The monetary analysis of hyperinflation and the appropriate specification of the demand for money

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Abstract: The paper emerges from the failure of the traditional models of hyperinflation with rational expectations or perfect foresight. Using the insights from two standard optimizing monetary settings the paper shows that the possibility of perfect foresight monetary hyperinflation paths depends robustly on the essentiality of money. We show that the popular semilogarithmic form of the demand for money is not appropriate to analyse monetary hyperinflation with perfect foresight. We propose a simple test of money essentiality for the appropriate specification of the demand for money equation in empirical studies of hyperinflation.

Keywords: monetary hyperinflation, inflation tax, money essentiality

JEL Code: E31, E41

1. Introduction

The paper emerges from the well known failure of the traditional monetary models of hyperinflation. Since the 'surprising monetarist arithmetic' analysed in Buiter (1987) it is known that under rational expectations or perfect foresight this class of models is fundamentally flawed because it is not able to generate the process it is designed to characterize - monetary hyperinflation[1] that is a speeding up inflation unstable dynamic process driven by monetary growth with real cash balances tending to vanish. This traditional class of models, such as Evans and Yarrow (1981) or Bruno and Fischer (1990), relies on the famous Cagan (1956) money demand and considers the monetization of a large fiscal deficit as the driving force of hyperinflation.

These models are so influential in the literature that small variations of them can be found in the major books on macroeconomics or monetary economics, such as Walsh (2003) for instance. Moreover most of the large empirical literature on hyperinflation (Petrovic and Mladenovic (2000), Slavova (2003) or Georgoutsos and Kouretas (2004) among others) relies on these traditional models with the Cagan money demand and rational expectations. The failure of this class of influential models with rational expectations or perfect foresight may

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[1] This paper is not about speculative hyperinflations which are the focus of other works such as Obstfeld and Rogoff (1983), Barbosa and Cunha (2003), or Arce (2009) for instance. Speculative hyperinflations, as defined by Obstfeld and Rogoff (1983), are explosive price-level paths unrelated to monetary growth.

cast doubt on part of the vast hyperinflation empirical studies. Although hyperinflationary episodes are rare they regularly generate a significant amount of empirical studies. The recent Zimbabwean experience, as the second most extreme hyperinflation in monetary history after Hungary in 1946 (see, for example, Hanke and Kwok (2009) and Pilossof (2009)), will surely stimulate interesting new empirical studies. The aim of this paper is twofold. First, the paper aims at understanding the failure of Cagan based inflationary finance models with perfect foresight. Second, it aims at providing empirical studies of hyperinflation with a test for the appropriate specification of the demand for money equation.

Traditional monetary models of hyperinflation based on Cagan (1956) imply the possibility of dual equilibria and the existence of an inflation tax Laffer curve. All models which generate the high inflation trap defined by Bruno and Fischer (1990) fail to produce monetary hyperinflation[2]. Evans and Yarrow (1981) and Bernholz and Gersbach (1992) already pointed out that the crucial condition for generating hyperinflation is that real money balances should not decrease more than inflation increases with high rates of inflation. This problem has given rise to a significant amount of new literature and some new specifications of this class of models have emerged. These new specifications can be mainly separated in two different approaches depending on the kind of feature included in the basic inflationary finance model to guarantee the former crucial condition.

The first approach includes in the models a mechanism of sluggish adjustment of some nominal variable like expected inflation, money holdings or the exchange rate. Sufficiently slow adaptive expectations, as in Evans and Yarrow (1981) or Bruno and Fischer (1990), learning as in Marcet and Nicolini (2003) or Adam et al (2006), a crawling peg rule for the exchange rate as in Bruno (1989), or a sufficiently slow adaptive adjustment on the money market as in Kiguel (1989) can restore the correct running of this class of models. However, even if one can find arguments in favour of the use of adaptive expectations during hyperinflationary episodes, as Bruno and Fischer (1990) do for instance, it is hard to justify the persistent presence of behaviours involving either systematic forecast mistakes or maladjustments resulting in prohibitive costs for the agents in a hyperinflationary context. The second approach maintains perfect foresight assuming that agents respond most likely instantaneously to changes in inflation during hyperinflation but abandons the Cagan money demand function. Ashworth and Evans (1998) look for empirical support for other functional forms than the Cagan money demand. Vazquez (1998), Gutierrez and Vazquez (2004) or Barbosa et al (2006) develop inflationary finance monetary optimizing models to obtain a demand for real cash balances compatible with explosive hyperinflation and perfect foresight. This paper follows this second approach.

