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MNB WORKING PAPERS 4
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MNB Working Papers 2011/4

Labor Market Participation, Unemployment and Monetary Policy*

(Munkapiaci részvétel, munkanélküliség és monetáris politika)

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Published by the Magyar Nemzeti Bank

Publisher in charge: dr. András Simon, Head of Communications

8-9 Szabadság tér, H-1850 Budapest

www.mnb.hu

ISSN 1585-5600 (online)

*Stefano Gnocchi gratefully acknowledges financial support from the Spanish Ministry of Education and Science through grant ECO2009-09847, the support of the Barcelona GSE Research Network and of the Government of Catalonia. This project was started while Alessia Campolmi was visiting Universitat Autònoma de Barcelona in May 2009. Alessia is grateful to the Economics Department for their hospitality. We thank seminar participants at LUISS, Valencia University, Goethe University Frankfurt, Norges Bank, University of Vienna, EUI, Queen Mary University and University of Milano-Bicocca for useful comments.

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Contents

Abstract

In the present paper we examine how the introduction of endogenous participation in an otherwise standard DSGE model with matching frictions and nominal rigidities affects business cycle dynamics and monetary policy. The contribution of the paper is threefold: first, we show that the model provides a good fit for employment and unemployment volatility, as well as participation volatility and its correlation with output for US data. Second, we show that in such a model, *and contrary to a model with exogenous participation*, a monetary authority that becomes more aggressive in fighting inflation decreases the volatility of employment and unemployment. Finally, we show the role of search costs in shaping those results.

JEL: E24, E32, E52.
Keywords: matching frictions, endogenous participation, monetary policy.

Összefoglalás

Ebben a tanulmányban azt vizsgáljuk, hogy egy legtöbb tekintetben standard, matching súrlódásokkal és nominális merevségekkel rendelkező DSGE modellben a munkapiaci részvételi döntés endogenizálása miként befolyásolja az üzleti ciklusok dinamikáját és a monetáris politikát. A tanulmány három eredményt tartalmaz: (1) Megmutatjuk, hogy a modell jól magyarázza az Egyesült Államok foglalkoztatási és munkanélküliségi adatainak volatilitását és ezen változóknak a kibocsátással való korrelációját. (2) Megmutatjuk, hogy ebben a modellben, ellentétben azokkal a modellekkel, amelyek a munkapiaci részvételi döntést exogénnek veszik, egy agresszívebben infláció-ellenes monetáris politika csökkenti a foglalkoztatási és a munkanélküliségi ráta változékonyságát. (3) Végül megvizsgáljuk, hogy az előző eredmény mennyire függ a keresési költségek szintjétől.

1 Introduction

The labor market participation margin over the business cycle has received surprisingly little attention by most of the literature. The assumption of inelastic labor force has become common practice since Andolfatto (1996) and Merz (1995), the first invoking matching frictions to explain the aggregate fluctuations of labor market variables. Furthermore, the assumption has been imported by most of the recent vintage of business cycle models featuring both nominal rigidities and matching frictions. However, the participation rate in the labor market fluctuates at business cycle frequencies and it is pro-cyclical. For instance, over the period 1964-2007 in the United States, the standard deviation[1] of the participation rate relative to GDP is 0.20, while its correlation with GDP is 0.42. In addition, as pointed out by Barnichon and Figura (2010), participation accounts for one-fourth of the variance of the unemployment rate over the business cycle.

In this paper we argue that models neglecting the participation margin lead to incorrect conclusions about the effect of monetary policy on the volatility of employment and unemployment. To do so, we first build an otherwise standard model with matching frictions and nominal rigidities allowing for endogenous fluctuations of the labor force. Then, we show that our model is able to account remarkably well for US business cycle evidence. Although we do not target the second moments of the participation rate, our model predicts a standard deviation relative to output of 0.22 and a correlation with output of 0.56, both well in line with the evidence. Finally, we study the effects of strict inflation targeting predicted by our model and we document that the presence of the participation margin overturns the conventional conclusions one obtains with a model featuring exogenous participation. For instance, it is well known that switching from a flexible to a strict inflation targeting regime (the latter defined as complete stabilization of inflation around its target) magnifies the volatility of the unemployment rate, conditionally on technology shocks: see, for example, Blanchard and Galí (2010). We show that the surge in volatility due to the policy change is considerably overstated, if the participation margin is not taken into account. Under our calibration, which allows for other shocks in addition to productivity, such as demand shocks and labor supply shocks, unemployment volatility falls when the central bank switches to a strict inflation targeting policy. This is the opposite of what a model abstracting from endogenous fluctuations of the labor force would predict. Our finding has interesting implications. If a policymaker is concerned about the consequences of being tough in stabilizing inflation in terms of unemployment rate volatility, a model accounting for the participation margin offers a more reassuring picture. More broadly, macro policies addressing the issue of unemployment fluctuations cannot abstract from the fact that incentives to substitute involuntary unemployment with voluntary non-employment are not invariant to policy. The intuition is straightforward: the incentives driving participation in the labor force respond to frictions. In turn, monetary policy affects the relevance of frictions over the business cycle. Therefore, the monetary policy regime affects households' willingness to move from unemployment to voluntary non-employment and the other way around.

We address our questions in a standard model featuring matching frictions à la Diamond (1982) and Mortensen and Pissarides (1999) and nominal price rigidity à la Calvo (1983), where we make the entry to the labor market costly by modeling home production activity and search activity as both requiring time. Hence, entering the labor market entails a cost in terms of home production paid in exchange for a chance to be matched with a job.

We calibrate the full model, as well as a version assuming exogenous participation rates (baseline NK-DMP model henceforth) to the US data. Finally, we use the two calibrated versions of the model to draw our policy implications.

We also contribute to the literature by showing the quantitative importance of households' search costs in two respects: in shaping the role of matching frictions in the participation decision and in affecting the strength of the policy transmission

[1] We extract the business cycle component from the data in logs by applying the Hodrick-Prescott filter with a conventional smoothing parameter of 1600. Further details are provided in Section 3.

channel acting through the labor force. In our model households' search costs are captured by a parameter governing the loss in terms of time devoted to home production due to a movement from non-participation to the unemployment pool, relative to the movement from non-participation to the employment pool. The way we calibrate the model ensures that the value of the search cost is in line with the microeconomic evidence coming from the American Time Use Survey. In order to assess the importance of the search cost for our results, we repeat the policy experiment in a counterfactual world where the search cost is implausibly high and close to 1, implying that the time the unemployed spend searching is the same as the time spent working by those employed. Therefore, the home production time forgone to enter the labor force does not depend on the employment status. Under this scenario, the exogenous and the endogenous participation models are virtually indistinguishable and they deliver the same predictions. The result cannot be understood if one abstracts from the incentives driving the participation decision that can be summarized as follows. First, the household aims at replicating the level of home production that would be achieved in a model without labor market frictions. It follows that the household's main incentive is to keep the marginal rate of transformation between market and non-market consumption at the ideal level that would prevail in the absence of labor market frictions. Equivalently, matching frictions open a home production gap and participation is chosen so as to close the gap. Second, when the search cost is calibrated to 1, matching frictions cannot distort the allocation of time provided that employment and unemployment are roughly equally expensive in terms of non-market activity as emphasized above. Therefore, neither does the household have the incentive to adjust the labor force to shocks, nor can monetary policy affect employment and unemployment fluctuations through the participation margin. This is the reason why the difference in the policy outcome predicted by our model and by the baseline NK-DMP version falls in the search cost.

Our findings are relevant for and related to different strands of the literature on aggregate fluctuations. On the one hand, several papers focus on the behavior of labor market variables at business cycle frequencies[2]. On the other hand, only few recent contributions focused on the participation margin. A representative, though not exhaustive, sample includes Garibaldi and Wasmer (2005), Ebell (2008), Haefke and Reiter (2006), Brückner and Pappa (2010), Galí (2010) and Christiano, Trabandt and Walentin (2010). However, all those papers are either real models or they abstract from the issue we address, i.e. the implications of the participation margin for the transmission channel of monetary policy. In addition, to the best of our knowledge, we are the first to tackle the importance of the household's search cost for the participation decision and its relevance for monetary policy.

Finally, it is worth pointing out that the participation decision in our framework is substantially different from the intensive margin of hours worked, such as the one studied in Sveen and Weinke (2008) or Trigari (2009). This is because the choice of hours is made *conditionally* on having entered the labor force and being in a match. It follows that the finding rate does not affect the hours decision, while it does affect the participation choice, since the job finding probability determines the expected marginal gain associated with entering the labor force. This is different from the case of endogenous search intensity, where intensity is a function of the finding rate. This is a point made by Ravn (2006) as well as by Krause and Lubik (2010), who build a real business cycle model with on-the-job search. However, these papers abstract from monetary policy and they do not analyze the role of households' search costs, either. Finally, our contribution may be viewed as complementary to Krussel, Mukoyama, Rogerson and Sahin (2010), who reassess the impact of distortionary taxation on labor supply decisions in a model with labor market frictions and incomplete financial markets.

The paper is organized as follows. In Section 2 we describe our model economy, which is calibrated to US data in Section 3. Section 4 investigates the incentives driving the participation decision, with an emphasis on the role of frictions. Finally, we conduct our policy experiment in Section 5, while Section 6 assesses the importance of the household's search cost for our results.

[2] Typical references are Shimer (2005), Hall (2005), Chéron and Langot (2000) and Walsh (2005). See also Trigari (2009) on the persistence of monetary policy shocks, Christoffel and Linzert (2010) on inflation dynamics, Gertler and Trigari (2009) and Krause and Lubik (2007) on the role of wage rigidity and Thomas (2008) and Faia (2008) on optimal monetary policy.

2 The Model

The representative household consists of a continuum $[0,1]$ of family members. Each of them can be employed, unemployed or non-participant. Non-participant family members allocate all their time to home production. Employed members spend all their time at work receiving a salary in exchange. Unemployed workers spend some of their time actively searching for a new job while the rest is used for home production. While unemployed, they are entitled to unemployment benefits. Wages, unemployment benefits and home production are pulled together and redistributed equally among family members so that they all enjoy the same level of consumption and home production[3]. Consumption and savings are decided at the household level, together with the choice of how many family members are let to participate in the labor market.

The economy is characterized by two sectors[4]. In the final sector there is a continuum of retailers, each selling a differentiated good under monopolistic competition and using intermediated goods as the only input in production. Calvo price stickiness is assumed in this sector. In the intermediate sector infinitely many firms produce a homogeneous good under perfect competition and flexible prices. In order to produce, each firm has to be matched with a worker. Firms are subject to a vacancy posting cost when searching for a worker. Existing matches can be exogenously discontinued at any time.

