Modeling phase changes of road networks

Arthur Huang[*]and David Levinson[†]

Abstract

Adopting an agent-based approach, this paper explores the topological evolution of road networks from a microscopic perspective. We assume a decentralized decision-making mechanism where roads are built by self-interested land parcel owners. By building roads, parcel owners hope to increase their parcels' accessibility and economic value. The simulation model is performed on a grid-like land use layer with a downtown in the center, whose structure is similar to the early form of many Midwestern and Western (US) cities. The topological attributes for the networks are evaluated by multiple centrality measures such as degree centrality, closeness centrality, and betweenness centrality. Our findings disclose that the growth of road network experiences an evolutionary process where tree-like structure first emerges around the centered parcel before the network pushes outward to the periphery. In addition, road network topology undergoes obvious phase changes as the economic values of parcels vary. The results demonstrate that even without a centralized authority, road networks have the property of self-organization and evolution; furthermore, the rise-and-fall of places in terms of their economic/social values may considerably impact road network topology.

Keywords: road network, land parcel, network evolution, network growth, phase change

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[*] University of Minnesota, Department of Civil Engineering, 500 Pillsbury Drive SE, Minneapolis, MN 55455 USA, Email: huang284@umn.edu

[†] RP Braun-CTS Chair of Transportation Engineering, University of Minnesota, Department of Civil Engineering, Email: dlevinson@umn.edu, web: `http://nexus.umn.edu`

1 Introduction

Road networks, as artifacts of human activities, display interesting patterns and order. While order of roads is often created hierarchically (for instance, since its inception in 1921, federal financial aid has funded improvements of the most important roads in the US (Rae, 1971)), order can also emerge from completely decentralized and spontaneous interactions of individuals (Ben-Joseph, 2005). Many of the modern roads in their earliest incarnation, for example, were constructed by individuals. As Powers (1910) indicated:

> "Our public roads are an evolution from the primary paths made by animals and by men. Of the identity of the first beings who made paths in the wilderness we are uncertain. Whatever their character and origin, we may be reasonably certain that they had roads of some sort."

The emergence of the "roads of some sort", without a centralized plan, must involve numerous discrete decisions. So what are the incentives that beget the interactions of individuals and ultimately produce the road patterns we see today?

Graphically, for road networks, intersections can be seen as nodes and roads as links. Based on this structure, the models to examine road network growth can probably be cataloged into three streams distinguished by modeling perspective.

First, in *probabilistic network growth models*, each link is presumably born with a probability. A notable example is the random graph model, arguably the first application of modern graph theory to explain real-world networks (Erdös and Rényi, 1959). Other approaches include the exponential model (Dorogovtsev and Mendes, 2002), preferential attachment model(Price, 1965; Barabási and Albert, 1999), Markov graph (Frank and Strauss, 1986; Wasserman and Pattison, 1996), and Newman-Gastern model (Gastner and Newman, 2006).

Second, in *network design models*, a link is built to optimize a centralized objective, such as minimizing the the Euclidean distance(Gastner and Newman, 2006), minimizing detour (Schweitzer et al., 1998), or maximize transportation potential bewteen two locations (Yamins et al., 2003).

Third, in *agent-based discrete choice models*, agents construct links with local objectives. For example, Helbing et al. (1997, 1998) adopts an active walker model to model the evolution of trails in urban green spaces. Yerra and Levinson (2005) models network growth with localized investment rules. Levinson and Yerra (2006) investigates the self-organization of road networks using a travel demand model coupled with revenue, cost, and investment models. Xie and Levinson (2009) adopts the approach of iterative process of network loading, traffic demand dynamics, investment, and disinvestment. Such decentralized agent-based approaches provide a down-to-top perspective to examine phase changes of network growth, path dependency (Arthur, 1989) and multiple equilibria (Yang, 1998; Correa et al., 2004).

