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Inflation and Endogenous Growth in Underground Economies

The wiiw Balkan Observatory

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Inflation and Endogenous Growth in Underground Economies*

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Abstract

The paper examines the effect of inflation on the growth rate in economies with underground, or "non-market", sectors. The model incorporates a non-market good into an endogenous growth cash-in-advance economy with human capital. Taxes on labor and capital induce substitution into the non-market sector which avoids such taxes. However the non-market sector uses only cash for exchange and cannot avoid the inflation tax, while the market sector allows costly credit use. We estimate a MIMIC model for latent underground economy using monthly data for Bulgaria, Croatia and Romania. Furthermore, we estimate a dynamic structural equation model and investigate short-run effects of the underground economy on output growth and test for Granger causality and long-run cointegrating relationships using bivariate Granger-causality tests and Johansen's maximum likelihood technique. The result indicate different shares of underground economies across the three countries and a positive long-run effect of underground economy on output growth.

JEL Classification: *E31, E13, O42, C31, C51, C52*
Keywords: *Shadow economy, endogenous growth, dynamic structural equation modelling, latent variables.*

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1 Introduction

Schneider (2000, 2003) and Schneider and Enste (2000) document the significant size of the shadow economies internationally, and how the sizes differ. For example, Schneider (2000) reports that the shadow output equals some 39% of the actual magnitude of reported GDP in developing countries, 23% in transition countries and 14% in OECD countries. Meanwhile the labor force, as a percent of the official labor force is about 50% in developing and transition countries and 17% in OECD countries. This makes potential tax collection across regions quite different. A country with a large labor supply in the shadow economy may find a tax on labor income inferior to alternatives such as use of the so-called inflation tax.

Using the inflation tax as a means of revenue collection can adversely affect growth. There is a substantial body of evidence now of such a negative growth effect especially in large panel studies. Moreover there is evidence that the magnitude of the negative effect of inflation on growth can differ among developing and industrial countries. For example Gillman, Harris and Matyas (2004) find that inflation has a larger marginal negative effect in a OECD sample as compared to an APEC sample at all levels of the inflation rate. Harris and Gillman (2003) control for standard measures of financial development and find further evidence supporting a more significant decrease in the growth rate, from an inflation rate increase, the more developed is the economy.

The literature on inflation and growth does not consider the possible role that the differing size of the shadow economies may play in explaining differences across regions. And the literature on the shadow economies tends not to investigate how the shadow economies affect the ability of countries to raise taxes and to achieve growth. But there is the fundamental issue that reliance on the inflation tax in countries with big shadow economies can significantly lower the growth rate. Since the inflation tax is deemed a relatively inefficient tax, such findings if supported empirically would suggest that there may be better ways to raise such taxes.

The contribution of this paper is to present a dynamic general equilibrium

analysis of how inflation can effect growth in economies with shadow sectors. Then the analysis is backed up with extensive econometric testing using transition country data. This enables us to reach some conclusions relative to how inflation can be viewed relative to other tax policies across regions in which the shadow sector plays varying roles.

The departure point for the analysis is the model of Parente, Rogerson and Wright (1999,2000). This is an exogenous growth economy without money or any taxes, but with a role for development through a differing cost of capital depending on development level. We take this model and set it in an endogenous growth monetary framework with a variety of taxes. This provides a rigorous setting for determining the role of the shadow sector.[1]

As in Piggott, J. and Whalley, J. (1996), Parente, Rogerson and Wright (1999, 2000) introduce a non-market Beckerian (1965)-like sector with a specific production function. They start with Parente and Prescott (1994)-type distortions that increase the cost of new physical capital above one, whereby there is a higher such cost of new investment for developing versus developed countries. They make the case that introduction of the non-market sector makes it easier to to match the empirical facts regarding income inequality along the exogenous growth path. In particular a smaller magnitude of the distortion to the cost of physical capital investment is required, making the model more plausible in reconciling income levels between developed and developing countries.

Using a model that can explain income differentials on the basis of a non-market sector is a natural basis for investigating the nature of shadow economies across regions that are characterized by different income levels. However for an examination of how different taxes including inflation affect the shadow economy, the market sector needs to include various taxes while the non-market sector needs to include some type of ability to avoid such taxes. The problem is that for example with a tax on labor and capital in the market sector, and no such taxes in non-market sector, the agent would want to supply all labor and capital to the non-market sector. A way to

[1]For alternative modelling approaches see for example Loayza (2003), Azuma and Grossman (2002), or Dabla-Norris and Feltenstein (2003).

model taxes and their evasion, while having the agent willing to supply labor to both market and non-market sectors, is to model the evasion activity explicitly.[2]

Modeling corruption explicitly requires a further change for Parente, Rogerson and Wright (1999,2000). There the non-market sector cannot add its goods output to the total income of the economy that is in turn divided into consumption and investment. This creates some divergences in the return on capital from that found in a standard economy. By explicitly modelling the corruption, the output of the non-market sector can be added to the output of the market sector to give the total output that is then divided between consumption and investment. This is natural since the market and non-market goods are substitutes. And this symmetric treatment of the sectors eliminates the discrepancies in the equilibrium conditions that exist in Parente, Rogerson and Wright, making for a set of standard equilibrium conditions regarding the allocation of labor and capital.

An "Islands"government (as in the Isle of Mann, the Bahamas, and Switzerland - the island in the EU- for capital taxes, and Sicily for labor taxes) is postulated that keeps the non-market sector from having to pay taxes but that charges a competitive based fee for this service, depending on the tax being avoided. Then since the agent is acting in part as the Islands government, the profit of the tax evasion is transferred in a lump sum back to the consumer. This production of this corruption is modelled using only labor, and this labor activity takes away from the labor available for human capital investment. The balanced-path growth rate of the economy can thereby fall as the corruption activity is larger.

Introducing money into the economy requires taking a stand on whether the market sector and non-market sectors differ in there use of money versus credit. Credit use typically leaves a "paper trail"that can be incriminating and so would be avoided (Dabla-Norris and Feltenstein 2003). The assumption here that accommodates the notion that the shadow economy is a cash-

[2]See Einarsson and Marquis (2001) for an endogenous growth, non-monetary, economy with a non-market sector and taxes in the market sector. The study the welfare effects of distortionary taxation on labor, capital, and the market good relative to the exogenous growth economy.

only economy so as to avoid the incriminating "paper trail"is that only fiat money (cash) is used in the shadow economy, while the agent in buying the market goods can use any combination of cash and credit.

The endogenous growth setting with human capital is necessary in order to model a way in which inflation can lower the return to human capital, and thus the growth rate. This is because the growth rate depends on the return to capital, physical and human capital returns are equal in equilibrium, and both returns are thereby lowered by increases in the inflation rate. An important difference from Parente, Rogerson and Wright (1999,2000) is that in going to an endogenous growth model with human capital, instead of assuming exogenous growth, differences in the cost of investment as in Parente and Prescott (1994) can now be modelled as differences in the productivity parameter of the human capital investment function. The switch to introducing differences in the cost of new capital in terms of the human capital accumulation process instead of the physical accumulation process follows the respected literature of Schultz (1964) and Lucas (2002) that emphasizes an increased return on human capital can explain the transition from developing to developed economies.

With these dimensions of the problem endogenized - a non-market sector, taxes on the market good, inflation as a tax on money use, competitive corruption, human capital, and endogenous growth - we study how the inflation effect on growth differs between developed and developing economies based on the nature of the explicit taxes, the size of their non-market sectors, the taste for corruption, and the degree of cash across these sectors.

2 The endogenous growth monetary economy

The model is an extension of Gillman and Kejak's (2004a) monetary economy with endogenous growth. The non-market good is combined with the market good in a constant elasticity of substitution fashion. Let the market consumption good be denoted at time t by c_{mt}, and the non-market good by c_{nt}. The aggregate consumption good is denoted by c_t, and with ν and ε utility function parameters, it is given to the representative agent as the

following combination of market and non-market goods

$$c_t = \left[\nu c_{mt}^{\varepsilon} + (1-\nu)\, c_{nt}^{\varepsilon}\right]^{1/\varepsilon}. \tag{1}$$

2.1 The Representative Consumer Problem

The consumer has a preference for both the market and the non-market goods, as well as leisure, denoted by x_t. With parameters μ and ε determining the relative preference for the market versus non-market good, the current period utility function is given by

$$u_t = \ln\left(\left[\nu c_{mt}^{\varepsilon} + (1-\nu)\, c_{nt}^{\varepsilon}\right]^{1/\varepsilon}\right) + \varrho \ln x_t \tag{2}$$

2.1.1 Capital and time allocation, and human capital investment

The consumer rents labor and capital for use in the production functions of the market and non-market goods. Let the shares of the physical capital stock in each sector be denoted by s_{mt}, and s_{nt} where

$$s_{mt} + s_{nt} = 1. \tag{3}$$

The agent accumulates physical and human capital, denoted by k_t and h_t, using household production of the human capital investment, denoted by $\dot{h}_t$, with a constant returns to scale function in only effective labor, as in Lucas (1988), where the effective labor is the raw labor multiplied by the human capital stock. The consumer also household produces a credit service, assumed also to use only effective labor.

Let the raw labor allocations to the same sectors be given by l_{mt}, l_{nt}, and l_{ht}, with the labor allocated to the credit (exchange finance) sector and to leisure denoted by l_{dt} and x_t, respectively. There is also labor time used in the corruption activity of the Islands government, denoted by l_{ct}, whereby

$$l_{mt} + l_{nt} + l_{ht} + l_{Ft} + x_t + l_{ct} = 1. \tag{4}$$

The human capital investment production function with $A_h > 0$ is given by

$$\dot{h}_t = A_h l_{ht} h_t - \delta_h h_t. \tag{5}$$

The consumer receives capital and labor income from working in the sectors of the market good and the non-market good, and receives labor income from working to provide the Islands corruption service; plus their are the receipts of the lump sum transfer from the government V_t and the return of profit (kickbacks) from the Islands corruption service $\Pi_{ct} P_t$. Expenditures are made on the market and non-market good plus physical, money stock, and bond investments. The resulting current income budget constraint is

$$\begin{aligned} 0 \;=\; & (1-\tau_l)\, w_t P_t l_{mt} h_t + (1-c_l)\, w_t P_t (l_{nt} + l_{ct}) h_t + \\ & (1-\tau_k)\, r_t P_t s_{mt} k_t + (1-c_k)\, r_t P_t\, (1-s_{mt})\, k_t \qquad (6) \\ & + V_t + \Pi_{ct} P_t - (1+\tau_c)\, P_t c_{mt} - (1+c_c)\, P_{nt} c_{nt} \\ & - \dot{k}_t - \delta_k k_t - \dot{m}_t - \pi_t m_t - \dot{b}_t + b_t (R_t - \pi_t). \qquad (7) \end{aligned}$$

2.1.2 Exchange

The goods output forms an input into the Becker (1965)- type household production of each of the two consumption good c_{mt} and c_{nt}. The goods used as an input for producing the output are denoted by y_{cmt} and y_{cnt} . The other input is exchange, denoted by y_{emt} and y_{ent}, which enters the production function $f_c(\cdot)$

$$c_{mt} \;=\; f_c(y_{cmt}, y_{emt}), \tag{8}$$

$$c_{nt} \;=\; f_c(y_{cnt}, y_{ent}) \tag{9}$$

The production function for the consumption good is assumed to be Leontieff, whereby the isoquant ray from the origin has a slope of one. This implies, where the relative price of the inputs is between zero and infinity, that

$$c_{mt} = y_{cmt}, \tag{10}$$

$$c_{mt} = y_{emt;} \tag{11}$$

$$c_{nt} = y_{cnt,} \tag{12}$$

$$c_{nt} = y_{ent}. \tag{13}$$

The exchange in turn is produced using two inputs: real money balances, denoted by m_t , and real credit, denoted by d_t. These inputs are perfect substitutes. Let P_t denote the nominal price of the market good, with it serving as the numeraire. Then the total exchange value is given by

$$y_{cmt} + (P_{nt}/P_t)\, y_{cnt} = m_t + d_t. \tag{14}$$

Real money balances are defined as the nominal money stock, denoted by M_t , divided by the nominal price of goods output, denoted by P_t; $m_t \equiv M_t/P_t$. The initial nominal money stock M_0 is given to the consumer. Additional money stock is transferred to the consumer exogenously in a lump sum fashion by an amount V_t. The consumer buys some fraction of the output goods with money, and the rest buys with credit. Let $a_t \in (0, 1]$ denote the fraction of output goods bought with money.[3] Then the agents demand for money is constrained to be this fraction of goods purchased. In real terms,

$$m_t = a_t y_{cmt} + (P_{nt}/P_t)\, y_{cnt}, \tag{15}$$

which by substitution from equation (10) gives a Clower (1967)-type constraint of

$$m_t = a_t c_{mt} + (P_{nt}/P_t)\, c_{nt}; \tag{16}$$

or in nominal terms,

$$M_t = a_t P_t c_{mt} + P_{nt} c_{nt}. \tag{17}$$

Credit demand is the residual fraction of output goods purchases. In real terms,

$$d_t = (1 - a_t) y_{cmt}, \tag{18}$$

or substituting in from equation (10) gives that

[3] An equilibrium with $a = 0$ does not have well-defined nominal prices.

$$d_t = (1 - a_t)c_{mt}, \tag{19}$$

where c_t can be viewed as the scale factor of a derived demand for the input.

The credit, per unit of consumption goods output, is produced with a function $f_d(\cdot)$ in a separate sector using the labor, denoted by l_{Ft}, factored by the human capital h_t, and normalized per unit of goods output

$$d_t/c_{mt} = f_d(l_{dt}h_t/c_{mt}). \tag{20}$$

In particular, it is assumed that the normalized credit output of d_t/c_{mt} is produced with the normalized effective labor in a diminishing returns fashion. With $\gamma \in (0, 1)$, and $A_F \in \mathbb{R}_{++}$ a shift parameter, the production function is given by

$$d_t/c_{mt} = A_d(l_{dt}h_t/c_{mt})^{\gamma}. \tag{21}$$

This function is homogenous of degree one in $l_{Ft}h_t$ and in c_{mt} , more easily seen when it is written as

$$d_t = A_d(l_{dt}h_t)^{\gamma}c_{mt}^{1-\gamma}. \tag{22}$$

There is a ready interpretation of equation (22). A credit card company such as American Express, in a decentralized setting, would maximize profit while taking as given how much is spent on goods for consumption. American Express would not try to change this goods expenditure but must consider it in making its optimal credit supply available to the consumer. From the consumer point of view, to increase the share of goods bought with credit, the consumer must increase the time allocation l_{dt} and faces diminishing returns to such effort. Thus making its inputs proportional to such consumption goods is a natural way to supply the credit. A similar decentralized problem is made explicit in Gillman and Kejak (2004b); this reveals that the explicit price of credit is the nominal interest rate but otherwise changes no equilibrium conditions, and it is foregone here for the sake of brevity of presentation.