We choose to consider three shocks in the model. Disturbances to market goods production technology and to preferences are considered, as is standard in the literature[5]. In addition, we include shocks to home goods production technology that are correlated with market technology shocks. This is because we want to allow for the possibility that an improvement in market technology spreads to the home production sector. In the calibration exercise we then leave the data to choose the variances and the cross-correlation between home and market technology shocks so as to match unconditional simulated moments with their observed counterparts. Preference and home production shocks are observationally equivalent in a model with exogenous participation. However, they are not if the labor force is endogenous. We discuss the identification restrictions when we calibrate the model to the US data.

2.1 HOUSEHOLDS

A household is made up by a continuum $[0,1]$ of family members. Let E_{t-1} be the employed members in period $t-1$. When entering period t, a fraction ρ of those jobs will be exogenously discontinued. Among those, some may drop out of the labor force, if the household decides to reduce labor market participation, while the others will search for a new job. We assume instantaneous hiring[6] i.e. searching workers matched with a firm will start working already in period t. Searching workers who are not be matched, will receive the unemployed status and will be entitled to unemployment benefit and take part in some home production. Intuitively, we are assuming that the search process takes place at the beginning of the period so that, if matched with a vacancy, workers can produce immediately. Otherwise, they can use the rest of their

[3] See Andolfatto (1996) and Merz (1995).

[4] We use the two-sector setup in order to keep the matching frictions separated from the price rigidity. See, for example, Sveen and Weinke (2008).

[5] Sveen and Weinke (2008) were the first to emphasize that demand shocks are needed to match the volatility of the unemployment rate.

[6] Because of the assumption of sticky prices, production is demand driven in the short run. Therefore, firms need to have a margin of adjustment to supply as many goods as demanded at the prevailing price. In a model without capital, as standard in this literature, there are two possible options. Either introducing endogenous job destruction or allowing for instantaneous hiring. We decided for the second in order to keep the model as simple as possible. Since we calibrate the model at quarterly frequency, it also seems reasonable.

time for home production. Therefore, if N_t is the fraction of family members participating in the labor market, searching workers in period t are defined as:

$$S_t = N_t - (1-\rho)E_{t-1} \tag{1}$$

We are implicitly assuming that when reducing participation, there are always enough unemployed workers to choose from so that all workers who were employed in the previous period and whose jobs were not exogenously discontinued, will keep their jobs. Non-participant members are given by:

$$L_t = 1 - N_t \tag{2}$$

Let f_t be the job finding rate, that will be endogenously defined when solving the search and matching problem in the intermediate sector. Then, the evolution of employment reads as follows:

$$E_t = (1-\rho)(1-f_t)E_{t-1} + f_t N_t \tag{3}$$

Let $C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ be a Dixit-Stiglitz aggregator of different varieties of goods. The optimal allocation of expenditure on each variety is given by $C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$ where $P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$. The representative household then maximizes the expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[Z_t \log(C_t) + \phi \frac{h_t^{1+\nu}}{1+\nu} \right] \tag{4}$$

subject to:

$$P_t C_t + R_t^{-1} D_t \leq D_{t-1} + W_t E_t + P_t b U_t + T_t \tag{5}$$

$$E_t = (1-\rho)(1-f_t)E_{t-1} + f_t N_t \tag{6}$$

$$N_t = E_t + U_t \tag{7}$$

$$h_t = \left[\xi_t (1 - E_t - \Gamma U_t)\right]^{1-\alpha_h} \tag{8}$$

taking as given the nominal interest rate R_t, the nominal wage W_t, the aggregate price of goods P_t, the probability of finding a job f_t and T_t, including lump-sum taxes and profits. b is the real unemployment benefit, D_t is a risk-free nominal bond paying one unit of currency in the following period, h_t represents the home production activity, $\nu < 0$ is the inverse of the home production elasticity, $0 < \Gamma < 1$ is the fraction of time that unemployed workers devote to the search activity and $\alpha_h \in [0, 1)$ allows for decreasing returns in the home production technology. Finally, Z_t is a shock to preferences and ξ_t is a shock to home production technology. Optimization implies a conventional Euler equation:

$$\beta R_t E_t \left\{ \frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t} \frac{P_t}{P_{t+1}} \right\} = 1 \tag{9}$$

and the following equation:

$$\left[\frac{1-f_t}{f_t}\right]\left(\frac{\phi\Gamma h_t^{\nu}C_t}{Z_t}\Xi_t(1-\alpha_h)h_t^{-\frac{\alpha_h}{1-\alpha_h}}-b\right)=\frac{W_t}{P_t}-\frac{\phi h_t^{\nu}C_t}{Z_t}\Xi_t(1-\alpha_h)h_t^{-\frac{\alpha_h}{1-\alpha_h}} + \beta E_t\left\{\frac{C_t}{C_{t+1}}\frac{Z_{t+1}}{Z_t}\frac{(1-\rho)(1-f_{t+1})}{f_{t+1}}\left(\frac{\phi\Gamma h_{t+1}^{\nu}C_{t+1}}{Z_{t+1}}\Xi_{t+1}(1-\alpha_h)h_{t+1}^{-\frac{\alpha_h}{1-\alpha_h}}-b\right)\right\} \quad (10)$$

2.1.1 Endogenous Participation

Rearranging the optimality condition (10) allows us to gain some insight into the key determinants of the participation decision. After defining:

$$\Omega_t \equiv \frac{(1-f_t)}{f_t}\left[\frac{\phi\Gamma h_t^{\nu}C_t}{Z_t}\Xi_t(1-\alpha_h)h_t^{-\frac{\alpha_h}{1-\alpha_h}}-b\right] \quad (11)$$

(10) can be rewritten recursively as:

$$\Omega_t = \frac{W_t}{P_t}-\frac{\phi h_t^{\nu}C_t}{Z_t}\Xi_t(1-\alpha_h)h_t^{-\frac{\alpha_h}{1-\alpha_h}}+E_t\left\{\frac{\beta C_t(1-\rho)}{C_{t+1}}\frac{Z_{t+1}}{Z_t}\Omega_{t+1}\right\} \quad (12)$$

Note that $\frac{\phi\Gamma h_t^{\nu}C_t}{Z_t}\Xi_t(1-\alpha_h)h_t^{-\frac{\alpha_h}{1-\alpha_h}}-b$ is the flow benefit of withdrawing one unemployed worker from the labor force and reallocating him/her to home production in terms of consumption, net of the unemployment benefit. Also, the term $\left[\frac{1-f_t}{f_t}\right]$ is a wedge introduced by matching frictions capturing the extra change in home production, relative to a frictionless labor market, needed to increase employment by one unit. In fact, by manipulating the law of motion of employment, it is straightforward to get:

$$E_t = (1-\rho)E_{t-1}+\frac{f_t}{1-f_t}U_t \quad (13)$$

Not surprisingly, matching frictions introduce a wedge between employment and the participation decision. Such a wedge decreases in the job finding rate and it is strictly positive.

We interpret (12) as the optimality condition for labor market participation: it states that the marginal benefit of increasing employment has to equalize its marginal cost, once the wedge due to frictions is taken into account. On the one hand, Ω_t is the utility loss implied by diverting from home production the extra fraction of population frictions require to marginally increase employment. On the other, the right hand side of (12) represents the household's marginal benefit, adding the wage premium over the marginal rate of substitution to the option value of getting an additional member into employment, Ω_{t+1}. A positive option value arises as long as a match realized in the current period allows the household to save on the future search cost with a positive probability $1-\rho$. Finally, note that if the wedge vanishes, the marginal rate of substitution between consumption and home production equals the real wage. We define such a situation as full participation, since non-employment is entirely voluntary.

Condition (12) links the participation decision to the job finding rate through home production. A raise in the finding rate shifts downwards the marginal cost of increasing employment for any given level of the marginal rate of substitution. Therefore, everything else equal, home production has to fall the same way as leisure would do in the baseline business cycle model with endogenous labor supply.

2.2 FIRMS

2.2.1 Intermediate Good Producers

There are infinitely many firms $j \in [0, 1]$ producing a homogeneous good under perfect competition and flexible prices using labor as the only input in production. The labor market is characterized by matching frictions in the standard Mortensen and Pissarides (1999) framework. Firms have to search for a worker in the pool of searching workers. Posting a vacancy costs κ units of the final good C_t in each period. When the vacancy is filled, it produces:

$$X_t(j) = A_t \tag{14}$$

where the (log of) technology A_t is assumed to follow an AR(1) process: $\log(A_t) = \rho_a \log(A_{t-1}) + \xi_t^a$ with ξ_t^a being an i.i.d. shock with zero mean and variance σ_a.

We use a standard constant returns to scale technology converting searching workers S_t and vacancies V_t into new matches M_t:

$$M_t = \omega V_t^{1-\gamma} S_t^{\gamma} \tag{15}$$

We define labor market tightness as $\theta_t \equiv \frac{V_t}{S_t}$, the job filling rate (i.e. the rate at which vacancies are filled) as $q_t \equiv \omega \theta_t^{-\gamma}$, and the job finding rate (i.e. the rate at which searching workers meet a vacancy) as $f_t \equiv \theta_t q_t$. Because of instantaneous hiring, once the vacancy is filled it is immediately productive. Let P_t^x be the price at which firms sell the homogeneous good to the final goods producers. The value of a filled vacancy, V_t^J expressed in terms of the final consumption bundle P_t, is given by:

$$V_t^J = \frac{P_t^x}{P_t} A_t - \frac{W_t}{P_t} + (1-\rho) E_t \left\{ Q_{t,t+1} V_{t+1}^J \right\} \tag{16}$$

where $Q_{t,t+1} \equiv \beta \frac{C_t}{Ct+1} \frac{Z_{t+1}}{Z_t}$. The free entry condition ensures that:

$$\frac{\kappa}{q_t} = V_t^J \tag{17}$$

Substituting (17) into (16) gives the job creation condition:

$$\frac{\kappa}{q_t} = \frac{P_t^x}{P_t} A_t - \frac{W_t}{P_t} + (1-\rho) E_t \left\{ Q_{t,t+1} \frac{\kappa}{q_{t+1}} \right\} \tag{18}$$

Finally, the wage is determined by solving a Nash bargaining problem between the firm and the worker. In order to do that we have to compute the surplus from employment keeping participation constant. This is given by[7]:

$$V_t^w = \frac{W_t}{P_t} - b - \frac{\phi h_t^{\nu} (1-\Gamma) C_t}{Z_t} \xi_t (1-\alpha_h) h_t^{-\frac{\alpha_h}{1-\alpha_h}} + E_t \left\{ Q_{t,t+1} (1-\rho)(1-f_{t+1}) V_{t+1}^w \right\} \tag{19}$$

[7] See Appendix for the derivation.

