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Dependent Revenues, Capital Risk and Credit Rationing

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Dependent Revenues, Capital Risk and Credit Rationing

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Abstract

Much of the literature on financial markets has not dealt with dependency of project revenues. In a setup similar to the seminal SW model, we show that the type of equilibrium can crucially depend on the degree of project dependency. By making aggregate payoffs risky, households face capital risk. Therefore, risk aversion and households' consumption-savings decision become very important. Capital risk deters households from saving so that there might be a credit rationing equilibrium. Defining the social optimum, we find that project dependency might reduce the number of safe projects in equilibrium in a socially harmful way. Thus, project dependency can aggravate adverse selection. In three extensions, we will show how risk aversion, imperfect revenue dependency and a different modelling of dependency influence our results. Our analysis points out that project dependency is an important factor in the determination of credit market outcomes.

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Contents

1 Introduction

Credit rationing has become a very important issue recently. In reality, firms complain that credit is not available. Secretaries blame banks to hoard money. On theoretical grounds, further research seems to be necessary as the forthcoming publication of Arnold and Riley (2009) (AR, henceforth) shows. It makes an important contribution but leaves some space for interesting further considerations. An important fact to analyze is dependency of project revenues,[1] whose consequences for one of the banks' most important tasks, diversification, can be tremendous - as dramatically shown by the current financial crisis. Revenue dependency within a single bank's credit portfolio is one of the main research topics in financial risk management. However, to our best knowledge, there is no literature on the effects of dependency of project revenues on credit market outcomes with the possibility of credit rationing.

The concept of credit rationing has already been known to Keynes (1930) in his Treatise on Money (Vol. 1). A seminal paper is Stiglitz and Weiss (1981) (SW, henceforth) who pointed out the important role of asymmetric information in credit rationing.[2] They used the term credit rationing "...for circumstances in which either a) among loan applicants who appear to be identical some receive a loan and others do not, and the rejected applicants would not receive a loan even if they offered to pay a higher interest rate; or b) there are identifiable groups of individuals in the population who, with a given supply of credit, are unable to obtain loans at any interest rate, even though with a larger supply of credit, they would" (pp. 394-395). SW argue that an increase of the loan rate might decrease banks' expected returns since some "good" borrowers do not demand credit any more, notwithstanding the remaining borrowers paying a higher rate (if they are able to pay back). They get an equilibrium with credit rationing if the decrease is such that the global maximum of the return function does not occur at the highest possible loan rate. However, the credit rationing result is inconsistent with the very SW assumptions, as pointed out by AR. They show that the natural outcome of the SW model is a two-price equilibrium in which only safe firms are rationed.

[1] Instead of "dependency", some readers might prefer to speak of "correlation". We use "dependency" since the statistical concept of correlation cannot be applied to our model.

[2] Other important early contributions are Jaffee and Russell (1976) and Bester (1985). Note that the definitions of credit rationing slightly differ between all the papers mentioned.

AR present two modifications of the SW assumptions which make credit rationing à la SW possible again: either a cost for seizing collateral or "fraudulent" borrowers (cf. the AR paper for details).

We build on the model of SW and AR and introduce dependent project revenues as a central assumption. We show that dependency might cause credit rationing à la SW, i.e., at a single interest rate in equilibrium. An implication of project dependency (and the assumption that banks pass through risk) is that households face capital risk in their consumption-savings decision. As a consequence, we have to explicitly analyze households' behavior,[3] thereby combining the credit rationing literature with a wholly different strand of literature, namely savings under capital risk.

We choose the easiest modelling approach with only two types of firms: risky and safe ones. We assume that only the revenues of the risky are dependent. In our eyes, it seems to be plausible to make this assumption for the following reasons. Low-risk firms can be thought of as producers of goods which meet physiological needs. Examples for such low-risk projects are investments in industries such as foods and beverages, utilities, health care, and so on. In these industries, risk is fairly low, and so is dependency. We resort to portfolio theory and the separation between market risk and idiosyncratic risk to make the argument clearer. Idiosyncratic risk is present in every firm. By definition, this kind of risk is independent between firms. The exposure to market risk, however, is highly unequal. We suggest to interpret low-risk firms as the ones which do not face (high) market risk. Hence, dependency between low-risk firms is low, but idiosyncratic risk is prevalent. In contrast, due to market risk, project revenues in high-risk industries are much more likely to be highly dependent since they frequently depend on some sort of breakthrough, which might be technological, political or social in nature.[4]

The structure of the paper is this. In section 2, we present the assumptions of our model (in 2.1) and put some emphasis on a bank's return function (in 2.2), which significantly differs from the one resulting from independent revenues. We specify the households' consumption-savings decision in a standard expected utility setup, and analyze the firms' investment decision (in 2.3). The central section of the paper describes different equilibrium cases

[3] SW and AR assume an exogenous, increasing capital supply.