Setting the credit demand equal to credit supply, from equations (19) and (22),

$$(1 - a_t) = A_F(l_{dt}h_t/c_{mt})^{\gamma}. \tag{23}$$

and substituting into equation (17) for a_t from equation (23), the money and credit constraints can be written as

$$M_t = \left[1 - A_d \left(\frac{l_{dt}h_t}{c_{mt}}\right)^{\gamma}\right] P_t c_{mt} + P_{nt}c_{nt}. \tag{24}$$

2.2 Goods Production

The output of the market and non-market goods are each produced by a representative firm using CRS technologies in capital and effective labor. With $A_m > 0$, $A_n > 0$, $\beta \in (0, 1)$, and $\alpha \in (0, 1)$, the production technologies are

$$c_{mt} = A_m \left(s_{mt}k_{\,t}\right)^{\beta} \left(l_{mt}h_t\right)^{1-\beta}, \tag{25}$$

$$c_{nt} = A_n \left(s_{nt}k_{\,t}\right)^{\alpha} \left(l_{nt}h_t\right)^{1-\alpha}. \tag{26}$$

Given the explicit taxes on goods, labor, and capital for the market good and the corruption fees on goods, labor and capital for the non-market goods, and with $p_{nt} \equiv P_{nt}/P_t$, the first-order conditions of the firms's profit maximization implies that

$$w_t = (1 - \beta) A_n \left(s_{mt}k_{\,t}\right)^{\beta} \left(l_{mt}h_t\right)^{-\beta}, \tag{27}$$

$$r_t = \beta A_n \left(s_{mt}k_{\,t}\right)^{\beta-1} \left(l_{mt}h_t\right)^{1-\beta}, \tag{28}$$

$$w_t = p_{nt} (1 - \alpha) A_n \left(s_{nt}k_{\,t}\right)^{\alpha} \left(l_{nt}h_t\right)^{-\alpha}, \tag{29}$$

$$r_t = p_{nt}\alpha A_n \left(s_{nt}k_{\,t}\right)^{\alpha-1} \left(l_{nt}h_t\right)^{1-\alpha}. \tag{30}$$

2.3 Government Budget Constraint

The agent faces proportional taxes on the labor, capital and goods in the market sector, denoted by τ_l, τ_k, and τ_c, and receives from the government a nominal lump sum transfer of the tax revenue denoted by V_t. Also the government can supply new money through open market operations in which it buys nominal bonds, denoted by B_t, and pays nominal interest on the bonds of R_t. The government budget constraint is given by

$$V_t = \tau_l w_t P_t l_{mt} h_t + \tau_k r_t P_t s_{mt} k_t + \tau_c P_t c_{mt} + \dot{M}_t + \dot{B}_t - B_t R_t. \quad (31)$$

It is assumed that the money supply grows at a constant rate of σ

$$\dot{M}_t = \sigma M_t. \quad (32)$$

In real terms, dividing equation (32) by P_t implies that the government's investment rate in real money is the supply growth rate minus the inflation-based deprecation of $\dot{P}_t/P_t \equiv \pi$

$$\dot{m}_t = (\sigma - \pi) m_t. \quad (33)$$

Defining $B_t/P_t \equiv b_t$, then $(\dot{B}_t - B_t R_t)/P_t = \dot{b}_t - b_t(R_t - \pi_t)$, and the government constraint in real terms is

$$V_t/P_t = \tau_l w_t l_{mt} h_t + \tau_k r_t s_{mt} k_t + \tau_c c_{mt} + \dot{m}_t + \pi m_t + \dot{b}_t - b_t(R_t - \pi_t). \quad (34)$$

2.4 Islands Government

The Islands government produces the corruption that is necessary to enable the consumer to avoid paying explicit labor, capital and goods taxes when supplying labor or capital to the non-market sector, or when buying the non-market good. However there is a proportional fee for the Islands corruption service, denoted by c_l, c_k, and c_c, that is levied on the labor and capital rentals to the non-market sector, and on the market good. The sum $c_l w_t P_t l_{nt} h_t + c_k r_t P_t s_{nt} k_t + c_c P_t c_{nt}$ comprises the real quantity of corruption

services, denoted by κ_t, that the Islands government supplies. This implies that

$$\kappa_t = c_l w_t l_{nt} h_t + c_k r_t s_{nt} k_t + c_c p_{nt} c_{nt} + c_l w_t l_{ct} h_t. \tag{35}$$

The quantity of the corruption service, denoted by $\kappa_t \equiv S_t/P_t$, is equal to the quantity of the non-market good. This implies that the corruption service and the non-market good are assumed to be Leontieff combined in a one-to-one fashion when the consumer buys the non-market good. And the consumer must have both the corruption and the non-market good in order to consume it.

The Islands maximizes real per period profit, denoted as Π_{ct}/P_t. The price received by the Islands government for their effort to produce corruption is denoted by p_{ct}. This makes the profit maximization problem

$$\Pi_{ct}/P_t = p_{ct}\kappa_t - w_t l_{ct}, \tag{36}$$

subject to the production function with $\omega \in (0,1)$ of

$$\kappa_t = A_c \left(l_{ct} h_t / c_{nt}\right)^{\omega}. \tag{37}$$

The production function implies that there are diminishing returns to increasing κ_t by increasing the share of labor devoted to corruption activity l_{ct}. The Islands government can solve its demand for corruption labor from its first order condition as

$$p_{ct} = \left(w_t l_{ct}\right) / \left(\omega \kappa_t\right) \equiv w_t / MP_{l_c}, \tag{38}$$

or that the relative price is the marginal factor cost w_t divided by the marginal product of labor.

2.5 Equilibrium

The consumer maximizes the following Hamiltonian with respect to c_{mt}, $c_{nt,}$ x_t, l_{mt}, l_{nt}, l_{dt}, l_{ct}, s_{mt}, m_t, b_t, k_t, and h_t :

$$
\begin{aligned}
\mathcal{H} \;=\; & e^{-\rho t}\left[\ln\left(\left[\nu c_{mt}^{\varepsilon}+(1-\nu)\,c_{nt}^{\varepsilon}\right]^{1/\varepsilon}\right)+\varrho\ln x_t\right] \\
& +\lambda_t[(1-\tau_l)\,w_t l_{mt}h_t+(1-c_l)\,w_t\,(l_{nt}+l_{ct})\,h_t+(1-\tau_k)\,r_t s_{mt}k_t \\
& +(1-c_k)\,r_t\,(1-s_{mt})\,k_t+V_t/P_t+\Pi_{ct}/P_t-(1+\tau_c)\,c_{mt}-(1+c_c)\,p_{nt}c_{nt} \\
& -\dot{k}_t-\delta_k k_t-\dot{m}_t-\pi_t m_t-\dot{b}_t+b_t(R_t-\pi_t)] \\
& +\eta_t\left[A_h\,(1-l_{mt}-l_{nt}-l_{dt}-x_t-l_{ct})\,h_t-\delta_h h_t\right] \\
& +\mu_t\left[M_t-(1+\tau_c)\left[P_t c_{mt}-P_t A_d\left(\frac{l_{dt}h_t}{c_{mt}}\right)^{\gamma}c_{mt}\right]-(1+c_c)\,P_{nt}c_{nt}\right].
\end{aligned}
\tag{39}
$$

A reduced set of twenty-two equilibrium conditions in twenty-two unknowns, and in the given Greek parameters that lack a time subscript, are

$$
\left(\frac{c_{mt}}{c_{nt}}\right)^{\varepsilon-1}\frac{\nu}{1-\nu}=\frac{1+a_tR_t+(1-a_t)\gamma R_t}{p_{nt}(1+R_t)}, \tag{40}
$$

$$
\frac{\frac{\nu c_{mt}^{\varepsilon-1}}{c}}{\varrho\left(\frac{1}{x}\right)}=\frac{1+a_tR_t+(1-a_t)\gamma R_t}{(1-\tau_l)\,w_th_t}
$$

$$
c_t=\left[\nu c_{mt}^{\varepsilon}+(1-\nu)\,c_{nt}^{\varepsilon}\right]^{1/\varepsilon}. \tag{41}
$$

$$
R_t=r_t\,(1-\tau_k)-\delta_k+\pi_t, \tag{42}
$$

$$
\frac{m_t}{c_{mt}}=\left[a_t+\frac{p_{nt}c_{nt}}{c_{mt}}\right](1+\tau_c)\,, \tag{43}
$$

$$
1-a_t=\left(\frac{\gamma R_t(1+\tau_c)}{w_t(1-\tau_l)}\right)^{\gamma/(1-\gamma)}A_d^{1/(1-\gamma)}, \tag{44}
$$

$$
(1-a_t)=A_d(l_{dt}h_t/c_{mt})^{\gamma}, \tag{45}
$$

$$
g_t \;=\; r(1-\tau_k)-\delta_k-\rho, \tag{46}
$$

$$
g_t \;=\; A_h(1-x_t)-\delta_h-\rho, \tag{47}
$$

$$g_t = A_h l_{ht} - \delta_h, \tag{48}$$
$$g_t = \sigma - \pi_t, \tag{49}$$

$$g_t = A_h l_{ht} - \delta_h, \tag{50}$$
$$g_t = \sigma - \pi_t, \tag{51}$$

$$p_{ct} = (w_t l_{ct}) / (\omega \kappa_t) \equiv w_t / MP_{l_c}, \tag{52}$$

$$\kappa_t = \tau_l w_t l_{nt} h_t + \tau_k r_t [1 - s_{mt}] k_t + \tau_c p_{nt} c_{nt}, \tag{53}$$

$$\kappa_t = A_c (l_{ct} h_t / c_{nt})^{\omega}. \tag{54}$$

$$\begin{aligned} & \left[A_m (s_{mt} k_t)^{\beta} (l_{mt} h_t)^{1-\beta} / k_t\right] + \left[A_n ([1 - s_{mt}] k_t)^{\alpha} (l_{nt} h_t)^{1-\alpha} / k_t\right] \quad (55) \\ = \; & (c_{mt}/k_t) + (p_{nt} c_{nt}/k_t) + g_t + \delta_k, \end{aligned}$$

$$w_t = (1 - \beta) A_n (s_{mt} k_t)^{\beta} (l_{mt} h_t)^{-\beta}, \tag{56}$$
$$r_t = \beta A_n (s_{mt} k_t)^{\beta - 1} (l_{mt} h_t)^{1-\beta}, \tag{57}$$
$$w_t = p_{nt} (1 - \alpha) A_n (s_{nt} k_t)^{\alpha} (l_{nt} h_t)^{-\alpha}, \tag{58}$$
$$r_t = p_{nt} \alpha A_n (s_{nt} k_t)^{\alpha - 1} (l_{nt} h_t)^{1-\alpha}, \tag{59}$$
$$p_{nt} = \left[(1 - \beta) A_n (s_{mt} k_t)^{\beta} (l_{mt} h_t)^{-\beta}\right] / \left[(1 - \varepsilon) A_n (s_{nt} k_t)^{\alpha} (l_{nt} h_t)^{-\alpha}\right], \tag{60}$$

$$l_{mt} + l_{nt} + l_{ht} + l_{dt} + x_t + l_{ct} = 1, \tag{61}$$

2.6 The Effect of the Shadow Economy on the BGP Equilibrium

A closed-from solution of the economy is not possible. For example, the value of the non-market sector output cannot be solved, but some relations are implied. From equation (40) plus equation (44), it is unambiguous that the higher is the nominal interest rate (or inflation rate if also using equation (42)) then the lower is the non-market goods consumption relative to the

market goods consumption. Inflation causes substitution to the market good from the non-market good. This results because the average exchange cost of market goods, using some credit, goes up by less than R when R increases while the average exchange cost of non-market goods equals R and goes up one for one with R.

Proposition 2.6.1 *The rate of return of capital is not affected by the size of the market sector.*

Proposition 2.6.2 *The growth rate is decreased unambiguously by a larger non-market sector, given that all other parameters are equal except for ν.*

As the preference ν for the non-market sector is made larger, then the sector increases in size relative to the market sector. This causes more money to be used overall in the economy, and makes the interest elasticity of money demand more inelastic. When the inflation rate increases, the growth rate then decreases by more. Thus with all factors equal except for the taste for corruption being greater, or the respect for the government with taxing authority to be lessor, the growth rate will be lower for any given inflation rate.

Other factors also affect the growth rate through the non-market sector. The greater is the efficiency of production of the Islands government, the less labor that will be used up in non-productive corruption-producing activity, and the higher will be the time available for all other activities, which acts as a small stimulant to growth relative to an inefficient Islands government.

Taxes affect the equilibrium in several ways. There is neutrality in terms of the ratio of market to non-market consumption with respect to taxes because of the structure of the equilibrium, whereby the Islands government must charge the same implicit taxes on goods, labor and capital. A higher tax on capital directly lowers the growth rate. A higher tax on goods, and a higher tax on labor, causes more money use, and makes money demand more inelastic. This in itself causes a more severely negative inflation-growth effect. Countries that rely on VATs more than others thereby tend to increase the negative effect of inflation on growth, and it may be that economies with big non-market sectors rely on the VAT more than others.

An extension would be to allow the preference parameter ν depend on the average tax rate. Then clearly the average tax rate would cause a larger non-market sector. Given that some of this was higher average tax was in terms of goods and labor taxes, the money demand would be more inelastic and the inflation-growth effect more severe. With some in capital taxes, then the growth rate would of course be lower as well.

Another effect is through the capital intensity of the market versus the non-market sector. As the inflation rate increases, the human capital is taxed and there is substitution from labor to capital. This favors expansion of the capital intensive sector. Assuming the market sector is more capital intensive, then the increase in the inflation rate will cause contraction in the non-market sector.

Some of these model-implied relationships can be tested empirically, and to this end we formulate several simple hypotheses.

Hypothesis $\mathbf{H_1}$: *The relationship between the nominal interest rate and the share of the underground economy is negative.*

Hypothesis $\mathbf{H_2}$: *The relationship between the inflation rate and the share of the underground economy is negative.*

Hypothesis $\mathbf{H_3}$: *The output growth is negatively related to the share of the underground economy.*

While $\mathbf{H_1}$–$\mathbf{H_3}$ formulate the key implications from the theoretical model in an empirically testable way, not all theoretically-implied relations can be straightforwardly tested due to data limitations. For example, having no data on taxes with sufficient time-variability, the effects of taxes cannot be modelled. Similarly, we are not able to empirically test the implications of the Proposition (2.6.1).