Let η be the firm's bargaining power. Then, the total surplus form the match is split according to the optimal sharing rule:

$$\eta V_t^w = (1-\eta)V_t^J \tag{20}$$

Using the definitions of V_t^J and V_t^w in (20), together with the free entry (17) and the job creation condition (18), it is possible to derive the wage equation:

$$\frac{W_t}{P_t} = (1-\eta)\frac{P_t^x}{P_t}A_t + \eta\left[b + \frac{\phi h_t^{\nu}(1-\Gamma)C_t}{Z_t}\xi_t(1-\alpha_h)h_t^{-\frac{\alpha_h}{1-\alpha_h}}\right] + (1-\eta)(1-\rho)E_t\left\{Q_{t,t+1}\kappa\theta_{t+1}\right\} \tag{21}$$

2.2.2 Final Goods Retailers

In the final good sector there are infinitely many producers of differentiated goods. Each is producing a variety $i \in [0,1]$ using the following technology:

$$Y_t(i) = X_t(i)^{1-\alpha} \tag{22}$$

They face a downward sloping demand function[8]:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\varepsilon}[C_t + \kappa V_t] \tag{23}$$

Under flexible prices the optimal pricing rule is given by:

$$\frac{P_t^*(i)}{P_t} = \frac{\varepsilon}{\varepsilon-1}\frac{MC_t(i)}{P_t} \tag{24}$$

where $P_t^*(i)$ is the optimal price and $MC_t(i) = \frac{1}{1-\alpha}P_t^x X_t(i)^{\alpha}$ is the nominal marginal cost. Imposing symmetry equation (24) becomes:

$$1 = \frac{\varepsilon}{\varepsilon-1}\frac{1}{1-\alpha}\frac{P_t^x}{P_t}X_t^{\alpha} \tag{25}$$

When price rigidity à la Calvo (1983) is assumed, the pricing first order condition for a firm allowed to reoptimize in t is given by:

$$\sum_{T=0}^{\infty}\xi^T E_t\left\{Q_{t,t+1}\frac{Y_{t+T}(i)}{P_{t+T}}\left[P_t^*(i) - \frac{\varepsilon}{\varepsilon-1}MC_{t+T}(i)\right]\right\} = 0 \tag{26}$$

where ξ represents the probability of not changing the price in a given period. Log-linearizing (26) around the zero inflation symmetric steady state we obtain the New Keynesian Phillips Curve (NKPC):

$$\widehat{\pi}_t = \beta\widehat{\pi}_{t+1} + \lambda\widehat{mc}_t \tag{27}$$

[8] Remember that intermediate firms pay the vacancy posting cost in terms of final goods and therefore solve an expenditure minimization problem like the household.

where $\lambda = \frac{(1-\xi)(1-\beta\xi)}{\xi}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$ and lower case variables with a hat represent log-deviations from steady state.

2.3 MARKET CLEARING CONDITIONS

The aggregate production of the intermediate sector is given by:

$$X_t = \int_0^1 X_t(j)dj = A_t E_t \tag{28}$$

Integrating the demand for good i, (23) yields the conventional aggregate resource constraint:

$$Y_t = C_t + \kappa V_t \tag{29}$$

after defining aggregate output as:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{30}$$

Combining the demand for final goods (23) with their production function and integrating delivers the aggregate production function:

$$Y_t = X_t^{1-\alpha}\Delta_t^{\alpha-1} \tag{31}$$

where the following definition applies:

$$\Delta_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\varepsilon}{1-\alpha}} di \tag{32}$$

and Δ_t, bounded by 1 from below, is a measure of price dispersion.

2.4 MONETARY POLICY

We assume that the monetary policy follows a simple interest rate rule:

$$\log(R_t) = -\log(\beta) + \phi_\pi \widehat{\pi}_t \tag{33}$$

3 Calibration

The conditional evidence on the response of the participation rate to productivity shocks is controversial. For example, Galí (2010) finds a negative response of the participation rate to productivity shocks identified by using conventional long-run restrictions in a five variable VAR including labor productivity, employment, the unemployment rate, price inflation and the average price mark-up. In contrast, Christiano et al. (2010) find the opposite result under the same identification strategy but a different specification of the VAR. Hence, we opt for assessing the model by looking at the unconditional evidence.

We calibrate three versions of our model: a version with Walrasian labor markets and variable home production, which we refer to as the frictionless labor market case (or frictionless for short); a version with matching frictions but exogenous labor market participation; and our full model with both matching frictions and endogenous labor market participation[9]. All versions feature the same steady state, apart from unemployment, vacancies and labor market tightness in the frictionless labor market model. Indeed, the first one is constantly equal to zero while the other two are not present in the frictionless model. It will be clear below that the calibration delivers the same primitives in the three versions of the model, the only differences concerning the parameters of the stochastic processes. Given that we focus on conditional standard deviations relative to output, this fact obviously implies that different policy predictions cannot be due to the calibration. Hence, they are entirely attributable to the propagation channel created by the participation decision.

We parameterize a subset of parameters to their conventional values in the literature. We set the discount factor β equal to 0.99. The elasticity of substitution among varieties of the final good is set to 6 and the Calvo parameter to $\xi = 2/3$. We also maintain $\alpha = 1/3$ in the production function. We restrict to the case of a deterministic steady state where inflation and productivity are constant and normalized to 0 and 1 respectively. It follows that the relative price dispersion of the final goods is zero, while the relative price of the intermediate good is distorted only by monopolistic competition in the final good sector. We choose $\nu = -5$ implying a Frisch elasticity of labor supply equal to 0.2, keeping comparability with Galí (2010). Finally, the Taylor rule coefficient is set to 1.5.

All remaining parameters are calibrated to U.S. quarterly data over the period 1964:1-2006:3. The sample start coincides with Gertler and Trigari (2009) and Krause and Lubik (2007) and the whole sample is the same as that of Christoffel and Kuester (2008). All data are from the Federal Reserve Bank of St. Louis' database FRED II. We apply a Hodrick-Prescott filter with a conventional smoothing parameter of 1600 to extract the business cycle component from the data in logs. Seasonality has been removed before filtering. We also need data on home production activity to calibrate Γ. For this purpose, we use the American Time Use Survey (ATUS). The ATUS provides nationally representative estimates of how Americans spend their time supplying data on a wide range of non-market activities, from child-care to volunteering. ATUS individuals are randomly selected from a subset of households that have completed their eight and final month of interviews for the Current Population Survey (CPS). In the sample we can observe minutes per day devoted to paid activities and home production for a cross-section of approximately 98000 individuals over the period 2003-2009. We also observe the employment status, i.e. whether the individual is employed, unemployed or out of the labor force.

We calibrate as many parameters as possible so as to match the steady state of labor market variables with their observed unconditional mean. The law of motion of employment (3) gives a relation between the steady state employment rate, the finding rate and the exogenous separation rate. We set the separation rate, ρ, to 0.1 following Shimer (2005) and by targeting an employment rate of 0.9411 we recover the implied finding rate, 0.6572 per quarter, which is lower than in Shimer (2005). A lower finding rate is explained by the assumption of instantaneous hiring. In fact, workers can be matched in the same period they start to search, so that the model needs a lower f to replicate the same employment rate. The

[9] A detailed description of the two alternative models is provided in Appendix B.

scaling parameter of the matching function, ω, is chosen in such a way that the job filling rate q is equal to 2/3. This implies a steady state labor market tightness of about 1. These values are conventionally used in the literature, though it is worth noticing that all our results are robust to changes of the steady state of q and θ. As pointed out by Shimer (2005), the value of those variables is simply a matter of normalization. Following Hagedorn and Manovskii (2008) and Galí (2010), we calibrate the cost of posting a vacancy κ by targeting vacancy costs per filled job as a fraction of the real wage. We choose 0.045 as a target, as in Galí (2010). To this end we use the job creation condition and the target to solve for the real wage at the steady state and parameter κ. From Petrongolo and Pissarides (2001) and Mortensen and Nagypal (2007) we know that η has to lie on the interval $(0.3, 0.5)$ therefore we choose the midpoint 0.4, while $\gamma = 0.6$ ensures that the Hosios (1990) condition holds[10]. Given the bargaining power of workers and the unemployment benefit, Γ is pinned down by the wage equation, after replacing the marginal rate of substitution with its steady state value. Hence, we choose the replacement rate from its admissible range[11] $(0.2, 0.4)$ to match Γ consistently with the micro evidence from the ATUS. We interpret Γ as the time devoted to home production that a household member forgoes when moving from out of the labor force to the unemployment pool, relatively to a member moving from out of the labor force to the employment pool. The first three rows of Table 1 show time devoted to home production, measured in minutes per day, depending on the employment status. We report averages for the periods 2003-2009 and 2003-2006. The last row computes Γ consistently with the ATUS evidence. As our sample ends in 2006, we choose to target 0.44. We obtain as a result $\frac{b}{W/P} = 0.4$, the upper bound value. ϕ is determined ex-post to implement the observed participation rate, $N_t = 0.6394$.

We are left with the parameters of the stochastic processes. We select them to match key moments of the U.S. economy. We set the serial correlation of all shocks to 0.9 and then we calibrate the standard deviation of market technology, home production technology and preference shocks, and the cross-correlation of market and home technology so as to minimize the average distance of the simulated unconditional moments from their empirical counterparts. We consider the following targets: the standard deviation of output, the standard deviation of employment and that of the unemployment rate relative to output and the correlation of the unemployment rate with output. We determine the parameters simultaneously by performing a grid search.

Table 2 presents the results of our calibration exercise by showing the empirical moments, the simulated moments for the model with endogenous participation and the simulated moments for the model with exogenous participation. It is evident that both models account well for business cycle fluctuations, though the version with exogenous participation performs slightly worse in terms of employment volatility relative to output. To evaluate the goodness of fit of the baseline version with endogenous participation, we look at the standard deviation of the participation rate relative to output and the correlation of the participation rate with output, two moments that we have *not* targeted. It is clear that the predictions of the model in terms of participation are well in line with the evidence.

