The Incentives of Intermediaries in Financial Markets: A Critical Analysis

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Contents

Preface

Intermediaries, such as credit rating agencies and fund managers, play an important role in financial markets. Credit rating agencies assess the probability that issuers repay their debt. Many investors pay attention to the ratings of the major credit rating agencies. Some fund managers are even restricted to invest in bonds with a specific rating. In addition, many regulators use the ratings of the major credit rating agencies to evaluate the amount of capital which financial institutions are required to hold. And fund managers manage a significant amount of assets.

Yet, the recent financial crisis reminds us that intermediaries, such as credit rating agencies and fund managers, may not always act in the best interest of investors, regulators and, eventually, tax payers. The major credit rating agencies, for instance, have assigned favorable ratings to structured debt products, such as mortgage-backed securities and collateralized-debt obligations. Favorable ratings facilitated the sale of structured debt products. The default rates, however, have been much higher than their initial rating would suggest. As a result, the major credit rating agencies have been blamed for contributing to the recent financial crisis.

It is often argued that credit rating agencies have an incentive to assign inflated ratings. The problem lies in the business model. The major credit rating agencies are paid by issuers who are interested in favorable ratings. This problem is exacerbated by the possibility of a collusive agreement. If credit rating agencies collude to offer inflated ratings, it may be impossible to detect whether high default rates are due to a collusive agreement or a common shock. In the wake of the recent financial crisis, the major credit rating agencies often pointed out that they all had used similar data, models and assumptions, and claimed that these had been the best available.

If credit rating agencies assign inflated ratings, the consequences may be serious. Many regulators, for instance, use the ratings of the major credit rating agencies to determine minimum capital requirements. The Securities and Exchange Commission, for example, designates Nationally Recognized Statistical Rating Organizations, and uses their ratings to determine how much capital financial institutions are required to hold. If the ratings then turn out to be inflated, minimum capital requirements turn out to be too low. And if the ratings turn out to be inflated only once default rates rise, asset prices fall, and a lot of capital is needed to absorb the losses, the consequences may be disastrous.

In the first chapter, we study a model in which a regulator approves credit rating agencies, which are paid by issuers. The regulator cannot observe whether ratings are correct. The regulator can only observe the default rate within a rating category for each credit rating agency. The default rate may, however, be influenced by a common shock, and the credit rating agencies may collude to assign inflated ratings. As a result, if all approved credit rating agencies collude to assign inflated ratings, the regulator cannot detect whether high default rates are due to a collusive agreement or a common shock.

We suggest that a regulator should hence not only deter a credit rating agency from unilaterally offering inflated ratings, but also provide an incentive to deviate from a collusive agreement to offer inflated ratings. In the model, the regulator can do so by denying approval to a credit rating agency with a worse performance than its competitors, and rewarding a credit rating agency which deviates from a collusive agreement by reducing the number of approved credit rating agencies in future periods.

In the wake of the subprime crisis, fund managers sometimes claimed that they had invested in structured debt products because they had relied on the ratings of the major credit rating agencies, and were surprised that the default rates were much higher than their initial rating would suggest. If they had known about the risks, they argued, they would not have invested in these assets.

In the second chapter, we argue, however, that fund managers might even had an incentive to invest in structured debt products if they knew about the risks. A fund's performance is usually compared to the performance of an index or other funds. If a fund trails the benchmark, the fund manager is often replaced. If investment restrictions are appropriate, this may sort out low-ability from high-ability fund managers. If, however, investment restrictions are inappropriate, this may lead to excessive risk-taking. To match the benchmark, fund managers may increase the risk of their portfolio even if this decreases the expected return on their portfolio.

Some fund managers, for example, are restricted to invest in AAA-rated bonds. If all AAA-rated bonds offer similar yields at a similar risk, benchmarking may sort out low-ability from high-ability fund managers. Suppose, however, some AAA-rated bonds, such as AAA-rated mortgage-backed securities, offer higher yields at a higher risk. Then, to match the benchmark, fund managers may invest in these bonds even if the expected return on these bonds is lower than the expected return on other, less risky AAA-rated bonds.

In the second chapter, we study a model in which fund managers differ in ability, and are fired if the realized return is lower than the average realized return. Fund managers can create a perfectly diversified portfolio. High-ability fund managers can create such a portfolio with a higher return than low-ability fund managers. In addition, however, fund managers have the opportunity to gamble. They can invest in a risky asset which increases the risk of the overall portfolio and decreases the expected return on the overall portfolio. We find that if the costs of being fired are sufficiently large, fund managers may invest in this risky asset to match the benchmark.

Though it is often argued that certifiers, such as credit rating agencies and auditors, may have an incentive to offer inflated certificates, they may not profit from offering inflated certificates. The reason is twofold. First, certifiers typically incur costs if they offer inflated certificates. Often they have to spend time and money to obscure that they offer inflated certificates. In addition, certifiers are usually caught with some probability if they offer inflated certificates. If they are caught, they often have to pay a fine. Moreover, they occasionally lose some of their business. In extreme cases, they even lose all of their business, as in the case of Arthur Andersen for its role in the Enron scandal. Secondly, the market may take into account that certifiers have an incentive to offer inflated certificates.

In the third chapter, we study a model in which a certifier is paid by sellers, and may offer them inflated certificates, but incurs costs if doing so. In contrast to the certifier, the buyers cannot observe the type of a good. The buyers can only observe whether the seller owns a certificate, and if so, the type of the certificate. The sellers are hence interested in favorable certificates.

We show that the certifier may face a commitment problem. The certifier always offers at least some inflated certificates if the costs of offering the first inflated certificate are lower than the sellers' willingness-to-pay for it. However, the certifier does not profit from offering inflated certificates, because in equilibrium, the buyers cannot be fooled. They anticipate correctly that the certifier offers a certain amount of inflated certificates, and take this into account in their willingness-to-pay for a good. The sellers, in turn, take this into account in their willingness-to-pay for a certificate. By offering inflated certificates, the certifier thus only incurs costs, but cannot increase its revenue as compared to the situation in which the certifier does not offer inflated certificates, and the buyers believe it. However, if the buyers actually believed the certifier did not offer inflated certificates, the certifier would have an incentive to do so.

Though the number of inflated certificates depends on the costs the certifier incurs, a certifier may yet oppose an increase in the costs of offering inflated certificates. On the one hand, an increase in the costs reduces the number of inflated certificates, and thus indirectly increases the certifier's profit. On the other hand, however, an increase in the costs directly reduces the certifier's profit. As a result, whether a certifier welcomes tighter regulation or lobbies against it, may depend on whether the new regulation only imposes higher costs on the certifier, or also helps to reduce the certifier's commitment problem significantly.

The following three chapters can be read independently of each other. Each chapter has its own introduction, appendix and references.

Chapter 1

The Regulation of Credit Rating Agencies*

1.1 Introduction

Financial regulators recognize certain credit rating agencies for regulatory purposes. The Securities and Exchange Commission (SEC), for instance, designates Nationally Recognized Statistical Rating Organizations (NRSROs) and uses the ratings of NRSROs to evaluate the amount of capital which financial institutions are required to hold.[1] In addition, many pension funds and other investors restrict their bond investments to bonds rated by a NRSRO.

However, it is often argued that credit rating agencies have an incentive to assign inflated ratings. Credit rating agencies assess the probability that issuers will default on their bonds. However, the major credit rating agencies are not paid by investors, but by issuers who are interested in high ratings. In addition, claiming that their ratings are independent expressions of opinion, credit rating

*A slightly different version of this chapter has been published in the Journal of Banking and Finance, 2009, Vol. 33 Issue 7, pp. 1266-1273.

[1] See e.g. SEC (2003, 2007).

agencies are immune to legal challenge. Lately, default rates of structured products, such as mortgage-backed securities and collateralized-debt obligations, have been much higher than their initial rating would suggest. As a result, credit rating agencies have been accused of assigning inflated ratings to structured products.

This paper studies a repeated principal-agent problem in which a regulator approves credit rating agencies. While credit rating agencies can observe an issuer's type, the regulator cannot. Credit rating agencies offer each issuer a rating and are paid by the issuers who demand a rating. The regulator cannot observe whether a credit rating agency assigns correct ratings. The regulator can only observe the default rate within a rating category for each credit rating agency. The default rate within a rating category does not only depend on whether a credit rating agency assigns correct ratings. The default rate can also be influenced by a common shock. Credit rating agencies may collude to offer inflated ratings. Yet we show that there exists an approval scheme which induces credit rating agencies to offer correct ratings.

The model shows that if credit rating agencies do not collude to offer inflated ratings, the regulator can filter out the common shock by evaluating the relative performance of credit rating agencies. If the credit rating agencies' discount factor is sufficiently high, the threat to deny approval in future periods can deter credit rating agencies from offering inflated ratings.

However, if all approved credit rating agencies collude to offer inflated ratings, the regulator cannot detect whether high default rates are due to collusion or the common shock. As a result, credit rating agencies may collude to offer inflated ratings.

The model shows that the regulator can prevent a collusive agreement to offer inflated ratings by providing an incentive to deviate. The model suggests that the regulator may reward a credit rating agency which deviates from such a collusive agreement by reducing the number of approved credit rating agencies in future periods.

The paper is related to the literature on relative performance evaluation first analyzed by Holmström (1982) and to the literature on collusion of certification intermediaries (e.g. Strausz, 2005; Peyrache and Quesada, 2010). Strausz (2005) and Peyrache and Quesada (2010) study incentives of a certification intermediary to collude with a seller of a product. Strausz (2005) derives conditions under which reputation enables certification intermediaries to resist capture and shows for instance that honest certification requires high prices and constitutes a natural monopoly. In contrast, Peyrache and Quesada (2010) focus on an equilibrium in which collusion may occur. They show that impatient intermediaries set lower prices in order to attract sellers with whom stakes for collusion are large. In contrast to our paper, Strausz (2005) and Peyrache and Quesada (2010) do not consider collusion between certification intermediaries. Strausz (2005) and Peyrache and Quesada (2010) moreover assume that buyers detect any collusion ex post. In our model, the regulator cannot observe whether a credit rating agency assigns correct ratings. The regulator can only observe the default rate within a rating category, which may be influenced by a common shock.

There is a growing theoretical literature on credit rating agencies. For example, Mählmann (2008) studies implications of rating publication rights and finds that there exists an equilibrium with partial nondisclosure of low ratings. Skreta and Veldkamp (2009) study rating shopping. They show that when issuers can choose from several ratings which rating to disclose, product complexity can lead to rating inflation, even if credit rating agencies produce unbiased ratings. In contrast to our paper, Mählmann (2008) and Skreta and Veldkamp (2009) do not focus on credit rating agencies' conflict of interest. Bolton et al. (2010) and Mathis et al. (2009) study this conflict. Bolton et al. (2010) find that credit rating agencies may assign inflated ratings when there are many naive investors or when (exogenous) reputation costs are low. Mathis et al. (2009) model reputation costs endogenously. They find that a credit rating agency may assign inflated ratings when a large fraction of the credit rating agency's income stems from rating complex products. In contrast to our paper, Bolton et al. (2010) and Mathis et al. (2009) do not consider collusion between credit rating agencies.

The rest of the paper is organized as follows. Section 1.2 presents the model. Section 1.3 shows that there exists an approval scheme which induces all credit rating agencies to offer correct ratings. Section 1.4 concludes. Proofs are provided in the appendix.

1.2 The Model

Consider a model with a regulator, several credit rating agencies (CRAs), and many issuers. While CRAs can observe an issuer's type, the regulator cannot.

In period 0, the regulator chooses an approval scheme. The approval $w_i^t \in \{0, 1\}$ of CRA i in period t, $t = 1, 2, \ldots$, can be made contingent on the default rates which the regulator has observed previously. If the regulator approves CRA i in period t, $w_i^t = 1$. If the regulator does not approve CRA i in period t, $w_i^t = 0$. Let n^t denote the number of approved CRAs in period t ($n^t = \sum_i w_i^t$).

Each period t, $t = 1, 2, \ldots$, consists of 3 stages. At stage 1, the regulator decides on the approval of CRAs according to the approval scheme. At stage 2, each CRA chooses a fee and offers a rating to each issuer. At stage 3, issuers decide whether and from which CRA to demand a rating. Figure 1.1 illustrates the time structure in period t, $t = 1, 2, \ldots$.

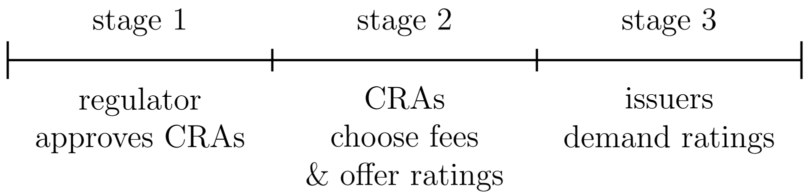

Figure 1.1: Time structure in period t, $t = 1, 2, \ldots$

At the beginning of period t, $t = 1, 2, \ldots$, a continuum of issuers enters. To simplify notation, its mass is normalized to 1. There are two types of issuers, A and B. If no shock occurs, type-A issuers have a low default probability d_A and type-B issuers have a high default probability d_B, where $0 < d_A < d_B < 1$. Let m denote the mass of type-A issuers, where $0 < m < 1$. In each period, issuers are uniformly located along the unit interval according to their type. Figure 1.2 illustrates this. While CRAs can observe an issuer's type and an issuer's location on the unit interval, the regulator cannot.

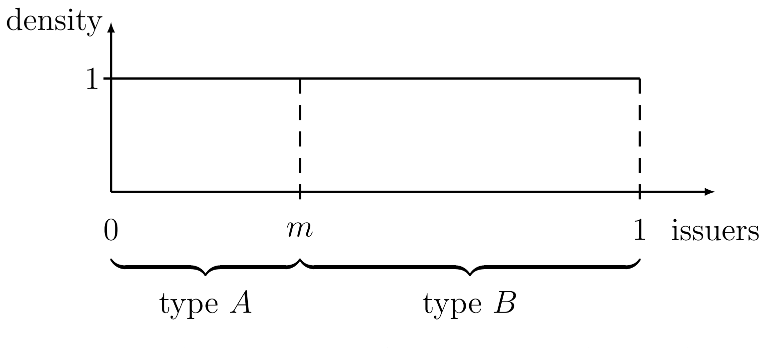

Figure 1.2: Issuers are located along the unit interval according to their type.

At stage 1, the regulator decides on the approval $w_i^t \in \{0, 1\}$ of CRA i according to the approval scheme. Approving a CRA for the first time generates approval costs c_A. Approval costs c_A may be interpreted as costs to establish a

CRA. Let $z_i^t \in \{0, 1\}$ denote whether the regulator approves CRA i for the first time in period t.

$$z_i^t = \begin{cases} 1 & \text{if } w_i^t = 1 \text{ and } w_i^{t-1} = 0 \\ 0 & \text{otherwise.} \end{cases} \tag{1.1}$$

At stage 2, each CRA chooses a fee and offers a rating to each issuer. There are two rating categories, again $\mathcal{A}$ and $\mathcal{B}$. Rating category $\mathcal{A}$ indicates that an issuer is of type A and rating category $\mathcal{B}$ indicates that an issuer is of type B. CRA i chooses fee $f_i^t \in \mathbb{R}_0^+$ and rating threshold $a_i^t \in [0, 1]$. CRA i offers issuers, who are located on or to the left of a_i^t on the unit interval, an $\mathcal{A}$ rating, and issuers, who are located to the right of a_i^t on the unit interval, a $\mathcal{B}$ rating. If $a_i^t = m$, CRA i offers all type-A issuers an $\mathcal{A}$ rating and all type-B issuers a $\mathcal{B}$ rating. If $a_i^t > m$, CRA i offers some issuers inflated ratings. Figure 1.3 illustrates this. Issuers are uniformly located along the unit interval on the horizontal axis according to their type. m issuers are of type A. If $a_i^t > m$, CRA i offers type-B issuers an $\mathcal{A}$ rating. We assume that CRAs publish a rating only if an issuer demands a rating.