3 Empirical modelling

3.1 Econometric methodology

The main characteristic of the "underground" or "shadow" economy is the unobservability. Thus the unofficial part of the economy is a classical example of a *latent* variable. Given the latent nature of this important economic quantity, the issue of its empirical measurement becomes exceptionally important in econometric models that include underground economy as endogenous or exogenous variable (Giles 1998b, Giles and Caragata 1999, Giles and Johnson 2000).

Literature on the methods for empirical measurement of the underground economy is extensive and several approaches are present in the literature. Schneider (2000) and Schneider and Enste (2000) review the existing approaches and apply several different methods to estimate the share of the underground economy in the OECD and developing countries.[4] Johnson et al. (1997) attempted to provide similar underground economy estimates for transitional countries, and Johnson et al. (1998) extend this analysis to the OECD and Latino-American countries.

However, most of these methods use various proxies and indirect measurement and thus fail to model the underground economy explicitly, as a latent variable. Frey and Weck (1983) termed such approaches "naive", and in a later paper they made a pioneering attempt to model the shadow economy using structural equation methods (Frey and Weck-Hanneman 1984, Helberger and Knepel 1988). The structural equation modelling (SEM)[5] was later implemented by Aigner et al. (1988), Loayza (1996), and Giles (1999). These authors used a special case of the general structural equation model with a single latent variable and multiple observed causes known as the "MIMIC"

[4]Schneider identifies six main approaches to empirical measurement of the underground economy: a) discrepancy between national expenditure and income statistics, b) discrepancy between official and actual labour force, c) transactions approach, d) currency demand approach, e) physical input, and e) model approach. See also Giles (1998a) for further literature review.

[5]The structural equation model with latent variables is often referred to as "LISREL model" in conjecture with the computer programme LISREL 8.54 (see e.g. Cziráky (2004a)).

model.[6]

The MIMIC model of Zellner (1970), Goldberger (1972a), Goldberger (1972b), and Jöreskog and Goldberger (1975) is a special case of the general structural equation model with latent variables (Jöreskog and Sörbom 1996), though proposed before the "LISREL" model of Jöreskog (1973). Jöreskog and Sörbom (1996) further generalised this model allowing for multiple observable indicators of multiple latent variables. The model with a single latent variable (Zellner 1970, Goldberger 1972b) turned out to be a special case of the general covariance structure model of Jöreskog (1970), and was analysed in detail in this framework by Jöreskog and Goldberger (1975).

The MIMIC model (Jöreskog and Goldberger 1975) is most frequently estimated by Gaussian maximum likelihood (ML) procedure of Jöreskog (1970), which requires assumptions that are seldom satisfied in the shadow economy measurement models such as those estimated by Giles (1999). The requirement of independence of observations and the static nature of the model are among the major weaknesses of the MIMIC models estimated by the covariance structure ML methods. In addition, MIMIC models enable merely estimation of the latent underground economy series and do not permit structural modelling of the effects of the underground economy on other economic variables such as output growth in the market sector.

An additional weakness of the MIMIC models of the Frey and Weck-Hanneman (1984) –type is that the supposedly error-free "causes" usually represent variables that are either observed with considerable error or that are themselves latent constructs. An example such construct is the "tax immorality" variable from the Frey and Weck-Hanneman (1984) model which is, no doubt, a latent construct, however in the existing literature on that uses the MIMIC approach this variable is assumed to be a perfectly observed "cause" of the latent hidden economy. Similar examples are the "labour market restrictions" and the "strength of the enforcement system" variables from the MIMIC model estimated by Loayza (1996); or "degree of economic regulation", "development of taxation", and "tax burden" variables used as "causes" of the underground economy in the Giles (1999) model.

[6]The abbreviation "MIMIC" stands for Multiple Indicators Multiple Causes.

The assumption of independent observations needed for the validity of the Jöreskog (1970) estimation procedure generally excludes time series and panel data, nevertheless the underground economy literature mainly uses this procedure (Frey and Weck-Hanneman 1984, Aigner et al. 1988, Loayza 1996, Giles 1999).

Methods for estimation of factor analytic and latent variable models that are appropriate for time series data exist in the literature, though they were not so far used for the underground economy estimation. For example, methods for estimation of dynamic factor models were proposed by Geweke and Singleton (1981) and Singleton (1980) and Engle and Watson (1981). Pena and Box (1987) proposed a procedure for dimension-reduction in time series data and, more recently, Bai and Ng (2002) and Bai (2003) developed procedures for determining the number of factors in approximate factor models and inferential theory for factor models of large dimension.

These methods could be used for estimation of the shadow economy measurement models, but they do not alow causal or structural relationships in the model. In order to incorporate shadow economy in macroeconometric models it is necessary to use full structural equation models with latent variables and estimation techniques suitable for time series data. MIMIC models could be then used merely as an auxiliary tool for delivering descriptive estimates of the latent underground economy series.

The aim of the methods proposed in this paper is to enable both measurement of the latent underground economy series and estimation of the dynamic structural equation models that include underground economy as latent variable. In such models the underground economy latent variable might affect, or be affected by, other modelled variables.

The measurement of the underground economy is relevant more for descriptive and comparative purposes then for econometric modelling of the underground economy's impact on e.g. market-sector output growth. Hence for the purpose of measuring the underground economy we specify a MIMIC–type model using the available macroeconomic data rather then measurement-error-prone constructs commonly used in the literature. Subsequently, we estimate a dynamic structural equation model that captures the impact of

the underground economy's share on the growth rate of the market-sector output as well as the relation between the underground economy and the other macroeconomic variables.

3.2 Estimation of the latent underground economy

The idea that certain model-implied relationships can be used to estimate the level of the underground economy by substituting observed variables for the latent ones is not new in the literature and is usually related to currency-demand models (see e.g. Bhattacharyya (1990). Giles (1999) uses a similar approach to set the scale of the latent underground economy by estimating a "long-run average" or expected value of the underground economy. This approach avoids arbitrary assumptions about the long-run average or a start year without underground economy.

Using the equilibrium conditions from the theoretical model we aim to estimate the mean of the underground economy, which is needed for setting the scale and range of the estimated latent underground economy variable. We show that in addition to the mean level it is also possible to calculate the underground economy time series using econometric estimation equations derived directly from the theoretical model, however possibly more efficient estimates can be obtained from a latent variable measurement model such as the MIMIC model (multiple indicators multiple causes). The estimates of the latent underground economy from a MIMIC model, however, would be scale-free and should be normalised according to information that is not obtainable from the MIMIC model itself.

We use model-implied equilibrium conditions (40), (43) and (44) to estimate the expected level (long-run average) of the underground economy.

Firstly, we need to make few simplifying assumptions in order to deal with the otherwise untractable non-linearities in the equilibrium conditions. Note that in (1) and (2) the aggregate consumption good and the current period utility function were parameterised with the utility function parameters ν and ε. Without loss of generality, we can assume an admissible value for ε, namely $\varepsilon = 2$, in which case the aggregate consumption good $c_t = [\nu c_{mt}^{\varepsilon} + (1-\nu) c_{nt}^{\varepsilon}]^{1/\varepsilon}$

assumes a convenient form of a weighted Euclidean norm in c_{mt} and c_{nt}, i.e. $c_t = \sqrt{\nu c_{mt}^2 + (1-\nu)c_{nt}^2}$. In addition, we set the γ parameter in the production function (21) to its mean admissible value of 0.5, recalling that $\gamma \in (0,1)$.

Specifically, we obtain an estimation equation of the form

$$\ln(y_t p_{mt}) = \alpha_0 + \ln\left(\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 - c_{mt}\right), \tag{62}$$

where $\alpha_0 \equiv -\ln[(1-\nu)/\nu]$, $\alpha_1 \equiv A^2/\beta$, $\alpha_2 \equiv -A^2/\delta$, $\alpha_3 \equiv \beta/\delta$, $\alpha_4 \equiv -\beta^2/\delta^2$. Similarly we made use of the following definitions

$$x_1 \equiv \frac{1}{2}m_t,\ x_2 \equiv \frac{m_t R_t^2}{4w_t(1+R_t)},\ x_3 \equiv \frac{c_{mt}(1+R_t)R_t + \frac{1}{2}c_{mt}R_t^2}{w_t(1+R_t)},\ x_4 \equiv \frac{c_{mt}R_t^3}{2w_t^2(1+R_t)}.$$

The above equation can be estimated with non-linear least squares (NLS) or maximum likelihood (ML) techniques.

The relevance of the equation (62) is twofold. By estimating α_0 and exponentiating the negative of its estimated value we can obtain the long-run or time-average value of the underground economy (see Appendix A for details).

Firstly, from (62) we specify a slightly modified estimation equation that includes a time-trend.[7]

$$\ln(y_t p_{mt}) = \alpha_0 + \ln\left(\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 - c_{mt} + \alpha_5 t^{1/3}\right) + \varepsilon. \tag{63}$$

The results from non-linear least squares (NLS) estimation of the equation (63) for Bulgaria, Croatia, and Romania are shown in Table 3.2. We use seasonally adjusted monthly data spanning from July 1995 to February 2003 for Bulgaria ($N = 91$), from June 1994 to May 2003 for Croatia ($N = 108$) and from January 1994 to May 2003 for Romania ($N = 113$).[8]

The long-run average level or expected value of the underground economy can be obtained by recalling that $\alpha_o = -ln\left(\frac{1-\nu}{\nu}\right)$ which implies that the

[7]We constrain the trend within the logarithm and raise it to the power 1/3, which is an arbitrary modification not implied by the theoretical model.

[8]The data span is shorter for Bulgaria due to unavailability of data on $M3$ before mid 1995.

share of the underground economy in the form of the ration of non-market and market consumption is given by $e^{-\alpha_o} = \frac{1-\nu}{\nu}$. From the results reported in Table 3.2 we obtain the following results

$$\begin{aligned}
\text{Bulgaria}: \quad \alpha_o &= 0.6225 = -\ln\left(\frac{1-\nu}{\nu}\right) \Rightarrow \frac{1-\nu}{\nu} = 0.5366 \\
\text{Croatia}: \quad \alpha_o &= 0.6376 = -\ln\left(\frac{1-\nu}{\nu}\right) \Rightarrow \frac{1-\nu}{\nu} = 0.5286 \\
\text{Romania}: \quad \alpha_o &= 0.1892 = -\ln\left(\frac{1-\nu}{\nu}\right) \Rightarrow \frac{1-\nu}{\nu} = 0.8276
\end{aligned}$$

These estimates appear higher then the previously available figures in the literature (see e.g. Johnson et al. (1997), Schneider (2000), and Schneider and Klinglmair (2004)) and suggest that non-market output is about half of the market output in Bulgaria and Croatia, and as much as 80% in Romania. Note however, that in estimation we used industrial production which was a monthly proxy for the GDP; thus a further normalisation needs to be applied if underground economy is to be expressed as the percentage of the reported GDP.

Table 1: The mean level of the underground economy

	NLS estimates					
	Bulgaria		Croatia		Romania	
Coefficient	Estimate	(SE)	Estimate	(SE)	Estimate	(SE)
α_0	0.6255	(0.0199)	0.6376	(0.0536)	0.1892	(0.0116)
α_1	–0.0456	(0.0109)	–0.0341	(0.1469)	0.0534	(0.0305)
α_2	0.0355	(0.0200)	1.5431	(3.7783)	0.1189	(0.3614)
α_3	–0.0398	(0.0287)	0.2787	(1.8306)	0.2658	(0.3858)
α_4	0.0175	(0.0062)	10.9660	(9.9026)	1.1642	(0.5199)
α_5	–56.2631	(12.8820)	–34.5590	(15.9330)	–0.4871	(0.5021)

Irrespective of the actual GDP-share that underground economy accounts for, our estimates indicate different magnitude of the non-market sectors among the three countries. Namely, it is explicit from the above results that Romanian non-market sector has higher magnitude relative to the market sector then the non-market sectors in Bulgaria and Croatia.

3.3 Structural equation modelling

The first aim of the structural equation modelling is estimation of the latent underground economy time series, and the second aim is evaluation of its impact on the rate of growth of the market-sector output. We address these issues in the structural equation modelling framework using certain special cases of the general dynamic structural equation model (SEM).

In brief, the static structural equation model with latent variables (SEM) (Jöreskog and Sörbom 1996, Jöreskog et al. 2001) is specified with three matrix equations–the structural equation, the measurement equation for latent exogenous variables, and the measurement equation for latent endogenous variables

$$\boldsymbol{\eta} = \boldsymbol{\alpha}_\eta + \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta}, \quad \mathbf{x} = \boldsymbol{\alpha}_x + \boldsymbol{\Lambda}_x\boldsymbol{\xi} + \boldsymbol{\delta}, \quad \mathbf{y} = \boldsymbol{\alpha}_y + \boldsymbol{\Lambda}_y\boldsymbol{\eta} + \boldsymbol{\varepsilon}, \quad (64)$$

where $\boldsymbol{\eta}$ is a $(m \times 1)$ matrix of endogenous latent variables; $\boldsymbol{\xi}$ is a $(g \times 1)$ matrix of exogenous latent variables; $\mathbf{B}$ and $\boldsymbol{\Gamma}$ are $(m \times m)$ and $(m \times g)$ matrices of structural coefficients, respectively; $\boldsymbol{\Lambda}_x$ and $\boldsymbol{\Lambda}_y$ are $k \times g$ and $l \times m$ matrices of factor loadings, respectively; $\boldsymbol{\alpha}_\eta$, $\boldsymbol{\alpha}_x$, and $\boldsymbol{\alpha}_y$ are $(m \times 1)$, $(k \times 1)$, and $(l \times 1)$ matrices of intercepts, respectively.

The static SEM model, aside of lacking dynamics relevant in time series models, is based on the multivariate normality assumption. This is a problem for dynamic models and time series data since N observations on a multivariate normal vector $\mathbf{x} \in \mathbb{R}^k$ with the density function $f(\mathbf{x}) = (2\pi)^{-k/2} |\boldsymbol{\Sigma}|^{-1/2} \exp\left(-\frac{1}{2}\mathbf{x}'\boldsymbol{\Sigma}^{-1}\mathbf{x}\right)$ will have a joint multivariate normal distribution with the sample likelihood in the form $L = (2\pi)^{-Nk/2} |\boldsymbol{\Sigma}|^{-N/2} \times \exp\left(-\frac{1}{2}\sum_{i=1}^{N} \mathbf{x}'\boldsymbol{\Sigma}^{-1}\mathbf{x}\right)$ if and only if their joint density can be written as a product of their individual densities, which requires independence of consecutive observations. For example, in a simple regression model $y_t = \gamma x_t + \varepsilon_t$ making the assumption that $\varepsilon_t \sim N$ implies that $y_t \sim N$, however serial dependence of y_t in general does not correspond to that of ε_t as the later can be serially uncorrelated conditional on the former. For example, in a correctly specified dynamic model e.g. $y_t = \beta_1 y_{t-1} + \gamma_0 x_t + \gamma_1 x_{t-1} + \varepsilon_t$ it is possible that ε_t is a white noise process while y_t is serially correlated or even

a long-memory process. Thus, if the dynamics in y_t are correctly specified, making the assumption that $\varepsilon \sim i.i.d.$ is reasonable; the same assumption about y_t however would be incorrect.

