MECHANISM DESIGN WITH LIMITED INFORMATION: THE CASE OF NONLINEAR PRICING

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Mechanism Design with Limited Information: The Case of Nonlinear Pricing*

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Abstract

We analyze the canonical nonlinear pricing model with limited information. A seller offers a menu with a finite number of choices to a continuum of buyers with a continuum of possible valuations. By revealing an underlying connection to quantization theory, we derive the optimal finite menu for the socially efficient and the revenue-maximizing mechanism. In both cases, we provide an estimate of the loss resulting from the usage of a finite n-class menu. We show that the losses converge to zero at a rate proportional to $1/n^2$ as n becomes large.

Keywords: Mechanism Design, Limited Information, Nonlinear Pricing, Quantization, Lloyd-Max Optimality.

JEL Classification: D82, D83, D86.

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1 Introduction

The theory of mechanism design addresses a wide set of questions, ranging from the design of markets and exchanges to the design of constitutions and political institutions. A central result in the theory of mechanism design is the "revelation principle" which establishes that if an allocation can be implemented incentive compatible in any mechanism, then it can be truthfully implemented in the direct revelation mechanism, where every agent reports his private information, his type, truthfully. Yet, when the private information (the type space) of the agents is large, then the direct revelation mechanism requires the agents to have abundant capacity to communicate with the principal, and the principal to have abundant capacity to process information. By contrast, the objective of this paper is to study the performance of optimal mechanisms, when the agents can communicate only limited information or equivalently when the principal can process only limited information. We pursue the analysis in the context of a representative, but suitably tractable, mechanism design environment, namely the canonical problem of nonlinear pricing. Here the principal, the seller, is offering a variety of choices to the agent, the buyer, who has private information about his willingness-to-pay for the product.

The distinct point of view, relative to the seminal analysis by Mussa and Rosen (1978) and Maskin and Riley (1984), resides in the fact that the information conveyed by the agents, and subsequently the menu of possible choices offered by the seller, is finite, rather than uncountable as in the earlier analysis. The limits to information may arise for various, direct or indirect, reasons. On the demand side, it may be too difficult or too complex for the buyer to communicate his exact preferences and resulting willingness to pay to the seller. On the supply side, it may be too time-consuming for the seller to process the fine detail of the consumer's preferences, or to identify the consumer's preferences across many goods with close attributes and only subtle differences.

Our analysis adopts a linear-quadratic specification (analogous to that of Mussa and Rosen (1978)) in which the consumer's gross utility is the product of his willingness-to-pay (his type θ) and the consumed quantity q of the product, whereas the cost of production cost is quadratic in the quantity. For this important case, we reveal an interesting connection between the problem of optimal nonlinear pricing with limited information to the problem of optimally quantizing a source signal by using a finite number of representation levels in information theory. In our setting, the socially efficient quantity q for a customer should be equated to his valuation θ if a continuum of choices were available. In the case where a finite number of choices are accessible q can take on only some values. If we see θ as the source signal and q as the representation level, then the total social welfare can be written as the mean square error between the source signal and the representation signal. Given this, the welfare maximization problem can be characterized by the Lloyd-Max optimality conditions, a well-established result in the theory of quantization. Furthermore, we can extend the analysis to the revenue maximization problem, after replacing the customer's true valuation by the corresponding virtual valuation, as defined by Myerson (1979). We estimate the welfare and revenue loss resulting from the use of a finite n-class contract (relative to the continuum contract). In particular, we characterize the rate of convergence for the welfare and revenue loss as a function of n. We examine this problem first for a given distribution on the customer's type, and then over all possible type distributions with finite support. We establish that the maximum welfare loss shrinks towards zero at the rate proportional to $1/n^2$.

The role of limited information in mechanism design has recently attracted increased attention. In a seminal paper, Wilson (1989) considers the impact of a finite number of priority classes on the efficient rationing of services. His analysis is less concerned with the optimal priority ranking for a given finite class, and more with the approximation properties of the finite priority classes. McAfee (2002) rephrases the priority rationing problem as a two-sided matching problem (between consumer and services) and shows that already binary priority contract ("coarse matching") can achieve at least half of the social welfare that could be generated by a continuum of priorities. Hoppe, Moldovanu, and Ozdenoren (2010) extend the matching analysis and explicitly considers monetary transfers between the agents. In particular, they present lower bounds on the revenue which can be achieved with specific, not necessarily optimal, binary contracts. By contrast, Madarasz and Prat (2010) suggest a specific allocation, the "profit-participation" mechanism to establish approximation results, rather than finite optimality results, in the nonlinear pricing environment. While the above contributions are concerned with single agent environments, there have been a number of contributions to multi-agent mechanisms, specifically single-item auctions among many bidders. Blumrosen, Nisan, and Segal (2007) consider the effect of restricted communication in auctions with either two agents or binary messages for every agent. Kos (2010) generalizes the analysis by allowing for a finite number of messages and agents. In turn, their equilibrium characterization in terms of partitions shares features with the optimal information structures in auctions as derived by Bergemann and Pesendorfer (2007).

