VaRs (parametric and non-parametric) with the Basel risk measures (Basel I, Basel II and the adjusted Basel II for market-based duration parameters). The last three columns of the table report the effective time horizon underlying the Basel risk measures. Using the square root of time rule, this is obtained by dividing the squared values from each Basel risk measure to those from the non-parametric principal component VaR (squared values).

Table.9B Comparison of different risk measures: ranking of banks' riskiness.

Banks ranked from the riskiest to the least risky (according to the non-parametric approach)	One-year principal component VaR		Basel Accord risk measure (240 working days)		
	Non-parametric distribution	Normal distribution	Basel I	Basel II	Adjusted Basel II
Bank 5	bank a	bank a	bank a	bank a	bank a
Bank 7	bank b	bank b	bank c	bank b	bank c
Bank 14	bank c	bank c	bank i	bank c	bank b
Bank 1	bank d	bank d	bank b	bank d	bank d
Bank 13	bank e	bank e	bank f	bank f	bank e
Bank 4	bank f	bank f	bank g	bank e	bank f
Bank 8	bank g	bank g	bank d	bank g	bank g
Bank 6	bank h	bank h	bank e	bank i	bank i
Bank 16	bank i	bank i	bank l	bank h	bank h
Bank 12	bank j	bank j	bank h	bank j	bank j
Bank 9	bank k	bank k	bank j	bank k	bank k
Bank 15	bank l	bank l	bank k	bank m	bank m
Bank 10	bank m	bank m	bank q	bank o	bank l
Bank 2	bank n	bank n	bank n	bank l	bank o
Bank 3	bank o	bank o	bank m	bank n	bank n
Bank 11	bank p	bank p	bank o	bank p	bank p
Bank 18	bank q	bank q	bank r	bank q	bank q
Bank 17	bank r	bank r	bank p	bank r	bank r

Table 10. Empirical distribution: worst cases.
(240-day interest rate changes in basis point)

Maturity	Worst case	
	1st percentile	99th percentile
3 months	-165	213
6 months	-182	211
1 year	-197	209
2 years	-202	212
3 years	-201	203
4 years	-193	195
5 years	-183	188
7 years	-164	183
10 years	-142	179
15 years	-118	166
20 years	-102	154
25 years	-92	145
30 years	-87	138

Table.11 Value At Risk on annual basis (percent of supervisory capital).

Banks	Basel II proposal		Parametric approach	Non-parametric approach	Historical simulation
	Actual version	Adjusted version (*)	One-year holding period	Daily VaR for SQRT(240)	
Bank 1	-5.5	-6.4	-5.0	-6.9	-5.1
Bank 2	-8.4	-10.2	-12.3	-13.9	-10.8
Bank 3	-9.4	-10.8	-9.9	-11.7	-10.4
Bank 4	-22.6	-27.0	-23.4	-27.2	-22.4
Bank 5	-7.0	-8.0	-6.2	-7.9	-6.5
Bank 6	-1.2	-1.5	-1.4	-2.1	-1.2
Bank 7	-5.8	-6.5	-4.9	-6.8	-5.4
Bank 8	-5.1	-6.2	-6.0	-6.6	-5.2
Bank 9	-4.1	-4.7	-4.1	-5.3	-4.5
Bank 10	-2.6	-3.0	-2.7	-3.4	-2.8
Bank 11	-1.8	-1.8	-2.8	-2.9	-2.4
Bank 12	-19.1	-22.1	-20.8	-25.9	-20.8
Bank 13	-6.8	-7.9	-6.8	-9.3	-6.9
Bank 14	-7.8	-10.2	-12.2	-10.2	-9.8
Bank 15	-0.6	-0.8	-2.3	-1.0	-1.7
Bank 16	-6.2	-8.6	-12.8	-6.6	-9.3
Bank 17	-8.1	-11.7	-17.4	-8.5	-12.7
Bank 18	-1.3	-2.4	-8.3	-3.3	-5.9
Mean	-6.9	-8.3	-8.9	-8.9	-8.0

(*) In the adjusted Basel II the duration coefficients have been changed to reflect conditions prevailing on the EU market at the end of September 2003 (see Table 6).

Table 12. Separate and combined shocks (basis points): 99th percentile of the simulated distribution at each maturity.

Maturity	Fixed volatility and correlation (A)	30% increase in volatility and fixed correlation (B)	Separate shocks				Combined shocks	
			Perfect* correlation on the first PC and fixed volatility (C)	Smaller** correlation on the first PC and fixed volatility (D)	80 % decrease of the second PC and fixed volatility (E)	80 % increase of the second PC and fixed volatility (F)	Smaller correlation on the first PC, 80% increase of the second PC and fixed volatility (G)	High correlation on the first PC, 80% decrease of the second PC and fixed volatility (H)
3 months	275	386	293	212	220	388	346	277
6 months	278	382	297	197	252	359	316	313
1 year	267	363	295	171	272	321	246	341
2 years	253	342	286	138	276	279	155	346
3 years	233	321	269	119	262	246	115	328
4 years	223	314	252	118	243	231	114	306
5 years	211	291	235	116	223	226	123	281
7 years	194	255	209	116	192	214	144	244
10 years	170	223	181	108	159	196	151	204
15 years	146	189	151	95	128	180	145	163
20 years	124	164	133	85	108	162	134	137
25 years	112	147	122	78	96	146	124	121
30 years	102	135	114	73	92	135	115	110

(*) The factor loadings on the first PC are all set to 1: the overall variance is entirely explained by the first factor.