This turns out to be an important issue in the ML estimation technique developed by Jöreskog and Sörbom (1996). Namely, this approach makes distributional assumptions about the data vector $\mathbf{Y} \equiv (\mathbf{x} : \mathbf{y})$, requiring $\mathbf{Y}$ to be i.i.d. multivariate normal and makes use of the model-implied covariance matrix $\mathbf{\Sigma}_{YY} = E[\mathbf{Y}\mathbf{Y}']$, where making use of (64) the data vectors are parameterised as $\mathbf{y} = \mathbf{\Lambda}_y \left[(\mathbf{I} - \mathbf{B})^{-1}(\mathbf{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta})\right] + \boldsymbol{\varepsilon}$ and $\mathbf{x} = \mathbf{\Lambda}_x\boldsymbol{\xi} + \boldsymbol{\delta}$. The maximum likelihood estimates of the model parameters are obtained by minimising the likelihood function $L = \ln|\mathbf{\Sigma}_{YY}| + \mathrm{tr}(\mathbf{S}\mathbf{\Sigma}_{YY}^{-1}) - \ln|\mathbf{S}| - k$, where $\mathbf{Y} \in \mathbb{R}^k$ and $\ln|\mathbf{S}| - k$ was subtracted from the ordinary multivariate normal likelihood to obtain a discrepancy function.

On the other hand, if we deal with a dynamic structural equation model model that includes lags of $\boldsymbol{\eta}_t$ and $\boldsymbol{\xi}_t$ variables, the i.i.d. assumption about $\mathbf{Y}$ could be replaced with the assumption that $\boldsymbol{\zeta}_t$ is i.i.d. conditional on correctly specified dynamics of $\mathbf{Y}$. In the context of maximum likelihood estimation with serially dependent data, the ML estimator would be actually a quasi-ML estimator. The quasi-ML estimator for dependent data is known to be consistent and asymptotically normal regardless of the treatment of the pre-sample observations (Wooldridge 1994).

Estimation of general dynamic structural equation models (DSEM) can be done by limited- and full-information instrumental variables methods (Cziráky 2003). However, as the IV methods generally require longer time series then those available for transitional countries, in this paper we propose an alternative full-information maximum likelihood method for the estimation of the general DSEM model.

Cziráky (2003) formulated a dynamic structural equation model with latent variables (DSEM) as a time series generalisation of the static SEM model.[9] The DSEM model is specified as a structural autoregressive distributed lag model of the form

[9] A static version of this model can be easily estimated by software packages such as LISREL 8.54 (see e.g. Cziráky, 2004).

$$\boldsymbol{\eta}_t = \boldsymbol{\alpha}_\eta + \sum_{j=0}^{p} \mathbf{B}_j \boldsymbol{\eta}_{t-j} + \sum_{j=0}^{q} \boldsymbol{\Gamma}_j \boldsymbol{\xi}_{t-j} + \boldsymbol{\zeta}_t, \tag{65}$$

where $\boldsymbol{\alpha}_\eta$, $\mathbf{B}_0$, and $\boldsymbol{\Gamma}_0$ are coefficient matrices from the static model (64), and $\mathbf{B}_1$, $\mathbf{B}_2$,..., $\mathbf{B}_p$, $\boldsymbol{\Gamma}_1$, $\boldsymbol{\Gamma}_2$,..., $\boldsymbol{\Gamma}_q$ are the additional $p+q$ matrices that contain coefficients of the lagged endogenous and exogenous latent variables.[10] Note that the specification (65) is simultaneous because contemporaneous endogenous latent variables might be included as regressors (i.e. $\mathbf{B}_0 \neq \mathbf{0}$). If we assume time-invariance of the measurement model, the usual specification for $\mathbf{x}_t$ and $\mathbf{y}_t$ applies. Hence the structural part of the model (65) can be augmented with the measurement equation for the latent exogenous variables

$$\mathbf{x}_t = \boldsymbol{\alpha}_x + \boldsymbol{\Lambda}_x \boldsymbol{\xi}_t + \boldsymbol{\delta}_t \tag{66}$$

and for the latent endogenous variables

$$\mathbf{y}_t = \boldsymbol{\alpha}_y + \boldsymbol{\Lambda}_y \boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \tag{67}$$

The matrix equations (65)-(67) provide full specification of the general DSEM model directly extending the static structural equation model with latent variables (SEM) to the time series case. It follows that the static SEM is a special case of the DSEM model.

However, the DSEM model from (65)–(67) cannot be directly estimated due to the presence of unobserved latent components. To solve this problem and enable estimation of the model parameters, we rewrite the latent variable specification in terms of the observed variables and latent errors only, following the approach used by Bollen (1996; 2001; 2002). Bollen used such specification to enable non-parametric estimation of standard (cross-sectional) structural equation models with an aim of achieving greater robustness to misspecification and non-normality.

A similar approach can be used to re-write the DSEM model in the *observed form specification* (OFS) and to subsequently estimate all model pa-

[10]Note that (65) does not require specification of lagged latent variables as separate variables; rather each vector containing all modelled and exogenous latent variables is written for each included lag separately, with a separate coefficient matrix. Also note that (65) allows different lag lengths for different latent variables (i.e., elements of $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ vectors) by appropriate specification of $\mathbf{B}_j$ and $\boldsymbol{\Gamma}_j$ matrices (e.g., zero elements).

rameters (except latent error terms) by generalised instrumental variables methods (see Cziráky (2004b)).

By ignoring the specific structure of the measurement error terms, let $\mathbf{u}_{1t} \equiv \boldsymbol{\zeta}_t + \boldsymbol{\varepsilon}_{1t} - \sum_{j=0}^{p} \mathbf{B}_j \boldsymbol{\varepsilon}_{1t-j} - \sum_{j=0}^{q} \boldsymbol{\Gamma}_j \boldsymbol{\delta}_{1t-j}$, $\mathbf{u}_{2t} \equiv \boldsymbol{\varepsilon}_{2t} - \boldsymbol{\Lambda}_2^{(y)} \boldsymbol{\varepsilon}_{1t}$, and $\mathbf{u}_{3t} \equiv \boldsymbol{\delta}_{2t} - \boldsymbol{\Lambda}_2^{(x)} \boldsymbol{\delta}_{1t}$ the structural OFS equations can then be written as

$$\mathbf{y}_{1t} = \boldsymbol{\alpha}_\eta + \sum_{j=0}^{p} \mathbf{B}_j \mathbf{y}_{1t-j} + \sum_{j=0}^{q} \boldsymbol{\Gamma}_j \mathbf{x}_{1t-j} + \mathbf{u}_{1t}, \tag{68}$$

with the measurement models

$$\mathbf{y}_{2t} = \boldsymbol{\alpha}_2^{(\mathrm{y})} + \boldsymbol{\Lambda}_2^{(\mathrm{y})} \mathbf{y}_{1t} + \mathbf{u}_{2t}, \tag{69}$$

and

$$\mathbf{x}_{2t} = \boldsymbol{\alpha}_2^{(x)} + \boldsymbol{\Lambda}_2^{(x)} \mathbf{x}_{1t} + \mathbf{u}_{3t}. \tag{70}$$

The estimation of the OFS equations can be done using the instrumental variable (IV) methods (Cziráky 2003). In this paper we consider maximum likelihood (ML) estimation, which is likely to have better small sample properties. The ML estimators for the DSEM model might be constructed either as full information (FIML) or limited information maximum likelihood (LIML) estimators, depending on the treatment of certain restrictions in the covariance matrix of the latent errors. In this sense LIML would not necessarily imply that we are not estimating all equations of the system, rather that information contained in these equations might be incomplete. However, we will use the term "full information" to refer to estimation of all equations in the system. In particular, we will refer to the ML estimator of the OFS system of equations as "OFS-FIML" in this context.

If we assume that $\mathbf{u} \sim \text{i.i.d.}\, N(\mathbf{0}, \boldsymbol{\Omega}_u)$, where $\mathbf{u} = (\mathbf{u}_{1t}' : \mathbf{u}_{3t}' : \mathbf{u}_{3t}')'$ it follows that $\mathbf{z}_t \sim \text{i.i.d.}\, N(\mathbf{0}, \boldsymbol{\Lambda}\boldsymbol{\Omega}_u\boldsymbol{\Lambda}')$, where

$$\boldsymbol{\Lambda} \equiv \begin{pmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{\Lambda}_2^{(y)} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \left(\mathbf{I} - \boldsymbol{\Lambda}_2^{(x)}\right)^{-1} \end{pmatrix}.$$

To simplify the notation, we define $\mathbf{\Omega}_z \equiv \mathbf{\Lambda}\mathbf{\Omega}_u\mathbf{\Lambda}'$. The equations (**??**)–(69) can be estimated by maximising the (conditional) multivariate Gaussian log likelihood function given by

$$L_{DSEM} = -\frac{m}{2}\ln(2\pi) + \frac{1}{2}\ln\left(\left|\mathbf{\Omega}_z^{-1}\right|\right) - \frac{1}{2T}\sum_{t=1}^{T}(\mathbf{y}_t - \mathbf{\Pi}\mathbf{x}_t)'\mathbf{\Omega}_z^{-1}(\mathbf{y}_t - \mathbf{\Pi}\mathbf{x}_t). \tag{71}$$

or equivalently by

$$L_{DSEM} = -\frac{m}{2}\ln(2\pi) + \frac{1}{2}\ln\left(\left|\mathbf{\Omega}_z^{-1}\right|\right) - \frac{1}{2T}\sum_{t=1}^{T}\mathbf{z'}_t\mathbf{\Omega}_z^{-1}\mathbf{z}_t, \tag{72}$$

where

$$\mathbf{y}_t = \begin{pmatrix} \mathbf{y}_{1t} \\ \mathbf{y}_{2t} \\ \mathbf{x}_{2t} \end{pmatrix}, \quad \mathbf{z}_t \equiv \begin{pmatrix} \mathbf{u}_{1t} \\ \mathbf{\Lambda}_2^{(y)}\mathbf{u}_{1t} + \mathbf{u}_{2t} \\ \left(\mathbf{I} - \mathbf{\Lambda}_2^{(x)}\right)^{-1}\mathbf{u}_{3t} \end{pmatrix},$$

$$\mathbf{\Pi} \equiv \begin{pmatrix} \boldsymbol{\alpha}_\eta & \mathbf{B}_1 & \cdots & \mathbf{B}_p & \mathbf{\Gamma}_1 & \cdots & \mathbf{\Gamma}_q \\ \mathbf{\Lambda}_2^{(y)}\boldsymbol{\alpha}_\eta + \boldsymbol{\alpha}_2^{(y)} & \mathbf{\Lambda}_2^{(y)}\mathbf{B}_1 & \cdots & \mathbf{\Lambda}_2^{(y)}\mathbf{B}_p & \mathbf{\Lambda}_2^{(y)}\mathbf{\Gamma}_1 & \cdots & \mathbf{\Lambda}_2^{(y)}\mathbf{\Gamma}_q \\ \left(\mathbf{I} - \mathbf{\Lambda}_2^{(x)}\right)^{-1}\boldsymbol{\alpha}_2^{(x)} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix},$$

and

$$\mathbf{x}_t = \begin{pmatrix} \mathbf{y'}_{1t-1} & \cdots & \mathbf{y'}_{1t-p} & \mathbf{x'}_{1t-1} & \cdots & \mathbf{x'}_{1t-q} \end{pmatrix}'.$$

See Appendix C for technical details and specification of the $\mathbf{\Omega_z}$ matrix.

3.3.1 A MIMIC model for the underground economy

The problem of measuring the latent underground economy is twofold. The first problem is to fix or calibrate the scale of the underground economy latent variable thus enabling estimation of the underground economy time series scores. To solve this problem we took a model-derived estimation approach utilizing our theoretical model and subsequently deriving an econometric estimation equation that allowed estimation of the time average of the underground economy share.

The second problem is how to measure the latent underground economy with the observable macroeconomic indicators, which poses even greater methodological challenges. Our solution to the measurement problem involves a latent variable measurement model, which we specify as a multiple indicator multiple causes (MIMIC) model following the contemporary literature (see Giles (1999) for a review).

We specify our MIMIC model as a special case of the general DSEM model and use OFS-FIML estimation procedure to obtain the coefficient estimates. Subsequently, we estimate scale-free latent scores from the MIMIC model and calibrate them with the model-derived scale.

