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Nonlinear Interest Rate Reaction Functions for the UK*

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Abstract

We empirically analyze Taylor-type equations for short-term interest rates in the United Kingdom using quarterly data from 1970Q1 to 2006Q2. Starting from strong evidence against a simple linear Taylor rule, we model nonlinearities using logistic smooth transition regression (LSTR) models. The LSTR models with time-varying parameters consistently track actual interest rate movements better than a linear model with constant parameters. Our preferred LSTR model uses lagged interest rates as a transition variable and suggests that in times of recessions the Bank of England puts more weight on the output gap and less so on inflation. A reverse pattern is observed in non-recession periods. Parameters of the model change less frequently after 1992, when an inflation target range was announced. We conclude that for the analysis of historical monetary policy, the LSTR approach is a viable alternative to linear reaction functions.

Keywords: interest rate reaction functions, smooth transition regression model, monetary policy

JEL classification: C51, C53, E43, E52, E58

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1 Introduction

Following the work of Taylor (1993), fairly simple linear interest rate reaction functions have been used to analyze and evaluate monetary policy of central banks. There is, however, an ongoing debate on how to model these decisions empirically. In line with the New Keynesian theory discussed in Clarida, Galí & Gertler (1999), a forward-looking approach to estimate a central bank's reaction function is widely used. Alternatives include backward-looking and contemporaneous Taylor rules (see e.g. Gerlach & Schnabel (2000), Gerlach-Kristen (2003), Gerdesmeier & Roffia (2004) and Surico (2003)). Typically, empirical results depend to some extent on the used estimation techniques and sample period. Another serious problem with the empirical results reported in the literature is that parameters from linear models seem to be rather unstable over time (see e.g. Judd & Rudebusch (1998) for the US economy).

A look at the history of monetary policy in the United Kingdom illustrates that the Bank of England's (BoE) policy towards inflation and interest rate setting has quite likely changed over time. Although an inflation reducing policy has been announced in 1976, a specific inflation target range was only introduced after the pound crisis that led to the breakdown of the European Monetary System (EMS) in 1992. While an average target for inflation of 2.5 percent was already officially announced in 1995, the BoE gained operational autonomy to fulfill the inflation target set by the Her Majesty's Treasury (HMT) in May 1997. Since the beginning of 2004, the point target is set to 2 percent. Recent empirical literature (see e.g. Martin & Milas (2004), Kesriyeli, Osborn & Sensier (2004), Dolado, Pedrero & Ruge-Murcia (2004), Assenmacher-Wesche (2006), Cukierman & Muscatelli (2008), Qin & Enders (2008) and Castro (2008)) points out time-varying and nonlinear evidence in the relationship between nominal interest rates set by central banks and deviations of output and inflation from their corresponding target values. Thus, a strictly linear rule-based approach may not adequately reflect the actual interest rate setting behavior of the Bank of England.

To analyze possible changes in monetary policy, Assenmacher-Wesche (2006) uses a Markov-switching approach to estimate central banks' reaction functions of the US, UK and Germany. She models abrupt switches, indicating different reactions dependent on existing inflationary pressure. For the UK, the only regime shift occurs in 1984. Before this, interest rate smoothing and output stabilization characterize the high inflation regime, whereas afterwards high weight is put on inflation stabilization. To incorporate slowly changing behavior, some authors estimate nonlinear Taylor rules using smooth transition regressions (STRs). For instance, Kesriyeli et al. (2004) conduct an analysis for the US, Germany and the United Kingdom using monthly data starting in 1984. In their backward-looking Taylor rules, the variable that governs the speed of transition between monetary regimes (the transition variable) is either the first difference of lagged interest rates or a linear trend. Qin & Enders (2008) analyze univariate, linear and nonlinear Taylor rules (the latter using STR and again lagged interest rates as transition variable) for US real-time data. Their exercise reveals that nonlinearities matter over different subsamples. Martin & Milas (2004) estimate a logistic smooth transition regression (LSTR) model and focus on determining varying behavior of the Bank of England induced by inflation changes. They find stronger reactions on increasing inflation if inflation is above the target than on decreasing inflation below the target after 1992. Moreover, they detect a smaller influence of inflation on the interest rate before 1992. Castro

(2008) also uses an LSTR model with inflation as transition variable to confirm that the BoE keeps inflation in between a target range of 1.8 - 2.4 percent rather than following a point target. In a recent paper, Cukierman & Muscatelli (2008) model the central bank's loss function theoretically by allowing for asymmetric preferences. For the period from 1979-2005, they use an STR-type model and confirm recession avoidance preferences before the target was introduced and inflation avoidance preferences afterwards. Dolado et al. (2004) demonstrate theoretically that nonlinearities in the central bank's reaction function occur if either the loss function is non-quadratic or the aggregate supply curve is convex. Similarly, Surico (2004) derives and estimates a model in which nonlinearity arises due to asymmetric central bank's preferences by using a cubic loss function. Thus, nonlinearities in interest rate reaction functions may occur for a number of reasons. For instance, in the UK these nonlinearities may be due to the structural changes related to monetary policy changes that have been going on in the past decades.

The main contribution of this paper is a detailed and thorough empirical analysis on the adequacy of linear and nonlinear Taylor-type rules for interest rate setting in the UK. Our approach differs from the ones mentioned above in various respects. The studies cited above typically estimate one form of a nonlinear reaction function with data from different countries and focus on a comparison of the results across countries. In contrast, we focus on one country and look at a variety of different nonlinear reaction functions. We investigate the evidence for nonlinearities and changing Taylor rule parameters empirically by employing smooth transition regression models (see e.g. Teräsvirta (1998, 2004) and van Dijk, Teräsvirta & Franses (2002)) to estimate the interest rate reaction functions. The STR modeling class allows for smoothly changing parameters and is therefore more flexible than pure switching models. When and how parameters change is governed by the so-called transition variable. In contrast to the existing literature, we document and compare results from different models using a large range of transition variables and base the choice for the transition variables on thorough specification and diagnostic testing. Moreover, unlike most other studies we include Generalized Methods of Moments (GMM) estimation results for nonlinear forward-looking interest rate reaction functions (in the spirit of Clarida, Galí & Gertler (1998)) to reflect that future inflation is the relevant quantity for today's interest rate decision. Finally, compared to the existing literature we cover a longer sample period by using data from 1970Q1-2006Q2.

Our empirical analysis suggests that LSTR nonlinearities are present in the data. Our preferred model specification is an LSTR model where the lagged interest rate is used as a transition variable. The parameter changes then essentially depend on the level of interest rates, which in turn can be viewed as an indicator of overall economic conditions and the monetary policy stance in the UK. From this model we indeed find evidence for changing parameters on both, inflation and the output gap. In periods of recessions, the BoE seems to have put more weight on the output gap and less so on inflation. A reverse pattern is observed for non-recession periods. Another interesting observation from our empirical models is that large changes in the parameters occurred more frequently prior to 1992. After this date, which coincides with the introduction of inflation targeting, the parameters of the nonlinear Taylor-type relation change less often. Thus our empirical model is consistent with the fact that monetary policy has not greatly changed after 1992. We also present results of an LSTR model in which the transition depends on a trend term. This type of model captures changes in the central bank's relative preference on inflation and on output over time. Indeed, we find evidence for changing parameters and a tendency towards a more stringent monetary policy since the beginning of the nineties. Finally, we consider a model with the output gap as a transition variable, explicitly reflecting the fact that monetary policy may be different in expansionary and recessionary periods. Again, similar to the findings for the lagged interest rate model, we find evidence that the BoE's policy depends on the state of the economy. Our results indicate that most considered nonlinear models outperform the simple linear specification in terms of model fit and the ability to track the actual interest rate. Overall, we find for the UK, that the smooth transition regression approach of this paper is a viable alternative to the widely used linear Taylor-type rules if interest is in the analysis of historical monetary policy.

The remainder of the paper is organized as follows. Section 2 introduces the smooth transition regression modeling framework and the empirical equations for nonlinear interest rate reaction functions. Section 3 presents the empirical results for UK data together with the economic interpretation of our results. Finally, Section 4 concludes.

