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PUBLIC AND PRIVATE HEALTH CARE FINANCING WITH ALTERNATE PUBLIC RATIONING RULES

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ABSTRACT. We develop a model to analyze parallel public and private health care financing under two alternative public sector rationing rules: needs-based rationing and random rationing. Individuals vary in income and severity of illness. There is a limited supply of health care resources used to treat individuals, causing some individuals to go untreated. Insurers (both public and private) must bid to obtain the necessary health care resources to treat their beneficiaries. Given individuals' willingnesses-to-pay for private insurance is increasing in income, the introduction of private insurance diverts treatment from relatively poor to relatively rich individuals. Further, the impact of introducing parallel private insurance depends on the rationing mechanism in the public sector. We show that the private health insurance market is smaller when the public sector rations according to need than when allocation is random.

KEYWORDS: health care financing, rationing rules

JEL CLASSIFICATIONS: I11, I18

1. Introduction

Pressure on public health care budgets has prompted increasing calls in many countries for an expanded role for private health care finance (see, e.g., Mossialos et al. 2002; OECD 2004), including parallel private insurance that guarantees treatment and covers the costs of obtaining (publicly insured) care outside the public plan. The effects of parallel finance on equity and efficiency are debated. Advocates of parallel private finance argue that it can reduce wait times within the public system, reduce financial pressure within the public system, increase access to needed care, and increase quality of care (Globerman and Vining 1998; Crowley 2003; Montreal Economic Institute 2005; Esmail 2006). Opponents argue that by drawing both resources and support away from the publicly financed system, parallel private finance can increase wait times in the public system, reduce access for many in society, and reduce quality in the public system (Yalnizyan 2006; Canadian Health Coalition 2006).

This debate has taken on increased salience in a number of countries. A 2005 Canadian Court decision, for instance, struck down a law prohibiting parallel private insurance based in part on the argument that such insurance would increase access to care without harming the public insurance system (Chaoulli vs Government of Quebec 2005; Flood et al. 2005). Australia and Portugal subsidize the purchase of parallel private insurance in the belief that such a policy reduces public-sector wait times, increases access and reduces public expenditures (Healy et al. 2006; Mossialos and Thomson 2004), while other countries (e.g., Austria, Greece, Ireland, Italy, Spain, and the United Kingdom) have reduced or eliminated tax subsidies to such private insurance in the belief that they are not effective (Mossialos and Thomson 2004).

Increasing our understanding of the effects of parallel finance is of considerable importance. The empirical literature investigating the effects of parallel finance provides limited guidance because it often lacks rigorous design and questionable generalizability across systems that differ in institutional design (see Tuohy et al. 2004 for survey). A small but growing analytic literature investigates a number of aspects of parallel finance. Iversen (1997) and

Olivella (2003), for example, analyze how public sector wait times change when a parallel system of finance is instituted and the private sector is regulated. Hoel and Sæther (2003) argue that public-sector waiting lists may be an effective sorting device facilitating income redistribution. Similarly, Marchand and Schroyen (2005) show that if income inequality is sufficiently large, a mixed health care system (with a large public system) where the 'rich' opt for private care in order to avoid waiting can be socially desirable. Several papers have also examined the behavioral responses by health care providers under parallel finance.[1]

This paper contributes to the analytic literature on public and private roles in health care financing by developing a simple model to investigate the effects of parallel public and private financing under different public-sector rationing rules. In the model individuals differ with respect to both health status and income-earning potential, thereby capturing two dimensions of concern for equity in health care. Previous literature has generally only considered heterogeneity along one dimension. Society does not have enough health care resources (e.g., physicians) to treat all those who are ill, so each period some individuals must go without treatment. Public and private insurers compete for these limited health care resources to treat beneficiaries of their respective plans. *Ex ante* individuals do not observe their illness severity. The desire to ensure access to treatment (and thereby avoid the loss of income associated with being ill and unable to work) creates demand for private health insurance alongside the public insurance system.

Under these assumptions, we examine outcomes (e.g., the average severity of the treated and untreated; the average income of the treated and untreated; the size of the private sector) under two regimes defined by the public-sector rationing rules: needs-based allocation and random allocation.[2] Many public health care systems strive to allocate resources according to need but do so only imperfectly so that allocation inevitably includes a random element. Our

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[1] See, for example, Gonzalez (2004, 2005), Biglaiser and Ma (2006), Brekke and Sørgard (2007), and Barros and Olivella (2005).

[2] Our approach is positive. Gravelle and Siciliani (2008) examine the optimality of different public sector rationing rules and show that the welfare effects of prioritization in the public health care system depend on the assumed distribution of health gains from treatment. They do not, however, focus on the determination of a health care market equilibrium under alternate financing arrangements.

two rationing rules represent the extreme cases of ideal allocation according to need and a complete breakdown of systematic allocation, providing insight into the effects of deviations from needs-based rationing. We demonstrate that outcomes – including the scope for a private health care system – depend crucially on the allocation rule adopted in the public system.

Section 2 below presents our basic framework; Section 3 presents the rationing rules; Section 4 characterizes the health care allocations with public financing only and with private financing only, which serve as useful benchmarks for the mixed financing system analyzed in Section 5. Finally, Section 6 discusses our results. All proofs are in the Appendix I.

2. The Model

Everyone in the population (which we normalize to size one) is sick, but individuals differ in the severity of their illness and their income. There is a continuum of individuals. Incomes, Y, are distributed in the population on the interval $[\underline{Y}, \overline{Y}]$ according to the cumulative distribution function $G(Y)$ with $G(\underline{Y}) = 0$, $G(\overline{Y}) = 1$, and $G'(Y) = g(Y) > 0$ for all Y. Severity levels, s, are distributed in the population on the interval $[0, 1]$ according to the cumulative distribution function $F(s)$ with $F(0) = 0$, $F(1) = 1$, and $F'(s) = f(s) > 0$ for all s. The total population can be represented as follows:

$$\int_{\underline{Y}}^{\overline{Y}} \int_0^1 dF dG = 1.$$

We assume that income and severity are independently distributed, which makes the model tractable. Later, we discuss how a negative correlation between income and severity affects our results. Individuals know that they are sick but they do not know the severity of their illness prior to any decision regarding the purchase of private health care insurance.

Illness, if not treated, affects an individual's income and health status. Individuals all have the same potential health status if treated, which we denote by h. The more severe the illness the more time the individual will be unable to work if not treated, and the greater the income loss. Without loss of generality, we assume that the income loss from not being treated is

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sY, so the individual's severity level can be interpreted as the fraction of the individual's income that is lost if the individual does not receive treatment.[3] Consequently, for a given severity level the income loss from not being treated is greater for higher income individuals. In addition, the more severe the illness the greater is the health loss from going untreated (sh) or, equivalently, the greater is the health benefit from treatment. This non-monetary loss associated with severity is independent of income and is the same for all individuals with the same severity level.

An individual's utility is separable in income and health status. Therefore, the expected utility of an individual with income Y and without access to treatment is:

$$(1) \qquad \int_0^1 \left[u((1-s)Y) + v((1-s)h) \right] dF.$$

We make two further simplifying assumptions that make our analysis more tractable but that do not affect our qualitative results. First, we assume that the marginal utility of income is unity so the individual's utility from income is simply given by income. Second, we suppress the part of utility arising from the individual's health status. Given our assumptions, the expected utility of an individual with income Y and without access to treatment is $\int_0^1 (1-s)Y \, dF$.

Individuals can be treated and cured immediately, regardless of their severity, by the receipt of one unit of a health care service. Treatment ensures that the patient suffers no income loss and is restored to full health. It will be convenient to view this health care service as a specialized service rather than a service offered by a general practitioner. In fact, parallel private insurance is most commonly purchased to gain better access to specialist services (Mossialos and Thomson 2004; Foubister et al. 2006). One unit of the health care service is produced using one unit of a health care resource, but there are not enough health care resources to treat everyone.

