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On the (Mis-) Alignment of Consumer and Social Welfare in Markets with Network Effects*

Irina Suleymanova† Christian Wey‡

April 2008

Abstract

We analyze duopoly Bertrand competition under network effects. We consider both incompatible and compatible products. Our main result is that network effects create a fundamental conflict between the maximization of social welfare and consumer surplus whenever products are incompatible. While consumer surplus is highest in the symmetric equilibrium, social welfare is highest in the asymmetric equilibrium. We also show that both consumer surplus and social welfare are higher in any equilibrium under compatibility when compared with incompatible products. However, firms never have strict incentives to achieve compatibility. Finally, we show the robustness of our results when products are horizontally differentiated.

JEL-Classification: D43, L13

Keywords: Bertrand Duopoly, Network Effects, (In-) Compatibility, Welfare

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1 Introduction

In many industries consumer demand is characterized by pronounced network effects, as e.g., in software and telecommunications markets. With network effects consumers' utility is increasing in the total number of consumers adopting the same (and hence, compatible) product. Network effects have produced intense debates in policy circles concerning the appropriate application of traditional competition policy concepts (see, e.g., OECD, 1997, and FTC, 1996). Put simply, a consensus has been reached concerning the desirability of compatibility (besides possibly adverse dynamic effects), whereas the assessment of market outcomes when products are incompatible remains largely unresolved (see also Klemperer, 2005). Ambiguities under incompatibility arise as on the one hand pronounced network effects may tip the market into a monopoly equilibrium (which appears to be an unfortunate outcome from a traditional competition policy point of view) while on the other hand a market sharing outcome where incompatible products compete head-to-head necessarily involves substantial incompatibilities among consumers (an outcome being obviously inefficient).

Our paper is largely supportive of those considerations. Our main contribution is to show that (at least some) of the ambiguities concerning the policy assessment of competition under incompatible products can be attributed to a fundamental conflict between consumer welfare and social welfare. We consider a simple Bertrand duopoly model with positive network effects and analyze both compatible and incompatible products. We search for fulfilled expectation Bertrand equilibria where consumers hold rational expectations. If products are incompatible, a symmetric equilibrium (where firms share the market equally) and two asymmetric equilibria (where one of the firms becomes the monopolist) coexist. While consumers prefer the symmetric equilibrium (where price competition is most intense), a social planer would prefer either one of the monopoly equilibria (where network effects are maximized). Moreover, the fundamental conflict between consumer surplus and social welfare maximization becomes stronger the larger the network effects in the industry. We also analyze the case of compatible products where a continuum of equilibria (ranging from complete monopolization to equal market sharing) emerge. Consumer surplus and social welfare are the same in all equilibria under compatibility. Moreover, consumer surplus and social welfare is always (weakly) higher under compatibility when compared with the equilibrium outcomes under incompatibility. Unfortunately, firms never have strict incentives to achieve compatibility (irrespectively of whether side payments are allowed or not). We extend our analysis by considering horizontal product differentiation which resolves the multiplicity of equilibria under compatibility such that a unique symmetric equilibrium emerges. We show that our results concerning the fundamental conflict between consumer surplus and social welfare maximization under incompatibility and vis-à-vis firms' insufficient compatibility incentives remain valid, whenever network effects are sufficiently large.

Taking a policy making perspective, our results are reassuring for governmental intervention that aims at increasing compatibility of firms' (otherwise incompatible) products. Our results also highlight the ambiguity involved with those governmental interventions which aim at picking a winning proprietary technology out of incompatible competitors (e.g., by committing governmental procurement or standard setting to a single technology).[1] While such a policy can be advisable from a social welfare perspective, consumers may be substantially hurt. We speculate that our results are somehow supported by the fact that policy makers taking an industrial policy perspective (i.e., focus primarily on profits) tend to prefer to pick a winning technology (out of a set of incompatible alternatives) while in competition policy circles (which are supposed to focus primarily on consumer surplus) a more reticent attitude appears to have gained control (as, e.g., expressed in FTC, 1996). Our model may explain those differences by the different weights the involved parties put on the elements of the social welfare function.