2 The Modeling Framework

Since the seminal paper by Taylor (1993) the nominal interest rate set by central banks is often assumed to depend on the output gap and on inflation. Our starting point is the forward-looking Taylor-type reaction function (see e.g. Clarida et al. (1998)), where the nominal interest rate r_t^* depends on the deviation from an inflation target, $\mathrm{E}[\pi_{t+1}|\Omega_t]-\pi^*$, and on the output gap, $\mathrm{E}[y_t|\Omega_t]-y_t^*$. Let $\bar{r}$ denote the long-run equilibrium rate, then r_t^* can be expressed as

$$r_t^* = \bar{r} + \beta(\mathrm{E}[\pi_{t+1}|\Omega_t] - \pi^*) + \gamma(\mathrm{E}[y_t|\Omega_t] - y_t^*) \tag{2.1}$$

Following Clarida et al. (1998), we define $\alpha = \bar{r} - \beta\pi^*$ and $y_t^{\text{gap}} = y_t - y_t^*$ to write (2.1) as

$$r_t^* = \alpha + \beta\mathrm{E}[\pi_{t+1}|\Omega_t] + \gamma\mathrm{E}[y_t^{\text{gap}}|\Omega_t] \tag{2.2}$$

In empirical specifications, additional terms are needed to account for the fact that interest rate changes are smooth. Thus a typical specification assumes that the actual rate adjusts only partially according to

$$r_t = \sum_{j=1}^{J} \rho_j r_{t-j} + \left(1 - \sum_{j=1}^{J} \rho_j\right) r_t^* + e_t \tag{2.3}$$

in which e_t is an iid innovation that is assumed to represent exogenous shocks to the interest rates and $J = 1$ or $J = 2$ is chosen depending on the particular empirical implementation. For estimation, equations (2.2) and (2.3) are combined to obtain the so-called reduced form model

$$r_t = \sum_{j=1}^{J} \rho_j r_{t-j} + \alpha^* + \beta^*\pi_{t+1} + \gamma^* y_t^{\text{gap}} + e_t, \tag{2.4}$$

in which $\alpha^* = (1 - \sum_{j=1}^{J} \rho_j)\alpha$, $\beta^* = (1 - \sum_{j=1}^{J} \rho_j)\beta$, $\gamma^* = (1 - \sum_{j=1}^{J} \rho_j)\gamma$, and inflation and output gap expectations have been replaced by realized values. In this paper, we extend this linear specification by introducing a smooth transition regression model. The STR model is discussed in detail in Teräsvirta (1998, 2004) and applied to our setting allows to model smooth changes in the reaction function of the central bank. To be more specific, we start with a model where we allow all coefficients to vary over time, including those of lagged interest rates. For this purpose, we introduce a nonlinear term in (2.4) such that the model is written as

$$r_t = \sum_{j=1}^{J} \rho_{0j} r_{t-j} + \alpha_0^* + \beta_0^* \pi_{t+1} + \gamma_0^* y_t^{\text{gap}} + \left[\sum_{j=1}^{J} \rho_{1j} r_{t-j} + \alpha_1^* + \beta_1^* \pi_{t+1} + \gamma_1^* y_t^{\text{gap}} \right] G(s_t, \theta, c) + \varepsilon_t. \tag{2.5}$$

α_0^*, β_0^*, γ_0^* and $\rho_{0j}, j = 1, 2$ are the parameters in the linear part of the model and α_1^*, β_1^*, γ_1^* and $\rho_{1j}, j = 1, 2$ are the parameters in the nonlinear part of the model. The error terms ε_t are assumed to be iid $(0, \sigma^2)$. The transition function

$$G(s_t, \theta, c) = \left\{ 1 + \exp \left[-\theta \prod_{k=1}^{K} (s_t - c_k) \right] \right\}^{-1}, \quad K = 1, 2 \tag{2.6}$$

is a logistic function, in which s_t denotes a particular transition variable. Typically, s_t is either an economic variable or a linear deterministic trend. The c_k denote particular threshold values to be determined from the data. Thus, changes in the parameter take place whenever the (economic) transition variable falls above or below a certain threshold. We consider models with $K = 1$ and $K = 2$. For $K = 1$ the parameters may change monotonically depending on the variable s_t. For instance, the parameter on inflation may change from β_0^* to $\beta_0^* + \beta_1^*$. We refer to this model as an LSTR1. For $K = 2$, the parameters change when the transition variable is either below c_1 or above c_2. This model is called the LSTR2 in the following. The parameter $\theta > 0$ governs the speed of transition between two regimes. The smaller θ in equation (2.6), the smoother is the transition between regimes. The speed of transition is not predetermined in this model but estimated from the data. Note that this specification also nests the linear model for the case when the transition function is constant. In the empirical specification, tests for linearity are conducted and for this purpose, it is convenient to rewrite the model (2.5) in compact notation as

$$r_t = \phi' \mathbf{z}_t + \psi' \mathbf{z}_t G(s_t, \theta, c) + \varepsilon_t, \tag{2.7}$$

where for instance for $J = 2$ one has $\mathbf{z}_t = (1, r_{t-1}, r_{t-2}, \pi_{t+1}, y_t^{\text{gap}})'$ as the vector of regressors and the vectors $\phi = (\alpha_0^*, \rho_{01}, \rho_{02}, \beta_0^*, \gamma_0^*)'$ and $\psi = (\alpha_1^*, \rho_{11}, \rho_{12}, \beta_1^*, \gamma_1^*)'$ contain the parameters from the linear and the nonlinear part, respectively. This modeling framework provides a fairly flexible way to model possible nonlinearities in the central bank's reaction function. The choice of the transition variable s_t as well as the number of regimes is an empirical question and is therefore discussed in the empirical analysis in the Section 3.2. LSTR models are estimated by conditional maximum likelihood (CML), where a grid search determines initial values for the coefficients θ and c_k. Since we estimate forward-looking Taylor rules, we check the robustness of our results by also estimating models by instrumental variable estimation via GMM.[1] The estimated LSTR models are evaluated using a battery of standard diagnostic tests to check for remaining autocorrelation,

[1] JMulTi (www.jmulti.com) and OxMetrics 6.0 have been used for the computations of the empirical analysis.

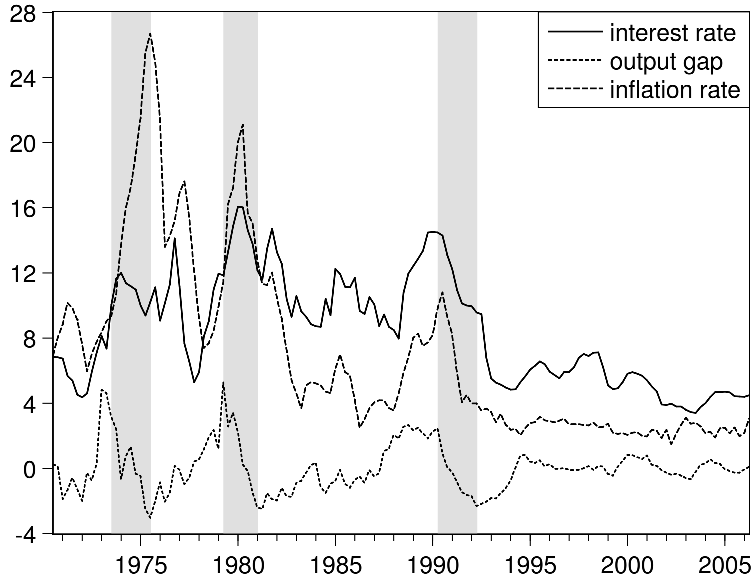

Figure 1: Plots of the Treasury bill rate (solid line), the inflation rate (long-dashed line) and the output deviations from the HP-filtered real GDP (short-dashed line) for the UK, 1970Q1-2006Q2. The shaded areas indicate times of recession following Krolzig & Toro (2002).

ARCH effects and non-normality in the residuals. Moreover, we check the model adequacy by tests for remaining additive nonlinearity and parameter constancy tests (see e.g. Teräsvirta (2004) for a detailed account on these tests).

3 Empirical Analysis

3.1 The Data

To estimate the interest rate setting rule discussed above for the UK, we use quarterly data for a sample period of 1970Q1-2006Q2. The three-month Treasury bill rate provided by the IMF-IFS database is used as the short-run nominal interest rate r_t. Inflation π_t is calculated as a year-to-year change $\pi_t = 100 \cdot (P_t - P_{t-4})/P_{t-4}$ of the retail price index (RPI), denoted by P_t. Since 1992 the BoE reports the retail price index without mortgage prices (RPIX). Thus, we construct the inflation series using the RPI until 1992Q3 and the RPIX afterwards. Both series are taken from the EcoWin Economics database. The output gap y_t^{gap} is constructed by using HP-filtered real GDP as measure for trend output (smoothing parameter λ =1600) and substracting it from the actual GDP series. Quarterly real GDP data are taken from the OECD Main Economic Indicators. Figure 1 shows the seasonally adjusted series for interest and inflation rate as well as for the output gap. While these series show some persistence, we follow the standard practice in this literature and do not consider the possibility of unit roots in the interest rate and the inflation rate as they do not seem to be plausible from an economic point of view. Besides domestic variables, we also include the US federal funds rate ffr_t in some of our models to account for foreign effects.

3.2 Testing for STR nonlinearities

Table 1 shows least squares (LS) and GMM estimation results for the linear interest rate rule given in (2.4), which has been augmented by the federal funds rate. For GMM estimation we follow the standard practice and include lagged values of the regressors as instrumental variables, which is also supported by the corresponding J-test.[2] To get a first impression of whether parameters vary over time we report estimation results for two different periods. Besides using the full sample period, we include a reduced sample starting in 1978Q1 as the period before is characterized by stagflation and large interest rate fluctuations due to the first oil crisis. The estimated coefficients in Table 1 for the full sample (1970Q1-2006Q2) and the reduced sample (1978Q1-2006Q2) indicate that the BoE has looked at both, inflation and output gaps. Moreover, external influences (picked up by the federal funds rates) have also some importance.[3] However, the estimated structural coefficients on the inflation rate are below unity and thus imply no influence of inflation on the real interest rate decision of the BoE (see e.g. Clarida et al. (1998)). Moreover, the reduced form inflation coefficients are no longer significant if GMM is used. Apart from this, LS and GMM estimates are fairly similar. Interestingly, the coefficients on both, inflation and output gap seem to be different in the two considered subperiods, where e.g. the output coefficient γ has increased considerably in the second subperiod. This may be a first indication of the fact that the parameters of the estimated linear models are not time-invariant over the entire sample period. This view is also supported by bootstrap versions of a Chow sample split and break point tests (see Candelon & Lütkepohl (2001) for details) when a one-time change in the parameters is tested in 1978Q1. The null hypothesis of constant parameters in the linear model is clearly rejected. Therefore, we take a closer look at these parameter changes and explore whether they may be captured by nonlinear LSTR models with constant parameters.