Although a spectrum of sources can lead to the birth of roads, we desire to understand the economic incentives for road network growth from a microscopic view. The objective of this research is to model the impact of individual land owners' behavior on road network patterns. The idea is consistent with Powers's review of the history of road building in the US. First, according to Powers (1910), the early roads were built due to a call for communication and navigation. In this research, we assume that roads are built by self-interested land developers who aim to increase their own land parcels' accessibility. Second, "road building began at centers and spread out with the spread of population" (Powers, 1910). So in this paper a center with the highest economic value of accessibility is presumed to exist (and therefore other land owners most want to connect to it). This represents for instance the location of a port or railroad station that provides accessibility to the outside world. Third, as the anecdotal evidence about early roads in Massachusetts Bay Colony depicts, "in 1636 a measure was passed in the Massachusetts Bay Colony which provided that two or three men from adjacent towns get together and lay out proper roads...provided they did not necessitate pulling down a man's house or going through his garden or orchard" (Powers, 1910). To replicate this scenario, the road network is thus modeled as an undirected graph on a land-use layers comprising a grid of land parcels which roads cannot cross. While the gridiron pattern is idealized, it has been de facto widely adopted in many places of the US [1] and elsewhere (Ben-Joseph, 2005); a case in point is Minneapolis-St. Paul (Twin Cities), MN, which have a typical grid-like pattern (see the 1906 map in Fig. 1). Given the historical accounts, we endeavor to make our model close to the real environment yet also simple enough to convey the results and implications most clearly. In this research, we are interested in exploring the topological properties of road networks from our model and the dynamic process wherein they are generated.

The rest of the paper is organized as follows. Section 2 sketches the history of urban road network growth in Minneapolis-St. Paul, MN. Section 3 introduces our agent model. The network measures are described in Section 4. Section 5 presents the results and analysis, following which the implication of the results are discussed. The last section summarizes our findings.

2 Minneapolis-St. Paul: a tale of two cities

The Minneapolis-St. Paul metropolitan area is the largest metro area in Minnesota, and is ranked No.16 in the US in terms of metropolitan population (U.S.Census, 2009). Adams and VanDrasek (1993) has portrayed the evolution, geography, social fabric, and economic life of the Twin Cities. Based on this work, we briefly review the historical path of urban road network of the Twin Cities, which also backdrops our modeling methodology.

In both cities, the oldest neighborhoods and roads lie within a mile or two of the downtowns

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[1] The wide embracement of the grid-like pattern was mainly due to the history of land survey practice in the US in the 18th century, although whether its benefit outweighs its cost is still in question.

formed in the late nineteenth century. As shown in Fig. 1, the road networks in 1906 near the downtowns (centers) of the Twin Cities are remarkably dense and the street are narrow, which probably suggests the early settlement of the two cities. Yet there are many vacant rectangle-shaped areas in the South. As the amenities of the neighborhoods in the downtowns deteriorate after World War II, people of a higher social and economic class gradually move out of the neighborhood. The road network also starts to spread out in many directions and ultimately covers the vacant lots of the metro area and beyond. The tale of the two cities indicates that the road network experiences an evolutionary growth process, both temporally and geographically. Our interest in this process leads us to investigate network topological changes in a context allowing for interactions between road builders.

3 The Model

3.1 Assumptions

In this research, we define a road (link) as a physical connection between two adjacent parcels. The road network to be built overlays a grid-like land layer of N land parcels, respectively owned by N land owners. The value of a land parcel is determined by its accessibility to other land parcels. Land owners build roads to increase the accessibility of their own parcels (and thus increase parcel values). Roads (links) can only run parallel to the x-axis or y-axis, with no overpasses. In addition, Road construction is irreversible; once a road is built, it cannot be severed. Multiple iterations are run until a stable road pattern emerges (i.e., no new links are built).

The agent model is programmed on the Netlogo platform (Wilensky, 1999). In programming, we adopt a square-like region as the basic layer with $k \times k$ (which equals N) land parcels. In our outputs, a non-centered parcel is symbolized by a green circle, and the centered parcel is marked by a red circle.