Our paper contributes to the industrial organization literature that analyzes how positive network effects affect competitive behavior and market performance (for a recent survey, see Farrell and Klemperer, 2007). Our paper builds on the seminal paper by Katz and Shapiro (1985) who examine network effects in a Cournot model both under incompatibility and compatibility. We adopt their concept of a fulfilled expectation equilibrium to our set-up of duopolistic price competition. While we obtain similar equilibrium patterns, our contribution is to sharply highlight the described conflict between consumer surplus and social welfare maximization when products are incompatible.[2] Closely related to our analysis is also Farrell and Saloner (1992)

[1] A recent example for this kind of intervention can be seen in the announcement of the EU to support DVB-H as the mobile-television standard over rival technologies, as e.g., Qualcomm's MediaFLO ("EU Opts for DVB-H as Mobile-TV Standard," The Wall Street Journal Europe, March 18, 2008, p. 5).

[2] Katz and Shapiro (1985) do not comment on the comparison of social welfare and consumer surplus under the different equilibria when products are incompatible. While such a comparison is certainly feasible within their

who analyze how the presence of (imperfect) converters affects equilibrium outcomes in a model of horizontally differentiated products and network effects under different market structures. Farrell and Saloner (1992) consider both standardization and incompatibility outcomes (which correspond to the asymmetric and symmetric equilibrium outcomes, respectively, in our model) when product supply is perfectly competitive. However, they focus exclusively on the "conversion equilibrium" (i.e., the equilibrium where some consumers buy converters) under duopoly competition. In contrast, our main concern is the comparison of the asymmetric (standardization) equilibria with the symmetric (incompatibility) equilibrium when two incompatible proprietary technologies compete against each other. Farrell and Saloner (1992) argue that the existence of (imperfect) converters makes a standardization outcome less likely, so that overall incompatibilities tend to be larger with converters. They interpret their finding as an inefficiency due to the *irresponsibility of competition*. In those instances, "[i]t might be better if some good were not offered at all, or were offered only at a high price, because consumers use it 'irresponsibly'; but with competition, no agent can decide that a good will not be offered, or that its price shall be high" (Farrell and Saloner, 1992, p. 13). Accordingly, our model also contributes to that literature which highlights conflicts between the maximization of social welfare and consumer welfare, an issue which is important as competition policy tends to be preoccupied with protecting consumer surplus, and thereby, either assumes that consumer protection should be aligned with social welfare maximization or simply neglects overall efficiency. Similar to excessive entry results (e.g., in the standard Cournot model or under monopolistic competition, Salop, 1979) we obtain that competition between incompatible products can give rise to market outcomes where consumers prefer a market sharing outcome (with substantial incompatibilities prevailing) which is inefficient from a social welfare perspective when compared with the monopoly outcome.

We proceed as follows. In Section 2 we present our basic Bertrand duopoly model with network effects. Section 3 presents the analysis and the main results of our basic model. In Section 4 we examine the case of horizontally differentiated products. Finally, Section 5 concludes.

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set-up, it is also blurred by the features of the underlying Cournot model (in particular, the dependence of total demand on firms' quantity choices).

2 The Model

We consider a Bertrand duopoly, where products exhibit positive network effects. Products may be either compatible or incompatible with each other. Each firm i ($i = A, B$) produces with constant marginal cost (which we normalize to zero). Firms compete in prices p_i ($i = A, B$) which they determine simultaneously. We assume consumer utility to be linearly increasing in the network size, so that each additional consumer creates a constant positive externality, $b > 0$, to the utility of the users of the same product under incompatibility or all consumers under compatibility. We assume a continuum of consumers with a mass of one. The firms' market shares are denoted by $\alpha_i \in [0, 1]$ ($i = A, B$). The utility of a consumer from buying product $i = A, B$ when products are incompatible is given by $U(p_i, \alpha_i) = v + b\alpha_i - p_i$ if nonnegative, with $v > 0$ denoting the stand-alone value of the product. Similarly, the utility from buying product $i = A, B$ is given by $U(p_i, \alpha_A + \alpha_B)$ when products are compatible.