[4] An example where a technological breakthrough caused a whole industry to flourish is the IT sector. A case in point for a social breakthrough is web technology which flourished, too. Genetic engineering is an example where we do not know yet if a political breakthrough occurs or not.

and sets up a condition for social optimality in order to find out equilibrium inefficiencies caused by asymmetric information (in 2.4). We do comparative statics (in 2.5) before extending the model threefold. First, in section 3, we present a non-expected utility setup which allows us, amongst other things, to make our propositions from section 2 more concrete by attributing results to different preference components. Second, section 4 generalizes the concept of project dependency and shows that the main results do not rely on the extreme assumption of perfectly dependent revenues. We implement imperfect dependency as deterministic (in 4.1), stochastic (in 4.2) and, going a little further, as stochastic and uncertain (in 4.3). As a further robustness test, section 5 describes another sort of project dependency. We add dependency of the safe firms' revenues (intra-type) and an inter-type dependency in that risky firms can only succeed if safe firms do. The final section will give some concluding remarks.

2 The model

2.1 Assumptions

There are two types of firms: risky and safe ones, with success probability p_R and p_S, respectively. Note that "safe" means relatively safe, that is, $p_R < p_S < 1$. We have a continuum of mass N_S of safe firms and a continuum N_R of risky firms. We define $\beta \equiv \frac{N_S}{N_S+N_R}$ as the economy's share of safe firms. Project revenues $\tilde{\mathcal{R}}$ are a binary random variable: If successful, revenues are $\mathcal{R}_S$ and $\mathcal{R}_R$, respectively, where $\mathcal{R}_S < \mathcal{R}_R$.[5] In case of default, the payoff is zero. Both types of firms have the same expected project revenue $p_R\mathcal{R}_R = p_S\mathcal{R}_S = E[\tilde{\mathcal{R}}]$. Projects require B ($< E[\tilde{\mathcal{R}}]$) units of capital which cannot be brought up internally by firms: They must rely on outside funding by banks which require C ($< B$) units of collateral.

There is asymmetric information: Firms know their type, but banks cannot observe it. However, banks know the distribution of types in the economy. Firms have one and only one project to invest in, which is either a risky or

[5]There are two different types of "returns" in our model: firms' project revenues and the rate of return of a bank. Since we use the latter far more often, we reserve the more common symbol $\tilde{R}$ for it.

a safe one.[6] This means that there is no moral hazard (no hidden actions). Furthermore, we assume that revenues are observable ex post (costless state verification), i.e., asymmetric information only appears in the guise of hidden information. It is the only market friction since we also abstain from enforcement problems.

A central assumption concerns the dependency of project revenues: In the main part of the paper, we assume that project revenues of risky firms are perfectly dependent (either all firms succeed or none does), whereas revenues of safe firms are independent.[7]

We assume that there are many banks whose intermediation is costless,[8] and that these banks have no equity.[9] Therefore, they go bankrupt (which we model as a disutility of minus infinity) if they cannot serve a stipulated claim from a deposit contract. As a consequence, banks do not take risks. Instead, they pass it through to households. They are intermediaries and, thus, active in two markets: the credit market, where they lend out funds to firms on the one hand, and the deposit market, where they collect funds from households on the other. We assume that banks set prices on the credit market, whereas they are price takers on the deposit market. A reader might wonder about the use of these two different concepts of market structure. The assumption is mainly technical in that it facilitates equilibrium argumentation. Assuming price setting on both markets would require a game theoretic foundation which would tremendously enhance complexity.[10] If banks are price takers, a single bank can choose an arbitrary price for its own goods (interest rate on money) without influencing any other bank's price.

The attitudes towards risk are crucial: Households are assumed to be risk-averse, whereas firms and banks are risk-neutral.[11]

There are H homogeneous households whose utility is assumed to exhibit

[6] Thus, we can speak of project type and firm type interchangeably.

[7] We relax both the assumption that only revenues of the risky are dependent and the assumption about perfect dependency, in sections 4 and 5.