The MIMIC model can be specified as a special case of the general DSEM model where $\mathbf{B}_j = \mathbf{0}, \forall j > 0$, $\mathbf{\Gamma}_j = \mathbf{0}$ for $j > 0$, $\mathbf{\Lambda}_x = \mathbf{I}$, and $\mathbf{\Theta}_\delta = \mathbf{0}$. Hence, the MIMIC model can be specified as

$$\begin{aligned} \boldsymbol{\eta}_t &= \mathbf{\Gamma}_0 \boldsymbol{\xi}_t + \boldsymbol{\zeta}_t & (73) \\ \mathbf{y}_t &= \mathbf{\Lambda}_y \boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t & (74) \\ \mathbf{x}_t &= \boldsymbol{\xi}_t. & (75) \end{aligned}$$

Note that this model has the covariance structure given by

$$\mathbf{\Sigma}_{YY} = \begin{pmatrix} E[\mathbf{y}_t \mathbf{y}'_t] & E[\mathbf{y}_t \mathbf{x}'_t] \\ E[\mathbf{x}_t \mathbf{y}'_t] & E[\mathbf{x}_t \mathbf{x}'_t] \end{pmatrix}, \tag{76}$$

where $E[\mathbf{y}_t \mathbf{y}'_t] = E[(\mathbf{\Lambda}_y \mathbf{\Gamma}_0 \boldsymbol{\xi}_t + \mathbf{\Lambda}_y \boldsymbol{\zeta}_t + \boldsymbol{\varepsilon}_t)(\boldsymbol{\xi}'_t \mathbf{\Gamma}'_0 \mathbf{\Lambda}'_y + \boldsymbol{\zeta}'_t \mathbf{\Lambda}'_y + \boldsymbol{\varepsilon}'_t)] = \mathbf{\Lambda}_y (\mathbf{\Gamma}_0 \mathbf{\Phi} \mathbf{\Gamma}'_0 + \mathbf{\Psi}) \mathbf{\Lambda}'_y + \mathbf{\Theta}_\varepsilon$, $E[\mathbf{y}_t \mathbf{x}'_t] = E[(\mathbf{\Lambda}_y \mathbf{\Gamma}_0 \boldsymbol{\xi}_t + \mathbf{\Lambda}_y \boldsymbol{\zeta}_t + \boldsymbol{\varepsilon}_t) \boldsymbol{\xi}'_t] = \mathbf{\Lambda}_y \mathbf{\Gamma}_0 \mathbf{\Phi}$, and $E[\mathbf{x}_t \mathbf{x}'_t] = E[\boldsymbol{\xi}_t \boldsymbol{\xi}'_t] = \mathbf{\Phi}$. Therefore the model-implied covariance matrix (**??**) is given by

$$\mathbf{\Sigma}_{YY} = \begin{pmatrix} \mathbf{\Lambda}_y \left(\mathbf{\Gamma}_0 \mathbf{\Phi} \mathbf{\Gamma}'_0 + \mathbf{\Psi} \right) \mathbf{\Lambda}'_y + \mathbf{\Theta}_\varepsilon & \mathbf{\Lambda}_y \mathbf{\Gamma}_0 \mathbf{\Phi} \\ \mathbf{\Phi} \mathbf{\Gamma}'_0 \mathbf{\Lambda}'_y & \mathbf{\Phi} \end{pmatrix}. \tag{77}$$

Note that $E[\mathbf{x}_t \mathbf{y}'_t] = E[\mathbf{y}_t \mathbf{x}'_t]' = \mathbf{\Phi} \mathbf{\Gamma}'_0 \mathbf{\Lambda}'_y$, $\mathbf{\Sigma}_{YY} = E[\mathbf{Y}\mathbf{Y}']$ and $\mathbf{Y} = (\mathbf{y}_t : \mathbf{x}_t)$, $\mathbf{\Phi} = E[\boldsymbol{\xi}_t \boldsymbol{\xi}'_t]$, $\mathbf{\Psi} = E[\boldsymbol{\zeta}_t \boldsymbol{\zeta}'_t]$, and $\mathbf{\Theta}_\varepsilon = E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_t]$.

We specify the MIMIC model for the underground economy using the available monthly time series data for the West-Balkans countries (Bulgaria, Croatia, and Romania). Preliminary unit-root tests indicated that most

variables are $I(1)$, hence we first-differenced them before estimation. Variable description and notation are given in Table 2.

Table 2: Variables and notation

Variable	Symbol
Output growth	ΔY_t
M0/M3 ratio	ΔCM_t
Money	$\Delta M1$
Unemployment	ΔU_t
Wages	ΔW_t
Inflation	ΔP_t
Consumption	ΔC_t
Interest rate	ΔR_t
Underground economy	ΔUE_t

The model is specified in two parts. Underground economy is assumed to be caused by wages, inflation, interest rates, and money (M1), and it is specified as

$$\Delta UE_t = \begin{pmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \end{pmatrix} \begin{pmatrix} \Delta W_t \\ \Delta P_t \\ \Delta R_t \\ \Delta M1_t \end{pmatrix} + \zeta_t. \tag{78}$$

Furthermore, underground economy is measured by the currency/money ratio, unemployment, and consumption (where market sector retail sales are used as proxy). Formally, we specify the measurement model as

$$\begin{pmatrix} \Delta CM_t \\ \Delta U_t \\ \Delta C_t \end{pmatrix} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} \Delta UE_t + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix}, \tag{79}$$

where the variable definitions are given in Table 2. We normalise the measurement model (79) by setting $\lambda_1 = 1$. The coefficient matrices from (78) and (79) are therefore

$$\mathbf{\Gamma}_0 = \begin{pmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \end{pmatrix}, \quad \mathbf{\Lambda}_y = \begin{pmatrix} 1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}, \quad \mathbf{\Psi} = \omega$$

$$\mathbf{\Theta}_\varepsilon = \begin{pmatrix} \theta_{11}^{(\varepsilon)} & & \\ 0 & \theta_{22}^{(\varepsilon)} & \\ 0 & 0 & \theta_{33}^{(\varepsilon)} \end{pmatrix}, \quad \mathbf{\Phi} = \begin{pmatrix} \phi_{11} & & & \\ \phi_{21} & \phi_{22} & & \\ \phi_{31} & \phi_{32} & \phi_{33} & \\ \phi_{41} & \phi_{42} & \phi_{43} & \phi_{44} \end{pmatrix}$$

We estimate the model (78)–(79) by OFS-FIML methods. The OFS equations corresponding to the model (78)–(79) are given by

$$\begin{aligned} \Delta UE_t &= \alpha_\eta + \gamma_1 \Delta W_t + \gamma_2 \Delta P_t + \gamma_3 \Delta R_t + \gamma_4 \Delta M1_t + u_{1t} \\ \Delta U_t &= \alpha_{21}^{(y)} + \lambda_2 \Delta UE_t + u_{2t} \\ \Delta C_t &= \alpha_{22}^{(y)} + \lambda_3 \Delta UE_t + u_{3t} \end{aligned}$$

where $u_{1t} = (\zeta_t + \varepsilon_{1t})$, $u_{2t} = (\varepsilon_{2t} - \lambda_2 \varepsilon_{1t})$, and $u_{3t} = (\varepsilon_{3t} - \lambda_3 \varepsilon_{1t})$. The typical MIMIC restriction placed on the $\mathbf{\Theta}_\varepsilon$ matrix require $\mathbf{\Theta}_\varepsilon = \text{diag}(\theta_1, \theta_2, \theta 3)$, which places non-linear restrictions to the $\mathbf{\Omega}_u$, where

$$\mathbf{\Omega}_u = \begin{pmatrix} \text{var}(u_{1t}) & & \\ \text{cov}(u_{2t}, u_{1t}) & \text{var}(u_{2t}) & \\ \text{cov}(u_{3t}, u_{1t}) & \text{cov}(u_{3t}, u_{2t}) & \text{var}(u_{3t}) \end{pmatrix}.$$

Namely, for the model (78)–(79), these restrictions are based on the assumption that $E[\zeta_t \varepsilon_i] = 0$, $E[\varepsilon_i \varepsilon_j] = 0$ for $i \neq j$. Therefore, the covariances in $\mathbf{\Omega}_0$ are calculated as

$$
\begin{aligned}
\mathrm{Var}(u_{1t}) &= E[(\zeta_t + \varepsilon_{1t})(\zeta_t + \varepsilon_{1t})] \\
&= E[\zeta_t^2 + 2\varepsilon_{1t}\zeta_t + \varepsilon_{1t}^2] = \omega + \theta_1 \\
\mathrm{Var}(u_{2t}) &= E[(\varepsilon_{2t} - \lambda_2\varepsilon_{1t})(\varepsilon_{2t} - \lambda_2\varepsilon_{1t})] \\
&= E[\varepsilon_{2t}^2 - 2\lambda_2\varepsilon_{1t}\varepsilon_{2t} + \lambda_2^2\varepsilon_{1t}^2] = \theta_2 - \lambda_2^2\theta_1 \\
\mathrm{Var}(u_{3t}) &= E[(\varepsilon_{3t} - \lambda_3\varepsilon_{1t})(\varepsilon_{3t} - \lambda_3\varepsilon_{1t})] \\
&= E[\varepsilon_{3t}^2 - 2\lambda_3\varepsilon_{3t}\varepsilon_{1t} + \lambda_3^2\varepsilon_{1t}^2] = \theta_3 + \lambda_3^2\theta_1 \\
\mathrm{Cov}(u_{2t}, u_{1t}) &= E[(\varepsilon_{2t} - \lambda_2\varepsilon_{1t})(\zeta_t + \varepsilon_{1t})] \\
&= E[\varepsilon_{2t}\zeta_t - \lambda_2\varepsilon_{1t}\zeta_t + \varepsilon_{2t}\varepsilon_{1t} - \lambda_2\varepsilon_{1t}\varepsilon_{1t}] = -\lambda_2\theta_1 \\
\mathrm{Cov}(u_{3t}, u_{1t}) &= E[(\varepsilon_{3t} - \lambda_3\varepsilon_{1t})(\zeta_t + \varepsilon_{1t})] \\
&= E[\varepsilon_{3t}\zeta_t - \lambda_3\varepsilon_{1t}\zeta_t + \varepsilon_{3t}\varepsilon_{1t} - \lambda_3\varepsilon_{1t}\varepsilon_{1t}] = -\lambda_3\theta_1 \\
\mathrm{Cov}(u_{3t}, u_{2t}) &= E[(\varepsilon_{3t} - \lambda_3\varepsilon_{1t})(\varepsilon_{2t} - \lambda_2\varepsilon_{1t})] \\
&= E[\varepsilon_{3t}\varepsilon_{2t} - \lambda_3\varepsilon_{1t}\varepsilon_{2t} - \lambda_2\varepsilon_{3t}\varepsilon_{1t} + \lambda_3\lambda_2\varepsilon_{1t}\varepsilon_{1t}] = \lambda_3\lambda_2\theta_1.
\end{aligned}
$$

Therefore, the OFS-FIML restrictions placed on $\mathbf{\Omega}_u$ for this model are

$$
\mathbf{\Omega}_u = \begin{pmatrix}
\omega + \theta_1 & & \\
-\lambda_2\theta_1 & \theta_2 - \lambda_2^2\theta_1 & \\
-\lambda_3\theta_1 & \lambda_3\lambda_2\theta_1 & \theta_3 + \lambda_3^2\theta_1
\end{pmatrix}.
$$

Note that ignoring the above restrictions placed on the $\mathbf{\Omega}_u$ matrix in the OFS-FIML estimation would be equivalent to specifying $\mathbf{\Theta}_\varepsilon$ as a full symmetric matrix, which in turn implies correlated errors in the measurement part of the MIMIC model. In fact, the restriction that $\mathbf{\Theta}_\varepsilon$ must be diagonal comes from the classical factor analysis while, in principle, general SEM and DSEM models allow for non-diagonal $\mathbf{\Theta}_\varepsilon$.

OFS-FIML estimates of the MIMIC model coefficients are shown in Table 3.3.1. Recalling that we fixed the scale of ΔUE_t by fixing $\lambda_1 = 1$ which assumes positive link between the underground economy and the currency/money ratio.

The bottom part of the Table 3.3.1 gives several measures of the approximate fit. The reported χ^2 goodness-of-fit tests are strictly valid only for models with independent data and in should be interpreted with caution when time series data are used. Amemiya and Anderson (1990) however show that this test is asymptotically distributed χ^2 under some very general

Table 3: MIMIC model estimates

	Bulgaria		Croatia		Romania	
Coefficient	ML	GLS	ML	GLS	ML	GLS
λ_2	–9.0809	–8.7336	10.8295	8.1138	8.3391	8.3343
(SE)	(4.2937)	(3.6004)	(5.19081)	(4.0576)	(4.5942)	(5.5267)
λ_3	9.1889	11.4370	–20.4402	–15.1907	0.5302	0.3528
(SE)	(3.9653)	(5.9996)	(13.6183)	(7.2057)	(0.2383)	(0.1399)
γ_1	0.0039	0.0035	–0.0013	–0.0016	–0.0106	–0.0104
(SE)	(0.0013)	(0.0012)	(0.0008)	(0.0009)	(0.0019)	(0.0019)
γ_2	–0.0002	–0.0005	0.0048	0.0063	–0.0034	–0.0035
(SE)	(0.0003)	(0.0004)	(0.0035)	(0.0040)	(0.0027)	(0.0017)
γ_3	–0.0001	–0.0001	0.0009	0.0014	–0.0050	–0.0006
(SE)	(0.0001)	(0.0001)	(0.0013)	(0.0016)	(0.0014)	(0.0003)
γ_4	0.0003	0.0003	–0.0003	–0.0004	–0.0025	–0.0025
(SE)	(0.0001)	(0.0001)	(0.0002)	(0.0003)	(0.0003)	(0.0003)
$\theta_1^{(\varepsilon)}$	0.0059	0.0058	0.0030	0.0028	0.0277	0.0260
(SE)	(0.0038)	(0.0037)	(0.0004)	(0.0004)	(0.0129)	(0.0122)
$\theta_2^{(\varepsilon)}$	16.5769	14.2697	5.6995	4.5283	45.9634	43.3586
(SE)	(2.5009)	(2.3469)	(0.9833)	(0.8363)	(6.2291)	(6.0501)
$\theta_3^{(\varepsilon)}$	25.4738	20.6782	29.5228	24.2119	101.3173	94.2274
(SE)	(3.8275)	(3.5212)	(4.5451)	(4.0499)	(13.5998)	(13.1308)
χ^2	11.5185	14.2235	14.5612	14.3637	6.9738	7.3949
(d.f.)	8	8	8	8	8	8
SRMR	0.5413	0.0772	0.0654	0.0802	0.0474	0.0522
GFI	0.9693	0.9549	0.9628	0.9613	0.9827	0.9810

assumptions. In addition we report the standardised root-mean-square error of approximation (SRMR) and the goodness-of-fit index (GFI). These fit statistics generally indicate relatively good fit in all three countries.

The MIMIC results indicate negative relationship between the underground economy and the nominal interest rate for Bulgaria and for Romania (γ_3), which is compatible with the hypothesis $\mathbf{H_1}$. For Croatia, on the other hand, the relationship between the underground economy and the nominal interest rate appears to be positive, however the estimates of the γ_3 coefficient are significant only for Romania.

The relationship between the inflation rate and the underground economy (γ_2) is also negative (hypothesis $\mathbf{H_2}$) for Bulgaria and Romania, while the hypothesized sign does not appear to hold for Croatia. Again, the estimated γ_2 coefficient seems to be of significant magnitude only for Romania.

Interesting differences can be observed in the measurement models among the three countries. With the unit-loading restriction imposed on ΔMCt, the estimated loadings (λ_2 and λ_3) differ both in sign and in magnitude (the reported estimates are unstandardized). Unemployment (ΔU_t) appears to be a negative indicator of the latent underground economy only in Bulgaria, while it is positive in Croatia and Romania, with magnitudes being similar across the countries. On the other hand, the market-sector consumption (ΔC_t) has negative loading only in Croatia and, while positive, the loading is of very small magnitude in Romania. These observed differences are indicative of different measurement models and thus of likely different composition of the underground economy across the three Balkans' countries.