3.2 Micro-economic principle of road construction

Parcel owner m (which also indicates parcel m) builds road link k in iteration t to maximize the value of its parcel:

$$p_m(k,t) = \sum_{j=1}^{J} d_{mj}^{-\delta} \cdot w_j - (d + \sum_{i \in R} d_i) \cdot c \tag{1}$$

where d_{mj} is the shortest path between parcel m and parcel j; δ represents the distance decay parameter; w_j refers to the value of accessing land j, which takes on a pre-determined value (so that w_{center}, the value of accessing j if it is the centered parcel (or an important locale such as downtown) is higher than $w_{noncenter}$. The first part of this function, a gravity model, refers to the value based on accessibility measures (Levinson et al., 1994), meaning that the benefit deteriorates geometrically with the distance. The second part of this function represents the total cost of building roads by land owner m in all iterations. The length of the newly-built link k is represented by d (the value is also pre-determinded). R is the road set built by parcel owner m in previous iterations.

Parcel owners take turns to build roads; the sequence is randomly decided. Each parcel owner can make two choices at one time: (1) building one link between two adjacent land parcels which are not yet connected. Moreover, a new road can only parallel the x-axis or y-axis. (2) building no links. Out of all possible links to be built, if the maximum benefit of $p_m(k,t)$ that can be obtained in iteration t is larger than the benefit of its previous iteration, parcel owner m then will build link k. This is thus a locally selfish, myopic optimization, maximizing short term benefit for the agent itself, similar to the greedy algorithm.

4 Measures of the topological attributes

After road networks are generated, some topological measures are used to evaluate the networks: degree centrality (D), closeness centrality (C), and betweenness centrality (B). While these concepts are originally proposed to measure certain properties for each node, here we calculate their mean values for all nodes to assess the collective structural feature.

Let's assume undirected graph G of J nodes (potential junctions) and K links; the graph can be represented by $J \times J$ matrix, where an element, if equaling 1, indicates the existence of a link and zero otherwise. This is a sparse matrix because links can only be constructed parallel to the x and y axis. Degree centrality is based on the idea that important nodes have the largest number of ties to other nodes in the graph. Based on Wasserman and Faust (1994), the degree centrality of node i is defined as:

$$D_i = \frac{\sum_{j=1}^{J} a_{ij}}{J-1} = \frac{k_i}{J-1} \tag{2}$$

where k_i is the degree of node i, i.e., the number of nodes adjacent to i.

Closeness centrality, C, is used to measure to which extent a node i is near to all the other nodes along the shortest paths (Sabidussi, 1966). The closeness centrality of node i is calculated as:

$$C_i = \frac{J-1}{\sum_{j \in G, j \neq i} d_{ij}} \tag{3}$$

where d_{ij} is the shortest path length between i and j, the smallest sum of the edges length throughout all the possible paths in the graph between i and j.

Betweenness, a measure of centrality of a node in a network, is the fraction of shortest paths between node paris that pass through the node of interest. Nodes that occur on more shortest paths between other nodes have higher betweenness centrality. The betweenness centrality of node i is:

$$B_i = \frac{1}{(J-1)(J-2)} \sum_{j,k \in G, j \neq k \neq i} n_{jk}(k)/n_{jk} \tag{4}$$

where n_{jk} is the number of shortest paths between j and k, and $n_{jk}(i)$ represents the shortest paths between j and k which contain node i.

In this research, the multiple centrality measures are calculated through the UCINET software (Borgatti et al., 2002).

5 Results and analysis

Our basic experiment is performed in the context of a 9×9 evenly-spaced grid in the form of a square, each point of which stands for a parcel (or land owner). In the beginning there is no road. The parameter values used in the basic simulation are shown in Table 1. Our results find multiple stable road network patterns given different sequences of decision-making for parcel owners. Some exemplary patterns are illustrated in Fig. 2. In Fig.2-1 to Fig.2-3, while not all potential roads are built, the road patterns are close to full connectivity; moreover, Fig.2-4 is fully-connected. Different sequences of decision-making for parcel owners lead to different network topologies in both temporal and spatial terms; this phenomenon is addressed as path dependency by Arthur (1988). It should be noted that were it not for the centered parcel (the red dot in the center), there would have been no roads because the cost of building a new road is higher than the benefit ($900 > 90 \times 4^{-0.3}$). Yet thanks to the existence of the valuable central parcel, roads are first paved around it and then spread out to other areas. With more parcels are connected to the network, the value of connecting to the whole network ascends, and ultimately all parcels are connected to the network. Fig. 3 displays the evolution of network patterns in different iterations until equilibrium. We can see that, at the end of the first iteration, the network pattern is tree-like. At the end of the second round, the network expands to parcels on the periphery; some redundant links are added to the tree-like structure. From the third iteration to the fourth iteration, the network gradually become fully-connected. This evolutionary path reveals that road network growth is a dynamic process where new roads are first built to connect to important parcels before they expand to less important parcels. The tree-like structure emerges first; yet later redundant links are added to the networks, which render multiple traveling paths from one parcel to another.