As consumers' utilities are interdependent because of positive network effects, they have to form expectations about the other consumers' purchasing decisions, and hence, about each firm i's future market share which we denote by α_i^e. For given consumer expectations and prices, we can express the demand for product i under incompatibility as

$$q_i(p_i, p_j, \alpha_i^e) =$$
$$\begin{cases} 1 & \text{if} \quad U(p_i, \alpha_i^e) > U(p_j, \alpha_j^e) \\ \alpha_i \in [0,1] & \text{if} \quad U(p_i, \alpha_i^e) = U(p_j, \alpha_j^e) \\ 0 & \text{if} \quad U(p_i, \alpha_i^e) < U(p_j, \alpha_j^e), \end{cases} \tag{1}$$

with $i, j = A, B$ and $i \neq j$. The demand function for product i when products are compatible follows from replacing $U(p_i, \alpha_i^e)$ and $U(p_j, \alpha_j^e)$ by $U(p_i, \alpha_A^e + \alpha_B^e)$ and $U(p_j, \alpha_A^e + \alpha_B^e)$, respectively.

The timing of our basic market game is as follows. In the first stage, consumers form expectations and firms set prices. In the second stage, consumers observe firms' pricing decisions and make their purchasing decisions. We search for fulfilled expectation Bertrand equilibria.[3] In a fulfilled expectation Bertrand equilibrium each firm's price maximizes its profit $\pi_i(p_i, p_j, \alpha_i^e) = p_i q_i(p_i, p_j, \alpha_i^e)$ ($i, j = A, B,\ i \neq j$) for a given price of the rival firm and for given consumer expectations. In addition, we require rational expectations so that each firm's equilibrium market

[3] Our concept of a fulfilled expectations equilibrium is borrowed from Katz and Shapiro (1985).

share equals its expected one. More precisely, in a fulfilled expectation Bertrand equilibrium, consumer expectations are fulfilled (i.e., $\alpha_i^e = \alpha_i^*$ for $i = A, B$) and equilibrium prices p_i^* follow from

$$p_i^* = \arg\max_{p_i \geq 0} \ \pi_i(p_i, p_j^*, \alpha_i^*) \text{ for } i, j = A, B \text{ and } i \neq j. \tag{2}$$

In the following we simply refer to the fulfilled expectation Bertrand equilibrium as to the equilibrium.

3 Analysis and Main Results

We now present the main results of our equilibrium analysis. The following proposition characterizes the equilibrium when products are incompatible.

Proposition 1. *If products are incompatible, then a symmetric and two asymmetric equilibria exist. In the asymmetric equilibria one of the firms gains the whole market, sets its price equal to b, while the other firm cannot do better than setting its price equal to zero. In the symmetric equilibrium firms share the market equally and set their prices equal to zero.*

Proof. We consider all possible market sharing outcomes and ask whether a particular outcome can be supported as an equilibrium. We first analyze the asymmetric outcome where one firm becomes the monopolist, then the symmetric outcome, and finally all other constellations.

Case i) Suppose that $\alpha_i^* = 1$ $(i = A, B)$ constitutes an equilibrium outcome. As expectations are supposed to be fulfilled, $\alpha_i^e = 1$ must also hold. From (1) we obtain that $\alpha_i^* = 1$ is only feasible if $U(p_i^*, 1) \geq U(p_j^*, 0)$ for $i \neq j$. Note that p_i^* must be the solution of the maximization problem (2) so that $U(p_i^*, 1) = U(p_j^*, 0)$ must hold, as otherwise (if $U(p_i^*, 1) > U(p_j^*, 0)$), firm i could increase its profit by decreasing its price. Accordingly, p_j^* must also be the solution of the maximization problem (2). Hence, it must hold that $p_j^* = 0$, as otherwise (if $p_j^* > 0$), firm j could increase its profit by setting the price $p_j = p_i - b - \epsilon$, with $\epsilon > 0$. From $U(p_i^*, 1) = U(p_j^*, 0)$ and $p_j^* = 0$ it follows that $p_i^* = b$. As $U(b, 1) = v$ we conclude that $\alpha_i^* = 1$ $(i = A, B)$ constitutes an equilibrium outcome with equilibrium prices $p_i^* = b$ and $p_j^* = 0$ $(j \neq i)$.