(**) The factor loadings on the first PC are diminished by 50 per cent.

Figure 1a. 3-month euro yield and volatility

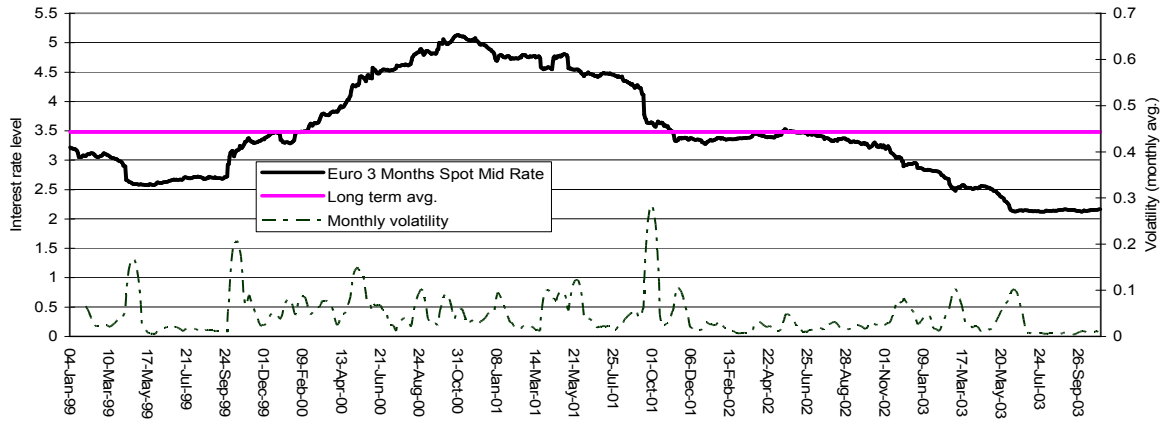


Figure 1b. 5-year euro yield and volatility.

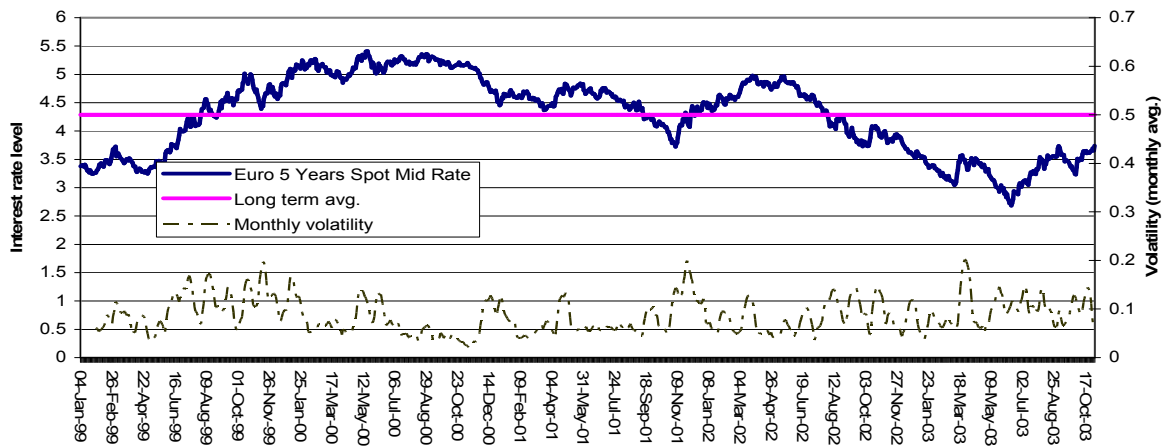
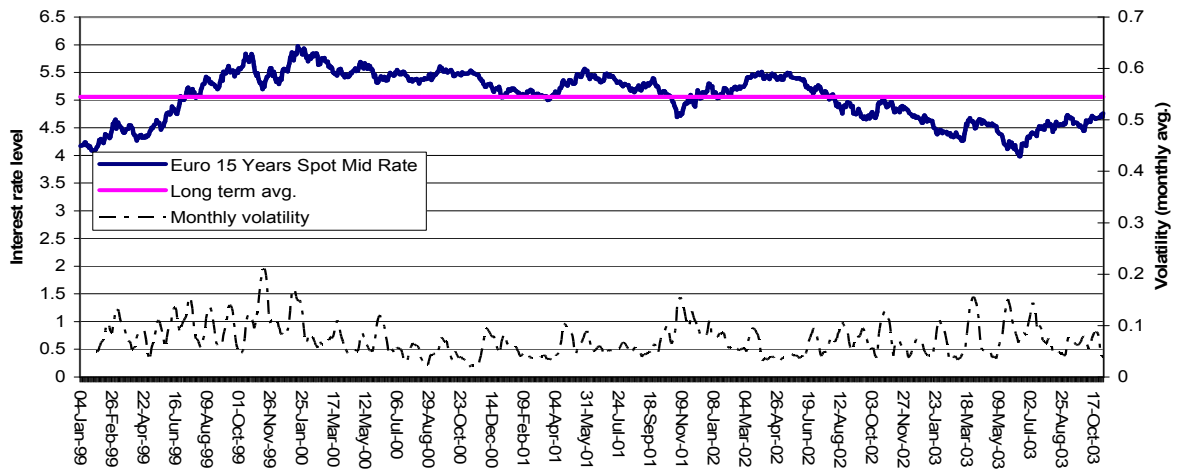
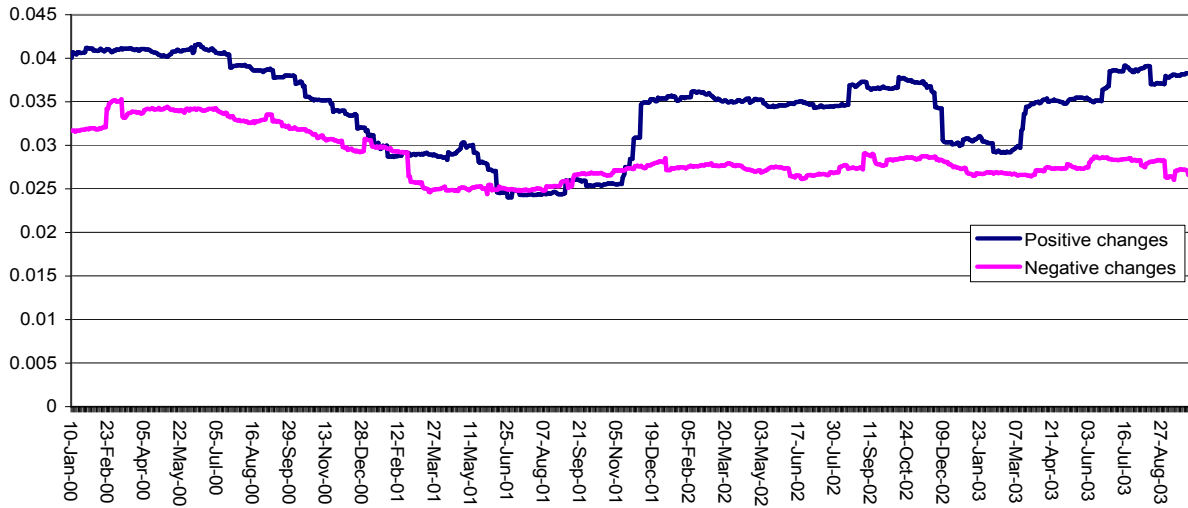