What then are the impacts of different values of the key parameters on road network patterns? First of all, we perform a sensitivity test by changing the value of $w_{noncenter}$ from 0 to 100, while keeping other parameters fixed. Our hypothesis is that as $w_{noncenter}$ gets larger, the network becomes denser. As expected, the simulation results disclose obvious phase changes for road networks given different values of $w_{noncenter}$ (see Fig. 4). For example, if $w_{noncenter} < 47$, only four links emerge which all connect to the centered parcel. As $w_{noncenter}$ becomes larger than 46, the threshold, all parcels are connected to the network (see Fig. 4-2, 4-3). Moreover, when $w_{noncenter}$ is larger than 100, the grid-like network is fully connected.

Furthermore, Fig. 5 shows the mean degree centrality, closeness centrality, and betweenness centrality for all connected parcels. All the centrality measures witness a sharp phase change when $w_{noncenter}$ rises above 46. Also, when $w_{noncenter} > 90$, the network switches to be fully connected, and the centrality measures show no change thereafter. This phenomenon may suggest that as the social, economic, or cultural values of neighborhoods surpass certain thresholds, the "invisible hand"—people's motivation to access such areas—will induce road network growth substantially.

Second, we fix the value of $w_{noncenter}$ to be 90, yet change the value of w_{center} from 1200 to 1800. We find that when $w_{center} \leq 1364$, there will be no network (because $900 > 1364 \times 4^{-0.3}$, i.e., the cost of building a link to the centered parcel is higher than the benefit). As w_{center} becomes larger than this threshold, the whole network becomes nearly fully-connected; when $w_{center} \geq 1367$, the road network turns out to have full connectivity. Further, Fig. 6 illustrates the sharp changes of centrality measures for the network patterns given different values of w_{center} .

6 Discussion

Although the growth of road networks in the real world are impacted by almost numerous factors, this research aims to shed light on the effect of a possible economic incentive—the value of accessibility. Our simulation results replicate the dynamic growth of road networks and their phase changes in different economic conditions. Two major implications can be derived.

First, road networks have the property of self-organization and evolution. Even without a central authority or following an optimal design, interesting road network patterns emerge out of individual parcel owners' road-building behavior. When certain economic conditions are met, roads are first built around the central parcel, and then gradually cover the parcels on the periphery. The tree-like (non-redundant) structure is the emergent topological characteristic in the first stage; as more iterations are run, the network not only reaches other parcels farther from the center, but also provides multiple paths for already-connected parcels. Meanwhile the value of the whole network for each parcel increases.

Second, the growth of road network also features path dependency and phase changes. Regarding path dependency, our results uncover that different sequences of decisions lead to different network topologies; moreover, the degrees of connectivity for individual parcels can be different. For phase changes, as the values of some parameters in the model exceed certain threshold, road network topology experiences a clear-cut transformation. This implies that even a small variation of certain economic conditions for places may trigger fundamental changes for road network in the long run.

7 Conclusions

In this paper, an agent model is developed to illustrate the dynamics of road network growth. The model is based on the assumption that self-interested land parcel owners build roads to increase the accessibility of each's parcel and thereby to enhance parcel value. After reviewing the development of early roads in Minneapolis-St. Paul since the late 19th century, we simulate network growth on a grid-like land use layer with a downtown (the central parcel with high value of accessibility) in the center. The network topologies are evaluated by three centrality measures (degree centrality, closeness centrality, and betweenness centrality). We first find that networks evolve from a simple tree-like structure to a more connected network which provides multiple paths from one destination to another. Our simulation results also support that the development of road network experiences an evolutionary process, and that when the economic or social conditions of places reach certain thresholds, network patterns could go through spectacular phase changes.