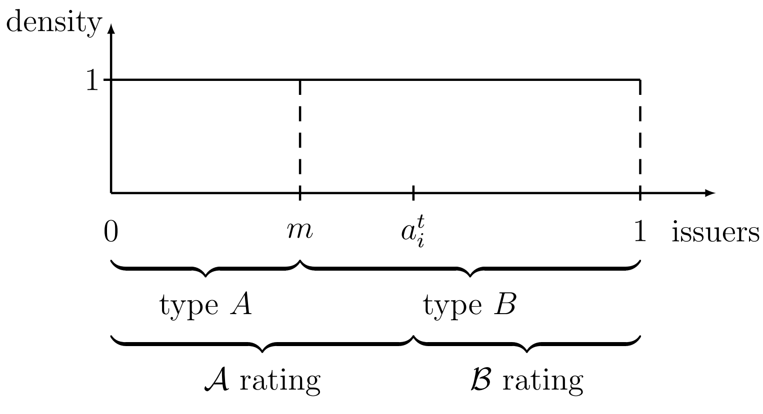

Figure 1.3: If $a_i^t > m$, CRA i offers type-B issuers an $\mathcal{A}$ rating.

At stage 3, issuers decide whether and from which CRA to demand a rating. We assume that issuers demand a rating only from an approved CRA. An issuer's utility of an $\mathcal{A}$ rating is Δ, where $\Delta > 0$. An issuer's utility of a $\mathcal{B}$ rating is 0. Δ could easily be endogenized, for instance, as the difference between the price of a bond with an $\mathcal{A}$ rating and the price of a bond with a $\mathcal{B}$ or no rating. An issuer's utility is hence given by

$$U_{Is} = \begin{cases} \Delta - f_i^t & \text{if it demands a rating from CRA } i \\ & \text{and receives an } \mathcal{A} \text{ rating} \\ 0 - f_i^t & \text{if it demands a rating from CRA } i \\ & \text{and receives a } \mathcal{B} \text{ rating} \\ 0 & \text{if it does not demand a rating.} \end{cases} \tag{1.2}$$

Therefore, each issuer demands a rating from the approved CRA which offers the issuer an $\mathcal{A}$ rating at the lowest fee, provided this fee does not exceed Δ. If several approved CRAs offer an issuer an $\mathcal{A}$ rating at the same fee and this fee is the lowest fee and does not exceed Δ, the issuer randomizes among them. If no approved CRA offers an issuer an $\mathcal{A}$ rating at a fee which does not exceed Δ, the issuer does not demand a rating.

Let $D_i^t \in [0, 1]$ denote the mass of issuers who demand a rating from CRA i in period t. The mass of issuers who demand a rating from CRA i thus depends on whether the regulator approves CRA i, the number of approved CRAs, the rating thresholds and the fees:

$$D_i^t = D_i^t(w_i^t, n^t, a_i^t, a_{-i}^t, f_i^t, f_{-i}^t) \tag{1.3}$$

If the regulator does not approve CRA i in period t ($w_i^t = 0$), no issuer demands a rating from CRA i by assumption, so

$$D_i^t = 0. \tag{1.4}$$

If the regulator approves only CRA i in period t ($w_i^t = 1$ and $n^t = 1$), the mass of issuers who demand a rating from CRA i is given by

$$D_i^t = \begin{cases} a_i^t & \text{if } f_i^t \leq \Delta \\ 0 & \text{if } f_i^t > \Delta. \end{cases} \tag{1.5}$$

Suppose now that the regulator approves CRA i ($w_i^t = 1$) and at least one other CRA in period t ($n^t \geq 2$). Suppose further that all other approved CRAs choose the same fee f_{-i}^t and the same rating threshold a_{-i}^t. The mass of issuers who demand a rating from CRA i is then given by

$$D_i^t = \begin{cases} a_i^t & \text{if } f_i^t < f_{-i}^t \text{ and } f_i^t \leq \Delta \\ \frac{1}{n^t} a_i^t & \text{if } f_i^t = f_{-i}^t \leq \Delta \text{ and } a_i^t \leq a_{-i}^t \\ a_i^t - \frac{n^t-1}{n^t} a_{-i}^t & \text{if } f_i^t = f_{-i}^t \leq \Delta \text{ and } a_i^t > a_{-i}^t \\ 0 & \text{else.} \end{cases} \tag{1.6}$$

At the end of period t, $t = 1, 2, \ldots$, the regulator observes the default rate within each approved CRA's rating category $\mathcal{A}$. Let x_i^t denote the default rate within CRA i's rating category $\mathcal{A}$ in period t. The regulator incurs monitoring costs c_M for each approved CRA. Results do not change, if the regulator can also observe the default rate within CRA i's rating category $\mathcal{B}$.

The default rate x_i^t does not only depend on rating threshold a_i^t. It can also be influenced by a common shock η^t. If CRA i rates a continuum of issuers, an individual shock, which affects the default probability of just one issuer, does not influence the default rate within CRA i's rating category $\mathcal{A}$. A common shock, by contrast, which affects many or all issuers (e.g. an industry- or economy-wide shock), has an effect on the default rate within CRA i's rating category $\mathcal{A}$.

With probability $1-p$, the common shock $\eta^t = 0$. With probability p, the common shock η^t is positive and uniformly distributed on the interval $(0, 1]$. Results do not change, if the common shock η^t can take on negative values as well.

The common shock η^t increases the default probability of types-A and -B issuers to $d_A + \eta^t_A$ and $d_B + \eta^t_B$, where $\eta^t_A = (1-d_A)\eta^t$ and $\eta^t_B = (1-d_B)\eta^t$. This assumption has two implications. First, because the common shock $\eta^t < 1$, the default probability of type-A issuers is lower than the default probability of type-B issuers: Because $\eta^t < 1$, $d_A + \eta^t_A < d_B + \eta^t_B$. Second, the default probability of type-A issuers can take on any value in the interval $[d_A, 1]$. Apart from these two implications, results do not depend on the functional form of η^t_A and η^t_B.

To simplify the analysis, we assume that defaults are uncorrelated. As a result, $Var(x^t_i) = Var(\eta^t)$. Let q^t_i denote the proportion of type-A issuers in CRA i's rating category $\mathcal{A}$ in period t. The default rate x^t_i is then given by

$$x^t_i = q^t_i(d_A + \eta^t_A) + (1 - q^t_i)(d_B + \eta^t_B), \tag{1.7}$$

where $\eta^t_A = (1 - d_A)\eta^t$ and $\eta^t_B = (1 - d_B)\eta^t$. This assumption has again two implications. First, if the proportion of type-A issuers in CRA i's rating category $\mathcal{A}$ is larger than the proportion of type-A issuers in CRA j's rating category $\mathcal{A}$, the default rate within CRA i's rating category $\mathcal{A}$ is smaller than the default rate within CRA j's rating category $\mathcal{A}$ for any common shock $\eta^t < 1$: If $q^t_i > q^t_j$, $x^t_i < x^t_j \ \forall \eta^t < 1$. Second, the regulator cannot detect a collusive agreement to offer inflated ratings, if no CRA deviates from it. If CRAs collude to offer inflated ratings, default rates increase, but do not differ from each other. Since the default probability of type-A issuers can take on any value in the interval $[d_A, 1]$, the regulator cannot detect whether high default rates are due to a collusive agreement or the common shock. If, however, a CRA deviates from such a collusive agreement, default rates differ and the regulator can detect the collusive agreement by evaluating the relative performance of CRAs.

CRAs are only interested in their monetary payoff. Let δ_{RA} denote a CRA's discount factor. CRA i's utility function is given by

$$U_{RA,i} = (1 - \delta_{RA}) \sum_{t=1}^{\infty} \delta_{RA}^{t-1} D_i^t(\cdot) f_i^t. \tag{1.8}$$

The regulator wants to induce all approved CRAs to offer correct ratings. This assumption is motivated by the observation that financial regulators often rely on ratings of certain recognized CRAs, for instance to evaluate the amount of capital which financial institutions are required to hold. However, we do not assume that the regulator wants to influence the CRAs' pricing behaviour, as the price-setting has only distributive effects in this model. Let δ_{Re} denote the regulator's discount factor. The regulator incurs costs of

$$(1 - \delta_{Re}) \sum_{t=1}^{\infty} \delta_{Re}^{t-1} \Big[\underbrace{c_A \sum\nolimits_i z_i^t}_{\text{approval costs}} + \underbrace{c_M \sum\nolimits_i w_i^t}_{\text{monitoring costs}} \Big]. \tag{1.9}$$

In period $t = 0$, the regulator hence chooses an approval scheme which minimizes approval and monitoring costs subject to two constraints. First, the regulator has to induce all approved CRAs to offer correct ratings. Second, the regulator has to approve at least one CRA in each period.

1.3 Inducing Correct Ratings

This section considers approval schemes which can induce all approved CRAs to offer correct ratings. First, we assume that CRAs do not collude to offer inflated ratings. Then, we relax this assumption.

1.3.1 In Absence of Collusion

If CRAs do not collude to offer inflated ratings, the regulator can easily induce all approved CRAs to offer correct ratings. Consider the following approval scheme.

Approval scheme 1 *Approve two CRAs in period 1. Replace CRA i in subsequent periods, if and only if $x_i > x_j$.*

According to approval scheme 1, the regulator approves two CRAs and evaluates the relative performance of approved CRAs. If the default rate within CRA i's rating category $\mathcal{A}$ is larger than the default rate within CRA j's rating category $\mathcal{A}$, the regulator denies approval to CRA i in all future periods and grants approval to another CRA. As the following proposition shows, approval scheme 1 can induce each approved CRA to offer correct ratings.

Proposition 1.1 *Suppose the regulator chooses approval scheme 1. If δ_{RA} is sufficiently high, there exists a subgame perfect equilibrium in which both approved CRAs offer correct ratings.*

The intuition for Proposition 1.1 is as follows. CRAs can collude to choose fee Δ using trigger strategies. If the CRAs' discount factor δ_{RA} is sufficiently high, there exists a punishment phase which is sufficiently long, such that an approved CRA has no incentive to deviate from choosing fee Δ, and sufficiently short, such that an approved CRA has no incentive to deviate from the punishment phase. If CRAs collude to choose fee Δ, CRAs make a profit Δ per issuer who demands a rating. If CRAs do not collude to offer inflated ratings, the regulator can filter out the common shock η by evaluating the relative performance of approved CRAs. If, in addition, the CRAs' discount factor δ_{RA} is sufficiently high, the threat to deny approval in all future periods deters CRAs from offering inflated ratings.

The regulator could also approve just one CRA and induce this CRA to offer correct ratings. Consider the following approval scheme.

Approval scheme 2 *Approve one CRA in period 1. Replace this CRA in subsequent periods, if and only if $x_i > d_A$.*

According to approval scheme 2, the regulator approves one CRA. Therefore, the regulator cannot filter out the common shock η by evaluating the relative performance of approved CRAs. If the default rate within rating category $\mathcal{A}$ is larger than default probability d_A, the regulator denies approval to the CRA in all future periods and grants approval to another CRA. As the following proposition shows, approval scheme 2 can also induce the approved CRA to offer correct ratings.

Proposition 1.2 *Suppose the regulator chooses approval scheme 2. If δ_{RA} is sufficiently high in relation to p, the approved CRA offers correct ratings.*

The intuition for proposition 1.2 is as follows. The approved CRA can again make a profit Δ per issuer who demands a rating. If the approved CRA offers correct ratings, the regulator only replaces the CRA, if the common shock η is positive. If the approved CRA offers inflated ratings, the regulator replaces the CRA in any case. The approved CRA hence has no incentive to offer inflated ratings, if the CRAs' discount factor δ_{RA} is sufficiently high in relation to the probability p that the common shock η is positive. If the approved CRA chooses a lower rating threshold and offers some type-A issuers a $\mathcal{B}$ rating, less issuers demand a rating. The default rate however does not decrease, as the approved CRA still offers only type-A issuers an $\mathcal{A}$ rating. The approved CRA hence has no incentive to choose a lower rating threshold.

However, as the following proposition shows, approval scheme 2 may be inefficient.

Proposition 1.3 *If δ_{Re} is sufficiently high and monitoring costs c_M are sufficiently smaller than approval costs c_A, approval scheme 1 generates less costs than approval scheme 2.*

The intuition is straightforward. If the regulator chooses approval scheme 1, the regulator approves two CRAs and replaces CRA i, if $x_i > x_j$. Given that both CRAs offer correct ratings, the regulator does not have to replace a CRA. If the regulator however chooses approval scheme 2, the regulator approves one CRA and replaces the CRA, if $x_i > d_A$. The regulator hence has to replace the approved CRA, if the common shock η is positive, even if the CRA offers correct ratings. Thus, approval scheme 2 generates lower monitoring costs than approval scheme 1, but may require the regulator to replace the approved CRA, even if the CRA offers correct ratings. If the regulator's discount factor δ_{Re} is sufficiently high, approval scheme 2 hence generates higher approval costs than approval scheme 1.

It is plausible that monitoring costs c_M are sufficiently smaller than approval costs c_A. Monitoring costs c_M represent costs to monitor default rates. Approval costs c_A may be interpreted as costs to establish a CRA.

1.3.2 Preventing Collusion

Approval scheme 1 however does not provide an incentive to deviate from a collusive agreement to offer inflated ratings. As the following proposition shows, CRAs may hence collude to offer inflated ratings.

Proposition 1.4 *If the regulator chooses approval scheme 1, CRAs may collude to offer inflated ratings.*

The intuition for proposition 1.4 is twofold. First, each approved CRA gets a higher payoff, if both approved CRAs collude to offer inflated ratings. Suppose both approved CRAs collude to offer inflated ratings. As a result, more issuers demand a rating from each approved CRA and default rates increase. However, as default rates do not differ ($x_i = x_j$), the regulator does not deny approval to the colluding CRAs in future periods. Second, approval scheme 1 does not provide an incentive to deviate from such a collusive agreement. Suppose CRA i offers less inflated ratings than CRA j. As a result, less issuers demand a rating from CRA i, and the default rate within CRA i's rating category $\mathcal{A}$ decreases. The default rate within CRA j's rating category $\mathcal{A}$ is thus larger than the default rate within CRA i's rating category $\mathcal{A}$ ($x_j > x_i$). Consequently, the regulator denies approval to CRA j in all future periods and approves another CRA to replace CRA j. Since the regulator immediately approves another CRA to replace CRA j, CRA i does not benefit from deviating. Given that CRA j sticks to the collusive agreement, it is optimal for CRA i to stick to the collusive agreement.

The regulator can again approve two CRAs and prevent the approved CRAs from colluding to offer inflated ratings by providing an incentive to deviate from such a collusive agreement. Consider the following approval scheme.

Approval scheme 3 *Approve two CRAs in period 1. Deny approval to CRA i in all future periods, if and only if $x_i > x_j$. Approve another CRA to replace CRA i, if and only if in the following periods $x_j > d_A$.*

Approval scheme 3 is similar to approval scheme 1 in some respects. The regulator again approves two CRAs and evaluates the relative performance of approved CRAs. If $x_i > x_j$, the regulator again denies approval to CRA i in all future periods.

Unlike approval scheme 1, approval scheme 3 however provides an incentive to deviate from a collusive agreement to offer inflated ratings. If $x_j > x_i$, the regulator does not immediately approve another CRA to replace CRA j. The regulator only approves another CRA, if in the following periods $x_i > d_A$. As the following proposition shows, approval scheme 3 can hence prevent the approved CRAs from colluding to offer inflated ratings.

Proposition 1.5 *Suppose the regulator chooses approval scheme 3. If δ_{RA} is sufficiently high, there exists a subgame perfect equilibrium, in which both approved CRAs offer correct ratings. In addition, approval scheme 3 prevents the approved CRAs from colluding to offer inflated ratings.*

The intuition is as follows. First, given CRA j offers correct ratings, it is again optimal for CRA i to offer correct ratings. Second, each CRA has an incentive to deviate from a collusive agreement to offer inflated ratings. Suppose CRA i and CRA j collude to offer inflated ratings and CRA j sticks to the collusive agreement. If CRA i offers less issuers an inflated rating than CRA j, there are two effects. First, the mass of issuers who demand a rating from CRA i decreases. Second, the default rate within CRA j's rating category $\mathcal{A}$ is larger than the default rate within CRA i's rating category $\mathcal{A}$ ($x_j > x_i$). The regulator consequently denies approval to CRA j and grants CRA i a monopoly, until $x_i > d_A$. If CRA i offers marginally less issuers an inflated ratings than CRA j, the mass of issuers who demand a rating from CRA i decreases only marginally. Still $x_j > x_i$. Thus, given CRA j offers inflated ratings, it is optimal for CRA i to offer marginally less issuers an inflated rating.

