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Macroeconomic Interdependence in a Two-Country DSGE Model under Diverging Interest-Rate Rules

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Abstract: The present article extends a variant of the Obstfeld/Rogoff (2001) two-country DSGE model by introducing Calvo (1983) pricing. It is possible to collapse the model into a canonical log-linear representation consisting of two dynamic IS and two New Keynesian Phillips curves. Reflecting the differing statutes of the ECB and the Fed, two diverging interest-rate rules are introduced. For a sensible calibration of the model we can derive a locally unique rational expectations equilibrium. Furthermore, we find that aggregate productivity shocks, which are assumed to be positively correlated across countries, have a negative impact on domestic and foreign output, a phenomenon already described for the closed economy by Galí (2002). Cost-push as well as contractionary monetary policy shocks, which are assumed to be country-specific, also have a negative impact on domestic and foreign output since the economies are interdependent due to terms-of-trade externalities. Contrary to Corsetti/Pesenti (2001), expansionary monetary policy shocks always have a "prosper thyself" and "beggar thy neighbor" effect since they influence the terms of trade beneficially for the respective country's resident households. Finally, if the ECB implemented the interest-rate rule proposed in the present article, it would encounter lower fluctuations in European producer price inflation compared to an interest-rate rule as proposed for the Fed. This is consistent with the ECB's paramount objective of price stability. However, this advantage only holds at the expense of relatively high fluctuations in the European output gap.

Keywords: two-country DSGE model, diverging interest-rate rules, equilibrium determinacy, calibration, impulse-response analysis.

JEL classification codes: E12 E52 E58 F41 F42 F47

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1. Introduction

If one studies scholarly articles that deal with macroeconomic models of two countries such as, e.g., Corsetti/Pesenti (2001), Obstfeld/Rogoff (2001), Clarida et al. (2002), Pappa (2004), or Benigno/Benigno (2006), one usually encounters that the countries' monetary authorities, if explicitly modeled at all, are modeled as perfectly symmetric institutions.

This gives rise to the question to which extent these models are able to capture real-world features and if policy recommendations based on these models' results are applicable. The reason why this is questionable is that, in general, two different central banks might each obey a differing and legally binding statute. Particularly, let us think of the two monetary authorities under examination as the European Central Bank (ECB) on the one hand and the Federal Reserve System (Fed) on the other.

Article 2 of the Protocol on the Statute of the European System of Central Banks and of the European Central Bank (1992, 2004) states the following:[2]

> *"In accordance with Article 105(1) of this Treaty, the primary objective of the ESCB shall be to maintain price stability. Without prejudice to the objective of price stability, it shall support the general economic policies in the Community with a view to contributing to the achievement of the objectives of the Community as laid down in Article 2 of this Treaty. The ESCB shall act in accordance with the principle of an open market economy with free competition, favouring an efficient allocation of resources, and in compliance with the principles set out in Article 4 of this Treaty."*

However, Section 2a of the Federal Reserve Act (1977, 2000) reads:

> *"The Board of Governors of the Federal Reserve System and the Federal Open Market Committee shall maintain long run growth of the monetary and credit aggregates commensurate with the economy's long run potential to increase production, so as to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates."*

As one can conclude from these diverging statutes, the paramount objective of the ECB is price stability, whereas for the Fed this goal is just one out of many. Therefore, in order to model monetary policy of each central bank consistent with their diverging statutes, one should try and incorporate these institutional features into their respective policy functions.

The analysis shall be carried out by introducing diverging interest-rate rules into a log-linear representation of a variant of the dynamic stochastic general equilibrium (DSGE) framework by Obstfeld/Rogoff (2001), which is extended by Calvo (1983) pricing, a more subtle form of nominal rigidities than the one used in the original article. The reasoning of some parts of Obstfeld/Rogoff (2001) itself is based on Corsetti/Pesenti (2001).

At first sight, the DSGE framework might mislead the reader to the Real Business Cycle (RBC) literature. Although the structure of the subsequent model is somewhat similar to RBC models, introducing the assumptions of [1] monopolistic competition on goods markets and [2] some form of nominal rigidity into the set-up will rebut this preliminary speculation. In contrast to the typical RBC model, monetary policy will not be neutral in the short run because of these two departing assumptions (see Galí 2008, pp. 4-5). This is the basic reason why the present model class is usually referred to as "New Keynesian".

The main results of a calibrated version of the model under scrutiny, for which a determinate rational expectations equilibriums exists, are summarized in the following.

- Simulated aggregate productivity shocks, which are assumed to be positively correlated across countries, have a negative impact on domestic and foreign output, a result already described for the closed economy by Galí (2002). The positive correlation of these shocks can be interpreted as exogenous R&D spill-over effects associated with "technology sourcing" as laid out by Griffith et

[2] More precisely, the responsible body for the monetary policy of the EU is the European System of Central Banks (ESCB), which comprises the ECB and the national central banks of all 27 EU member states (in 2009).

al. (2006).

- Simulated cost-push as well as monetary policy shocks, which are assumed to be country-specific, also have a negative impact on domestic and foreign output since both economies are interdependent due to terms-of-trade externalities.

- In contrast to CORSETTI/PESENTI (2001), expansionary monetary policy shocks always have a "prosper thyself" and "beggar thy neighbor" effect since they influence the terms of trade beneficially for the home (foreign) country's resident households by decreasing them below (raising them above) their zero-inflation steady-state value. In addition, this effect would induce a rise of both domestic and foreign output above their flexible-price values.

- If the ECB implemented the interest-rate rule proposed in the present article, it would encounter lower fluctuations in European producer price inflation compared to an interest-rate rule as proposed for the Fed. This is consistent with the ECB's paramount objective of price stability. However, this advantage only holds at the expense of relatively high fluctuations in the European output gap; a trade-off commonly observed in literature on monetary policy.

The remainder of this article is structured as follows: Section 2 gives a short review of current literature on the topic, Section 3 outlines the basic discrete-time, two-country DSGE model, Section 4 presents the equilibrium conditions on all markets under flexible prices, Section 5 introduces the New Keynesian framework, and Section 6 derives a locally unique rational expectations equilibrium for a calibrated version of the model. The analysis is completed by an impulse-response analysis in Section 7. Finally, Section 8 concludes. Lengthy derivations are given in respective appendices.

2. Related Literature

In general, the present article can be embedded in the so-called New Open Economy Macroeconomic literature pioneered by OBSTFELD/ROGOFF (1995), which has been enriching the traditional Open Economy Macroeconomic literature by its rigorous microeconomic foundation. There has been done a lot of research on this topic so far and it seems to continue to appeal to a wide range of scholars. There are quite numerous survey articles and survey book chapters on this issue, e.g., OBSTFELD/ROGOFF (1996, chapter 10), LANE (2001), ENGEL (2002), WALSH (2003, chapter 6), or GALÍ (2008, chapter 7). Of course, this parsimonious overview remains incomprehensive.

In particular, the present paper deals with a *two-country* DSGE model such that it is noteworthy to briefly summarize the content of similar articles besides OBSTFELD/ROGOFF (2001) and CORSETTI/PESENTI (2001) such as, e.g., CLARIDA et al. (2002), PAPPA (2004), or BENIGNO/BENIGNO (2006).

- CLARIDA et al. (2002) investigate whether there are gains from cooperation between two monetary authorities in case of discretionary monetary policy. The authors find that in case of non-cooperation the structure of the policy problem is isomorphic to the closed-economy case. Unless there is logarithmic utility of consumption, there are gains from monetary policy cooperation as the optimizing monetary authorities are willing to internalize possible spill-over effects from the terms of trade.

- PAPPA (2004) finds that for the case of central banks that are committed to their policy rules there are no gains from cooperation for the special parameter constellation, for which the cooperative and non-cooperative regimes coincide, since in this case independent central banks would not face any spill-over effects from the terms of trade. For this parameter constellation a monetary union is clearly suboptimal as the supranational authority cannot replicate the first-best allocation when the nominal exchange rate is fixed. For any other parameter constellations, however, there may be welfare losses from deviating from the cooperation benchmark.

- Benigno/Benigno (2006) contribute to the literature as they manage to show that it is possible to design "specific targeting rules" for non-cooperating central banks, which have the property to assign the incentive to independent central banks to replicate the cooperative allocation such that possible welfare losses from non-cooperation can be avoided.

There are also numerous articles dealing with *small open economy* DSGE models such as, e.g., Clarida et al. (2001) or Galí/Monacelli (2005), where the model of the latter coincides with the one discussed in Galí (2008, chapter 7). Nonetheless, some of their features are useful for two-country frameworks, too, such that these articles shall also be briefly summarized.

- Clarida et al. (2001) show that the log-linear representation of a small open economy is isomorphic to a closed economy since all structural equations of the small open economy are identical to their closed-economy counterparts, except that they are related to the terms of trade. As a result, the policy problem of the small open economy is isomorphic to the policy problem of the closed economy. Moreover, also the optimal monetary policies under discretion as well as under commitment are analogous to the closed-economy case, but they affect the terms of trade.

- Galí/Monacelli (2005) explore the size of welfare losses of suboptimal monetary policies compared to the benchmark case of optimal monetary policy, which is associated with the complete stabilization of the output gap and producer prices. The suboptimal monetary policies under scrutiny are a (stylized) producer price inflation-based Taylor rule, a (stylized) consumer price inflation-based Taylor rule and a credible peg of the nominal exchange rate. The authors find that the producer price inflation-based Taylor rule features the lowest welfare losses, followed by the consumer price inflation-based Taylor rule and the peg of the nominal exchange rate.

In Galí (2008, chapter 8) one can find possible extensions to the basic, closed-economy New Keynesian framework, which might also be intriguing to open economy researchers.

3. New Open Economy Macroeconomic Model

The subsequent model is based on the Obstfeld/Rogoff (2001) two-country DSGE framework, which extends the basic Obstfeld/Rogoff (1995) model by introducing uncertainty.

3.1. Preferences, Consumption and Price Indexes

Suppose world population is constant over time and consists of a continuum of measure 1 of infinitely lived atomistic households characterized by identical preferences. Assume further perfect information and rational expectations on part of all agents. There are two countries, where domestic households live on the segment $[0, n]$ of the unit interval while foreign households live on the remaining segment $(n, 1]$.

The discounted stream of expected period utilities of the representative domestic household reads as follows:[3]

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\rho}}{1-\rho} + \frac{\chi}{1-\varepsilon} \left(\frac{M_s}{P_s} \right)^{1-\varepsilon} - \frac{\gamma}{1-\xi} L_s^{1-\xi} \right] \right\}. \tag{1}$$

The above utility function is a constant elasticity of substitution (CES) composite separable in its arguments real consumption C, real money balances M/P (where P denotes the domestic consumer price index (CPI)), and leisure $-L$ such that the partial derivatives of the utility function with respect to one variable are independent of all other variables. β denotes an intertemporal discount factor ($0 < \beta < 1$).

[3]Note that a possible superscript i to distinguish individual variables is suppressed throughput the analysis for legibility reasons.

Moreover, the following shall hold for the various parameters: $\chi, \gamma > 0$, $0 < \rho, \varepsilon < 1$, and $\xi < 0$.[4]

Since (1) is a function in real money balances, the model is a variant of the SIDRAUSKI (1967) and BROCK (1974) money-in-the-utility-function (MIU) models, in which putting real money balances into the utility function is justified by assuming that the use of money facilitates transactions. This modeling shortcut guarantees the usage of money even though holding money per se does not yield a positive real return.[5]

The utility function of the representative foreign household is the same as (1), except that C^* may differ from C, as well as M^* from M, P^* from P, χ^* from χ, γ^* from γ, and L^* from L.[6]

Moreover, the total domestic consumption index C from above is defined as a population-weighted per-capita Cobb-Douglas composite of domestic and foreign commodity bundles, which implicitly assumes that all consumption goods are tradable and that there are no trading costs:[7]

$$C_t := \frac{C_{t,H}^n C_{t,F}^{1-n}}{n^n (1-n)^{1-n}}. \tag{2}$$

The commodity bundles C_H and C_F are CES composites of differentiated final goods produced at home (C_H) and abroad (C_F) as in DIXIT/STIGLITZ (1977):[8]

$$C_{t,H} := \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

$$C_{t,F} := \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 C_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}. \tag{4}$$

The preference for differentiated goods expresses a love of variety on part of the households. As one can see from (2), the elasticity of substitution between domestic and foreign commodity bundles σ_{C_H,C_F} equals 1 (Cobb-Douglas specification). One gets from (3) and (4) that the elasticity of substitution across

[4]Therefore, we obtain the subsequent first and second partial derivatives of the utility function (1) with respect to the single variables:

$$\begin{aligned}
\frac{\partial U}{\partial C} &= C^{-\rho} > 0, \quad \frac{\partial^2 U}{\partial C^2} = (-\rho) C^{-\rho-1} < 0, \\
\frac{\partial U}{\partial (\frac{M}{P})} &= \chi \left(\frac{M}{P}\right)^{-\varepsilon} > 0, \quad \frac{\partial^2 U}{\partial (\frac{M}{P})^2} = (-\varepsilon)\chi \left(\frac{M}{P}\right)^{-\varepsilon-1} < 0, \\
\frac{\partial U}{\partial (-L)} &= \gamma L^{-\xi} > 0, \quad \frac{\partial^2 U}{\partial (-L)^2} = \xi \gamma L^{-\xi-1} < 0.
\end{aligned}$$

1 minus each of these parameters represents the elasticity of the partial utility function in one of the three arguments, denoted by the respective subscript, with respect to this very argument:

$$\begin{aligned}
\epsilon_{U_C,C} &:= \frac{\partial U}{\partial C}\frac{C}{U_C} = C^{-\rho}\frac{C}{\frac{C^{1-\rho}}{1-\rho}} = 1-\rho, \\
\epsilon_{U_{\frac{M}{P}},\frac{M}{P}} &:= \frac{\partial U}{\partial (\frac{M}{P})}\frac{\frac{M}{P}}{U_{\frac{M}{P}}} = \chi\left(\frac{M}{P}\right)^{-\varepsilon}\frac{\frac{M}{P}}{\frac{\chi}{1-\varepsilon}\left(\frac{M}{P}\right)^{1-\varepsilon}} = 1-\varepsilon, \\
\epsilon_{U_{(-L)},(-L)} &:= \frac{\partial U}{\partial (-L)}\frac{(-L)}{U_{(-L)}} = \gamma L^{-\xi}\frac{L}{\frac{\gamma}{1-\xi}L^{1-\xi}} = 1-\xi.
\end{aligned}$$

[5]Note, however, that some New Open Economy Macroeconomic models abstract from explicitly modeling liquidity services provided by the use of money (see, e.g., CLARIDA et al. 2002, p. 882). Note further that domestic households are assumed to derive utility from holding domestic money only, whereas foreign households are assumed to derive utility from using foreign money only.

[6]As one can see here, real foreign variables are denoted by a superscript asterisk. The same holds for nominal foreign variables in foreign currency, except for nominal internationally traded bonds, which will be discussed in more detail below.

[7]Hence, there is no source for the Harrod-Balassa-Samuelson effect as described in OBSTFELD/ROGOFF (1996, pp. 210-216). Furthermore, the total domestic consumption index (2) is population-weighted for the CPI (5) below to have the usual form rather than a form such as, e.g., in CLARIDA et al. (2002, p. 882).