We consider two standard continuous time and non-stochastic optimizing monetary settings representing alternative ways of modelling the transaction role of money: a money-in-the-utility-function model (henceforth called MIUF model) and a cash-in-advance model (henceforth called CIA model). We build on Gutierrez and Vazquez (2004) but consider the two monetary optimizing setups with general household's preferences which is something new. The aim is to examine the possibility of monetary hyperinflation with perfect foresight using the insights from these two monetary optimizing settings. Using the insights from a monetary optimizing model to address one specific issue concerning hyperinflation dynamics may be a useful approach as shown by Arce (2009)[3] for instance. We work with two different

[2] See Evans (1995) or Vazquez (1998) for a survey of this literature.
[3] Arce (2009) focuses on the explanation of the hysteresis in the stock of real cash balances after the end of hyperinflations using a cash-and-credit model.

monetary settings to look for robustness of the results. We show that in both settings monetary hyperinflation can arise consistently with perfect foresight under a similar condition stating that the households should consider the money sufficiently essential to the system. In this respect the paper contributes to the understanding of the well known Cagan inflationary finance models failure with perfect foresight and provides a test for the specifications of the demand for money equation designed for empirical studies of monetary hyperinflation.

The paper is organized in the following way. Section 2 considers a version of a MIUF economy with a general specification of the representative agent's preferences. It provides a general characterization of agents' preferences compatible with perfect foresight monetary hyperinflation equilibria relying on the essentiality of the money. Section 3 studies a version of a CIA model with a general specification of the representative agent's preferences and shows again the dependence of perfect foresight monetary hyperinflation paths on a sufficient level of money essentiality. Section 4 relates money essentiality to money demand inelasticity and provides specific theoretical support to the double-log functional form of the money demand during hyperinflation. Section 5 concludes.

2. MIUF economy, hyperinflation and money essentiality

We extend the basic setup of Gutierrez and Vazquez (2004) first, by considering general utility functions and, second, by taking into account the goods market equilibrium condition. The optimizing monetary model is a continuous time model (assumption [A1]) where the economy consists of a large number of identical infinitely-lived forward looking households endowed with perfect foresight (assumption [A2]). Population is constant and its size is normalized to unity for convenience (assumption [A3]). There is no uncertainty (assumption [A4]). Each household has a non-produced endowment $y_t > 0$ of the non-storable consumption good per unit of time (assumption [A5]).

In the MIUF model the role of money as a medium of exchange is assumed to be captured by introducing real money balances into the household utility function. Our MIUF framework considers households preferences represented by a general class of utility functions (assumption [A6]). Therefore, the representative household utility at time 0 is

$$\int_0^\infty U\left(c_t, m_t\right) e^{-rt} dt \,. \qquad (1)$$

The instantaneous utility function has standard properties (assumption [A7]): it is continuous, twice differentiable on $\mathbb{R}_+^2$, increasing and strictly concave in c_t, the household's consumption at time *t*, and $m_t = \dfrac{M_t}{P_t}$ his holdings of real monetary balances (*M* is the nominal stock of money, *P* is the price level). The rate r is the subjective discount rate which is assumed to be equal to the real rate of interest (assumption [A8]). Households can hold money and bonds paying a nominal interest i_t (assumption [A9]). Real per capita financial wealth and the nominal interest rate are defined as

$$w_t = m_t + b_t \,,$$

$i_t = r + \pi_t$,

respectively, where b_t denotes real per capita government debt, π_t is the inflation rate. The household allocates its resources between consumption, gross accumulation of real money balances and bonds. The household's budget constraint is

$$\dot{w} = y_t - \tau_t + rw_t - \left(c_t + i_t m_t\right), \qquad (2)$$

where τ_t is a lump-sum tax assumed to be constant. The household's optimization problem leads to the following first-order condition:

$$r + \pi_t = \frac{U'_m\left(c_t, m_t\right)}{U'_c\left(c_t, m_t\right)}, \qquad (3)$$

where $U'_c\left(c_t, m_t\right)$ is constant with respect to time because the instantaneous rate of time preference is equal to the real rate of interest. Condition (3) requires that at each moment the nominal rate of interest be equal to the marginal rate of substitution of consumption for money. It implicitly defines a demand for money as a function of the nominal interest rate *i*. The optimum solution is completed by the transversality condition:

$$\lim_{t \to \infty}\left[e^{-rt} U'_c\left(c_t, m_t\right) w_t\right] = 0. \qquad (4)$$