1.4 Conclusion

The model shows that there exists an approval scheme which can induce credit rating agencies to offer correct ratings. The model suggests that a regulator should both deter a credit rating agency from unilaterally offering inflated ratings, and provide an incentive to deviate from a collusive agreement to offer inflated ratings. The model indicates that a regulator should both threaten to deny approval in future periods if a credit rating agency's performance is worse than its competitors', and reward a credit rating agency which deviates from a collusive agreement to offer inflated ratings by reducing the number of approved credit rating agencies in future periods.

Financial regulators recognize certain credit rating agencies and rely on their ratings. However, it is unclear whether the current regulatory approach induces credit rating agencies to assign correct ratings. Currently, financial regulators neither explicitly threaten to deny recognition in future periods nor explicitly offer to reward a credit rating agency which deviates from a collusion to offer inflated ratings.

1.5 Appendix

Proof of Proposition 1.1

Suppose the regulator chooses approval scheme 1.

Consider the following strategy.

> Choose rating threshold m in each period. In period 1, choose fee Δ. In period $t > 1$, choose fee Δ, if both approved CRAs chose fee Δ in the previous period. Also choose fee Δ, if both approved CRAs chose fee 0 in the previous τ periods, where
>
> $$\frac{\ln(2\delta_{RA} - 1)}{\ln \delta_{RA}} - 1 \leq \tau \leq \frac{\ln[(1 - \delta_{RA})\frac{2}{m}(1 - m)]}{\ln \delta_{RA}}.$$
>
> Otherwise, choose fee 0.

The strategy of an approved CRA hence involves two phases: a (potentially infinite) collusive phase in which the CRA chooses rating threshold m and fee Δ and a (τ-period) punishment phase in which the CRA chooses rating threshold m and fee 0. If both approved CRAs play the two-phase strategy, they only collude on prices, not on rating thresholds.

Suppose that CRA j chooses rating threshold m and fee Δ. The mass of issuers who demand a rating from CRA i is then given by

$$D_i = \begin{cases} a_i & \text{if } f_i < \Delta \\ \frac{1}{2}a_i & \text{if } f_i = \Delta \text{ and } a_i \leq m \\ a_i - \frac{1}{2}m & \text{if } f_i = \Delta \text{ and } a_i > m \\ 0 & \text{else.} \end{cases} \tag{1.10}$$

If both approved CRAs choose rating threshold m (and thus offer correct ratings) and fee Δ, each type-A issuer randomizes among them, while type-B issuers do not demand a rating. As a result, issuers of mass $\frac{m}{2}$ demand a rating from CRA i ($D_i = \frac{m}{2}$). The default rates within both approved CRA's rating category $\mathcal{A}$ do not differ ($x_i = x_j$). Each approved CRA hence gets the payoff

$$(1-\delta_{RA})\sum_{t=1}^{\infty}\delta_{RA}^{t-1}\ \frac{m}{2}\Delta = \frac{m}{2}\Delta. \tag{1.11}$$

If CRA i instead chooses a higher rating threshold $a_i > m$ (and thus offers inflated ratings), the mass of issuers who demand a rating from CRA i increases for any given fee $f_i \leq \Delta$. (If $f_i > \Delta$, no issuer requests a rating from CRA i irrespective of its rating threshold a_i.) The default rate within its rating category $\mathcal{A}$ increases as well. As the default rate within CRA i's rating category $\mathcal{A}$ is consequently larger than the default rate within CRA j's rating category $\mathcal{A}$ ($x_i > x_j$), the regulator denies approval to CRA i in all future periods. If a single approved CRA chooses a higher rating threshold $a_i > m$, it hence maximizes its payoff by maxizing its payoff in this period. An approved CRA maximizes its payoff in this period, if it chooses rating threshold $a_i = 1$ (and thus offers all issuers an $\mathcal{A}$ rating) and fee $f_i = \Delta - \epsilon$, where ϵ is arbitrarily small. In this case, all issuers demand a rating from the CRA ($D_i = 1$). The deviation yields the payoff

$$(1-\delta_{RA})\Big[1\,(\Delta-\epsilon) + \sum_{t=2}^{\infty}\delta_{RA}^{t-1}\,0\Big] \tag{1.12}$$

An approved CRA has no incentive to choose a higher rating threshold $a_i > m$, if

$$\frac{m}{2}\Delta \geq (1-\delta_{RA})(\Delta-\epsilon), \tag{1.13}$$

or

$$\delta_{RA} \geq 1 - \frac{m}{2}. \qquad (1.14)$$

If CRA i instead chooses a lower rating threshold $a_i < m$ (and thus offers (some) type-A issuers a $\mathcal{B}$ rating), the mass of issuers who demand a rating from CRA i decreases for any given fee $f_i \leq \Delta$. (If $f_i > \Delta$, no issuer requests a rating from CRA i irrespective of its rating threshold a_i.) Since CRA i still offers only type-A issuers an $\mathcal{A}$-rating, the default rate within CRA i's rating category $\mathcal{A}$ does not decrease. An approved CRA hence has no incentive to choose a lower rating threshold $a_i < m$.

If CRA i only deviates from choosing fee Δ, it maximizes its payoff by choosing fee $f_i = \Delta - \epsilon$, where ϵ is arbitrarily small. In this case, all type-A issuers demand a rating from CRA i ($D_i = m$). If either approved CRA deviates from choosing fee Δ, the (τ-period) punishment phase begins. The deviation hence yields the payoff

$$(1 - \delta_{RA})\Big[m(\Delta - \epsilon) + \sum_{t=2}^{\tau+1} \delta_{RA}^{t-1} 0 + \sum_{t=\tau+2}^{\infty} \delta_{RA}^{t-1} \frac{m}{2}\Delta\Big]. \qquad (1.15)$$

An approved CRA has no incentive to deviate from choosing fee Δ, if

$$\frac{m}{2}\Delta \geq (1 - \delta_{RA})\Big[m(\Delta - \epsilon) + \sum_{t=\tau+2}^{\infty} \delta_{RA}^{t-1} \frac{m}{2}\Delta\Big]. \qquad (1.16)$$

This is equivalent to

$$\sum_{t=1}^{\tau+1} \delta_{RA}^{t-1} \geq 2, \qquad (1.17)$$

or

$$\tau \geq \frac{ln(2\delta_{RA} - 1)}{ln\delta_{RA}} - 1. \qquad (1.18)$$

An approved CRA hence has no incentive to deviate from choosing fee Δ, if the punishment phase is sufficiently long.

$$2\delta_{RA} - 1 > 0, \tag{1.19}$$

if

$$\delta_{RA} > \frac{1}{2}, \tag{1.20}$$

which holds, if

$$\delta_{Ra} \geq 1 - \frac{m}{2} \tag{1.21}$$

and

$$m < 1. \tag{1.22}$$

Note that $m < 1$ holds by assumption. For $\delta_{RA} \in \left]\frac{1}{2}, 1\right]$,

$$\frac{\partial}{\partial \delta_{RA}} \Big(\frac{ln(2\delta_{RA} - 1)}{ln\delta_{RA}} - 1 \Big) \quad < \quad 0 \tag{1.23}$$

$$\lim_{\delta_{RA} \to \frac{1}{2}} \Big(\frac{ln(2\delta_{RA} - 1)}{ln\delta_{RA}} - 1 \Big) \quad \to \quad \infty \tag{1.24}$$

$$\lim_{\delta_{RA} \to 1} \Big(\frac{ln(2\delta_{RA} - 1)}{ln\delta_{RA}} - 1 \Big) \quad = \quad 1. \tag{1.25}$$

The (lower) threshold for the length of the punishment phase decreases with the CRAs' discount factor δ_{RA}. If δ_{RA} converges to $\frac{1}{2}$, the threshold goes to ∞. If δ_{RA} converges to 1, the threshold converges to 1.

If at least one approved CRA deviates from choosing fee Δ, the two-phase strategy prescribes choosing rating threshold m and fee 0 for τ periods.

Suppose that CRA j chooses rating threshold m and fee 0. The mass of issuers who demand a rating from CRA i is then given by

$$D_i = \begin{cases} \frac{1}{2}a_i & \text{if } f_i = 0 \text{ and } a_i \leq m \\ a_i - \frac{1}{2}m & \text{if } f_i = 0 \text{ and } a_i > m \\ a_i - m & \text{if } 0 < f_i \leq \Delta \text{ and } a_i > m \\ 0 & \text{else.} \end{cases} \tag{1.26}$$

Let us consider the first period of the punishment phase. If neither approved CRA deviates from the punishment phase, each approved CRA gets the payoff

$$(1-\delta_{RA})\Big[\sum_{t=1}^{\tau}\delta_{RA}^{t-1}\,0 + \sum_{t=\tau+1}^{\infty}\delta_{RA}^{t-1}\frac{m}{2}\Delta\Big] = \delta_{RA}^{\tau}\frac{m}{2}\Delta. \tag{1.27}$$

Given that CRA j does not deviate from the punishment phase, CRA i can only increase its payoff in this period by choosing a higher rating threshold $a_i > m$ (i.e. by offering inflated ratings). As the default rate within CRA i's rating category $\mathcal{A}$ is consequently larger than the default rate within CRA j's rating category $\mathcal{A}$ ($x_i > x_j$), the regulator denies approval to CRA i in all future periods. If CRA i deviates from the punishment phase by choosing a higher rating threshold $a_i > m$, it hence maximizes its payoff by choosing rating threshold $a_i = 1$ (i.e. by offering all issuers an $\mathcal{A}$ rating) and fee $f_i = \Delta$. In this case, all type-B issuers demand a rating from CRA i ($D_i = 1 - m$). All type-A issuers demand a rating from CRA j ($D_j = m$). The deviation yields the payoff

$$(1-\delta_{RA})\Big[(1-m)\,\Delta + \sum_{t=2}^{\infty}\delta_{RA}^{t-1}\,0\Big]. \tag{1.28}$$

An approved CRA has no incentive to deviate in the first period of the punishment phase, if

$$\delta_{RA}^{\tau}\frac{m}{2}\Delta \geq (1-\delta_{RA})(1-m)\Delta \tag{1.29}$$

or

$$\tau \leq \frac{\ln[(1-\delta_{RA})\frac{2}{m}(1-m)]}{\ln \delta_{RA}}. \tag{1.30}$$

Hence, an approved CRA has no incentive to deviate in the first period of the punishment phase, if the punishment phase is sufficiently short.

$$\frac{\ln[(1-\delta_{RA})\frac{2}{m}(1-m)]}{\ln \delta_{RA}} \geq 1, \tag{1.31}$$

if

$$\delta_{RA} \geq \frac{2-2m}{2-m}, \tag{1.32}$$

which holds, if

$$\delta_{Ra} \geq 1-\frac{m}{2} \tag{1.33}$$

and

$$m \geq 0. \tag{1.34}$$

Note that $m > 0$ holds by assumption. For $\delta_{RA} \in \left[\frac{2-2m}{2-m}, 1\right]$,

$$\frac{\partial}{\partial \delta_{RA}}\Big(\frac{\ln[(1-\delta_{RA})\frac{2}{m}(1-m)]}{\ln \delta_{RA}}\Big) > 0 \tag{1.35}$$

$$\lim_{\delta_{RA}\to\frac{2-2m}{2-m}}\Big(\frac{\ln[(1-\delta_{RA})\frac{2}{m}(1-m)]}{\ln \delta_{RA}}\Big) = 1 \tag{1.36}$$

$$\lim_{\delta_{RA}\to 1}\Big(\frac{\ln[(1-\delta_{RA})\frac{2}{m}(1-m)]}{\ln \delta_{RA}}\Big) \to \infty. \tag{1.37}$$

The (upper) threshold for the length of the punishment phase increases with the

CRAs' discount factor δ_{RA}. If δ_{RA} converges to $\frac{2-2m}{2-m}$, the threshold converges to 1. If δ_{RA} converges to 1, the threshold goes to ∞.

Let us now consider the t-th period of the punishment phase. If neither approved CRA deviates from the punishment phase, each approved CRA gets the payoff

$$\delta_{RA}^{\tau-(t-1)}\frac{m}{2}\Delta. \tag{1.38}$$

Given that CRA j does not deviate from the punishment phase, CRA i maximizes its payoff in this period by choosing rating threshold $a_i = 1$ (i.e. by offering all issuers an $\mathcal{A}$ rating) and fee Δ. The deviation again yields the payoff

$$(1-\delta_{RA})(1-m)\Delta. \tag{1.39}$$

An approved CRA has no incentive to deviate in the t-th period of the punishment phase, if

$$\delta_{RA}^{\tau-(t-1)}\frac{m}{2}\Delta \geq (1-\delta_{RA})(1-m)\Delta \tag{1.40}$$

Since

$$\delta_{RA}^{\tau-(t-1)} > \delta_{RA}^{\tau} \tag{1.41}$$

for

$$t > 1, \tag{1.42}$$

an approved CRA has no incentive to deviate in any period of the punishment phase, if it has no incentive to deviate in the first period of the punishment phase.

There exists a punishment phase, which is sufficiently long, such that an approved CRA has no incentive to deviate from choosing fee Δ, and sufficiently short, such that an approved CRA has no incentive to deviate from the punishment phase, if there exists a $\tau \in \mathbb{N}$, which satisfies inequality (1.18) and (1.30).

$\exists\tau \in \mathbb{N}$, which satisfies inequality (1.18) and (1.30), if

$$\frac{ln(2\delta_{RA}-1)}{ln\delta_{RA}} - 1 + 1 \leq \frac{\ln[(1-\delta_{RA})\frac{2}{m}(1-m)]}{\ln\delta_{RA}}, \tag{1.43}$$

or

$$\delta_{RA} \geq 1 - \frac{m}{2}. \tag{1.44}$$

That is, there exists a punishment phase, which is sufficiently long, such that an approved CRA has no incentive to deviate from choosing fee Δ, and sufficiently short, such that an approved CRA has no incentive to deviate from the punishment phase, if $\delta_{RA} \geq 1 - \frac{m}{2}$.

Thus, if the regulator chooses approval scheme 1 and if $\delta_{RA} \geq 1 - \frac{m}{2}$, there exists a subgame perfect equilibrium, in which both approved CRAs offer correct ratings. ∎

Proof of Proposition 1.2

Suppose that the regulator chooses approval scheme 2.

The mass of issuers who demand a rating from the approved CRA is given by

$$D = \begin{cases} a & \text{if } f \leq \Delta \\ 0 & \text{if } f > \Delta. \end{cases} \tag{1.45}$$

It is hence optimal for the approved CRA to choose fee Δ. Let us now consider the CRA's choice of rating threshold a.

If the approved CRA chooses rating threshold $a = m$ (and thus offers correct ratings), issuers of mass m demand a rating. As the CRA only offers type-A issuers an $\mathcal{A}$ rating, the regulator only denies approval to the CRA in all future periods, if the common shock is positive ($\eta > 0$). The CRA hence gets the payoff

$$(1-\delta_{RA})\Big[m\Delta + \sum_{t=2}^{\infty} \delta_{RA}^{t-1}(1-p)^t m\Delta\Big]. \tag{1.46}$$

If the approved CRA instead chooses a higher rating threshold $a > m$ (and thus offers inflated ratings), the mass of issuers who demand a rating increases and the default rate within rating category-$\mathcal{A}$ is larger than default probability d_A. The regulator hence denies approval to the CRA in all future periods. If the approved CRA chooses a higher rating threshold $a > m$, it hence maximizes its payoff by maximizing its payoff in this period. The approved CRA maximizes its payoff in this period, if it chooses rating threshold $a = 1$ (and thus offers all issuers an $\mathcal{A}$ rating). In this case, all issuers demand a rating ($D = 1$). The CRA hence gets the payoff

$$(1-\delta_{RA})\Big[1\Delta + \sum_{t=2}^{\infty} \delta_{RA}^{t-1} 0\Big]. \tag{1.47}$$

The approved CRA has no incentive to choose a higher rating threshold $a > m$, if

$$(1-\delta_{RA})\Big[m\Delta + \sum_{t=2}^{\infty} \delta_{RA}^{t-1}(1-p)^t m\Delta\Big] \geq (1-\delta_{RA})\Delta \tag{1.48}$$

or

$$\delta_{RA} \geq \frac{1-m}{1-p}. \tag{1.49}$$

If the approved CRA instead chooses a lower rating threshold $a < m$ (and thus offers (some) type-A issuers a $\mathcal{B}$ rating), the mass of issuers who demand a rating decreases. Since the CRA still offers only type-A issuers an $\mathcal{A}$-rating, the default rate within rating category $\mathcal{A}$ does not decrease. The approved CRA hence has no incentive to choose a lower rating threshold $a < m$.