Case ii) Suppose the symmetric outcome $\alpha_A^* = \alpha_B^* > 0$ constitutes an equilibrium. With fulfilled expectations, $\alpha_i^e = \alpha_i^*$ $(i = A, B)$ must also hold. From (1) it follows that $\alpha_A^* = \alpha_B^* > 0$ is only feasible if $U(p_A^*, \alpha_A^*) = U(p_B^*, \alpha_B^*)$. Hence, it follows that $p_A^* = p_B^*$ must hold in a symmetric equilibrium. Solving the corresponding maximization problems (2) it must hold that $p_A^* = p_B^* = 0$, as otherwise (if $p_A^* = p_B^* > 0$) one of the firms can increase its profit by decreasing its price slightly. As $U(0, \alpha_A^*) > v$ holds, the market is covered in the symmetric equilibrium, and thus, $\alpha_A^* = \alpha_B^* = 1/2$ must hold. Hence, $\alpha_i^* = 1/2$ is an equilibrium outcome with equilibrium prices $p_i^* = 0$ $(i = A, B)$.

Case iii) Assume now all asymmetric outcomes with $\alpha_i^* > \alpha_j^* > 0$ for $i, j = A, B$ and $i \neq j$. As expectations must be fulfilled $\alpha_i^e = \alpha_i^*$ and $\alpha_j^e = \alpha_j^*$ must then hold as well. Applying the demand function (1) we obtain that $\alpha_i^* > \alpha_j^* > 0$ can only hold if $U\ (p_i^*, \alpha_i^*) = U(p_j^*, \alpha_j^*)$. It is now easily checked that no prices $p_i^*, p_j^* \geq 0$ exist which can support such an equilibrium outcome. Note first that $p_i^* > p_j^*$ must hold, as otherwise, $U(p_i^*, \alpha_i^*) = U(p_j^*, \alpha_j^*)$ cannot hold as well (as $\alpha_i^* > \alpha_j^*$). But $p_i^*, p_j^* > 0$ cannot hold, as then any of the two firms can gain the whole market by slightly decreasing its price. Finally, $p_i^* > p_j^* = 0$ can also not support the proposed outcome as an equilibrium as firm i could then gain the whole market by slightly decreasing the price. Hence, there does not exist a pair of prices p_i^* and p_j^* which would support an outcome with $\alpha_i^* > \alpha_j^* > 0$ for $i = A, B$ and $i \neq j$. *Q.E.D.*

Proposition 1 states that three equilibria exist under incompatibility: two asymmetric equilibria where one firm gains the entire market (with $\alpha_i^* = 1$, $i = A, B$) and a symmetric equilibrium where both firms share the market equally (with $\alpha_i^* = 1/2$, $i = A, B$). In the asymmetric equilibria the monopolist sets a price of $p_i^* = b$ while the losing competitor cannot do better than setting $p_j^* = 0$ $(j \neq i)$. In the symmetric equilibrium we obtain the Bertrand paradox, such that both firms set their prices equal to their marginal costs (i.e., $p_A^* = p_B^* = 0$).

With Proposition 1 at hand we can now evaluate consumer surplus and social welfare in the different equilibria under incompatibility. We denote consumer surplus (i.e., the integral over consumers' utilities) by CS and social welfare (i.e., the sum of consumer surplus and firms' profits) by SC. In the following we use the superscript "a" to indicate the asymmetric equilibrium and the superscript "s" to indicate the symmetric equilibrium under incompatibility.

Proposition 2. *Consumer surplus in the symmetric (asymmetric) equilibrium is given by $v + b/2$ (v) and social welfare in the symmetric (asymmetric) equilibrium is given by $v + b/2$ $(v + b)$. Hence, social welfare is highest in the asymmetric equilibria whereas consumer surplus* *is highest in the symmetric equilibrium. Moreover, the total value of the differences of social welfare and consumer surplus under the symmetric and the asymmetric equilibrium is strictly increasing in the level of the network effect, b.*

Proof. In the asymmetric equilibrium we obtain $CS^a = v$ and $SW^a = v + b$, where in the latter expression b is the profit of the firm which gains the entire market. In the symmetric equilibrium we obtain $CS^s = v + b/2$ and $SW^s = v + b/2$. Calculating the differences we get $CS^s - CS^a = SW^a - SW^s = b/2$, which are both increasing in the level of the network effect, b. *Q.E.D.*