2 Model

We consider a monopolist facing a continuum of heterogeneous consumers. Each consumer is characterized by a quasi-linear utility function: $u(\theta, q, t) = \theta q - t$, where q is the quantity of his consumption purchased from the monopolist, θ describes his willingness-to-pay for the good (his "type"), and t is the transfer paid by the agent. The monopoly seller offers q units of products at a cost $c(q) = \frac{1}{2}q^2$. Consequently, the net utility of the buyer and seller are given by $\theta q - t(q)$ and $t(q) - \frac{1}{2}q^2$ respectively, where $t(q)$ is the transfer price that the buyer has to pay the seller for a quantity q of the product. The specific parameterization of the utility function and the cost function is referred to as the "linear-quadratic model" and has been extensively studied in the literature beginning with Mussa and Rosen (1978). The prior distribution of θ is given by F and has compact support on $\mathbb{R}$. Without loss of generality we normalize it to the unit interval $[0, 1]$. We denote the set of all distribution on the unit interval by $\Delta \equiv \Delta[0, 1]$.

3 Welfare Maximization

We first consider the social welfare maximization problem in the absence of private information by the agent. That is, the willingness-to-pay of the buyer, his type, is publicly known. Moreover, as the transfer t does not determine the level of the social surplus, but rather its distribution between buyer and seller, it does not enter the social welfare problem. In the absence of communication constraints, $n = \infty$, the social surplus, denoted by SW_∞ is then determined as the solution to the allocation problem:

$$SW_\infty \triangleq \max_{q(\theta)} \mathbb{E}\left[\theta q(\theta) - \frac{1}{2}q^2(\theta)\right]. \tag{1}$$

In the absence of private information, the optimal solution for every type θ can be obtained pointwise, and is given by $q^*(\theta) = \theta$. In other words, the socially optimal menu $M^*_\infty = \{q^*(\theta) = \theta\}$ offers a continuum of choices and assigns each consumer the quantity of the good which is equal to his willingness-to-pay. The resulting social surplus is given by $SW_\infty = \frac{1}{2}\mathbb{E}\left[\theta^2\right]$. Importantly, given its linear-quadratic structure, the welfare maximizing problem is equivalent to minimizing the mean square error (MSE), $\mathbb{E}_\theta[(\theta - q)^2]$. We shall use this equivalent representation of the problem as we now consider the problem with communication constraints.

By contrast, we seek to determine the optimal menu when we can offer only a finite number of choices, and we denote by $\mathcal{M}_n$ the set of contracts which offer at most a finite number n of quantity choices. Henceforth, such a discretized contract $M_n = \{q_k\}_{k=1}^n$ is called an $n-$class contract or $n-$class menu. The socially optimal assignment rule then seeks to assign to each buyer with type θ a specific quantity $q(\theta)$ with the property that the quantity $q(\theta)$ represents an element in the $n-$class contract. For a given number of choices n, the social welfare problem is:

$$SW_n = \max_{q(\theta)} \mathbb{E}_\theta \left[\theta q(\theta) - \frac{1}{2}(q(\theta))^2\right] \quad \text{subject to } \{q(\theta)\}_{\theta=0}^1 \in \mathcal{M}_n. \tag{2}$$