Thus, if the CRAs' discount factor $\delta_{RA} \geq \frac{1-m}{1-p}$, approval scheme 2 can induce the approved CRA to offer correct ratings. ∎

Proof of Proposition 1.3

Approval scheme 1 incurs costs of

$$(1-\delta_{Re})\Big[2c_A + \sum_{t=1}^{\infty} \delta_{Re}^{t-1} 2c_M\Big]. \tag{1.50}$$

Approval scheme 2 incurs costs of

$$(1-\delta_{Re})\Big[c_A + \sum_{t=2}^{\infty} \delta_{Re}^{t-1} pc_A + \sum_{t=1}^{\infty} \delta_{Re}^{t-1} c_M\Big]. \tag{1.51}$$

Approval scheme 1 incurs less costs than approval scheme 2, if

$$c_M < [(1+p)\delta_{Re} - 1]c_A. \tag{1.52}$$

$$(1+p)\delta_{Re} - 1 > 0, \tag{1.53}$$

if

$$\delta_{Re} > \frac{1}{1+p}. \tag{1.54}$$

Note that

$$\frac{\partial}{\partial \delta_{Re}}\Big([(1+p)\delta_{Re} - 1]c_A\Big) > 0. \tag{1.55}$$

If the regulator's discount factor δ_{Re} converges to 1, approval scheme 1 incurs less costs than approval scheme 2, if

$$c_M < pc_A. \tag{1.56}$$

Thus, if the regulator's discount factor δ_{Re} is sufficiently high and monitoring costs c_M are sufficiently smaller than approval costs c_A, approval scheme 1 incurs less costs than approval scheme 2. ∎

Proof of Proposition 1.4

Suppose the regulator chooses approval scheme 1.

As shown in the proof of proposition 1.1, if $\delta_{RA} \geq 1 - \frac{m}{2}$, there exists a subgame perfect equilibrium in which both approved CRAs offer correct ratings. In this equilibium, each approved CRA gets the payoff

$$(1-\delta_{RA})\sum_{t=1}^{\infty}\delta_{RA}^{t-1}\,\frac{m}{2}\Delta = \frac{m}{2}\Delta. \tag{1.57}$$

Consider again the strategies which form the equilibrium. Suppose now that both approved CRAs collude to choose rating threshold $a^H > m$ instead of rating threshold m. As a result, more issuers demand a rating from each approved CRA ($D_i = D_j = \frac{a^H}{2}$) and default rates increase. However, as default rates do not differ ($x_i = x_j$), the regulator does not deny approval to the colluding CRAs in future periods. Each approved CRA hence gets the payoff

$$(1-\delta_{RA})\Big[\frac{a^H}{2}\Delta + \sum_{t=2}^{\infty}\delta_{RA}^{t-1}\,\frac{m}{2}\Delta\Big]. \tag{1.58}$$

Since

$$(1-\delta_{RA})\Big[\frac{a^H}{2}\Delta + \sum_{t=2}^{\infty}\delta_{RA}^{t-1}\,\frac{m}{2}\Delta\Big] > \frac{m}{2}\Delta, \tag{1.59}$$

each approved CRA gets a higher payoff, if both approved CRAs collude to offer inflated ratings.

Approval scheme 1 does not provide an incentive to deviate from such a collusive agreement. Suppose CRA i and CRA j collude to choose rating threshold $a^H > m$ and fee Δ and suppose CRA j sticks to the collusive agreement. If CRA i deviates from the collusive agreement by choosing rating threshold $a^H - \epsilon$, the default rate within CRA j's rating category $\mathcal{A}$ is larger than the default rate within CRA i's rating category $\mathcal{A}$ ($x_j > x_i$). The regulator consequently denies approval to CRA j in all future periods and approves another CRA to replace CRA j. CRA i hence gets the payoff

$$(1-\delta_{RA})\Big[\frac{a^H-\epsilon}{2}\Delta + \sum_{t=2}^{\infty}\delta_{RA}^{t-1}\ \frac{m}{2}\Delta\Big]. \tag{1.60}$$

Since

$$(1-\delta_{RA})\Big[\frac{a^H}{2}\Delta + \sum_{t=2}^{\infty}\delta_{RA}^{t-1}\ \frac{m}{2}\Delta\Big] > (1-\delta_{RA})\Big[\frac{a^H-\epsilon}{2}\Delta + \sum_{t=2}^{\infty}\delta_{RA}^{t-1}\ \frac{m}{2}\Delta\Big], \tag{1.61}$$

CRA i has no incentive to deviate from the collusive agreement by choosing rating threshold $a^H - \epsilon$.

Moreover, if the CRA's discount factor δ_{RA} is sufficiently high, CRA i has no incentive to deviate from the price collusion. Suppose again that CRA j sticks to the price collusion and chooses rating threshold a^H and fee Δ. If CRA i deviates from the price collusion by choosing fee $\Delta - \epsilon$, the punishment phase begins. Proceeding analogously as in the proof of proposition 1.1, it can be shown that there exists a punishment phase, which is sufficiently long, such that an approved CRA has no incentive to deviate from choosing fee Δ, and sufficiently short, such that an approved CRA has no incentive to deviate from the punishment phase, if

$$\delta_{RA} \geq \frac{2+a^H-2m}{2+a^H-m}. \tag{1.62}$$

If $a^H = m$, this condition is equivalent to the prior condition

$$\delta_{RA} \geq 1 - \frac{m}{2}. \tag{1.63}$$

For $m > 0$ (which holds by assumption),

$$\frac{\partial}{\partial a^H}\left(\frac{2 + a^H - 2m}{2 + a^H - m}\right) > 0. \tag{1.64}$$

If $a^H = 1$, the condition is equivalent to

$$\delta_{RA} \geq \frac{3 - 2m}{3 - m}. \tag{1.65}$$

If $m > 0$,

$$\frac{3 - 2m}{3 - m} < 1. \tag{1.66}$$

Thus, if the regulator chooses approval scheme 1, CRAs may collude to offer inflated ratings. ∎

Proof of Proposition 1.5

Suppose the regulator chooses approval scheme 3.

If $\delta_{RA} \geq 1 - \frac{m}{2}$, there exists a subgame perfect equilibrium in which both approved CRAs offer correct ratings. The proof proceeds along the same lines as the proof of proposition 1.1. It is therefore omitted.

Moreover, approval scheme 3 provides an incentive to deviate from a collusive agreement to offer inflated ratings. Suppose CRA i and CRA j collude to choose rating threshold $a^H > m$ and fee Δ. If both CRAs stick to the collusive agreement, each approved CRA gets the payoff

$$(1-\delta_{RA})\Big[\frac{a^H}{2}\Delta+\sum_{t=2}^{\infty}\delta_{RA}^{t-1}\ \frac{m}{2}\Delta\Big]. \tag{1.67}$$

If CRA i deviates from the collusive agreement and chooses rating threshold $a^H-\epsilon$, the mass of issuers who demand a rating from CRA i decreases by $\frac{\epsilon}{2}$. The default rate within CRA i's rating category $\mathcal{A}$ decreases as well. As a result $x_j > x_i$. The regulator consequently denies approval to CRA j in all future periods and approves only CRA i until the default rate within CRA i's rating category $\mathcal{A}$ is larger than default probability d_A. CRA i hence gets the payoff

$$(1-\delta_{RA})\Big[\frac{a^H-\epsilon}{2}\Delta+\delta_{RA}m\Delta+\sum_{t=3}^{\infty}\delta_{RA}^{t-1}(1-p)^{t-2}m\Delta+\sum_{t=3}^{\infty}\delta_{RA}^{t-1}p^{t-2}\frac{m}{2}\Delta\Big]. \tag{1.68}$$

Since the payoff from deviating is larger than the payoff from sticking to the collusive agreement, each CRA has an incentive to deviate from a collusive agreement to offer inflated ratings.

If the regulator approves only CRA i until the default rate within CRA i's rating category $\mathcal{A}$ is larger than default probability d_A, CRA i offers correct ratings, if the CRAs' discount factor δ_{RA} is sufficiently high. The proof proceeds along the same lines as the proof of proposition 1.2. It is therefore omitted.

Thus, if δ_{RA} is sufficiently high, there exists a subgame perfect equilibrium, in which both approved CRAs offer correct ratings. In addition, approval scheme 3 prevents the approved CRAs from colluding to offer inflated ratings. ∎

1.6 References

Bolton, Patrick, Xavier Freixas and Joel Shapiro, 2010. The Credit Ratings Game. Economics Working Papers 1221, Universitat Pompeu Fabra.

Holmström, Bengt, 1982. Moral Hazard in Teams. Bell Journal of Economics 13, 324-340.

Mathis, Jérôme, James McAndrews and Jean-Charles Rochet, 2009. Rating the Raters: Are Reputation Concerns Powerful Enough to Discipline Rating Agencies? Journal of Monetary Economics 56, 657-674.

Mählmann, Thomas, 2008. Rating Agencies and the Role of Rating Publication Rights. Journal of Banking and Finance 32, 2414-2422.

Peyrache, Eloïc and Lucía Quesada, 2010. Intermediaries, Credibility and Incentives to Collude. Working paper, HEC Paris and Universidad Torcuato Di Tella.

SEC, 2003. Report on the role and function of credit rating agencies in the operation of the securities markets. http://www.sec.gov/news/studies/credratingreport0103.pdf.

SEC, 2007. Final rule: Oversight of credit rating agencies registered as nationally recognized statistical rating organizations. 17 CFR Parts 240 and 249b, Release No. 34-55857; http://www.sec.gov/rules/final/2007/34-55857.pdf.

Skreta, Vasiliki and Laura Veldkamp, 2009. Ratings Shopping and Asset Complexity: A Theory of Ratings Inflation. Journal of Monetary Economics 56, 678-695.

Strausz, Roland, 2005. Honest certification and the threat of capture. International Journal of Industrial Organization 23, 45-62.

Chapter 2

The Appeal of Risky Assets

2.1 Introduction

In the wake of the recent financial crisis, it has often been argued that fund managers did not know about the risks in their portfolio; otherwise, it has been argued, they would not have invested in such risky assets. We argue, however, that fund managers might have known about the risks and invested in the risky assets nonetheless. We suggest that it might even have been individually optimal to invest in the risky assets if the expected return on these assets was lower than the expected return on other, less risky assets.

A fund's performance is usually compared to the performance of an index or other funds. If a fund trails the benchmark, the fund manager is often replaced. We argue that this may lead to excessive risk-taking if fund managers differ in ability and investment restrictions are inadequate. If investment restrictions are adequate, benchmarking may sort out low-ability from high-ability fund managers. If, however, investment restrictions are inadequate, benchmarking may lead to excessive risk-taking. To match the benchmark, fund managers may increase the risk of their portfolio even if this decreases the expected return on the portfolio.

Some fund managers, for instance, are restricted to invest in AAA-rated bonds. If all AAA-rated bonds offer similar yields at a similar risk, benchmarking may sort out low-ability from high-ability fund managers. If, however, some AAA-rated bonds (e.g. AAA-rated mortgage-backed securities) offer higher yields at a higher risk, fund managers may invest in these bonds to match the benchmark even if their expected return is lower than the expected return on other, less risky AAA-rated bonds.

We consider a model in which fund managers differ in ability and are fired if they miss the benchmark. Fund managers can create a perfectly diversified portfolio and, in addition, gamble. High-ability fund managers can create a perfectly diversified portfolio with a higher return than low-ability fund managers.[1] In addition, fund managers have the opportunity to invest in a risky asset which increases the risk of the overall portfolio and decreases the expected return on the overall portfolio. Fund managers are fired if the realized return is lower than the average realized return.[2] If a fund manager is fired, the fund manager incurs costs. Besides, fund managers get a fixed wage, receive a share of profits and also bear a share of losses.

We find that if the costs of being fired are sufficiently large in relation to the share of the realized return, there exists no equilibrium in which no fund

[1]Suppose, for instance, high-ability managers may create a perfectly diversified portfolio at lower costs than low-ability managers, and investors receive the return of the portfolio net of these costs. Then, high-ability managers can create such a portfolio with a higher net return than low-ability managers.

[2]The firing rule is taken as given. If investment restrictions were adequate and fund managers could not gamble, such a firing rule would sort out low-ability from high-ability fund managers. The main purpose here, however, is not to design an optimal incentive or sorting scheme; it is to show that benchmarking may lead to excessive risk-taking if investment restrictions are inadequate.

manager invests in the risky asset. If there exists a symmetric[3] equilibrium in pure strategies, low-ability fund managers, at least, invest in the risky asset. We find that there exists at least one such equilibrium if and only if the returns of the risky asset or the probability that the risky asset generates a high return are sufficiently large.[4] (If fund managers do not receive a share of the realized return, our results hold if fund managers incur positive costs if they are fired.)

There is a substantial literature on incentives in funds management.[5] Brown et al. (1996), for instance, show empirically that fund managers with a bad relative performance increase risk relative to fund managers with a good relative performance. They informally argue that fund managers may increase risk because fund inflows are convex in relative performance.[6] Taylor (2003) develops a theoretical model in which fund managers compete for inflows in a winner-takes-all tournament. Taylor finds, however, that in such a setting, fund managers with a good relative performance are more likely to increase risk than managers with a bad relative performance. Our model differs from the prevalent point of view. We do not consider a situation in which fund managers try to be a 'winner'. We consider a situation in which fund managers try not to be a 'loser'. Khorana (1996) documents that fund managers with a bad relative performance are more likely to be replaced. We suppose that this influences investment decisions. And in contrast to the predictions of Taylor's model, the predictions of our model are consistent with Brown et al.'s empirical finding.

There is a number of models in which low-ability fund managers try to mimic the actions of high-ability fund managers. Trueman (1988) and Dasgupta and

[3]That is, all low-ability fund managers make the same investment decision and all high-ability fund managers make the same investment decision.

[4]The assumption, however, that an investment in the risky asset decreases the expected return on the overall portfolio still holds.

[5]Bhattacharya et al. (2008) provide a good literature overview.

[6]Sirri and Tufano (1998) among others, show empirically that fund inflows are indeed convex in relative performance.

Prat (2006), for instance, show that fund managers may trade excessively (termed 'noise trading' or 'churning') because of career concerns. They argue that low-ability fund managers (who are uninformed about the future return of a risky asset) may trade in order to appear to have high ability (i.e. to be informed about the future return of a risky asset). In our model, however, fund managers do not try to mimic portfolio choices. Instead, they try to mimic portfolio returns.

The rest of the paper is organized as follows. In section 2.2, we present a model in which fund managers differ in ability, have the opportunity to take excessive risk, and are fired if they miss the benchmark. In section 2.3, we examine investment decisions and characterize possible equilibria. In section 2.4, we conclude and highlight the importance of adequate investment restrictions. Proofs are provided in the appendix.

2.2 The Model

Consider a model with many fund managers. The managers differ in ability, have the opportunity to take excessive risk, and are fired if they miss the benchmark.

There are two types of managers who differ in their ability θ, $\theta \in \{\theta_L, \theta_H\}$, $\theta_H > \theta_L$. Of each type, there is a continuum of mass 1. (That is, half of managers have high ability. Qualitative results do not change if, instead, a fraction λ, $\lambda \in (0, 1)$, of managers have high ability.) Managers with ability θ_L are indexed by l, $l \in [0, 1]$. Managers with ability θ_H are indexed by h, $h \in [0, 1]$.