We conclude by investigating the restrictions imposed by the model that allow us to identify the shocks from the data. For this purpose, impulse responses are helpful, though we leave the discussion of the economic intuition to the following section. Figures 1-3 display the response of selected macro variables to market technology, preference and non-market technology shocks. Also, we discuss identification restrictions by calibrating the model to our targets allowing for only two shocks at a time. Table 3 reports the best fit in terms of moments for the model with endogenous participation.

It is well known that business cycle models need preference shocks to match the volatility of employment relative to output. In fact, the volatility of employment relative to output increases in the fraction of output variance explained by non-technology shocks. Sveen and Weinke (2008) pointed out that preference shocks may also help in matching the volatility of the unemployment rate relative to output. The third column of Table 3 confirms this fact. In addition, the shock induces a negative correlation between the unemployment rate and output, as it is in the data. In fact, as Figure 2 makes it clear, despite a surge in the participation rate, employment increases and the unemployment rate falls after a preference shock. However, the preference shock matches the three moments at the cost of overstating the volatility of output.

The second column of Table 3 shows that the home production TFP shock also increases significantly the volatility of employment and that of the unemployment rate at the cost of inducing a positive correlation between unemployment

[10] We show in Appendix C that the conventional Hosios condition applies also to our model.

[11] As for the bargaining power, see Petrongolo and Pissarides (2001) and Mortensen and Nagypal (2007) for a discussion.

and output. It is immediate to see from Figure 3 that this is due to the response of participation: despite the fact that employment increases on impact, the large inflow into the labor force drives up the unemployment rate. This is the reason why in the endogenous participation model non-market technology shocks are separately identified from preference shocks. Note also that, in the absence of the preference shock, the data pick a zero cross-correlation between home and market technology.

Hence, the last column of Table 3 analyzes the role of cross-correlation between home and market TFP by calibrating the model with all shocks but restricting the cross-correlation to zero. Though the performance of the model is better when both shocks are included, relatively to the second and the third columns, the volatility of employment and of the unemployment rate are matched at the cost of an excessively high volatility of output, which is brought in line with the data by high cross-correlation, as in our baseline calibration reported in Table 2. The positive correlation allows indeed to improve the trade-off induced by preference shocks between matching the volatility of output and employment. In fact, for a given standard deviation of the market TFP, a positively correlated non-market technology shock amplifies the volatility of output. It follows that a lower market TFP standard deviation is needed, therefore the preference shock is more effective in matching the volatility of employment. This intuition is confirmed by the fact that in our baseline calibration the standard deviation of market TFP, 0.0070, is lower than in the last column of Table 3, 0.0105, while the standard deviation of the other shocks is unchanged.

4 Participation and frictions

The object of this section is to disentangle the incentives driving the participation decision. The intuition goes as follows. A shock can affect the marginal rate of substitution between market consumption and home production directly (e.g. preference shock) and/or indirectly through the presence of price rigidity and matching frictions by changing the job finding rate and then the allocation of time between market and non-market activity. Households use the participation margin to keep the marginal rate of substitution between consumption and home production as close as possible to the one that would arise in a model with Walrasian labor markets.

We make the point by comparing (Figure 4) the impulse responses of home production and participation to a positive one percent market TFP shock for the three versions of our model. The frictionless model constitutes a useful benchmark. Conditionally on prices being flexible, with log utility, the income and substitution effects generated by a positive TFP shock exactly offset each other so that home production and participation do not move. Instead, under sticky prices the smaller reduction in prices induces the household to substitute less between market good and home production, relatively to the flexible price case and, as a result, participation declines in equilibrium while home production increases.

The introduction of matching frictions creates an important link between home production and the finding rate. For a constant level of participation, a higher finding rate shifts a fraction of unemployed family members into the employment pool.[12] Consider first the case of flexible prices. Home production under exogenous participation has to fall after a positive productivity shock due to the higher finding rate. This opens a gap with respect to the constant level that would be observed in the frictionless model. Then, if participation is endogenous, the household withdraws some unemployed members from the labor force, lowering participation and reducing the home production gap. When prices are sticky the finding rate still goes up, though by less than with flexible prices, reducing home production under exogenous participation. However, in the frictionless model the desired level of non-market activity increases. With endogenous participation the household reduces it responding both to the undesirable reduction of home activity due to the higher finding rate and the surge of the frictionless level of home production.

To sum up, in response to a market technology shock, the finding rate always increases, be prices sticky or not, so that home production falls if participation does not adjust. Given that the desired level of home production is constant under flexible prices and increases under sticky prices, participation always declines to replicate the flexible labor market outcome, and it does so by more when prices are sticky.

Now that the incentives behind the participation decision have been clarified, we explore in detail the transmission of the different shocks to macro variables.

Figure 1 shows the responses of several variables to a positive market productivity shock. Consistent with the discussion outlined above, participation falls and the household substitutes unemployment with voluntary non-employment to increase the level of home production. Both the increase of the finding rate and the outflow from the labor force drive the unemployment rate down, making it more responsive than it would be under exogenous participation. Note that, conditionally on a market TFP shock, participation is counter-cyclical in the model. Also, as usual for matching models, employment reacts very little. As in Sveen and Weinke (2008), a demand shock helps solving this problem.

Figure 2 considers the case of a positive preference shock to the market good. This shock influences directly the marginal rate of substitution inducing the household to demand less of home production and more of the market good. As a conse-

[12] The loss in terms of home production declines in the search cost. We show in Section 6 how the simplifying, but unrealistic, assumption of $\Gamma = 1$ crucially breaks this relation by making the movement from unemployment to employment status costless in terms of home production.

quence, participation increases and the unemployment rate falls, though by less than it would with exogenous participation. In fact, the surge in participation counterbalances the rise in the finding rate. Now movements in the participation margin dampen the reaction of labor market variables to a preference shock, as opposed to the case of a market productivity shock. Participation is now pro-cyclical and employment rises more than under the previous shock.

Figure 3 displays the case of higher productivity in the home production technology. In terms of the marginal rate of substitution, this shock is very similar to the positive preference shock as it induces households to demand more of the market good, thus pushing output up. However, the improved technology makes it feasible to produce more of the home goods with the same number of non-participants/unemployed. Participation increases by more than with the previous shock, since it entails a lower loss in terms of home production. The increased labor supply pushes wages and prices down. Employment increases but not enough for compensate the surge in participation; the unemployment rate increases as well, even though unemployment falls.

5 Participation and Monetary Policy

Now that we have clarified how shocks are transmitted when participation is endogenous, we assess the relevance of the participation margin for predictions about the effects of monetary policy on volatilities and co-movements of macro variables. This is an interesting exercise, since the incentives driving participation interact with frictions. In turn, monetary policy affects the role of frictions in shaping the response of macro variables to shocks. As a consequence, it is natural to expect that the presence of the participation margin creates an additional transmission channel of monetary policy overlooked by the current literature.

To this end we compare the predictions of the endogenous and exogenous participation models for two values of ϕ_π, 1.5 and 100. Hence, we focus on the effect on macro moments of a policy switch from a flexible to a strict inflation targeting regime, where $\phi_\pi = 100$ implements the flexible price allocation. For each of the two models we keep structural parameters at the value minimizing the distance from the data of the model predicted moments. This is because we want to give both models the same chance to fit the data unconditionally. However, note that the parameters across the two models only differ in terms of standard deviations of the shocks and of the cross correlation between home and market technology. As an implication, *conditionally* on each of the shocks, differences in the predicted moments across models do not depend on the calibration. Therefore, when looking at the conditional moments, differences in the impact of the policy switch are *entirely* due to the presence or the lack of the participation margin.

The experiment shows that, indeed, a change in the monetary policy rule affects business cycle moments in a way that is overlooked by models abstracting from the participation margin. In fact, when people can optimally reallocate time between market and home production activity, they choose to do so in such a way that the effect of a monetary policy regime change may be dampened, as in the case of market technology shocks, or magnified, as in the case of preferences or home technology shocks.

Tables 4-6 report the moments of macro variables for the case of $\phi_\pi = 1.5$ and strict inflation targeting, conditioning on one shock at a time, market technology, home technology and preference shocks respectively. Table 7 reports the same moments unconditionally, when all shocks hit.

It is evident from Table 4 that conditionally on market productivity shocks, strict inflation targeting magnifies the volatility of employment and unemployment rates in both models. This is because replicating the flexible price equilibrium eliminates inefficient fluctuations in price mark-ups and boosts the sensitivity of aggregate demand to market productivity. It follows that the positive response of vacancy posting and of the finding rates are higher. However, the exogenous participation model over-predicts the surge in the volatility of labor market variables. When the household indeed chooses the participation rate, the volatility of the labor force also increases. Constant the participation rate, the stronger response of the finding rate induced by the regime switch would lead to a reallocation of time from home to market that the household dislikes. Hence, for the household it is optimal to substitute some of the unemployed with non-participant members and she does so, to a greater extent when monetary policy is more aggressive. The fall in the number of searching workers counterbalances the rise in vacancy posting. As a result, employment, the employment rate and the unemployment rate are less volatile, relative to a world where the household cannot adjust the participation margin.

Table 5 displays the case of home productivity shocks. In both models the volatility of output is higher under strict inflation targeting. As with market technology shocks, when price rigidity vanishes, demand and thus output are more responsive to productivity due to the elimination of mark-ups time variation. However, under exogenous participation, the volatility of all macro variables varies proportionally with output volatility, so that the standard deviation of labor market variables relative to output does not change. When the participation margin is active, the picture is different. Table 5 makes it easy to see that the different reaction of employment and unemployment rates to the policy change across models is entirely due to the participation margin. In fact, just like in the case of exogenous participation, when the size of the labor force can be adjusted, the volatility of employment relative to output does not change. Even so, employment and unemployment rates behave differently. A positive home production shock increases participation and the unemployment rate more than proportionally relative to output, so that the unemployment rate is always pro-cyclical irrespectively of the monetary policy regime. Since more aggressive monetary policy reduces the response of participation to the shock, the employment and unemployment rate fluctuations are also dampened.

As reported in Table 6, under preference shocks and absent the participation margin, once again the policy rule does not affect macro moments relative to output. In contrast, when the household can choose the participation rate, the shock triggers a flow into the labor force and, under the baseline policy, an increase of output and vacancies, which will drive down the unemployment rate. Strict inflation targeting magnifies the volatility of participation. As an implication, the larger flows to the labor force make the volatility of the employment rate and its correlation with output smaller. The effect is so large that switching from flexible to strict inflation targeting changes the sign of the correlation between employment rate and output from positive to negative.