To detect nonlinear patterns in the form of equation (2.5), we perform LM-type linearity test. Details on this approach are given in Teräsvirta (1998, 2004). Since the model is only identified under the alternative of nonlinearity as written in equation (2.5), a third-order Taylor approximation around $\theta = 0$ is done for $G^*(\cdot) = G(\cdot) - 1/2$ because for $\theta \to 0$, $G(\cdot) \to 1/2$. From the Taylor expansion one obtains

$$r_t = \delta_0' \mathbf{z}_t + \sum_{j=1}^{3} \delta_j' \tilde{\mathbf{z}}_t s_t^j + \varepsilon_t^*, \quad \varepsilon_t^* = \varepsilon_t + remainder. \tag{3.1}$$

$\tilde{\mathbf{z}}_t$ denotes the vector of variables in $\mathbf{z}_t$ without the constant when s_t is an element of $\mathbf{z}_t$. Under the null hypothesis of linearity, $\delta_j = 0 \; \forall \; j$; under the alternative $\delta_j \neq 0$ for at least one j. The test procedure considers each regressor as a candidate transition variable and is implemented as an F-significance test. In case of rejecting the null for several specifications, we tend to use the variable with the strongest rejection of the null (with the lowest p-value). The test results for two different subsamples are given in Table 2.

[2]Forward-looking Taylor rules are commonly estimated using GMM estimation due to the involved expectations. However, GMM results reported in the literature depend to some extent on the choice of instruments. Moreover, LS results are often very similar to results from IV estimation (see for example Clarida et al. (1998)). Therefore, we focus on the discussion of LS results in the following but point out when important differences occur.

[3]We have also tried models that include the Dollar exchange rate, the German call money market rate, German M3 and the conditional variance of inflation. These additional variables turned out not to be very important in our models.

The p-values of the joint significance test are given in the first column denoted by F. To make a decision on the number of regimes K, we consider three other hypotheses for which the p-values of the corresponding F-statistics are given in the columns labeled F4, F3 and F2 in Table 2, respectively. Following Teräsvirta (2004), the three hypothesis are $H_{04} : \ \delta_3 = 0, \ H_{03} : \ \delta_2 = 0|\delta_3 = 0$ and $H_{02} : \ \delta_1 = 0|\delta_2 = \delta_3 = 0$ from equation (3.1), an LSTR1 model would be proposed by the strongest rejection in either H_{04} or H_{02}, whereas the smallest p-value being the one for H_{03} would imply to model nonlinearities via LSTR2 (or exponential STR) models.

We conduct the nonlinearity tests for the two sample periods and report the results in Table 2. They suggest clear evidence of STR-type nonlinearities for a number of transition variables. For both samples, the null of linearity is rejected when lagged interest rate, the output gap or a simple trend is used as a transition variable, respectively. The test strategy described above suggests LSTR1 models (i.e. $K = 1$) for $s_t = r_{t-1}$, and $s_t = t$, while a LSTR2 model with $K = 2$ is suggested for $s_t = r_{t-2}$ in the full sample and for $s_t = y_t^{\text{gap}}$ in the reduced sample. The evidence is mixed for the inflation rate as a transition variable: For the full sample, the tests suggest a linear specification if the inflation rate is used as a transition variable whereas an LSTR1 model is suggested for the shorter subsample. If the federal funds rate is chosen as the transition variable the tests do not find evidence for STR nonlinearities. Note that when a trend is used as a transition variable the corresponding nonlinearity test may be viewed as a type of parameter constancy test. In contrast to a Chow-type test with the alternative of a one-time sudden change of the parameters, here the alternative is of LSTR form. Thus, the rejection of linearity when the trend is used as a transition variable (see last line of Table 2) may indicate model misspecifcation of the linear model in the sense that parameter changes are present that can be captured by LSTR models.

We follow the recommendation of Teräsvirta (2004) and estimate some of the candidate models and check whether they adequately describe the data at hand. Results from some alternative LSTR models are described in the following subsections.

3.3 LSTR Model Results

3.3.1 Lagged Interest Rates as Transition Variables

We first consider LSTR models with lagged interest rates as the transition variable. In these models, parameter changes set in whenever the interest rate drops below or rises above some threshold value. Thus, the timing of the parameter changes depends on the level of interest rates, which may be viewed as an indicator of the state of the economy.

Column (1) in Table 3 shows conditional ML estimates for an LSTR1 model with r_{t-1} as a transition variable and estimation period 1970Q1-2006Q2. Both Taylor rule determinants, inflation and the output gap, enter the nonlinear part significantly. In addition inflation is significant in the linear part. Note, however, that the coefficient on one-period ahead inflation is quite small in the linear part and has a negative sign in the nonlinear part. Very similar coefficient estimates on output and inflation are obtained when considering an LSTR2 model with r_{t-2} as a transition variable (see column (2) of Table 3). Interestingly, in both models the US federal funds rate enters significantly in the linear part. When considering the sample 1978Q1-2006Q2 (see column (3) of Table 3), the estimates of the inflation coefficient increase drastically in the linear part, which may reflect the fact that the BoE has put more weight on inflation in the later sample. In line with the estimates for the full sample, the output deviations are - although with smaller coefficient estimates - still important in the nonlinear part. Interest rate smoothing is significant with similar sum of smoothing coefficients in the linear part for all three model specifications. Note that we have never deleted the key variables, output gap and inflation, from the models when their coefficient estimates turned out to be insignificant in order to facilitate the comparison across model specifications and to avoid problems related to sequential model selection. The estimates of the threshold parameters are informative about interest rate levels where the nonlinear part of the model becomes relevant. For instance, in the LSTR1 model for the full sample (column (1)) the nonlinear part of the model comes into play whenever the interest rate is above 12.2%. In all three models from Table 3 the (standardized) slope coefficient is in the same order of magnitude and thus the speed of transition in the models is similar.[4] The results of the diagnostic tests in Table 4 suggest that apart from some evidence of residual non-normality and the rejection of the null hypothesis in the ARCH-LM test, the LSTR1 and LSTR2 models are all reasonably well specified.[5] In particular, there is not much evidence for remaining nonlinearity of STR-type and parameters seem to be constant. These results indicate that both, the Chow stability test and the linearity tests with a trend as a transition variable (reported in Section 3.2) pick up model misspecification of the linear model and reject stability due to parameter changes that can indeed be captured by LSTR-type models. Based on the results of tests for remaining nonlinearity and parameter constancy tests, we prefer to use the LSTR2 model for the full sample in the following as it seems to be slightly better specified.

To get a better understanding of the nonlinear dynamics in the estimated models we provide a graphical representation of the linear and nonlinear parts of the interest rate as well as of the transition function. For the estimated LSTR2 (full sample) and the LSTR1 (reduced sample) from Table 3 the linear parts of the interest rate together with the actual rate are shown in Panel (a) and (b) of Figure 2. Panel (c) shows the corresponding nonlinear parts from both models, while Panel (d) graphs the values of the transition functions. In both models the transition sets in at the beginning of the second recession in 1979 and shortly before the third one in 1989. Note that the transition starting points are almost identical from both models. In the LSTR2 model regime changes occur regularly and are also relevant after 1992Q3, while in the LSTR1 for the reduced sample no parameter changes occur after 1992. The difference can be explained by the structure of the LSTR2 model, in which the nonlinear part becomes also relevant when the interest rate drops below 4.4% (see $\hat{c}_1$ in Table 3) as in the later part of our sample.

To account for possible endogeneity of regressors we additionally present nonlinear GMM estimates in Table 5.[6] They underline the main findings from Table 3. Compared to that table there are small changes in some of the coefficients. For instance, the inflation parameter estimate $\hat{\beta}_1^*$ is slightly higher in absolute values and is no longer significant in the LSTR1 model for the full

[4]We do not report the standard errors for the slope and threshold parameters as they are typically not accurately estimated when θ is large and thus only a few observations lie in the neighborhood of c_k. Note that even for smaller θ the corresponding standard error does not have the usual 't-ratio' type interpretation due to the mentioned identification problem. See e.g. Teräsvirta (1994) for a discussion of this problem.

[5]The rejection of H_0 in the normality and ARCH-LM test is likely due to some unusually large residuals.

[6]In nonlinear GMM estimation the instrument variables have to be correlated with the gradient of the regression function. Therefore, the set of instruments includes lagged regressors and squared values of these lags.