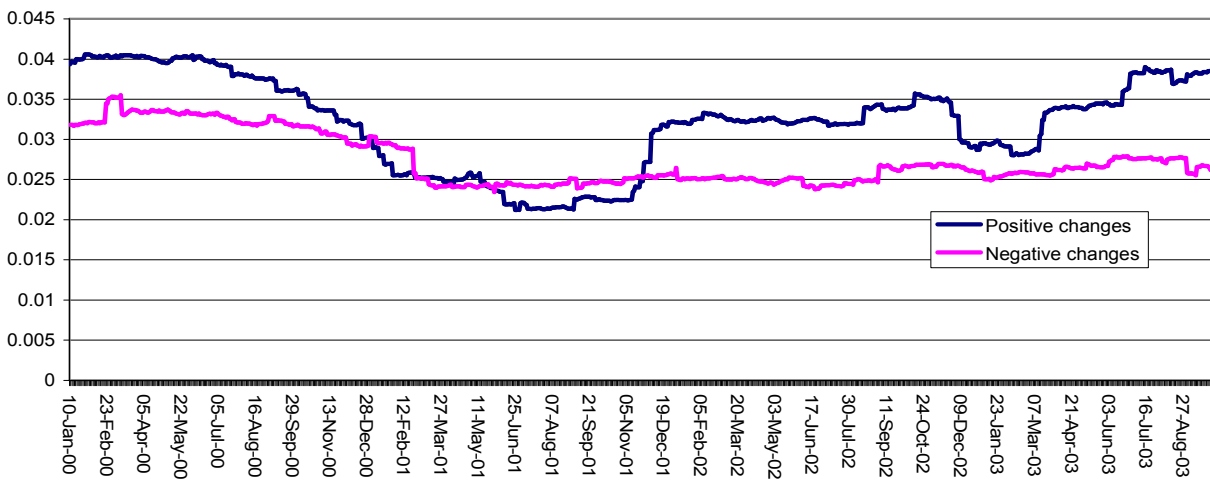
Figure 1c. 15-year euro yield and volatility.