Finally, Table 7 replicates the experiment when all shocks hit. In this case the endogenous participation model predicts a fall in unemployment rate volatility when switching from flexible to strict inflation targeting. The exogenous participation model predicts the opposite. The result follows directly from the conditional analysis performed above. Under market technology shocks the exogenous participation model over-predicts the surge in unemployment volatility. In addition, it overlooks its fall, conditionally on home technology shocks. These facts explain the difference in the policy evaluation of unconditional moments across the two versions of the model.

6 The role of the search cost

As underlined in the introduction, a common feature of other models that have recently introduced the endogenous participation margin is the assumption of high search cost (e.g. Ebell (2008) and Galí (2010)). However, the interaction between the marginal rate of substitution, matching frictions and nominal rigidities crucially depends on the cost of search, Γ and so does the participation decision. Hence, we consider, alongside the baseline calibration, an alternative one with $\Gamma = 0.99$, implying that unemployed workers spend all their time searching for a job and therefore cannot contribute to home production.[13]

The search cost shapes the equilibrium relation between home production and participation when frictions are introduced. In the extreme case of $\Gamma = 1$, home production moves one to one with participation as

$$h_t = \Xi_t(1 - N_t)^{1-\alpha_h}$$

It follows that if $\Gamma = 1$, given the participation decision, finding rates cannot affect the allocation of time. Participation in the market indeed is costly irrespectively of the employment status.

Figure 5 replicates Figure 4 under the new calibration of the search cost. When the cost of search is high, movements in the finding rate do not affect home production that is therefore always constant in the model with exogenous participation. Hence, under flexible prices a constant home production level, as in the case of the frictionless model, is achieved without the need to move participation. When prices are flexible and the cost of search is high the participation margin does not matter. When prices are sticky, the desired level of home production in the frictionless model increases thus inducing a decline in participation after a market TFP shock. Still, participation moves less than under our baseline calibration since the higher finding rate does not decrease home production.

To sum up, when the search cost is high, participation always moves less, because finding rates play no role and the only driving force is price stickiness. Intuitively, by introducing endogenous participation alongside the assumption of high search cost, we are reducing by construction the role played by the newly introduced margin.

We conclude this section by repeating our previous monetary policy experiment under the assumption of high search cost (Table 8). For the sake of concision, we restrict our attention to a few moments, the volatility of employment, unemployment and participation rates. Also, we only focus on market technology shocks. However, our conclusions are general and carry over to other moments and shocks. Differently from the results under the baseline calibration, now the introduction of endogenous participation only marginally reduces the volatility of employment and unemployment rate, and this is true under both monetary policy rules. It follows that the exogenous participation model over-predicts the impact of the policy change on the employment rate, but by much less than under the baseline calibration. This is no surprise since under this calibration the participation margin is less relevant compared to our benchmark.

Overall, there are two main conclusions to our experiment. First, neglecting participation leads to an incorrect assessment of the impact on macro variables of policy. Second, the role of the participation margin is decreasing in households' search cost. The latter conclusion strengthens the former one. In fact, the exogenous and the endogenous participation margins

[13] The calibration strategy creates a link between the worker's bargaining power and the cost of search. To maintain the same strategy, we consistently set $1 - \eta = 0.95$. For example, this calibration is close to the one considered by Galí (2010). To meet the Hosios (1990) condition we also change the matching function parameter to $\gamma = 0.95$. Keeping $\gamma = 0.6$ and $1 - \eta = 0.6$, as in the baseline calibration, does not change the results.

behave similarly only for an implausibly high value of households' search costs, i.e. when it is assumed that moving from non-participation to unemployment implies a loss in home production which is as large as the one suffered by members moving from non-participation to employment. But this is at odds with the survey evidence.

7 Conclusions

We introduced endogenous participation in an otherwise standard New Keynesian model with matching frictions. We used this laboratory economy to study how the introduction of the participation margin changes the way shocks are transmitted to the economy compared to two other cases: a frictionless labor market with endogenous participation; and matching frictions with exogenous participation. In particular, we showed that switching from a flexible to a strict inflation targeting regime has remarkably different implications on second moments once the participation margin is introduced. It increases employment and unemployment rate volatilities, conditionally on a TFP shock. However, the introduction of endogenous participation dampens such a surge in volatility compared to a model with (constant) exogenous participation. The same switch in monetary policy decreases the volatility of employment and unemployment rate conditionally on a home productivity or a preference shock, while it does not when participation is exogenous. Finally, once all shocks are considered, a policy of strict inflation targeting decreases the volatility of employment, unemployment rate and employment rate in our model. The opposite obtains if participation is exogenous and constant.

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Appendix A Value of Employment

Let us rewrite utility recursively:

$$U_t = Z_t \log(C_t) + \phi \frac{[\Xi_t(1 - E_t - \Gamma(N_t - E_t))]^{(1-\alpha_h)(1+\nu)}}{1+\nu} + \beta E_t \{U_{t+1}\} \tag{34}$$

Compute $\frac{\partial U_t}{\partial E_t}$ taking into account (3), (5) and (7):

$$\frac{\partial U_t}{\partial E_t} = \frac{Z_t}{C_t}\left[\frac{W_t}{P_t} - b\right] - \phi h_t^{\nu}(1-\Gamma)\Xi_t(1-\alpha_h)h_t^{-\frac{\alpha_h}{1-\alpha_h}} + \beta E_t\left\{\frac{\partial U_{t+1}}{\partial E_t}\right\} \tag{35}$$

Note that:

$$\begin{aligned} \frac{\partial U_{t+1}}{\partial E_t} &= (1-\rho)(1-f_{t+1})\left[\frac{Z_{t+1}}{C_{t+1}}\left(\frac{W_{t+1}}{P_{t+1}} - b\right) - \phi h_{t+1}^{\nu - \frac{\alpha_h}{1-\alpha_h}}(1-\Gamma)\Xi_{t+1}(1-\alpha_h) + \beta E_{t+1}\left\{\frac{\partial U_{t+2}}{\partial E_{t+1}}\right\}\right] \\ &= (1-\rho)(1-f_{t+1})\frac{\partial U_{t+1}}{\partial E_{t+1}} \end{aligned} \tag{36}$$

Therefore, we can rewrite (35) as:

$$\frac{\partial U_t}{\partial E_t} = \frac{Z_t}{C_t}\left[\frac{W_t}{P_t} - b\right] - \phi h_t^{\nu}(1-\Gamma)\Xi_t(1-\alpha_h)h_t^{-\frac{\alpha_h}{1-\alpha_h}} + E_t\left\{(1-\rho)(1-f_{t+1})\beta\frac{\partial U_{t+1}}{\partial E_{t+1}}\right\} \tag{37}$$

Let $V_t^w \equiv \frac{\partial U_t}{\partial E_t}/U_{c,t} = \frac{\partial U_t}{\partial E_t}\frac{C_t}{Z_t}$ be the surplus from employment in terms of current consumption of the final good. Then,

$$V_t^w = \frac{W_t}{P_t} - b - \phi h_t^{\nu}(1-\Gamma)\frac{C_t}{Z_t}\Xi_t(1-\alpha_h)h_t^{-\frac{\alpha_h}{1-\alpha_h}} + E_t\left\{(1-\rho)(1-f_{t+1})\beta\frac{C_t}{C_{t+1}}\frac{Z_{t+1}}{Z_t}V_{t+1}^w\right\} \tag{38}$$

that coincides with equation (19) in the text.

Appendix B Alternative Models

In the paper we have often compared the results of our model to those obtained under two alternative specifications: a model with endogenous participation but no matching frictions (No Frictions); and a model with matching frictions but exogenous participation (Exogenous). Below we lay down both models.

B.1 NO FRICTIONS

All family members participating in the labor market are employed ($N_t = E_t$) due to the lack of matching frictions. Thus, the representative households chooses $\{C_t, N_t, D_t\}$ so as to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[Z_t \log(C_t) + \phi \frac{h_t^{1+\nu}}{1+\nu} \right] \tag{39}$$

subject to:

$$P_t C_t + R_t^{-1} D_t \leq D_{t-1} + W_t N_t + T_t \tag{40}$$

and where $h_t = [\xi_t (1 - N_t]^{1-\alpha_h}$. Optimization implies the same Euler equation (9) plus a labor supply equation that replace the participation condition (10):

$$\frac{\phi h_t^{\nu} C_t}{Z_t} \xi_t (1 - \alpha_h) h_t^{-\frac{\alpha_h}{1-\alpha_h}} = \frac{W_t}{P_t} \tag{41}$$

In the intermediate good sector we keep the one worker - one firm modeling assumption used when matching frictions are in place. Thus, there are as many firms as participating workers, i.e. $X_t = N_t A_t$. The profit maximization problem gives the labor demand equation:

$$\frac{P_t^x}{P_t} A_t = \frac{W_t}{P_t} \tag{42}$$

The final good sector is the same as in the main model, thus (24) is the optimal pricing rule under flexible prices while we have the NKPC (27) when prices are sticky.

Finally, the aggregate resource constraint (29) simplifies to $Y_t = C_t$.

B.2 EXOGENOUS

The only difference between this model and the one developed in the paper is that participation is exogenous and constantly equal to the steady state value of participation in the endogenous model, i.e. $N = 0.6394$ substitutes the participation condition (10).

Appendix C Hosios Condition

In this section we show that in our model with endogenous participation the condition under which the Nash bargaining sustains the efficient allocation coincides with the one derived by Hosios (1990), i.e. $\eta = 1 - \gamma$.