Our results indicate the basic trade-off between the maximization of social welfare and consumer surplus in the presence of the network effects, whenever products are incompatible. While social welfare maximization requires consumers to coordinate on a single product, consumers are better off when both products compete head-to-head. In an asymmetric equilibrium the expected monopolist has a competitive advantage vis-à-vis its rival given by the amount of the overall network effects it provides, namely, b. That advantage gives the monopolist the opportunity to extract all the consumer surplus generated by the network effects, b. In contrast, in the symmetric equilibrium none of the firms has a similar (expectational) advantage and thus both firms compete all profits away. The resulting lower price in the symmetric equilibrium overcompensates consumers' losses from lower network effects. The higher social surplus in the asymmetric equilibrium is due to that fact that network effects are maximized in that case. Proposition 2 also states that the conflict between consumer surplus and social welfare maximization becomes more severe with increasing levels of the network effect, b. Therefore, when network effects are large then any equilibrium involves considerable losses either from a consumer surplus or social welfare point of view.

We now turn to the equilibrium analysis under compatibility (where we indicate equilibrium values by the superscript "c").

Proposition 3. *Under compatibility the market is always covered and there exists a continuum of equilibria with $\alpha_i^c \in [0, 1]$ $(i = A, B)$ and unique equilibrium prices $p_A^c = p_B^c = 0$. In all equilibria, consumer surplus and social welfare are given by $v + b$.*

Proof. Under compatibility consumer expectations about network effects are the same for both products and the expected utility from buying from firm $i = A, B$ is given by $U(p_i, \alpha_A^e + \alpha_B^e) =$

$v + b(\alpha_A^e + \alpha_B^e) - p_i$. We show that p_A^c, $p_B^c = 0$ must hold in equilibrium. Assume to the contrary that equilibrium prices fulfill $p_i > p_j > 0$. Then firm i could increase its profit by setting $p_i = p_j - \epsilon$, $\epsilon > 0$. Assume next that $p_i > p_j = 0$. Then firm j could increase its profit by setting $p_j = p_i - \epsilon$, $\epsilon > 0$. Assume finally that $p_i = p_j > 0$. Then at least one of the firms can increase its profit by slightly reducing its price.[4] If $p_i = p_j = 0$, then none of the firms has a strict incentive to alter its price. As expectations must be fulfilled, the market share of firm i is given by $\alpha_i^e = \alpha_i^c \in [0, 1]$ $(i = A, B)$. Moreover, the market is covered in equilibrium as for any $\alpha_i^e \geq 0$ it holds that $U(p_A, \alpha_A^e + \alpha_B^e) = U(p_B, \alpha_A^e + \alpha_B^e) = v + b(\alpha_A^e + \alpha_B^e) \geq v$. Hence, under compatibility there are infinitely many equilibria with $\alpha_i^c \in [0, 1]$ $(i = A, B)$ and $\alpha_A^c + \alpha_B^c = 1$ where firms' prices fulfill $p_A^c = p_B^c = 0$. Consumer surplus is then given by $CS^c = v + b$ and social welfare is given by $SW^c = v + b$ (as firms' profits are equal to zero). *Q.E.D.*

Proposition 3 states that under compatibility a continuum of equilibria emerges, where consumers' expectations pin down the equilibrium fully. Firms compete all profits away so that prices are equal to marginal costs. From a consumer surplus and social welfare perspective, indifference holds everywhere. Comparison of consumer surplus and social welfare under compatibility and incompatibility gives rise to the following corollary.

Corollary 1. *Comparison of consumer surplus and social welfare under compatibility and incompatibility yields the ordering $CS^c > CS^s > CS^a$ and $SW^c = SW^a > SW^s$, respectively.*

Corollary 1 states that both consumer surplus and social welfare are always maximized under compatibility. Social welfare and consumer surplus are, therefore, perfectly aligned under compatibility. As products are compatible, network effects are always maximized, and consumers always enjoy the benefits from homogenous goods Bertrand competition. According to Corollary 1 a switch from incompatibility to compatibility is beneficial from a consumer surplus as well as from a social welfare perspective. We note that our results give a sharper prediction than Katz and Shapiro (1985) who obtain an ambiguous comparison of social welfare under incompatibility and compatibility which is due to the fact that firms' profits may decrease under compatibility (and that this decrease is not necessarily compensated by the unambiguously increasing consumer surplus).

[4] Note that if $\alpha_j^e = 1$, then only firm i has an incentive to reduce its price (with $i, j = A, B$ and $i \neq j$).