The setup is completed by considering the equilibrium condition in the goods market. Following Barbosa et al (2006) or Vazquez (1998, p. 438) "in the spirit of the traditional approach to the study of hyperinflationary phenomena, we assume that output and government expenditures are constant". Therefore, the market for goods is in equilibrium when constant supply of good *y* equals household consumption and constant per capita government expenditures (*g*):

$$y = c_t + g. \qquad (5)$$

In usual inflationary finance models a constant per capita government's budget deficit, *d*, is financed by issuing high-powered money (assumption [A10]):

$$d = \frac{\dot{M}_t}{P_t} = \dot{m}_t + \pi_t m_t. \qquad (6)$$

Substituting the value of π extracted from first-order equation (3) in the latter expression leads to the inflationary finance model dynamics described by the following law of motion for real cash balances:

$$\dot{m}_t = d - \left(\frac{U'_m\left(c_t, m_t\right)}{U'_c\left(c_t, m_t\right)} - r\right) m_t. \qquad (7)$$

Differential equation (7) provides a complete characterization of real per-capita money balances dynamics which will be studied by using the technique of phase diagram on $[0;+\infty[$. The main interesting point here is to examine whether this law of motion for real cash balances is able to produce hyperinflation paths. An explosive hyperinflation path will be observed if the law of motion presents a path leading to a zero level of real cash balances. Therefore, the conditions for this kind of paths should be identified. As the mathematical function representing the law of motion is continuous (which is true with standard assumptions on *U*) this kind of paths will be observed as long as (dropping index time for convenience):

$$\lim_{m \to 0_+} \dot{m} < 0 . \qquad (8)$$

The calculation of $\lim_{m \to +\infty} \dot{m}$ will assess the existence of any steady state. Nevertheless, whatever the number of steady states, since we focus on possible explosive hyperinflation paths we are only interested in the paths starting at the left of the first possible steady state when the condition $\lim_{m \to 0_+} \dot{m} < 0$ is met.

According to Obstfeld and Rogoff (1983) in the context of speculative hyperinflations issue, any path leading to a zero value of real cash balances and crossing eventually the vertical axis at some finite point should be ruled out on grounds that such paths would not be feasible because the real stock of money would eventually become negative. However, we would rather follow the point made by Barbosa and Cunha (2003, p. 192) who contested the Obstfeld and Rogoff (1983) approach by arguing that on such hyperinflationary paths “when the real quantity of money reaches zero hyperinflation would have wiped out the value of money, the opportunity cost of holding money would have become infinite”, and “the economy would no longer be a monetary economy”. Therefore, we follow the point made by Barbosa and Cunha (2003) and consider the explosive hyperinflation paths corresponding to the condition $\lim_{m \to 0_+} \dot{m} < 0$ as perfect foresight competitive equilibrium paths.

Moreover, it’s important to stress that the possible explosive hyperinflationary paths are explosive monetary hyperinflations because along these paths the rate of growth of the money supply explodes. Rewriting government budget constraint as

$$\frac{\dot{M}}{M} = \frac{d}{m},$$

we see that along the paths of continuously declining *m*, given that $d > 0$, the growth rate of money supply increases continuously.

In this respect, according to the law of motion (7), the possibility of explosive hyperinflation will depend on the condition

$$\lim_{m \to 0_+} \left[\frac{U'_m(c,m)}{U'_c(c,m)} m \right] > d . \qquad (9)$$

The latter condition is basically a condition about a sufficient level of the essentiality of money along a hyperinflationary path. Scheinkman (1980) considered money as essential in a hyperinflation context if the inflation tax collected by the government does not tend to zero when the rate of inflation explodes. The interpretation of such a condition is that "no matter how expensive it becomes to hold money people still hold a large quantity of it; that is money is very necessary to the system" (Scheinkman, 1980, p. 96)[4]. From (6) we see that seigniorage obtained by printing money can be decomposed into two components, the change in the real stock of money and the inflation tax πm which can be written, according to equation (3):

$$\pi m = \left(\frac{U'_m(c,m)}{U'_c(c,m)} - r \right) m = \frac{U'_m(c,m)}{U'_c(c,m)} m - rm \, .$$

Then, when the rate of inflation explodes we consider

$$\lim_{m \to 0_+} [\pi m] = \lim_{m \to 0_+} \left[\frac{U'_m(c,m)}{U'_c(c,m)} m \right].$$

Therefore, when $\lim_{m \to 0_+} \left[\frac{U'_m(c,m)}{U'_c(c,m)} m \right] > 0$ then $\lim_{m \to 0_+} [\pi m] > 0$ and money is essential. These findings enable to formulate a first proposition.