Given that the valuation of the buyer is supermodular, i.e. $\partial^2 u(\theta, q)/\partial\theta\partial q > 0$, it follows that the optimal assignment of types to quantities has a partitional structure. Let $\{A_k = [\theta_{k-1}, \theta_k)\}_{k=1}^n$ represent a partition of the set of consumer types where $0 = \theta_0 < \cdots < \theta_{k-1} < \theta_k < \cdots < \theta_n = 1$. A consumer with type $\theta \in A_k$ will be assigned $q^*(\theta) = q_k^*$, and the socially optimal menu $M_n^* = \{q_k^*\}_{k=0}^n$ is increasing in k, so that $q_1^* < q_2^* < \cdots < q_k^*$. Now, given the relationship to the mean square error problem discussed above, if we view θ as the source signal and q_k as the representation points of θ on the quantization intervals $A_k = [\theta_{k-1}, \theta_k)$, then the solution to the social welfare maximizing contract is given by the n-level quantization problem, where both the quantization intervals A_k and the corresponding representation points q_k are chosen to minimize the mean square error (MSE):

$$MSE_n \equiv \min_{q(\theta)} \mathbb{E}_\theta \left[(\theta - q)^2\right], \quad \text{subject to } \{q(\theta)\}_{\theta=0}^1 \in \mathcal{M}_n. \tag{3}$$

Hence, the optimal solution must satisfy the Lloyd-Max optimality conditions, see Lloyd (1982) and Max (1960).

Proposition 1 (Lloyd-Max-Conditions) *The optimal menu M_n^* of the social welfare problem (2) satisfies:*

$$\theta_k^* = \frac{1}{2}\left(q_k^* + q_{k+1}^*\right), \qquad q_k^* = \mathbb{E}_\theta\left[\theta | \theta \in \left[\theta_{k-1}^*, \theta_k^*\right)\right], \qquad k = 0, \ldots, n. \tag{4}$$

That is, q_k^*, the production level for the interval $A_k^* = \left[\theta_{k-1}^*, \theta_k^*\right)$, must be the conditional mean for θ given that θ falls in the interval A_k^* and θ_k^*, which separates two neighboring intervals A_k^* and A_{k+1}^*, must be the arithmetic average between q_k^* and q_{k+1}^*. One can observe immediately that q_k^* is actually determined by the first-order condition with respect to (3) because MSE_n in (3) is convex in q_k when taking θ_k and θ_{k+1} as given. Similarly, θ_k^* is determined by the first-order condition when q_k and q_{k+1} are given because MSE_n in (3) is convex in θ_k when taking q_k and q_{k+1} as given. For certain family of distributions (e.g., uniform distribution and some discrete distributions) we can obtain closed-form solutions from the Lloyd-Max optimality conditions. We are interested in the relative performance of finite contracts and evaluate the difference between SW_∞^* and SW_n^*.

Definition 1 *Given any $F \in \Delta$, the welfare loss of an n-class contract compared with the optimal continuous contract is defined by $L(F;n) \equiv SW_\infty^* - SW_n^*$.*

It is easy to see that the lower bound over all densities is zero, i.e. $\inf_{F\in\Delta} L(F;n) = 0$. This can be achieved by a categorical distribution, i.e., $\Pr\left(\theta = \frac{k}{n}\right) = \frac{1}{n}$ for $k = 1, ..., n$. Our main task is to provide an upper bound over all distributions, i.e., the worst-case scenario from the point of view of total social welfare.

Definition 2 *The maximum welfare loss of an n-class contract over all $F \in \Delta$ is given by $L(n) \equiv \sup_{F\in\Delta} L(F;n)$.*

We first consider a simple example, and show in detail how to use the Lloyd-Max conditions to obtain the optimal discretized contract and measure the resulting welfare loss.

Example 1 *Suppose that θ is uniformly distributed over $[0,1]$. The optimization problem (2) has a unique optimal solution given by $\theta_k^* = \frac{k}{n}$, $\quad q_k^* = \frac{(k-1/2}{n}$, $k = 0, 1, \ldots, n$. The expected social welfare is $SW_n^* = \frac{1}{6} - \frac{1}{24n^2}$ and the welfare loss is $SW_\infty^* - SW_n^* = \frac{1}{24n^2}$.*