Turning finally to firm's incentives to achieve compatibility in the first place we obtain the following result.

Corollary 2. *Firms never have a strict incentive to achieve compatibility independently on whether or not side payments are feasible.*

Corollary 2 follows immediately from comparing firms' profits in the compatibility and incompatibility equilibria. The corollary states that firms can never do better under compatibility when compared with incompatibility. The statement holds for all possible equilibrium outcomes under incompatibility and compatibility. In fact, the expected monopolist under incompatibility has a strict incentive to block any move towards compatibility. Moreover, this result does not depend on whether or not side payments are admissible. A similar result has been obtained in Katz and Shapiro (1985) when an asymmetric equilibrium emerges under incompatibility. However, they also show significant incentives to achieve compatibility if the symmetric equilibrium is valid under incompatibility. Our model, therefore, gives a much gloomier picture on firms' compatibility incentives. We may interpret our results such that an active role of the government to achieve more compatibility among firms' (proprietary) technologies can be advisable if network effects are substantial. If, however, increasing compatibility is not a viable policy option, then picking a winning technology involves a fundamental trade-off between social welfare (or, equivalently, producer surplus) maximization (i.e., industrial policy concerns) and consumer surplus maximization (i.e., competition policy concerns); a conflict that becomes more pronounced the larger network effects become.

4 Horizontal Product Differentiation

In this section we consider the case where firms' products are horizontally differentiated à la Hotelling. The timing of the market game is the same as in our basic Bertrand duopoly model. Consumers are assumed to be uniformly distributed on the unit interval such that each consumer obtains an address $x \in [0, 1]$. The utility a consumer with address x derives from consuming product A is given by $U_x^A(p_A, \alpha_A) = v + b\alpha_A - tx - p_A$ and from consuming product B is given by $U_x^B(p_B, \alpha_B) = v + b\alpha_B - t(1-x) - p_B$, where α_i and p_i $(i = A, B)$ stand for firm i's market share and price, respectively, while v is the stand-alone value which is assumed to be sufficiently large, so that the market is always covered in equilibrium. We may then express the demand for product $i = A, B$ for given consumer expectations and firms' prices as

$$q_i(p_i, p_j, \alpha_i^e) = \begin{cases} 1 & \text{if} \quad p_j - p_i \geq b(1 - 2\alpha_i^e) + t \\ \frac{1}{2} + \frac{b(2\alpha_i^e - 1) - p_i + p_j}{2t} & \text{if} \quad b(1 - 2\alpha_i^e) - t < p_j - p_i < b(1 - 2\alpha_i^e) + t \\ 0 & \text{if} \quad p_j - p_i \leq b(1 - 2\alpha_i^e) - t, \end{cases} \tag{3}$$

with $i, j = A, B$ and $i \neq j$. We start with the analysis of the symmetric equilibrium. Given the demand for firm i's ($i = A, B$) product and the price of the rival firm j ($j \neq i$) each firm i in a market sharing equilibrium sets its price according to the maximization problem (2) which yields $p_i(\alpha_i^e) = t + b(2\alpha_i^e - 1)/3$. Imposing our requirement that expectations are fulfilled in equilibrium so that $\alpha_i^e = q_i(p_i(\alpha_i^e), p_j(\alpha_j^e), \alpha_i^e)$ for $i, j = A, B$ and $i \neq j$ must hold, we obtain the equilibrium output levels and prices with $q_i^* = 1/2$ and $p_i^* = t$, respectively. As each firm's maximization problem is strictly concave and q_i^* and p_i^* are positive for any values of b and t we conclude that a unique symmetric equilibrium exists for any $t, b > 0$.

Let us now turn to the asymmetric equilibria where firm i ($i = A, B$) becomes the monopolist. As we assume that the market is always covered (i.e., we pose v to be sufficiently large) prices must fulfill $p_i^* = b - t$ and $p_j^* = 0$ ($j \neq i$) in an asymmetric equilibrium when firm i ($i = A, B$) becomes the monopolist. Those prices constitute an equilibrium only if firm i does not have an incentive to increase its price, so that

$$\left.\frac{\partial \pi_i}{\partial p_i}\right|_{p_i = b - t,\ p_j = 0,\ \alpha_i = 1} \leq 0 \tag{4}$$

must hold in an asymmetric equilibrium. Evaluating Condition (4) we obtain the parameter restriction $b \geq 3t$. We are now in a position to state the following proposition.