The "causes"of the underground economy also differ across countries, but their effect seems to be very small in terms of coefficients' magnitude and statistical significance. However, it is interesting to note that wages (ΔW_t) seem to negatively affect the underground economy both in Croatia and Romania, while they have a significant and positive effect in Bulgaria. Inflation (ΔP_t), on the other hand, affects underground economy positively only in Croatia while its effect is negative in the other two countries. Differences across countries in signs and magnitudes of the effects are notable also for the interest rate (ΔR_t) and money (ΔM_t).

Scaling the latent scores with the estimated long-run or expected value of the underground economy share (Table 3.2), and expressing them as % of the market sector output, we obtain time series plots shown in Figure 1.

3.3.2 Underground economy and output growth

We specify an empirical model that captures dynamic impact of the underground economy on the output growth in the market sector as a special case of the general DSEM model. The structural part of the model includes $p = 4$ lags of output growth and a first difference of the underground economy

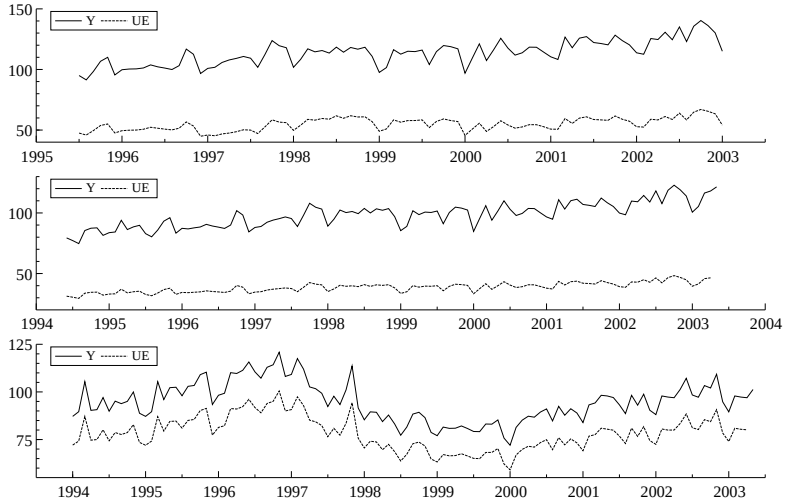

Figure 1: Market and non-market output

share. The "causes"of the underground economy enter without lags ($q = 0$). The measurement model for the underground economy is the same as in the MIMIC model where time-invariance is assumed and output growth is treated as perfectly observed. Therefore, we specify the structural part of the model in the latent form as

$$\begin{pmatrix} \Delta Y_t \\ \Delta UE_t \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \sum_{j=1}^{p} \begin{pmatrix} \beta_{11}^j & \beta_{12}^j \\ 0 & \beta_{22}^j \end{pmatrix} \begin{pmatrix} \Delta Y_{t-j} \\ \Delta UE_{t-j} \end{pmatrix} + \sum_{j=0}^{q} \begin{pmatrix} 0 & \gamma_{12}^j & \gamma_{13}^j & \gamma_{14}^j \\ \gamma_{21}^j & \gamma_{22}^j & \gamma_{23}^j & \gamma_{24}^j \end{pmatrix} \begin{pmatrix} \Delta W_{t-j} \\ \Delta P_{t-j} \\ \Delta R_{t-j} \\ \Delta M1_{t-j} \end{pmatrix} + \begin{pmatrix} \zeta_{1t} \\ \zeta_{2t} \end{pmatrix},$$

and the measurement part of the model as

$$\begin{pmatrix} \Delta CM_t \\ \Delta U_t \\ \Delta C_t \\ \Delta Y_t \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha_3 \\ \alpha_4 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & \lambda_{22} \\ 0 & \lambda_{32} \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \Delta Y_t \\ \Delta UE_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ 0 \end{pmatrix}.$$

The corresponding OFS form of this model is thus given by the following equations

$$\begin{aligned}
\Delta Y_t &= \alpha_1 + \sum_{i=1,j=0}^{p} \left(\beta_{11}^{i}\Delta Y_{t-i} + \beta_{12}^{j}\Delta CM_{t-j}\right) \\
&\quad +\gamma_{12}^{0}\Delta P_t + \gamma_{13}^{0}\Delta R_t + \gamma_{14}^{0}\Delta M1_t + u_{1t} \\
\Delta CM_t &= \alpha_2 + \sum_{j=1}^{p} \beta_{22}^{j}\Delta CM_{t-j} \\
&\quad +\gamma_{21}^{0}\Delta W_t + \gamma_{22}^{0}\Delta P_t + \gamma_{23}^{0}\Delta R_t + \gamma_{24}^{0}\Delta M1_t + u_{2t} \\
\Delta U_t &= \alpha_3 + \lambda_{22}\Delta CM_t + u_{3t} \\
\Delta C_t &= \alpha_4 + \lambda_{32}\Delta CM_t + u_{4t}
\end{aligned}$$

We estimate the OFS equations with the OFS-FIML method described above. The estimation results are given in Table 3.3.2. The factor-loadings from the previously estimated MIMIC model (λ_2 and λ_3) are very similar to those from the MIMIC model which implies that the inclusion of dynamics and structural effects did not affect the measurement model. Similarly, the "causes"from the MIMIC model also have similar coefficients in the structural model.

The DSEM estimates confirm the previously reported MIMIC results regarding the hypotheses $\mathbf{H_1}$ and $\mathbf{H_2}$. Namely, the contemporaneous effects of the nominal interest rate (γ_{23}^{0}) on the underground economy is negative (hypothesis $\mathbf{H_1}$) for Bulgaria and Romania, while the sign is positive for Croatia. Schneider and Klinglmair (2004) report panel results where the sign of the inflation effect might differ between transitional and other countries, but they merely estimate the inflation effect on growth and do not consider its effect on the underground economy. Here we find that the effect of inflation (or interest rates) on the underground economy might be different among transitional countries.

The important inside from the DSEM results are reflected in the ability to test the hypothesis $\mathbf{H_3}$, which includes dynamic effects of up to four lags. The hypothesized negative effect of the underground economy on output growth ($\mathbf{H_3}$) is found for Croatia and Romania (coefficients β_{22}^{1}–β_{22}^{4}), while this effect appears to be generally positive for Bulgaria.

Table 4: DSEM model estimates

	FIML estimates					
	Bulgaria		Croatia		Romania	
Coefficient	Estimate	(SE)	Estimate	(SE)	Estimate	(SE)
β_{11}^{1}	–0.4640	(0.1017)	–0.5998	(0.1090)	–0.3362	(0.1202)
β_{11}^{2}	–0.5148	(0.1048)	–0.6119	(0.1079)	–0.3965	(0.1154)
β_{11}^{3}	–0.4140	(0.1070)	–0.4013	(0.1068)	–0.0968	(0.1196)
β_{11}^{4}	–0.3124	(0.0997)	–0.2201	(0.0966)	–0.2254	(0.1167)
β_{12}^{0}	–2.7930	(1.9947)	–28.2280	(10.1520)	5.8369	(3.8616)
β_{12}^{1}	2.4853	(1.8460)	–6.3435	(3.3250)	1.4059	(1.1458)
β_{12}^{2}	8.5366	(3.7199)	16.7490	(7.0060)	–7.5948	(3.1355)
β_{12}^{3}	–10.6530	(4.8818)	23.4910	(11.4100)	–7.5940	(3.5974)
γ_{12}^{0}	–0.0210	(0.0216)	0.0239	(1.0422)	0.3294	(0.1866)
γ_{13}^{0}	0.0058	(0.0048)	–0.1772	(0.5421)	–0.0334	(0.1006)
γ_{14}^{0}	0.0040	(0.0033)	0.1163	(0.0636)	–0.0064	(0.0099)
β_{22}^{1}	0.0476	(0.0851)	–0.1324	(0.0736)	–0.3293	(0.0837)
β_{22}^{2}	0.1639	(0.0837)	–0.0819	(0.0722)	–0.2986	(0.0875)
β_{22}^{3}	–0.0030	(0.0822)	0.0607	(0.0691)	–0.0535	(0.0822)
β_{22}^{4}	0.1410	(0.0836)	–0.1618	(0.0777)	–0.1403	(0.0772)
γ_{21}^{0}	0.0045	(0.0013)	–0.0018	(0.0009)	–0.0010	(0.0009)
γ_{22}^{0}	–0.0004	(0.0003)	0.0049	(0.0057)	–0.0014	(0.0025)
γ_{23}^{0}	–0.0001	(0.0001)	0.0028	(0.0030)	–0.0006	(0.0014)
γ_{24}^{0}	0.0002	(0.0000)	–0.0004	(0.0003)	–0.0007	(0.0002)
λ_{22}	–7.0331	(5.7756)	13.5410	(17.0030)	8.8110	(4.9852)
λ_{32}	14.7880	(7.5364)	–19.7070	(42.0220)	0.5386	(0.8504)
χ^2	43.3215		74.3573		52.7598	
(d.f.)	27		27		27	

The overall significance of these dynamic effects can be obtained by testing for Granger causality. For the given data length we can only use simple (bivariate) Granger causality test applied on the factor scores of the latent underground economy estimated from the MIMIC model. Such approach was proposed by Giles et al. (1999), with the difference that we compute latent scores using a formal procedure described in Jöreskog (2000).

Results from the pairwise Granger causality tests using latent scores and including 10 lags for testing the temporal causal effect of the underground

economy on output growth (using first differences) indicate highly significant effect in Croatia, and somewhat less significant effect in Romania, while in Bulgaria there does not appear to be any evidence of Granger causality (see Table 5). Note that these results thus support the hypothesis $\mathbf{H_3}$

Table 5: Granger causality tests

Country	F-test (d.f.)	F-value	F-probability
Bulgaria	F(10, 59)	1.0869	0.1869
Croatia	F(10, 75)	4.3711	0.0001
Romania	F(10, 80)	1.1646	0.0269

In the literature, generally the effect of the underground economy on growth is considered to be ambiguous. Loayza (1996) points out that this effect might be negative in the economies with larger then optimal statutory tax burden and where the enforcement of compliance is weak. Transitional and developing countries would therefore be expected to have such negative relationship between the underground economy and growth (e.g. Loayza (1996) finds a negative relationship among Latino-American countries). However, this cannot easily explain the empirical lack of a significantly negative effect in Bulgaria as opposite to Croatia and Romania, as it would be difficult to argue that Bulgarian statutory tax burden and enforcement system are substantially better then those in Croatia and Romania.

Schneider and Klinglmair (2004), on the other hand, find Loayza (1996)'s conclusions overly dependent on unrealistic assumptions[11] and suggest a possibility that the underground economy responds to the economic environment, namely to the demand for urban services and small-scale manufacturing. Given a substantial small and medium enterprizes (SME) sector in transitional countries[12] and its varying magnitude across the countries, the possible link between the underground economy's effect on growth and

[11]For example, Loayza (1996) assumes dependence of the production technology on tax-financed public services, which are in turn subject to congestion and which are in not financed by penalties paid by the informal sector.

[12]This is especially true in Croatia, see e.g. Cziráky et al. (2003a))

enterprize-type structure of the economy is interesting. Furthermore, Schneider and Klinglmair (2004) suggest that the voluntary self-selection between the formal and informal sectors might induce higher growth by creating a higher potential for economic development.

While ambiguous on the ultimate sign of the effect of the underground economy on growth, this literature does suggest a possibility that the sign of the effect depends on the development level and/or enterprize structure of the economy. In a large panel of both developed and developing countries Schneider and Klinglmair (2004) find a negative effect of the underground economy on growth in developing and transitional countries, while the effect is positive for the developed countries.

However, there are two important aspects that need to be taken into consideration. Firstly, the estimates Schneider and Klinglmair (2004) report use a panel with a dominant cross-sectional dimension (104 countries over ten annual time points) thus any time dynamics and differences between long- and short-run effects cannot be taken into consideration. However, the temporal dimension would have to be a long-run one due to the used data frequency, yet the time span is limited to 10 years. Due to similar data limitations but with monthly frequency we were able to estimate short-run effects up to one quarter (four monthly lags), but extending our results to the long-run would be rather informal.

Secondly, the underground economy estimates used by Schneider and Klinglmair (2004) were obtained from cross-sectional data generally treating the analysed countries as random observations, which is highly problematic. Namely, the underground economy estimates were used to test for the possible differences among countries in terms of the underground economy's effect on growth, and indeed such differences were found. Yet, the estimates of the underground economy used by Schneider and Klinglmair (2004) were not obtained from time series data for each country separately. This problem is particularly acute when structural equation models (e.g. MIMIC) are used. In addition, a general problem with "growth regressions" using estimated latent scores for the underground economy share is that the estimated standard errors are incorrect as they do not take into account the fact that the scores

were estimated in another model. We note that the DSEM estimates reported in this paper avoid the problem with incorrect standard errors, however the Granger-causality tests do use estimated scores and are thus somewhat less formal.

4 Conclusion

The paper investigated the effect of inflation on the growth rate in economies with underground, or "non-market", sectors. A model that incorporates a non-market good into an endogenous growth cash-in-advance economy with human capital was used where taxes on labor and capital induced substitution into the non-market sector.

The paper presented a dynamic general equilibrium analysis of how inflation can effect growth in economies with shadow sectors. Then the analysis was backed up with extensive econometric testing using transition country data.

The departure point for the analysis was an exogenous growth economy without money or any taxes, but with a role for development through a differing cost of capital depending on development level.

We adopt an econometric framework that enables both measurement of the latent underground economy series and estimation of the dynamic structural equation models that include underground economy as a latent variable. Moreover, we use the theoretically-derived equilibrium conditions to estimate the mean of the underground economy, which is needed in setting the scale and range of the estimated latent underground economy variable. Furthermore, we estimate a dynamic structural equation model and investigate the effects of the underground economy on output growth and test for Granger causality using bivariate Granger-causality tests. We find some evidence in support of the theoretically-implied negative sign of the effect of the nominal interest rates and the inflation rate for Bulgaria and Romania. The negative effect of the underground economy on output growth implied by our theoretical model was supported in the dynamic context and we find significant evidence of Granger causality for Croatia and Romania, while this effect

failed to reach statistical significance for Bulgaria.

Finally, we note that full implications of the theoretical model require further empirical testing, which calls for more detailed data that would include information on taxes and capital. This would require a large panel data set since time series data generally does not provide sufficient variability in taxes, thus suggesting an interesting extension of the empirical results reported in this paper.