[8]Alternatively, one could treat imported goods as production factors rather than consumption goods as in MCCALLUM/NELSON (2001). This formulation shall not be adopted, however.

two individual goods z, z' produced within a country $\sigma_{C(z),C(z')}$ equals θ (CES specification, $\theta > 1$ for an equilibrium to exist).[9]

The total domestic CPI is again a Cobb-Douglas composite of domestic and foreign producer price indexes (PPIs):

$$P_t = P_{t,H}^{n} P_{t,F}^{1-n}, \tag{5}$$

whereupon these subindexes are CES composites of domestic and foreign final goods prices:

$$P_{t,H} = \left[\frac{1}{n}\int_0^n P_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}, \tag{6}$$

$$P_{t,F} = \left[\frac{1}{1-n}\int_n^1 P_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}. \tag{7}$$

For a derivation of the domestic CPI (5), the domestic PPI for domestic goods (6) as well as the domestic demand functions for individual and composite domestic goods, which will be introduced below, see Appendix A.1. The domestic PPI for foreign goods (7), domestic demand curves for individual and composite foreign goods as well as all foreign indexes can be derived analogously.

Assume that the law of one price holds for consumers across all individual goods at all times:

$$P_t(z) = S_t P_t^*(z) \forall z \in [0,1], \tag{8}$$

where S denotes the endogenously determined nominal exchange rate in price quotation (domestic currency units in terms of foreign currency units).

Thus, as domestic and foreign households are characterized by identical preferences, the law of one price implies that absolute purchasing power parity (PPP) always holds for the total CPI, even if relative PPP (stating that *changes* in domestic and foreign price levels should be equal in the long run) would be the more realistic statement (see OBSTFELD/ROGOFF 1996, pp. 200-202):

$$P_t = S_t P_t^*. \tag{9}$$

The demand functions of the representative domestic household for individual domestic $C(h)$ and foreign goods $C(f)$ read as follows:

$$C_t(h) = \frac{1}{n}\left[\frac{P_t(h)}{P_{t,H}}\right]^{-\theta} C_{t,H}, \tag{10}$$

$$C_t(f) = \frac{1}{1-n}\left[\frac{P_t(f)}{P_{t,F}}\right]^{-\theta} C_{t,F}, \tag{11}$$

where $z = h \in [0,n]$ denotes a typical differentiated good z produced at home and $z' = f \in (n,1]$ another typical differentiated good z' produced abroad.

As one can see from equations (10) and (11), demand for individual goods is decreasing in its own price

[9]

$$|\sigma_{C_H,C_F}| := \left|\frac{\mathrm{d}\left(\frac{C_H}{C_F}\right)}{\mathrm{d}\left(\frac{\mathrm{d}C_H}{\mathrm{d}C_F}\right)} \frac{\frac{\mathrm{d}C_H}{\mathrm{d}C_F}}{\frac{C_H}{C_F}}\right| = 1,$$

$$|\sigma_{C(z),C(z')}| := \left|\frac{\mathrm{d}\left[\frac{C(z)}{C(z')}\right]}{\mathrm{d}\left[\frac{\mathrm{d}C(z)}{\mathrm{d}C(z')}\right]} \frac{\frac{\mathrm{d}C(z)}{\mathrm{d}C(z')}}{\frac{C(z)}{C(z')}}\right| = \theta.$$

relative to the respective domestic or foreign PPI.[10] Note that θ does not only denote the elasticity of substitution between any two individual goods, but also the price elasticity of demand for any individual good faced by each producer.[11] Equation (2) implies that the demand curves for the composite domestic and foreign goods, C_H and C_F, are given by:

$$C_{t,H} = n\left(\frac{P_{t,H}}{P_t}\right)^{-1} C_t, \tag{12}$$

$$C_{t,F} = (1-n)\left(\frac{P_{t,F}}{P_t}\right)^{-1} C_t. \tag{13}$$

Now we should make use of the fact that world consumption C^w equals the population weighted sum of total domestic and total foreign consumption, where C^w then denotes per capita as well as total world consumption as world population is normalized to 1:

$$C_t^w := nC_t + (1-n)C_t^*. \tag{14}$$

Combining (14) with equations (8), (10), (11), (12), and (13) one finally obtains the *global* demand functions for individual domestic and foreign goods in terms of (total) world consumption:[12]

$$C_t^w(h) = \left[\frac{P_t(h)}{P_{t,H}}\right]^{-\theta}\left(\frac{P_{t,H}}{P_t}\right)^{-1} C_t^w, \tag{15}$$

$$C_t^w(f) = \left[\frac{P_t(f)}{P_{t,F}}\right]^{-\theta}\left(\frac{P_{t,F}}{P_t}\right)^{-1} C_t^w. \tag{16}$$

3.2. Households

The representative domestic household maximizes its objective functional (1) subject to the following sequence of intertemporal budget constraints (in nominal terms) with respect to the decision variables C_t, M_t, B_t, and L_t:

$$W_tL_t + (1+i_{t-1})B_{t-1} + M_{t-1} + \Gamma_t(h) \geq P_tC_t + M_t + B_t + P_t\tau_t. \tag{17}$$

As an example for a typical flow budget constraint, inequality (17) states that the household's period t expenditure must not exceed period t income.[13] W denotes the endogenously determined nominal wage being the remuneration for supplying labor, which is identical across households ($L = L(h)$) on the assumed to be perfectly competitive labor market, an assumption differing from Clarida et al. (2002). i_{t-1} denotes the nominal interest rate between period $t-1$ and period t on riskless one-period non-government bonds B_{t-1} carried over from period $t-1$. These nominal bonds are denominated in

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10

$$\frac{\partial C(h)}{\partial P(h)} = (-\theta)\frac{1}{n}\left[\frac{P(h)}{P_H}\right]^{-\theta-1}\frac{C_H}{P_H} < 0.$$

11

$$\epsilon_{C(h),P(h)} := \frac{\partial C(h)}{\partial P(h)}\frac{P(h)}{C(h)} = (-\theta)\frac{1}{n}\left[\frac{P(h)}{P_H}\right]^{-\theta-1}\frac{C_H}{P_H}P(h)n\left[\frac{P(h)}{P_H}\right]^{\theta}C_H^{-1} = -\theta.$$

12

$$C_t^w(h) = nC_t(h) + (1-n)C_t^*(h) = \left[\frac{P_t(h)}{P_{t,H}}\right]^{-\theta}\left(\frac{P_{t,H}}{P_t}\right)^{-1}[nC_t + (1-n)C_t^*] = \left[\frac{P_t(h)}{P_{t,H}}\right]^{-\theta}\left(\frac{P_{t,H}}{P_t}\right)^{-1}C_t^w.$$

13

$$P_tC_t = P_{t,H}C_{t,H} + P_{t,F}C_{t,F} = \int_0^n P_t(h)C_t(h)dh + \int_n^1 P_t(f)C_t(f)df.$$

domestic currency and are supposed to be internationally tradable.[14]

Money holdings M_{t-1} can also be transferred from $t-1$ to t but yield no nominal return. Consumption goods, however, are perishable and cannot be stored. $\Gamma_t(h)$ are instantaneous profits of the representative household acting as a producer of an individual, differentiated domestic good h, which will be explained in more detail below. Finally, let τ denote non-distortionary real lump-sum taxes.

Again, for the representative foreign household the intertemporal budget constraint is the same as (17). Since internationally traded bonds are supposed to be denominated in domestic currency, foreign bond holdings in domestic currency B^* first have to be divided by the nominal exchange rate before they enter the foreign intertemporal budget constraint: B^*/S. Moreover, W^* may differ from W, i^* from i, $\Gamma^*(f)$ from $\Gamma(h)$, as well as τ^* from τ.

The maximization of the utility function (1) subject to the budget constraint (17) then holding with equality is undertaken by maximizing the corresponding Lagrangian and yields the subsequent first order conditions for a utility maximum:

$$\frac{C_t^{-\rho}}{P_t} = \beta(1+i_t)E_t\left[\frac{C_{t+1}^{-\rho}}{P_{t+1}}\right]. \tag{18}$$

This is the intertemporal Euler equation for total real consumption stating that the marginal rate of substitution between total real consumption in t and in $t+1$ equals their discounted relative prices.

Moreover, one obtains that in a utility maximum the marginal rate of substitution between real money balances and total real consumption equals the opportunity costs of holding money:

$$\chi\frac{\left(\frac{M_t}{P_t}\right)^{-\varepsilon}}{C_t^{-\rho}} = \frac{i_t}{1+i_t}. \tag{19}$$

Note that equation (19) can be rearranged in order to get the following money demand equation:

$$\left(\frac{M_t}{P_t}\right)^{\varepsilon} = \chi\frac{1+i_t}{i_t}C_t^{\rho},$$

where one can see that the higher total real consumption, the higher is demand for real money balances.

Finally, one also gets the subsequent labor supply equation:

$$\gamma\frac{L_t^{-\xi}}{C_t^{-\rho}} = \frac{W_t}{P_t}, \tag{20}$$

which states that the marginal rate of substitution between labor and total real consumption equals their relative prices, the real consumer wage. For a derivation of conditions (18), (19), and (20) see Appendix A.2.

Note that analogous equations to (18), (19), and (20) also hold abroad.

3.3. Firms

Let us assume further that agents at home and abroad do not only act as utility maximizing households, but also as profit maximizing firms of final goods, which shall be producible without the input of intermediate goods. In contrast to their role as households whose preferences are assumed to be identical, all commodities are differentiated goods satisfying the households' love of variety.

Hence, there is the possibility to raise individual goods' prices, $P(h), P(f)$, above marginal cost without

[14]Note that one could generalize this formulation by assuming that domestic and foreign households had access to a complete portfolio of state-contingent Arrow-Debreu securities, both domestically and internationally tradable, as in Clarida et al. (2002) in order to guarantee for the completeness of (international) financial markets.

the risk of dropping out of the market. In other words, non-zero profits are feasible in this model of monopolistic competition.

Let individual domestic output be produced according to the following linear production function:

$$Y_t(h) = A_t L_t(h). \tag{21}$$

This is a production function in labor only. For the sake of simplicity, real capital shall be omitted as additional input factor throughout the analysis. This step can be justified by the short- to medium-run perspective of the model. A shall be a random variable denoting an exogenous aggregate productivity shock, interpretable as a transitory process innovation. This productivity shock shall be positively correlated with its foreign equivalent A^*, whereby the positive correlation may be interpreted as exogenous R&D spill-over effects associated with "technology sourcing" as described in GRIFFITH et al. (2006, pp. 1859-1861). In the present set-up, technology sourcing would then have a mutual nature.

Households need not be self-employed, but it is assumed that domestic firms can employ domestic labor only as well as foreign firms shall be allowed to employ foreign labor only. In other words, there is no migration in this world.

Individual foreign output is produced using the same technology (21) as at home. Nonetheless, $Y^*(f)$ may differ from $Y(h)$, A^* from A, as well as $L^*(f)$ from $L(h)$.

Producers' instantaneous profits $\Gamma_t(h)$, which have already been introduced above, are then given by:

$$\Gamma_t(h) = P_t(h)Y_t(h) - W_t L_t(h). \tag{22}$$

Relative to the producer's own price, equation (22) rearranges to:

$$\frac{\Gamma_t(h)}{P_t(h)} = Y_t(h) - \frac{W_t}{P_t(h)} L_t(h) = Y_t(h) - \frac{W_t}{P_t(h)} \frac{Y_t(h)}{A_t} = Y_t(h) - \kappa_t Y_t(h), \tag{23}$$

where one has made use of the production function (21). In (23) $\kappa := W/[P(h)A]$ is defined as individual real marginal production cost.

For now assume all goods prices to be flexible. Then each domestic producer charges the same price denoted by the domestic PPI ($P_H = P(h)$). Thus, instantaneous profits rearrange to:

$$\Gamma_t(h) = P_{t,H} Y_t(h) - W_t L_t(h). \tag{24}$$

Maximizing equation (24) with respect to $Y(h)$ and using the fact that in case of goods market clearing output of a single producer equals global demand for the differentiated good ($Y(h) = C^w(h)$), we get the standard first order condition for a profit maximum in a model of monopolistic competition:

$$\begin{aligned} \frac{\partial \Gamma_t(h)}{\partial Y_t(h)} &= P_{t,H} + Y_t(h)\frac{\partial P_{t,H}}{\partial Y_t(h)} - W_t \frac{\partial L_t(h)}{\partial Y_t(h)} = P_{t,H}\left(1 + \frac{1}{\epsilon_{C(h),P(h)}}\right) - W_t \frac{1}{A_t} \\ &= P_{t,H}\left(1 + \frac{1}{-\theta}\right) - W_t \frac{1}{A_t} = 0 \\ \Rightarrow \frac{W_t}{P_{t,H} A_t} &= \frac{\theta - 1}{\theta} := \kappa_t^{flex}. \end{aligned} \tag{25}$$

Note that an analogous equation to (25) also holds abroad and that $\kappa^{flex} = (\kappa^*)^{flex} = (\theta - 1)/\theta$.

Equation (25) states that in a profit maximum associated with flexible prices, the corresponding real marginal production cost defined as κ^{flex} equals $(\theta - 1)/\theta$.[15]

[15] Note that if one solved equation (25) for P_H, one would obtain the domestic PPI as a mark-up on marginal unit labor costs W/A: $P_H = [\theta/(\theta - 1)]W/A$ with $\theta/(\theta - 1) = 1/\kappa^{flex}$ denoting the flexible-price mark-up factor.

4. Market Clearing under Flexible Prices

Before introducing nominal rigidities in Section 5, one should first consider the benchmark case of market equilibria in a world with completely flexible prices.

4.1. World Bond and Goods Markets

For the derivation of the subsequent equations and their relation to one another see Appendix A.3.

Begin with the equilibrium conditions on the world markets for domestic and foreign goods denoted in domestic currency:

$$\begin{aligned} P_{t,H}Y_t &= P_t C_t^w, && (26) \\ P_{t,F}Y_t^* &= P_t C_t^w, && (27) \end{aligned}$$

where the left-hand side of equation (26) denotes global supply of and the right-hand side global demand for domestic goods.

Note that an analogous interpretation for (27) also holds abroad.

Equations (26) and (27) immediately collapse to the definition of the terms of trade (TOT):

$$T_t := \frac{P_{t,F}}{P_{t,H}} = \frac{S_t P_{t,F}^*}{P_{t,H}} = \frac{Y_t}{Y_t^*}, \qquad (28)$$

which is the ratio of imported goods' and exported goods' prices from the home country's perspective.

Using the domestic intertemporal budget constraint (17) plus further manipulations eventually yield the domestic and foreign balance of payment identities:

$$\begin{aligned} P_{t,H}Y_t - P_t C_t + i_{t-1}B_{t-1} &\equiv B_t - B_{t-1}, && (29) \\ P_{t,F}Y_t^* - P_t C_t^* + i_{t-1}B_{t-1}^* &\equiv B_t^* - B_{t-1}^* && (30) \end{aligned}$$

with the left-hand side of equation (29) representing the home country's current account and the right-hand side its capital account.

Note that an analogous interpretation for (30) also holds abroad.

Internationally tradable bonds are supposed to be in zero net world supply:

$$nB_t + (1-n)B_t^* = 0. \qquad (31)$$

Assuming that international bond holdings have initially been zero $B_0 = B_0^* = 0$ together with (14), (29), (30), and (31) implies that $B_t = B_t^* = 0$ at all times according to Corsetti/Pesenti (2001, pp. 430-432) and Obstfeld/Rogoff (2001, p. 8). Then equations (29) and (30) simplify to the following:

$$\begin{aligned} C_t &= \frac{P_{t,H}Y_t}{P_t}, && (32) \\ C_t^* &= \frac{P_{t,F}Y_t^*}{P_t}. && (33) \end{aligned}$$

Using the definition of the TOT (28) the preceding equations can be rewritten as:

$$C_t = T_t^{n-1} Y_t, \quad (34)$$
$$C_t^* = T_t^n Y_t^*. \quad (35)$$

These are the conditions for domestic and foreign goods market clearing, which imply that households across countries always consume exactly their real incomes (see Obstfeld/Rogoff 2001, p. 8).