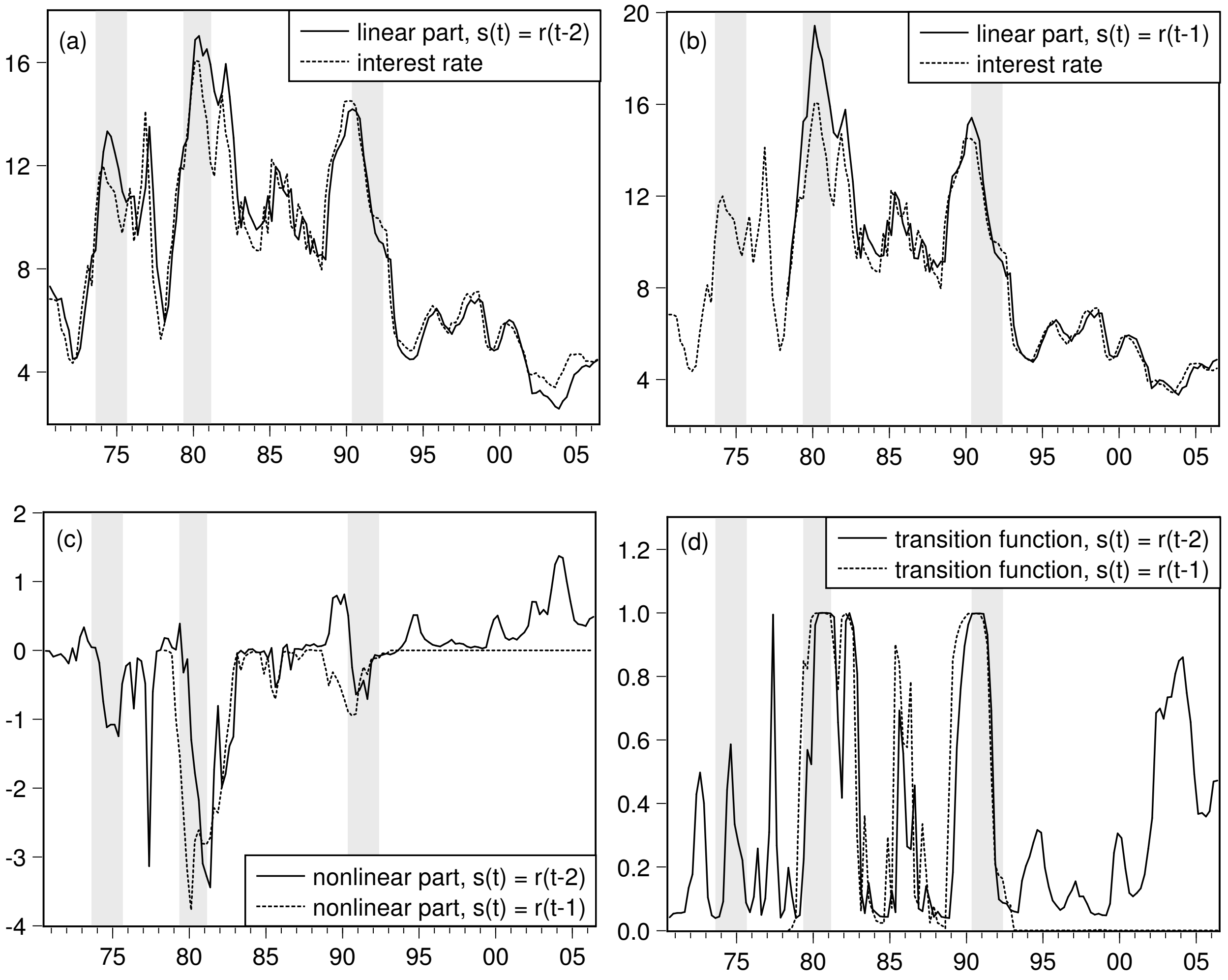

Figure 2: Top row: Linear parts of the interest rate together with actual interest rates from estimated LSTR2 (Panel (a)) and LSTR1 (Panel (b)) models reported in column (3) and (2) of Table 3. Bottom row: Nonlinear parts of the interest rate (Panel (c)) and transition functions (Panel (d)) from estimated LSTR models. Solid line: LSTR2 for 1970Q1-2006Q2, column (2), dashed line: LSTR1 for 1978Q1-2006Q2, column (3) of Table 3.

sample. Moreover, the output coefficient is no longer significant in the LSTR2 specification for the full sample. The role of interest rate smoothing diminishes in the linear parts compared to Table 3, but lagged interest rates enter the nonlinear part significantly in columns (1) and (3). Moreover, the threshold parameters $\hat{c}_1$ and $\hat{c}_2$ are close to the results above in any specification. In all three models the transitions between regimes are steeper than in those estimated with conditional maximum likelihood. This is reflected in larger slope coefficient estimates $\hat{\theta}/\hat{\sigma}_s$. Thus, the corresponding graphical representation of the GMM results indicates almost identical regimes with, however, much steeper transitions. The corresponding graphs are not shown to conserve space.

To shed some more light on the relative gains from using the nonlinear models, we report relative root mean squared errors (RMSEs) to compare linear and nonlinear specifications. To be more precise, we compute

$$\text{Rel.RMSE} = \sqrt{\frac{\sum_{t=1}^{T}(r_t - \hat{r}_{t,\text{nonlin}})^2}{\sum_{t=1}^{T}(r_t - \hat{r}_{t,\text{lin}})^2} \cdot \frac{(T - k_{\text{lin}})}{(T - k_{\text{nonlin}})}}. \tag{3.2}$$

r_t denotes the actual interest rate, $\hat{r}_{t,\text{nonlin}}$ and $\hat{r}_{t,\text{lin}}$ denote the fitted values for a particular nonlinear

STR model and the linear model, respectively. To make a fair comparison, we also correct for the different numbers of parameters to be estimated in linear and nonlinear models. The results for all three nonlinear LSTR models are quite clear-cut. The RMSEs relative to their linear counterparts for models (1), (2) and (3) of Table 3 are given by 0.955, 0.923 and 0.949, respectively. In other words all values are below 1. Thus, we conclude that the in-sample model fit of our nonlinear specifications is superior to the simple linear model. A possible reason is, of course, that the linear model does not capture parameter changes due to structural breaks.

Next we turn to the economic interpretation of our results. We consider the evolution over time of the key parameters in the Taylor-type equation. For this purpose we focus on the evolution of the coefficients on inflation and the output gap in the structural form of our Taylor specifications.[7] Thus, we need to calculate back the structural form parameters from the reduced form estimation results. Due to the nonlinearity of the model, the structural coefficients are made up of the linear and the nonlinear part. Note that in contrast to equation (2.4), in (2.5) the interest rate smoothing term is extended by a nonlinear part, too, for which has to be accounted in the structural coefficients such that

$$\beta_{0t} = \frac{\beta_0^*}{(1 - \sum_j \rho_{0j} - \sum_j \rho_{1j} G(s_t, \theta, c))}, \quad \beta_{1t} = \frac{\beta_1^*}{(1 - \sum_j \rho_{0j} - \sum_j \rho_{1j} G(s_t, \theta, c))} \tag{3.3}$$

and

$$\gamma_{0t} = \frac{\gamma_0^*}{(1 - \sum_j \rho_{0j} - \sum_j \rho_{1j} G(s_t, \theta, c))}, \quad \gamma_{1t} = \frac{\gamma_1^*}{(1 - \sum_j \rho_{0j} - \sum_j \rho_{1j} G(s_t, \theta, c))}. \tag{3.4}$$

β_{0t} and γ_{0t} are the parameters from the linear part, which are now time-varying due to time variation in interest rate smoothing. β_{1t} and γ_{1t} denote the parameters from the nonlinear part of the model and finally the overall coefficients are given by

$$\beta_t = \beta_{0t} + \beta_{1t} G(s_t, \theta, c) \tag{3.5}$$

and

$$\gamma_t = \gamma_{0t} + \gamma_{1t} G(s_t, \theta, c). \tag{3.6}$$

Considering now the results based on the preferred LSTR models with lagged interest rate as a transition variable, we give the evolution of the parameters over time in Figure 3. The inflation coefficient is rather small in the beginning of recessionary periods, which are periods characterized by fairly high interest rates. In these times the weight on the output gap increases substantially. Thus, in recessionary periods the BoE seems to put more weight on output, while weight on inflation increases in non-recession periods. The level of the computed structural coefficient estimates is lower considering the full sample period including the struggling times in the 70s. This could be interpreted as hint for a more stringent policy after the time of high inflation. Note that considering the reduced sample period, the coefficients on inflation and output gap fluctuate only in the time before 1992, indicating ongoing changes in the preferences of the central bank (possibly due to

[7]Note that the values of the structural coefficients at time t depend on the realization of the transition variable at this point in time. This is inherent in the structure of the LSTR model and should be considered in the coefficient interpretation.