Appendix

A Derivation of the scale-calibration equation

Firstly, we need to make few simplifying assumptions in order to deal with the otherwise untractable non-linearities in the equilibrium conditions. Note that in (1) and (2) the aggregate consumption good and the current period utility function were parameterised with the utility function parameters ν and ε. Without loss of generality, we can assume an admissible value for ε, namely $\varepsilon = 2$, in which case the aggregate consumption good $c_t = [\nu c_{mt}^{\varepsilon} + (1-\nu)c_{nt}^{\varepsilon}]^{1/\varepsilon}$ assumes a convenient form of a weighted Euclidean norm in c_{mt} and c_{nt}, i.e. $c_t = \sqrt{\nu c_{mt}^2 + (1-\nu)c_{nt}^2}$. In addition, we set the γ parameter in the production function (19) to its mean admissible value of 0.5, recalling that $\gamma \in (0,1)$.

Now, we derive the expression for the underground economy using the equilibrium condition (40). We have

$$\left(\frac{c_{mt}}{c_{nt}}\right)^{\varepsilon-1}\left(\frac{\nu}{1-\nu}\right) = \frac{1 + a_t R_t + (1-a_t)\gamma R_t}{p_{nt}(1+R_t)},$$

which can be re-written by setting $\varepsilon = 2$ as

$$c_{nt} = \left(\frac{1-\nu}{\nu}\right)\frac{c_{mt}p_{nt}(1+R_t)}{1 + a_t R_t + (1-a_t)\gamma R_t}.$$

To write this expression in terms of the total underground economy we need to multiply it with p_{nt}, which gives

$$c_{nt}p_{nt} = \left(\frac{1-\nu}{\nu}\right)\frac{c_{mt}p_{nt}^2(1+R_t)}{1+a_tR_t+(1-a_t)\gamma R_t}.$$

This, however, leaves us with the square of p_{nt}, which is a latent quantity. To proceed we need another assumption, namely that the price level in the non-market sector is approximately equal to the price level in the market sector. Alternatively, we might assume that these two variables are linearly related i.e. $p_{nt} = \alpha p_{mt}$. Using the simplest case with $\alpha = 1$ and defining $c_{mt}p_{mt} \equiv y_t$, where y_t denotes output in the market sector, we get

$$c_{nt}p_{nt} = \left(\frac{1-\nu}{\nu}\right)\frac{y_tp_{mt}(1+R_t)}{1+a_tR_t+(1-a_t)\gamma R_t},$$

which gives us the expression for the total non-market good i.e. underground economy. Now we need to substitute the equation for a_t from (44), which we simplify by setting $\gamma = 0.5$, hence $\frac{\gamma}{1-\gamma} = 1$ and $\frac{1}{1-\gamma} = 2$ and therefore

$$\begin{aligned} 1-a_t &= \left(\frac{\gamma R_t(1+\tau_c)}{w_t(1-\tau_l)}\right)^{\gamma/(1-\gamma)} A^{1/(1-\gamma)} \\ &= \frac{\frac{1}{2}R_t(1+\tau_c)}{w_t(1-\tau_l)}A^2 \\ &= \frac{\beta R_t}{\delta w_t}, \end{aligned}$$

where $\beta \equiv \frac{1}{2}A^2(1+\tau_c)$ and $\delta \equiv (1-\tau_l)$, i.e., we treat τ_c and τ_l as constants.[13]

We can now complete the expression for the non-market output as

[13]Note that tax rates generally do not differ across time within particular countries, and since our focus is on time series data the assumption of constant or time-invariant τ_c and τ_l is reasonable.

$$
\begin{aligned}
c_{nt}p_{nt} &= \left(\frac{1-\nu}{\nu}\right)\frac{y_t p_{mt}(1+R_t)}{1+a_t R_t+(1-a_t)\gamma R_t} \\
&= \left(\frac{1-\nu}{\nu}\right)\frac{y_t p_{mt}(1+R_t)}{1+\left(1-\frac{\beta R_t}{\delta w_t}\right)R_t+\frac{\beta R_t^2}{2\delta w_t}} \\
&= \left(\frac{1-\nu}{\nu}\right)\frac{y_t p_{mt}(1+R_t)}{1+R_t-\frac{\beta R_t^2}{\delta w_t}+\frac{\beta R_t^2}{2\delta w_t}} \\
&= \left(\frac{1-\nu}{\nu}\right)\frac{y_t p_{mt}(1+R_t)}{1+R_t-\frac{\beta R_t^2}{2\delta w_t}} \\
&= \left(\frac{1-\nu}{\nu}\right)\frac{y_t p_{mt}}{1-\frac{\beta R_t^2}{2\delta w_t(1+R_t)}}
\end{aligned}
$$

The above derived equation, while providing an expression for the non-market goods output, does not have an additive intercept that would mimic its average or long-run value. This problem can be solved by taking logarithms which yields

$$
\ln(c_{nt}p_{nt}) = \ln\left(\frac{1-\nu}{\nu}\right)+\ln(y_t p_{mt})-\ln\left(1-\frac{\beta R_t^2}{2\delta w_t(1+R_t)}\right). \tag{80}
$$

We can solve the equilibrium condition (43) for $c_{nt}p_{nt}$ in a similar fashion using the above expression for a_t recalling that $\beta \equiv \frac{1}{2}A^2(1+\tau_c)$ hence $(1+\tau_c) = 2\beta/A^2$. Therefore,

$$
\begin{aligned}
\frac{m_t}{c_{mt}} &= \left(a_t+\frac{p_{nt}c_{nt}}{c_{mt}}\right)(1+\tau_c) \\
&\Rightarrow \quad m_t = \frac{2\beta}{A^2}\left[c_{mt}\left(1-\frac{\beta R_t}{\delta w_t}\right)+c_{nt}p_{nt}\right] \\
&\Rightarrow \quad \frac{2\beta}{A^2}c_{nt}p_{nt} = m_t-\frac{2\beta}{A^2}c_{mt}\left(1-\frac{\beta R_t}{\delta w_t}\right) \\
&\Rightarrow \quad c_{nt}p_{nt} = \frac{A^2}{2\beta}m_t-c_{mt}\left(1-\frac{\beta R_t}{\delta w_t}\right).
\end{aligned}
$$

Taking logarithms this becomes

$$
\ln(c_{nt}p_{nt}) = \ln\left[\frac{A^2}{2\beta}m_t-c_{mt}\left(1-\frac{\beta R_t}{\delta w_t}\right)\right]. \tag{81}
$$

Finally, by equating (80) to (81) we can derive an econometric estimation equation that does not contain any unobservable variables, i.e.,

$$\ln\left(\frac{1-\nu}{\nu}\right)+\ln(y_t p_{mt})-\ln\left(1-\frac{\beta R_t^2}{2\delta w_t(1+R_t)}\right)=\ln\left[\frac{A^2}{2\beta}m_t-c_{mt}\left(1-\frac{\beta R_t}{\delta w_t}\right)\right],$$

which can be simplified as follows

$$\begin{aligned}
\ln(y_t p_{mt}) &= \ln\left[\frac{A^2}{2\beta}m_t-c_{mt}\left(1-\frac{\beta R_t}{\delta w_t}\right)\right]+\ln\left(1-\frac{\beta R_t^2}{2\delta w_t(1+R_t)}\right)-\ln\left(\frac{1-\nu}{\nu}\right)\\
&= \ln\left\{\left[\frac{A^2}{2\beta}m_t-c_{mt}\left(1-\frac{\beta R_t}{\delta w_t}\right)\right]\left(1-\frac{\beta R_t^2}{2\delta w_t(1+R_t)}\right)\right\}-\ln\left(\frac{1-\nu}{\nu}\right)\\
&= \ln\left[\frac{A^2}{2\beta}m_t-\frac{A^2 m_t R_t^2}{4\delta w_t(1+R_t)}-c_{mt}\left(1-\frac{\beta R_t}{\delta w_t}\right)+c_{mt}\left(1-\frac{\beta R_t}{\delta w_t}\right)\frac{\beta R_t^2}{2\delta w_t(1+R_t)}\right]\\
&- \ln\left(\frac{1-\nu}{\nu}\right)\\
&= \ln\left[\frac{A^2}{2\beta}m_t-\frac{A^2 m_t R_t^2}{4\delta w_t(1+R_t)}-c_{mt}+\frac{\beta c_{mt}R_t}{\delta w_t}+\left(c_{mt}-\frac{\beta c_{mt}R_t}{\delta w_t}\right)\frac{\beta R_t^2}{2\delta w_t(1+R_t)}\right]\\
&- \ln\left(\frac{1-\nu}{\nu}\right)\\
&= \ln\left[\frac{A^2}{2\beta}m_t-\frac{A^2 m_t R_t^2}{4\delta w_t(1+R_t)}-c_{mt}+\frac{\beta c_{mt}R_t}{\delta w_t}+\frac{\beta c_{mt}R_t^2}{2\delta w_t(1+R_t)}-\frac{\beta^2 c_{mt}R_t^3}{2\delta^2 w_t^2(1+R_t)}\right]\\
&- \ln\left(\frac{1-\nu}{\nu}\right)\\
&= \ln\left[\frac{A^2}{2\beta}m_t-\frac{A^2 m_t R_t^2}{4\delta w_t(1+R_t)}-c_{mt}+\frac{\beta\left(c_{mt}(1+R_t)R_t+\frac{1}{2}c_{mt}R_t^2\right)}{\delta w_t(1+R_t)}-\frac{\beta^2 c_{mt}R_t^3}{2\delta^2 w_t^2(1+R_t)}\right]\\
&- \ln\left(\frac{1-\nu}{\nu}\right),
\end{aligned}$$

which gives an econometric estimation equation. We can simplify the notation by defining the following coefficients $\alpha_0 \equiv -\ln[(1-\nu)/\nu]$, $\alpha_1 \equiv A^2/\beta$, $\alpha_2 \equiv -A^2/\delta$, $\alpha_3 \equiv \beta/\delta$, $\alpha_4 \equiv -\beta^2/\delta^2$. Similarly, define

$$x_1 \equiv \frac{1}{2}m_t,\ x_2 \equiv \frac{m_t R_t^2}{4w_t(1+R_t)},\ x_3 \equiv \frac{c_{mt}(1+R_t)R_t+\frac{1}{2}c_{mt}R_t^2}{w_t(1+R_t)},\ x_4 \equiv \frac{c_{mt}R_t^3}{2w_t^2(1+R_t)}.$$

Using the above definitions, the derived estimation equation becomes

$$\ln(y_t p_{mt}) = \alpha_0 + \ln\left(\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 - c_{mt}\right). \tag{82}$$

B Derivation of the OFS equations

We show that the general DSEM model (68)–(70) can be written in the *observed form specification* (OFS) that consists of the observed variables and latent errors only.

The OFS uses the fact that in the measurement model for each latent variable one loading can be fixed to one without loss of generality. Thus, we can re-write the measurement models for $\mathbf{x}_t$ and $\mathbf{y}_t$ as

$$\mathbf{x}_t = \begin{pmatrix} \mathbf{x}_{1t} \\ \mathbf{x}_{2t} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \boldsymbol{\alpha}_2^{(x)} \end{pmatrix} + \begin{pmatrix} \mathbf{I} \\ \boldsymbol{\Lambda}_2^{(x)} \end{pmatrix} \boldsymbol{\xi}_t + \begin{pmatrix} \boldsymbol{\delta}_{1t} \\ \boldsymbol{\delta}_{2t} \end{pmatrix} \tag{83}$$

and

$$\mathbf{y}_t = \begin{pmatrix} \mathbf{y}_{1t} \\ \mathbf{y}_{2t} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \boldsymbol{\alpha}_2^{(y)} \end{pmatrix} + \begin{pmatrix} \mathbf{I} \\ \boldsymbol{\Lambda}_2^{(y)} \end{pmatrix} \boldsymbol{\eta}_t + \begin{pmatrix} \boldsymbol{\varepsilon}_{1t} \\ \boldsymbol{\varepsilon}_{2t} \end{pmatrix} \tag{84}$$

Note that the observed indicators with unit loadings were placed in the top part of the vectors for $\mathbf{x}_t$ and $\mathbf{y}_t$ and thus the upper part of the lambda matrix is an identity matrix. Having divided $\mathbf{x}_t$ into $\mathbf{x}_{t1}$ and $\mathbf{x}_{t2}$, note that for $\mathbf{x}_{t1}$ it holds that

$$\mathbf{x}_{1t} = \boldsymbol{\xi}_t + \boldsymbol{\delta}_{1t} \Rightarrow \boldsymbol{\xi}_t = \mathbf{x}_{1t} - \boldsymbol{\delta}_{1t} \tag{85}$$

and, similarly, for $\mathbf{y}_{t1}$ we can replace the latent variable with its unit-loading indicators

$$\mathbf{y}_{1t} = \boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_{1t} \Rightarrow \boldsymbol{\eta}_t = \mathbf{y}_{1t} - \boldsymbol{\varepsilon}_{1t} \tag{86}$$

It is now possible to use the relations in (85) and (86) to re-write the measurement model for $\mathbf{x}_t$ as

$$\begin{aligned} \mathbf{x}_{2t} &= \boldsymbol{\alpha}_2^{(x)} + \boldsymbol{\Lambda}_2^{(x)} \left(\mathbf{x}_{1t} - \boldsymbol{\delta}_{1t}\right) + \boldsymbol{\delta}_{2t} \\ &= \boldsymbol{\alpha}_2^{(x)} + \boldsymbol{\Lambda}_2^{(x)} \mathbf{x}_{1t} + \left(\boldsymbol{\delta}_{2t} - \boldsymbol{\Lambda}_2^{(x)} \boldsymbol{\delta}_{1t}\right) \end{aligned} \tag{87}$$

and for $\mathbf{y}_t$ as

$$\begin{aligned}\mathbf{y}_{2t} &= \boldsymbol{\alpha}_2^{(y)} + \boldsymbol{\Lambda}_2^{(y)}\left(\mathbf{y}_{1t} - \boldsymbol{\varepsilon}_{1t}\right) + \boldsymbol{\varepsilon}_{2t} \\ &= \boldsymbol{\alpha}_2^{(y)} + \boldsymbol{\Lambda}_2^{(y)}\mathbf{y}_{1t} + \left(\boldsymbol{\varepsilon}_{2t} - \boldsymbol{\Lambda}_2^{(y)}\boldsymbol{\varepsilon}_{1t}\right)\end{aligned} \tag{88}$$

Following the same principle it is possible to re-write the structural part of the model using definitions (85) and (86) as follows

$$\mathbf{y}_{1t} - \boldsymbol{\varepsilon}_{1t} = \boldsymbol{\alpha}_\eta + \sum_{j=0}^{p}\mathbf{B}_j(\mathbf{y}_{1t-j} - \boldsymbol{\varepsilon}_{1t-j}) + \sum_{j=0}^{q}\boldsymbol{\Gamma}_j(\mathbf{x}_{1t-j} - \boldsymbol{\delta}_{1t-j}) + \boldsymbol{\zeta}_t. \tag{89}$$

Separating the observed part of the model from the latent errors we obtain

$$\mathbf{y}_{1t} = \boldsymbol{\alpha}_\eta + \sum_{j=0}^{p}\mathbf{B}_j\mathbf{y}_{1t-j} + \sum_{j=0}^{q}\boldsymbol{\Gamma}_j\mathbf{x}_{1t-j} + \left(\boldsymbol{\zeta}_t + \boldsymbol{\varepsilon}_{1t} - \sum_{j=0}^{p}\mathbf{B}_j\boldsymbol{\varepsilon}_{1t-j} - \sum_{j=0}^{q}\boldsymbol{\Gamma}_j\boldsymbol{\delta}_{1t-j}\right), \tag{90}$$

with the measurement model for the latent endogenous variables

$$\mathbf{y}_{2t} = \boldsymbol{\alpha}_2^{(y)} + \boldsymbol{\Lambda}_2^{(y)}\mathbf{y}_{1t} + \left(\boldsymbol{\varepsilon}_{2t} - \boldsymbol{\Lambda}_2^{(y)}\boldsymbol{\varepsilon}_{1t}\right), \tag{91}$$

and for the latent exogenous variables

$$\mathbf{x}_{2t} = \boldsymbol{\alpha}_2^{(x)} + \boldsymbol{\Lambda}_2^{(x)}\mathbf{x}_{1t} + \left(\boldsymbol{\delta}_{2t} - \boldsymbol{\Lambda}_2^{(x)}\boldsymbol{\delta}_{1t}\right). \tag{92}$$

Aside of the specific structure of the latent error terms, (90)–(92) present a classical structural equation system with observed variables. This completes the derivation of the OFS equations.