At date 1, managers make their investment decisions. Managers can create a perfectly diversified portfolio and, in addition, gamble. They have the opportunity to invest in a risky asset which increases the risk of the overall portfolio and decreases the expected return on the overall portfolio. (For instance, a manager who is restricted to invest in AAA-rated bonds may have the opportunity to invest in AAA-rated mortgage-backed securities.) To simplify notation, we normalize the amount of capital available to a manager to 1. Manager l invests a_l, $a_l \in [0,1]$, in the risky asset. Manager h invests a_h, $a_h \in [0,1]$, in the risky asset.

Managers differ in their ability to create a perfectly diversified portfolio. (Suppose, for instance, high-ability managers may create a perfectly diversified portfolio at lower costs than low-ability managers, and investors receive the return of the portfolio net of these costs. Then, high-ability managers can create such a portfolio with a higher net return than low-ability managers.) To simplify the model, we assume managers with ability θ_L can create a perfectly diversified portfolio which generates a return $r^0 + \theta_L$. Managers with ability θ_H can create a perfectly diversified portfolio which generates a return $r^0 + \theta_H$. (Results do not change if the perfectly diversified portfolios are not risk-free, as long as the returns are perfectly correlated.)

Managers do not, however, differ in their ability to gamble. (Qualitative results do not change if managers who have a higher ability to create a perfectly diversified portfolio also have a higher ability to gamble.) Each manager can invest in a risky asset with a return R,

$$R = \begin{cases} r^+ & \text{with probability } p \\ r^- & \text{with probability } 1-p. \end{cases} \tag{2.1}$$

We assume

$$r^+ > r^0 + \theta_H \tag{2.2}$$

and

$$E[R] = pr^+ + (1-p)r^- < r^0 + \theta_L. \tag{2.3}$$

That is, if the risky asset generates a high return, then the return on the risky asset is higher than the return on the perfectly diversified portfolio for each manager. (For instance, AAA-rated mortgage-backed securities offered a higher yield than other, less risky AAA-rated bonds.) However, the expected return on the risky asset is lower than the return on the perfectly diversified portfolio for each manager. (The purpose here is to show that it may be individually optimal to increase the risk of the portfolio *even if* this decreases the expected return on the portfolio. For instance, we want to argue that it might even have been individually optimal to invest in AAA-rated mortgage-backed securities if the expected return on these bonds was lower than on other, less risky bonds.)

The return on a manager's overall portfolio depends on the return on the perfectly diversified portfolio, the return on the risky asset and the composition of the overall portfolio. Manager l's portfolio generates a return Π_l,

$$\Pi_l = r^0 + \theta_L + [R - (r^0 + \theta_L)]a_l. \tag{2.4}$$

Manager h's portfolio generates a return Π_h,

$$\Pi_h = r^0 + \theta_H + [R - (r^0 + \theta_H)]a_h. \tag{2.5}$$

The average portfolio return $\bar{\Pi}$ depends on the managers' investment decisions. An individual manager's investment decision, however, has no influence on the average portfolio return because there is a continuum of managers. The average portfolio return is given by

$$\bar{\Pi} = \frac{1}{2}\left(\int_{l=0}^{1} \Pi_l dl + \int_{h=0}^{1} \Pi_h dh\right) \tag{2.6}$$

or

$$\bar{\Pi} = r^0 + \frac{1}{2}\left\{\theta_L + \theta_H + [R - (r^0 + \theta_L)]\int_{l=0}^{1} a_l dl + [R - (r^0 + \theta_H)]\int_{h=0}^{1} a_h dh\right\}. \tag{2.7}$$

At date 2, returns and payoffs are realized. Managers receive a fixed wage w and a share s, $s \in [0,1]$, of the realized return. Manager l (h) is fired, if the realized return π_l (π_h) is lower than the average realized return $\bar{\pi}$. If a manager is fired, the manager incurs costs c, $c \in \mathbb{R}_0^+$. (These costs may be interpreted as costs of finding a new job.)

The compensation contract and the firing rule are taken as given. If investment restrictions were adequate and managers could not gamble, the firing rule would sort out low-ability from high-ability managers. The main purpose here, however, is not to design an optimal incentive or sorting scheme; it is to show that if investment restrictions are inadequate and managers can gamble, such a firing rule may lead to excessive risk taking.

The probability that manager l (h) misses the benchmark depends on the amount a_l (a_h) the manager invests in the risky asset, and the amount a_{-l} (a_{-h}) the other managers invest in the risky asset. Let $P[\Pi_l < \bar{\Pi}|a_l, a_{-l}]$ ($P[\Pi_h < \bar{\Pi}|a_h, a_{-h}]$) denote the probability that manager l (h) misses the benchmark.

Managers are risk-neutral and maximize their expected utility. Manager l chooses a_l to solve

$$\max_{a_l \in [0,1]} E[U] = w + s\{r^0 + \theta_L + [E[R] - (r^0 + \theta_L)]a_l\} - P[\Pi_l < \bar{\Pi}|a_l, a_{-l}]c. \tag{2.8}$$

Manager h chooses a_h to solve

$$\max_{a_h \in [0,1]} E[U] = w + s\{r^0 + \theta_H + [E[R] - (r^0 + \theta_H)]a_h\} - P[\Pi_h < \bar{\Pi}|a_h, a_{-h}]c. \tag{2.9}$$

2.3 The Appeal of Risky Assets

Managers may not maximize the expected return on the overall portfolio because they incur costs if the realized return is lower than the average realized return. We now examine investment decisions and characterize possible equilibria. We focus on symmetric[7] equilibria in pure strategies.

Proposition 2.1 *If costs c are sufficiently large in relation to the share s of the realized return, there exists no equilibrium in which no manager invests in the risky asset.*

The intuition is straightforward. Consider a manager with low ability and suppose all other managers do not invest in the risky asset. If the manager does not invest in the risky asset, the manager misses the benchmark with probability 1. If, by contrast, the manager invests sufficiently in the risky asset, there are two effects: On the one hand, the expected return on the overall portfolio decreases. On the other hand, the probability of matching the benchmark increases. The manager matches the benchmark if the risky asset generates a high return. If costs c are sufficiently large in relation to the share s of the realized return, the manager hence invests in the risky asset.

Note that the threshold for costs c decreases with the share s of the realized return. If managers do not receive a share of the realized return ($s = 0$), a low-ability manager always invests in the risky asset (i.e. if $c \geq 0$ which is satisfied by assumption).

Suppose costs c are sufficiently large in relation to the share s of the realized return and a symmetric equilibrium in pure strategies exists. Then, there is a

[7]That is, we examine equilibria in which all managers with low ability θ_L make the same investment decision and all managers with high ability θ_H make the same investment decision.

linear relationship between the amount managers with low ability and managers with high ability invest in the risky asset:

Proposition 2.2 *Suppose costs c are sufficiently large in relation to the share s of the realized return, and a symmetric equilibrium in pure strategies exists. Then,*

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)} a_h. \tag{2.10}$$

The intuition is as follows: Suppose costs c are sufficiently large in relation to the share s of the realized return and a symmetric equilibrium in pure strategies exists. Then, both low- and high-ability managers must invest just as much in the risky asset as is necessary to match the benchmark if the risky asset generates a high return.

There may exist an equilibrium in which only low-ability managers invest in the risky asset. If high-ability managers invest

$$a_h = 0 \tag{2.11}$$

in the risky asset, and low-ability managers invest

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} \tag{2.12}$$

in the risky asset, both low- and high-ability managers match the benchmark if the risky asset generates a high return.

However, there may also exist equilibria in which high-ability managers invest in the risky asset. If high-ability managers invest

$$a_h = x \tag{2.13}$$

in the risky asset, where $0 < x \leq 1$, low-ability managers have to invest

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)} x \tag{2.14}$$

in the risky asset to match the benchmark if the risky asset generates a high return. And if, in turn, low-ability managers invest

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)} x \tag{2.15}$$

in the risky asset, high-ability managers have to invest

$$a_h = x \tag{2.16}$$

in the risky asset to match the benchmark if the risky asset generates a high return.

Hence, if costs c are sufficiently large in relation to the share s of the realized return and a symmetric equilibrium exists, there is a linear relationship between the amount low- and high-ability managers invest in the risky asset:

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)} a_h. \tag{2.17}$$

The more high-ability managers invest in the risky asset, the more low-ability managers invest in the risky asset, and vice versa.

Possible symmetric equilibria in pure strategies are depicted in figure 2.1. Note that low-ability managers, at least, invest in the risky asset ($a_l > 0$), and they invest at least as much as high-ability managers ($a_l \geq a_h$).

If costs c are sufficiently large in relation to the share s of the realized return, at least one symmetric equilibrium exists under certain conditions:

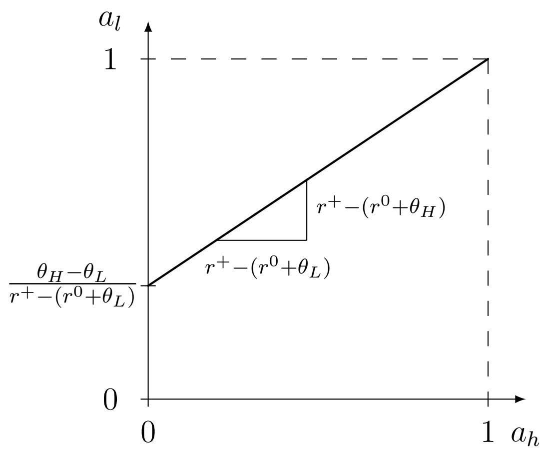

Figure 2.1: Possible equilibria.

Proposition 2.3 *Suppose costs c are sufficiently large in relation to the share s of the realized return. Then, there exists at least one symmetric equilibrium in pure strategies if and only if*

- *the returns r^+ and r^- of the risky asset are sufficiently large, or*
- *the probability p that the risky asset generates a high return is sufficiently large in relation to the share s of the realized return.*

The intuition is as follows. There exists at least one equilibrium if and only if there exists an equilibrium in which high-ability managers invest

$$a_h = 0 \tag{2.18}$$

in the risky asset, and low-ability managers invest

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} \tag{2.19}$$

in the risky asset. The 'if' part is obvious. For the 'only if' part, note that if there does not exist an equilibrium in which only low-ability managers invest in the risky asset, there does not exist an equilibrium in which both low- and high-ability managers invest in the risky asset.

There exists an equilibrium in which high-ability managers invest 0 in the risky asset, and low-ability managers invest $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset, if and only if the returns r^+ and r^- are sufficiently large, or the probability p is sufficiently large in relation to the share s of the realized return.[8] To see this, consider the investment decisions of high- and low-ability managers.

First, consider the investment decision of a high-ability manager. Suppose all low-ability managers invest $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset and all other high-ability managers invest 0 in the risky asset. If the high-ability manager also invests 0 in the risky asset, the manager misses the benchmark with probability 0. If, by contrast, the high-ability manager invests more than 0 in the risky asset, there are two negative effects. First, the expected return on the overall portfolio decreases. And second, the probability of missing the benchmark may increase. Hence, the high-ability manager also invests 0 in the risky asset.

Now, consider the investment decision of a low-ability manager. Suppose all high-ability managers invest 0 in the risky asset and all other low-ability managers invest $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset. If the low-ability manager also invests $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset, the manager matches the benchmark if the risky asset generates a high return, but misses the benchmark if the risky asset generates a low return. That is, the manager misses the benchmark with probability $1 - p$. If, by contrast, the low-ability manager invests 0 in the risky asset, there are again two effects. First, the expected return on the overall portfolio increases. Second, the probability of missing the benchmark changes. The probability of missing the benchmark now depends on the returns r^+ and r^- of the risky asset.

[8] The assumption, however, that the expected return on the risky asset is lower than the return on the perfectly diversified portfolio still holds.

If r^+ and r^- are sufficiently large, the low-ability manager misses the benchmark with probability 1. If r^+ and r^- are not sufficiently large, the low-ability manager only misses the benchmark with probability p.

The reason is as follows. (Consider again the investment decision of a low-ability manager, and suppose again the manager invests 0 in the risky asset, while all other low-ability managers invest $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ and all high-ability managers invest 0 in the risky asset.) If the risky asset generates a high return, the low-ability manager misses the benchmark in any case. If the risky asset generates a low return, the manager also misses the benchmark if the average portfolio return does not suffer much. This is the case if r^+ and r^- are sufficiently large. If the return r^+ is large, the other low-ability managers do not have to invest much in the risky asset to match the benchmark if the risky asset generates a high return (i.e. $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ is small). And if the return r^- is large, portfolio returns do not suffer much for a given amount invested in the risky asset if the risky asset generates a low return. Hence, if r^+ and r^- are sufficiently large, the low-ability manager misses the benchmark with probability 1 if the manager does not invest in the risky asset. If r^+ and r^- are not sufficiently large, the low-ability manager only misses the benchmark with probability p if the manager does not invest in the risky asset.

Suppose now the returns r^+ and r^- are sufficiently large. Then, the low-ability manager misses the benchmark with probability 1 if the manager does not invest in the risky asset. Hence, the low-ability manager invests $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset if costs c are sufficiently large in relation to the share s of the realized return (which is satisfied by assumption).

Note that the threshold for costs c decreases with the share s of the realized return. If managers do not receive a share of the realized return ($s = 0$), a low-ability manager invests in the risky asset if managers incur positive costs if they are fired ($c \geq 0$).

Now, suppose the returns r^+ and r^- are not sufficiently large. Then, the low-ability manager misses the benchmark with probability p if the manager does not invest in the risky asset. Hence, the low-ability manager invests $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset if the probability p that the risky asset generates a high return is sufficiently large in relation to the share s of the realized return.

Note that the threshold for probability p decreases with the share s of the realized return. If managers do not receive a share of the realized return ($s = 0$), a low-ability manager invests in the risky asset if a high return is at least as likely as a low return ($p \geq \frac{1}{2}$).

There may also exist equilibria in which both low- and high-ability managers invest in the risky asset if the returns r^+ and r^- are sufficiently large, or the probability p is sufficiently large in relation to the share s of the realized return. If high-ability managers invest more than 0 in the risky asset, low-ability managers face a trade-off similar to those described above. If low-ability managers invest more than $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset, high-ability managers also face a similar trade-off. Suppose low-ability managers invest more than $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset. Then, if a high-ability manager invests 0 in the risky asset, the manager misses the benchmark if the risky asset generates a high return. If, by contrast, the high-ability manager invests sufficiently in the risky asset, the manager misses the benchmark with probability 0. If costs c are sufficiently large in relation to the share s of the realized return, the high-ability manager hence invests in the risky asset.

As a result, managers may face a coordination problem. They would be best off if only low-ability managers invested in the risky asset. However, they may end up in an equilibrium in which both low- and high-ability managers invest in the risky asset.

2.4 Conclusion

We argue that fund managers may take excessive risk if they differ in ability, are fired if they miss the benchmark, and have the opportunity to gamble. If investment restrictions are adequate, benchmarking may sort out low-ability from high-ability fund managers. If, however, fund managers have the opportunity to gamble, they may take excessive risk to match the benchmark.

Investment restrictions are often based on ratings. It can be argued that inflated ratings increase fund managers' ability to gamble. Hence, inflated ratings may lead to excessive risk-taking even if fund managers do not take ratings at face value and know about the risks. Therefore, it is indeed important that credit rating agencies assign correct ratings.

2.5 Appendix

Proof of Proposition 2.1

Consider manager l's investment decision (choice of a_l) and suppose all other managers do not invest in the risky asset ($a_{-l} = 0$). Then, the average portfolio return is

$$\bar{\Pi} = r^0 + \frac{1}{2}(\theta_L + \theta_H). \tag{2.20}$$

If the manager does not invest in the risky asset ($a_l = 0$), the portfolio return is

$$\Pi_l = r^0 + \theta_L. \tag{2.21}$$

The manager hence misses the benchmark with probability 1 and gets the utility

$$U = w + s(r^0 + \theta_L) - c. \tag{2.22}$$

The manager's utility decreases with the amount a_l invested in the risky asset until a_l is sufficiently large such that the portfolio return matches the average portfolio return if the risky asset generates a return r^+. The portfolio return matches the average portfolio return if $R = r^+$ and

$$r^0 + \theta_L + [r^+ - (r^0 + \theta_L)]a_l = r^0 + \frac{1}{2}(\theta_L + \theta_H) \tag{2.23}$$

or

$$a_l = \frac{\theta_H - \theta_L}{2[r^+ - (r^0 + \theta_L)]}. \tag{2.24}$$

Let

$$\hat{a} \equiv \frac{\theta_H - \theta_L}{2[r^+ - (r^0 + \theta_L)]} \tag{2.25}$$

and note that

$$0 < \hat{a} < 1. \tag{2.26}$$

If the manager chooses $a_l = \hat{a}$, the manager only misses the benchmark with probability $1 - p$. The manager hence gets the utility

$$E[U] = w + s\{r^0 + \theta_L + [E[R] - (r^0 + \theta_L)]\hat{a}\} - (1 - p)c. \tag{2.27}$$

Starting from $a_l = \hat{a}$, the manager's utility again decreases with the amount a_l invested in the risky asset.