**Figure 2a. 5-year euro yield:
volatility of positive and negative changes**



**Figure 2b. 7-year euro yield:
volatility of positive and negative changes**



**Figure 2c. 10-year euro yield:
volatility of positive and negative changes**

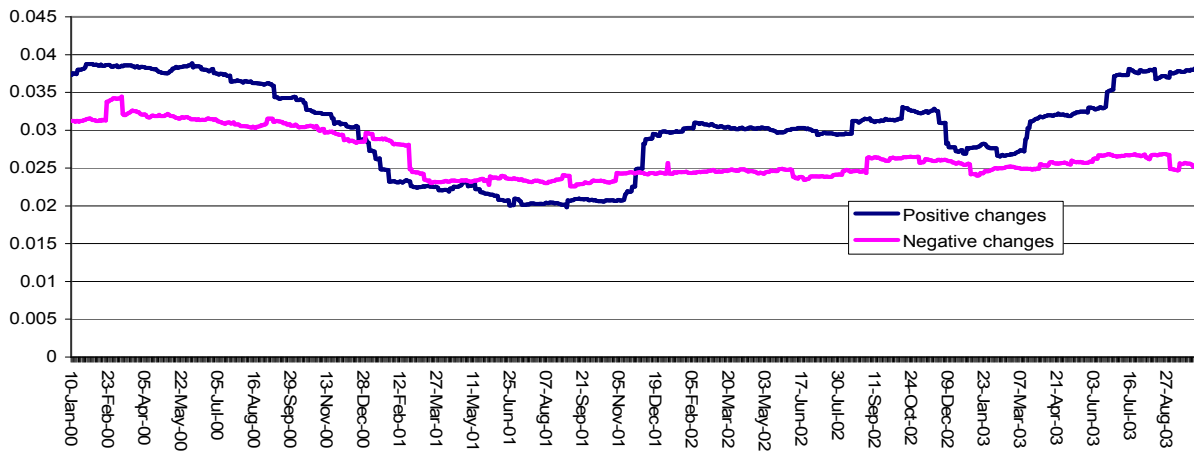


Figure 3. Principal components: level, tilt and curvature

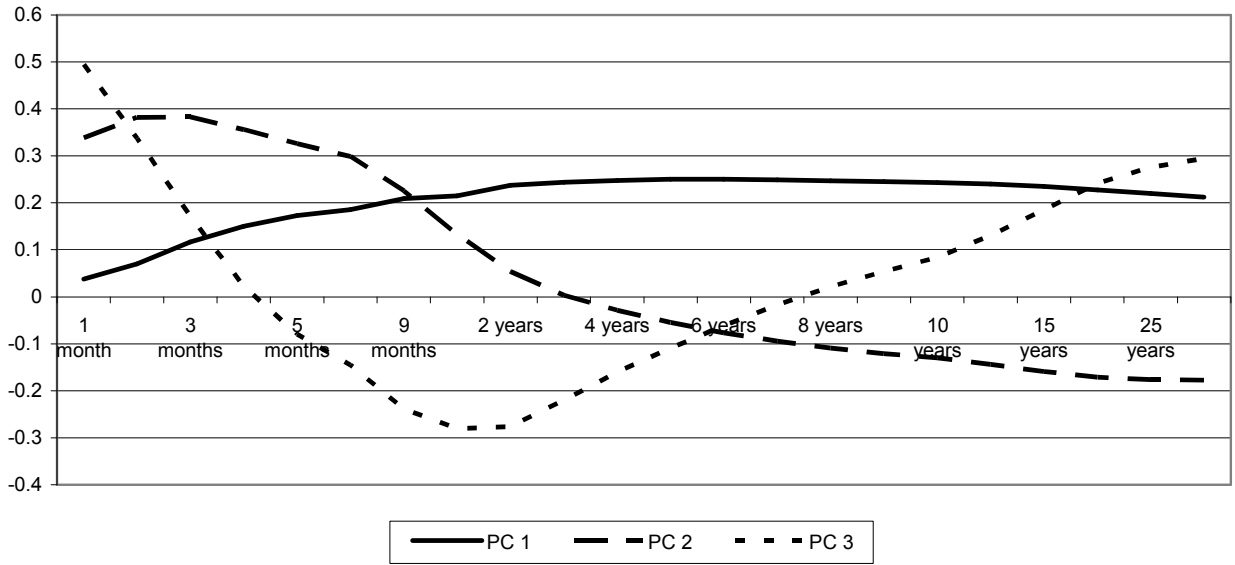


Figura 4a. PC non-parametric densities.

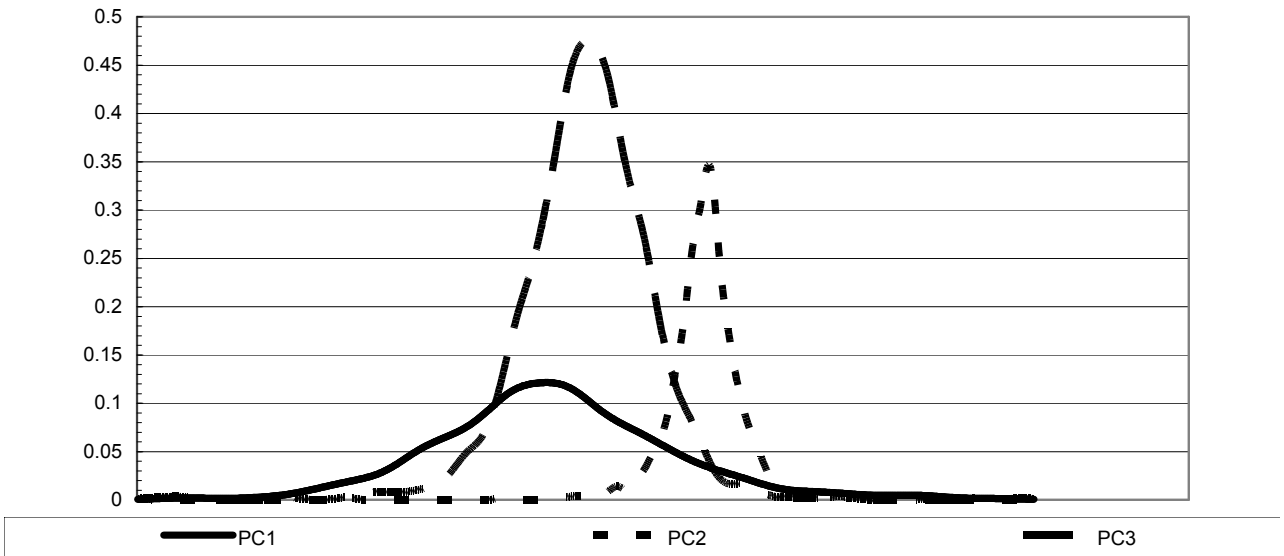


Figure 4b. PC non-parametric probability functions.