C Derivation of the OFS-FIML estimator

We consider estimation of the OFS equations (68)–(70) with full-information maximum likelihood methods based on the multivariate Gaussian likelihood.

To set the notation first let $\mathbf{u}_{1t} \equiv \boldsymbol{\zeta}_t + \boldsymbol{\varepsilon}_{1t} - \sum_{j=0}^{p}\mathbf{B}_j\boldsymbol{\varepsilon}_{1t-j} - \sum_{j=0}^{q}\boldsymbol{\Gamma}_j\boldsymbol{\delta}_{1t-j}$, $\mathbf{u}_{2t} \equiv \boldsymbol{\varepsilon}_{2t} - \boldsymbol{\Lambda}_2^{(y)}\boldsymbol{\varepsilon}_{1t}$, and $\mathbf{u}_{3t} \equiv \boldsymbol{\delta}_{2t} - \boldsymbol{\Lambda}_2^{(x)}\boldsymbol{\delta}_{1t}$. Then the structural OFS equations can be written as

$$
\begin{aligned}
\mathbf{y}_{1t} &= \boldsymbol{\alpha}_\eta + \sum_{j=1}^{p} \mathbf{B}_j \mathbf{y}_{1t-j} + \sum_{j=0}^{q} \boldsymbol{\Gamma}_j \mathbf{x}_{1t-j} + \mathbf{u}_{1t} \\
\mathbf{y}_{2t} &= \boldsymbol{\alpha}_2^{(y)} + \boldsymbol{\Lambda}_2^{(y)} \mathbf{y}_{1t} + \mathbf{u}_{2t} \\
\mathbf{x}_{2t} &= \boldsymbol{\alpha}_2^{(x)} + \boldsymbol{\Lambda}_2^{(x)} \mathbf{x}_{1t} + \mathbf{u}_{3t}.
\end{aligned}
$$

This can be re-written as

$$
\begin{aligned}
\begin{pmatrix} \mathbf{y}_{1t} \\ \mathbf{y}_{2t} \\ \mathbf{x}_{2t} \end{pmatrix} &= \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{\Lambda}_2^{(y)} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Lambda}_2^{(x)} \end{pmatrix} \begin{pmatrix} \mathbf{y}_{1t} \\ \mathbf{y}_{2t} \\ \mathbf{x}_{2t} \end{pmatrix} \\
&+ \begin{pmatrix} \boldsymbol{\alpha}_\eta & \mathbf{B}_1 & \cdots & \mathbf{B}_p & \boldsymbol{\Gamma}_1 & \cdots & \boldsymbol{\Gamma}_q \\ \boldsymbol{\alpha}_2^{(y)} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{\alpha}_2^{(x)} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{I} \\ \mathbf{y}_{1t-1} \\ \vdots \\ \mathbf{y}_{1t-p} \\ \mathbf{x}_{1t-1} \\ \vdots \\ \mathbf{x}_{1t-q} \end{pmatrix} + \begin{pmatrix} \mathbf{u}_{1t} \\ \mathbf{u}_{2t} \\ \mathbf{u}_{3t} \end{pmatrix},
\end{aligned}
$$

which has a reduced form given by

$$
\begin{aligned}
\begin{pmatrix} \mathbf{y}_{1t} \\ \mathbf{y}_{2t} \\ \mathbf{x}_{2t} \end{pmatrix} &= \begin{pmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{\Lambda}_2^{(y)} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \left(\mathbf{I} - \boldsymbol{\Lambda}_2^{(x)}\right)^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{\alpha}_\eta & \mathbf{B}_1 & \cdots & \mathbf{B}_p & \boldsymbol{\Gamma}_1 & \cdots & \boldsymbol{\Gamma}_q \\ \boldsymbol{\alpha}_2^{(y)} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{\alpha}_2^{(x)} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{I} \\ \mathbf{y}_{1t-1} \\ \vdots \\ \mathbf{y}_{1t-p} \\ \mathbf{x}_{1t-1} \\ \vdots \\ \mathbf{x}_{1t-q} \end{pmatrix} \\
&+ \begin{pmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{\Lambda}_2^{(y)} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \left(\mathbf{I} - \boldsymbol{\Lambda}_2^{(x)}\right)^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{u}_{1t} \\ \mathbf{u}_{2t} \\ \mathbf{u}_{3t} \end{pmatrix}.
\end{aligned}
$$

Simplifying and multiplying out the matrices, the reduced form becomes

$$\begin{pmatrix} \mathbf{y}_{1t} \\ \mathbf{y}_{2t} \\ \mathbf{x}_{2t} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\alpha}_\eta & \mathbf{B}_1 & \cdots & \mathbf{B}_p & \boldsymbol{\Gamma}_1 & \cdots & \boldsymbol{\Gamma}_q \\ \boldsymbol{\Lambda}_2^{(y)}\boldsymbol{\alpha}_\eta + \boldsymbol{\alpha}_2^{(y)} & \boldsymbol{\Lambda}_2^{(y)}\mathbf{B}_1 & \cdots & \boldsymbol{\Lambda}_2^{(y)}\mathbf{B}_p & \boldsymbol{\Lambda}_2^{(y)}\boldsymbol{\Gamma}_1 & \cdots & \boldsymbol{\Lambda}_2^{(y)}\boldsymbol{\Gamma}_q \\ \left(\mathbf{I} - \boldsymbol{\Lambda}_2^{(x)}\right)^{-1}\boldsymbol{\alpha}_2^{(x)} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{I} \\ \mathbf{y}_{1t-1} \\ \vdots \\ \mathbf{y}_{1t-p} \\ \mathbf{x}_{1t-1} \\ \vdots \\ \mathbf{x}_{1t-q} \end{pmatrix}$$

$$+ \begin{pmatrix} \mathbf{u}_{1t} \\ \boldsymbol{\Lambda}_2^{(y)}\mathbf{u}_{1t} + \mathbf{u}_{2t} \\ \left(\mathbf{I} - \boldsymbol{\Lambda}_2^{(x)}\right)^{-1}\mathbf{u}_{3t} \end{pmatrix}.$$

We now make the following definitions;

$$\mathbf{y}_t = \begin{pmatrix} \mathbf{y}_{1t} \\ \mathbf{y}_{2t} \\ \mathbf{x}_{2t} \end{pmatrix}, \quad \mathbf{z}_t \equiv \begin{pmatrix} \mathbf{u}_{1t} \\ \boldsymbol{\Lambda}_2^{(y)}\mathbf{u}_{1t} + \mathbf{u}_{2t} \\ \left(\mathbf{I} - \boldsymbol{\Lambda}_2^{(x)}\right)^{-1}\mathbf{u}_{3t} \end{pmatrix}$$

$$\boldsymbol{\Pi} \equiv \begin{pmatrix} \alpha_\eta & \mathbf{B}_1 & \cdots & \mathbf{B}_p & \boldsymbol{\Gamma}_1 & \cdots & \boldsymbol{\Gamma}_q \\ \boldsymbol{\Lambda}_2^{(y)}\alpha_\eta + \alpha_2^{(y)} & \boldsymbol{\Lambda}_2^{(y)}\mathbf{B}_1 & \cdots & \boldsymbol{\Lambda}_2^{(y)}\mathbf{B}_p & \boldsymbol{\Lambda}_2^{(y)}\boldsymbol{\Gamma}_1 & \cdots & \boldsymbol{\Lambda}_2^{(y)}\boldsymbol{\Gamma}_q \\ \left(\mathbf{I} - \boldsymbol{\Lambda}_2^{(x)}\right)^{-1}\alpha_2^{(x)} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}$$

$$\text{and} \quad \mathbf{x}_t = \begin{pmatrix} \mathbf{y}'_{1t-1} & \cdots & \mathbf{y}'_{1t-p} & \mathbf{x}'_{1t-1} & \cdots & \mathbf{x}'_{1t-q} \end{pmatrix}'.$$

If we assume that $\mathbf{u} \sim$ i.i.d. $N(\mathbf{0}, \boldsymbol{\Omega}_u)$, where $\mathbf{u} = (\mathbf{u}'_{1t} : \mathbf{u}'_{3t} : \mathbf{u}'_{3t})'$ it follows that $\mathbf{z}_t \sim$ i.i.d. $N(\mathbf{0}, \boldsymbol{\Lambda}\boldsymbol{\Omega}_u\boldsymbol{\Lambda}')$, where

$$\boldsymbol{\Lambda} \equiv \begin{pmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{\Lambda}_2^{(y)} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \left(\mathbf{I} - \boldsymbol{\Lambda}_2^{(x)}\right)^{-1} \end{pmatrix}.$$

To simplify the notation, we define $\boldsymbol{\Omega}_z \equiv \boldsymbol{\Lambda}\boldsymbol{\Omega}_u\boldsymbol{\Lambda}'$. Therefore, the (conditional) OFS-FIML log likelihood is given by

$$L_{DSEM} = -\frac{m}{2}\ln(2\pi) + \frac{1}{2}\ln\left(\left|\boldsymbol{\Omega}_z^{-1}\right|\right) - \frac{1}{2T}\sum_{t=1}^{T}\mathbf{z}'_t\boldsymbol{\Omega}_z^{-1}\mathbf{z}_t.$$

or equivalently by

$$L_{DSEM} = -\frac{m}{2}\ln(2\pi) + \frac{1}{2}\ln\left(\left|\mathbf{\Omega}_z^{-1}\right|\right) - \frac{1}{2T}\sum_{t=1}^{T}(\mathbf{y}_t - \mathbf{\Pi}\mathbf{x}_t)'\mathbf{\Omega}_z^{-1}(\mathbf{y}_t - \mathbf{\Pi}\mathbf{x}_t).$$

$$\mathbf{\Omega}_z = \begin{pmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{\Lambda}_2^{(y)} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \left(\mathbf{I} - \mathbf{\Lambda}_2^{(y)}\right) \end{pmatrix} \begin{pmatrix} \mathbf{u}_{1t}\mathbf{u}'_{1t} & \mathbf{u}_{1t}\mathbf{u}'_{2t} & \mathbf{u}_{1t}\mathbf{u}'_{3t} \\ \mathbf{u}_{2t}\mathbf{u}'_{1t} & \mathbf{u}_{2t}\mathbf{u}'_{2t} & \mathbf{u}_{2t}\mathbf{u}'_{3t} \\ \mathbf{u}_{3t}\mathbf{u}'_{1t} & \mathbf{u}_{3t}\mathbf{u}'_{2t} & \mathbf{u}_{3t}\mathbf{u}'_{3t} \end{pmatrix} \begin{pmatrix} \mathbf{I} & \mathbf{\Lambda}_2^{(y)'} & \mathbf{0} \\ \mathbf{\Lambda}_2^{(y)} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \left(\mathbf{I} - \mathbf{\Lambda}_2^{(y)'}\right) \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{u}_{1t}\mathbf{u}'_{1t} & \mathbf{u}_{1t}\mathbf{u}'_{1t}\mathbf{\Lambda}_2^{(y)'} + \mathbf{u}_{1t}\mathbf{u}'_{2t} & \mathbf{u}_{1t}\mathbf{u}'_{3t} \quad \mathbf{I} - \mathbf{\Lambda}_2^{(y)'} \\ \mathbf{\Lambda}_2^{(y)}\mathbf{u}_{1t}\mathbf{u}'_{1t} + \mathbf{u}_{2t}\mathbf{u}'_{1t} & \mathbf{\Lambda}_2^{(y)}\mathbf{u}_{1t}\mathbf{u}'_{1t} + \mathbf{u}_{2t}\mathbf{u}'_{1t} \quad \mathbf{\Lambda}_2^{(y)'} + \mathbf{\Lambda}_2^{(y)}\mathbf{u}_{1t}\mathbf{u}'_{2t} + \mathbf{u}_{2t}\mathbf{u}'_{2t} & \mathbf{\Lambda}_2^{(y)}\mathbf{u}_{1t}\mathbf{u}'_{3t} + \mathbf{u}_{2t}\mathbf{u}'_{3t} \quad \mathbf{I} - \mathbf{\Lambda}_2^{(y)'} \\ \mathbf{I} - \mathbf{\Lambda}_2^{(y)} \quad \mathbf{u}_{3t}\mathbf{u}'_{1t} & \mathbf{I} - \mathbf{\Lambda}_2^{(y)} \quad \mathbf{u}_{3t}\mathbf{u}'_{1t}\mathbf{\Lambda}_2^{(y)'} + \mathbf{u}_{3t}\mathbf{u}'_{2t} & \mathbf{I} - \mathbf{\Lambda}_2^{(y)} \quad \mathbf{u}_{3t}\mathbf{u}'_{3t} \quad \mathbf{I} - \mathbf{\Lambda}_2^{(y)'} \end{pmatrix}$$

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