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[3]A more general monetary loss function from not being treated could be assumed, for example, $L(s, Y)$. Provided $L_{sY} > 0$ all of our results hold.

The supply of health care resources is fixed at H. Empirical elasticity estimates for both physician labour supply curves and service supply curves are generally inelastic (e.g., Rizzo and Blumenthal 1994; Thornton and Eakin 1997; Kantarevic et al. 2008) which is not completely inconsistent with our assumption of fixed supply but we also discuss the implications of relaxing our assumption below. We also assume that $H < 1$ so only a fraction of the total population can be treated. Given our population normalization, H can be interpreted as the number of individuals treated. Health care resources are offered on a competitive market. Health care is financed by both the public insurer and private insurers who contract with suppliers of health care resources to provide services to their respective beneficiaries.[4] The public insurer bids for resources according to its ability-to-pay, as determined by the public health care budget. Private insurers bid for health care resources according to their willingnesses-to-pay, which are based on individuals' willingnesses-to-pay for private insurance that guarantees access to care regardless of severity level.[5] The two sectors compete directly for the limited health care resources and there is a market-clearing equilibrium price for the health care resource, P, at which all of the health care resource, H, is allocated to insurers and thereby across the population.

Our assumed market structure for private insurance is formally identical to one with no private insurance organization in which individuals bid *ex ante* in a competitive auction for contracts with the suppliers of health care resources to guarantee treatment.[6] Both characterizations capture the essential problem faced by an individual concerned about access within a public system who must decide *ex ante* (e.g., because of pre-existing-condition exclusions) whether to purchase private insurance. An individual's *ex ante* willingness-to-pay for insurance is positive because without insurance she cannot be guaranteed access to care *ex post*: even if she tries to purchase care paying out-of-pocket the price will exceed her

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[4]Our model of mixed financing can be viewed as a simple two-sided market (Rochet and Tirole 2003). Health insurers are the platform so to speak. Once they sell an insurance contract, they guarantee treatment, which is why they contract with a physician to eventually deliver health care.

[5]Private insurers are unable to price discriminate; they charge a single price for private insurance.

[6]In a competitive auction, all individuals purchase the contract at the same price. The equivalence between the two market structures would not necessarily hold under alternative auction mechanisms.

ability-to-pay. In this sense, as Nyman (2003) has argued, insurance does not just buy risk reduction: it also buys access to services a person could not otherwise afford to purchase.

While, recognizing the equivalence of the model with an insurance organization and the model with decentralized contracting in a competitive environment, for the remainder of the paper we emphasize the institutional structure with private insurance organizations.

3. Public Rationing Rules

Given the limited supply of health care resources, the public insurer must ration access to publicly insured services. Most public health care systems ration through a combination of criteria for treatment eligibility and wait times. Our analysis focuses on the former, which we call rationing rules, fixing public wait times to zero for those who are treated right away and infinity for those who do not receive treatment. Our analysis takes the public insurer's rationing rule as given and investigates how the public rationing rule interacts with financing arrangements to determine who receives treatment and the size of the private insurance sector.

We consider two rationing rules. Under one rule the public insurer rations health care according to need, treating the most severe cases that present to the public system. Rationing by need requires that the public insurer observes individual severity levels prior to treatment. Many health care systems, for example, attempt to ensure allocation according to need by having General Practitioners (GPs) function as gatekeepers to many types of services, including specialist care, diagnostic services, and non-medical services such as home care, based on the nature and severity of the patient's condition. Specialists can analogously ration access to advanced procedures on the basis of patient need. Allocation according to need is the stated objective of many publicly financed health care systems (van Doorslaer et al. 1993). In our environment, allocation by severity levels is consistent with allocation according to need under commonly cited interpretations of need (Culyer and Wagstaff 1993) and with allocation of health services to reduce (avoidable) inequality in the distribution of health in the population (Hauck et al. 2002).

The second public-sector rationing rule we consider is random access to services. Under this rationing rule, the probability of treatment is independent of both income and severity. No system deliberately rations randomly, but every system contains elements of random rationing, so that in reality allocation within publicly financed health systems lies between the two extremes of allocation according to need and random allocation. Some randomness of access can arise even in systems that strive to allocate health care according to need because, for instance, GP gatekeeping is always imperfect; specialists often manage their own wait lists so that, in the absence of system-wide coordination, less-severe patients of specialists with shorter lists may have better access than other, more-severe cases of other specialists; and need is often ranked only within service areas (e.g., heart disease, cancer), with limited scope to rank need across conditions.[7] The allocation process ends up being a mixture of our two extreme rules — needs-based allocation and random allocation — where the weight given to each depends on the emphasis placed on need and the specific institutional arrangements of a health care system.

Before examining a mixed system of parallel public and private finance, we briefly consider the cases of public health care finance only and private health care finance only which serve as useful benchmarks.

4. Benchmark Health Care Allocations

4.1. **Public Health Care Finance Only.** Assume first that only public insurance is available and that there are no user fees in the public system. The public insurer has an exogenously determined budget, B, measured in dollars, that is independent of the public rationing rule. The public insurer would like to treat as many people as possible. Denote M as the number (or, fraction of population) treated by the public sector. The public insurer's

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[7]A recent survey of MRI clinics in Canada, for instance, found that: "... patients with the same medical indication for an MRI examination, at the same centre, could be placed in different prioritization categories, with very different wait times ... This inconsistency in defining prioritization categories and the considerable variation in the number of categories leads to significant inconsistencies in access to MRI from site to site even within a given province." Emery et al. 2009, p. 82.

ability-to-pay (ATP) per treatment is given by

$$(2) \qquad\qquad ATP = B/M.$$

Since the public insurer is the only demander of health care resources, the equilibrium price per unit of resource, P_b, that clears the health care resource market is[8]

$$(3) \qquad\qquad P_b = \frac{B}{H}.$$

The public insurer purchases all available health care resources, so H individuals are treated and $1 - H$ individuals remain untreated.[9] Who receives treatment from the public insurer depends on how the public insurer rations health care.

Let π_b denote the proportion of individuals treated (or, the probability of treatment) in the public sector when public health care is allocated randomly. Then,

$$(4) \qquad\qquad \pi_b = H.$$

The expected severity level of those treated (and of those untreated) is simply the unconditional expected severity level, $E(s)$, which by definition is

$$(5) \qquad\qquad E(s) = \int_0^1 s dF.$$

When the public insurer rations health care according to need, it specifies a severity threshold $s_b > 0$ such that all individuals with severity $s \geq s_b$ are treated and all those with $s < s_b$ are not treated. The severity threshold is given by

$$(6) \qquad\qquad s_b = F^{-1}(1 - H).$$

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[8]We assume the health care providers' reservation price is below the equilibrium price.
[9]This holds regardless of the size of the public insurer's budget. If the supply of health care resources was not fixed and depended on the price, then an increase in the public insurer's budget would increase the number of individuals treated.

The expected severity conditional on being treated is equal to

$$(7) \qquad E(s|s \geq s_b) = \frac{\int_{s_b}^1 sdF}{1 - F(s_b)},$$

where $1 - F(s_b)$ is the probability of having a severity greater than the threshold, i.e., being treated under needs-based allocation. Since the public sector targets resources at high-severity patients, the expected severity of the treated is greater than the unconditional mean, $E(s|s \geq s_b) > E(s)$. Under either public rationing rule, the likelihood of treatment in a pure public system is independent of income.