Moreover, $B_0 = B_0^* = 0$ together with (14), (29), (30), and (31) also implies that $C_t = C_t^* = C_t^w$ at all times such that

$$C_t = C_t^* = C_t^w = nC_t + (1-n)C_t^* = nT_t^{n-1}Y_t + (1-n)T_t^n Y_t^* = Y_t^n (Y_t^*)^{1-n},$$

while making use of (34) and (35).

In consequence, consumption shares across countries are not only time-constant but even equal (see Obstfeld/Rogoff 2001, p. 8). Since current and capital accounts between the two countries are in balance at all times and in all possible states of the world, the mechanism of adjustment to shocks in the world economy will only be represented by movements in the TOT, but not by changes in the countries' net asset positions. Hence, international financial markets are redundant anyway such that explicitly modeling financial market completeness by introducing Arrow-Debreu securities can be waived.

4.2. National Money Markets and World Currency Market

The government is assumed to set its expenditures equal to its revenues at all times such that the government budget is always in balance and no seignorage can occur (see Obstfeld/Rogoff 1996, p. 523):[16]

$$M_t - M_{t-1} + P_t \tau_t = 0. \quad (36)$$

Note that an analogous equation to (36) also holds abroad.

Equation (36) describes domestic money supply. Combining (36) with (19) and using the condition for domestic goods market clearing (34), one obtains two equations in M, which can be set equal and eventually solved for P:

$$P_t = \frac{M_{t-1}}{\chi^{\frac{1}{\varepsilon}} \left(\frac{1+i_t}{i_t}\right)^{\frac{1}{\varepsilon}} (T_t^n Y_t^*)^{\frac{\rho}{\varepsilon}} + \tau_t}.$$

Making use of (9), an analogous equation in P can be computed abroad such that both equations can again be set equal and finally solved for S:

$$S_t = \frac{M_{t-1} \left[(\chi^*)^{\frac{1}{\varepsilon}} \left(\frac{1+i_t^*}{i_t^*}\right)^{\frac{1}{\varepsilon}} (T_t^n Y_t^*)^{\frac{\rho}{\varepsilon}} + \tau_t^*\right]}{M_{t-1}^* \left[\chi^{\frac{1}{\varepsilon}} \left(\frac{1+i_t}{i_t}\right)^{\frac{1}{\varepsilon}} (T_t^{n-1} Y_t)^{\frac{\rho}{\varepsilon}} + \tau_t\right]}.$$

As one can see from the above formula, the current equilibrium nominal exchange rate S_t positively depends on past domestic nominal money balances M_{t-1}, current domestic opportunity costs of holding money $i_t/(1+i_t)$, current foreign output Y_t^*, and current foreign real lump-sum taxes τ_t^*. The dependence on the remaining variables is of opposite sign, except for the current TOT T_t, whose influence is ambiguous. An increase of S illustrates a depreciation of the domestic currency, whereas a decrease characterizes an appreciation.

[16]One could extend the model by introducing government spending (shocks) (see Obstfeld/Rogoff 2001, pp. 37-38), which shall be waived for this analysis however.

4.3. National Labor Markets

Notice from equations (20) and (25) that the real wage differs between consumers and producers because they use different price indexes. The ratio between real producer and real consumer wage is known as one type of "wedge" in Labor Market Economics (see, e.g., LANDMANN/JERGER 1999, pp. 136-138) and equals $P_H/P_H^n P_F^{1-n} = (P_H/P_F)^{1-n} = T^{n-1}$ in the present set-up.

Nonetheless, by combining (20), (25), and (34) with the CPI (5) one obtains two equations in $W/P = (W/P_H)T^{n-1}$ which can be solved for L:

$$L_t = T_t^{\frac{(n-1)(\rho-1)}{\xi}} \left(\frac{A_t}{\gamma}\right)^{-\frac{1}{\xi}} \left(\frac{\theta-1}{\theta}\right)^{-\frac{1}{\xi}} Y_t^{\frac{\rho}{\xi}}. \tag{37}$$

Equation (37) states that in an equilibrium on the perfectly competitive labor market, domestic employment positively depends on the aggregate productivity shock A and flexible-price real marginal production cost $(\theta-1)/\theta$, but negatively on the TOT T and domestic output Y.

Note that an analogous equation to (37) also holds abroad.

Combining equation (37) with the production function (21) and solving for Y, one finally obtains the domestic flexible-price equilibrium output Y^{flex}:

$$Y_t^{flex} = T_t^{\frac{(n-1)(\rho-1)}{\xi-\rho}} A_t^{\frac{\xi-1}{\xi-\rho}} \left(\frac{\theta}{\theta-1}\right)^{\frac{1}{\xi-\rho}} \gamma^{\frac{1}{\xi-\rho}}. \tag{38}$$

The domestic flexible-price equilibrium output positively depends on the aggregate productivity shock A, yet negatively on the TOT T and the flexible-price mark-up factor $\theta/(\theta-1)$.

Note that an analogous equation to (38) also holds abroad.

5. New Keynesian Framework

After having drawn the DSGE set-up and derived optimality conditions for both households and firms (Section 3) as well as market clearing conditions under flexible prices (Section 4), let us now turn to the New Keynesian framework. In order to establish such a framework, one has to introduce some form of nominal rigidity in addition to the assumption of monopolistic competition. In the present case, we will concentrate on sticky prices and forego sticky nominal wages as done, for instance, by CORSETTI/PESENTI (2001).

Log-linearizing the alternative market clearing and optimality conditions in the neighborhood of a non-stochastic zero-inflation steady state will lead to a canonical representation of the equilibrium of the model consisting of a dynamic IS curve, a New Keynesian Phillips curve (NKPC), and some form of monetary policy rule, both at home and abroad, as well as an equation for the TOT. This makes it possible for the fully micro-founded New Open Economy Macroeconomic literature to tie in with traditional Open Economy Macroeconomic models of the Mundell-Fleming-Dornbusch type.

As there are two countries, altogether we will obtain a system of seven log-linear equations. This form makes the model analytically tractable, especially for empirical applications: LEITH/MALLEY (2007), e.g., estimate NKPCs for the G7 economies by using the generalized method of moments (GMM) estimator based on log-linear equations. RUMLER (2007) applies a similar approach for the Euro area countries.[17]

Finally, the monetary policy rules which will be introduced below shall be *different* across countries. This is one of the crucial assumptions of this article.

[17] In contrast to CORSETTI/PESENTI (2001) who present a closed-form solution of their (deterministic) model, the log-linear approximation used here is considered to be advantageous since the link to empirical applications is immediate.

5.1. Dynamic IS Curves

It is straightforward to derive the dynamic IS curves for both countries by log-linearizing the domestic intertemporal Euler equation for real consumption (18) and its foreign analog around the non-stochastic zero-inflation steady state as shown in Appendix A.4. Accordingly, one obtains:[18]

$$\hat{y}_t = E_t[\hat{y}_{t+1}] + \frac{1}{\rho}\{E_t[\pi_{t+1}] - \hat{i}_t\} - (1-n)E_t[\Delta t_{t+1}], \quad (39)$$

$$\hat{y}_t^* = E_t[\hat{y}_{t+1}^*] + \frac{1}{\rho}\{E_t[\pi_{t+1}^*] - \hat{i}_t^*\} + nE_t[\Delta t_{t+1}]. \quad (40)$$

These two dynamic IS curves represent aggregate demand in both countries, where (39) can be interpreted as follows: current domestic demand is higher than its zero-inflation steady-state value if the expected domestic output deviation $E_t[\hat{y}_{t+1}]$ is positive (interpretable as an expected boom at home). There is also a clear positive relation of current demand to expected CPI inflation $E_t[\pi_{t+1}]$ (households consume more today if prices are expected to augment in the future) and a negative relation to current deviations from the zero-inflation steady-state nominal interest rate $\hat{i}_t$ (investing in nominal bonds is relatively attractive compared to buying consumption goods).

Moreover, there are also spill-over effects from abroad, which affect current domestic demand through expected movements in the TOT $E_t[\Delta t_{t+1}]$: current domestic demand negatively depends on an expected increase in the TOT since TOT expected to augment mean that imported goods become more expensive relative to domestic goods.[19] $1-n$ denotes the degree of openness of the home country to the foreign country (see Galí 2008, pp. 155-156). Since the degree of openness coincides with the size of the foreign country due the definition of the domestic CPI (2), there is no home bias in consumption, different to what is discussed in Pappa (2004, pp. 770-771).

Note that an analogous interpretation for (40) also holds abroad.

It may also be useful to introduce the domestic real interest rate r, which can be obtained via the Fisher relation:

$$(1+i_t) \equiv \frac{E_t[P_{t+1}]}{P_t}(1+r_t),$$

whose log-linear version reads

$$i_t \equiv E_t[\pi_{t+1}] + r_t. \quad (41)$$

Note that an analogous equation to (41) also holds for the foreign real interest rate r^*.

Note further that $\bar{i} = \bar{i}^* = \bar{r} = \bar{r}^* = (1-\beta)/\beta$ denotes the zero-inflation steady-state nominal and real interest rates, both at home and abroad, which can easily be obtained by solving the zero-inflation steady-state version of the domestic intertemporal Euler equation for real consumption (18) and its foreign analog for $\bar{i}$ and $\bar{i}^*$, respectively ($C_t^{-\rho} = E_t[C_{t+1}^{-\rho}] = \bar{C}^{-\rho}, P_t = E_t[P_{t+1}] = \bar{P}$).

5.2. New Keynesian Phillips Curves

The NKPCs for both countries can be derived by log-linearizing the price-setting equations of domestic and foreign firms around the non-stochastic zero-inflation steady-state as shown in Appendix A.5. In order to obtain the short-run "trade-off" between PPI inflation and the output gap represented by a

[18]Note that except for all types of interest rates, lower-case Latin letters denote natural logarithms of the corresponding variables. The hats above these log variables signify, except for all types of interest rates, percentage deviations from their zero-inflation steady-state values. In case of any interest rate these hats denote deviations measured in percentage points. The zero-inflation steady-state values themselves are denoted by upper bars.

[19]The TOT are expected to increase over time if either the domestic currency is expected to depreciate or if expected foreign PPI inflation will be higher than expected domestic PPI inflation, where these rates of inflation will be discussed below in more detail.

Phillips curve it is necessary to assure for price stickiness in addition to monopolistic competition. This shall be done by introducing Calvo (1983) contracts, which means that each producer is only allowed to reset her price with probability $1-\delta$ in any given period, independent of the time since the last adjustment. Therefore, a measure of $1-\delta$ of firms reset theirs prices each period, while a measure of δ of firms keep their prices constant and simply adjust their individual output in order to meet demand. $1/(1-\delta)$ then captures the average duration of a price (see Galí 2008, p. 43):

$$\pi_{t,H} = \beta E_t[\pi_{t+1,H}] + \frac{(1-\delta)(1-\delta\beta)}{\delta}\hat{\kappa}_t, \tag{42}$$

$$\pi^*_{t,F} = \beta E_t[\pi^*_{t+1,F}] + \frac{(1-\delta^*)(1-\delta^*\beta)}{\delta^*}\hat{\kappa}^*_t. \tag{43}$$

In equation (42) $\pi_{t,H} := \hat{p}_{t,H} - \hat{p}_{t-1,H}$ is defined as current domestic PPI inflation, which typically differs from domestic CPI inflation. The NKPC (42) states that current domestic PPI inflation $\pi_{t,H}$ is an increasing function of both expected domestic PPI inflation $E_t[\pi_{t+1,H}]$ and the deviation of current domestic real marginal production cost from its zero-inflation steady-state value $\hat{\kappa}_t := \kappa_t - \kappa_t^{flex}$.

Note that an analogous interpretation for (43) also holds abroad. However, δ^* is assumed to differ from δ and κ^* also from κ, although $\kappa^{flex} = (\kappa^*)^{flex} = (\theta-1)/\theta$. Furthermore, let us assume that "setting a new price at home" and "setting a new price abroad" are stochastically independent events. As domestic and foreign firms both set theirs prices in the currency of the countries where they are located, the present model features producer-currency pricing, which is one of the possible occurrences of pricing to market.[20]

Nonetheless, it would be desirable to express equations (39), (40), (42), and (43) in terms of the output gap, which shall be defined as the difference between the sticky-price and the flexible-price output deviations: $x_t := \hat{y}_t - \hat{y}_t^{flex}$ and $x_t^* := \hat{y}_t^* - (\hat{y}_t^*)^{flex}$. If we want to implement this we have to investigate the ratio of the sticky-price real marginal production cost κ_t and its flexible-price counterpart κ_t^{flex} given by (25):

$$\frac{\kappa_t}{\kappa_t^{flex}} = \frac{\frac{W_t}{P_{t,H}A_t}}{\frac{\theta-1}{\theta}} = \frac{\theta W_t T_t^{1-n}}{(\theta-1)P_t A_t}. \tag{44}$$

Combining equation (44) with the labor supply curve (20), the production function (21), and the condition for domestic goods market clearing (34), we obtain:

$$\frac{\kappa_t}{\kappa_t^{flex}} = \frac{\theta\gamma\left(\frac{Y_t}{A_t}\right)^{-\xi} T_t^{1-n}}{(\theta-1)(T_t^{n-1}Y_t)^{-\rho}A_t} = \frac{\theta}{\theta-1}\gamma A_t^{\xi-1} T_t^{(n-1)(\rho-1)} Y_t^{\rho-\xi} = \left(\frac{Y_t}{Y_t^{flex}}\right)^{\rho-\xi}, \tag{45}$$

where Y_t^{flex} denotes the domestic flexible-price equilibrium output as given by equation (38). Log-linearizing this expression around the zero-inflation steady-state yields:

$$\hat{\kappa}_t = (\rho-\xi)(\hat{y}_t - \hat{y}_t^{flex}) = (\rho-\xi)x_t. \tag{46}$$

Hence, by using (46) equations (39), (40), (42), and (43) rearrange to:

$$x_t = E_t[x_{t+1}] + \frac{1}{\rho}\{E_t[\pi_{t+1}] - \hat{i}_t\} - (1-n)E_t[\Delta t_{t+1}] + E_t[\hat{y}_{t+1}^{flex}] - \hat{y}_t^{flex}, \tag{47}$$

$$x_t^* = E_t[x_{t+1}^*] + \frac{1}{\rho}\{E_t[\pi_{t+1}^*] - \hat{i}_t^*\} + nE_t[\Delta t_{t+1}] + E_t[(\hat{y}_{t+1}^*)^{flex}] - (\hat{y}_t^*)^{flex}, \tag{48}$$

$$\pi_{t,H} = \beta E_t[\pi_{t+1,H}] + \mu x_t + u_t, \tag{49}$$

$$\pi^*_{t,F} = \beta E_t[\pi^*_{t+1,F}] + \mu^* x_t^* + u_t^* \tag{50}$$

[20] This specification has already been adopted in the theoretical literature (see, e.g., Clarida et al. 2002, p. 885) and can also be justified by empirical evidence for most of the G7 countries (see Leith/Malley 2007, p. 420).

with $\mu := [(1-\delta)(1-\delta\beta)(\rho-\xi)]/\delta$ ($\mu > 0$) and $\mu^* := [(1-\delta^*)(1-\delta^*\beta)(\rho-\xi)]/\delta^*$ ($\mu^* > 0$) representing the slope coefficients of the NKPCs with respect to the domestic (foreign) output gap. In addition, u_t shall denote an exogenously given, stationary AR(1) process of the form $u_t = \zeta_u u_{t-1} + \eta_{u,t}$ $(0 < \zeta_u < 1)$ with the exogenous error term η_u assumed to be i.i.d. $\sim N(0, \sigma^2_{\eta_u})$. This AR(1) process can be interpreted as a transitory cost-push shock reflecting determinants of real marginal production cost which do not move proportionally with the output gap (see Clarida et al. 2001, pp. 250-251).