C.1 THE PLANNER'S PROBLEM

The planner faces the following constraints:

$$E_t = (1-\rho)(1-f_t)E_{t-1} + f_t N_t \tag{43}$$

$$S_t = N_t - (1-\rho)E_{t-1} \tag{44}$$

$$N_t = E_t + U_t \tag{45}$$

$$M_t = \omega V_t^{1-\gamma} S_t^{\gamma} \tag{46}$$

$$(A_t E_t)^{1-\alpha} = C_t + \kappa V_t \tag{47}$$

and uses the definitions

$$f_t = \omega \theta_t^{1-\gamma} \tag{48}$$

$$h_t = [\xi_t(1 - E_t - \Gamma U_t)]^{1-\alpha_h} \tag{49}$$

$$\theta_t = \frac{V_t}{S_t} \tag{50}$$

We can use (45) and (46) to eliminate N_t and M_t. Thus, the planner chooses $\{C_t, E_t, U_t, V_t, S_t, f_t\}$ in order to maximize:

$$E_0 \sum_{t=0}^{\infty} \left[Z_t \log(C_t) + \frac{\phi[\xi_t(1 - E_t - \Gamma U_t)]^{(1-\alpha_h)(1+\nu)}}{1+\nu} \right] \tag{51}$$

subject to:

$$E_t = (1-\rho)(1-f_t)E_{t-1} + f_t(E_t + U_t) \tag{52}$$

$$f_t = \omega \left(\frac{V_t}{S_t} \right)^{1-\gamma} \tag{53}$$

$$S_t = E_t + U_t - (1-\rho)E_{t-1} \tag{54}$$

$$(A_t E_t)^{1-\alpha} = C_t + \kappa V_t \tag{55}$$

Let $\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}$ be the Lagrange multipliers of the 4 constraints. The first order conditions are given by:

$$\frac{Z_t}{C_t} = \lambda_{4,t} \tag{56}$$

$$\begin{aligned} -\, \Xi_t(1-\alpha_h)\phi h_t^{\nu - \frac{\alpha_h}{1-\alpha_h}} - \lambda_{1,t}(1-f_t) + \beta(1-\rho)E_t\{\lambda_{1,t+1}(1-f_{t+1})\} + \\ \lambda_{3,t} - \beta(1-\rho)E_t\{\lambda_{3,t+1}\} + \lambda_{4,t}(1-\alpha)(A_t E_t)^{-\alpha} A_t = 0 \end{aligned} \tag{57}$$

$$-\Gamma \Xi_t(1-\alpha_h)\phi h_t^{\nu - \frac{\alpha_h}{1-\alpha_h}} + \lambda_{1,t} f_t + \lambda_{3,t} = 0 \tag{58}$$

$$(1-\gamma)\omega V_t^{-\gamma} S_t^{\gamma-1} \lambda_{2,t} - \kappa \lambda_{4,t} = 0 \tag{59}$$

$$-(1-\gamma)\lambda_{2,t}\omega V_t^{1-\gamma} S_t^{\gamma-2} - \lambda_{3,t} = 0 \tag{60}$$

$$\lambda_{1,t} S_t - \lambda_{2,t} = 0 \tag{61}$$

First order conditions simplify to:

$$mrs_t = \frac{\gamma}{1-\gamma} \frac{\kappa}{\Gamma} \theta_t \tag{62}$$

$$\Omega_t^* + \frac{\kappa \theta_t^\gamma}{\omega} = mrpl_t - mrs_t + (1-\rho)E_{t+1}Q_{t,t+1} \left\{ \Omega_{t+1}^* + \frac{\kappa \theta_{t+1}^\gamma}{\omega} \right\} \tag{63}$$

where

- $$mrs_t = \frac{C_t}{Z_t} \phi h_t^\nu \Xi_t (1-\alpha_h) h_t^{\frac{-\alpha_h}{1-\alpha_h}} \tag{64}$$
- $$Q_{t,t+1} = \beta \frac{C_t Z_{t+1}}{C_{t+1} Z_t} \tag{65}$$

- $$mrpl_t = (1-\alpha_h)A_t^{1-\alpha_h}E_t^{-\alpha} \tag{66}$$

- $$\Omega_t^* \equiv \frac{(1-f_t)}{f_t}\left[\frac{\phi\Gamma h_t^{\nu}C_t}{Z_t}\Xi_t(1-\alpha_h)h_t^{-\frac{\alpha_h}{1-\alpha_h}}\right] = \Gamma\frac{(1-f_t)}{f_t}mrs_t \tag{67}$$

It is convenient to note that (63) can be rewritten as

$$S_t^{H,*} + S_t^{F,*} = mrpl_t - mrs_t \tag{68}$$

where

$$S_t^{H,*} = \Omega_t^* - (1-\rho)E_tQ_{t,t+1}\left\{\Omega_{t+1}^*\right\} \tag{69}$$

$$S_t^{F,*} = \frac{\kappa\theta_t^{\gamma}}{\omega} - (1-\rho)E_tQ_{t,t+1}\left\{\frac{\kappa\theta_{t+1}^{\gamma}}{\omega}\right\} \tag{70}$$

may be interpreted as the flow values of a match for the household and firms respectively, at the efficient equilibrium. Finally, (67), (68), (69) and (62) imply

$$S_t^{F,*} = (1-\gamma)mrpl_t - (1-\gamma)(1-\Gamma)mrs_t - \gamma(1-\rho)E_t\{Q_{t,t+1}\kappa\theta_{t+1}\} \tag{71}$$

C.2 DECENTRALIZING THE EFFICIENT ALLOCATION

Prerequisites for efficiency in the market allocation are: flexible prices in both the final and intermediate sector; a production subsidy in the final sector eliminating the distortion due to monopolistic competition; no unemployment benefit (i.e. $b = 0$). Assuming that those hold, we check under which conditions Nash bargaining sustains the efficient allocation.

Under those assumptions we have:

$$\frac{P_t^x}{P_t} = (1-\alpha)x_t^{-\alpha} = (1-\alpha)(A_tE_t)^{-\alpha} = \frac{mrpl_t}{A_t} \tag{72}$$

Also, the lack of unemployment benefit implies:

$$\Omega_t^* = \Omega_t \tag{73}$$

First, the job creation condition and the participation equation can be combined to form:

$$\Omega_t = mrpl_t - mrs_t - \frac{\kappa}{\omega}\theta_t^{\gamma} + (1-\rho)E_t\left\{Q_{t,t+1}\frac{\kappa}{\omega}\theta^{\gamma}\right\} + (1-\rho)E_t\left\{Q_{t,t+1}\Omega_{t+1}\right\} \tag{74}$$

$$S_t^H + S_t^F = mrpl_t - mrs_t \tag{75}$$

Hence, the condition (68) is satisfied independently of the wage bargaining process. Combining job creation, the value of employment, the free entry and the Nash sharing rule we obtain

$$S_t^F = \eta mrpl_t - \eta(1-\Gamma)mrs_t - (1-\eta)(1-\rho)E_t\{Q_{t,t+1}\kappa\theta_{t+1}\} \tag{76}$$

By comparing (76) and (71) we can see that the two equations are equivalent if

$$\eta = 1-\gamma \tag{77}$$

i.e. the Hosios condition holds even for our model.