Proposition 1: In a MIUF economy, characterized by the set of assumptions [A1] to [A10], explosive monetary hyperinflations are possible only if money is sufficiently essential that is if $\lim_{m \to 0_+} \left[\frac{U'_m(c,m)}{U'_c(c,m)} m \right] > d$.

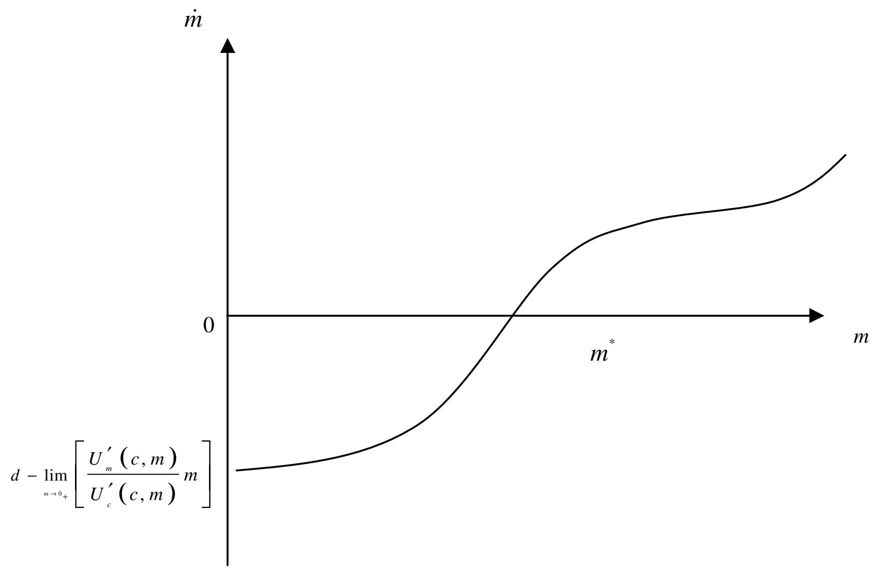
Proof: The proof relies on the previous arguments and can be illustrated by the phase diagram depicted on Figure 1. The precise shape of the phase diagram depends on the first and second derivative of $\dot{m}$ with respect to m. Other shapes than that depicted on Figure 1 could be possible for the phase locus. However, as the important point for the analysis conducted here insists on the condition for $\lim_{m \to 0_+} \dot{m} < 0$, our analysis focuses only on the paths leading to a zero value of real cash balances. If $\lim_{m \to +\infty} \dot{m} > 0$, the locus $\dot{m}$ will cross the horizontal axis at least once. We consider here a unique unstable steady state m^* but the qualitative analysis for explosive hyperinflationary paths doesn't change in the case of more steady states. All paths originating at the right of m^* are hyperdeflationary paths that can be ruled out because violating the transversality condition (4). All paths starting to the left of m^* are explosive hyperinflations paths.■

Using a similar MIUF framework with a particular constant-relative-risk-aversion utility function Gutierrez and Vazquez (2004) point out that explosive hyperinflationary dynamics are more likely when the transaction role of money becomes important. Our results confirm and extend to more generality the point made by Gutierrez and Vazquez (2004) by relating the

[4] This definition of the essentiality of money is also used in Sturzenegger (1994) or Barbosa and Cunha (2003) for example in the context of speculative hyperinflationary paths.

possibility of monetary explosive hyperinflations to a sufficient level of money essentiality in the model.

Figure 1.
Monetary dynamics in a MIUF economy with money sufficiently essential



Explosive hyperinflation paths starting at the left of m^* are equilibrium paths since they are consistent with equilibrium condition on the goods market (5). Along these paths of declining real cash balances real per capita consumption will remain constant at $c_t = y - g$ but households will suffer from an increasing loss of welfare representing the harmful effect of hyperinflation on the economy.