The two NKPCs represent aggregate supply in both countries and are isomorphic to their closed-economy counterparts, where (49) can be interpreted as follows: the positive short-run "trade-off" between current domestic PPI inflation $\pi_{t,H}$ and the current domestic output gap x_t can easily be seen.[21] However, this is not really a trade-off to be exploited since $\pi_{t,H}$ is also positively related to (discounted) expected domestic PPI inflation $\beta E_t[\pi_{t+1,H}]$.[22]

Note that an analogous interpretation for (50) also holds abroad. However, u^* shall be uncorrelated with u such that domestic and foreign cost-push shocks are country-specific.

It will turn out to be convenient that the following holds for $E_t[\hat{y}^{flex}_{t+1}] - \hat{y}^{flex}_t$ in case one makes use of the log-linear version of the current domestic flexible-price equilibrium output according to (38) and its expected counterpart:

$$\begin{aligned} E_t[\hat{y}^{flex}_{t+1}] - \hat{y}^{flex}_t &= E_t[y^{flex}_{t+1}] - y^{flex}_t \\ &= \frac{(n-1)(\rho-1)}{\xi-\rho}E_t[t_{t+1}] + \frac{\xi-1}{\xi-\rho}E_t[a_{t+1}] + \frac{1}{\xi-\rho}\ln\left(\frac{\theta}{\theta-1}\right) + \frac{1}{\xi-\rho}\ln\gamma \\ &- \frac{(n-1)(\rho-1)}{\xi-\rho}t_t - \frac{\xi-1}{\xi-\rho}a_t - \frac{1}{\xi-\rho}\ln\left(\frac{\theta}{\theta-1}\right) - \frac{1}{\xi-\rho}\ln\gamma \\ &= \frac{(n-1)(\rho-1)}{\xi-\rho}E_t[\Delta t_{t+1}] + \frac{\xi-1}{\xi-\rho}E_t[\Delta a_{t+1}], \end{aligned} \tag{51}$$

where a_t is assumed to obey an exogenously given, stationary random process of the form $a_t = \zeta_a a_{t-1} + \varphi_a a^*_t + \eta_{a,t}$ $(0 < \zeta_a, \varphi_a < 1)$ with the exogenous error term η_a assumed to be i.i.d. $\sim N(0, \sigma^2_{\eta_a})$.

Note that an analogous equation to (51) also holds abroad.

In consequence, the dynamic IS curves (47) and (48) rearrange to:

$$x_t = E_t[x_{t+1}] + \frac{1}{\rho}\{E_t[\pi_{t+1}] - \hat{i}_t\} + \frac{(n-1)(\xi-1)}{\xi-\rho}E_t[\Delta t_{t+1}] + \frac{\xi-1}{\xi-\rho}E_t[\Delta a_{t+1}], \tag{52}$$

$$x^*_t = E_t[x^*_{t+1}] + \frac{1}{\rho}\{E_t[\pi^*_{t+1}] - \hat{i}^*_t\} + \frac{n(\xi-1)}{\xi-\rho}E_t[\Delta t_{t+1}] + \frac{\xi-1}{\xi-\rho}E_t[\Delta a^*_{t+1}]. \tag{53}$$

It would also be preferable to express these dynamic IS curves in terms of PPI rather than CPI inflation, which can be achieved by using the subsequent log-linear representation of the TOT (28): $t_t := s_t + p^*_{t,F} - p_{t,H}$. Subtracting this expression from its expected analog one gets: $E_t[\Delta t_{t+1}] = E_t[\Delta s_{t+1}] + E_t[\pi^*_{t+1,F}] - E_t[\pi_{t+1,H}] = E_t[\pi_{t+1,F}] - E_t[\pi_{t+1,H}]$. Combining this outcome with the log-linear versions of the domestic CPI (5) and its foreign equivalent, one obtains the following relations between expected CPI and expected PPI inflation at home and abroad:

$$E_t[\pi_{t+1}] \equiv E_t[\pi_{t+1,H}] - (n-1)E_t[\Delta t_{t+1}], \tag{54}$$

$$E_t[\pi^*_{t+1}] \equiv E_t[\pi^*_{t+1,F}] - nE_t[\Delta t_{t+1}]. \tag{55}$$

[21] Note that NKPCs such as (49) and (50) in terms of the output gap sometimes are referred to as aggregate supply (AS) curves (see Clarida et al. 2001, p. 250).

[22] If, for instance, some institution had the power to raise domestic output above its flexible-price value (given the deviations from its zero-inflation steady-state value) by raising $\pi_{t,H}$, not only $\pi_{t,H}$ but also $\beta E_t[\pi_{t+1,H}]$ would have to rise for (49) to hold with equality. This means that if output were kept on this artificially high level for an extended period of time, the respective expected inflation rates would continue to rise at accelerating speed (acceleration theorem).

Substituting for $E_t[\pi_{t+1}]$ by (54) and for $E_t[\pi^*_{t+1}]$ by (55), the dynamic IS curves (52) and (53) change to the following:

$$x_t = E_t[x_{t+1}] + \frac{1}{\rho}\{E_t[\pi_{t+1,H}] - \hat{i}_t\} + \vartheta E_t[\Delta t_{t+1}] + \frac{\xi-1}{\xi-\rho}E_t[\Delta a_{t+1}], \quad (56)$$

$$x^*_t = E_t[x^*_{t+1}] + \frac{1}{\rho}\{E_t[\pi^*_{t+1,F}] - \hat{i}^*_t\} + \vartheta^* E_t[\Delta t_{t+1}] + \frac{\xi-1}{\xi-\rho}E_t[\Delta a^*_{t+1}], \quad (57)$$

where $\vartheta := [(n-1)(\xi\rho-\xi)]/[(\xi-\rho)\rho]$ $(\vartheta > 0)$ and $\vartheta^* := [n(\xi\rho-\xi)]/[(\xi-\rho)\rho]$ $(\vartheta^* < 0)$ holds for the slope coefficients of the dynamic IS curves with respect to the expected movements in the TOT.

Therefore, we need an equation that expresses these movements as a function of the remaining endogenous variables. In order to do so, let us introduce uncovered interest rate parity as condition for avoiding currency arbitrage:

$$(1+i_t) \equiv \frac{E_t[S_{t+1}]}{S_t}(1+i^*_t),$$

where its log-linear version reads

$$i_t \equiv E_t[\Delta s_{t+1}] + i^*_t. \quad (58)$$

By solving the period $t-1$ equivalent of equation (58) for $E_{t-1}[\Delta s_t] = \Delta s_t$ (assuming that past expectations have been correct) and plugging the result into the log-linear representation of current movements in the TOT $\Delta t_t = \Delta s_t + \pi^*_{t,F} - \pi_{t,H}$ one obtains:

$$\Delta t_t = \hat{i}_{t-1} - \hat{i}^*_{t-1} + \pi^*_{t,F} - \pi_{t,H}. \quad (59)$$

5.3. Monetary Policy Rules

With the derivation of equations (49), (50), (56), (57), and (59) one has obtained a system of five log-linear expectational difference equations. However, with $x, x^*, \pi_H, \pi^*_F, \Delta t, \hat{i}, \hat{i}^*$ one has seven endogenous variables, two more variables than equations. Therefore, we need two more equations which represent domestic and foreign monetary policy as Taylor (1993) type interest-rate rules in order to obtain a determined system of equations. Following Woodford (2003, pp. 90-101), these interest-rate rules shall comprise a feedback from (some of) the endogenous variables. There, those interest-rate rules are first incorporated into a Neo-Wicksellian cashless economy, but Woodford (2003, pp. 101-106) also shows that rules of this form produce equivalent results in case of monetary frictions, e.g., in the MIU model given by equation (1). Even though the Woodford (2003) results have been derived for the closed economy, they are supposed to hold for the open economy, too, which is due to the isomorphism of the models.

The feedback is introduced to circumvent price level (and inflation) indeterminacy as shown by Sargent/Wallace (1975), which is typically associated with purely exogenous interest-rate targets (see Woodford 2003, p. 86). In case the latter type of modeling is avoided, the monetary aggregate is not a superior policy instrument compared to the short-run nominal interest rate. Moreover, it is assumed that the central banks are committed to their rules rather than they implement new "rules" on a period-by-period basis. This is done in order to overcome time inconsistency of monetary policy. For a debate on discretion versus commitment in monetary policy and possible welfare gains from the latter see, e.g., Clarida et al. (1999, pp. 1670-1671) or Galí (2008, chapter 5).

Hereinafter, the two monetary authorities shall be called ECB at home and Fed abroad as already foreshadowed in the introductory Section 1. In consequence, the home "country" will be denoted European Union (EU) and the foreign country United States (US).

These two central banks shall be assumed to conduct their monetary policies autonomously, which means that they take as given the policy actions of the respective other monetary authority. In other words, the two central banks are assumed to be non-cooperating. This assumption differs, for instance, from CLARIDA et al. (2002), PAPPA (2004), or BENIGNO/BENIGNO (2006) who consider cooperative equilibria among other possibilities. A particular reason why cooperative solutions can be ignored is the finding that there are only quantitatively negligible welfare gains from cooperation between the ECB and the Fed for empirically plausible parameter constellations (see PAPPA 2004, pp. 770-774).

Even though the central banks' targeting rules fulfill a similar purpose in the EU and the US, it can be justified to assume that they differ to a certain extent. This is due to the diverging statutes of the ECB and the Fed, which have also been stated in the introductory Section 1.

Therefore, the interest rate rules shall differ such that the Fed is supposed to conduct its monetary policy by considering current US PPI inflation $\pi^*_{t,F}$ *and* the current US output gap x^*_t, while, for the sake of simplicity, the ECB is supposed to impose its monetary policy taking into account current EU PPI inflation $\pi_{t,H}$ *only*. This difference is due to the fact that all conceivable policy goals of the ECB besides price stability can be interpreted as secondary.

Hence, the two interest-rate rules read:

$$
\begin{aligned}
\hat{i}_t &= \alpha\pi_{t,H} + \omega\hat{i}_{t-1} + v_t && (60)\\
\Leftrightarrow i_t &= \alpha\pi_{t,H} + \omega i_{t-1} + (1-\omega)\bar{i} + v_t, \\
\hat{i}^*_t &= \alpha^*\pi^*_{t,F} + \iota^* x^*_t + \omega^*\hat{i}^*_{t-1} + v^*_t && (61)\\
\Leftrightarrow i^*_t &= \alpha^*\pi^*_{t,F} + \iota^* x^*_t + \omega^* i_{t-1} + (1-\omega^*)\bar{i}^* + v^*_t.
\end{aligned}
$$

The ECB's interest rate rule (60) can be interpreted as follows: α ($\alpha > 0$) denotes the sensitivity of the ECB to current domestic PPI inflation $\pi_{t,H}$. Since past decisions cannot be ignored under commitment (see PAPPA 2004, p. 754), the rule incorporates some degree of inertia of the monetary policy instrument i itself as in WOODFORD (2003, pp. 95-96), which is measured by the parameter ω ($0 < \omega < 1$). The parameter $1 - \omega$, however, measures the degree of adjustment to the zero-inflation steady-state value of the nominal interest rate $\bar{i}$.[23]

In (60), v_t shall denote an exogenously given, stationary AR(1) process of the form $v_t = \zeta_v v_{t-1} + \eta_{v,t}$ ($0 < \zeta_v < 1$) with the exogenous error term η_v assumed to be i.i.d. $\sim N(0, \sigma^2_{\eta_v})$. This AR(1) process can be interpreted as a transitory monetary policy shock, where a positive realization of η_v would denote a contractionary shock (see GALÍ 2008, p. 51).

Note that an analogous interpretation for (61) also holds for the US. However, α^* may differ from α as well as ω^* from ω. Moreover, v^* shall be uncorrelated with v such that domestic and foreign monetary policy shocks are country-specific. ι^* ($\iota^* > 0$) denotes the sensitivity of the Fed to the current foreign output gap x^*_t.[24] Since the signs of the elasticities of the central banks' policy instruments to endogenous variables are all positive so that they react anti-cyclically to their changes, the policies could alternatively be characterized to have a "lean against the wind" property as in CLARIDA et al. (1999, p. 1672).

Altogether, (49), (50), (56), (57), (59), (60) and (61) form a determined system of six log-linear expectational difference equations.

6. Determinacy of the Rational Expectations Equilibrium

In order to investigate whether there is a determinate rational expectations equilibrium to the system of expectational difference equations (49), (50), (56), (57), (59), (60) and (61) it is advantageous to rearrange it in matrix form.

[23] Following GALÍ/MONACELLI (2005, p. 723), both rules (60) and (61) could also be denoted PPI (or domestic) inflation-based Taylor rules (DITR) as opposed to CPI inflation-based Taylor rules (CITR) or a credible peg for the nominal exchange rate. However, we will not take up these other possibilities of monetary policy design here.

[24] Note that $\iota = 0$ is assumed to hold for the ECB.

In consequence, we obtain the subsequent matrix form for the system of equations:

$$\mathbf{A}\mathbf{y} = \mathbf{B}\mathbf{x} + \mathbf{u}, \tag{62}$$

where the vectors of unknowns $\mathbf{y}, \mathbf{x}$ and the vector of disturbance terms $\mathbf{u}$ read as follows:

$$\mathbf{y} := \begin{bmatrix} x_t \\ x_t^* \\ \pi_{t,H} \\ \pi_{t,F}^* \\ \Delta t_t \\ \hat{i}_{t-1} \\ \hat{i}_{t-1}^* \end{bmatrix}, \mathbf{x} := \begin{bmatrix} E_t[x_{t+1}] \\ E_t[x_{t+1}^*] \\ E_t[\pi_{t+1,H}] \\ E_t[\pi_{t+1,F}^*] \\ E_t[\Delta t_{t+1}] \\ \hat{i}_t \\ \hat{i}_t^* \end{bmatrix}, \mathbf{u} := \begin{bmatrix} \frac{\xi-1}{\xi-\rho} E_t[\Delta a_{t+1}] \\ \frac{\xi-1}{\xi-\rho} E_t[\Delta a_{t+1}^*] \\ u_t \\ u_t^* \\ 0 \\ -\omega^{-1} v_t \\ -(\omega^*)^{-1} v_t^* \end{bmatrix}.$$

The coefficient matrices $\mathbf{A}, \mathbf{B}$, however, read:

$$\mathbf{A} := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -\mu & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\mu^* & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & -1 & 1 \\ 0 & 0 & \frac{\alpha}{\omega} & 0 & 0 & 1 & 0 \\ 0 & \frac{\iota^*}{\omega^*} & 0 & \frac{\alpha^*}{\omega^*} & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{B} := \begin{bmatrix} 1 & 0 & \rho^{-1} & 0 & \vartheta & -\rho^{-1} & 0 \\ 0 & 1 & 0 & \rho^{-1} & \vartheta^* & 0 & -\rho^{-1} \\ 0 & 0 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (\omega^*)^{-1} \end{bmatrix}.$$

In order to determine the eigenvalues of the system of equations (62), it has to be rearranged in the following form:

$$\mathbf{y} = \mathbf{M}\mathbf{x} + \mathbf{v}, \tag{63}$$

where $\mathbf{M} := \mathbf{A^{-1}B}$ and $\mathbf{v} := \mathbf{A^{-1}u}$. Moreover, $\mathbf{A^{-1}}$ denotes the inverse of $\mathbf{A}$, which exists because $\det(\mathbf{A}) = 1 \neq 0$.