This research provides insights into the formation of early roads, the foundation of today's hierarchical transportation systems. While fully recognizing that central authorities have played an important role in advancing current road networks, we study the dynamics of roads out of individuals' spontaneous behavior. In the future, we will quantitatively analyze the road networks in the Twin Cities in the last 50 years to validate the parameters in the model. Such a model may help explain the change from grid-like networks constructed in the pre-World War II era to the more hierarchical networks constructed in the post-War era.

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Table 1: Values of parameters in the basic simulation

Variable	Description	Value
δ	distance decay parameter	0.3
d	length of a new link (mi)	4
c	cost of building a new link between two adjacent nodes ($)	900
w_{center}	value of connecting to the center land parcel ($)	1500
$w_{noncenter}$	value of connecting a non-center land parcel ($)	90
N	total land parcels (owners)	81

Note: d also equals the distance between two adjacent parcels.

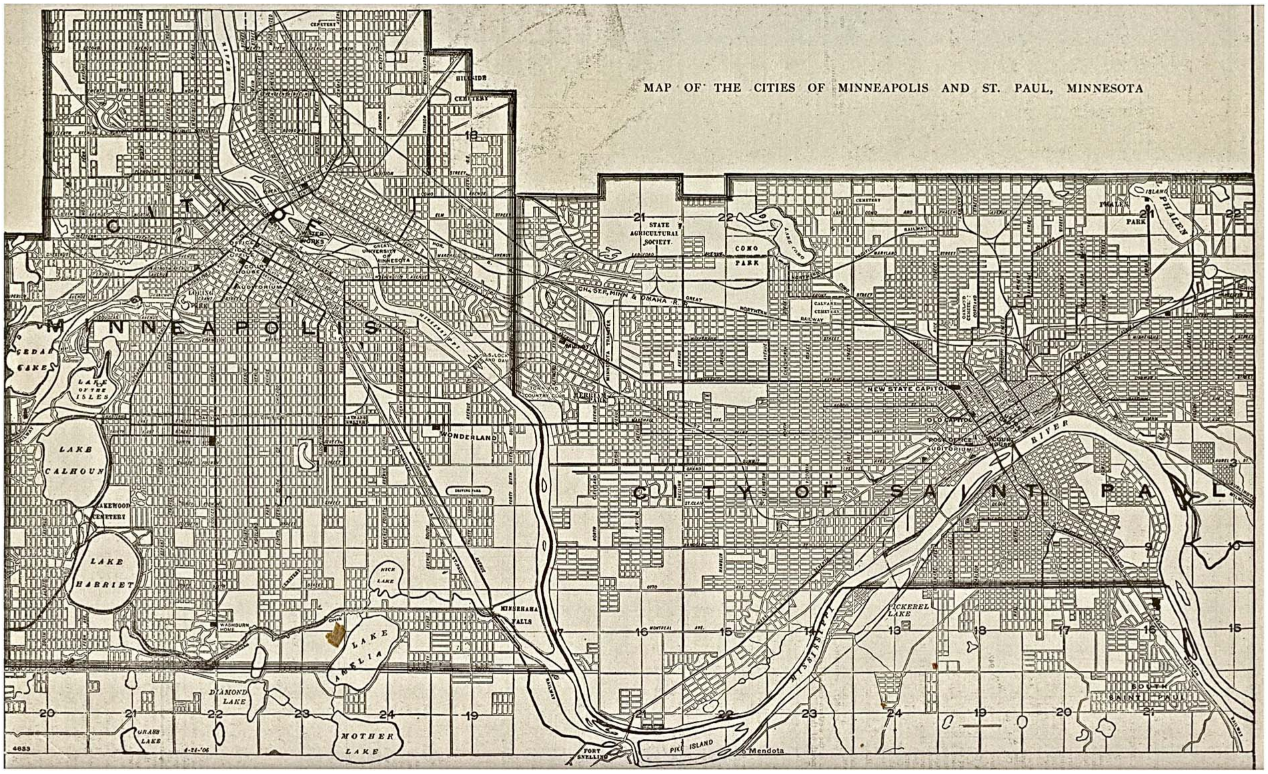

Figure 1: Road map of Minneapolis-St.Paul, MN, 1906 (The New Encyclopedic Atlas and Gazetteer of the World. Edited and Revised by Francis J. Reynolds, 1917) Source: `http://www.lib.utexas.edu/maps/historical/minneapolis_1906.jpg`.