Appendix D Figures

Figure 1
Impulse responses to a market production TFP shock

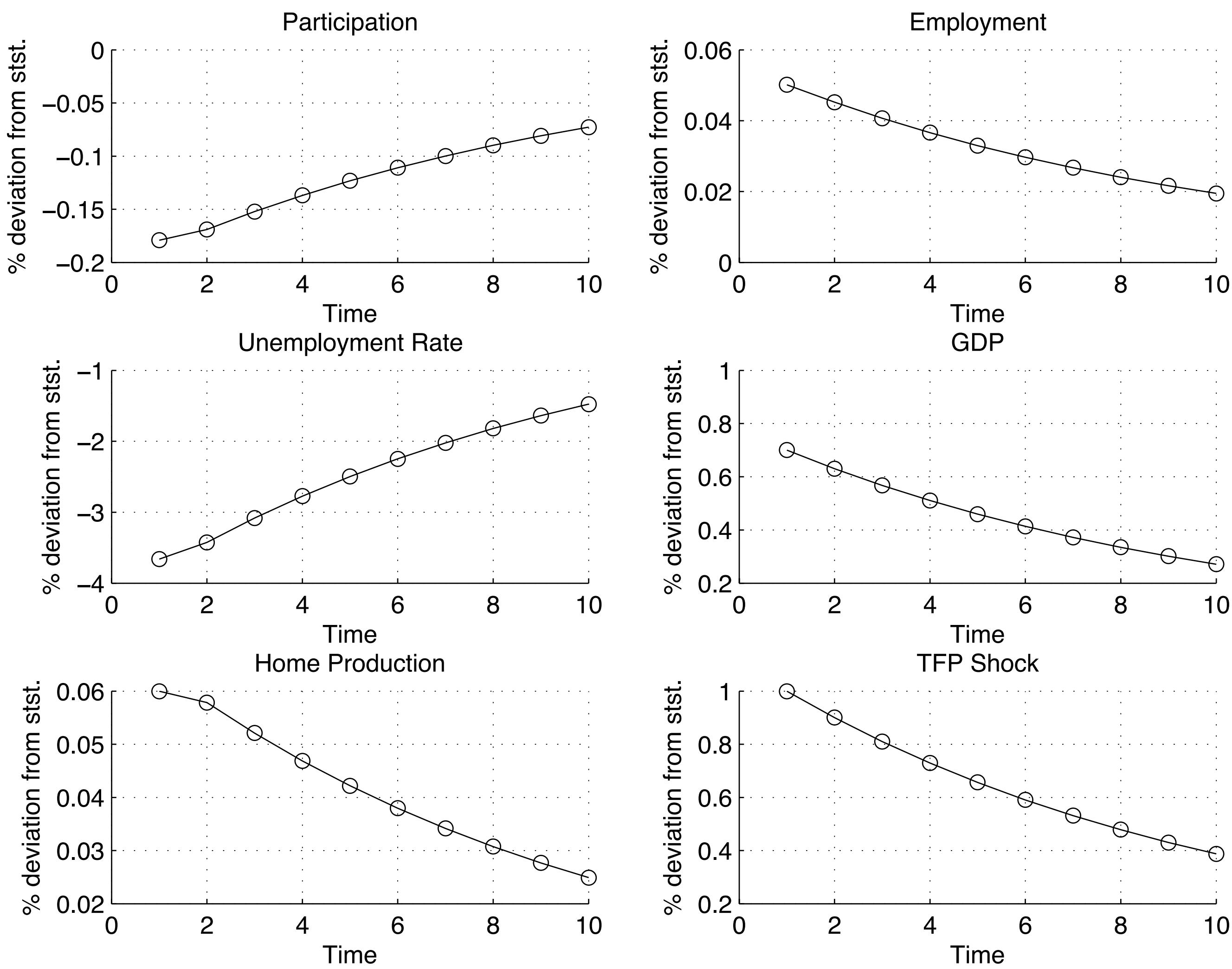


Figure 2
Impulse responses to a preference shock

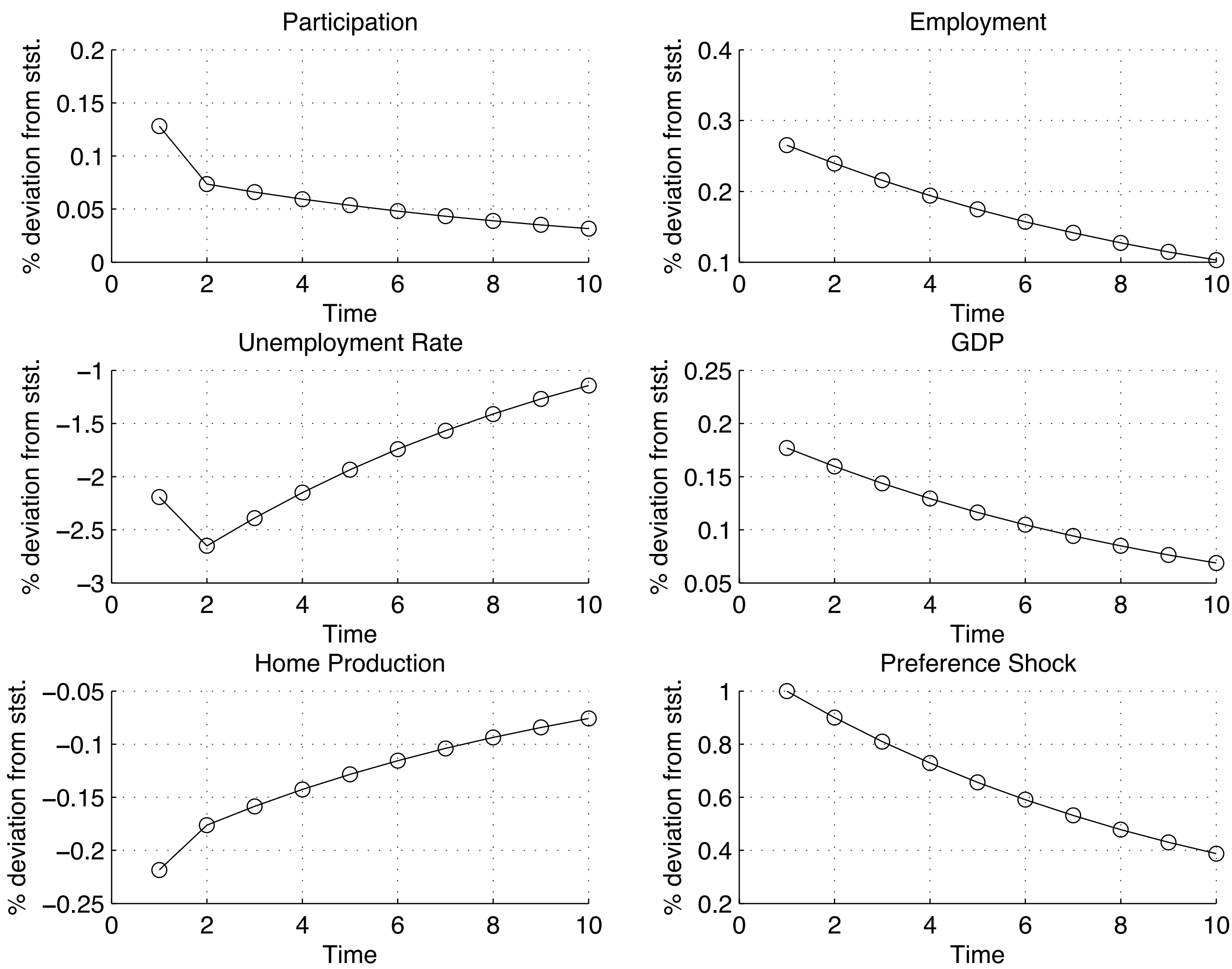


Figure 3
Impulse responses to a home productivity shock

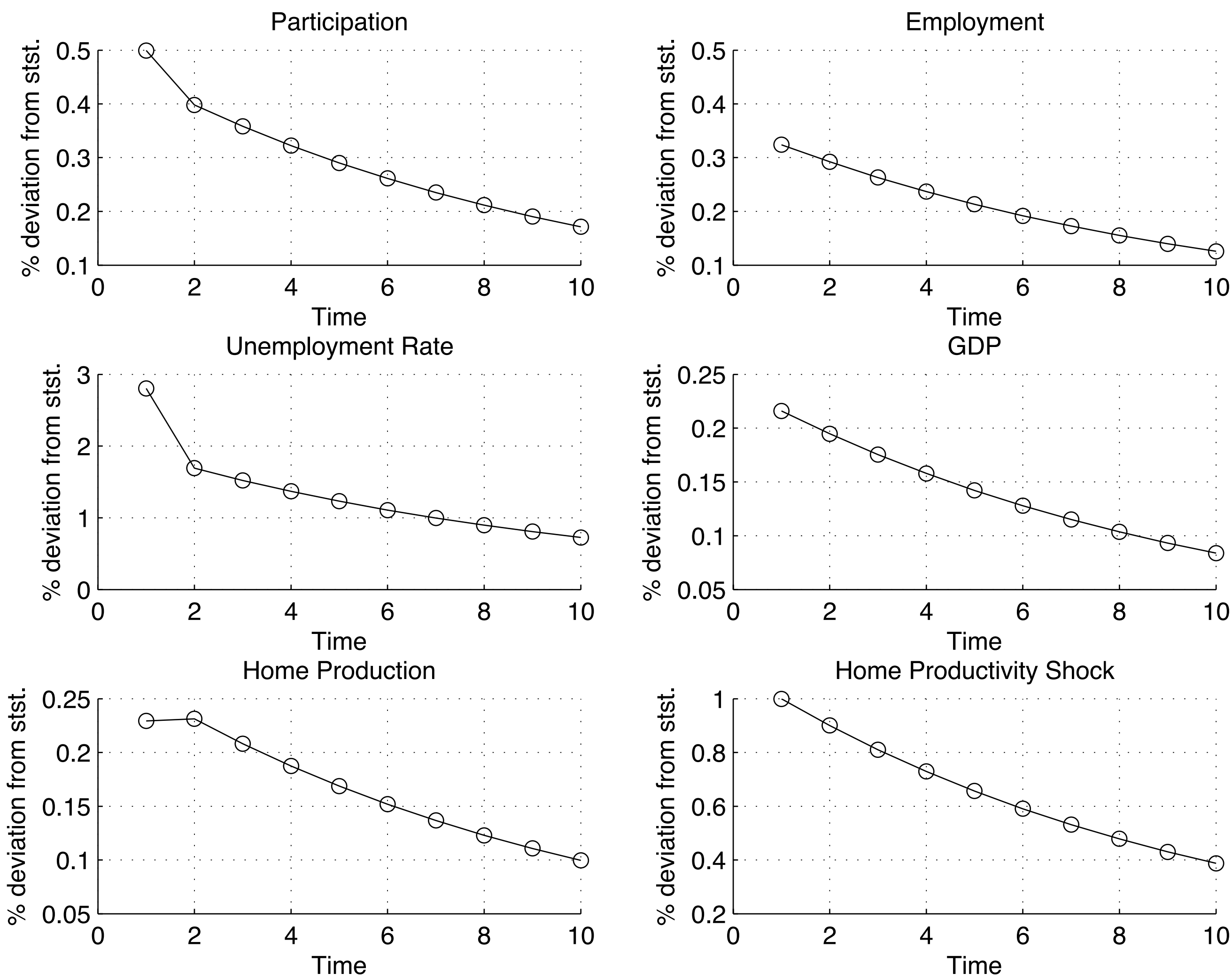


Figure 4
Impulse responses to a market production TFP shock.

Comparison between endogenous participation with matching frictions (CG), endogenous participation without matching frictions (No Frictions) and exogenous participation with matching frictions (Exogenous) under both Flexible and Sticky Prices. Baseline calibration $\Gamma = 0.44$.