The matrices $\mathbf{A^{-1}}$ and $\mathbf{M}$ and the vector $\mathbf{v}$ read as follows:

$$\mathbf{A^{-1}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \mu & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \mu^* & 0 & 1 & 0 & 0 & 0 \\ -\frac{\mu\,(\omega+\alpha)}{\omega} & \frac{\mu^*\,\omega^*+\mu^*\,\alpha^*+\iota^*}{\omega^*} & -\frac{\omega+\alpha}{\omega} & \frac{\omega^*+\alpha^*}{\omega^*} & 1 & 1 & -1 \\ -\frac{\mu\,\alpha}{\omega} & 0 & -\frac{\alpha}{\omega} & 0 & 0 & 1 & 0 \\ 0 & -\frac{\mu^*\,\alpha^*+\iota^*}{\omega^*} & 0 & -\frac{\alpha^*}{\omega^*} & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & \rho^{-1} & 0 & \vartheta & -\rho^{-1} & 0 \\ 0 & 1 & 0 & \rho^{-1} & \vartheta^* & 0 & -\rho^{-1} \\ \mu & 0 & \frac{\mu}{\rho}+\beta & 0 & \mu\,\vartheta & -\frac{\mu}{\rho} & 0 \\ 0 & \mu^* & 0 & \frac{\mu^*}{\rho}+\beta & \mu^*\,\vartheta^* & 0 & -\frac{\mu^*}{\rho} \\ -\frac{\mu\,(\omega+\alpha)}{\omega} & \frac{\mu^*\,\omega^*+\mu^*\,\alpha^*+\iota^*}{\omega^*} & -\frac{\mu\,(\omega+\alpha)}{\omega\,\rho}-\frac{(\omega+\alpha)\beta}{\omega} & \frac{\mu^*\,\omega^*+\mu^*\,\alpha^*+\iota^*}{\omega^*\,\rho}+\frac{(\omega^*+\alpha^*)\beta}{\omega^*} & -\frac{\mu\,(\omega+\alpha)\vartheta}{\omega}+\frac{(\mu^*\,\omega^*+\mu^*\,\alpha^*+\iota^*)\,\vartheta^*}{\omega^*} & \frac{\mu\,(\omega+\alpha)}{\omega\,\rho}+\omega^{-1} & -\frac{\mu^*\,\omega^*+\mu^*\,\alpha^*+\iota^*}{\omega^*\,\rho}-(\omega^*)^{-1} \\ -\frac{\mu\,\alpha}{\omega} & 0 & -\frac{\mu\,\alpha}{\omega\,\rho}-\frac{\alpha\,\beta}{\omega} & 0 & -\frac{\mu\,\alpha\,\vartheta}{\omega} & \frac{\mu\,\alpha}{\omega\,\rho}+\omega^{-1} & 0 \\ 0 & -\frac{\mu^*\,\alpha^*+\iota^*}{\omega^*} & 0 & -\frac{\mu^*\,\alpha^*+\iota^*}{\omega^*\,\rho}-\frac{\alpha^*\,\beta}{\omega^*} & -\frac{(\mu^*\,\alpha^*+\iota^*)\,\vartheta^*}{\omega^*} & 0 & \frac{\mu^*\,\alpha^*+\iota^*}{\omega^*\,\rho}+(\omega^*)^{-1} \end{bmatrix},$$

$$\mathbf{v} = \begin{bmatrix} \frac{(\xi-1)E_t[\Delta a_{t+1}]}{\xi-\rho} \\ \frac{(\xi-1)E_t[\Delta a^*_{t+1}]}{\xi-\rho} \\ \frac{\mu\,(\xi-1)E_t[\Delta a_{t+1}]}{\xi-\rho}+u_t \\ \frac{\mu^*\,(\xi-1)E_t[\Delta a^*_{t+1}]}{\xi-\rho}+u^*_t \\ -\frac{\mu\,(\omega+\alpha)(\xi-1)E_t[\Delta a_{t+1}]}{\omega\,(\xi-\rho)}+\frac{(\mu^*\,\omega^*+\mu^*\,\alpha^*+\iota^*)(\xi-1)E_t[\Delta a^*_{t+1}]}{\omega^*\,(\xi-\rho)}-\frac{(\omega+\alpha)u_t}{\omega}+\frac{(\omega^*+\alpha^*)u^*_t}{\omega^*}-\frac{v_t}{\omega}+\frac{v^*_t}{\omega^*} \\ -\frac{\mu\,\alpha\,(\xi-1)E_t[\Delta a_{t+1}]}{\omega\,(\xi-\rho)}-\frac{\alpha\,u_t}{\omega}-\frac{v_t}{\omega} \\ -\frac{(\mu^*\,\alpha^*+\iota^*)(\xi-1)E_t[\Delta a^*_{t+1}]}{\omega^*\,(\xi-\rho)}-\frac{\alpha^*\,u^*_t}{\omega^*}-\frac{v^*_t}{\omega^*} \end{bmatrix},$$

where $\det(\mathbf{M}) = 0$.

The impact of the single disturbances contained in $\mathbf{v}$ on the structural equations of the model can be characterized as follows. The positively correlated aggregate productivity shocks $E_t[\Delta a_{t+1}], E_t[\Delta a^*_{t+1}]$ affect the respective dynamic IS curves, NKPCs, and interest rate rules. The country-specific cost-push shocks u, u^*, however, only influence the respective NKPCs and interest-rate rules. The country-specific monetary policy shocks v, v^* have a sole impact on the respective interest-rate rules. All macroeconomic shocks spill over abroad since they explicitly affect the TOT equation (59). In addition, there is an implicit spill-over effect due to the positive correlation of the aggregate productivity shocks.

6.1. General Case

The system of equations (63) consists of two predetermined variables $(\hat{i}_{t-1}, \hat{i}^*_{t-1})$ and five non predetermined ones $(x_t, x^*_t, \pi_{t,H}, \pi^*_{t,F}, \Delta t_t)$. Comparable to the case discussed for the closed economy in Galí (2008, p. 56), there is a unique stationary solution of (63) if and only if the coefficient matrix $\mathbf{M}$ has five eigenvalues k inside and two eigenvalues k on or outside the complex unit circle (sufficient condition for equilibrium determinacy). If there were more than five stable eigenvalues, there would be multiple stationary solutions. If there were more than two instable eigenvalues instead, no stationary solution would exist at all.

By computing the characteristic determinant $\det(\mathbf{M} - k\mathbf{I_7})$ one obtains one eigenvalue $k_1 = 0$ and a sixth-degree polynomial in k, which cannot be solved analytically. This polynomial is not displayed here, but its Matlab code is available on request.

In consequence, we have to assign sensible numerical values to the model parameters in order to determine the remaining eigenvalues of $\mathbf{M}$.

6.2. Calibration

The numerical exercise is carried out as follows, whereby the length of one period shall correspond to one quarter of a year. First, the EU and the US can be treated as approximately equal-sized countries such that $n = 1 - n = 0.5$. $\beta = 0.97$ is assumed to hold for the intertemporal discount factor, which implies $\bar{i} = \bar{i}^* = \bar{r} = \bar{r}^* = (1-\beta)/\beta \approx 0.03$ for the zero-inflation steady-state nominal and real interest rates across countries. Furthermore, $\xi = -1$ such that $\epsilon_{U_L,L} = \epsilon_{U^*_L,L^*} = 2$ holds for the partial elasticity of the utility function with respect to domestic (foreign) labor. The sensitivity of the Fed to the current foreign output gap shall be fixed ($\iota^* = 0.5$), where this number corresponds to the original value estimated by Taylor (1993) for the Fed for the time from 1987 to 1992. The Taylor principle, which states that the monetary authority ought to react to an increase in current PPI inflation by augmenting its policy instrument *more* than one for one in order to account for a determinate rational expectations equilibrium (see Woodford 2003, p. 40), shall be fulfilled by both central banks ($\alpha = \alpha^* = 1.5$).[25] The degrees of nominal interest-rate inertia across countries shall also be fixed ($\omega = \omega^* = 0.1$) implying that both monetary authorities are supposed to place relatively more weight ($1 - \omega = 1 - \omega^* = 0.9$) on the adjustment of their short-run policy instruments to their zero-inflation steady-state value.

Moreover, set $\rho = 0.8$ such that one gets the following for the slope coefficients ϑ, ϑ^* of the dynamic IS

[25] Note that the Taylor principle in its purest form is not a necessary condition for equilibrium determinacy for an interest-rate rule of type (61). Instead, the condition $\mu^*(\alpha^* - 1) + (1 - \beta)\iota^* > 0$ is a necessary and sufficient condition for equilibrium determinacy in case of contemporaneous data (see, e.g., Bullard/Mitra 2002, pp. 1125-1126).

curves (56) and (57) with respect to expected movements in the TOT:[26]

$$\vartheta = \frac{(0.5-1)[(-1)\text{x}0.8-(-1)]}{[(-1)-0.8]0.8} \approx 0.07,$$
$$\vartheta^* = \frac{0.5[(-1)\text{x}0.8-(-1)]}{[(-1)-0.8]0.8} \approx -0.07.$$

Set the degree of price stickiness to $\delta = \delta^* = 0.75$ across countries, which corresponds to an average duration of a price of 4 quarters. This implies the following for the slope coefficients μ, μ^* of the NKPCs (49) and (50) with respect to the domestic (foreign) output gap:

$$\mu = \mu^* = \frac{(1-0.75)(1-0.75\text{x}0.97)[0.8-(-1)]}{0.75} \approx 0.16.$$

Calculating the characteristic determinant $\det(\mathbf{M} - k\mathbf{I_7})$ while using the above parameter configuration then yields the subsequent numerical eigenvalues:

$$\begin{aligned} k_1 &= 0, \\ k_2 &= 0.5441, \\ k_3 &= 0.8176 + 0.2411i, \\ k_4 &= 0.8176 - 0.2411i, \\ k_5 &= 0.9775, \\ k_6 &= 13.3372, \\ k_7 &= 18.2548. \end{aligned}$$

Since $\mathbf{M}$ contains five stable (k_1 to k_5) and two unstable eigenvalues (k_6 and k_7), there is a unique stationary solution to the system of equations (63) such that the rational expectations equilibrium indeed is determinate.

Notice that the above results have been derived for a calibrated version of the two-country DSGE model only so that they may not necessarily be universally applicable.

7. Impulse-Response Analysis

After having assured for determinacy of the rational expectations equilibrium, it would be interesting to investigate how the endogenous variables of the model react to simulated transitory shocks at home and abroad. This impulse-response analysis can also be viewed as additional robustness test for the goodness of the present model specification.

For this purpose, let us assume the following autocorrelation coefficients of the domestic and foreign productivity, cost-push, and monetary policy shocks: $\zeta_a = \zeta_a^* = \zeta_u = \zeta_u^* = \zeta_v = \zeta_v^* = 0.8$.[27] Moreover, $\varphi_a = \varphi_a^* = 0.3$ shall be proposed for the correlation coefficients of the interdependent productivity shocks.

Using the above specification and starting from the non-stochastic zero-inflation steady state, we entail

[26] Note that if we examined the special case of $\rho = 1$, which corresponds to logarithmic utility of consumption, we would get three implications for the dynamic IS curves (56) and (57): [1] the intertemporal elasticity of substitution of real consumption $1/\rho$ is equal to 1 across countries, [2] the impact of the TOT vanishes since $\vartheta = \vartheta^* = 0$, and [3] the coefficients on the aggregate productivity shocks $E_t[\Delta a_{t+1}], E_t[\Delta a^*_{t+1}]$ simplify to 1. Hence, the dynamic IS curves would be isomorphic to their closed-economy counterparts. Let us remind the reader at this point that even though the TOT effects would disappear for this case, the two economies would remain interdependent because of the positively correlated aggregate productivity shocks $E_t[\Delta a_{t+1}], E_t[\Delta a^*_{t+1}]$.

[27] We propose this relatively high serial correlation of the transitory shock variables mainly for illustrative reasons. Qualitatively, we would obtain the same results if we used smaller autocorrelation coefficients.

impulses in period 1 on the exogenous error terms $\eta_a, \eta_a^*, \eta_u, \eta_u^*, \eta_v, \eta_v^*$ in terms of one standard deviation of $+\sqrt{0.4}$ on their expected value of 0. The impulse-response analysis is carried out by employing the DYNARE package for MATLAB while simulating over 2100 periods.[28] For the DYNARE program code see Appendix A.6.

The six figures below show the responses of the output gaps x, x^*, PPI inflation rates π_H, π_F^*, movements in the TOT Δt, nominal interest rates $\hat{i}, \hat{i}^*$, and the relevant shock variables themselves to (orthogonalized) impulses on the various exogenous error terms for a time range of 40 periods or 10 years.[29]

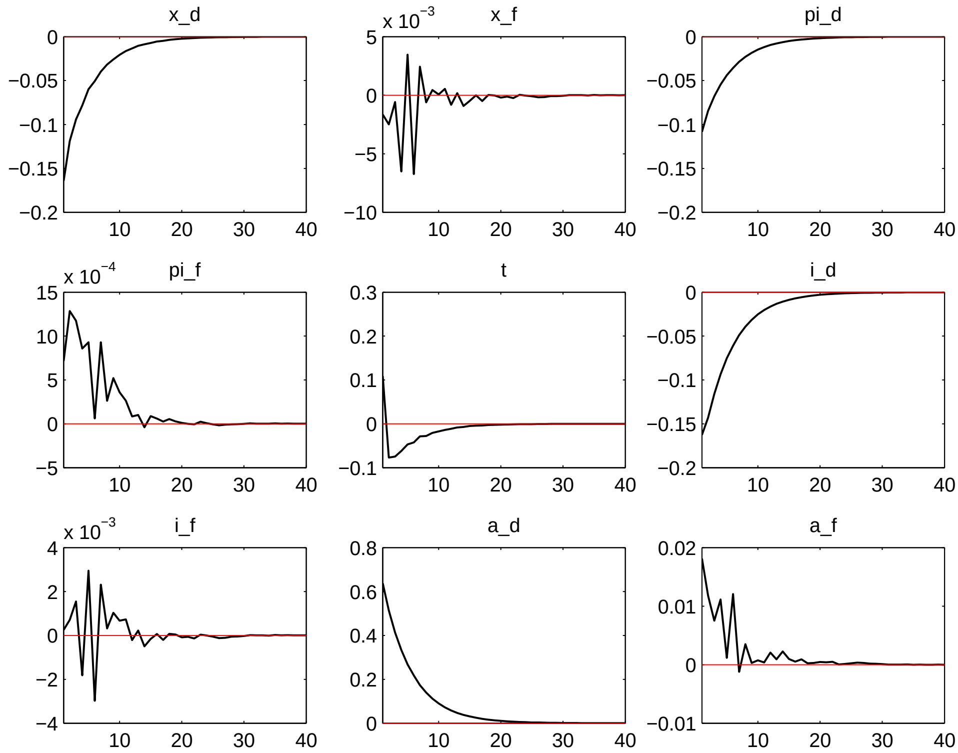

Figure 1: Responses to an impulse on the domestic productivity shock

[28]The software is downloadable from http://www.cepremap.cnrs.fr/dynare/. For all computations associated with the impulse-response analysis it uses the pure perturbation algorithm developed by SCHMITT-GROHÉ/URIBE (2004, pp. 764-765) as its default option.