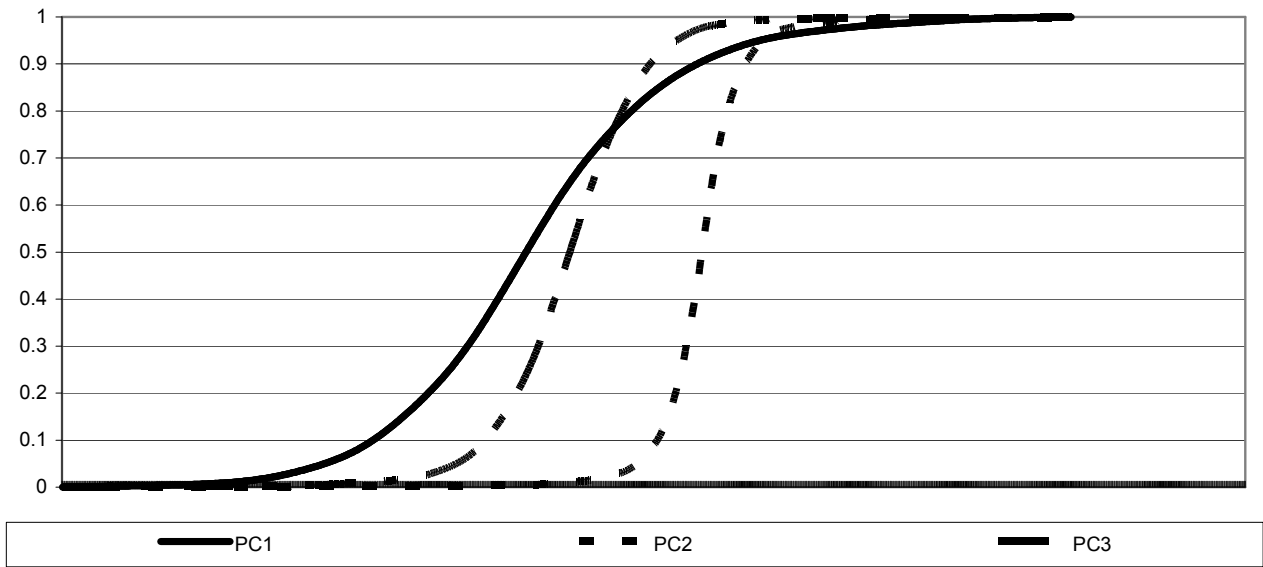


Figure 5b. Percentiles of interest rate change distribution in out-of-sample period (from Jan 2002 to Feb 2004) and worst case scenarios by 10,000 simulations from non-parametric distribution. (changes in basis point)

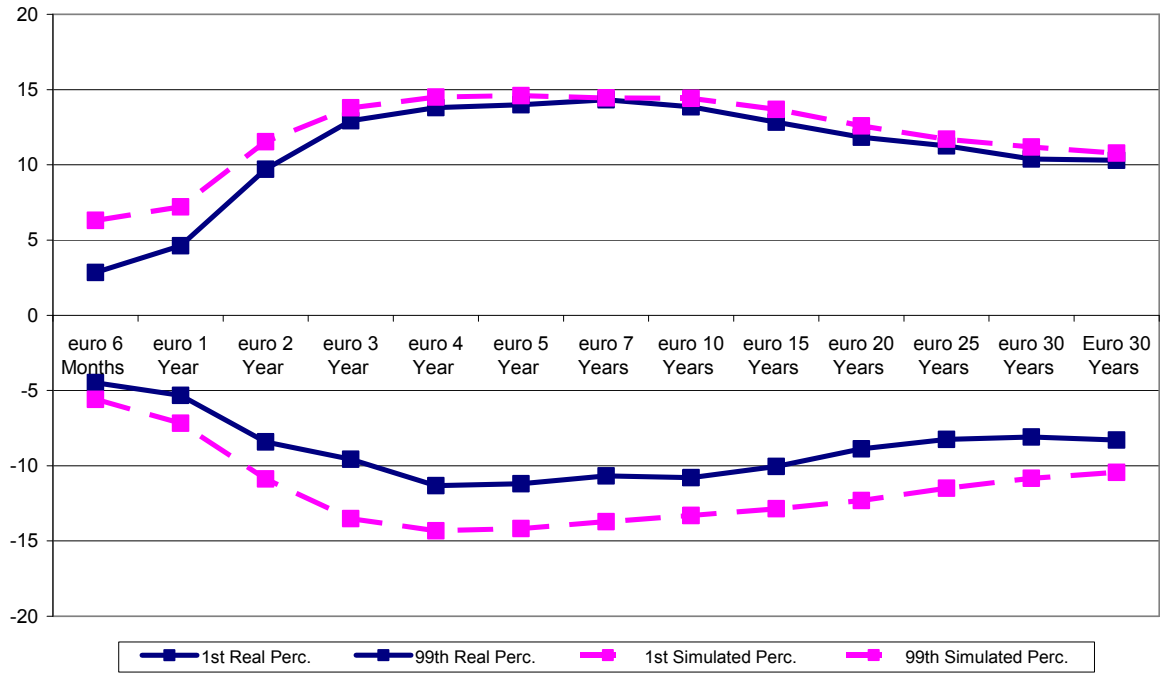


Figure 5a. Percentiles of interest rate change distribution in out-of-sample period (from Jan 2002 to Feb 2004) and worst case scenarios by 10,000 simulations from normal distribution. (changes in basis point)