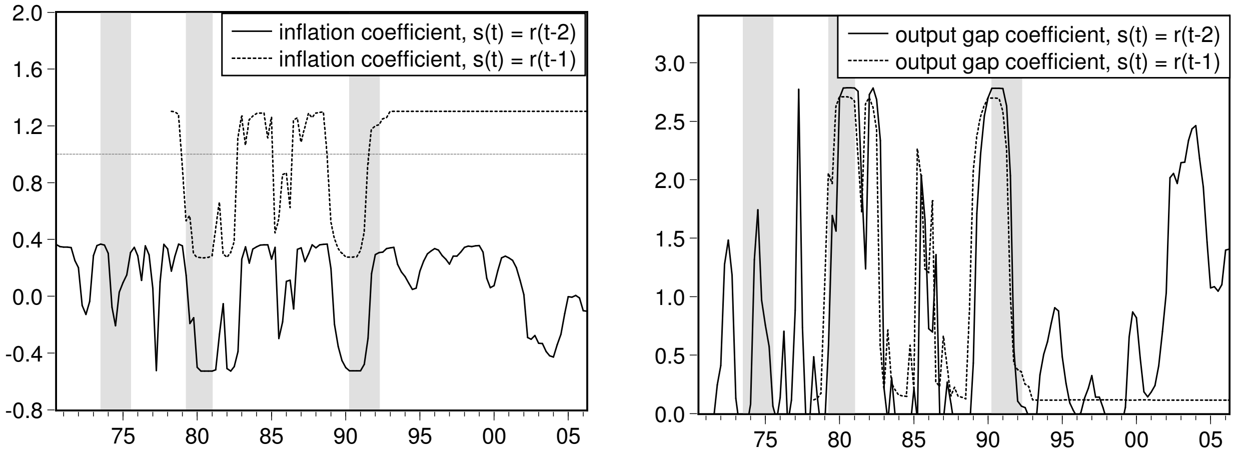

Figure 3: Time-varying coefficients for inflation (β_t) and output gap (γ_t) calculated from equations (3.5) and (3.6) of the estimated LSTR2 and LSTR1 models reported in columns (2) and (3) of Table 3. Sample periods: 1970Q1-2006Q2 (solid line) and 1978Q1-2006Q2 (dashed line).

volatile movements in the economy). Interestingly, after 1992/1993, the coefficients do not change anymore. Thus, in contrast to a linear model, our nonlinear model is able to capture the changing environment at the beginning of our sample period and at the same time also indicates more stable parameters in the recent years. Moreover, the implied weight on inflation for the later periods is such that the Taylor principle is satisfied. The overall results are generally in line with those from GMM estimation.

Related studies in the literature have used (future) inflation rate rather than interest rates as a transition variable. This and the different sample periods makes a direct comparison to e.g. the results of Martin & Milas (2004) and Castro (2008) difficult. Nevertheless, it may be worth pointing out that Martin & Milas (2004) and Castro (2008) find that the parameters of a Taylor rule change even in the time after 1992 in their LSTR2 type models. This seems similar to our results of the LSTR2 but in contrast to what is obtained from the LSTR1 specification. Clearly, the differences can be attributed to using different transition variables. For the sample periods considered in this paper, we have also tried to fit a model as in Martin & Milas (2004). This attempt did not result in an adequately specified model.

3.3.2 Additional Results

LSTR Models with a Trend as a Transition Variable

In addition to the models presented above, we specify LSTR models that allow for a one-time smooth transition between the parameters of two states. This is accomplished by using a trend as a transition function. Such a model allows e.g. for time-varying BoE's attitude towards inflation. We report results for the sample starting in 1978Q1 because fitting an LSTR model for the full sample period with $s_t = t$ did not result in an adequately specified model. In particular, after fitting LSTR models for the full sample period we still find evidence for misspecification such as remaining additive nonlinearities and parameter non-constancy. In the reported model we have excluded the US federal funds rate as it did not enter the model significantly. The results are given in columns

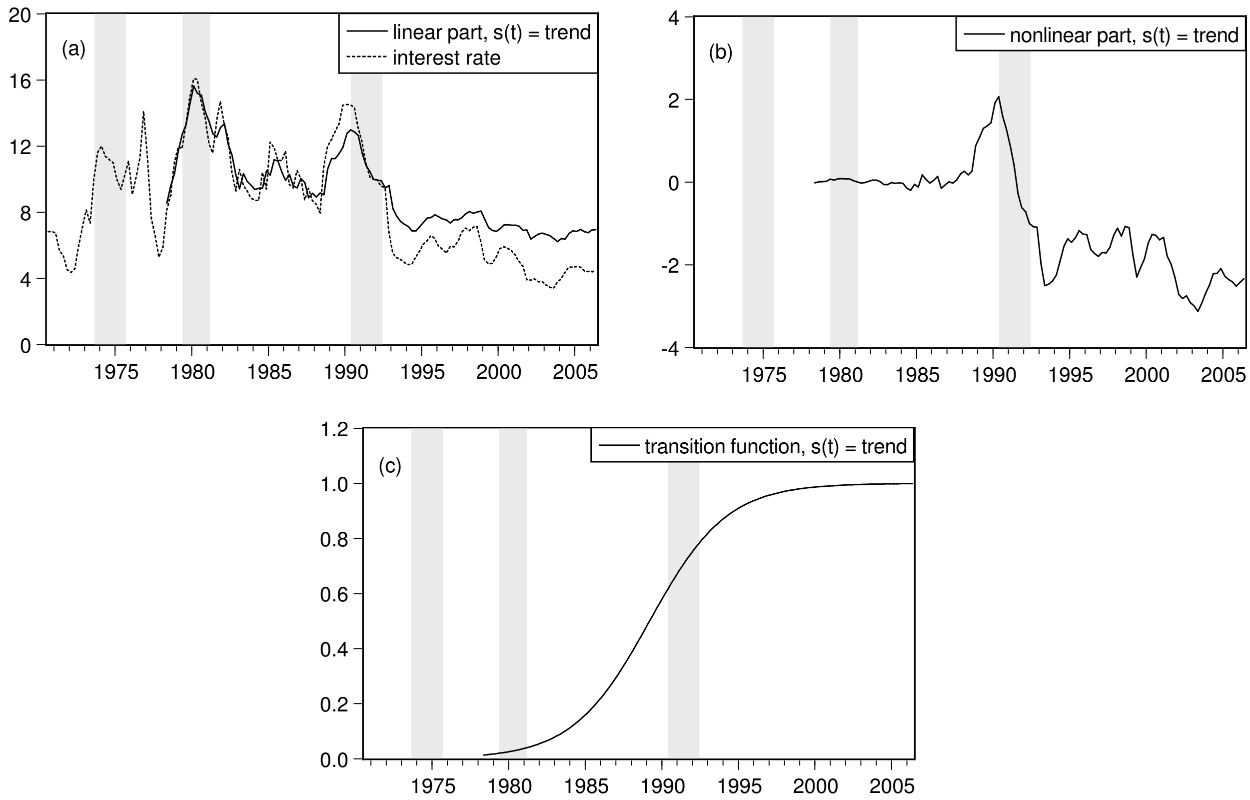

Figure 4: Linear part of the interest rate together with the actual interest rate (Panel (a)), nonlinear part of the interest rate (Panel (b)) and transition function (Panel (c)) of estimated LSTR1 with a trend as a transition variable reported in column (1) of Table 6. Sample period: 1978Q1-2006Q2.

(1) and (2) of Table 6.

Model estimates are reported in column (1) and (2) of Table 6. Focussing on the reported model of column (1), we do neither find evidence for remaining residual autocorrelation and remaining nonlinearity nor evidence against constant parameters (Detailed results are omitted to conserve space.). Thus, the model seems to be well specified. From the parameter estimates we find that the inflation rate enters significantly in the linear part of the model, while it is insignificant in the nonlinear part. In comparison to the linear model (see Table 1), the inflation coefficient in the linear part increased and the output gap coefficient is no longer significant in the linear part but enters the nonlinear part significantly. Thus, we conclude that the nonlinear part contains substantial information. This is also supported by the relative RMSE computed from (3.2) which is 0.885. The role of the nonlinear part is also obvious from Figure 4 in which we have plotted the linear part (with actual interest rate) together with the implied nonlinear part of $\hat{r}_t$ and the transition function. The transition function deviates from zero right at the time of the recession in 1979 and reaches its turning point shortly before the last recession (1990Q2). The final state is not reached before 1999, i.e. just after the introduction of inflation targeting and the EMS II system.

A graphical representation of the two time-varying fundamental parameters obtained from the estimated LSTR model given in column (1) of Table 6 is given in Figure 5. We show the time-varying coefficients on inflation and the output gap. The transition sets in during the mid 80s and the turning point of the coefficient function is exactly at the end of the last recession. The estimated inflation coefficient $\hat{\beta}_t$ increases to above unity (for most specifications), thus implying an

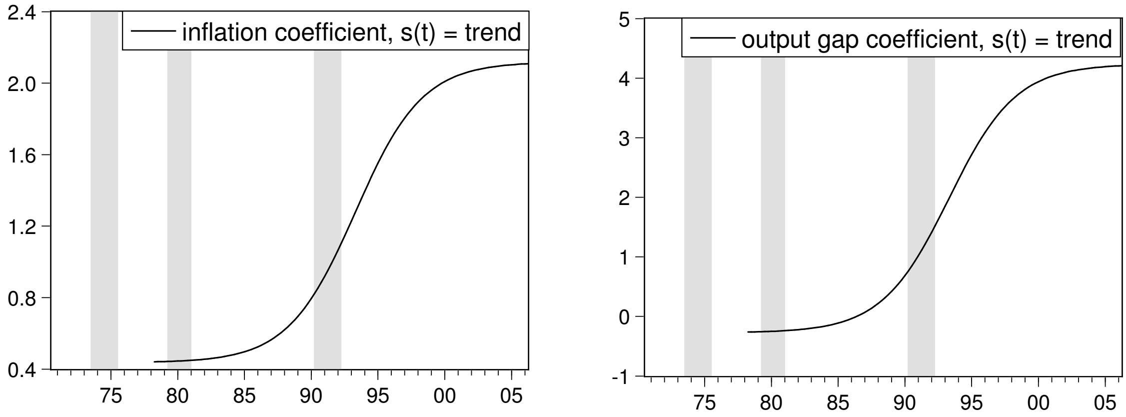

Figure 5: Time-varying coefficients for inflation (β_t) and output gap (γ_t) calculated from equations (3.5) and (3.6) from estimated LSTR1 with a time trend as a transition variable reported in column (1) of Table 6. Sample period: 1978Q1-2006Q2.