[29]Note that the following is assumed for the variance-covariance matrix of the various exogenous error terms:

$$Var(\eta) = \begin{bmatrix} \sigma^2_{\eta_a} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{\eta_a^*} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\eta_u} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{\eta_u^*} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{\eta_v} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{\eta_v^*} \end{bmatrix}.$$

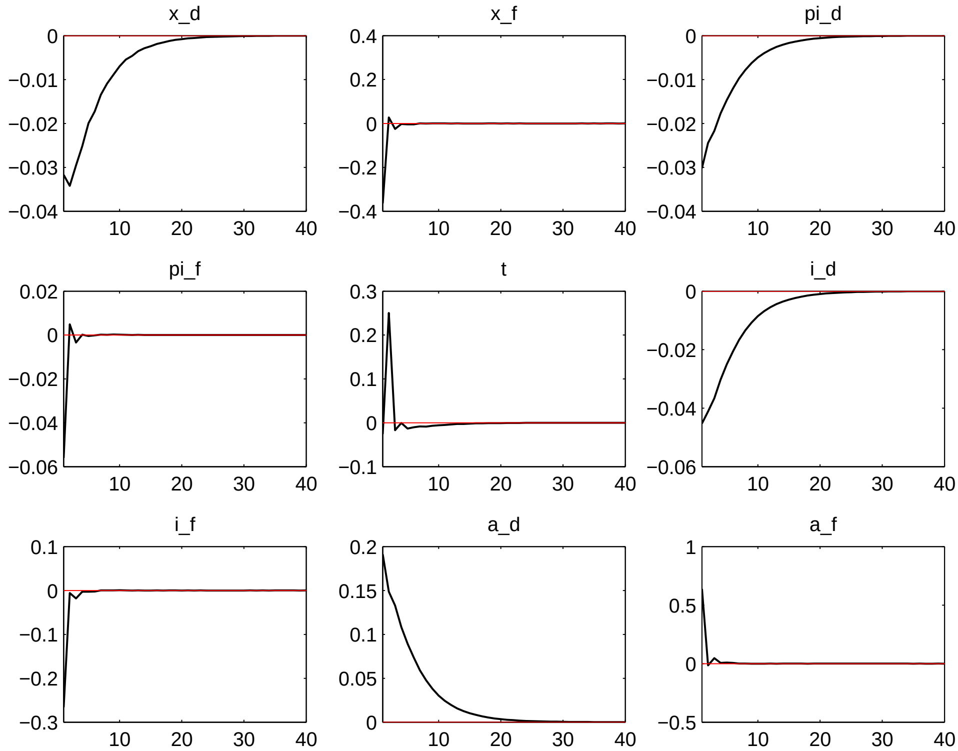

Figure 2: Responses to an impulse on the foreign productivity shock

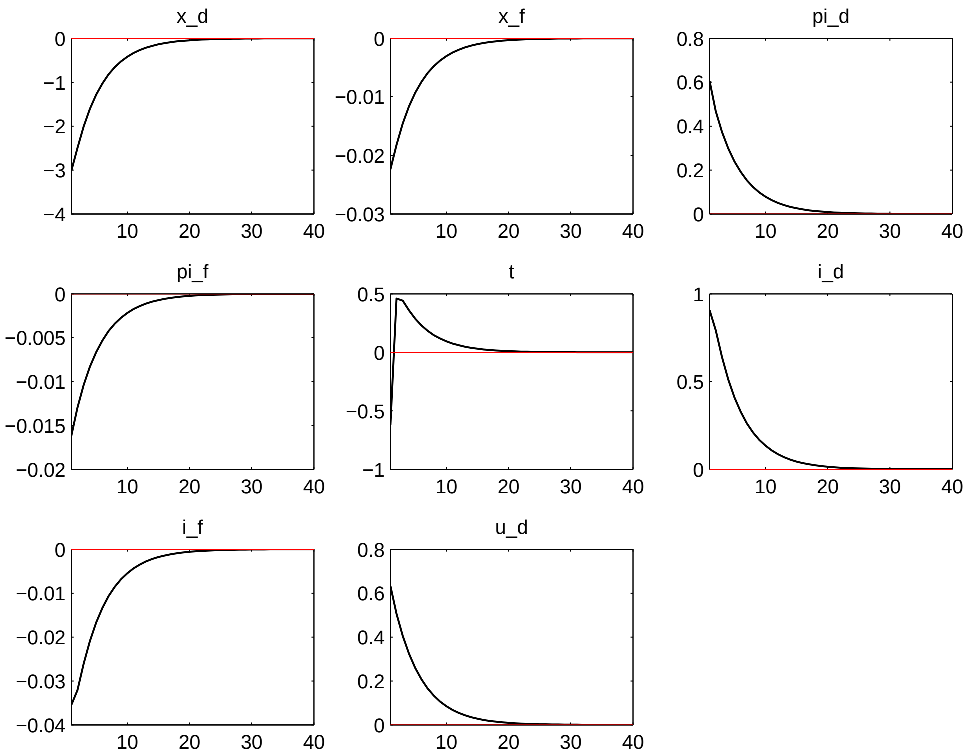

Figure 3: Responses to an impulse on the domestic cost-push shock

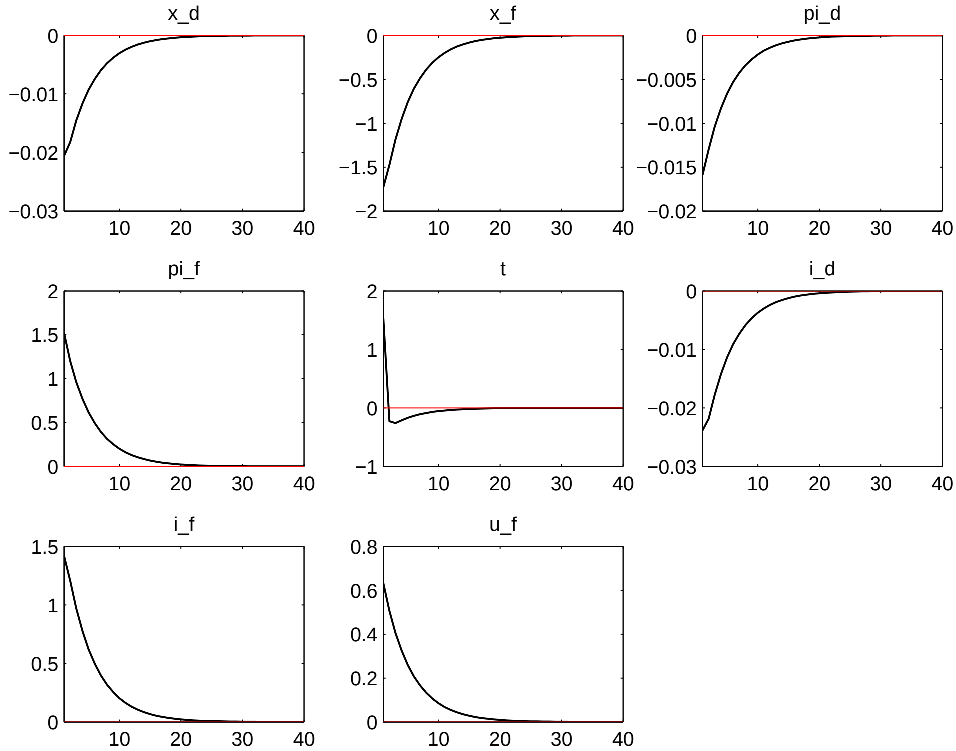

Figure 4: Responses to an impulse on the foreign cost-push shock

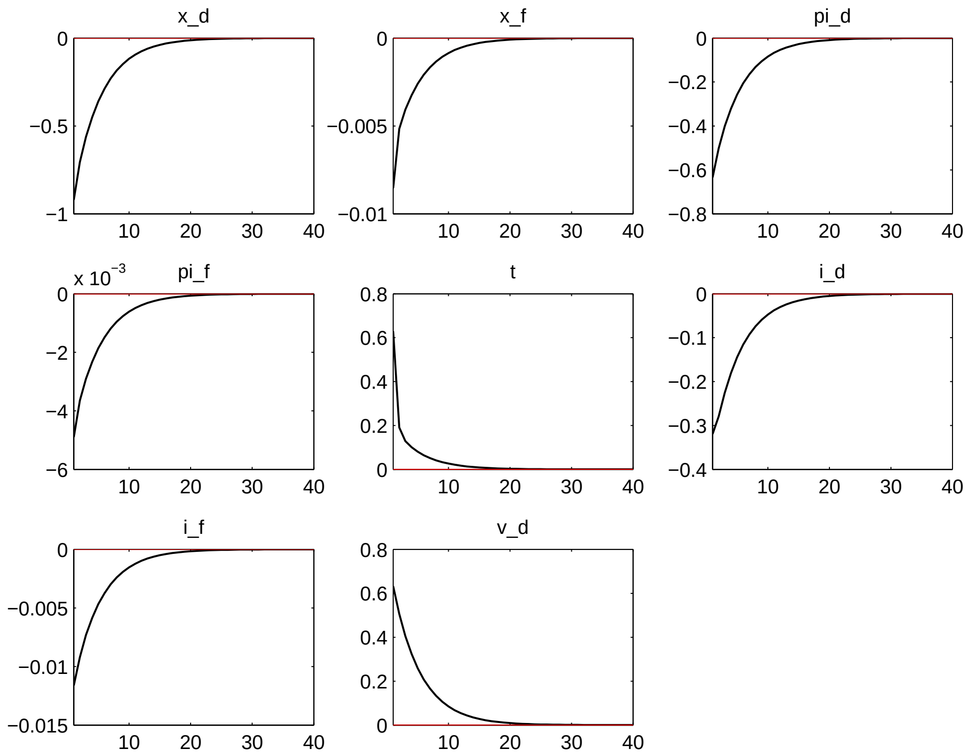

Figure 5: Responses to an impulse on the domestic monetary policy shock

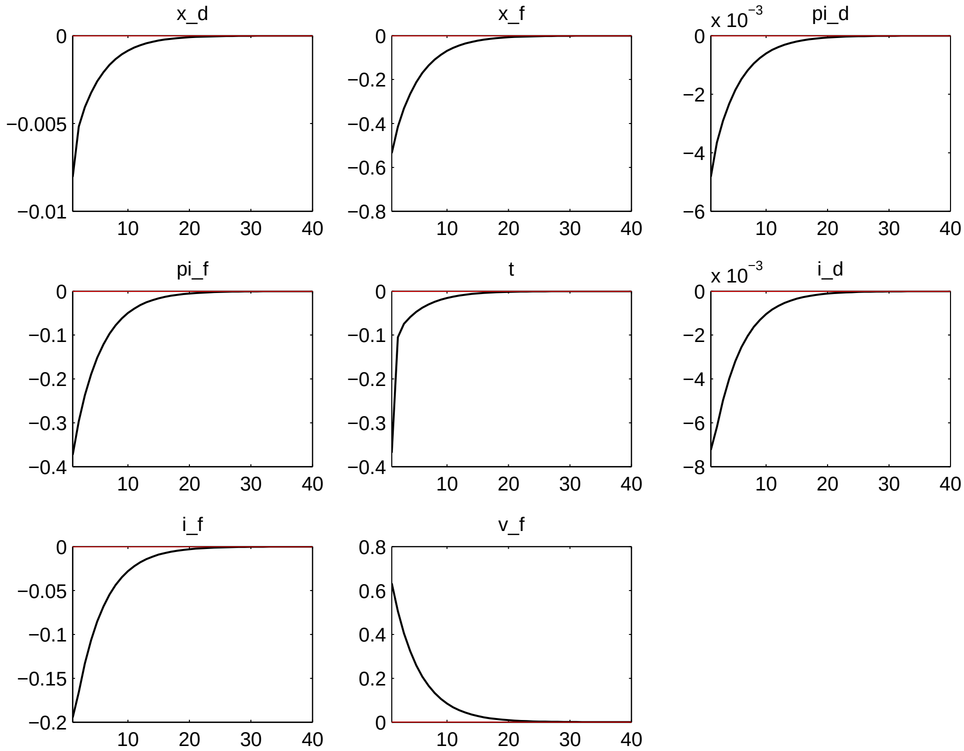

Figure 6: Responses to an impulse on the foreign monetary policy shock

An interpretation of the results that can be derived from Figures 1 to 6 can be found in the following.

1. The domestic output gap, PPI inflation and nominal interest rates decrease before they return to their zero-inflation steady-state values in response to an impulse on the domestic productivity shock (Figure 1). The TOT first augment, then drop below their zero-inflation steady-state value until they eventually converge. There is also an impact on all foreign endogenous variables, which is of the same sign except for the foreign PPI and nominal interest rates. Nonetheless, this impact is quantitatively small and induces fluctuations.

2. The foreign output gap, PPI inflation and nominal interest rates decrease before they return to their zero-inflation steady-state values in response to an impulse on the foreign productivity shock (Figure 2). The TOT first augment, then drop to their zero-inflation steady-state value. There is also an impact on all domestic endogenous variables, which barely fluctuate. The impact is of the same sign but quantitatively larger compared to the impact of the domestic productivity shock on foreign variables. In addition, the US recovers notably faster from a shock on its own productivity compared to the EU.

3. The domestic output gap decreases, yet the domestic PPI inflation and nominal interest rates increase before all endogenous variables return to their zero-inflation steady-state values in response to an impulse on the cost-push shocks (Figure 3). The TOT first plummet, then jump above their zero-inflation steady-state value until they eventually converge. There is also an impact on all foreign endogenous variables, which is of the same sign except for the foreign PPI and nominal interest rates.

4. The foreign output gap decreases, yet the foreign PPI inflation and nominal interest rates increase before all endogenous variables return to their zero-inflation steady-state values in response to an impulse on the cost-push shocks (Figure 4). The TOT first augment, then drop below their zero-inflation steady-state value until they eventually converge. There is also an impact on all domestic endogenous variables, which is of the same sign except for the foreign PPI and nominal interest rates.

5. The domestic output gap, PPI inflation and nominal interest rates decrease before all endogenous variables return to their zero-inflation steady-state values in response to an impulse on the monetary policy shocks (Figure 5). The TOT augment before they return to their zero-inflation steady-state value. There is also an impact on all foreign endogenous variables, which is of the same sign.

6. The foreign output gap, PPI inflation and nominal interest rates decrease before all endogenous variables return to their zero-inflation steady-state values in response to an impulse on the monetary policy shocks (Figure 6). The TOT plummet before they return to their zero-inflation steady-state value. There is also an impact on all domestic endogenous variables, which is of the same sign.

Except for the aggregate productivity shocks, the deviation of the domestic output gap from the zero-inflation steady state is higher than the one of the foreign output gap in case of home-made disturbances. However, the picture is ambiguous in case of PPI inflation and nominal interest rates. As expected, none of the shocks discussed above is able to raise output above its flexible-price equilibrium level, neither at home nor abroad. Instead, output in both countries drops in response to all entailed impulses before it eventually converges.

The negative influence of the productivity shocks on the economy contradicts one of the central implications of the standard RBC model, namely a positive correlation of productivity (shocks) and output. Findings for a closed-economy New Keynesian model, which are similar to the present results, are reported, e.g., in Galí (2002, pp. 17-18). Empirical studies support this view and show in addition that technology shocks do not seem to be a significant source for the creation of business cycles at all, which contradicts another central implication of the standard RBC model, namely that technology shocks ought to be the dominant driving force for the creation of business cycles (see Galí/Rabanal 2004, pp. 36-39).

The subsequent property of the monetary policy shocks is also worth mentioning. Contrary to Corsetti/Pesenti (2001, pp. 435-439), *negative* realizations of v, v^*, which correspond to expansionary shocks, always have a "prosper thyself" and "beggar thy neighbor" effect since they influence the TOT beneficially for the home (foreign) country's resident households by decreasing them below (raising them above) their zero-inflation steady-state values. In addition, this effect would induce a rise of both domestic and foreign output above their flexible-price values.[30]

Finally, statistical moments, correlations, and autocorrelations of the simulated endogenous variables are given in Tables 1 to 3 below.