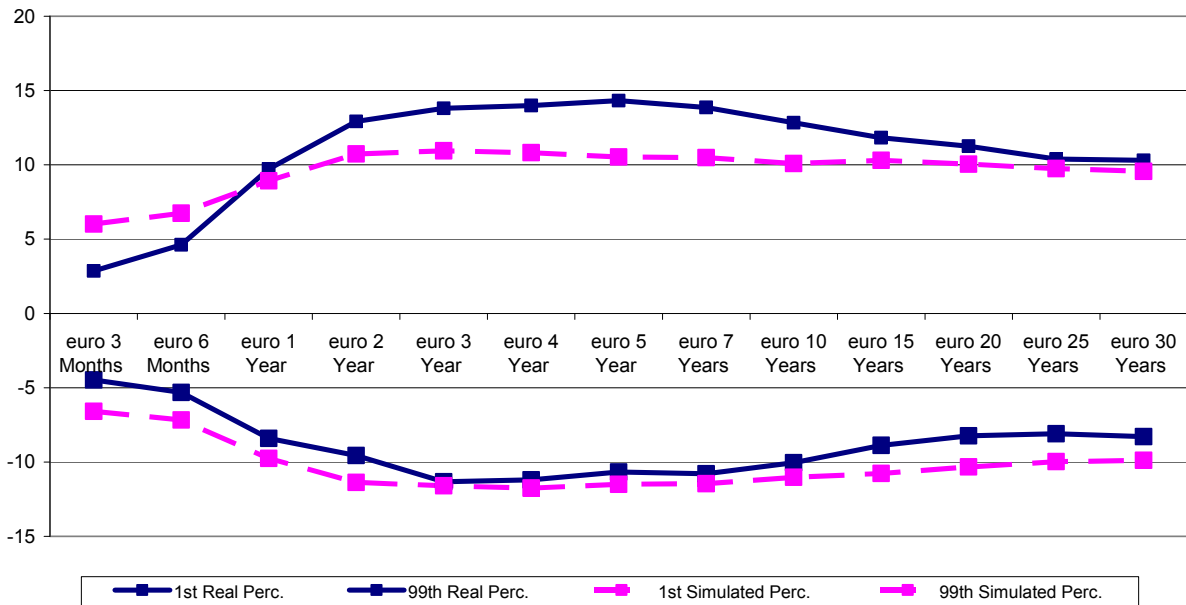


Figure 6a. Time series of daily profits and losses from January 2002 to February 2004 and parametric and non-parametric principal component VaR. (Bank 5: million of euros)

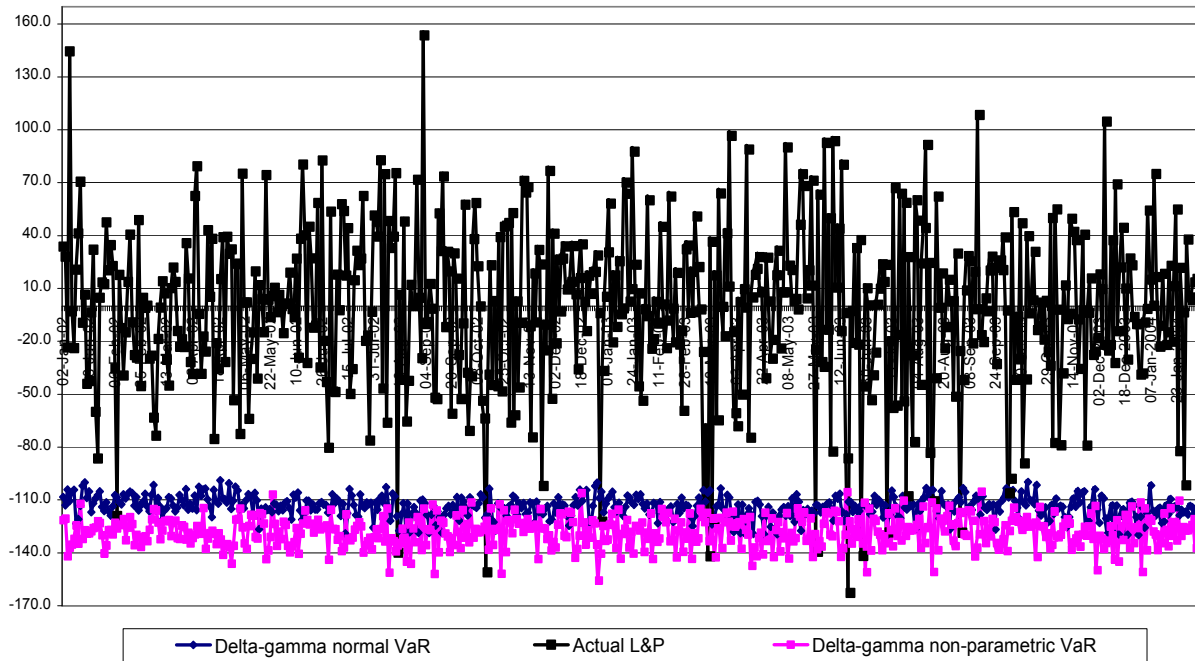


Figure 6b. Time series of daily profits and losses from January 2002 to February 2004 and parametric and non-parametric principal component VaR. (Bank 7: million of euros)