[8] Thus, we do not analyze the raison d'être of banks. A seminal paper where bank intermediation is endogenous is Diamond (1984).

[9] If we modelled banks with equity, we would have to say much more about their risk attitude and behavior in the market. However, this shall not be our focus.

[10] This has been done by Arnold (2007) for the case of independent project revenues and a continuum of borrower types. Results did not change qualitatively.

[11] Changing risk attitudes of banks and firms does not have significant consequences for the subsequent analysis. However, households risk attitude is of utmost importance as will become clear later on.

constant relative risk aversion (CRRA). They maximize expected utility in a two-period setup with income Y in period 1. Income in period 2 only comes from savings. If there are several contract offers by banks, we assume that households only invest in one contract.

Most of the assumptions are identical to a two-type version of the models of SW and AR with which we contrast our results. However, our assumption of project dependency has far-reaching consequences in that it introduces capital risk for households. Thus, our second main assumption, risk aversion of households, becomes very important, too. Taken together, explicit modelling of the consumption-savings decision is indispensable.

2.2 Return function

The return function of a bank is state-contingent due to dependent revenues of the risky firms. Dependency is perfect so that we have only two states of the world, where a state of the world is not defined by certainty about every random variable, but by the aggregate level of repayment. In the good state (probability p_R), all risky firms succeed and so does a share p_S of the safe firms (if they apply for capital in the first place). With probability $(1 - p_R)$, the bad state occurs. Then, from the risky firms, banks only get the collateral, whereas of the safe firms, the same share p_S is successful (if they apply).

In Figure 1, the two solid lines denote the bank's returns in the two states. The dashed line in between is expected return. Let us stress the crucial difference to AR and SW at this point. In their setup, households do not face risk because the expected returns, which are passed through to them, realized for sure (due to the law of large numbers (LLN) and a continuum of borrowers). Therefore, even though SW and AR talk of expected returns, households do not actually expect the deposit rate, they rather know it.

In the first interval ($r \in [0, r_S]$), both firm types demand credit. In the second interval ($r \in (r_S, r_R]$), only the risky do, i.e., there is adverse selection. This can be understood from expected profits of firm i ($i \in \{S, R\}$) as a function of the loan rate,

$$E\pi_i^{firm}(r) = (1 - p_i)(-C) + p_i \left[\mathcal{R}_i - (1 + r)B\right]. \tag{1}$$

Since firms are risk-neutral, they demand credit as long as their expected profits are non-negative. The respective breakeven loan rates are

$$r_S = \frac{p_S \mathcal{R}_S - (1 - p_S)C}{p_S B} - 1, \tag{2}$$

$$r_R = \frac{p_R \mathcal{R}_R - (1 - p_R)C}{p_R B} - 1. \tag{3}$$

It is instructive to look at the state-contingent return in the two intervals. In the first interval, both firm types are active so that we get

$$i_b(r) = \frac{N_S(p_S(1+r)B + (1-p_S)C) + N_R C}{B(N_R + N_S)},$$

$$i_g(r) = \frac{N_S(p_S(1+r)B + (1-p_S)C) + N_R(1+r)B}{B(N_R + N_S)}.$$

From the safe firms, a share p_S is successful and pays back principal plus interest, $(1+r)B$. The remaining share $1-p_S$ defaults and loses its collateral. This is the same in both states of the world. However, the repayment from a risky firm differs between states: $(1 + r)B$ in the good state and C in the bad state. In the second interval, only risky firms demand capital so that

$$i_b(r) = \frac{C}{B} - 1, \qquad i_g(r) = r.$$

Several properties of the rates of return of banks (state-contingent or expected) can be pointed out (cf. Figure 1):

i) The good state return $i_g(r)$ is monotonically increasing in r with a discontinuous upward jump at r_S.

ii) The bad state return $i_b(r)$ is monotonically increasing in the first interval, but constant and at its global minimum in the second.

iii) The expected return $E[i(r)]$ is monotonically increasing in r in both intervals, but discontinuously decreasing at r_S.

iv) $E[i(r)]$ attains its global maximum at r_R.

v) The variance of the return $Var[i(r)]$ is monotonically increasing in r.