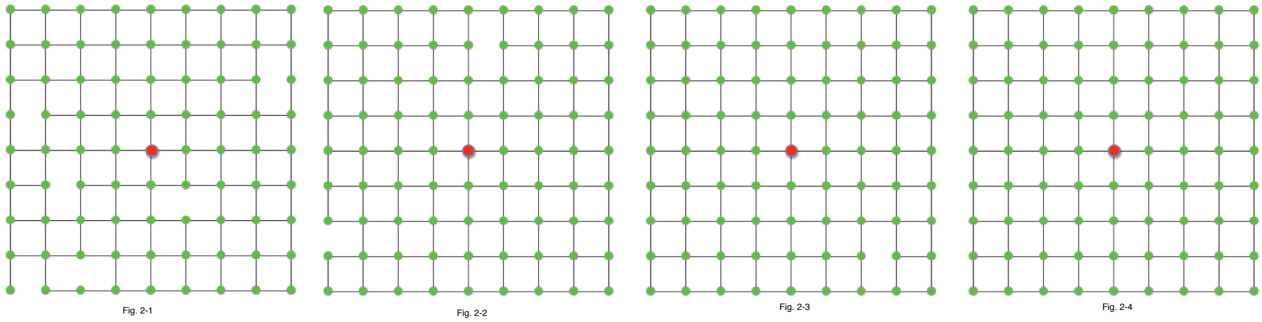
Fig. 2-1
Fig. 2-2
Fig. 2-3
Fig. 2-4

Figure 2: Exemplary resultant road network patterns given different sequences o decision making for parcel owners, with β = -0.3, c = 900, $w_{noncenter}$ = 90, w_{center} = 1500, N=81, and d = 4.