effect on the real interest rate in the later periods. The increase of the inflation coefficient over time may be interpreted as representing a more stringent policy of the BoE. Thus, our model reflects the changes in the bank's policy towards inflation and interest rate setting over time. This is for instance reflected by events like the announcement of an inflation target in 1992 after the pound crisis or operational autonomy of the BoE to fulfill the inflation target in 1997. These slow changes in preferences are captured by using a trend as transition variable. The right panel of Figure 5 plots the output gap coefficient obtained from equation (3.6). There is some evidence for the increasing importance of the output gap in the BoE's interest rate setting policy in the later periods. The turning point in the transition function is around the end of the recessionary period of 1990-1992. The GMM estimates reported in column (2) of Table 6 are different from the CML estimates in some respects. In particular, the GMM estimate on the output gap is now significant in the linear part and switches its sign in the nonlinear part. This together with the J-statistic p-value close to 10% gives rise to doubting the GMM estimates and we refrain from interpreting the results.

LSTR models with the Output Gap as a Transition Variable

We also consider LSTR models with the output gap as a transition variable. Such a transition variable may be used to model different interest rate reactions in recessionary and booming periods. Our preferred model is the LSTR2 for the full sample without the federal funds rate. Empirical results for equation (2.5) with $s_t = y_t^{\text{gap}}$ are given in columns (3) and (4) of Table 6. The inflation coefficient is relevant in the linear part and the output gap coefficient enters the linear and nonlinear part with significant coefficient estimates. Moreover, interest rate smoothing is quite important. The residuals of this model reported in column (3) are not autocorrelated but the null of the ARCH-LM test is rejected. Further diagnostic tests reveal no remaining nonlinearity but parameter constancy is rejected on the 5% but not on the 1% level. Thus, this model is not specified adequately in all respects.[8] The nonlinear behavior of this LSTR2 model is summarized

[8]We have also estimated a model for a sample starting in 1978Q1 but find evidence for both, remaining nonlinearities and parameter non-constancy (results not presented).

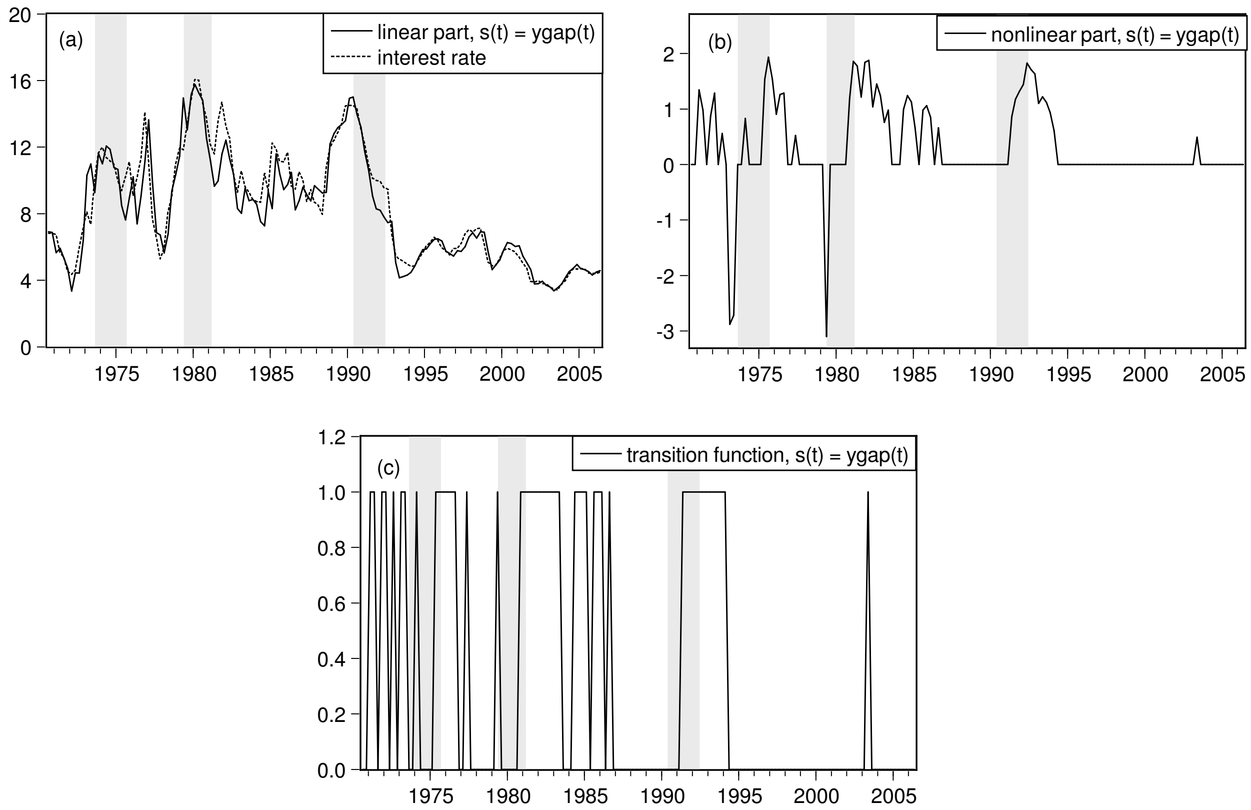

Figure 6: Linear part of the interest rate together with the actual interest rate (Panel (a)), nonlinear part of the interest rate (Panel (b)) and transition function (Panel (c)) of estimated LSTR2 with y_t^{gap} as a transition variable reported in column (3) of Table 6. Sample period: 1970Q1-2006Q2.

in Figure 6. Panel (c) indicates that nonlinearity sets in whenever the economy is right before a recessionary period - which is largely in line with the interest rate models presented above. In addition nonlinearities seem to be important during the eighties. A similar pattern is observed for the corresponding GMM estimates in column (4) but is not shown to conserve space.

In terms of model fit, the relative RMSE for the estimated model in column (3) is 0.942, which indicates that the nonlinear model using the output gap as a transition variable fits the data slightly better than the linear model.

Concerning the economic interpretation of our results we find that the estimated coefficient on inflation obtained from the LSTR2 model shows some small decreases whenever output falls below the trend and we observe in general higher coefficients since 1994. Since the coefficient is smaller than one there will be no influence on the real rate using this target. Regime shifts appear quite often during volatile times in the beginning of the sample until regimes become more stable at the end. The coefficient on output deviates symmetrically around the estimate from the linear model (result not shown). Thus, there is no sign of asymmetric preferences. Essentially, the output coefficient changes from values around zero to large positive values. The switches also occur in the middle of recession. In case of positive output deviations from the trend and increasing inflation, the BoE seems to put more weight on both, inflation and output gap. This is also supported by time-varying coefficients computed from GMM estimates.

To illustrate how different models track the actual interest rate, we recursively calculate the