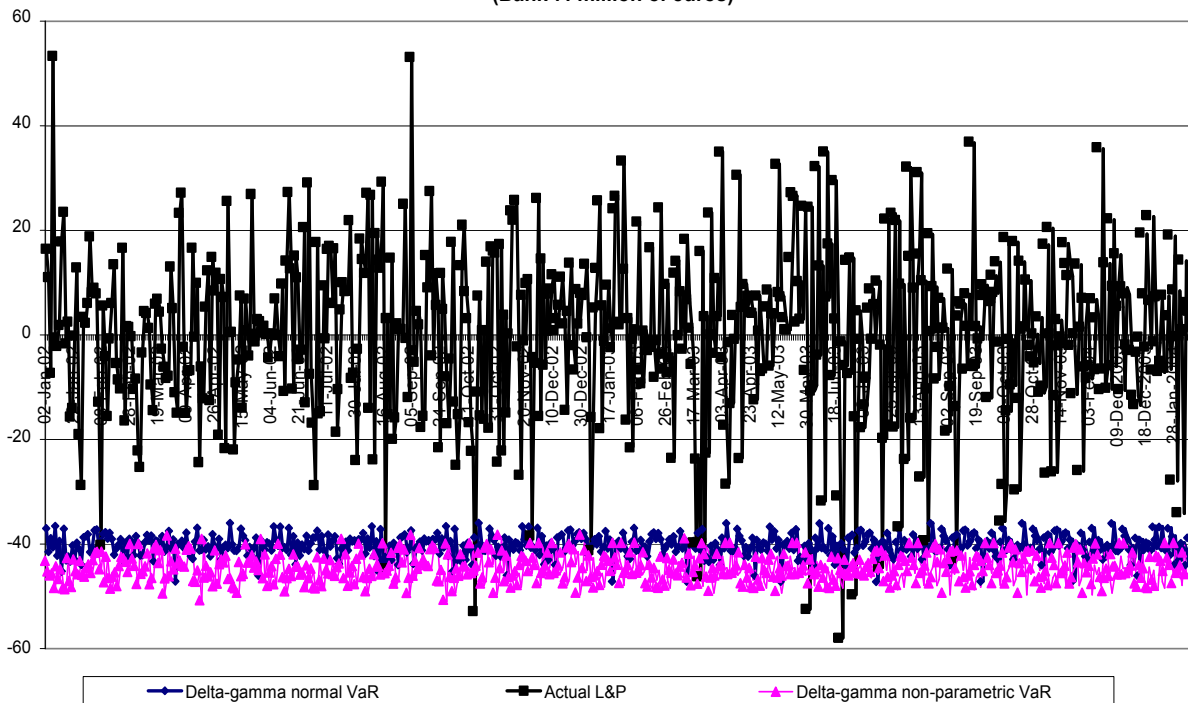


Figure 6c. Time series of daily profits and losses from January 2002 to February 2004 and parametric and non-parametric principal component VaR.
(Bank 14: million of euros)

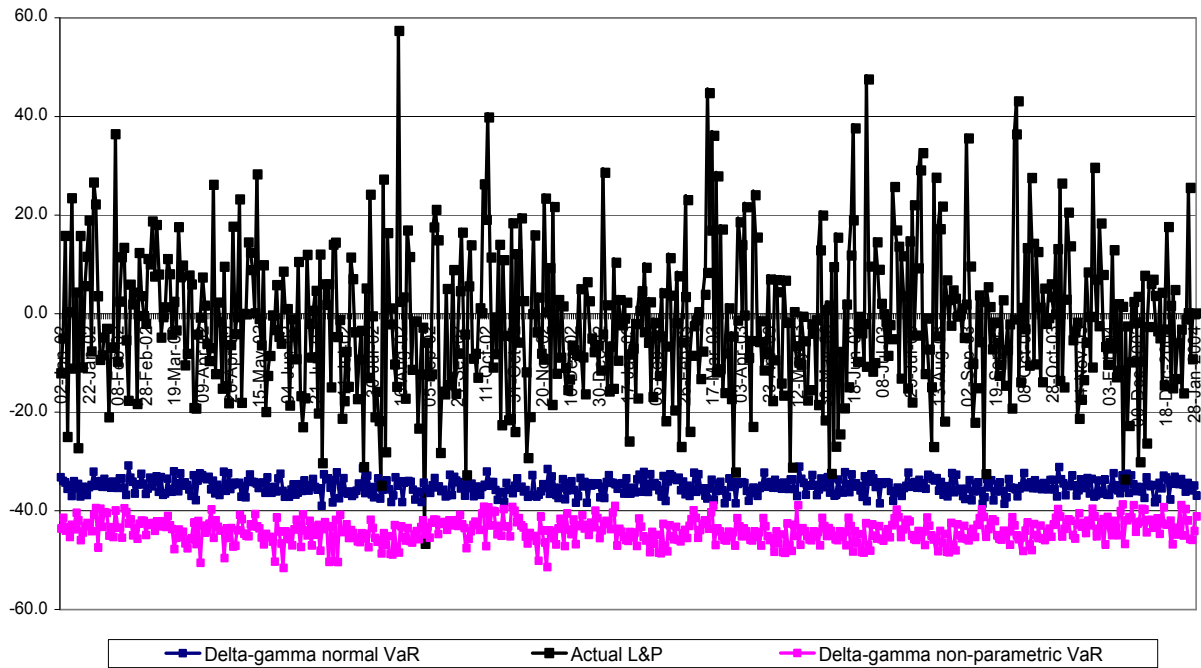


Figure 6d. Time series of daily profits and losses from January 2002 to February 2004 and parametric and non-parametric principal component VaR.
(Bank16, million of euros)