Considering the particular case where the utility function is additively separable in consumption and real cash balances:

$$U\left(c_t, m_t\right) = u(c_t) + v(m_t),$$

where the functions u and v are continuous, twice differentiable on $\mathbb{R}_+^2$, increasing in their respective argument, and strictly concave [assumption A7'], the condition (9) of Proposition 1 resumes to

$$\lim_{m\to 0_+}\left[mv'(m)\right] > du'(c). \qquad (10)$$

In the latter condition the value of $u'(c)$ is constant with respect to time and can be replaced by $u'(y-g)$ using the goods market equilibrium condition (5). Scheinkman (1980) related the condition $\lim_{m\to 0_+} mv'(m) > 0$ to the essentiality of money. The condition (10), as a particular case of Proposition 1, states that the possibility of explosive hyperinflation depends on a sufficient level of money essentiality which is conveyed by the utility function for money services.

According to Proposition 1, the failure of the Cagan inflationary finance model to produce explosive hyperinflations with perfect foresight is not surprising. The Cagan ad-hoc model relying on the Cagan money demand can be considered as a special case of the MIUF model developed here. Since Kingston (1982), it is known that the semi-log schedule is 'integrable'. In the terms of the latter it means that the schedule 'can be generated by at least one optimizing framework'. The 'integrability' of Cagan money demand was shown again later by Calvo and Leiderman (1992).

Proposition 2: In a MIUF economy, characterized by the set of assumptions [A1] to [A6], [A7'], and [A8] to [A10], the 'integrable' Cagan money demand with perfect foresight does not comply with money essentiality.

Proof: The 'integrability' of Cagan money demand is shown by using a utility function for money services *v(m)* such as :

$$v(m) = u'(y-g)\alpha^{-1}(1+\gamma+\alpha r - \log m)m \quad \text{for all } 0 < m < e^{\gamma+\alpha r}.$$

The latter utility function for money services will deliver through the first-order equation (3) the famous semi-logarithmic Cagan money demand ($\log m = \gamma - \alpha\pi$, with γ a constant and α a positive constant). The current MIUF model will resume in the inflationary finance Cagan model. However, such a utility function for money services doesn't comply with money essentiality requirement since for the latter utility function $\lim_{m \to 0_+} mv'(m) = 0$. Then, it won't allow the modelling of monetary hyperinflation as stated in Proposition 1.■

3. CIA economy, hyperinflation and money essentiality

We adopt the basic setup of section 2 keeping all assumptions from [A1] to [A5] and from [A8] to [A10]. It differs, however, from it in two aspects. First, assumption [A6] is replaced by assumption [A6b] stating that representative household's preferences are represented by a general class of utility function depending only on the level of real consumption. Accordingly, the household utility at time 0 is

$$\int_0^\infty e^{-rt} U(c_t)\,dt\,. \qquad (11)$$

The function *U* belongs to a general class of utility function increasing and strictly concave in its single argument, real good consumption: assumption [A7] is replaced by assumption [A7b]. Second, in a CIA economy the role of money as a medium of exchange is captured by a cash-in-advance constraint assuming that money holding is strictly essential to buy the consumption good (assumption [A7c]). In order to consume *c* units of the consumption good at time *t*, the household must hold a stock of real cash balances, *m*, greater or equal to *c*:

$$m_t \geq c_t\,.$$

In this non-stochastic environment assuming that the nominal interest rate i is greater than zero, meaning that money is return-dominated by government bond, it follows that CIA constraint must hold with equality:

$$m_t = c_t . \qquad (12)$$

The representative household optimization problem consisting of maximizing (11) subject to the constraints given by (2) and (12) leads to the following first order condition:

$$U'(m_t) = \lambda\left(1+i_t\right). \qquad (13)$$

The associated Lagrange multiplier λ is constant with respect to time because the agent's rate of time preference equals the real rate of interest, and real cash balances will indirectly enter the utility function according to (12). Equation (13) characterizes a demand for real money balances decreasing with respect to the rate of inflation (or the cost of holding cash balances) because the utility function U is strictly concave. The transversality condition implies that

$$\lim_{t\to\infty} e^{-rt}\lambda w_t = 0 . \qquad (14)$$

By using the definition of the nominal interest rate, the first order condition (13) can be written as follows:

$$\pi_t = \frac{U'(m_t) - \lambda\left(1+r\right)}{\lambda} . \qquad (15)$$

As in usual inflationary finance models a constant per capita government's budget deficit, d, is financed by issuing high-powered money, the law of motion for real money balances in this CIA inflationary finance model will be given by combining (6) and (15), leading to

$$\dot{m}_t = d - \frac{1}{\lambda}\left(U'(m_t) - \lambda\left(1+r\right)\right)m_t . \qquad (16)$$