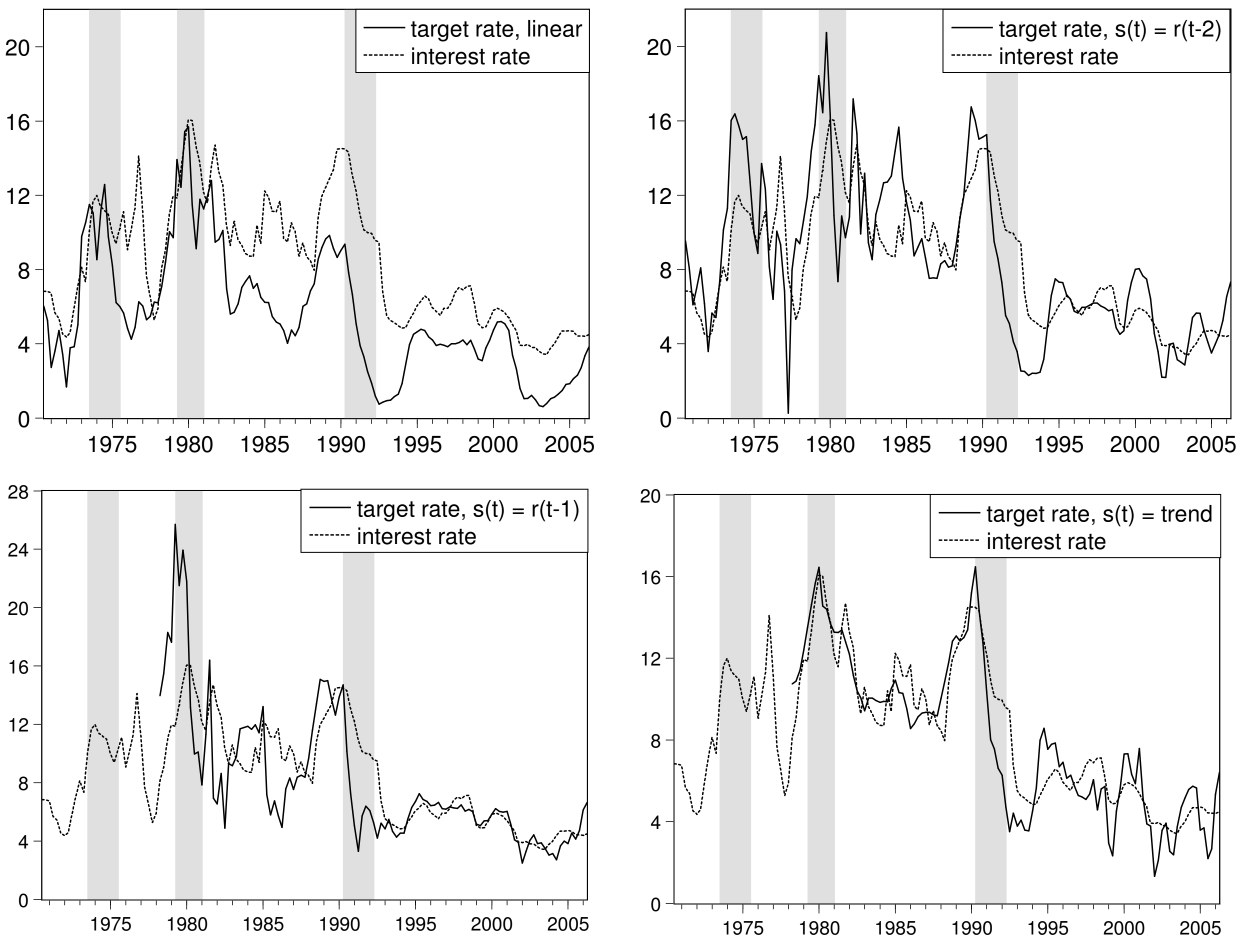

Figure 7: Treasury bill rate r_t (dotted line) and implied nominal target interest rate $\hat{r}_t^*$ (solid line) from linear model (upper left panel), LSTR2 with r_{t-2} as a transition variable (upper right), LSTR1 with r_{t-1} as a transition variable (lower left) and LSTR1 with a time trend as a transition variable (lower right panel). Sample periods: 1970Q1-2006Q2 and 1978Q1-2006Q2.

estimate for the implied target interest rate r_t^*. In analogy to the linear model the implied target rate is determined by plugging in the estimates to

$$r_t^* = \alpha_{0t} + \beta_{0t}\pi_{t+1} + \gamma_{0t}y_t^{\text{gap}} + (\alpha_{1t} + \beta_{1t}\pi_{t+1} + \gamma_{1t}y_t^{\text{gap}})G(s_t, \theta, c) \tag{3.7}$$

Here, following Assenmacher-Wesche (2006), we do not apply interest rate smoothing to make the differences easier to visualize. We find that the nonlinear models outperform the linear ones in the sense that the nonlinear specifications are able to track the actual rate better for most specifications. Figure 7 compares the implied target rates from the linear and nonlinear models with actual rates. The upper left panel shows the actual interest rate and the rate ‘predicted’ by the linear Taylor rule. For most periods the predicted rate from the linear Taylor rule is below the actual rate. In particular, the linear model does not track the actual target well in volatile times. The specifications using lagged interest rate as transition variable (upper right and lower left panel) track the actual interest rate better, except some overshooting visible using the full sample period. The results for the LSTR1 model with a trend as the transition variable (lower right panel) indicate that also this model tracks the actual interest rate quite well. To sum it up, the nonlinear models capture actual interest rate dynamics better than the linear model.

4 Conclusions

Using quarterly UK data for the period 1970Q1-2006Q2 we find strong evidence against a linear Taylor type relation. Nonlinearity tests indicate the possibility of nonlinearities in form of smooth transition regression models. Alternative logistic smooth transition regression models are specified which differ with respect to the chosen transition variable. Most of the considered nonlinear models outperform the simple linear specification in terms of model fit and the ability to track the actual interest rate.

Our overall preferred model specification (based on diagnostic tests) is a model with lagged interest rate as a transition variable. From this model we see changing parameters on both, inflation and the output gap. In periods of recessions, the BoE seems to have put more weight on the output gap and less so on inflation. A reverse pattern is observed for non-recessionary periods. Another interesting observation from this model is that changes in the parameters occur more frequently prior to 1992. After this date, the parameters of the Taylor-type relation have not changed greatly. This is consistent with the fact that monetary policy goals have not greatly altered after 1992.

To analyze the possibility of a more stringent behavior of the BoE when targeting inflation we also use a linear trend as a transition variable, thereby allowing a one-time gradual change of the parameters. From this model we again find evidence for the fact that the BoE's weights on inflation and the output gap have indeed changed over time. Against this background we interpret the Bank of England's failure to bring down inflation rates in the 1970s and 1980s as a result of very low weights on inflation during this period.

A more flexible model that allows for more than one gradual change in the parameters uses the output gap as a transition variable. We find that parameters on inflation and the output gap have changed more frequently in the first part of the sample. This reflects the more volatile economic environment of the 1970s and 1980s and the changing UK monetary policy during that time.

In summary, we find that estimating linear Taylor-type rules with constant parameters is not adequate for UK data. This is particular true for time spans with high interest rate and inflation volatility. Findings based on the linear model may therefore be quite misleading and may lead to inferior interest rate forecasts. For the UK case, the smooth transition regression approach followed in this paper may therefore be a viable alternative for the analysis of historical monetary policy and for forecasting interest rates.

Table 1: Estimation results for the linear reduced from models (2.4) and implied structural form coefficients

	1970Q1:2006Q2		1978Q1:2006Q2	
	LS	GMM	LS	GMM
	reduced form coefficients			
α^*	0.484**	0.443***	0.455**	0.322***
	(0.205)	(0.110)	(0.194)	(0.091)
ρ_1	1.003***	1.029***	0.826***	0.885***
	(0.083)	(0.098)	(0.036)	(0.030)
ρ_2	−0.191**	−0.186*	-	-
	(0.081)	(0.097)		
β^*	0.033**	0.020	0.068*	0.032
	(0.016)	(0.020)	(0.035)	(0.033)
γ^*	0.152***	0.194**	0.228***	0.342***
	(0.056)	(0.090)	(0.062)	(0.052)
η^*	0.129***	0.105***	0.096***	0.051
	(0.033)	(0.032)	(0.035)	(0.031)
	structural form coefficients			
α	2.576***	2.833***	2.611***	2.786***
	(0.884)	(0.606)	(0.854)	(0.604)
β	0.176**	0.129	0.389**	0.274***
	(0.080)	(0.116)	(0.180)	(0.072)
γ	0.807**	1.236*	1.310***	2.966***
	(0.361)	(0.644)	(0.490)	(0.684)
η	0.686***	0.674***	0.548***	0.446*
	(0.132)	(0.150)	(0.185)	(0.253)
$\bar{R}^2$	0.930	0.929	0.955	0.953
$\hat{\sigma}$	0.885	0.900	0.745	0.769
p(J-stat)	-	0.755	-	0.829

Note: The federal funds rate ffr_t is included as an additional regressor (coefficients η^* and η, respectively). Estimated standard errors are given in parentheses. Standard errors for structural coefficients are derived by the delta method. *, **, *** denote significance at the 10%, 5% and 1% level, respectively. Instruments used in GMM estimation: constant, r_{t-1}, r_{t-2}, r_{t-3}, r_{t-4}, π_{t-1}, π_{t-2}, π_{t-3}, π_{t-4}, y^{gap}_{t-1}, y^{gap}_{t-2}, y^{gap}_{t-3}, y^{gap}_{t-4}, ffr_{t-1}, ffr_{t-2}, ffr_{t-3}, ffr_{t-4}. $\bar{R}^2$ is the adjusted R^2, $\hat{\sigma}$ denotes the standard error of regression and p(J-stat) denotes the p-value for the corresponding J-test statistics.

Table 2: p-values of linearity tests

	1970Q1:2006Q2					1978Q1:2006Q2				
s_t	F	F4	F3	F2	model	F	F4	F3	F2	model
r_{t-1}	0.019	0.037	0.136	0.150	LSTR1	0.017	0.282	0.110	0.016	LSTR1
r_{t-2}	0.001	0.047	0.012	0.051	LSTR2	-	-	-	-	-
π_{t+1}	0.338	0.830	0.629	0.043	Linear	0.013	0.370	0.053	0.017	LSTR1
y_t^{gap}	0.025	0.071	0.178	0.083	LSTR1	0.014	0.065	0.050	0.148	LSTR2
ffr_{t-1}	0.217	0.637	0.125	0.220	Linear	0.108	0.727	0.134	0.050	Linear
trend	0.001	0.062	0.126	0.003	LSTR1	0.003	0.684	0.013	0.003	LSTR1

Note: The table shows p-values of linearity tests, for which equation (2.5) indicates the model under the alternative hypothesis of nonlinearity. LSTR1 and LSTR2 denote suggested logistic smooth transition models with $K = 1$ and $K = 2$, respectively. For details on nonlinearity tests see Teräsvirta (2004, Section 3).