4.2. **Private Health Care Finance Only.** Assume now that health care is financed wholly by private insurance. Prior to learning their severity, individuals can choose to purchase a private insurance policy which guarantees treatment. Consider individuals with income Y deciding whether to purchase private insurance. Recall that individuals obtain utility directly from their incomes and that the marginal utility of income is unity.[10] At the time they have to make an insurance decision individuals do not know their severity levels and would expect to lose $E(s)Y$ of their incomes if they do not purchase insurance. Therefore, without private insurance the individual's expected income is $(1 - E(s))Y$. It follows that an individual with given income Y will have a maximum willingness-to-pay for insurance of

$$(8) \qquad WTP = E(s)Y,$$

which is increasing in the expected loss without treatment, i.e., the higher the income and/or the unconditional expected severity.[11]

The private insurers bid for health care resources based on individuals' willingness-to-pay for private insurance. We can think about P, the price per unit of the health care resource,

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[10]Our qualitative results continue to hold with a strictly concave utility function provided the maximum willingness to pay (WTP) is increasing in income, where WTP is defined implicitly by $u(Y - WTP) = \int_0^1 u[(1 - s)Y]f(s)ds$. A sufficient condition for this is $u''' \leq 0$.

[11]An individual would be willing to pay more for private insurance if individual's experienced a non-monetary utility loss from not being treated, i.e., health loss. Since all individuals would have the same health-related expected utility loss (as it depends only on severity and not income) the following analysis would continue to hold albeit the equilibrium price would be higher.

as the price for private health insurance. Using equation (8), all individuals with income higher than $P/E(s)$ would like to purchase insurance. Health care services will be allocated according to maximum willingness-to-pay. But given competition among private insurers, individuals who purchase insurance pay the equilibrium price P rather than their maximum willingness-to-pay.[12] Thus, the number of individuals who purchase private insurance at a given price P is given by

$$(9) \qquad \int_{P/E(s)}^{\overline{Y}} dG = 1 - G\left(P/E(s)\right).$$

Only H individuals can be treated. The equilibrium price per unit of the resource (or, equivalently for private insurance), P_r, that clears the health care resource market is

$$(10) \qquad P_r = E(s)G^{-1}(1 - H).$$

Again, the proportion of individuals not being treated is $1 - H$. In this case, all individuals with incomes $Y \geq Y_r$ are treated and all those individuals with incomes $Y < Y_r$ are not treated, where

$$(11) \qquad Y_r = \frac{P_r}{E(s)} = G^{-1}(1 - H).$$

Higher income individuals receive treatment under private health care finance only. This is in contrast to a public only system where allocation is independent of income.

5. Mixed, Parallel Public and Private Finance

We now consider the case of mixed, parallel finance with both a public insurer and a private insurance sector. To determine willingness-to-pay for private insurance in a mixed system, an individual must form expectations about the probability of being treated by the public system and her severity level if left untreated by the public system. We assume that all individuals form the same expectations. These expectations depend on how the public

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[12]Alternatively, the single price could result from insurance regulation that requires community-rated premiums, as is found in a number of private (and public) insurance markets internationally.

insurer rations its resources. Consider first rationing by random allocation (denoted by the superscript "R").

5.1. **Rationing by Random Allocation.** Under rationing by random allocation, the expected severity conditional on not being treated in the public sector is *independent* of the probability of being treated. Hence, when an individual forms expectations about the probability of being treated in the public sector, denoted by π^e, she recognizes that her expected severity if not treated in the public sector is simply her unconditional expected severity level, $E(s)$, as given by (5).

Consider now an individual with income Y deciding whether to purchase private insurance. If she does not purchase private insurance her expected income is $\pi^e Y + (1 - \pi^e)(1 - E(s))Y$. It follows that her maximum willingness-to-pay for private insurance will be

$$(12) \qquad WTP^R = (1 - \pi^e)E(s)Y.$$

Using (12), the number of individuals who have a maximum willingness-to-pay greater than or equal to a given price of private insurance P is given by

$$(13) \qquad \int_{P/[(1-\pi^e)E(s)]}^{\overline{Y}} dG = 1 - G\left(\frac{P}{(1 - \pi^e)E(s)}\right).$$

The public insurer's ability-to-pay is still given by expression (2). Health care resources are allocated to insurers according to their willingness/ability-to-pay. The price paid by insurers (given expectations) that clears the health care resource market is implicitly defined by the following:

$$(14) \qquad 1 - G\left(\frac{P}{(1 - \pi^e)E(s)}\right) + \frac{B}{P} = H.$$

To have an equilibrium, expectations must be confirmed:

$$(15) \qquad \pi^e = \pi,$$

where π is the proportion of individuals in the public system who are actually treated. This proportion is given by the health care resources available to the public system, B/P, divided by the number of individuals who rely solely on the public insurer, $G(\cdot)$:[13]

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(16) \qquad\qquad \pi = \frac{B/P}{G\left(P/[(1-\pi^e)E(s)]\right)}.
$$

From (15) and (16), there is a unique π such that expectations are confirmed.[14] Therefore, the two conditions given by (14) and (16) with $\pi^e = \pi$ can be solved for the equilibrium values of the price of health care resources, $P_m^R(B,H)$, and the probability of treatment by the public insurer, $\pi_m^R(B,H)$, as functions of the size of the public budget and the fixed amount of health care resources.

To have an equilibrium with both an active public insurer and active private insurers, it must be the case that $B/H < (1-H)E(s)\overline{Y}$. If this condition does not hold, the public insurer would be able to contract with all available suppliers of the health care resource at a price greater than the highest-income individual's maximum willingness-to-pay, thereby outbidding private insurers for health care resources. In this case, the only equilibrium would be one with only the public insurer. An equilibrium with no public insurer ($\pi = 0$), on the other hand, can never occur. While the private maximum willingness-to-pay is always bounded from above, the public ability-to-pay is not. From equation (2) we have that $ATP \to \infty$ as $M \to 0$; that is, at $\pi = 0$ the public insurer would be willing to spend an infinite amount for the first marginal unit, contradicting $\pi = 0$ as an equilibrium. It can be shown (see Appendix I) that the equilibrium with both active public and private insurers will be unique.

Individuals with a willingness-to-pay equal to or greater than the equilibrium price of the resource purchase private insurance. Denote by Y_m^R the income level of the person with the

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[13]Privately insured individuals obtain services only through the private insurance contract. They still contribute to public financing so their purchase of private insurance does not affect the public insurer's budget.
[14]Note that in (π^e, π)-space, (15) is the 45 degree line and (16) is a strictly downward sloping curve. Evaluating (16) at $\pi^e = 0$ and $\pi^e = H$ yields $\pi > 0$ and $\pi < H$, respectively. Therefore, by continuity (16) will cross the 45 degree line only once in (π^e, π)-space.

maximum willingness-to-pay exactly equal to the equilibrium price, where

$$(17) \qquad Y_m^R = \frac{P_m^R(B,H)}{(1 - \pi_m^R(B,H))E(s)}.$$

Individuals with income $Y \geq Y_m^R$ purchase private insurance and those with $Y < Y_m^R$ are publicly insured, some of whom go untreated. Our model captures the realistic feature that the demand for supplemental private insurance is strongly positively correlated with income (Barret and Conlon 2001; Besley et al. 1999; Besley 2001; Propper 2000; Mossialos and Thomson 2004). In a mixed system with public rationing by random allocation, all treated individuals have the same average severity as the untreated individuals but the average income of those treated is higher than those untreated.