On the basis of the methodology and the argumentation developed in section 2, the possibility of explosive hyperinflation paths depends on condition (8) leading to the following condition for the CIA economy (dropping the time index for convenience)

$$\lim_{m\to 0_+}\left[mU'(m)\right] > \lambda d . \qquad (17)$$

In the same way as in section 2 in the framework of a MIUF economy, condition (17) relates the possibility of explosive hyperinflation to a sufficient level of money essentiality. Moreover, this sufficient level of money essentiality is conveyed by the agent's preferences. According to (15), inflation tax is given by

$$\pi m = \left(\frac{U'(m) - \lambda\left(1+r\right)}{\lambda}\right)m .$$

Then, when the rate of inflation explodes we consider

$$\lim_{m \to 0_+} \left[\pi m \right] = \frac{1}{\lambda} \lim_{m \to 0_+} \left[m U'(m) \right].$$

From the mathematical point of view it appears that condition (17) allowing the model to generate possible explosive hyperinflations paths is exactly of the same kind as condition (9) in the MIUF model. Condition (17) is particularly similar to condition (10) in the MIUF case with an additive separable utility function.

Proposition 3: In a CIA economy, characterized by the set of assumptions [A1] to [A5], [A6b], [A7b], [A7c], and [A8] to [A10], explosive hyperinflations are possible only if money is sufficiently essential that is if $\lim_{m \to 0_+} \left[m U'(m) \right] > \lambda d$.

Proof: The proof relies on previous arguments.■

The possibility of monetary hyperinflation paths is again a discussion about a sufficient level of money essentiality. The CIA model presents exactly the same kind of limitations as the MIUF model for characterizing explosive hyperinflation paths with perfect foresight. The CIA constraint doesn't convey by itself sufficient money essentiality even if it makes money necessary for the transactions.

Proposition 4: According to proposition 3, in a CIA economy, characterized by the set of assumptions [A1] to [A5], [A6b], [A7b], [A7c], and [A8] to [A10], the 'integrable' Cagan money demand with perfect foresight does not comply with money essentiality.

Proof: The 'integrability' of the Cagan money demand in the CIA setup is shown by using the following household's utility function:

$$U\left(c\right) = \lambda \left(1 + r + \frac{1+\gamma}{\alpha} - \frac{1}{\alpha} \log c \right) c, \quad \text{for all} \quad c < e^{\gamma + \alpha(1+r)} .$$

The latter household's utility function will deliver through the first-order equation (13) the famous semi-logarithmic Cagan money demand ($\log m = \gamma - \alpha\pi$, with γ a constant and α a positive constant). The current CIA model will resume in the inflationary finance Cagan model. However, such a utility function doesn't comply with the money essentiality requirement since for the latter utility function $\lim_{m \to 0_+} m U'(m) = 0$. Then, it won't allow the modelling of monetary hyperinflation as stated in Proposition 3.■

According to the CIA constraint (12), household real consumption will fall along explosive hyperinflation paths characterized by the declining value of real money balances. The fall of households' real consumption will cause an increasing loss of welfare and represent the harmful effect of hyperinflation on the CIA economy. There is some evidence supporting this result. As pointed out by Vazquez (1998), Webb (1989) in his Table 5.4 shows evidence that consumption fell dramatically during German hyperinflation. The recent collapse of the Zimbabwean economy illustrates the same point.

4. Money essentiality, money demand inelasticity and monetary hyperinflation

Money essentiality is closely related to the inelasticity of the demand for money with respect to the cost of holding cash balances. We define the function $f(m)$ measuring the cost of money services according to

$$f(m) = mi = m\left(r+\pi\right) = \begin{cases} m \cdot \dfrac{U'_m\left(c,m\right)}{U'_c\left(c,m\right)} \text{ in the MIUF economy} \\ m \cdot \left(\dfrac{U'\left(m\right)-\lambda}{\lambda}\right) \text{ in the CIA economy} \end{cases}.$$

Proposition 5: According to propositions 1 and 3, any differentiable money demand function inelastic with respect to the cost of holding cash balances is consistent with money essentiality. Moreover, if it complies with $\lim_{m\to 0_+} f(m) > d$ then it will allow the modelling of monetary hyperinflation under perfect foresight.