Proposition 4. *Suppose products are horizontally differentiated and incompatible. Then there exists a unique symmetric equilibrium in which each firm i ($i = A, B$) sets the price $p_i^s = t$ and serves half of the market. If network effects are large enough, i.e., $b \geq 3t$, then two asymmetric equilibria also exist in which firm i ($i = A, B$) gains the entire market and sets the price $p_i^a = b - t$ while the losing rival firm cannot do better than setting $p_j^a = 0$ (with $i \neq j$).*

If network effects are relatively small (or, conversely, products are quite differentiated), then only the symmetric equilibrium emerges (i.e., if $b < 3t$ holds), while for larger network effects (or, rather homogeneous products) also two asymmetric equilibria emerge (i.e., if $b \geq 3t$). Product differentiation, therefore, tends to make a symmetric equilibrium outcome more likely under incompatibility when compared with our previous basic model where products were assumed to be (inherently) symmetric. With Proposition 4 at hand, we can next compare consumer surplus and total welfare when both symmetric and asymmetric equilibria coexist (i.e., if $b \geq 3t$).[5]

Proposition 5. *Suppose products are horizontally differentiated and incompatible. Moreover, assume that $b \geq 3t$ so that both a unique symmetric and two asymmetric equilibria coexist. Then there exists a conflict between social welfare and consumer surplus if $b > 7t/2$, such that social welfare is highest in the asymmetric equilibria and consumer surplus is highest in the symmetric equilibrium. If, otherwise, $3t \leq b \leq 7t/2$, then no conflict arises such that consumer surplus and social welfare are highest in the asymmetric equilibrium.*

Proof. In the asymmetric equilibrium under incompatibility consumer surplus is given by $CS^a = v + t/2$ and social welfare is given by $SW^a = v + b - t/2$. In the symmetric equilibrium under incompatibility consumer surplus is given by $CS^s = v + b/2 - 5t/4$ while social welfare is given by $SW^s = v + b/2 - t/4$. It is easily checked that $v + b - t/2 > v + b/2 - t/4$ for any t and b if $b \geq 3t$, while from comparing CS^s and CS^a we obtain that $CS^s > CS^a$ if $b > 7t/2$, whereas the opposite holds for $b < 7t/2$. *Q.E.D.*

Considering the differences $CS^s - CS^a = (2b - 7t)/4$ and $SW^a - SW^s = (2b - t)/4$ we observe that both differences increase in b. We can, therefore, conclude that with increasing network effects the asymmetric equilibrium becomes less attractive from a consumer perspective but more attractive from a social welfare point of view. According to Proposition 5, if network effects are strong (or product differentiation is weak), such that $b > 7t/2$ holds, then the conflict between consumer surplus and social welfare is preserved under product differentiation. Interestingly enough, Proposition 5 also shows the existence of an intermediate parameter range ($3t \leq b \leq 7t/2$), where both social welfare and consumer surplus are aligned and jointly maximized in the asymmetric equilibrium. In that area, one firm, say firm A, can only gain the entire market with a "predatory" price which makes the consumer at the other end of the Hotelling line at least indifferent between buying firm A's product (which creates a disutility of t but gives rise to

[5] As in the previous section, we index in the following the symmetric equilibrium by the superscript "s" and the asymmetric equilibria by the superscript "a" when products are incompatible.

network utility b) or firm B's product which is offered at a price of zero (but lacks any network utility).

We now turn to the case when firms' products are compatible. In this case the utility from the product of firm i is given by $U^i(p_i, 1)$ $(i = A, B)$. The following proposition characterizes the associated equilibrium outcome.

Proposition 6. *Suppose products are horizontally differentiated and compatible. Then a unique symmetric equilibrium emerges in which each firm sets the price* $p_i^c = t$ $(i = A, B)$ *and serves half of the market. Consumer surplus and social welfare are given by* $CS^c = v + b - 5t/4$ *and* $SW^c = v + b - t/4$, *respectively.*

Proof. Let us first consider the symmetric equilibrium. In the symmetric equilibrium each firm maximizes its profit given by $[1/2 + (p_j - p_i)/2t]p_i$ for a given price of its competitor p_j $(i = A, B,\ i \neq j)$. We then obtain that each firm sets the price $p_i^c = t$ $(i = A, B)$ and firms share the market equally. Consumer surplus and social welfare are then given by $CS^c = v + b - 5t/4$ and $SW^c = v + b - t/4$, respectively.