5.2. **Rationing By Need.** Suppose now that the public insurer rations public health care resources according to need (denoted by the superscript "N"). We again begin by deriving the individual's maximum willingness-to-pay for private insurance. Under rationing by need, the expected severity conditional on not being treated is no longer independent of the probability of being treated in the public sector. An individual forms expectations about the severity threshold for treatment in the public system, denoted by s_m^e, which determines her expectations both about the probability of not being treated by the public insurer, $F(s_m^e)$, and her expected severity if left untreated in the public system, $E(s|s < s_m^e)$, where by definition

$$(18) \qquad E(s|s < s_m^e) = \frac{\int_0^{s_m^e} s\,dF}{F(s_m^e)}.$$

Consider an individual with income Y. If she does not purchase private insurance her expected income loss will be $F(s_m^e)E(s|s < s_m^e)Y$. Therefore, using equation (18), an individual's maximum willingness-to-pay is $\left(\int_0^{s_m^e} s\,dF \right) Y$. Therefore, an individual's maximum willingness-to-pay is

$$(19) \qquad WTP^N = \left(\int_0^{s_m^e} s\,dF \right) Y.$$

Using equation (19), the number of individuals who have a maximum willingness-to-pay greater than or equal to a given price of private insurance P is given by

$$(20) \qquad \int_{P/\int_0^{s_m^e} s\,dF}^{\overline{Y}} dG = 1 - G\left(\frac{P}{\int_0^{s_m^e} s\,dF}\right).$$

The public insurer's ability-to-pay is still given by expression (2). Health care resources are allocated according to insurers' willingness-to-pay and the equilibrium price (given expectations) that clears the health care resource market is implicitly defined by

$$(21) \qquad 1 - G\left(\frac{P}{\int_0^{s_m^e} s\,dF}\right) + \frac{B}{P} = H.$$

As there are not enough resources to treat everyone in the population, the number of individuals (given expectations) relying on the public sector, $G(\cdot)$, exceeds the publicly available resources, B/P. The public insurer adjusts the severity threshold, s_m, and with it the probability of public treatment, $1 - F(s_m)$, so as to use its available resources to treat those with the highest severity levels:

$$(22) \qquad \frac{B}{P} = (1 - F(s_m))G\left(\frac{P}{\int_0^{s_m^e} s\,dF}\right).$$

Again, to have an equilibrium, expectations must be confirmed, that is,

$$(23) \qquad s_m^e = s_m,$$

where s_m is implicitly defined by expression (22). As before, there is a unique s_m such that expectations are confirmed,[15] and the conditions (21) and (22) with $s_m^e = s_m$ can be solved for the equilibrium $P_m^N(B, H)$ and $s_m^N(B, H)$. We continue to assume that the public insurer is not able to contract with all the available suppliers of health care at a price greater than

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[15]Note that in (s_m^e, s_m)-space (23) is the 45 degree line and (22) is a strictly downward sloping curve. For there to be a unique s_m such that expectations are confirmed, we need to show that at some s_m^e (22) is above the 45 degree line and at some other s_m^e (22) is below the 45 degree line. Suppose $s_m^e = 1$, then from (22) $1 - F(s_m) = (B/P)/G(P/E(s)) > 0$ and $s_m < 1$. As s_m^e gets very close to zero, the term $P/(\int_0^{s_m^e} s\,dF)$ gets very large and $G(\cdot)$ approaches one. From (22), $1 - F(s_m)$ approaches B/P and since under a mixed, parallel finance system, $B/P \leq H < 1$, s_m will be strictly greater than zero.

the highest-income individual's maximum willingness-to-pay,[16] and again there cannot be an equilibrium with only private insurers. It can be shown that there will be an equilibrium with both active public and private insurers (see Appendix I). We assume that the equilibrium is stable.[17]

Individuals with $Y \geq Y_m^N$ will purchase private insurance and those with $Y < Y_m^N$ will not purchase private insurance where, using (19) and (23), Y_m^N is given by

$$(24) \qquad Y_m^N = \frac{P_m^N(B,H)}{\int_0^{s_m^N(B,H)} s\, dF}.$$

In a mixed system with public rationing by need, both the average severity and the average income of those treated will be higher than those untreated.

5.3. **Comparison of Public Rationing Rules.** We are interested in determining how the effects of introducing parallel financing of health care depend on the rationing rule used by the public sector. To do this, we have assumed that the supply of health care resources and the size of the public insurer's budget are both fixed and have characterized the equilibrium with mixed health care financing under each of the two rationing rules above. By comparing the systems of equations that characterize these two equilibria, we obtain the following result (see Appendix I for details):

Result 1. *In a mixed system of health care finance, the equilibrium price of health care is higher, the equilibrium probability of treatment in the public sector is lower, and the equilibrium number of individuals who purchase private insurance is greater when the public insurer rations by random allocation than when it rations by need.*

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[16]That is, $B/H < \left(\int_0^{F^{-1}(1-H)} s\, dF \right) \bar{Y}$.

[17]Stability requires that excess demand for health care resources is decreasing in the price. In Appendix I, we derive a condition under which a unique stable equilibrium exists. In the general case, we may have multiple stable equilibria. Note, however, that Result 1 holds for *all* equilibria under needs-based allocation so that there is no need to impose this condition as an assumption for this result.

Under a mixed, parallel system of finance the public rationing rule affects a number of outcomes, including the scope for the private insurance market. Compared to random rationing, under needs-based rationing: fewer people buy private insurance; the probability of treatment by those who rely on the public system is higher; and the rents earned by suppliers of the fixed health care resource are smaller. Consequently, the average income of those treated under random allocation will be higher than under needs-based allocation. Furthermore, the average severity of those treated will be lower under random allocation than under needs-based rationing.

The intuition for our main result is straightforward. In a pure public system, the number of individuals treated is the same across the two rationing rules. Now consider allowing individuals the choice to purchase private insurance (holding the number of individuals treated by the public insurer constant across the two rationing rules). Individuals will be willing to pay more for private insurance under random rationing because the expected loss if not treated by the public insurer is higher than it is under needs-based rationing. There will be greater demand for private insurance and therefore greater competition for the fixed health care resources. Under both forms of rationing, the initial competition by private insurers for health care resources will have a feedback effect on private demand through the probability of public treatment, but the key is that the competition will be greater under rationing by random allocation. Consequently, in equilibrium the price of the health care resource and the number of individuals purchasing private insurance will be higher under random rationing than needs-based rationing.

The result that the private sector will be larger under rationing by random allocation does not depend on having a fixed supply of health care resources. Consider, for example, the extreme case of a perfectly elastic supply of health care resources. In this case, the price of health care will be fixed, say at P_h. The number of individuals treated by the public insurer will also be fixed and given by B/P_h.[18] Now, if individuals could purchase private insurance,

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[18] It is assumed that B is not sufficient to treat the entire population and P_h is sufficiently high such that not all individuals can afford private insurance.

demand would once again be greater under rationing by random allocation than under by rationing by need (for a given probability of treatment by the public insurer); but of course there will again be feedback effects on the probability of public treatment. In equilibrium, the probability of public treatment under each of the two rationing rules will be determined by each of the following two conditions:

$$(25) \qquad \pi_h = \frac{B/P_h}{G\left(\frac{P_h}{(1-\pi_h)E(s)}\right)}; \quad 1 - F(s_h) = \frac{B/P_h}{G\left(\frac{P_h}{F(s_h)E(s|s \leq s_h)}\right)}.$$

By comparing these expressions, it follows that in equilibrium $\pi_h > 1 - F(s_h)$ and $Y_h^R < Y_h^N$.[19] More individuals purchase private insurance under rationing by random allocation than rationing by need. Since the number of individuals treated by the public insurer is fixed, the probability of treatment by the public insurer will actually be higher under rationing by random allocation than under rationing by need.