The manager strictly prefers $a_l = \hat{a}$ to $a_l = 0$ if

$$w + s\{r^0 + \theta_L + [E[R] - (r^0 + \theta_L)]\hat{a}\} - (1 - p)c > w + s(r^0 + \theta_L) - c. \tag{2.28}$$

This can be rearranged to get

$$pc > s(r^0 + \theta_L - E[R])\hat{a}. \tag{2.29}$$

Substituting $\hat{a}$ and rearranging gives

$$c > s\frac{\theta_H - \theta_L}{2p}\frac{r^0 + \theta_L - E[R]}{r^+ - (r^0 + \theta_L)}. \tag{2.30}$$

Hence, if costs c are sufficiently large in relation to the share s of the realized return, there exists no equilibrium in which no manager invests in the risky asset. ∎

Proof of Proposition 2.2

Suppose

$$c > s\frac{\theta_H - \theta_L}{2p}\frac{r^0 + \theta_L - E[R]}{r^+ - (r^0 + \theta_L)}, \tag{2.31}$$

and all managers with low ability θ_L invest the same amount a_l in the risky asset and all managers with high ability θ_H invest the same amount a_h in the risky asset.

First, consider managers with low ability θ_L. Note that they match the benchmark if they match managers with high ability θ_H. If the risky asset generates a high return r^+, managers with low ability θ_L match the benchmark if

$$r^0 + \theta_L + [r^+ - (r^0 + \theta_L)]a_l = r^0 + \theta_H + [r^+ - (r^0 + \theta_H)]a_h \tag{2.32}$$

or

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)}a_h. \tag{2.33}$$

Let

$$\hat{a}_l(a_h) \equiv \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)}a_h, \tag{2.34}$$

and note that

$$0 < \hat{a}_l(a_h) \leq 1. \tag{2.35}$$

There cannot be an equilibrium in which managers with low ability θ_L invest $0 < a_l < \hat{a}_l(a_h)$ in the risky asset. Each manager with low ability θ_L would be better off investing $a_l = 0$ in the risky asset.

There cannot be an equilibrium in which managers with low ability θ_L invest $a_l > \hat{a}_l(a_h)$ in the risky asset. Each manager with low ability θ_L would be better off investing $a_l = \hat{a}_l(a_h)$ in the risky asset.

Therefore, if there is an equilibrium, managers with low ability θ_L must invest either $a_l = 0$ or $a_l = \hat{a}_l(a_h)$ in the risky asset.

Now, consider managers with high ability θ_H. Note that they fall behind the benchmark if they fall behind managers with low ability θ_L. If managers with low ability θ_L invest $a_l \leq \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset, managers with high ability θ_H do not fall behind the benchmark if they invest $a_h = 0$ in the risky asset. If, by contrast, managers with low ability θ_L invest $a_l > \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset, managers with high ability θ_H fall behind the benchmark if they invest $a_h = 0$ in the risky asset and the risky asset generates a high return r^+.

If $a_l > \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ and $R = r^+$, managers with high ability θ_H do not fall behind the benchmark if

$$r^0 + \theta_H + [r^+ - (r^0 + \theta_H)]a_h = r^0 + \theta_L + [r^+ - (r^0 + \theta_L)]a_l \tag{2.36}$$

or

$$a_h = -\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_H)} + \frac{r^+ - (r^0 + \theta_L)}{r^+ - (r^0 + \theta_H)} a_l. \tag{2.37}$$

Let

$$\hat{a}_h(a_l) \equiv -\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_H)} + \frac{r^+ - (r^0 + \theta_L)}{r^+ - (r^0 + \theta_H)} a_l, \tag{2.38}$$

and note that

$$0 < \hat{a}_h(a_l) \leq 1. \tag{2.39}$$

If $a_l \leq \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$, there cannot be an equilibrium in which managers with high ability θ_H invest $a_h > 0$ in the risky asset. Each manager with high ability θ_H would be better off investing less in the risky asset.

Therefore, if $a_l \leq \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ and there is an equilibrium, managers with high ability θ_H must invest $a_h = 0$ in the risky asset.

If $a_l > \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$, there cannot be an equilibrium in which managers with high ability θ_H invest $0 < a_h < \hat{a}_h(a_l)$ in the risky asset. Each manager with high ability θ_H would be better off investing $a_h = 0$ in the risky asset.

If $a_l > \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$, there cannot be an equilibrium in which managers with high ability θ_H invest $a_h > \hat{a}_h(a_l)$ in the risky asset. Each manager with high ability θ_H would be better off investing $a_h = \hat{a}_h(a_l)$ in the risky asset.

Therefore, if $a_l > \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ and there is an equilibrium, managers with high ability θ_H must invest either $a_h = 0$ or $a_h = \hat{a}_h(a_l)$ in the risky asset.

By proposition 2.1, $a_l = 0$ and $a_h = 0$ cannot be an equilibrium. Thus, if there is an equilibrium, $a_l \geq \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$.

Note that

$$\hat{a}_h \left(\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} \right) = 0 \tag{2.40}$$

and

$$\hat{a}_h(a_l) = \hat{a}_l^{-1}(a_h). \tag{2.41}$$

Therefore, if there is an equilibrium, there is a linear relationship between the amount managers with low ability θ_L and those with high ability θ_H invest in the risky asset:

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)} a_h. \quad \blacksquare \tag{2.42}$$

Proof of Proposition 2.3

Suppose

$$c \geq s \frac{\theta_H - \theta_L}{p} \frac{r^0 + \theta_L - E[R]}{r^+ - (r^0 + \theta_L)}. \tag{2.43}$$

Let

$$\hat{a}_l(a_h) \equiv \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)} a_h \tag{2.44}$$

and

$$\hat{a}_h(a_l) \equiv -\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_H)} + \frac{r^+ - (r^0 + \theta_L)}{r^+ - (r^0 + \theta_H)} a_l. \tag{2.45}$$

There exists a symmetric equilibrium in pure strategies if and only if each manager with low ability θ_L prefers

$$a_l = \hat{a}_l(a_h) \tag{2.46}$$

to

$$a_l = 0, \tag{2.47}$$

and each manager with high ability θ_H prefers

$$a_h = \hat{a}_h(a_l) \tag{2.48}$$

to

$$a_h = 0. \tag{2.49}$$

First, consider the investment decision of a manager with high ability θ_H. Then, consider the investment decision of a manager with low ability θ_L.

If all managers with low ability θ_L invest $a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset, $\hat{a}_h = 0$. If all managers with low ability θ_L invest $a_l > \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset, $\hat{a}_h > 0$. In this case, each manager with high ability θ_H prefers $a_h = \hat{a}_h$ to $a_h = 0$ if

$$w + s\{r^0 + \theta_H + [E[R] - (r^0 + \theta_H)]\hat{a}_h\} \geq w + s(r^0 + \theta_H) - pc. \tag{2.50}$$

This can be rearranged to get

$$c \geq s\frac{r^0 + \theta_H - E[R]}{p}\hat{a}_h. \tag{2.51}$$

Now, consider a manager with low ability θ_L and suppose the manager invests $a_l = 0$ in the risky asset. Furthermore, suppose all other managers with low ability θ_L invest $\hat{a}_l$ in the risky asset and all managers with high ability θ_H invest $\hat{a}_h$ in the risky asset. Then, the low-ability manager misses the benchmark if the risky asset generates a high return r^+. If the risky asset generates a low return r^-, the manager also misses the benchmark if

$$r^0 + \theta_L < r^0 + \frac{1}{2}\left\{\theta_L + \theta_H + [r^- - (r^0 + \theta_L)]\hat{a}_l + [r^- - (r^0 + \theta_H)]\hat{a}_h\right\}. \tag{2.52}$$

This can be rearranged to get

$$(r^0 + \theta_L - r^-)\hat{a}_l + (r^0 + \theta_H - r^-)\hat{a}_h < \theta_H - \theta_L. \tag{2.53}$$

That is, if the risky asset generates a low return r^-, the low-ability manager misses the benchmark if the other managers do not invest 'too much' in the risky asset.

First, suppose

$$(r^0 + \theta_L - r^-)\hat{a}_l + (r^0 + \theta_H - r^-)\hat{a}_h < \theta_H - \theta_L. \tag{2.54}$$

Then, a manager with low ability θ_L misses the benchmark with probability 1. If, by contrast, the manager also invests $a_l = \hat{a}_l$ in the risky asset, the manager matches the benchmark if the risky asset generates a high return r^+. That is, the manager only misses the benchmark with probability $1 - p$. The manager prefers

$a_l = \hat{a}_l$ to $a_l = 0$ if

$$w + s\{r^0 + \theta_L + [ER - (r^0 + \theta_L)]\hat{a}_l\} - (1-p)c \geq w + s(r^0 + \theta_L) - c. \quad (2.55)$$

This can be rearranged to get

$$c \geq s\frac{r^0 + \theta_L - E[R]}{p}\hat{a}_l. \quad (2.56)$$

Note that $\hat{a}_h \geq 0$ and $\hat{a}_l \geq \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$. Therefore, there exists at least one symmetric equilibrium in pure strategies if

$$c \geq s\frac{\theta_H - \theta_L}{p}\frac{r^0 + \theta_L - E[R]}{r^+ - (r^0 + \theta_L)}, \quad (2.57)$$

which is satisfied by assumption, and

$$(r^0 + \theta_L - r^-)\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + (r^0 + \theta_H - r^-)0 < \theta_H - \theta_L. \quad (2.58)$$

The latter condition can be rearranged to get

$$r^+ + r^- > 2(r^0 + \theta_L). \quad (2.59)$$

Now, suppose

$$(r^0 + \theta_L - r^-)\hat{a}_l + (r^0 + \theta_H - r^-)\hat{a}_h \geq \theta_H - \theta_L. \quad (2.60)$$

Then, if a manager with low ability θ_L invests $a_l = 0$ in the risky asset, the manager only misses the benchmark if the risky asset generates a high return r^+. If, by contrast, the manager also invests $a_l = \hat{a}_l$ in the risky asset, the manager does not miss the benchmark if the risky asset generates a high return r^+. However, the manager misses the benchmark if the risky asset generates a low return r^-. The manager prefers $a_l = \hat{a}_l$ to $a_l = 0$ if

$$w + s\{r^0 + \theta_L + [E[R] - (r^0 + \theta_L)]\hat{a}_l\} - (1-p)c \geq w + s(r^0 + \theta_L) - pc. \quad (2.61)$$

This can be rearranged to get

$$p \geq \frac{1}{2} + s\frac{r^0 + \theta_L - E[R]}{2c}\hat{a}_l. \quad (2.62)$$

Note again that $\hat{a}_h \geq 0$ and $\hat{a}_l \geq \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$. Therefore, there exists at least one symmetric equilibrium in pure strategies if

$$p \geq \frac{1}{2} + s\frac{\theta_H - \theta_L}{2c}\frac{r^0 + \theta_L - E[R]}{r^+ - (r^0 + \theta_L)}. \quad (2.63)$$

Finally, note that if neither

$$r^+ + r^- > 2(r^0 + \theta_L) \quad (2.64)$$

nor

$$p \geq \frac{1}{2} + s\frac{\theta_H - \theta_L}{2c}\frac{r^0 + \theta_L - E[R]}{r^+ - (r^0 + \theta_L)}, \quad (2.65)$$

there exists no symmetric equilibrium in pure strategies. ∎

2.6 References

Bhattacharya, Sudipto, Amil Dasgupta, Alexander Guembel and Andrea Prat, 2008. Incentives In Funds Management: A Literature Overview, in: Thakor, A., Boot, A. (Eds.), Handbook of Financial Intermediation and Banking. North-Holland, Amsterdam, pp. 285-303.

Brown, Keith, W. V. Harlow and Laura Starks, 1996. Of Tournaments and Temptations: An Analysis of Managerial Incentives in the Mutual Fund Industry. Journal of Finance 51, 85-110.

Dasgupta, Amil, and Andrea Prat, 2006. Financial Equilibrium with Career Concerns. Theoretical Economics 1, 67-94.

Khorana, Ajay, 1996. Top Management Turnover: An Empirical Investigation of Mutual Fund Managers. Journal of Financial Economics 40, 403-427.

Sirri, Erik, and Peter Tufano, 1998. Costly Search and Mutual Fund Flows. Journal of Finance 53, 1589-1622.

Taylor, Jonathan, 2003. Risk-Taking Behavior in Mutual Fund Tournaments. Journal of Economic Behavior and Organization 50, 373- 383.

Trueman, Brett, 1988. A Theory of Noise Trading in Securities Markets. Journal of Finance 18, 83-95.

Chapter 3

The Credibility of Certifiers

3.1 Introduction

It is often argued that certifiers have an incentive to offer inflated certificates. Recently, credit rating agencies have attracted a lot of attention. They have been accused of assigning inflated ratings to structured products, such as mortgage-backed securities and collateralized-debt obligations. In the last decade, auditors have also been repeatedly in the spotlight. Especially in the wake of the Enron scandal, they have faced allegations of being lapdogs instead of watchdogs.

The problem, it is argued, lies in the business model. Certifiers are typically paid by sellers who are interested in favorable certificates. An old proverb says: 'He who pays the piper calls the tune.'

Yet, certifiers typically also incur costs if they offer inflated certificates. Often they have to spend time and money to obscure that they offer inflated certificates. In addition, an inflated certificate is usually detected with some probability. And if caught, certifiers often have to pay a fine. Moreover, they occasionally lose some of their business. Sometimes, they even lose all of their business, as in the case of Arthur Andersen for its role in the Enron scandal, though this was a rare event.

In this paper, we examine a certifier's incentive to offer inflated certificates. We consider a model in which a certifier is paid by sellers. While the certifier can observe the type of a good, the buyers cannot. The buyers can only observe whether the seller owns a certificate, and if so, the type of the certificate. The sellers are hence interested in favorable certificates. The certifier may offer them inflated certificates, but incurs costs if doing so. We suppose that the costs of offering inflated certificates increase with the number of inflated certificates.

We find that if the costs of offering the first inflated certificate are lower than the sellers' willingness-to-pay for it, the certifier always offers at least one inflated certificate. However, the certifier does not profit from offering inflated certificates, because in equilibrium, the buyers cannot be fooled: They believe correctly that the certifier offers a certain amount of inflated certificates, and take this into account in their willingness-to-pay for a good. And the sellers take this into account in their willingness-to-pay for a certificate. The certifier would hence make a higher profit if the certifier did not offer inflated certificates and the buyers believed it.

The certifier hence faces a commitment problem. It would be best off if it did not offer inflated certificates, and the buyers believed it. But if the buyers actually believed that the certifier does not offer inflated certificates, the certifier would have an incentive to do so.

The number of inflated certificates obviously depends on the costs the certifier incurs when offering inflated certificates. If the costs increase, the certifier offers less inflated certificates in equilibrium.

Yet, the certifier may oppose an increase in the costs of offering inflated certificates, because an increase in the costs has two opposing effects on the certifier's profit. On the one hand, an increase in the costs reduces the number of inflated certificates, and thereby indirectly increases the certifier's profit. On the other hand, however, an increase in the costs directly reduces the certifier's profit.

There is a growing literature on certification intermediaries. Several papers study under which conditions a certifier does not have an incentive to offer inflated certificates. However, there is also a growing number of papers which show that a certifier may offer inflated certificates in equilibrium.

Strausz (2005) studies a model in which buyers detect any inflated certificate ex post, and the certifier goes out of business if caught. He shows that honest certification requires a patient certifier, high prices, and constitutes a natural monopoly. In our model, we take a different approach. We consider a situation in which a certifier offers inflated certificates in equilibrium, and show that the certifier faces a commitment problem.