Proof:
The first derivative of $f(m)$ is

$$f'(m) = i\left(1+\frac{m}{i}\frac{\partial i}{\partial m}\right) = i\left(1-\frac{1}{|\varepsilon|}\right).$$

where ε represents the elasticity of the money demand with respect to the nominal interest rate. If the money demand is interest-rate inelastic, $|\varepsilon|<1$, then $f'(m)<0$.

Since $f(m)\geq 0$ and $f'(m)<0$ when the money demand is inelastic, it follows that $\lim_{m\to 0_+} f\left(m\right) > 0$. Then, we have

$$\lim_{m\to 0_+} f(m) = \begin{cases} \lim_{m\to 0_+} m \cdot \dfrac{U'_m\left(c,m\right)}{U_c^{'}\left(c,m\right)} > 0 \text{ in the MIUF economy} \\ \dfrac{1}{\lambda}\lim_{m\to 0_+} mU'\left(m\right) > 0 \text{ in the CIA economy} \end{cases},$$

implying that when money demand is interest rate-inelastic then money is essential. Proposition 1 and Proposition 3 complete the proof.■

Barbosa et al (2006), in a similar framework, point out the role of the inelasticity of money demand functions with respect to the nominal interest rate for the possibility of explosive inflation path but insist in the need of an increasing government deficit. Our results stress, rather, the role of money essentiality and are established with a constant government deficit without needing an increasing deficit.

Proposition 5 establishes that inelastic money demand function complying with a sufficient level of money essentiality can be candidates for replacing the famous Cagan money demand function to model successfully monetary hyperinflation under perfect foresight. Among them the double-log schedule with perfect foresight may be distinguished:

$$\log m = \delta - \beta \log \pi, \qquad 0 < \beta < 1,$$

with δ constant. This money demand functional form exhibits a constant elasticity lower than one with respect to the inflation rate.

Proposition 6: According to propositions 1 and 3, the 'integrable' double-log schedule with $0 < \beta < 1$ is an appropriate candidate functional form to replace the Cagan money demand function in the analysis of monetary hyperinflation with perfect foresight.

Proof: As shown by Kingston (1982), the double-log schedule is 'integrable' in a MIUF setup. Using the setup of a MIUF economy with additive-separable utility function for instance, one can easily verify that using a utility function for money services *v(m)* such as

$$v(m) = \left(rm + \frac{\beta e^{\frac{\delta}{\beta}}}{\beta - 1} m^{1-\frac{1}{\beta}} \right) \cdot u'\left(y-g\right),$$

will give microeconomic foundations to the double-log schedule. The money demand function described by the double-log schedule complies with Proposition 1 as shown by the following calculation:

$$\lim_{m \to 0_+} \frac{mv'(m)}{u'\left(y-g\right)} = +\infty > d .$$

The double-log schedule is also 'integrable' in the CIA setup of section 3. Using a utility function such as

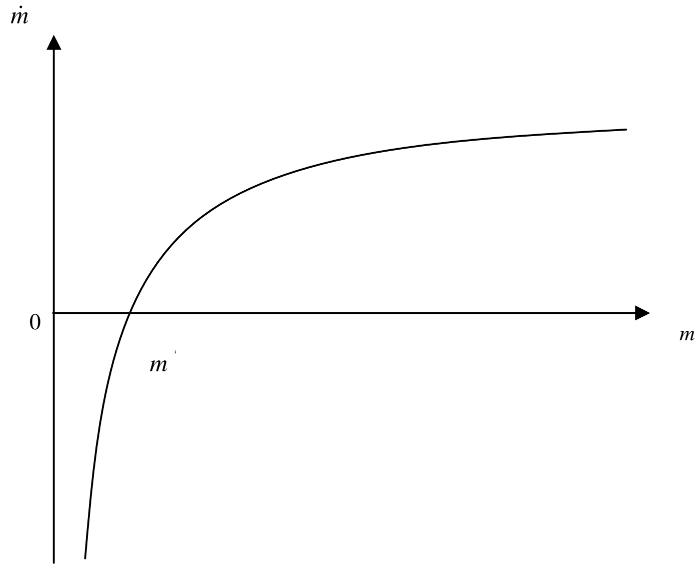
$$U\left(c\right) = \lambda\left(1+r\right)c + \lambda \frac{\beta}{\beta-1} e^{\frac{\delta}{\beta}} c^{1-\frac{1}{\beta}},$$