We have shown that for a given size of the public insurer's budget the impact of a parallel system of health care financing will depend on the allocation method used in the public sector. The public insurer's allocation method may also affect the determination of the size of the public insurer's budget. This raises some important questions regarding, for instance the size of the public budget in a pure public system under each type of rationing rule, the effect of parallel financing on public support for the public system, and how this support depends on the allocation method used in the public sector. The examination of these questions remain outside of the scope of this model and we leave them for future research. We can, however, comment on the possibility of the public insurer's health care budget exogenously differing across the two forms of rationing. The public budget available for treatment may differ, for example, if real resources are required to first assess an individual's health need under needs-based rationing.[20] Provided this resource cost for screening is not too high, then our main result would continue to hold.

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[19]Suppose $\pi_h \leq 1 - F(s_h)$. By (25), this leads to a contradiction since $E(s) > E(s|s \leq s_h)$.
[20]We thank a reviewer for suggesting this possibility.

5.4. **Supply of Health Care Resources and Size of Public Budget.** Using the above system of equations, we can also determine how changes in the fixed supply of health care resources and the size of the public budget will affect the equilibrium price and probability of treatment in the public sector under both types of rationing rules. Under rationing by random allocation, we obtain the following partial derivatives (see Appendix I for details):

$$(26) \qquad \frac{\partial P_m^R(B,H)}{\partial H} < 0, \frac{\partial P_m^R(B,H)}{\partial B} \gtreqless 0, \frac{\partial \pi_m^R(B,H)}{\partial H} > 0, \frac{\partial \pi_m^R(B,H)}{\partial B} > 0.$$

Only the effect of the public budget on the equilibrium price is ambiguous. The increase in the public insurer's ability-to-pay has a direct positive effect on the price; but also an indirect effect working in the opposite direction because the increase in the probability of treatment in the public sector reduces the expected loss of relying on the public system, and with it individuals' willingness-to-pay for private insurance. Because the bids of private insurers reflect individuals' willingness-to-pay, the higher public treatment probability softens competition for health care resources. Which of the two effects dominates depends on the assumed income distribution.

We obtain similar results in the case of a mixed system with needs-based public rationing (see Appendix I for details):

$$(27) \qquad \frac{\partial P_m^N(B,H)}{\partial H} < 0, \frac{\partial P_m^N(B,H)}{\partial B} \gtreqless 0, \frac{\partial s_m^N(B,H)}{\partial H} < 0, \frac{\partial s_m^N(B,H)}{\partial B} < 0.$$

Under needs-based rationing, an increase in public insurer's budget has a direct positive effect on the price as in the case above. The increase in B also reduces the public severity threshold. This increases the probability of treatment in the public system (as above) but also has an additional indirect negative effect on price by reducing the expected loss incurred by the individual if not treated in the public system and thus, the individual's maximum willingness-to-pay for private insurance. Again, the direct effect and indirect effects move in opposite directions. The additional indirect effect of a change in B under needs-based rationing, however, may cause a change in the size of the public insurer's budget under

mixed, parallel finance to have qualitatively different impacts on the equilibrium price of health care under the two different rationing rules. For example, with uniform distributions an increase in B under rationing by random allocation has no impact on the equilibrium price whereas under needs-based rationing it decreases the equilibrium price (see Appendix I).

Neither the fixed supply of health care resources nor the public budget has a direct effect on the size of the privately insured population under either type of rationing rule (see equations (17) and (24)). These variables only indirectly affect the size of this population through their effects on the equilibrium price and probability of treatment in the public system. Only in the case of rationing by random allocation are we able to say something conclusive about what happens to the number of privately insured individuals as we change the supply of health care resources or the size of the public budget.

Consider first what happens as the size of the public budget increases under rationing by random allocation. From (27), the equilibrium probability of treatment in the public system will go up and (for a given price) the demand for private insurance will go down. The equilibrium price, however, may increase or decrease and thus could have an offsetting effect on the demand for private insurance. It can be shown that the effect of the increase in π_m^R on demand for private insurance will always dominate and Y_m^R will go up with an increase in the public budget (see Appendix I).

Next, consider an increase in the supply of health care resources. This increase will have opposite effects on the equilibrium price and probability of public treatment and, consequently, opposing effects on the demand for private insurance. Which effect dominates (π or P) will depend on the equilibrium value of the probability of public treatment (as shown in Appendix I). To understand why, note that the number of individuals treated by the public insurer is given by B/P. An increase in H reduces the price and necessarily increases the number of individuals who are treated by the public insurer. This increase may be less than, equal to, or greater than the increase in H and thus the number of individuals treated through private insurance may increase, remain constant or decrease. For large values of π_m^R,

the demand for private insurance goes down with the increase in H (the effect of the change π dominates) and for small values of π_m^R the demand for private insurance goes up with the increase in H (the effect of the change in P dominates).

6. DISCUSSION

In our model the system of financing and the public-sector rationing rule interact to generate the ultimate effects on health production, need-related equity and income-related inequity of access. The average severity of those treated can serve as an index of both need-related distributional equity and, because treatment restores everyone to full health, of the total amount of health produced with the available resources. The average severity of treated, and the total amount of health produced, is highest under needs-based rationing. The average income of the treated provides insight into income-related equity of access. Higher average income among the treated implies that higher-income individuals have better access to services (unrelated to their need). The average income of the treated is higher under mixed financing with random rationing than mixed financing with needs-based rationing because the private insurance sector is larger under the former.

The introduction of parallel private insurance creates an income gradient in access as those with higher incomes have both greater means and greater incentive to purchase private insurance that guarantees access to care. These findings are broadly consistent with empirical analyses of income-related equity in the utilization of health care, which document that for specialist services countries with the largest parallel private sectors to the public or social insurance system exhibit some of the highest degrees of pro-rich income-related inequity in use (van Doorslaer et al. 2004). But our model highlights that the magnitude of this income-access gradient also depends in part on the public-sector rationing rule: other things equal, private insurance is more desirable for an individual under random rationing than under needs-based rationing. Consequently, both the size of the private insurance sector and the income gradient in access are larger under random rationing.

Our model suggests that the public-sector rationing rule can affect the demand for parallel private insurance. Anecdotal evidence indicates that individuals are, within limits, willing to wait for a service if they believe that the public system rations according to need and their wait reflects the more serious needs of others. However, a perception that some are getting quicker access for reasons other than need reduces tolerance for waiting and increases support for private options. Hence, demand for privately financed care depends not just on the length of the wait, but on the process for allocating access to public services. Consequently, the impact of parallel finance depends in part on the rationing rule used by the public sector.

Our model makes some strong assumptions. Perhaps the most notable of these is that severity and income are independently distributed, while in reality it is well-documented that illness and income are negatively correlated. A simple example available in Appendix II relaxes this assumption to allow for such negative correlation. The results do not differ substantially from those obtained under the assumption of independence. A key difference from the independent case is that under needs-based rationing each individual faces a different probability of treatment by the public insurer for a given severity threshold. But again, for a given probability of treatment by the public insurer, the willingness-to-pay for private insurance will be higher under rationing by random allocation than by need.

Our model also assumes that both suppliers of health care and private insurers are passive. In reality their behaviour can play an important role in determining outcomes under alternative financing and allocation arrangements. Strategic responses by these groups would likely exacerbate both the efficiency and equity concerns associated with parallel private insurance. For example, if suppliers were able to establish dual practices to treat patients in both the public system and a private-pay practice, the strategic incentive to selectively recommend privately paid care for those who can afford it would strengthen the income gradient in access to care and weaken the relationship between need and use. For example, private insurers that offer supplementary parallel insurance tend to focus on a small number of relatively simple procedures, leaving the complicated care for the public system (Mossialos

and Thomson 2004) and can attempt to cream-skim the relatively healthy within any risk category.