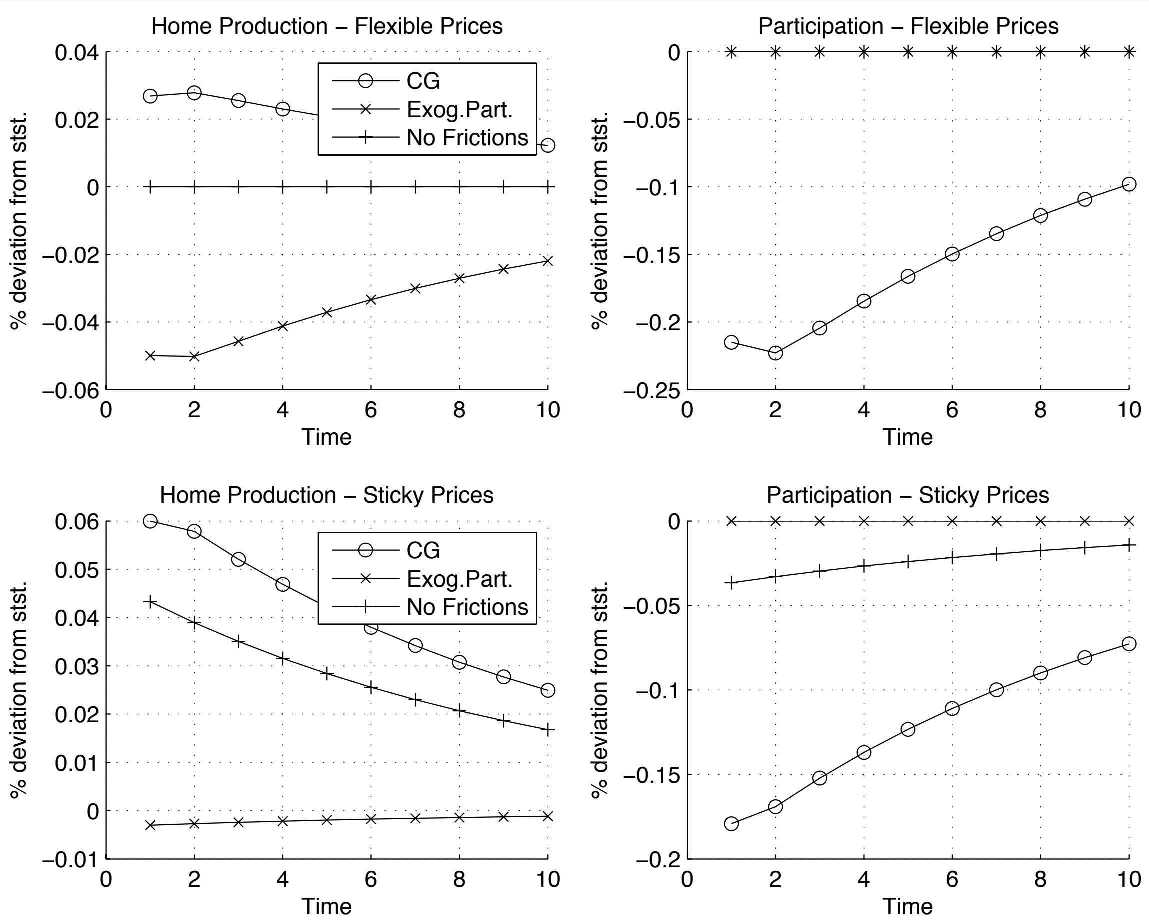
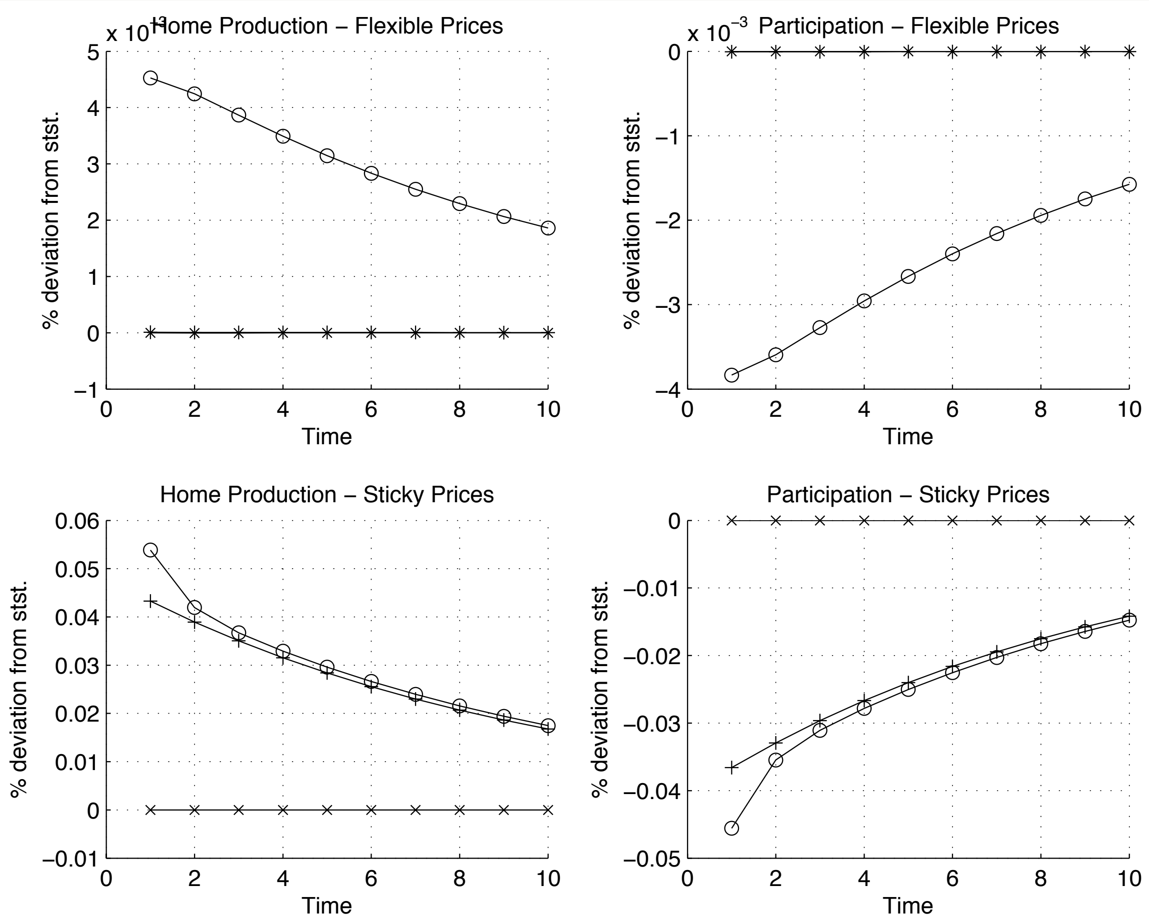


Figure 5
Impulse responses to a market production TFP shock.

Comparison between endogenous participation with matching frictions (CG), endogenous participation without matching frictions (No Frictions) and exogenous participation with matching frictions (Exogenous) under both Flexible and Sticky Prices. High search cost $\Gamma = 0.99$.



Appendix E Tables

Table 1
Time allocated to home production (minutes per day). Data are from the American Time Use Survey (ATUS) and were collected over the period 2003-2009.

Status	2003-2009	2003-2006
Employed	119	118
Unemployed	154	154
Not in labor force	178	183
Search cost Γ	0.41	0.44

Table 2
Selected unconditional moments in the data, the endogenous and the exogenous participation models. Employment and unemployment rate volatility are relative to output. Volatilities are expressed in percentage standard deviations. Both models have been calibrated so as to give the best possible fit for the first 4 moments.

Unconditional Moments	Data	Endogenous	Exogenous
Output volatility	1.53	1.43	1.56
Unemployment rate volatility	7.40	7.36	7.55
Employment volatility	0.63	0.67	0.47
Correlation of Unemployment rate with Output	-0.85	-0.75	-1
Participation rate volatility	0.20	0.24	-
Correlation of Participation with Output	0.42	0.56	-

Calibrated Parameters			
st.dev. market TFP		0.0070	0.0074
st.dev. home TFP		0.0037	0.0070
st.dev. preference shock		0.0147	0
$corr_{A,AH}$		0.9474	1

Table 3
Selected unconditional moments in the data (first column), in the endogenous participation model with only market and home TFP shock (second column), in the endogenous participation model with only market and preference shock (third column), in the endogenous participation model with all shocks but restricting to zero the cross-correlation. In each case the model has been calibrated so as to give the best possible fit for the 4 moments.

Unconditional Moments	Data	(A_t, ξ_t)	(A_t, Z_t)	(A_t, ξ_t, Z_t)
Output volatility	1.53	1.45	1.79	1.80
Unemployment rate volatility	7.40	7.38	7.37	7.39
Employment volatility	0.63	0.95	0.50	0.53
Correlation of Unemployment rate with Output	-0.85	0.072	-0.89	-0.86
Calibrated Parameters				
st.dev. market TFP		0.0070	0.0105	0.0105
st.dev. home TFP		0.0184	0	0.0037
st.dev. preference shock		0	0.0147	0.0147
$corr_{A,AH}$		0	0	0

Table 4
Selected moments in the endogenous and exogenous participation models, conditionally on market technology shocks. Employment and unemployment rate volatility are relative to output. Volatilities are expressed in percentage standard deviations. The table reports the value of moments under strict inflation targeting. In parentheses are the values for an inflation coefficient equal to 1.5 in the Taylor rule.

Moments Conditional on MTFP Shocks	Endogenous	Exogenous
Output volatility	1.24 (1.12)	1.17 (1.08)
Unemployment rate volatility	8.33 (5.40)	2.07 (0.12)
Employment volatility	0.20 (0.07)	0.13 (0.008)
Employment rate volatility	0.52 (0.34)	0.13 (0.008)
Participation rate volatility	0.32 (0.27)	0 (0)
Correlation of Participation with Output	-0.99 (-0.99)	0 (0)
Correlation of Unemployment rate with Output	-1 (-1)	-1 (-1)

Table 5

Selected moments in the endogenous and exogenous participation models, conditionally on home technology shocks. Employment and unemployment rate volatility are relative to output. Volatilities are expressed in percentage standard deviations. The table reports the value of moments under strict inflation targeting. In parentheses are the values for an inflation coefficient equal to 1.5 in the Taylor rule.

Moments Conditional on HTFP Shocks	Endogenous	Exogenous
Output volatility	0.20 (0.18)	0.52 (0.49)
Unemployment rate volatility	7.21 (9.63)	23.97 (23.97)
Employment volatility	1.5 (1.5)	1.5 (1.5)
Employment rate volatility	0.45 (0.60)	1.5 (1.5)
Participation rate volatility	1.89 (2.10)	0 (0)
Correlation of Participation with Output	0.99 (1)	0 (0)
Correlation of Unemployment rate with Output	0.84 (0.98)	-1 (-1)

Table 6

Selected moments in the endogenous and exogenous participation models, conditionally on preference shocks. Employment and unemployment rate volatility are relative to output. Volatilities are expressed in percentage standard deviations. The table reports the value of moments under strict inflation targeting. In parentheses are the values for an inflation coefficient equal to 1.5 in the Taylor rule.

Moments Conditional on Preference Shocks	Endogenous	Exogenous
Output volatility	0.29 (0.60)	0 (0)
Unemployment rate volatility	7.56 (15.91)	23.97 (23.97)
Employment volatility	1.5 (1.5)	1.5 (1.5)
Employment rate volatility	0.47 (1.00)	1.5 (1.5)
Participation rate volatility	1.92 (0.52)	0 (0)
Correlation of Participation with Output	0.99 (0.98)	0 (0)
Correlation of Unemployment rate with Output	0.85 (-0.99)	-1 (-1)

Table 7
Selected unconditional moments in the endogenous and exogenous participation models. Employment and unemployment rate volatility are relative to output. Volatilities are expressed in percentage standard deviations. The table reports the value of moments under strict inflation targeting. In parentheses are the values for an inflation coefficient equal to 1.5 in the Taylor rule.

Unconditional Moments	Endogenous	Exogenous
Output volatility	1.46 (1.43)	1.69 (1.56)
Unemployment rate volatility	6.52 (7.35)	8.79 (7.57)
Employment volatility	0.48 (0.67)	0.55 (0.47)
Employment rate volatility	0.41 (0.46)	0.55 (0.47)
Participation rate volatility	0.40 (0.24)	0 (0)
Correlation of Participation with Output	0.13 (0.56)	0 (0)
Correlation of Unemployment rate with Output	-0.90 (-0.76)	-1 (-1)

Table 8
Selected moments in the endogenous and exogenous participation models, conditionally on market technology shocks. Employment and unemployment rate volatility are relative to output. Volatilities are expressed in percentage standard deviations. Here we depart from the baseline calibration and assume a high households' search cost, i.e. $\Gamma = 0.99$. The table reports the value of moments under strict inflation targeting. In parentheses are the values for an inflation coefficient equal to 1.5 in the Taylor rule.

Moments Conditional on MTFP Shocks	Endogenous	Exogenous
	Values	Values
Employment rate volatility	0.1100 (0.0699)	0.1107 (0.0726)
Unemployment rate volatility	1.7682 (1.1606)	1.7580 (1.0689)
Participation rate volatility	0.0048 (0.0583)	0 (0)

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