In this example, the cutoff points are uniformly distributed, which is due to the fact that the underlying distribution of θ is uniform. In addition, the convergence rate of the welfare loss induced by discretized contracts is of the order $1/n^2$. Next, we provide a general estimate of the convergence rate of the welfare loss induced by discretized contracts as the number of classes tends to infinity. A direct approach to calculate the welfare loss for general distributions would require the explicit form of the optimal quantizer, determined by the Lloyd-Max conditions. But an explicit characterization of the optimal quantizer is not known, and thus we pursue an indirect approach to obtain a bound through a series of suboptimal quantizers. For any given $F \in \Delta$, we have:

$$SW_n = \mathbb{E}_\theta \left[\theta q - \frac{1}{2}q^2\right] = \frac{1}{2}\mathbb{E}\left(\theta^2\right) - \frac{1}{2}MSE_n,$$

and since the social welfare with the continuous contract is $SW_\infty = \frac{1}{2}\mathbb{E}\left(\theta^2\right)$, we obtain

$$SW_\infty - SW_n = \frac{1}{2}MSE_n. \tag{5}$$

Given the necessary conditions of Proposition 1, it will suffice to confine our attention to the set of finite menus $\mathcal{M}_n^*$ with the property that, given a distribution $F \in \Delta$, the menu $M_n = \{q_k\}_{k=1}^n$ can be generated by a finite partition A_k through $q_k = \mathbb{E}\left(\theta | \theta \in A_k\right),\ k = 1, \ldots, n$, so that $\mathcal{M}_n^*$ is the feasible set of menus $\mathcal{M}_n$ consistent with the optimality condition (4). For any $M_n \in \mathcal{M}_n^*$,

$$MSE_n = \mathbb{E}_\theta\left[(q-\theta)^2\right] = \sum_{k=1}^{n}\left(F\left(\theta_k\right) - F\left(\theta_{k-1}\right)\right) var\left(\theta | \theta \in A_k\right). \tag{6}$$

We can write $L\left(F; n\right)$ and $L\left(n\right)$, using (5) and (6) as follows:

$$L\left(F;n\right) = \inf_{M_n \in \mathcal{M}_n^*}\left[SW_\infty^* - SW_n\right] = \inf_{M_n \in \mathcal{M}_n^*}\frac{1}{2}\sum_{k=1}^{n}\left(F\left(\theta_k\right) - F\left(\theta_{k-1}\right)\right) var\left(\theta|\theta \in A_k\right), \tag{7}$$

and consequently:

$$L\left(n\right) = \sup_{F \in \Delta}\inf_{M_n \in \mathcal{M}_n^*}\frac{1}{2}\sum_{k=1}^{n}\left(F\left(\theta_k\right) - F\left(\theta_{k-1}\right)\right) var\left(\theta|\theta \in A_k\right). \tag{8}$$

It is then central to estimate the variance of θ conditional on the interval A_k to provide an upper bound on $L\left(n\right)$.

Proposition 2 *For $F \in \Delta$, and any $n \geq 1$, $L\left(F;n\right) \leq \frac{1}{8n^2}$.*

Proof. For any given $F \in \Delta$, let M_n be defined by $\theta'_k = k/n$, $q'_k = \mathbb{E}\left[\left(\theta|\theta \in \left[\theta_{k-1}, \theta_k\right)\right)\right]$, $k = 0, 1, \ldots, n$. Now, we have $L\left(F;n\right)$ given by:

$$\inf_{M_n \in \mathcal{M}_n^*}\frac{1}{2}\sum_{k=1}^{n}\left(F\left(\theta_k\right) - F\left(\theta_{k-1}\right)\right) var\left(\theta|\theta \in \left[\theta_{k-1}, \theta_k\right)\right) \leq \frac{1}{2}\sum_{k=1}^{n}\left(F\left(\theta'_k\right) - F\left(\theta'_{k-1}\right)\right) var\left(\theta|\theta \in \left[\theta'_{k-1}, \theta'_k\right)\right).$$

But the variance in any interval is bounded by the following elementary inequality:

$$var\left(\theta|\theta \in \left[\theta'_{k-1}, \theta'_k\right)\right) \leq \frac{1}{4}\left(\theta'_k - \theta'_{k-1}\right)^2 = \frac{1}{4n^2}.$$

It then follows that:

$$L(F;n) \leq \frac{1}{8n^2}\sum_{k=1}^{n}\left(F\left(\theta'_k\right) - F\left(\theta'_{k-1}\right)\right) = \frac{1}{8n^2},$$

which concludes the proof. ■

By considering the uniform distribution of Example 1, we can in fact show that the maximum welfare loss is bounded both above and below by $1/n^2$ (up to some constant).

Proposition 3 *For any* $n \geq 1$, $\frac{1}{24n^2} \leq L(n) \leq \frac{1}{8n^2}$, *i.e.* $L(n) = \Theta\left(\frac{1}{n^2}\right)$.