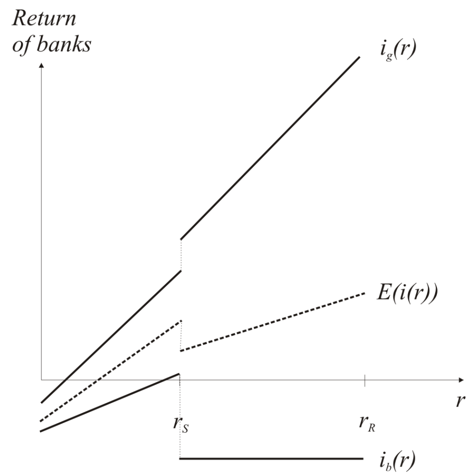

Figure 1: Banks' returns given dependent risky projects.

We prove these properties in appendix 7.1. For these proofs, as well as for an intuitive understanding, we need to know the expected returns of a bank in state k ($k \in \{good, bad\}$), which are given by

$$E\pi^{bank}(r|k) = \frac{E[p|k](1+r)B + (1 - E[p|k])C}{B} - 1. \tag{4}$$

The expectation $E[p|k]$ is the expected success probability in state k and equals the proportion of successful firms, due to the law of large numbers. It is a function of the loan rate r as can be seen in Figure 2. The thick line is the expected success probability in the good state, the dashed line represents the bad state probability. $E[p|k]$ differs in (two) states and (two) intervals and can thus take on four different values.

Property iv) is the result of AR. Intuitively, there are both risky and safe firms active at r_S, and the risky make strictly positive expected profits. At r_R, only risky firms are active and their expected profits are zero. Thus, expected returns of banks must be maximum at r_R.

2.3 Credit and deposit market

From the above, it is clear that credit demand, D, is a stepwise function of the loan rate r, equal to $(N_S + N_R)B$ in the first and $N_R B$ in the second interval, zero otherwise. This is because all firms have positive expected profits in the first interval, whereas in the second, only the risky have.

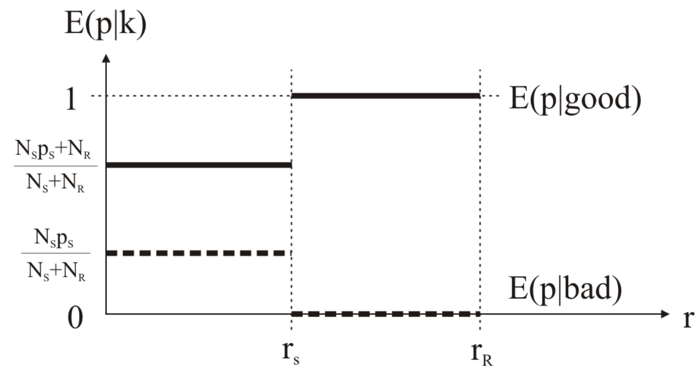

Figure 2: Expected success probabilities.

The description of the mechanisms on the deposit market are more complicated. Since households are homogeneous, capital supply is simply the number of households times savings of a representative household. A household's optimal savings depend on the deposit rate faced. Since banks have to make zero profits in a competitive equilibrium (due to the usual downbidding process) and pass through risk, any equilibrium deposit rate combination must equal the return rate combination, so that $i_b(r)$ and $i_g(r)$ denote both banks' rates of return and the state-contingent deposit rates offered to households at the same time. We will omit the argument r and write i_g and i_b unless talking about deposit rates at a particular loan rate, such as r_S or r_R, for instance. Households maximize expected utility. Let U denote overall utility and u be instantaneous utility in the respective period. Using additively-separable utility such that $U(c_1, c_2) \equiv u(c_1) + \delta u(c_2)$ (δ being the discount factor), optimal savings s^* solve

$$\max_s EU = E\{u(Y-s) + \delta u(s\tilde{R})\}, \tag{5}$$

where $\tilde{R}$ is the random gross interest rate on deposits (not to be mixed up with $\tilde{\mathcal{R}}$). The FOC is

$$u'(Y-s) = \delta E\left[u'(s\tilde{R})\tilde{R}\right]. \tag{6}$$

We use CRRA utility $u(c) = \frac{c^{1-\theta}}{1-\theta}$.[12] The parameter θ captures attitudes

[12] And $u(c) = \ln(c)$ for $\theta = 1$. We will focus on $\theta < 1$ later on so that we can omit this special case.

towards inequality both over states and in time. Since households are risk-averse, θ must always be positive. Optimal savings s^* can be derived from the FOC of the maximization problem,

$$s^* = \frac{Y}{1 + \left(\delta E[\tilde{R}^{1-\theta}]\right)^{-\frac{1}{\theta}}}. \tag{7}$$