Variable	Mean	Std. Dev.	Variance	Skewness	Kurtosis
x	0.074386	5.191024	26.946731	-0.075655	0.064088
x^*	-0.134416	3.104816	9.639881	-0.108630	-0.065179
π_H	-0.019727	1.537767	2.364727	0.026293	-0.077795
π_F^*	0.083086	2.605910	6.790766	0.059946	-0.110862
Δt	-0.025646	2.154687	4.642675	0.036449	-0.037179
$\hat{i}$	-0.034227	1.743373	3.039349	0.039592	0.133487
$\hat{i}^*$	0.094419	2.515244	6.326453	0.084558	-0.102184

Table 1: Moments of simulated variables

[30] Note, however, that monetary policy shocks in Corsetti/Pesenti (2001) are modeled in terms of permanent and unexpected changes in money supply.

Variable	x	x^*	π_H	π_F^*	Δt	$\hat{i}$	$\hat{i}^*$
x	1.0000	0.0566	-0.4303	-0.0157	-0.1783	-0.7667	-0.0062
x^*	0.0566	1.0000	0.0413	-0.8503	-0.2025	0.0165	-0.8834
π_H	-0.4303	0.0413	1.0000	-0.0426	-0.1487	0.8980	-0.0501
π_F^*	-0.0157	-0.8503	-0.0426	1.0000	0.3056	-0.0308	0.9894
Δt	-0.1783	-0.2025	-0.1487	0.3056	1.0000	0.0099	0.2668
$\hat{i}$	-0.7667	0.0165	0.8980	-0.0308	0.0099	1.0000	-0.0407
$\hat{i}^*$	-0.0062	-0.8834	-0.0501	0.9894	0.2668	-0.0407	1.0000

Table 2: Correlation of simulated variables

Variable	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
x	0.7948	0.6069	0.4625	0.3513	0.2574
x^*	0.7944	0.6279	0.5004	0.3857	0.2957
π_H	0.8091	0.6536	0.5277	0.4337	0.3558
π_F^*	0.7941	0.6264	0.4935	0.3789	0.2852
Δt	0.0887	0.0285	-0.0018	0.0046	0.0046
$\hat{i}$	0.8173	0.6288	0.4808	0.3745	0.2857
$\hat{i}^*$	0.8008	0.6337	0.4981	0.3878	0.2952

Table 3: Autocorrelation of simulated variables

As one can see from Table 1, the explanation why for almost any impulse the deviation of x is notably higher than of x^* may be found in the differing interest-rate rules for the ECB (60) and the Fed (61). The positive and fixed sensitivity of the Fed to the current US output gap may ceteris paribus absorb part of the impulses transmitted through the system of equations (63). This additional channel constitutes a dampening effect to any exogenous disturbance, which does not exist for the EU by assumption. In consequence, the simulated variance of the domestic output gap $\tilde{\sigma}_x^2$ is almost three times as high as the simulated variance of the foreign output gap $\tilde{\sigma}_{x^*}^2$:

$$\frac{\tilde{\sigma}_x^2}{\tilde{\sigma}_{x^*}^2} = \frac{26.946731}{9.639881} \approx 2.795339.$$

On the contrary, the simulated variance of the foreign PPI inflation rate $\tilde{\sigma}_{\pi_F^*}^2$ is almost three times as high as the simulated variance of the domestic PPI inflation rate $\tilde{\sigma}_{\pi_H}^2$:

$$\frac{\tilde{\sigma}_{\pi_F^*}^2}{\tilde{\sigma}_{\pi_H}^2} = \frac{6.790766}{2.364727} \approx 2.871691.$$

This means that if the ECB implemented its monetary policy by following the interest-rate rule (60), sustaining price stability, which is its primary objective according to its statute, would be better attainable than if, e.g., it were using an interest-rate rule as proposed for the Fed (61) instead. Nonetheless, this advantage can only be reached at the expense of relatively high fluctuations in the EU output gap, which is a trade-off commonly observed in literature on monetary policy.

Notice that again the above results have been derived for a calibrated version of the two-country DSGE model only so that they may not necessarily be universally applicable.

8. Concluding Remarks

The main results of the present article have already been stated in the introductory Section 1 such that there is no need to repeat them once more at this point. However, one might be interested in possible extensions and applications of the present model to be dealt with by future research.

In line with Obstfeld/Rogoff (2001, pp. 37-38) and as mentioned in Section 4, one could enrich this model, e.g., by introducing government spending (shocks). Or one could try and implement optimal monetary policy into the framework. Another possibility would be to alter the specification and correlation patterns of the various macroeconomic shocks. As already stated in Section 2, Galí (2008) gives various suggestions on how to extend a basic closed-economy New Keynesian model. Out of these suggestions we find that introducing labor market frictions, migration, and unemployment, imperfect information and learning, or the use of real capital as additional production factor would be particularly appealing (see Galí 2008, pp. 188-190).

An immediate application of the present framework, however, would be an empirical one. Similar to Rubaszek/Skrzypczynski (2008) who treat the US as a closed economy, one could test the forecasting performance of this model against an unconstrained vector autoregressive (VAR) model while using the same data.

A. Appendix

A.1. Consumption-based Consumer and Producer Price Indexes, Demand Curves for Individual and Composite Goods

The derivation of all price indexes and demand curves follows the ideas in Obstfeld/Rogoff (1996, pp. 662, 664) for the basic Obstfeld/Rogoff (1995) model.

Consumption-based Consumer Price Index, Demand Curves for Composite Goods The representative domestic household maximizes

$$C = \frac{C_H^n C_F^{1-n}}{n^n(1-n)^{1-n}}$$

with respect to C_H subject to the budget constraint

$$PC = P_H C_H + P_F C_F.$$

Hence,

$$\Lambda = \frac{C_H^n C_F^{1-n}}{n^n(1-n)^{1-n}} - \lambda(P_H C_H + P_F C_F - PC) \rightarrow \max_{C_H}$$

$$\Rightarrow \frac{\partial \Lambda}{\partial C_H} = \frac{n C_H^{n-1} C_F^{1-n}}{n^n(1-n)^{1-n}} - \lambda P_H = 0.$$

Solving this expression for C_H, one obtains the subsequent preliminary demand function for the composite domestic good:

$$C_H = \lambda^{\frac{1}{n-1}} P_H^{\frac{1}{n-1}} \frac{n}{1-n} C_F.$$

Multiplying the preceding equation with P_H, one obtains:

$$P_H C_H = \lambda^{\frac{1}{n-1}} P_H^{\frac{n}{n-1}} \frac{n}{1-n} C_F,$$

with $P_H C_H = nPC$. Now combine the preceding equation with the preliminary demand function from above. Then one gets for C_H equation (12):

$$C_H = n\left(\frac{P_H}{P}\right)^{-1} C.$$

Analogously, one gets for C_F equation (13):

$$C_F = (1-n)\left(\frac{P_F}{P}\right)^{-1} C.$$

Plugging these two equations into the definition of C, one gets:

$$C = \frac{\left[n\left(\frac{P_H}{P}\right)^{-1} C\right]^n \left[(1-n)\left(\frac{P_F}{P}\right)^{-1} C\right]^{1-n}}{n^n(1-n)^{1-n}} = \left(\frac{P_H}{P}\right)^{-n}\left(\frac{P_F}{P}\right)^{n-1} C.$$

Solving this for P, one finally obtains equation (5):

$$P = P_H^n P_F^{1-n}.$$

Consumption-based Producer Price Index, Demand Curves for Individual Goods The representative domestic household maximizes

$$C_H = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}$$

with respect to $C(h)$ subject to the budget constraint

$$P_H C_H = \int_0^n P(h)C(h)dh.$$

Hence,

$$\Lambda = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}} - \lambda\left[\int_0^n P(h)C(h)dh - P_H C_H\right] \rightarrow \max_{C(h)}$$

$$\Rightarrow \frac{\partial\Lambda}{\partial C(h)} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{1}{\theta-1}} \left(\frac{1}{n}\right)^{\frac{1}{\theta}} C(h)^{-\frac{1}{\theta}} - \lambda P(h) = 0.$$

Solving this expression for $C(h)$, one obtains the subsequent preliminary demand function for individual domestic goods:

$$C(h) = P(h)^{-\theta}\left\{\frac{\lambda}{\left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{1}{\theta-1}} \left(\frac{1}{n}\right)^{\frac{1}{\theta}}}\right\}^{-\theta}.$$

Multiplying the preceding equation with $P(h)$, one obtains:

$$P(h)C(h) = P(h)^{1-\theta}\left\{\frac{\lambda}{\left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{1}{\theta-1}} \left(\frac{1}{n}\right)^{\frac{1}{\theta}}}\right\}^{-\theta}.$$

Taking the integral from 0 to n over both sides of this equation, one gets:

$$\int_0^n P(h)C(h)dh = \int_0^n P(h)^{1-\theta}dh \left\{\frac{\lambda}{\left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{1}{\theta-1}} \left(\frac{1}{n}\right)^{\frac{1}{\theta}}}\right\}^{-\theta},$$

with $\int_0^n P(h)C(h)dh = P_H C_H$. Now combine the preceding equation with the preliminary demand function from above. Then one gets for $C(h)$:

$$C(h) = P(h)^{-\theta} \frac{P_H C_H}{\int_0^n P(h)^{1-\theta} dh}.$$

Plugging this into the definition of C_H, one gets:

$$C_H = \left\{ \left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n \left[P(h)^{-\theta} \frac{P_H C_H}{\int_0^n P(h)^{1-\theta} dh} \right]^{\frac{\theta-1}{\theta}} dh \right\}^{\frac{\theta}{\theta-1}}.$$

Dividing this formula by C_H and raising both sides of the resulting equation to the power of $(\theta-1)/\theta$, I obtain:

$$1^{\frac{\theta-1}{\theta}} = \left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n \left[P(h)^{-\theta} \frac{P_H}{\int_0^n P(h)^{1-\theta} dh} \right]^{\frac{\theta-1}{\theta}} dh,$$

which can be solved for P_H to finally obtain the domestic PPI given by equation (6):

$$P_H = \left[\frac{1}{n} \int_0^n P(h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}}.$$

Plugging this formula into the last given equation in $C(h)$, one eventually gets equation (10):

$$C(h) = \frac{1}{n} \left[\frac{P(h)}{P_H} \right]^{-\theta} C_H.$$

A.2. First Order Conditions for a Utility Maximum

The representative household maximizes

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\rho}}{1-\rho} + \frac{\chi}{1-\varepsilon} \left(\frac{M_s}{P_s}\right)^{1-\varepsilon} - \frac{\gamma}{1-\xi} L_s^{1-\xi} \right] \right\}$$

with respect to the decision variables C_t, M_t, B_t, L_t subject to the intertemporal budget constraint (in real terms)

$$\frac{W_t}{P_t} L_t + (1+i_{t-1}) \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \frac{\Gamma_t(h)}{P_t} = C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} + \tau_t.$$

Hence,

$$\begin{aligned} \Lambda_t \quad &= \quad E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\rho}}{1-\rho} + \frac{\chi}{1-\varepsilon} \left(\frac{M_s}{P_s}\right)^{1-\varepsilon} - \frac{\gamma}{1-\xi} L_s^{1-\xi} \right] \right. \\ &- \quad \left. \lambda_s \left[\frac{W_s}{P_s} L_s + (1+i_{s-1}) \frac{B_{s-1}}{P_s} + \frac{M_{s-1}}{P_s} + \frac{\Gamma_s(h)}{P_s} - C_s - \frac{M_s}{P_s} - \frac{B_s}{P_s} - \tau_s \right] \right\} \rightarrow \max_{C_t, M_t, B_t, L_t, \lambda_t} \end{aligned}$$

with $\{\lambda\}_{s=t}^{\infty}$ denoting a sequence of Lagrange multipliers.

$$
\begin{aligned}
\Rightarrow \frac{\partial \Lambda_t}{\partial C_t} &= C_t^{-\rho} - \lambda_t(-1) = 0, \\
\frac{\partial \Lambda_t}{\partial M_t} &= \chi \left(\frac{M_t}{P_t}\right)^{-\varepsilon} \frac{1}{P_t} - \lambda_t \left(-\frac{1}{P_t}\right) - \beta E_t \left[\frac{\lambda_{t+1}}{P_{t+1}}\right] = 0, \\
\frac{\partial \Lambda_t}{\partial B_t} &= -\lambda_t \left(-\frac{1}{P_t}\right) - \beta(1+i_t) E_t \left[\frac{\lambda_{t+1}}{P_{t+1}}\right] = 0, \\
\frac{\partial \Lambda_t}{\partial L_t} &= -\gamma L_t^{-\xi} - \lambda_t \frac{W_t}{P_t} = 0, \\
\frac{\partial \Lambda_t}{\partial \lambda_t} &= -\left[\frac{W_t}{P_t} L_t + (1+i_{t-1}) \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \frac{\Gamma_t(h)}{P_t} - C_t - \frac{M_t}{P_t} - \frac{B_t}{P_t} - \tau_t\right] = 0.
\end{aligned}
$$

From the first partial derivative one obtains $C_t^{-\rho} = -\lambda_t$ and therefore $C_{t+1}^{-\rho} = -\lambda_{t+1}$. Plugging this into the fourth one, one gets equation (20):

$$
\gamma \frac{L_t^{-\xi}}{C_t^{-\rho}} = \frac{W_t}{P_t}.
$$

Now, by using $C_t^{-\rho} = -\lambda_t$ and $C_{t+1}^{-\rho} = -\lambda_{t+1}$, one obtains from the third partial derivative equation (18):

$$
\frac{C_t^{-\rho}}{P_t} = \beta(1+i_t) E_t \left[\frac{C_{t+1}^{-\rho}}{P_{t+1}}\right].
$$

Finally, plugging the above expression into the second partial derivative, one obtains equation (19):

$$
\chi \frac{\left(\frac{M_t}{P_t}\right)^{-\varepsilon}}{C_t^{-\rho}} = \frac{i_t}{1+i_t}.
$$

A.3. Equilibrium Conditions on World Bond and Goods Markets

The subsequent derivation is based on Obstfeld/Rogoff (2001, pp. 7-9), which itself is based on reasoning by Corsetti/Pesenti (2001, pp. 430-433).

Start with the market clearing condition for a single good z:

$$
Y_t(z) = nC_t(z) + (1-n)C_t^*(z).
$$

Assuming, for instance, that good z is a typical domestic good such that $z = h \in [0, n]$ and multiplying the preceding equation with $P_t(h)$ one obtains:

$$
P_t(h)Y_t(h) = nP_t(h)C_t(h) + (1-n)P_t(h)C_t^*(h).
$$

Taking the integral from 0 to n and using equations (6) and (10) yields:

$$
\int_0^n P_t(h)Y_t(h)dh = nP_{t,H}C_{t,H} + (1-n)P_{t,H}C_{t,H}^*.
$$

Because of equations (12) and (14) this expression implies:

$$
\int_0^n P_t(h)Y_t(h)dh = n^2 P_t C_t + (1-n)nP_t C_t^* = nP_t C_t^w,
$$

where the right-hand side of the above equation denotes global demand for domestic goods in domestic currency. Since Y denotes domestic per-capita output, the left-hand side of the equation can alternatively be written as $nP_{t,H}Y_t$, which yields the subsequent equilibrium condition on the world market for domestic goods (26):

$$P_{t,H}Y_t = P_t C_t^w.$$

Note that the equilibrium condition on the world market for foreign goods (27) can be derived analogously.