Peyrache and Quesada (2010) also study a model in which an inflated certificate is detected with probability one, and the certifier goes out of business forever if caught. They focus, however, on an equilibrium in which a certifier may offer an inflated certificate: They find that an impatient certifier may offer an inflated certificate with some probability. However, in their model, even a very impatient certifier does not offer an inflated certificate with probability one. In our model, we find that a certifier always offers at least one inflated certificate if the costs of offering the first inflated certificate are lower than the sellers' willingness-to-pay for it.

Mathis et al. (2009) study a reputation model to analyze the incentive of a credit rating agency to assign inflated ratings. They assume that a credit rating agency is either committed to tell the truth, or maximizes its payoff. They show that a credit rating agency which maximizes its payoff may first build a reputation of being committed, and then 'cash in' on its reputation by assigning inflated ratings. Our model, however, does not rely on the assumption that the buyers believe that the certifier is committed to tell the truth with positive probability. In our model, the buyers know that the certifier maximizes its profit.

Bolton et al. (2010) also study the incentive of a credit rating agency to offer inflated ratings. They assume that a fraction of investors are naive and take ratings at face value. They find that a credit rating agency may offer inflated ratings if there are many naive investors and the expected punishment is light. In contrast to Bolton et al. (2010), our model does not rely on the assumption that a fraction of buyers are naive. We show that even if buyers cannot be fooled in equilibrium, a certifier may have an incentive to offer inflated certificates.

In Stolper (2009), we study a model in which a regulator can observe the default rate within a rating category for each credit rating agency. The default rate may, however, be influenced by a common shock, and the credit rating agencies may collude to offer inflated ratings. As a result, the regulator cannot detect whether high default rates are due to collusion or the common shock. In Stolper (2009), we hence suggest that a regulator should not only deter a credit rating agency from unilaterally offering inflated ratings, but also provide an incentive to deviate from a collusive agreement to offer inflated ratings. In our model of a certifier's credibility, we do not consider the possibility of a collusive agreement. Instead, we show that a certifier may face a commitment problem.

There are several papers which study related commitment problems in other settings. Barro and Gordon (1983), for instance, study a model in which a central bank tries to decrease unemployment by increasing inflation. When making its decision, the central bank takes inflation expectations as given, and chooses an excessive rate of inflation. However, in equilibrium, people correctly anticipate the central bank's decision. The central bank hence cannot reduce unemployment, and would be better off if both inflation and the expectation of inflation were zero. Yet, if the expectation of inflation was actually zero, the central bank would have an incentive to choose a positive rate of inflation.

Signal-jamming models (first analyzed by Holmström, 1982) often make a similar point. Stein (1989), for instance, studies a model in which a manager tries to increase the stock price by pumping up earnings. However, in equilibrium, the manager cannot fool investors because they take into account that earnings are inflated. The manager would hence be better off if he (or she) did not manipulate earnings, and the investors did not suspect that earnings are manipulated. But if the investors actually believed that the manager does not manipulate earnings, the manager would have an incentive to do so.

The rest of the paper is organized as follows. In section 3.2, we present the model. In section 3.3, we consider the certifier's decision whether to offer inflated certificates. We show that the certifier faces a commitment problem and analyze the effect of an increase in the costs of offering inflated certificates. In section 3.4, we conclude. Proofs are provided in the appendix.

3.2 The Model

Consider a model with many sellers, many buyers, and a certifier. Each seller owns one good, of which there are two types, A and B. Let a denote the number of sellers who own a type-A good, and let b denote the number of sellers who own a type-B good. Each seller faces one buyer. A buyer's utility of a type-A

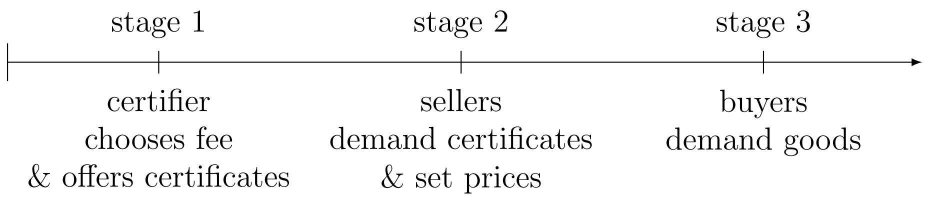

Figure 3.1: The time structure.

good is u_A, and a buyer's utility of a type-B good is u_B, where $u_A > u_B$. The buyers cannot, however, observe the type of a good. The certifier, by contrast, can observe the type of a good. The buyers can only observe whether the seller owns a certificate, and if so, the type of the certificate. There are two types of certificates, again $\mathcal{A}$ and $\mathcal{B}$. A type-$\mathcal{A}$ certificate indicates that a good is of type A, and a type-$\mathcal{B}$ certificate indicates that a good is of type B.

The time structure is as follows. At stage 1, the certifier chooses a (uniform) fee and offers each seller a certificate. (Results do not change if the certifier chooses different fees for different types of certificates.) At stage 2, each seller decides whether to buy the certificate and sets a price for his or her good. At stage 3, each buyer decides whether to buy the good. Figure 3.1 illustrates the time structure.

At stage 1, the certifier offers every seller, who owns a type-A good, a type-$\mathcal{A}$ certificate, but may offer sellers, who own a type-B good, a type-$\mathcal{B}$ as well as a type-$\mathcal{A}$ certificate. Let x denote the number of sellers who are offered an inflated certificate.

If the certifier offers sellers an inflated certificate, the certifier incurs costs. Typically, a certificate which indicates the wrong type is detected with some probability, and if detected, the certifier is punished. The certifier may, for instance, have to pay a fine or be ignored by buyers in future periods. In addition, the certifier may have to spend time and money to obscure that it offers inflated certificates. We suppose that the probability of detection, the punishment, or the cost of obfuscation increases with the number of inflated certificates. We hence assume that the certifier incurs costs $K(x)$, where $K(0) = 0$ and $K'(x) > 0$. Moreover, to keep the analysis simple, we assume $K''(x) \geq 0$. We suppose that if the probability of detection or the punishment varies for different sellers, a certifier first offers an inflated certificate to sellers, for whom the detection probability or the punishment is low. However, results do not change if $K''(x) < 0$ or $K(x)$ has an upper bound.

3.3 The Credibility of Certifiers

In this section, we consider the certifier's decision whether to offer inflated certificates. First, we show that the certifier faces a commitment problem. Then, we analyze the effect of an increase in the costs of offering inflated certificates.

3.3.1 A Commitment Problem

To consider the certifier's commitment problem, we solve the model backwards. We begin with the buyers' decision whether to buy the good.

At stage 3, the buyers buy the good if and only if the price is less than or equal to their willingness-to-pay, which depends on the certificate the seller owns. If a seller owns a type-$\mathcal{B}$ certificate, the buyer knows that the seller owns a type-B good,[1] and is hence only willing to pay u_B. If a seller either owns a type-$\mathcal{A}$ certificate or does not own a certificate at all, the buyer's willingness-to-pay depends on the buyer's beliefs.

[1] Because the certifier may only offer sellers, who own a type-B good, a type-$\mathcal{B}$ certificate.

Suppose the buyers believe that every seller, who is offered a type-$\mathcal{A}$ certificate, buys the certificate, and no seller, who is offered a type-$\mathcal{B}$ certificate, buys the certificate. Let x^e denote the buyers' belief about the number of sellers who are offered an inflated certificate. Then, if a seller owns a type-$\mathcal{A}$ certificate, the buyer believes that the seller owns a type-A good with probability $\frac{a}{a+x^e}$ and a type-$\mathcal{B}$ good with probability $\frac{x^e}{a+x^e}$, and is hence willing to pay $\frac{au_A+x^e u_B}{a+x^e}$ for the good. If a seller does not own a certificate, the buyer believes that the seller owns a type-B good, and is hence only willing to pay u_B.

At stage 2, each seller, with or without a certificate, sets a price equal to the buyer's willingness-to-pay. The difference in prices determines the sellers' willingness-to-pay for a certificate. The sellers are willing to pay zero for a type-$\mathcal{B}$ certificate. The willingness-to-pay for a type-$\mathcal{A}$ certificate again depends on the buyers' beliefs.

Suppose again the buyers believe that every seller, who is offered a type-$\mathcal{A}$ certificate, buys the certificate, and no seller, who is offered a type-$\mathcal{B}$ certificate, buys the certificate. Then, the sellers are willing to pay $\frac{au_A+x^e u_B}{a+x^e} - u_B$ for a type-$\mathcal{A}$ certificate, which is equivalent to $\frac{a(u_A-u_B)}{a+x^e}$.

The sellers buy the offered certificate if and only if the certifier chose a fee less than or equal to their willingness-to-pay. Sellers, who are offered a type-$\mathcal{A}$ certificate, buy the certificate if and only if the fee is less than or equal to their willingness-to-pay for the certificate. Sellers, who are offered a type-$\mathcal{B}$ certificate, buy the certificate if and only if the fee is zero.

At stage 1, the certifier hence chooses a fee equal to the sellers' willingness-to-pay for a type-$\mathcal{A}$ certificate. As a result, every seller, who is offered a type-$\mathcal{A}$ certificate, buys the certificate, and no seller, who is offered a type-$\mathcal{B}$ certificate, buys the certificate.

Suppose in the following that the buyers believe correctly that every seller, who is offered a type-$\mathcal{A}$ certificate, buys the certificate, and no seller, who is offered a type-$\mathcal{B}$ certificate, buys the certificate. Then, as shown above, the sellers are willing to pay $\frac{a(u_A-u_B)}{a+x^e}$ for a type-$\mathcal{A}$ certificate, and the certifier chooses a fee equal to this amount. Let $WTP(x^e)$ denote the sellers' willingness-to-pay for a type-$\mathcal{A}$ certificate, where

$$WTP(x^e) = \frac{a(u_A - u_B)}{a + x^e}. \tag{3.1}$$

The certifier's profit $\Pi(x|x^e)$ is given by

$$\Pi(x|x^e) = WTP(x^e)(a + x) - K(x). \tag{3.2}$$

The certifier chooses x to maximize its profit Π given the buyers' belief x^e. (In equilibrium, the buyers' beliefs are correct, but, when making its decision, the certifier takes the buyers' beliefs as given.)

Suppose there exists an equilibrium with an interior solution. Let x^* denote the number of sellers who are offered an inflated certificate in equilibrium. Then, x^* is given by

$$\Pi'(x = x^*|x^e = x^*) = 0. \tag{3.3}$$

x^* is hence given by

$$K'(x^*) = WTP(x^*). \tag{3.4}$$

That is, in an equilibrium with an interior solution, two conditions must hold. First, the marginal costs of offering an inflated certificate are equal to the sellers' willingness-to-pay for a certificate ($K'(x) = WTP(x^e)$). Secondly, the buyers believe correctly that the certifier offers a certain number of inflated certificates

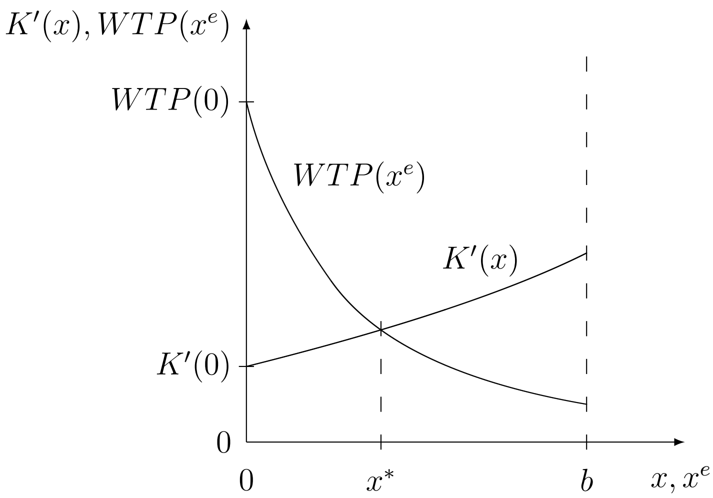

Figure 3.2: An equilibrium with an interior solution.

$(x^e = x)$. Figure 3.2 illustrates this. x^* is given by the intersection of $K'(x)$ and $WTP(x^e)$.

In figure 3.2, we can also see:

Proposition 3.1 *If* $K'(x = 0) < WTP(x^e = 0)$*, then, in equilibrium, the certifier offers at least one seller an inflated certificate* $(x^* > 0)$*.*

That is, if the costs of offering the first inflated certificate are lower than the sellers' willingness-to-pay for this certificate (given the buyers believe the certifier does not offer inflated certificates), then, in equilibrium, the certifier offers at least one seller an inflated certificate.

The certifier does not, however, profit from offering inflated certificates:

Proposition 3.2 *Suppose* $x^* > 0$*. Then, the certifier would make a higher profit than in equilibrium if the certifier did not offer inflated certificates* $(x = 0)$*, and the buyers believed it* $(x^e = 0)$*.*

The intuition for this result is straightforward. In equilibrium, the buyers cannot be fooled. They believe correctly that the certifier offers a certain number of inflated certificates, and take this into account in their willingness-to-pay for a good. The sellers, in turn, take this into account in their willingness-to-pay for a certificate. By offering inflated certificates, the certifier hence only incurs costs, but cannot increase its revenue as compared to the situation in which the certifier does not offer inflated certificates, and the buyers believe it.

If $x^* > 0$, the certifier hence faces a commitment problem. The certifier would make a higher profit if the certifier did not offer inflated certificates, and the buyers believed it. But if the buyers actually believed the certifier did not offer inflated certificates, the certifier would have an incentive to do so.

In addition, there may be a welfare loss. The certifier may, for instance, have to spend time and money to obscure that it offers inflated certificates. Moreover, if there is a fine for offering inflated certificates, the fine may not be a pure transfer. If there are costs of imposing the fine, a part of the fine is 'lost'. Besides, there may also be a welfare loss which is not captured by our simple model. For example, it is often argued that credit rating agencies contributed to the subprime crisis by assigning inflated ratings to mortgage-backed securities and collateralized debt obligations.

3.3.2 The Effect of an Increase in Costs

The number x^* of inflated certificates in equilibrium and the profit $\Pi(x = x^*|x^e = x^*)$ the certifier makes in equilibrium depend on the costs $K(x)$ of offering inflated certificates. Let

$$K(x) \equiv ck(x), \tag{3.5}$$

where $k(0) = 0$, $k'(x) > 0$ and $k''(x) \geq 0$. Suppose an equilibrium with an interior solution exists, and consider an increase in c.

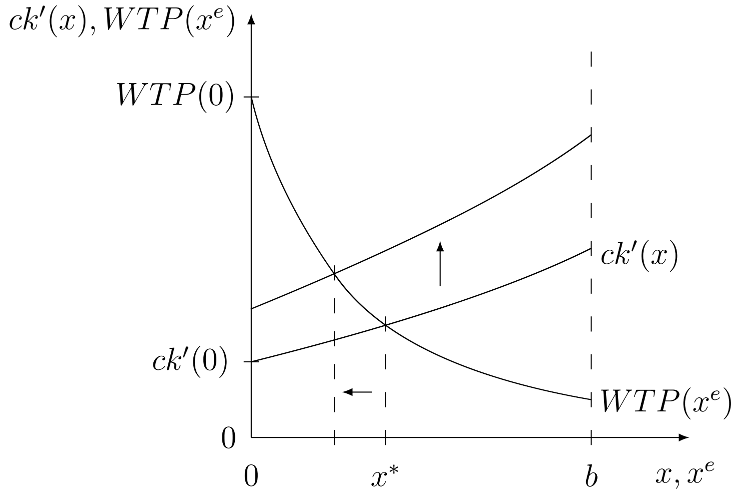

Figure 3.3: The effect of an increase in costs.

First, consider the effect of an increase in c on x^*. If c increases, the certifier obviously offers less inflated certificates in equilibrium.

Proposition 3.3 *Suppose* $0 < x^* < b$. *Then,* x^* *decreases if* c *increases.*

Figure 3.3 illustrates this. An increase in c shifts $ck'(x)$ up. As a result, the certifier offers less inflated certificates in equilibrium.