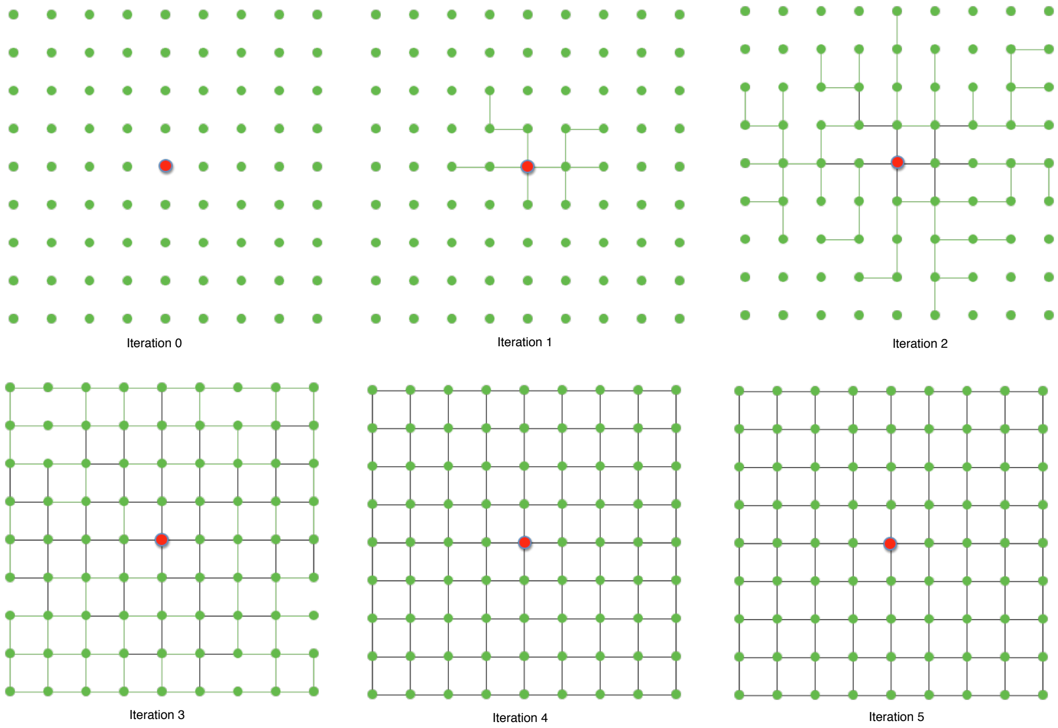

Figure 3: The evolution of road networks from Iteration 0 to Iteration 5, with β = -0.3, c = 900, $w_{noncenter}$ = 90, w_{center} = 1500, N=81, and d = 4. There is no road in the beginning. A tree-like structure emerges at the end of first iteration. At the end of Iteration 6, the network is fully-connected. The green links indicate the roads generated in the current iteration; the dark links stand for the roads emerged in previous iterations.

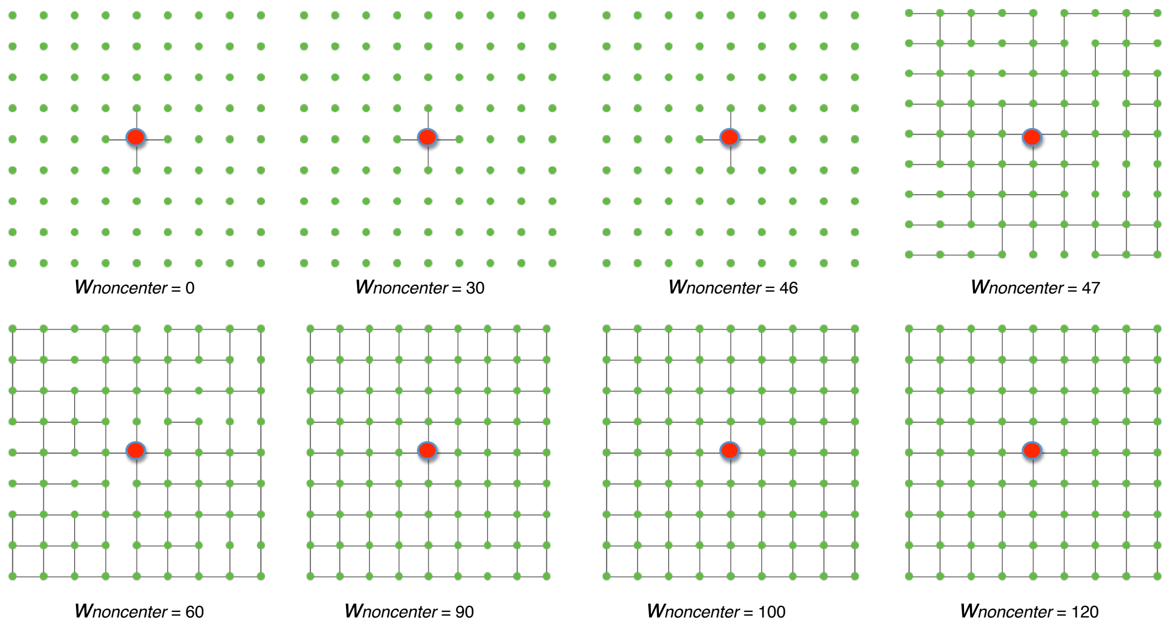

Figure 4: Road network patterns in equilibrium as $w_{noncenter}$ changes from 0 to 120, with β = -0.3, c = 900, w_{center} = 1500, N=81, and d = 4. The Road network experiences clear-cut phase changes when $w_{noncenter}$ becomes larger than 46.