Both equations immediately collapse to the definition of the TOT given by equation (28):

$$T_t := \frac{P_{t,F}}{P_{t,H}} = \frac{S_t P_{t,F}^*}{P_{t,H}} = \frac{Y_t}{Y_t^*}.$$

Furthermore, substituting equation (22) for the household's instantaneous profits into the intertemporal budget constraint (17) we get:

$$(1+i_{t-1})B_{t-1} + M_{t-1} + P_t(h)Y_t(h) = P_t C_t + M_t + B_t + P_t \tau_t.$$

Integrating from 0 to n and using $\int_0^n P_t(h)Y_t(h)dh = nP_{t,H}Y_t$ one obtains:

$$(1+i_{t-1})B_{t-1} + M_{t-1} + P_{t,H}Y_t = P_t C_t + M_t + B_t + P_t \tau_t.$$

Due to the government's budget constraint (36) the preceding equation rearranges to the domestic balance of payments identity (29):

$$P_{t,H}Y_t - P_t C_t + i_{t-1}B_{t-1} \equiv B_t - B_{t-1}.$$

Note that the foreign balance of payments identity (30) can be derived analogously.

A.4. Dynamic IS Curves

First rewrite the domestic Euler equation for real consumption (18) as follows:

$$C_t^{-\rho} = \beta(1+i_t)P_t E_t \left[\frac{C_{t+1}^{-\rho}}{P_{t+1}} \right].$$

After having done so, use the condition for domestic goods market clearing (34) into the preceding equation:

$$(T_t^{n-1}Y_t)^{-\rho} = \beta(1+i_t)P_t E_t \left[\frac{(T_{t+1}^{n-1}Y_{t+1})^{-\rho}}{P_{t+1}} \right].$$

The non-stochastic zero-inflation steady-state version of this equations reads as follows:

$$(\bar{T}^{n-1}\bar{Y})^{-\rho} = \beta(1+\bar{i})\bar{P}\frac{(\bar{T}^{n-1}\bar{Y})^{-\rho}}{\bar{P}}.$$

The ratio of the last two equations then reads:

$$\left(\frac{T_t^{n-1}Y_t}{\bar{T}^{n-1}\bar{Y}} \right)^{-\rho} = \frac{1+i_t}{1+\bar{i}} P_t \frac{E_t \left[\frac{(T_{t+1}^{n-1}Y_{t+1})^{-\rho}}{P_{t+1}} \right]}{(\bar{T}^{n-1}\bar{Y})^{-\rho}}.$$

By taking the natural logarithm of this ratio, one obtains:

$$-\rho[(n-1)t_t + y_t - (n-1)\bar{t} - \bar{y}] = \ln(1+i_t) - \ln(1+\bar{i}) + p_t - E_t[p_{t+1}] - \rho\{(n-1)E_t[t_{t+1}] + E_t[y_{t+1}]\} + \rho[(n-1)\bar{t} + \bar{y}].$$

Note that $\ln(1+i_t) \approx i_t$ and $\ln(1+\bar{i}) \approx \bar{i}$. Moreover, the approximation $\ln E_t[\Psi_{t+1}] \approx E_t[\Psi_{t+1}] - 1 = E_t[\Psi_{t+1} - 1] \approx E_t[\ln \Psi_{t+1}] = E_t \psi_{t+1}$ assures for the exchangeability of the ln and expectations operators for a generic random variable Ψ.

Subsequently, $E_t[\pi_{t+1}] := E_t[p_{t+1}] - p_t$ shall be defined as the expected CPI inflation rate in period $t+1$. In addition, let hatted variables denote the percentage deviations from their zero-inflation steady-state values ($\hat{y}_t := y_t - \bar{y}, E_t[\hat{y}_{t+1}] := E_t[y_{t+1}] - \bar{y}, \hat{i}_t := i_t - \bar{i}$).

Taking this into account and cancelling the term $\rho(n-1)\bar{t}$ on both sides, the last equation rearranges to:

$$-\rho(n-1)t_t - \rho y_t + \rho\bar{y} = \hat{i}_t - E_t[\pi_{t+1}] - \rho(n-1)E_t[t_{t+1}] - \rho E_t[y_{t+1}] + \rho\bar{y}.$$

Solving this for $\hat{y}_t$, one finally obtains the domestic dynamic IS curve (39):

$$\hat{y}_t = E_t[\hat{y}_{t+1}] + \frac{1}{\rho}\{E_t[\pi_{t+1}] - \hat{i}_t\} - (1-n)E_t[\Delta t_{t+1}].$$

Note that the foreign dynamic IS curve (40) can be derived analogously.

A.5. New Keynesian Phillips Curves

In period t, a domestic producer willing to reset her price maximizes her expected discounted future profits with respect to $P_t(h)$:

$$E_t\left\{\sum_{s=t}^{\infty}\delta^{s-t}\beta^{s-t}\left(\frac{C_s^w}{C_t^w}\right)^{-\rho}\left[\frac{P_t(h)}{P_{s,H}}Y_s(h) - \kappa_s Y_s(h)\right]\right\} \rightarrow \max_{P_t(h)}$$

$\beta^{s-t}(C_s^w/C_t^w)^{-\rho}$ is a stochastic discount factor, which denotes the marginal rate of substitution of real (world) consumption between periods s and t. Note that here one has made use of equation (23). In case of goods market clearing output of an individual producer equals global demand for the differentiated good ($Y(h) = C^w(h)$). Note further that the condition $P_t(h) = P_s(h)$ during the length of the contract implies for the global demand function (15) for a representative domestic good:

$$C_s^w(h) = \left[\frac{P_t(h)}{P_{s,H}}\right]^{-\theta}\left(\frac{P_{s,H}}{P_s}\right)^{-1}C_s^w.$$

Substituting this into the above equation yields:

$$E_t \quad \left\{\sum_{s=t}^{\infty}(\delta\beta)^{s-t}\left(\frac{C_s^w}{C_t^w}\right)^{-\rho}\left[\left(\frac{P_t(h)}{P_{s,H}}\right)^{1-\theta}\left(\frac{P_{s,H}}{P_s}\right)^{-1}C_s^w - \kappa_s\left(\frac{P_t(h)}{P_{s,H}}\right)^{-\theta}\left(\frac{P_{s,H}}{P_s}\right)^{-1}C_s^w\right]\right\} \rightarrow \max_{P_t(h)}$$

$$\Rightarrow \quad E_t\left\{\sum_{s=t}^{\infty}(\delta\beta)^{s-t}\left(\frac{C_s^w}{C_t^w}\right)^{-\rho}\frac{1}{P_{s,H}}\left[(1-\theta)\left(\frac{P_t(h)}{P_{s,H}}\right)^{-\theta}\left(\frac{P_{s,H}}{P_s}\right)^{-1} + \theta\kappa_s\left(\frac{P_t(h)}{P_{s,H}}\right)^{-\theta-1}\left(\frac{P_{s,H}}{P_s}\right)^{-1}\right]C_s^w\right\} = 0$$

Solving this for $P_t(h)/P_{t,H}$, one gets after some manipulation the subsequent price-setting equation:

$$\frac{P_t(h)}{P_{t,H}} = \frac{\theta}{\theta-1}\frac{E_t\left\{\sum_{s=t}^{\infty}(\delta\beta)^{s-t}\left[\kappa_s\left(\frac{P_{s,H}}{P_{t,H}}\right)^{\theta}\left(\frac{P_{s,H}}{P_s}\right)^{-1}(C_s^w)^{1-\rho}\right]\right\}}{E_t\left\{\sum_{s=t}^{\infty}(\delta\beta)^{s-t}\left[\left(\frac{P_{s,H}}{P_{t,H}}\right)^{\theta-1}\left(\frac{P_{s,H}}{P_s}\right)^{-1}(C_s^w)^{1-\rho}\right]\right\}}.$$

Now consider the case where everybody resets their prices ($\delta = 0$). As each producer charges the same price ($P_H = P(h)$), the above equation collapses to the following :

$$\frac{P_t(h)}{P_{t,H}} = \frac{\theta}{\theta-1}\kappa_t = 1.$$

Again we get the real marginal production cost associated with a flexible-price equilibrium κ^{flex}:

$$\kappa_t^{flex} = \frac{\theta-1}{\theta}.$$

Now let us return to the case of sticky prices ($\delta > 0$). From the domestic PPI (6) one gets the subsequent law of motion:

$$P_{t,H}^{1-\theta} = (1-\delta)P_t(h)^{1-\theta} + \delta P_{t-1,H}^{1-\theta}.$$

Log-linearizing the preceding formula around the zero-inflation steady-state price level $\bar{P}_H$ yields the following percentage deviations:

$$\hat{p}_{t,H} = (1-\delta)\hat{p}_t(h) + \delta\hat{p}_{t-1,H}.$$

Now reformulate the price-setting equation as follows:

$$\begin{aligned}
E_t \quad & \left\{\sum_{s=t}^{\infty}(\delta\beta)^{s-t}\left[\left(\frac{P_{s,H}}{P_{t,H}}\right)^{\theta-1}\left(\frac{P_{s,H}}{P_s}\right)^{-1}(C_s^w)^{1-\rho}\right]\right\}Q_t \\
= \quad & \frac{\theta}{\theta-1}E_t\left\{\sum_{s=t}^{\infty}(\delta\beta)^{s-t}\left[\kappa_s\left(\frac{P_{s,H}}{P_{t,H}}\right)^{\theta}\left(\frac{P_{s,H}}{P_s}\right)^{-1}(C_s^w)^{1-\rho}\right]\right\},
\end{aligned}$$

where $Q_t := P_t(h)/P_{t,H}$.

If one log-linearizes this equation around the zero-inflation steady-state, one finally obtains the subsequent percentage deviations ($\bar{Q} = 1, [\theta/(\theta-1)]\kappa_t^{flex} = 1$):

$$\begin{aligned}
\ln \quad & \left[\frac{(\bar{C}^w)^{1-\rho}}{1-\delta\beta}\right] \\
+ \quad & \frac{1}{\frac{(\bar{C}^w)^{1-\rho}}{1-\delta\beta}}\left\{\frac{(\bar{C}^w)^{1-\rho}}{1-\delta\beta}\hat{q}_t + \sum_{s=t}^{\infty}(\delta\beta)^{s-t}(\bar{C}^w)^{1-\rho}[(1-\rho)\hat{c}_s^w + (\theta-1)(E_t[\hat{p}_{s,H}] - \hat{p}_{t,H}) + (-1)(E_t[\hat{p}_{s,H}] - E_t[\hat{p}_s])]\right\} \\
= \quad & \ln\left[\frac{(\bar{C}^w)^{1-\rho}\frac{\theta}{\theta-1}\kappa_t^{flex}}{1-\delta\beta}\right] \\
+ \quad & \frac{1}{\frac{(\bar{C}^w)^{1-\rho}}{1-\delta\beta}}\left\{\sum_{s=t}^{\infty}(\delta\beta)^{s-t}(\bar{C}^w)^{1-\rho}[(1-\rho)\hat{c}_s^w + E_t[\hat{\kappa}_s] + \theta(E_t[\hat{p}_{s,H}] - \hat{p}_{t,H}) + (-1)(E_t[\hat{p}_{s,H}] - E_t[\hat{p}_s])]\right\},
\end{aligned}$$

where most of the terms cancel out.

Solving the remainder for $\hat{q}_t + \hat{p}_{t,H}$, one gets:

$$\begin{aligned}
\hat{q}_t + \hat{p}_{t,H} \quad &= \quad (1-\delta\beta)\sum_{s=t}^{\infty}(\delta\beta)^{s-t}\{E_t[\hat{p}_{s,H}] + E_t[\hat{\kappa}_s]\} \\
&= \quad (1-\delta\beta)(\hat{p}_{t,H} + \hat{\kappa}_t) + \delta\beta\{E_t[\hat{q}_{t+1}] + E_t[\hat{p}_{t+1,H}]\} \\
\Leftrightarrow \hat{q}_t \quad &= \quad (1-\delta\beta)\hat{\kappa}_t + \delta\beta\{E_t[\hat{q}_{t+1}] + E_t[\pi_{t+1,H}]\},
\end{aligned}$$

where $E_t[\pi_{t+1,H}] := E_t[\hat{p}_{t+1,H}] - \hat{p}_{t,H}$. Due to $\hat{q}_t := \hat{p}_t(h) - \hat{p}_{t,H}$ and $\hat{p}_t(h) = [1/(1-\delta)]\hat{p}_{t,H} - [\delta/(1-\delta)]\hat{p}_{t-1,H}$, it follows that $\hat{q}_t = [\delta/(1-\delta)]\pi_{t,H}$. Plugging this result into the above equation one finally obtains the domestic NKPC (42):

$$\pi_{t,H} = \beta E_t[\pi_{t+1,H}] + \frac{(1-\delta)(1-\delta\beta)}{\delta}\hat{\kappa}_t.$$

Note that the foreign NKPC (43) can be derived analogously.

A.6. Dynare Program Code for Matlab

```
periods 1000;

var x_d x_f pi_d pi_f t i_d i_f a_d a_f u_d u_f v_d v_f;

varexo e_af e_ad e_ud e_uf e_vd e_vf;

parameters ALPHA_D ALPHA_F MU_D MU_F OMEGA_D OMEGA_F THETA_D THETA_F IOTA_F XI RHO
BETA ZETA_AD ZETA_AF ZETA_UD ZETA_UF ZETA_VD ZETA_VF PHI_AD PHI_AF;

ALPHA_D=1.5;
ALPHA_F=1.5;
MU_D=.16;
MU_F=.16;
OMEGA_D=.1;
OMEGA_F=.1;
THETA_D=.07;
THETA_F=-.07;
IOTA_F=.5;
XI=-1;
RHO=.8;
BETA=.97;
ZETA_AD=.8;
ZETA_AF=.8;
ZETA_UD=.8;
ZETA_UF=.8;
ZETA_VD=.8;
ZETA_VF=.8;
PHI_AD=.3;
PHI_AF=.3;

model;
x_d=x_d(+1)+1/RHO*pi_d(+1)-1/RHO*i_d+THETA_D*t(+1)+(XI-1)/(XI-RHO)*a_d(+1)-(XI-1)/(XI-RHO)*a_d;
x_f=x_f(+1)+1/RHO*pi_f(+1)-1/RHO*i_f+THETA_F*t(+1)+(XI-1)/(XI-RHO)*a_f(+1)-(XI-1)/(XI-RHO)*a_f;
pi_d=BETA*pi_d(+1)+MU_D*x_d+u_d;
pi_f=BETA*pi_f(+1)+MU_F*x_f+u_f;
t=pi_f-pi_d+i_d(-1)-i_f(-1);
i_d=ALPHA_D*pi_d+OMEGA_D*i_d(-1)+v_d;
i_f=IOTA_F*x_f+ALPHA_F*pi_f+OMEGA_F*i_f(-1)+v_f;
a_d=ZETA_AD*a_d(-1)+PHI_AD*a_f+e_ad;
a_f=ZETA_AF*a_f(-1)*PHI_AF*a_d+e_af;
u_d=ZETA_UD*u_d(-1)+e_ud;
u_f=ZETA_UF*u_f(-1)+e_uf;
v_d=ZETA_VD*v_d(-1)+e_vd;
v_f=ZETA_VF*v_f(-1)+e_vf;
end;

initval;
x_d=0;
x_f=0;
pi_d=0;
pi_f=0;
t=0;
i_d=0;
i_f=0;
a_d=0;
a_f=0;
u_d=0;
```

```
u_f=0;
v_d=0;
v_f=0;
e_ad=0;
e_af=0;
e_ud=0;
e_uf=0;
e_vd=0;
e_vf=0;
end;

steady;

check;

shocks;
var e_ad=.4;
var e_af=.4;
var e_ud=.4;
var e_uf=.4;
var e_vd=.4;
var e_vf=.4;
end;

stoch_simul(periods=2100);
```

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