Mixed Finance with Rationing by Random Allocation. Define the implicit functions:

$$Z^1(P, \pi; B, H) = G\left(\frac{P}{(1-\pi)E(s)}\right) - \frac{B}{P} - (1 - H),$$

$$Z^2(P, \pi; B, H) = \pi - \frac{B}{P}\frac{1}{G\left(P/[(1-\pi)E(s)]\right)}.$$

The equilibrium is characterized by $Z^1(P, \pi; B, H) = 0$ and $Z^2(P, \pi; B, H) = 0$. Let $\pi = z(P; B, H)$ be the explicit solution to the second condition. We note that:

- $Z^1(P, z(P; B, H); B, H)$ is continuous in P.
- $Z^1(P, z(P; B, H); B, H)$ is monotically increasing in P.
- $Z^1(0, z(0; B, H); B, H) < 0$.
- $\lim_{P \to \infty} Z^1(P, z(P; B, H); B, H) > 0$.

It follows that there exists a unique equilibrium. Totally differentiating the two equilibrium conditions and using Cramer's Rule, we obtain

$$(A.1) \quad P_H^R = \frac{1}{\det J}\left[-Z_H^1 Z_\pi^2 + Z_H^2 Z_\pi^1\right] < 0,$$

$$(A.2) \quad P_B^R = \frac{1}{\det J}\left[-Z_B^1 Z_\pi^2 + Z_B^2 Z_\pi^1\right] = \frac{1}{\det J}\frac{1}{P}\left[1 - \frac{g\left(\frac{P}{(1-\pi)E(s)}\right)}{G\left(\frac{P}{(1-\pi)E(s)}\right)}\frac{P}{(1-\pi)E(s)}\right] \gtreqless 0,$$

$$(A.3) \quad \pi_H^R = \frac{1}{\det J}\left[-Z_H^2 Z_P^1 + Z_H^1 Z_P^2\right] > 0,$$

$$(A.4) \quad \pi_B^R = \frac{1}{\det J}\left[-Z_B^2 Z_P^1 + Z_B^1 Z_P^2\right] > 0.$$

where

$$Z_P^1 = \frac{g\left(\frac{P}{(1-\pi)E(s)}\right)}{(1-\pi)E(s)} + \frac{B}{P^2} > 0, \quad Z_\pi^1 = \frac{g\left(\frac{P}{(1-\pi)E(s)}\right)P}{(1-\pi)^2 E(s)} > 0,$$

$$Z_P^2 = \frac{B}{P^2}\frac{1}{G((P/[(1-\pi)E(s)])} + \frac{B}{P}\frac{g\left(\frac{P}{(1-\pi)E(s)}\right)}{[G((P/[(1-\pi)E(s)])]^2}\frac{1}{(1-\pi)E(s)} > 0,$$

$$Z_\pi^2 = 1 + \frac{B}{P}\frac{g\left(\frac{P}{(1-\pi)E(s)}\right)}{[G((P/[(1-\pi)E(s)])]^2}\frac{P}{(1-\pi)^2 E(s)} > 0,$$

$$Z_H^1 = 1, \quad Z_H^2 = 0,$$

$$Z_B^1 = -\frac{1}{P} < 0, \quad Z_B^2 = -\frac{1}{PG\left(P/[(1-\pi)E(s)]\right)} < 0,$$

and $\det J$ is the determinant of the Jacobian matrix

$$\det J = Z_P^1 Z_\pi^2 - Z_\pi^1 Z_P^2 = \frac{B}{P^2} + \frac{g}{E(s)} > 0,$$

where the second expression follows from substituting in $Z^2(P, \pi; B, H) = 0$.

The equilibrium price will be increasing (decreasing) in B if $[dG(y)/dy][y/G(y)]$ is less than (greater than) one. Under a uniform distribution, $[dG(y)/dy][y/G(y)] = 1$ so $P_B^R = 0$.

It follows from the above results and equation (17) that

$$Y_H^R = P_H^R \frac{1}{(1-\pi)E(s)} + \frac{P}{(1-\pi)^2 E(s)} \pi_H^R = \frac{1}{\det J} \frac{1}{(1-\pi)^2 E(s)} (2\pi - 1) \gtreqqless 0,$$

$$Y_B^R = P_B^R \frac{1}{(1-\pi)E(s)} + \frac{P}{(1-\pi)^2 E(s)} \pi_B^R = \frac{1}{\det J} \frac{1}{(1-\pi)E(s)} \frac{1}{P} > 0.$$

Mixed Finance with Need-Based Rationing. Define the implicit functions:

$$M^1(P, s_m; B, H) = G\left(\frac{P}{\int_0^{s_m} s \, dF}\right) - \frac{B}{P} - (1 - H),$$

$$M^2(P, s_m; B, H) = (1 - F(s_m))G\left(\frac{P}{\int_0^{s_m} s \, dF}\right) - \frac{B}{P}.$$

The equilibrium is characterized by $M^1(P, s_m; B, H) = 0$ and $M^2(P, s_m; B, H) = 0$. Let $s_m = m(P; B, H)$ be the explicit solution to the second condition. We then note:

- $M^1(P, m(P; B, H); B, H)$ is continuous in P.
- $M^1(0, m(0; B, H); B, H) < 0$.
- $\lim_{P \to \infty} M^1(P, m(P; B, H); B, H) > 0$.

It follows that there exists an equilibrium. We focus on a stable equilibrium in which excess demand for health care resources is decreasing in P or equivalently (given fixed supply) $M^1(P, s(P; B, H); B, H)$ is increasing in P at the equilibrium. A sufficient condition for

uniqueness is that $M^1(\cdot)$ is globally increasing in P. The unique equilibrium would also be stable. Totally differentiating the two equilibrium conditions and using Cramer's Rule, we obtain

(A.5) $$P_H^N = \frac{1}{\det J}\left[-M_H^1 M_{s_m}^2 + M_H^2 M_{s_m}^1\right] < 0,$$

(A.6) $$P_B^N = \frac{1}{\det J}\left[-M_B^1 M_{s_m}^2 + M_B^2 M_{s_m}^1\right] \gtreqless 0,$$

(A.7) $$s_H^N = \frac{1}{\det J}\left[-M_H^2 M_P^1 + M_H^1 M_P^2\right] < 0,$$

(A.8) $$s_B^N = \frac{1}{\det J}\left[-M_B^2 M_P^1 + M_B^1 M_P^2\right] < 0,$$

where

$$M_P^1 = \frac{g\left(\frac{P}{\int_0^{s_m} sdF}\right)}{\int_0^{s_m} sdF} + \frac{B}{P^2} > 0, \quad M_{s_m}^1 = -\frac{g\left(\frac{P}{\int_0^{s_m} sdF}\right)Ps_m f(s_m)}{\left(\int_0^{s_m} sdF\right)^2} < 0,$$

$$M_P^2 = (1 - F(s_m))\left[\frac{g\left(\frac{P}{\int_0^{s_m} sdF}\right)}{\int_0^{s_m} sdF}\right] + \frac{B}{P^2} > 0,$$

$$M_{s_m}^2 = -f(s_m)G\left(\frac{P}{\int_0^{s_m^e} sdF}\right) - (1 - F(s_m))\frac{g\left(\frac{P}{\int_0^{s_m} sdF}\right)Ps_m f(s_m)}{\left(\int_0^{s_m} sdF\right)^2} < 0,$$

$$M_H^1 = 1, \quad M_B^1 = M_B^2 = -\frac{1}{P} < 0, \quad M_H^2 = 0,$$

and

$$\det J = M_P^1 M_{s_m}^2 - M_{s_m}^1 M_P^2 < 0,$$

where the last inequality follows from the assumed stability of the equilibrium. The sign of P_B^* will be unambiguously negative when income and severity are uniformly distributed.