Similar to us, Wilson (1989) establishes that a finite priority ranking of order n induces a welfare loss of order $1/n^2$. His method of proof is different from ours, in that it does not use quantization explicitly, and in that for the limit results he proposes uniform quantization of the relevant distribution.

4 Revenue Maximization

We now analyze the problem of revenue maximization with limited information. In contrast to the social welfare maximizing problem, the seller wishes to maximizes his expected net revenue. The expected net revenue is the difference between the gross revenue that he receives from the buyer minus the cost of providing the demanded quantity. The contract offered by the principal now has to satisfy two sets of constraints, namely the participation constraint, $\theta q(\theta) - t(\theta) \geq 0$, for all $\theta \in [0,1]$, and the incentive constraints: $\theta q(\theta) - t(\theta) \geq \theta q\left(\theta'\right) - t\left(\theta'\right)$, of the buyer for all $\theta, \theta' \in [0,1]$. The participation constraint guarantees that the buyer receives a nonnegative net utility from his choice, and the incentive constraints account for the fact that the type θ is private information to the buyer, and hence the revelation of the information is required to be incentive compatible. The current problem is then identical to the seminal analysis by Mussa and Rosen (1978) and Maskin and Riley (1984) with one important exception: the buyer can only access a finite number of choices due to the limited communication with the seller. Now, a menu of quantity-price bundles is designed by the monopolistic seller to extract as much profit as possible.

The revenue maximization problem, finding the optimal solution for the allocation $q(\theta)$ and the transfer $t(\theta)$ simultaneously, then appears to be rather distinct from the welfare maximization problem, which only involved the allocation $q(\theta)$. However, we can use the above incentive constraints to eliminate the transfers and rewrite the problem in terms of the allocation alone. This insight appeared prominently in the analysis of revenue maximizing auction in Myerson (1979). He showed that the revenue maximization problem can be transformed into a welfare maximization problem (without incentive constraints) as long as we replace the *true valuation* θ of the buyer with the corresponding *virtual valuation*:

$$\hat{\theta} \equiv \psi(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}. \tag{9}$$

The virtual valuation is always below the true valuation, and the inverse of the hazard rate $(1-F(\theta))/f(\theta)$ accounts for the information rent, the cost of the private information, as perceived by the principal in the optimal mechanism. We shall follow Myerson (1979) and impose the regularity condition that $\psi(\theta)$ is strictly increasing in θ. With this standard transformation of the problem, the expected profit of the seller (without information constraints) is:

$$\Pi_\infty^* = \max_{q(\theta)} \mathbb{E}_\theta \left[q(\theta)\psi(\theta) - \frac{1}{2}q^2(\theta) \right]. \tag{10}$$

The resulting optimal contract exhibits $q^*(\theta) = \max\{\psi(\theta), 0\}$. Now, in the world with limited information, the seller can only offer a finite menu $\{(q_k, t_k), k = 1, \ldots, n\}$ to the buyer. After rewriting the revenue maximizing problem in terms of the virtual utility, we can omit the dependence on the transfers and rewrite the problem in terms of a choice over a finite set of allocations $\mathcal{M}_n$:

$$\Pi_n^* = \max_{q(\theta)\in\mathcal{M}_n} \mathbb{E}_\theta \left[q\psi(\theta) - \frac{1}{2}q^2 \right]. \tag{11}$$

We denote the distribution function and density function of $\hat{\theta}$ by G and g, respectively. We have $F(x) = \Pr(\theta \le x) = \Pr(\hat{\theta} \le \psi(x)) = G(\psi(x))$, and thus $f(x) = g(\psi(x))\psi'(x)$. Using the insights of the previous section, we observe that maximizing the seller's revenue is equivalent to minimizing the mean square error $\mathbb{E}_{\hat{\theta}}[(\hat{\theta}-q)^2]$, where the expectation is taking with respect to the new random variable $\widehat{\theta}$. We then appeal to the appropriately modified Lloyd-Max optimality conditions to characterize the revenue maximizing contract in the presence of information constraints:

Proposition 4 *The revenue maximizing solution (11) satisfies:*

$$\theta_k^* - \frac{1 - F(\theta_k^*)}{f(\theta_k^*)} = \frac{1}{2}\left(q_k^* + q_{k+1}^*\right) \qquad k = 0, \ldots, n-1, \tag{12}$$

and

$$q_k^* = \frac{\theta_{k-1}^*\left(1 - F\left(\theta_{k-1}^*\right)\right) - \theta_k^*\left(1 - F(\theta_k^*)\right)}{F(\theta_k^*) - F\left(\theta_{k-1}^*\right)} \qquad k = 1, \ldots, n. \tag{13}$$