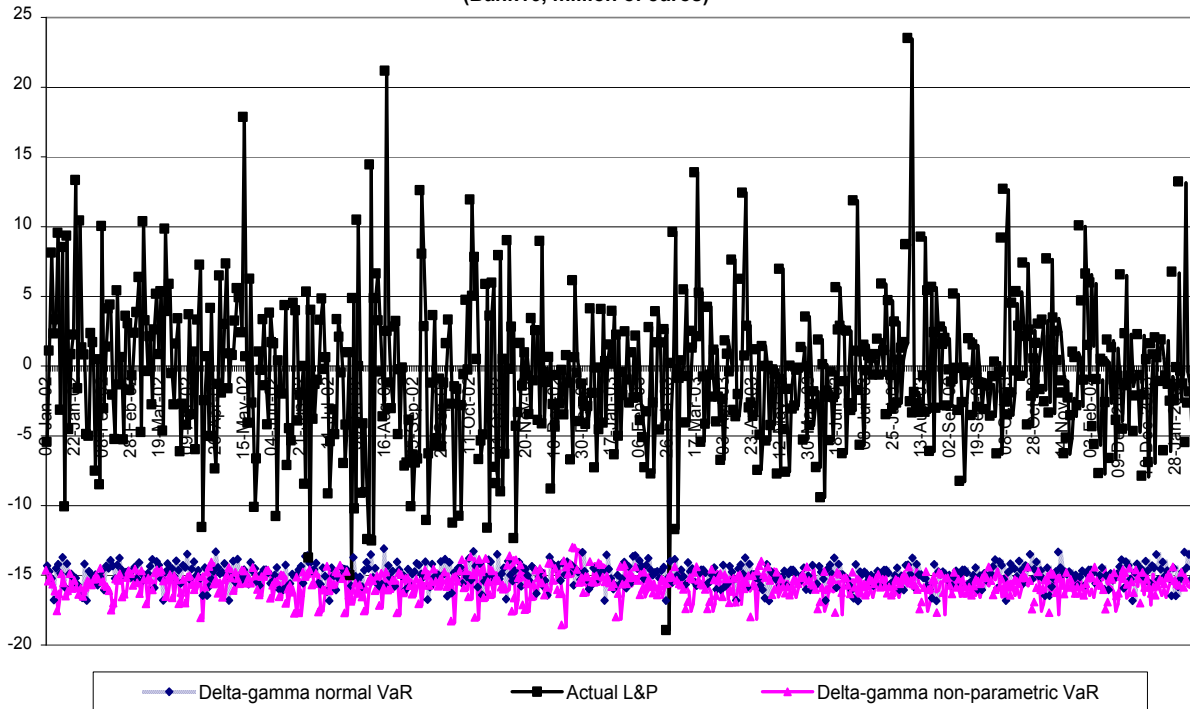
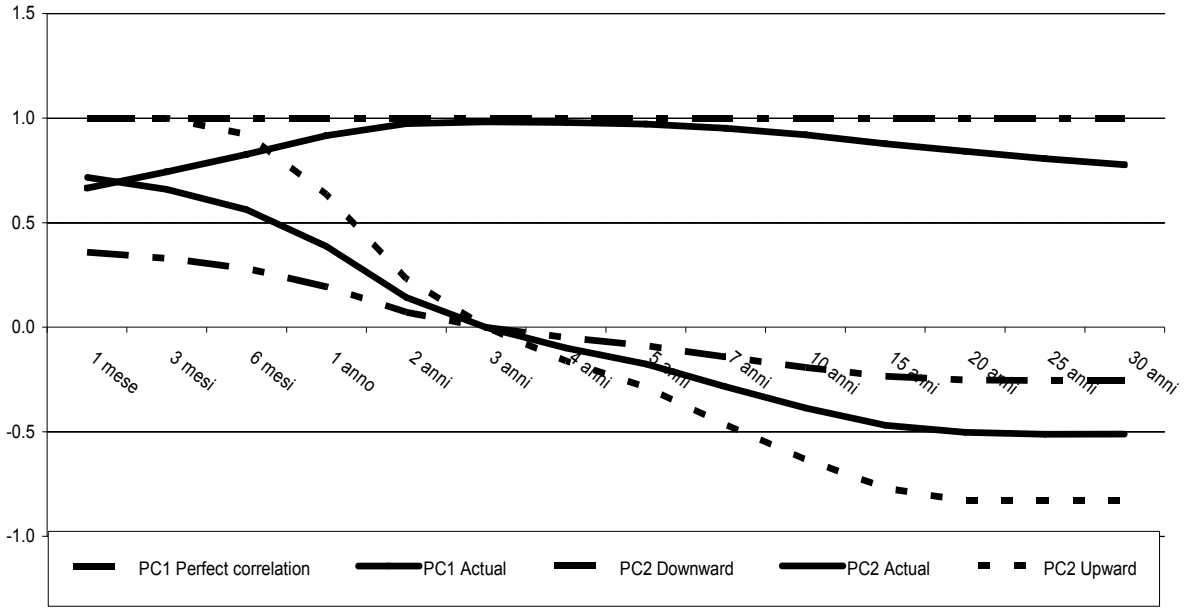


Figure 7. Separate scenarios for each PC: stressed correlation and fixed volatility.



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