will also give microeconomic foundations to the double-log schedule complying with Proposition 3 since for the latter we have:

$$\lim_{m \to 0_+} mU'\left(m\right) = +\infty > \lambda d .$$

Figure 2 represents the monetary dynamics derived from the double-log schedule under perfect foresight. All paths starting at the left of the unique unstable steady state m^* are monetary hyperinflations. The paths starting at the right of the unique steady state can be ruled out because violating the transversality condition (4) for the MIUF setup or (14) for the CIA setup.■

Figure 2.
Monetary dynamics with the double-log schedule



Proposition 6 provides theoretical support for the use of the double-log schedule for money demand in the modelling of explosive hyperinflation under perfect foresight.

5. Conclusion

The insights from the two monetary optimizing settings, the MIUF and the CIA setups, considered in this paper have been useful to show that the possibility of perfect-foresight explosive monetary hyperinflation paths requires the households to consider the money as sufficiently essential to the system. It is shown that, whether in the MIUF or in the CIA framework, the appropriate level of money essentiality is conveyed by the representative agent's preferences and does not depend on the specific way, CIA or MIUF, of modelling the role of money as a medium of exchange. Therefore the paper establishes a quite robust theoretical link between the possibility of monetary hyperinflation paths and the essentiality of money[5]. The importance of the issue of the essentiality of money has been already raised in the literature of speculative hyperinflations as in Barbosa and Cunha (2003). It is confirmed in this paper in the monetary model of hyperinflation with perfect foresight. Further research may be conducted to better assess the robustness of this result to alternative ways of modelling the transaction role of money as search-theoretic approaches for instance. Moreover, as the attention has been restricted to perfect-foresight equilibria, further research could be also conducted to deal with rational expectations equilibria in a stochastic environment.

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[5] The requirement of sufficient essentiality of money is relevant for hyperinflationary paths analyse beyond technical arguments. As pointed out by Gutierrez and Vazquez (2004), money becomes more essential for purchasing goods during hyperinflation than during stable periods "because extreme inflation dramatically decreases credit transactions and in general the use of long term contracts". Moreover, a sufficient level of money essentiality is crucial in inflationary finance models of hyperinflation since the government needs the money to be essential to the system in order to get sufficient inflation tax when inflation explodes.

The result of the theoretical link between the possibility of monetary hyperinflation paths and the essentiality of money leads to two contributions to the monetary analysis of hyperinflation. First, it contributes to the understanding of the well known failure of Cagan inflationary finance models with perfect foresight. The semi-log schedule of the famous Cagan money demand with perfect foresight is shown not to comply with money essentiality neither in the MIUF setup nor in the CIA one. This analysis may cast doubt on hyperinflation empirical studies that have adopted the monetary model of hyperinflation with the Cagan money demand and rational expectations. Second, it provides a test of the sufficient essentiality of money for the appropriate specification of the demand for money for the empirical studies of hyperinflation. A similar test of the money essentiality for empirical studies has been also suggested in Barbosa and Cunha (2003) to address the issue of the exclusion of speculative hyperinflation paths. This paper confirms the importance of such a test for the choice of appropriate functional forms of the demand for money in empirical studies of hyperinflation.

A particular class of inflation inelastic money demand functions has been shown to be appropriate candidates to replace the popular semilogarithmic functional form in the analysis of explosive hyperinflation in inflationary finance models. The paper provides a particular and robust theoretical justification to the double-log schedule with perfect foresight. Ashworth and Evans (1998) provided an empirical support to functional forms in which the absolute inflation elasticity is a decreasing function of inflation and particularly to the inflation inelastic double-log or log-linear schedule. Therefore, this paper may be complementary to Ashworth and Evans (1998) by giving the theoretical support to the log-linear specification of the demand for money in the analysis of explosive hyperinflation. Further research may be conducted for the choice of alternative appropriate forms of the demand for real cash balances in hyperinflation contexts for which microeconomic foundations should comply with the money essentiality requirement. The recent Zimbabwean experience may give rise to interesting empirical studies of hyperinflation using in particular the log-linear form for the specification of the demand for money.

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