Similar to the social welfare problem, we wish to evaluate the upper bound of $\Pi_\infty^* - \Pi_n^*$ across all possible distribution functions $F \in \Delta$. To this end, we define the revenue loss induced by an n-class contract compared with the optimal continuous contract, given a distribution $F \in \Delta$, as $\Lambda(F;n) \equiv \Pi_\infty^* - \Pi_n^*$ and the maximum revenue loss induced by an n-class contract across all $F \in \Delta$ as $\Lambda(n) \equiv \sup_{F \in \Delta} \Lambda(F;n)$. The example of the uniform distribution is again illustrative before turning to the general analysis.

Example 2 *Suppose that θ is uniformly distributed over $[0,1]$. The optimization problem (11) has a unique solution: $\theta_k^* = \frac{n+k+1}{2n+1}$, $\quad q_k^* = \frac{2k}{2n+1}$, $\quad k = 0, \ldots, n$. The maximum expected revenue is $\Pi_n^* = \frac{n(n+1)}{3(2n+1)^2}$ and revenue loss is $\Pi_\infty^* - \Pi_n^* = \frac{1}{12(2n+1)^2}$.*

It follows that the convergence rate of the revenue loss induced by discretized contracts is also of the order $1/n^2$. We find that the seller tends to serve fewer buyers as compared to the case when a continuous contract is used. This property holds for general distributions as the seller's ability of extracting revenue is limited. To compensate, the seller reduces the service coverage to pursue higher marginal revenues. We now provide the convergence rate of the revenue loss induced by discretized contracts as the number of intervals (classes) tends to infinity. Thus,

$$\Pi_\infty^* - \Pi_n = \frac{1}{2}\left[\int_0^{\hat{\theta}_0} \hat{\theta}^2 g\left(\hat{\theta}\right) d\hat{\theta} + \int_{\hat{\theta}_0}^1 \left(\hat{\theta} - q\right)^2 g\left(\hat{\theta}\right) d\hat{\theta}\right]. \tag{14}$$

The first term in the square bracket captures the revenue loss by reducing the service coverage. The second term in the square bracket and $L(F;n)$ in (7) are very much alike. One can immediately get this term by replacing θ by $\hat{\theta}$ and F by G in $L(F;n)$. We can then adapt Proposition 2 to the current environment.

Proposition 5 *For any $F \in \Delta$, and any $n \geq 1$, $\Lambda(f; n) \leq 1/8n^2$.*

The approximation result of the revenue maximizing problem is similar to the one of the social welfare program. Likewise, we can use the above uniform example to establish a lower bound for the revenue losses.

Proposition 6 *For any $n \geq 1$, $1/12\,(2n+1)^2 \leq \Lambda(n) \leq 1/8n^2$, and hence $\Lambda(n) = \Theta(1/n^2)$.*

5 Conclusion

We analyzed the role of limited information (or communication) in the context of the canonical nonlinear pricing environment. By focusing on the simple linear-quadratic specification of the utility and cost function, we were able to relate the limited information problem directly to the quantization problem in information theory. This allowed us to explicitly derive the optimal mechanism, both from a social efficiency as well as from a revenue-maximizing point of view. In either case, our analysis established that the worst welfare loss due to the limits of information, imposed by an n-class contract, is of the order of $1/n^2$.

While the nonlinear pricing environment is of interest by itself, it also represents an elementary instance of the general mechanism design environment. The simplicity of the nonlinear pricing problem arises from the fact that it can viewed as a relationship between the principal, here the seller, and a single agent, here the buyer, even in the presence of many buyers. The reason for the simplicity is that the principal does not have to solve allocative externalities. By contrast, in auctions, and other multi-agent allocation problems, the allocation (and hence the relevant information) with respect to a given agent constrains and is constrained by the allocation to the other agents. For an information-theoretic point of view, the ensuing multi-dimensionality would suggest that the methods of vector quantization rather than the scalar quantization employed here, would become relevant.

Finally, the current analysis focused on limited information, and the ensuing problem of efficient source coding. But clearly, from an information-theoretic as well as economic viewpoint, it is natural to augment the analysis to reliable communication between agent and principal over noisy channels, the problem of channel coding, which we plan to address in future work.

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