Comparison of Parallel Finance Outcomes under Different Rationing Rules. The equilibrium with rationing by random allocation can be characterized by the following:

(R.1) $$P^R = (1 - \pi^R)E(s)Y^R,$$

(R.2) $$H = 1 - G(Y^R) + \frac{B}{P^R},$$

(R.3) $$\frac{B}{P^R} = \pi^R G(Y^R).$$

An equilibrium with needs-based rationing can be characterized by:

(N.1) $$P^N = F(s^N)E(s|s \le s^N)Y^N,$$

(N.2) $$H = 1 - G(Y^N) + \frac{B}{P^N},$$

(N.3) $$\frac{B}{P^N} = (1 - F(s^N))G(Y^N),$$

where $1 - F(s^N)$ is the probability of being treated in the public system. We first show that equilibrium prices, probability of treatment in the public system and income cut-offs must differ under the two rationing rules.

Suppose $Y^R = Y^N$. From $(R.2)$ and $(N.2)$, we have $P^R = P^N$. Then, from $(R.3)$ and $(N.3)$, we have $\pi^R = 1 - F(s^N)$ and by $(R.1)$ and $(N.1)$ we have a contradiction since $E(s) > E(s|s \le s^N)$ (given $s^N < 1$) so $Y^R \ne Y^N$. By a similar argument, $P^R \ne P^N$. From $(R.2)$ and $(N.2)$, we have either (i) $Y^R < Y^N, P^R > P^N$ or (ii) $Y^R > Y^N, P^R < P^N$.

Suppose $\pi^R = 1 - F(s^N)$. Combining $(R.2)$ with $(R.3)$ and $(N.2)$ with $(N.3)$ yields

(R.4) $$H = 1 - (1 - \pi^R)G(Y^R),$$

(N.4) $$H = 1 - F(s^N)G(Y^N).$$

By $(R.4)$ and $(N.4)$, $Y^R = Y^N$ which has already shown not to be possible. Therefore, $\pi^R \ne 1 - F(s^N)$. Further, from $(R.4)$ and $(N.4)$, we have that either (iii) $\pi^R > 1 - F(s^N)$, $Y^R > Y^N$ or (iv) $\pi^R < 1 - F(s^N)$, $Y^R < Y^N$.

Combining $(i) - (iv)$, we have two possible orderings of the equilibria variables; (a) $\pi^R > 1 - F(s^N)$, $Y^R > Y^N$, $P^R < P^N$, or (b) $\pi^R < 1 - F(s^N)$, $Y^R < Y^N$, $P^R > P^N$.

Combining $(R.1)$ with $(R.3)$ and $(N.1)$ with $(N.3)$ yields

(R.5) $$B = \pi^R G(Y^R)(1 - \pi^R)E(s)Y^R,$$

(N.5) $$B = (1 - F(s^N))G(Y^N)F(s^N)E(s|s \leq s^N)Y^N.$$

Equating the left-hand sides of $(R.4)$ and $(N.4)$, we obtain that $(1-\pi^R)G(Y^R) = F(s^N)G(Y^N)$. Using this expression and equating the left-hand sides of $(R.5)$ and $(N.5)$, we obtain

$$\pi^R E(s)Y^R = (1 - F(s^N))E(s|s \leq s^N)Y^N.$$

Suppose we have ordering (a) so $\pi^R > 1 - F(s^N)$ and $Y^R > Y^N$. From the above expression, we have a contradiction since $E(s) > E(s|s \leq s^N)$. Therefore, we must have $\pi^R < 1 - F(s^N)$, $Y^R < Y^N$, and $P^R > P^N$ as stated in Result 1. Note that the above proof holds for all possible equilibria.

Appendix II

Our model has assumed (largely for reasons of tractability) that income and severity are independently distributed. It is well-documented that, in reality, income and health status are positively associated. Across many societies, the poor have lower average health status (Evans et al. 1994). We now allow for income and severity to be negatively correlated.

We assume that income is uniformly distributed on the interval $[0, \overline{Y}]$ where $\overline{Y} > 1$ but that severity is no longer independent of income. Each individual with a given income Y draws a severity level $s = \rho Y + \epsilon$, where the correlation coefficient is $\rho \in (-1/\overline{Y}, 0)$ and ϵ is a random variable distributed uniformly on the interval $[-\rho \overline{Y}, 1]$. The density of this random variable is $k(\epsilon) = 1/(1 + \rho \overline{Y})$, the cumulative distribution is $K(\epsilon) = (\epsilon + \rho \overline{Y})/(1 + \rho \overline{Y})$, and the mean is $E[\epsilon] = 1/2 - \rho \overline{Y}/2$. Consequently, severity is distributed on the interval $[0, 1]$ but is not independent of income (except in the special case of $\rho = 0$). The mean severity (in the total population) is equal to $1/2$ but the expected severity of each individual depends on their income.[21] The expected severity level of an individual with income Y, for example, is given by $E[s|Y] = \rho Y + E[\epsilon|Y]$, and because ϵ is independent of income, it holds that for any Y

$$(28) \qquad E[s|Y] = \rho Y + 1/2 - \rho \overline{Y}/2 > 0.$$

Differentiating the above expression with respect to Y, yields $dE[s|Y]/dY = \rho < 0$. Expected severity conditional on income is *decreasing* in income.

We now examine how this negative correlation between income and severity affects our results by considering in turn each of the three financing arrangements.

6.1. **Public Health Care Finance Only.** Allowing a negative correlation between income and severity leaves unchanged the following results derived under the independence assumption: the price of the health care resource is given by $P_b = B/H$; the probability of treatment for all individuals is equal to H; under random allocation the average severity of

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[21] To see this, note $E[s] = E[\rho Y + \epsilon] = \rho \overline{Y}/2 + 1/2 - \rho \overline{Y}/2 = 1/2$.

those treated is $E(s) = 1/2$, with no relationship between income and treatment; and, under needs-based allocation the average severity of those treated is higher and the average severity of those not treated is lower than under random allocation. In contrast, unlike the case with independence, a negative correlation between income and severity under needs-based rationing causes income and treatment to be negatively correlated (because lower-income individuals are more likely to have high severity levels).

6.2. **Private Health Care Finance Only.** An individual's maximum willingness-to-pay for private insurance equals the expected monetary loss if they do not purchase insurance:

$$(29) \qquad WTP = E(s|Y)Y = (\rho Y + 1/2 - \rho \overline{Y}/2)Y$$

Differentiating this term with respect to Y yields

$$(30) \qquad \frac{dE(s|Y)Y}{dY} = 2\rho Y + 1/2 - \rho \overline{Y}/2.$$

The individual's maximum willingness-to-pay could be increasing or decreasing in income. Evidence indicates that the demand for supplemental private insurance is strongly positively correlated with income (Barret and Conlon 2001; Besley et al. 1999; Besley 2001; Propper 2000; Mossialos and Thomson 2004). To be consistent with this evidence, we make the following assumption regarding the correlation coefficient between income and severity, which ensures that an individual's maximum willingness-to-pay strictly increases in income:

Assumption 1: $\rho > -1/(3\overline{Y})$.[22]

Consequently, as we saw under the independence assumption, the purchase of private insurance and treatment is positively associated with income. The negative correlation between income and severity, however, alters three results compared to the independent case: (1) the average severity of those treated is lower than the average severity of those

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[22]The expression $dE(s|Y)Y/dY$ is decreasing in Y since $\rho < 0$ which implies that the individual's maximum willingness-to-pay, $E(s|Y)Y$ is a strictly concave function in income. Define $\hat{Y}$ such that $1/2 - \rho \overline{Y}/2 + 2\rho \hat{Y} = 0$ or $\hat{Y} = \overline{Y}/4 - 1/(4\rho)$. Assumption 1 is equivalent to assuming that $\hat{Y} > \overline{Y}$. If $\hat{Y} < \overline{Y}$ or $\rho < -1/3\overline{Y}$, then the WTP will be strictly increasing in income up to $\hat{Y}$ and then strictly decreasing in income for $Y > \hat{Y}$. In this case, it would be those in the middle of the income distribution with the highest maximum willingness-to-pay.

who are not treated (before they were equal); and (2) an increase in the supply of the health care resource now increases the average severity of both those treated and those not treated (before it had no impact); (3) the average severity of those treated in the private-only system is lower than under a public-only system, regardless of the allocation rule (before the average severity of those treated under private-only finance equaled the average severity of those treated under public-only finance with random allocation).