We prove now that an asymmetric equilibrium cannot exist under compatibility. Assume, to the contrary, that firm A holds a monopoly position in equilibrium. Then, it must hold that $U^A(p_A, x = 1) = U^B(p_B, x = 1)$, as otherwise firm A cannot gain the entire market. It is then immediate that $p_B = 0$ must hold as well, as otherwise, firm B could increase its profit by decreasing its price slightly. Hence, it follows that $p_A = -t < 0$ must hold, an outcome obviously not admissible. *Q.E.D.*

We are now in a position to compare consumer surplus, social welfare and firms' profits in the different equilibria under incompatibility with the equilibrium under compatibility. Clearly, $SW^c = v + b - t/4$ is larger than both $SW^a = v + b - t/2$ and $SW^s = v + b/2 - t/4$, so that social welfare is (strictly) maximized whenever products are compatible. It is straightforward to check that consumer surplus under compatibility $CS^c = v + b - 5t/4$ is always larger than consumer surplus in the symmetric equilibrium under incompatibility $CS^s = v + b/2 - 5t/4$. Moreover, for those parameter constellations, where both two asymmetric equilibria and a symmetric equilibrium under incompatibility emerge (i.e., $b \geq 3t$) it holds that $CS^c > CS^a = v+t/2$ as $v + b - 5t/4 > v + t/2$ holds for any $b > 7t/4$. Hence, whenever an asymmetric equilibrium under incompatibility exists, then consumer surplus is higher under compatibility. The following corollary summarizes our results.

Corollary 3. *Suppose that products are differentiated and multiple equilibria emerge under incompatibility (i.e., $b \geq 3t$ holds). Then the ordering of consumer surplus and social welfare under compatible and incompatible products is such that $SW^c > SW^a > SW^s$ and $CS^c > \max\{CS^a, CS^s\}$ with $CS^a > CS^s$ for all $3t \leq b < 7t/2$ and $CS^s > CS^a$ for all $b > 7t/2$. If, otherwise, $b < 3t$, then $SW^c > SW^s$ and $CS^c > CS^s$.*

Corollary 3 states that both social welfare and consumer surplus are highest under compatibility independently of the type of equilibrium under incompatibility. In contrast to our basic model in the previous section, we also obtain that social welfare is now strictly higher under compatibility when compared with the asymmetric equilibrium outcome under incompatibility. While the asymmetric outcome under incompatibility still maximizes network effects it also entails welfare losses because of reduced product variety. As the latter loss is absent in the equilibrium under compatibility, social welfare is strictly higher under compatibility. We now turn to firms' incentives to achieve compatibility.

Corollary 4. *Suppose that products are differentiated. Then, firms never have strict incentives to achieve compatibility, irrespectively on whether or not transfers are feasible.*

Corollary 4 mirrors Corollary 2 such that product differentiation does not affect our result of our basic model that firms cannot unilaterally or jointly improve (strictly) their profits by making products compatible.

5 Conclusions

In this paper we have highlighted a fundamental conflict between consumer surplus and social welfare maximization whenever products are incompatible and network effects determine consumers' willingness to pay for a certain product. While consumers prefer market sharing because of the resulting lower prices, a monopoly outcome is preferable from a social welfare perspective as such an outcome maximizes overall network effects. At the same time, a monopoly outcome tends to take competitive pressure out of the market so that consumers are worse off when compared with the symmetric equilibrium where firms share the market equally. The conflict becomes more pronounced when network effects become more important; a fact which is especially true when products are differentiated. Our results also show that private incentives for compatibility are largely absent, so that governmental intervention in that regard may be advisable.

Governmental intervention if compatibility is not a feasible policy option is less simple. Public policy may try to tip the market into one of the monopoly equilibria (e.g., by committing public procurement or state-subsidized projects to a certain technology). While the resulting monopoly equilibrium may be preferable from a social welfare perspective, it may also unfold significant negative impacts on consumer surplus.

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