Now consider the effect of an increase in c on $\Pi(x = x^*|x^e = x^*)$. In equilibrium, the buyers cannot be fooled ($x^e = x^*$). The profit the certifier makes in equilibrium is hence given by

$$\Pi(x = x^*|x^e = x^*) = \frac{a(u_A - u_B)}{a + x^*}(a + x^*) - ck(x^*) \tag{3.6}$$

which is equivalent to

$$\Pi(x = x^*|x^e = x^*) = a(u_A - u_B) - ck(x^*). \tag{3.7}$$

The profit the certifier makes in equilibrium thus depends on the costs $ck(x^*)$ the certifier incurs in equilibrium.

An increase in c has two opposing effects on the costs $ck(x^*)$ the certifier incurs in equilibrium. On the one hand, an increase in c directly increases the costs the certifier incurs. On the other hand, an increase in c reduces the number of inflated certificates, and thus indirectly reduces the costs the certifier incurs. The effect of c on the costs $ck(x^*)$ is given by

$$\frac{\mathrm{d}[ck(x^*(c))]}{\mathrm{d}c} = k(x^*) + c\frac{\mathrm{d}k(x^*)}{\mathrm{d}x}\frac{\mathrm{d}x^*}{\mathrm{d}c}. \tag{3.8}$$

The effect of an increase in c on the profit the certifier makes in equilibrium hence depends on the properties of the cost function. We get:

Proposition 3.4 *Suppose* $0 < x^* < b$. *Then, the profit the certifier makes in equilibrium increases with* c *if*

$$\frac{k''(x^*)}{k'(x^*)^2} < \frac{1}{k(x^*)} - \frac{c}{a(u_A - u_B)}, \tag{3.9}$$

and decreases with c *if the inequality is reversed.*

The intuition for this result is as follows. If $k(x^*)$ is small, the direct effect of an increase in c on the costs $ck(x^*)$ is small. If $k'(x^*)$ is large, the effect of a decrease in x^* on the costs $ck(x^*)$ is large. And if $k''(x^*)$ is small, x^* decreases a lot if c increases. Hence, if $k(x^*)$ is small, $k'(x^*)$ is large, and $k''(x^*)$ is small, the profit the certifier makes in equilibrium increases with c. And vice versa.

Cost functions are often assumed to be linear, quadratic, or exponential. If the certifier faces a linear cost function, the marginal costs of offering inflated certificates are constant. If the certifier faces a quadratic or exponential cost function, the marginal costs of offering inflated certificates increase. We get:

Proposition 3.5 *Suppose* $0 < x^* < b$. *Then, if the cost function is linear or quadratic, the profit the certifier makes in equilibrium increases with* c. *If the cost function is exponential, the profit the certifier makes in equilibrium decreases with* c.

The costs $ck(x)$ may, for example, be interpreted as the expected fine the certifier has to pay. c may be interpreted as the fine the certifier has to pay if caught, and $k(x)$ may be interpreted as the probability of being caught (which increases with the number of inflated certificates). In this case, to use a more adequate notation, c could be substituted by f and $k(x)$ by $p(x)$. Then, the certifier would face an expected fine of $p(x)f$ for offering x inflated certificates. (Alternatively, c may be interpreted as the fine the certifier has to pay for each inflated certificate which is detected, and $k(x)$ may be interpreted as the expected number of inflated certificates which are detected. In this case, to use a more adequate notation, c could be substituted by f, and $k(x)$ by $n(x)$. Then, the certifier would expect to pay a total fine of $n(x)f$ for offering x inflated certificates.)

Suppose now that the certifier has to pay a fine f if caught, and that the certifier is caught with probability $p(x)$. (That is, the certifier faces an expected fine of $p(x)f$ for offering x inflated certificates.) Then, an increase in the fine has two effects. On the one hand, an increase in the fine increases the punishment if caught. On the other hand, an increase in the fine decreases the number of inflated certificates, and thus decreases the probability of being caught. Proposition 3.5 suggests that the certifier would welcome an increase in the fine if the probability of being caught increases linearly or quadratically with the number of inflated certificates. The certifier would, however, oppose an increase in the fine if the probability of being caught increases exponentially with the number of inflated certificates.

3.4 Conclusion

Certifiers often claim that they have no incentive to offer inflated certificates. The message of this paper, however, is that certifiers face a commitment problem if the costs of offering the first inflated certificate are lower than the sellers' willingness-to-pay for it: A certifier does not profit from offering inflated certificates, because in equilibrium, the buyers cannot be fooled. The certifier would make a higher profit if the certifier did not offer inflated certificates, and the buyers believed it. But if the buyers actually believed the certifier did not offer inflated certificates, the certifier would have an incentive to do so.

It seems that the costs of offering the first inflated certificate are indeed often lower than the sellers' willingness-to-pay for it. Typically, an inflated certificate is detected with some probability, and if detected, the certifier is usually punished. Yet, it seems that if a certifier is only a bit too lax, the probability of detection is low, and if the certifier is caught nonetheless, the punishment is not too harsh.

In the model, we find that the number of inflated certificates depends on the costs a certifier incurs if it offers inflated certificates. If the costs are low, a certifier offers a lot of inflated certificates in equilibrium.

It could be argued that credit rating agencies had a strong incentive to assign inflated ratings to mortgage-backed securities and collateralized-debt obligations, because they got off lightly for assigning inflated ratings. Granted, they now face (somewhat) tighter regulation and lost some business, but none of the major credit rating agencies went out of business, and it is unclear whether they lost more business than they attracted by offering inflated ratings in the first place.

In the model, we show that a certifier may yet oppose an increase in the costs of offering inflated certificates. On the one hand, an increase in the costs reduces the number of inflated certificates, and thus indirectly increases the certifier's profit. On the other hand, however, an increase in the costs directly reduces the certifier's profit. Hence, only if the increase in costs reduces the number of inflated certificates by a large amount, the certifier's profit increases with the costs of offering inflated certificates. If, however, the increase in costs does not reduce the number of inflated certificates significantly, the certifier's profit decreases with the costs of offering inflated certificates.

This suggests that whether a certifier welcomes tighter regulation or lobbies against it may depend on whether the new regulation only imposes higher costs on the certifier, or also helps to reduce the certifier's commitment problem significantly.

3.5 Appendix

Proof of Proposition 3.1

Suppose $K'(x = 0) < WTP(x^e = 0)$. Then, there does not exist an equilibrium in which the certifier does not offer inflated certificates: If the buyers believe the certifier does not offer inflated certificates ($x^e = 0$), the certifier offers at least one seller an inflated certificate ($x > 0$), because

$$\Pi'(x = 0|x^e = 0) > 0 \tag{3.10}$$

if

$$K'(x = 0) < WTP(x^e = 0). \tag{3.11}$$

There exists either an equilibrium with an interior solution (as characterized in section 3.3.1), or there exists an equilibrium with a corner solution, where $x^* = b$.

Hence, if $K'(x = 0) < WTP(x^e = 0)$, the certifier offers at least one seller an inflated certificate in equilibrium ($x^* > 0$). ∎

Proof of Proposition 3.2

$$\Pi(x = 0|x^e = 0) > \Pi(x = x^*|x^e = x^*), \tag{3.12}$$

if

$$\frac{a(u_A - u_B)}{a + 0}(a + 0) - K(0) > \frac{a(u_A - u_B)}{a + x^*}(a + x^*) - K(x^*) \tag{3.13}$$

which is equivalent to

$$K(x^*) > 0 \tag{3.14}$$

or

$$x^* > 0. \tag{3.15}$$

Hence, if $x^* > 0$, the certifier would make a higher profit than in equilibrium if the certifier did not offer inflated certificates ($x = 0$), and the buyers believed it ($x^e = 0$). ∎

Proof of Proposition 3.3

Suppose $0 < x^* < b$. Then, x^* is given by

$$ck'(x^*) = \frac{a(u_A - u_B)}{a + x^*}. \tag{3.16}$$

Applying the implicit function theorem yields

$$\frac{\mathrm{d}x^*}{\mathrm{d}c} = -\frac{k'(x^*)}{ck''(x^*) + \frac{a(u_A - u_B)}{(a+x^*)^2}}. \tag{3.17}$$

Because $k'(x) > 0$ and $k''(x) \geq 0$,

$$\frac{\mathrm{d}x^*}{\mathrm{d}c} < 0. \tag{3.18}$$

Hence, if $0 < x^* < b$, x^* decreases if c increases. ∎

Proof of Proposition 3.4

The certifier's profit is given by

$$\Pi = \frac{a(u_A - u_B)}{a + x^e}(a + x) - ck(x). \tag{3.19}$$

In equilibrium, the buyers cannot be fooled ($x^e = x^*$). Hence, in equilibrium, the certifier's revenue is given by

$$\frac{a(u_A - u_B)}{a + x^*}(a + x^*) = a(u_A - u_B). \tag{3.20}$$

The effect of c on $\Pi(x = x^*|x^e = x^*)$ thus depends on the effect of c on $ck(x^*(c))$, which is given by

$$\frac{\mathrm{d}[ck(x^*(c))]}{\mathrm{d}c} = k(x^*) + c\frac{\mathrm{d}k(x^*)}{\mathrm{d}x}\frac{\mathrm{d}x^*}{\mathrm{d}c}. \tag{3.21}$$

Suppose $0 < x^* < b$. Then, using (3.16) and (3.17) yields

$$\frac{\mathrm{d}[ck(x^*(c))]}{\mathrm{d}c} = k(x^*) - \frac{1}{\frac{k''(x^*)}{k'(x^*)^2} + \frac{c}{a(u_A - u_B)}}. \tag{3.22}$$

The effect of c on $\Pi(x = x^*|x^e = x^*)$ is thus given by

$$\frac{\mathrm{d}\Pi(x = x^*|x^e = x^*)}{\mathrm{d}c} = -k(x^*) + \frac{1}{\frac{k''(x^*)}{k'(x^*)^2} + \frac{c}{a(u_A - u_B)}}. \tag{3.23}$$

Hence,

$$\frac{\mathrm{d}\Pi(x = x^*|x^e = x^*)}{\mathrm{d}c} > 0 \tag{3.24}$$

if

$$\frac{k''(x^*)}{k'(x^*)^2} < \frac{1}{k(x^*)} - \frac{c}{a(u_A - u_B)}. \tag{3.25}$$

Note that

$$\frac{1}{k(x^*)} - \frac{c}{a(u_A - u_B)} > 0 \tag{3.26}$$

if

$$a(u_A - u_B) - ck(x^*) > 0 \tag{3.27}$$

or

$$\Pi(x = x^*|x^e = x^*) > 0. \tag{3.28}$$

Finally,

$$\frac{\mathrm{d}\Pi(x = x^*|x^e = x^*)}{\mathrm{d}c} < 0 \tag{3.29}$$

if

$$\frac{k''(x^*)}{k'(x^*)^2} > \frac{1}{k(x^*)} - \frac{c}{a(u_A - u_B)}. \tag{3.30}$$

Hence, if $0 < x^* < b$, the profit the certifier makes in equilibrium increases with c if

$$\frac{k''(x^*)}{k'(x^*)^2} < \frac{1}{k(x^*)} - \frac{c}{a(u_A - u_B)}, \tag{3.31}$$

and decreases with c if the inequality is reversed. ∎

Proof of Proposition 3.5

Suppose $0 < x^* < b$. First consider a linear cost function, then consider a quadratic and an exponential cost function.

First, suppose

$$K(x) \equiv cx. \tag{3.32}$$

Then, x^* is given by

$$c = \frac{a(u_A - u_B)}{a + x^*}, \tag{3.33}$$

or

$$x^* = \frac{a(u_A - u_B - c)}{c}. \tag{3.34}$$

$\Pi(x = x^*|x^e = x^*)$ is thus given by

$$\Pi = a(u_A - u_B) - c\frac{a(u_A - u_B - c)}{c} \tag{3.35}$$

or

$$\Pi = ac. \tag{3.36}$$

If $K(x) = cx$, $\Pi(x = x^*|x^e = x^*)$ hence increases with c.

Now, suppose

$$K(x) \equiv cx^2. \tag{3.37}$$

Then, x^* is given by

$$2cx = \frac{a(u_A - u_B)}{a + x^*}, \tag{3.38}$$

or

$$x^* = \sqrt{\left(\frac{a}{2}\right)^2 + \frac{a(u_A - u_B)}{2c}} - \frac{a}{2}. \tag{3.39}$$

$\Pi(x = x^*|x^e = x^*)$ is thus given by

$$\Pi = a(u_A - u_B) - c\left(\sqrt{\left(\frac{a}{2}\right)^2 + \frac{a(u_A - u_B)}{2c}} - \frac{a}{2}\right)^2 \tag{3.40}$$

or

$$\Pi = \frac{a(u_A - u_B)}{2} - \frac{a^2 c}{2} + \sqrt{\left(\frac{a^2 c}{2}\right)^2 + \frac{a^3 c(u_A - u_B)}{2}}. \tag{3.41}$$

$\frac{\mathrm{d}[\Pi(x=x^*|x^e=x^*)]}{\mathrm{d}c}$ is hence given by

$$\frac{\mathrm{d}\left[\Pi(x = x^*|x^e = x^*)\right]}{\mathrm{d}c} = -\frac{a^2}{2} + \frac{1}{2}\frac{\left(\frac{a^2}{2}\right)^2 2c + \frac{a^3(u_A - u_B)}{2}}{\sqrt{\left(\frac{a^2 c}{2}\right)^2 + \frac{a^3 c(u_A - u_B)}{2}}}. \tag{3.42}$$

Hence,

$$\frac{\mathrm{d}\left[\Pi(x = x^*|x^e = x^*)\right]}{\mathrm{d}c} > 0 \tag{3.43}$$

if

$$u_A > u_B, \tag{3.44}$$

which holds by assumption. If $K(x) = cx^2$, $\Pi(x = x^*|x^e = x^*)$ hence increases with c.

Finally, suppose

$$K(x) \equiv ce^x. \tag{3.45}$$

Then, using (3.23) yields

$$\frac{\mathrm{d}\Pi(x = x^*|x^e = x^*)}{\mathrm{d}c} = -e^{x^*} + \frac{1}{\frac{e^{x^*}}{(e^{x^*})^2} + \frac{c}{a(u_A - u_B)}}. \tag{3.46}$$

Hence,

$$\frac{\mathrm{d}\Pi(x = x^*|x^e = x^*)}{\mathrm{d}c} < 0 \tag{3.47}$$

if

$$\frac{1}{e^{x^*}} < \frac{e^{x^*}}{(e^{x^*})^2} + \frac{c}{a(u_A - u_B)} \tag{3.48}$$

or

$$\frac{c}{a(u_A - u_B)} > 0, \tag{3.49}$$

which holds by assumption. If $K(x) = ce^x$, $\Pi(x = x^*|x^e = x^*)$ hence decreases with c.

Hence, if $0 < x^* < b$ and the cost function is linear or quadratic, the profit the certifier makes in equilibrium increases with c. If the cost function is exponential, the profit the certifier makes in equilibrium decreases with c. ∎

3.6 References

Barro, Robert, and David Gordon, 1983. A Positive Theory of Monetary Policy in a Natural Rate Model. Journal of Political Economy 91, 589-610.

Bolton, Patrick, Xavier Freixas and Joel Shapiro, 2010. The Credit Ratings Game. Economics Working Papers 1221, Universitat Pompeu Fabra.

Holmström, Bengt, 1982. Moral Hazard in Teams. Bell Journal of Economics 13, 324-340.

Mathis, Jérôme, James McAndrews and Jean-Charles Rochet, 2009. Rating the Raters: Are Reputation Concerns Powerful Enough to Discipline Rating Agencies? Journal of Monetary Economics 56, 657-674.

Peyrache, Eloïc, and Lucía Quesada, 2010. Intermediaries, Credibility and Incentives to Collude. Working paper, HEC Paris and Universidad Torcuato Di Tella.

Stolper, Anno, 2009. Regulation of Credit Rating Agencies. Journal of Banking and Finance 33, 1266-1273.

Stein, Jeremy, 1989. Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior. Quarterly Journal of Economics 104, 655-669.

Strausz, Roland, 2005. Honest Certification and the Threat of Capture. International Journal of Industrial Organization 23, 45-62.

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