6.3. **Mixed, Parallel Public and Private Health Care Finance.**

6.3.1. *Rationing by Random Allocation.* An individual's maximum willingness-to-pay for treatment given the equilibrium probability of treatment in the public system, π, will be[23]

$$(31) \qquad\qquad WTP^R = (1 - \pi)E(s|Y)Y,$$

which is decreasing in the expected probability of being treated in the public sector and (by Assumption 1) increasing in income.

Again there will be an income cut-off for purchasing private insurance, denoted by Y^R, which is increasing in both P and π and implicitly defined by

$$(32) \qquad\qquad (1 - \pi)(\rho Y^R + 1/2 - \rho \overline{Y}/2)Y^R = P$$

such that all individuals with income greater than Y^R purchase private insurance and all those individuals with income less than Y^R rely on the public insurer.

The equilibrium price paid by insurers that clears the health care resource market is implicitly defined by the following:

$$(33) \qquad\qquad \frac{\overline{Y} - Y^R}{\overline{Y}} + \frac{B}{P} = H$$

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[23]We assume that all individuals form the same expectations about the probability of treatment by the public insurer and that these expectations are confirmed in equilibrium.

Finally, the equilibrium proportion of individuals in the public system who are actually treated is given by

$$(34) \qquad\qquad \pi = \frac{B/P}{Y^R/\overline{Y}}.$$

The non-linear system of equations given by (36), (37), and (38) yield equilibrium values for P, π, and Y^R. Although we cannot solve explicitly for the equilibrium values when $\rho < 0$, we can make some firm conclusions. In particular, unlike the independence case, the average severities of those treated differ across the public and private sectors: the average severity of those treated in public system is greater than $E(s) = 1/2$ because higher-income individuals who have, on average, lower severities purchase private insurance. In addition, unlike the independence case, the average severity of all those treated (publicly and privately) now depends on the fixed supply of health care resources and the public insurer's budget. The effect of a change in the fixed supply of health care resources and the public insurer's budget on the equilibrium probability of treatment, the equilibrium price, and the demand for private insurance are qualitatively the same as in the independence case except the equilibrium price is decreasing in the size of the public insurer's budget when $\rho < 0$.

Comparison of mixed finance with random allocation against public-only with random allocation reveals that, because higher-income, lower-severity individuals purchase private insurance in the mixed system, the average severity of those treated is lower under mixed finance than under public-only finance.

6.3.2. *Rationing by Need.* When the public insurer rations public health care resources according to need, and income and severity are negatively correlated, the probability of being treated by the public insurer depends on an individual's income level.[24] An individual will be treated if $s = \rho Y + \epsilon \geq s_m$ or if $\epsilon \geq s_m - \rho Y$. It is possible that with a sufficiently high negative correlation between income and severity some individuals with high (low) incomes

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[24]We again assume that all individuals form the same expectations about the public sector threshold s_m^e and that these expectations are confirmed in equilibrium.

will never (always) be treated. We are interested in investigating the case when *all* individuals have a positive probability of not being treated in the public system. We restrict ρ to conform to this.[25]

An individual's expected severity if not treated is given by

$$(35) \qquad E(s|s < s_m, Y) = \rho Y + E(\epsilon|\epsilon < s_m - \rho Y, Y) = \frac{\rho Y + s_m - \rho \overline{Y}}{2},$$

which is decreasing in income since $\rho < 0$.

The individual's maximum willingness-to-pay is

$$(36) \qquad WTP^N = Prob(\epsilon < s_m - \rho Y)E(s|s < s_m, Y)Y = \frac{s_m^2 - \rho^2(\overline{Y} - Y)^2}{2(1 + \rho \overline{Y})}Y,$$

Again, by assumption an individual's maximum willingness-to-pay is increasing in income.[26] Therefore, there will be an income cut-off, Y^N, such that all individuals with incomes greater than Y^N have a maximum willingness-to-pay greater than or equal to P, where Y^N is implicitly defined by

$$(37) \qquad \frac{s_m^2 - \rho^2(\overline{Y} - Y^N)^2}{2(1 + \rho \overline{Y})}Y^N = P$$

and is increasing in P and decreasing in s_m.

Health care resources are allocated according to insurers' willingness-to-pay and the equilibrium price that clears the health care resource market is implicitly defined by

$$(38) \qquad \frac{\overline{Y} - Y^N}{\overline{Y}} + \frac{B}{P} = H.$$

The public insurer's severity threshold s_m is implicitly defined by the following expression

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[25]The individual will not be treated if $s = \rho Y + \epsilon < s_m$ or if $\epsilon < s_m - \rho Y$. For this to happen with positive probability for all $Y \in [0, \overline{Y}]$, it must be that $s_m > -\rho(\overline{Y} - Y)$ which necessarily holds if $s_m > -\rho \overline{Y}$ which we assume to be the case.

[26]Individuals will only be willing to pay a positive price for insurance if $Prob(\epsilon < s_m - \rho Y) > 0$. This probability is increasing in Y. Therefore, an individual's maximum willingness-to-pay for private insurance will be increasing in income if the expression $E(s|s < s_m, Y)Y$ is increasing in Y. Our assumption that $\rho > -s_m/\overline{Y}$ ensures that this will be the case for all $Y \in [0, \overline{Y}]$.

$$(39) \qquad \frac{B}{P} = \int_0^{Y^N} Prob(\epsilon \geq s_m - \rho Y)(1/\overline{Y})dY = \frac{1}{1 + \rho\overline{Y}}\left[1 - s_m + \frac{\rho Y^N}{2}\right]\frac{Y^N}{\overline{Y}}$$

When choosing its threshold the public insurer has to take into account that, for any given s_m, each individual relying on the public system faces a different probability of having a severity greater than the threshold.

The equilibrium is again characterized by a non-linear system of equations that we are unable to solve explicitly for equilibrium values of P, s_m, and Y^N. Nor are we able to determine in this case how the equilibrium values change with an increase in the fixed supply of health care resources or the public insurer's budget. We can, however, make the following observations regarding the outcome with needs-based rationing in a mixed system of finance: like mixed finance with independence, the average severity of those treated is higher under needs-based rationing than under random rationing but is less than that for needs-based rationing with public-only finance.

Regardless of whether income and severity are independent or negatively correlated, an individual's maximum willingness-to-pay for private insurance depends on how the public insurer rations public health care. Further, this willingness-to-pay is always lower under needs-based rationing than under random rationing.[27] Consequently, both the outcomes that occur and the impact of a change from public-only to mixed, parallel finance differ under the two rationing rules.

Although in this example we have allowed income and severity to be negatively correlated, we have continued to assume that willingness-to-pay for private insurance is positively correlated with income, causing the average income of those treated to always be higher in the presence of private insurance than when there is only a public insurer.

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[27]To see this, suppose an individual with a given Y faced the same probability of not being treated in the public sector under each type of rationing; that is, $1 - \pi = Prob(\epsilon < s_m - \rho Y)$. The result follows directly from expressions (35) and (40) noting that $E(s|Y) > E(s|s < s_m, Y)$.

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