Table 3: Conditional ML estimates for LSTR model (2.5) with lagged interest rate as a transition variable

	(1)	(2)	(3)
Sample	70Q1:06Q2	70Q1:06Q2	78Q1:06Q2
Model	LSTR1	LSTR2	LSTR1
s_t	r_{t-1}	r_{t-2}	r_{t-1}
linear part			
α_0^*	-0.009	-0.672	-0.090
	(0.326)	(0.676)	(0.314)
ρ_{01}	1.097***	0.831***	0.785***
	(0.116)	(0.125)	(0.072)
ρ_{02}	-0.242**	0.009	-
	(0.097)	(0.141)	
β_0^*	0.051**	0.067*	0.280***
	(0.023)	(0.040)	(0.080)
γ_0^*	0.019	-0.048	0.025
	(0.077)	(0.109)	(0.108)
η_0^*	0.165***	0.251***	0.126**
	(0.051)	(0.060)	(0.056)
nonlinear part			
α_1^*	-5.027	1.986**	-
	(4.558)	(1.000)	
ρ_{11}	0.487	0.215	0.101
	(0.441)	(0.274)	(0.080)
ρ_{12}	-0.135	-0.276	-
	(0.263)	(0.268)	
β_1^*	-0.204***	-0.184***	-0.249**
	(0.094)	(0.069)	(0.099)
γ_1^*	0.656***	0.666***	0.283*
	(0.245)	(0.215)	(0.161)
η_1^*	0.105	-0.031	-0.044
	(0.108)	(0.097)	(0.098)
θ/σ_s	1.404	0.766	1.681
c_1	12.213	4.351	10.921
c_2	-	11.789	-
$\bar{R}^2$	0.943	0.947	0.964
$\hat{\sigma}$	0.845	0.817	0.707
Rel.RMSE	0.955	0.923	0.949

Note: The federal funds rate ffr_t is included as an additional regressor (coefficients η_0^* and η_1^*). Estimated standard errors are given in parentheses. *, **, *** denote significance at the 10%, 5% and 1% level, respectively. $\bar{R}^2$ is the adjusted R^2 and $\hat{\sigma}$ denotes the standard error of regression. Rel.RMSE denotes the root mean squared error relative to the RMSE of the corresponding linear model (see eqn. (3.2)).

Table 4: p-values of diagnostic tests for LSTR models from Table 3

	(1)	(2)	(3)
Sample	70Q1:06Q2	70Q1:06Q2	78Q1:06Q2
Model	LSTR1	LSTR2	LSTR1
s_t	r_{t-1}	r_{t-2}	r_{t-1}
Residual Tests			
JB	0.000	0.000	0.026
ARCH-LM(1)	0.253	0.002	0.027
LM(2)	0.232	0.565	0.775
LM(4)	0.421	0.387	0.755
Remaining Nonlinearity: H_0 : no			
r_{t-1}	0.694	0.565	0.525
r_{t-2}	0.153	0.243	-
π_{t+1}	0.330	0.806	0.207
y_t^{gap}	0.057	0.179	0.028
ffr_{t-1}	0.532	0.987	0.273
Parameter Constancy: H_0 : yes			
H1	0.089	0.154	0.072
H2	0.168	0.188	0.116
H3	0.352	0.518	0.358

Note: JB denotes the Jarque-Bera test for non-normality, LM(1) and LM(2) denote LM tests for first and second order autocorrelation, and ARCH-LM(1) denotes a first order ARCH tests. All tests are described in more detail in Section 6.3.3. of Teräsvirta (2004).

Table 5: GMM estimates for LSTR models (2.5) with lagged interest rate as a transition variable

	(1)	(2)	(3)
Sample	70Q1:06Q2	70Q1:06Q2	78Q1:06Q2
Model	LSTR1	LSTR2	LSTR1
s_t	r_{t-1}	r_{t-2}	r_{t-1}
linear part			
α_0^*	0.173	-2.742	0.035
	(0.166)	(1.871)	(0.158)
ρ_{01}	1.170***	0.671***	0.766***
	(0.102)	(0.128)	(0.050)
ρ_{02}	-0.332***	0.301	-
	(0.103)	(0.235)	
β_0^*	0.041**	0.166***	0.414***
	(0.020)	(0.041)	(0.086)
γ_0^*	-0.084	0.039	0.195**
	(0.109)	(0.200)	(0.076)
η_0^*	0.149***	0.328**	0.030
	(0.055)	(0.165)	(0.041)
nonlinear part			
α_1^*	-5.726***	3.128	-
	(2.097)	(1.988)	
ρ_{11}	-0.118	0.367	0.171***
	(0.281)	(0.262)	(0.059)
ρ_{12}	0.536**	-0.365	-
	(0.227)	(0.321)	
β_1^*	-0.096	-0.219***	-0.385***
	(0.089)	(0.049)	(0.099)
γ_1^*	0.792***	0.244	0.293**
	(0.182)	(0.250)	(0.140)
η_1^*	0.022	-0.336*	-0.037
	(0.078)	(0.199)	(0.077)
θ/σ_s	245.760	75.397	157.360
c_1	11.329	5.998	11.833
c_2	-	10.447	
$\bar{R}^2$	0.913	0.910	0.954
$\hat{\sigma}$	0.999	1.011	0.764
p(J-stat)	0.365	0.616	0.602
Rel.RMSE	1.110	1.123	0.993

Note: The federal funds rate ffr_t is included as an additional regressor (coefficients η_0^* and η_1^*). Estimated standard errors are given in parentheses. *, **, *** denote significance at the 10%, 5% and 1% level, respectively. Instruments: constant, r_{t-1}, r_{t-2}, r_{t-3}, r_{t-4}, π_{t-1}, π_{t-2}, π_{t-3}, π_{t-4}, y_{t-1}^{gap}, y_{t-2}^{gap}, y_{t-3}^{gap}, y_{t-4}^{gap}, ffr_{t-1}, ffr_{t-2}, ffr_{t-3}, ffr_{t-4} and their squares. $\bar{R}^2$ is the adjusted R^2, $\hat{\sigma}$ denotes the standard error of regression and p(J-stat) denotes the p-value for the corresponding J-test statistics. Rel.RMSE denotes the root mean squared error relative to the RMSE of the corresponding linear model (see eqn. (3.2)).

Table 6: Conditional ML and GMM estimates for LSTR models (2.5) with a trend and the output gap as transition variables

	(1)	(2)	(3)	(4)
Sample	78Q1:06Q2	78Q1:06Q2	70Q1:06Q2	70Q1:06Q2
Model	LSTR1	LSTR1	LSTR2	LSTR2
s_t	trend t	trend t	y_t^{gap}	y_t^{gap}
Estimation	CML	GMM	CML	GMM
linear part				
α_0^*	4.196***	0.485	0.653***	1.534***
	(0.828)	(0.539)	(0.240)	(0.413)
ρ_{01}	0.453***	0.829***	0.970***	0.610***
	(0.103)	(0.047)	(0.106)	(0.185)
ρ_{02}	-	-	−0.118	0.055
			(0.106)	(0.159)
β_0^*	0.239***	0.501***	0.039*	0.107*
	(0.050)	(0.041)	(0.023)	(0.059)
γ_0^*	−0.149	0.122***	0.639***	1.002***
	(0.125)	(0.037)	(0.106)	(0.205)
nonlinear part				
α_1^*	−4.283***	−0.173	−0.032	−3.129**
	(0.884)	(0.532)	(0.521)	(1.442)
ρ_{11}	0.444***	0.056	0.152	1.219***
	(0.133)	(0.084)	(0.167)	(0.462)
ρ_{12}	-	-	−0.111	−0.729**
			(0.162)	(0.364)
β_1^*	−0.022	−0.067	−0.013	−0.144
	(0.159)	(0.216)	(0.033)	(0.105)
γ_1^*	0.583***	−0.554***	−0.654***	−1.325***
	(0.209)	(0.161)	(0.134)	(0.350)
θ/σ_s	0.099	41.280	398.733	519.661
c_1	44.311	47.478	−0.620	−0.593
c_2			4.570	4.563
$\bar{R}^2$	0.966	0.948	0.938	0.883
$\hat{\sigma}$	0.679	0.809	0.877	1.152
p(J-stat)	-	0.137	-	0.981
Rel.RMSE	0.885	1.044	0.942	1.215

Note: Estimated standard errors are given in parentheses. *, **, *** denote significance at the 10%, 5% and 1% level, respectively. Instruments for GMM estimation: constant, r_{t-1}, r_{t-2}, r_{t-3}, r_{t-4}, π_{t-1}, π_{t-2}, π_{t-3}, π_{t-4}, y_{t-1}^{gap}, y_{t-2}^{gap}, y_{t-3}^{gap}, y_{t-4}^{gap} and their squares. Estimated standard errors are given in parentheses. $\bar{R}^2$ is the adjusted R^2, $\hat{\sigma}$ denotes the standard error of regression and p(J-stat) denotes the p-value for the corresponding J-test statistics. Rel.RMSE denotes the root mean squared error relative to the RMSE of the corresponding linear model (see eqn. (3.2)).

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