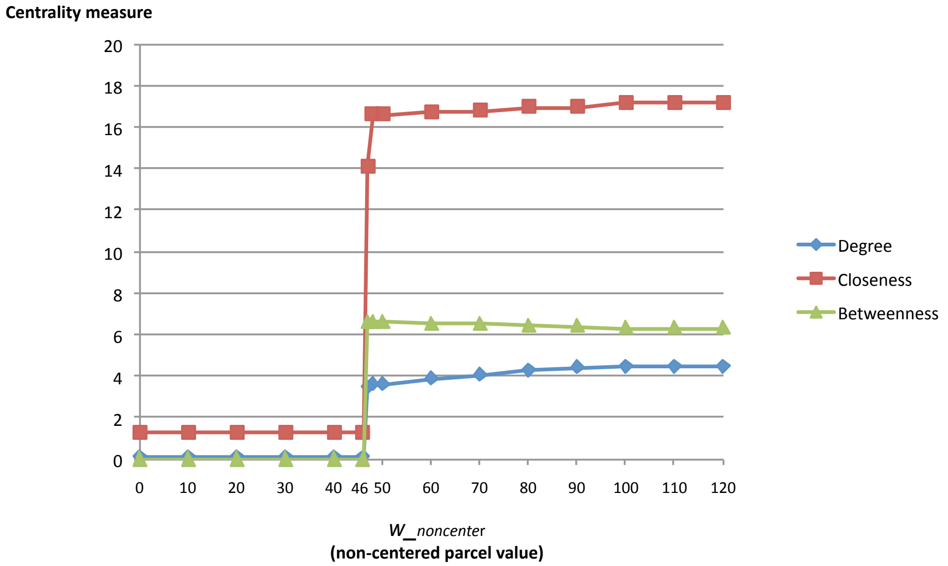

Figure 5: Network centrality measures as $w_{noncenter}$ changes from 0 to 120, with β = -0.3, c = 900, w_{center} = 1500, N=81, and d = 4. The centrality measures changes considerably when $w_{noncenter}$ becomes larger than 46. The centrality values turn to be stable when $w_{noncenter} > 90$.

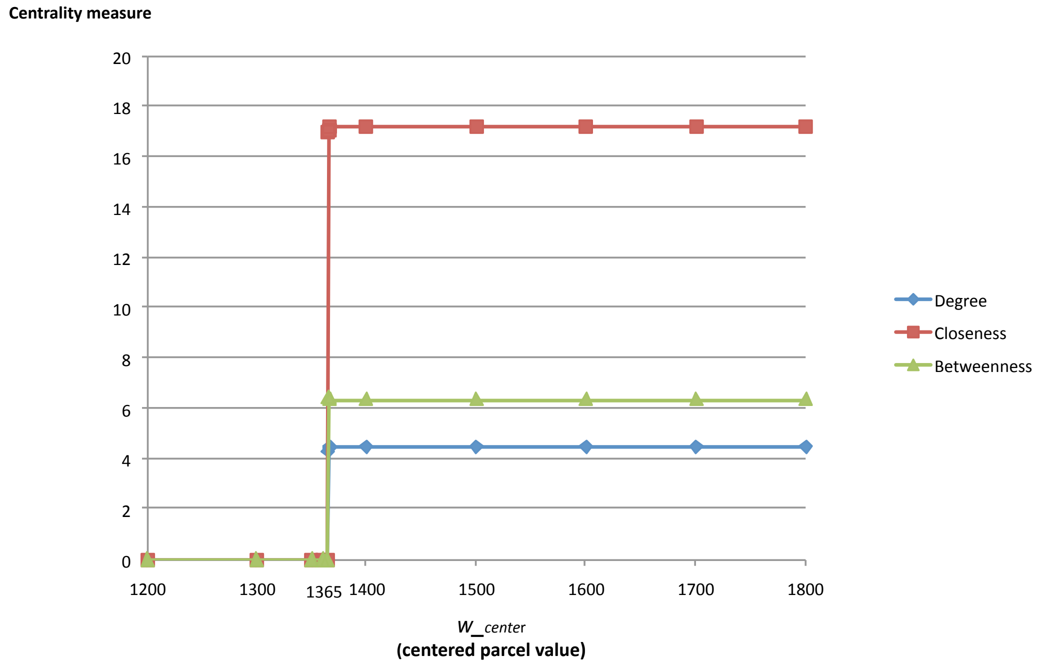

Figure 6: Network centrality measures as w_{center} changes from 1200 to 1800, with β = -0.3, c = 900, $w_{noncenter}$ = 90, N=81, and d = 4. The centrality measures equals 0 when $w_{center} < 1365$, for there is no network generated. When $w_{center} = 1366$, there is a drastic increase of the centrality measures, and the network becomes nearly fully-connected. When $w_{center} > 1367$, the network is fully-connected; the centrality values become steady thereafter.