We can replace $\tilde{R}$ using the fact that $\tilde{R} = 1 + i_g$ with probability p_R and $\tilde{R} = 1 + i_b$ otherwise.[13] Thus, equation (7) becomes

$$s^* = \frac{Y}{1 + \delta^{-\frac{1}{\theta}} \left[p_R(1+i_g)^{1-\theta} + (1-p_R)(1+i_b)^{1-\theta}\right]^{-\frac{1}{\theta}}} = \frac{Y}{1 + (\delta z)^{-\frac{1}{\theta}}}, \tag{8}$$

where we use the convenient definition

$$z \equiv E[\tilde{R}^{1-\theta}] = p_R(1+i_g)^{1-\theta} + (1-p_R)(1+i_b)^{1-\theta}. \tag{9}$$

We need the derivative of z w.r.t. r later on. It is

$$\frac{dz}{dr} = (1-\theta)\left(p_R(1+i_g)^{-\theta}\frac{di_g}{dr} + (1-p_R)(1+i_b)^{-\theta}\frac{di_b}{dr}\right) \gtrless 0 \Leftrightarrow \theta \lessgtr 1, \tag{10}$$

since $\frac{di_g}{dr} > 0$ and $\frac{di_b}{dr} \geq 0$ in each of the intervals. At r_S, $\frac{\Delta z}{\Delta r} \lessgtr 0 \Leftrightarrow \theta \lessgtr 1$. To see this, note that we have three discontinuous jumps at r_S: $1 + i_g$ goes up, $1 + i_b$ goes down and $E(\tilde{R})$ goes down. Since $\tilde{R}^{1-\theta}$ is a monotonically increasing concave transformation of the binary random variable $\tilde{R}$ if $\theta < 1$, its expectation $E[\tilde{R}^{1-\theta}] = z$ must decrease at r_S, then. For $\theta > 1$, the transformation is monotonically decreasing and convex such that $E[\tilde{R}^{1-\theta}]$ must increase in that case.

We get indirect lifetime utility (LTU) by inserting optimal savings from equation (8) into the objective function (5),

$$LTU = \frac{Y^{1-\theta}}{1-\theta}\left[(\delta z)^{\frac{1}{\theta}} + 1\right]^{\theta}. \tag{11}$$

Aggregate savings are given by $S = Hs^*$. Both S and LTU are composite functions and can be written as $S(z[i_b(r), i_g(r)])$ and $LTU(z[i_b(r), i_g(r)])$.

[13]It goes without saying that $\tilde{R}$ is a function of r, too.

Thus, they can be plotted in a graph with the loan rate on the abscissa. This will be important to follow our equilibrium argumentation graphically. The loan rate r determines the deposit rates i_b and i_g. The latter two determine optimal savings (and, thus, capital supply) and LTU. They also determine z, which we only introduce to ease some proofs. Regarding the difference between capital supply S and optimal savings s^*, note that one is only an upscaled version of the other, so that all relative relations are valid for both functions. For the equilibrium argumentation, we need capital supply, but the properties of capital supply can also be proven using optimal savings. The equilibrium analysis crucially depends on the shape of capital supply and, thus, on the value of θ. We focus on the case of $\theta < 1$, meaning that the substitution effect outweighs the income effect in the consumption-savings decision. Thus, if $\tilde{R}$ were deterministic, capital supply would increase in the deposit rate. However, since it is random, the shape (especially the slope) of capital supply depends on the change in the distribution of $\tilde{R}$.[14] A stylized graph of capital supply, LTU and the deposit rate combinations (equal to banks' state-contingent return rates) can be found in Figure 3. It clarifies the dependencies: Capital supply and LTU are both functions of $i_b(r)$ and $i_g(r)$. The above mentioned z is just a helpful mathematical construct for the proofs. There are some general properties of capital supply and LTU:

Lemma 1: *If $\theta < 1$, capital supply increases monotonically in r in each of the intervals $[0, r_S]$ and $(r_S, r_R]$ with a discontinuous downward jump at r_S.*

Proof:
The fact that optimal savings s^* increase in each of the two intervals is implied by a more general result, namely Proposition 2 in Basu and Ghosh (1993, p. 124). Transferring their result to the expected utility setup, their proposition states that savings are lower in a first-order stochastically dominated distribution (of deposit rates) if $\theta < 1$. From Figure 1 we can see that a higher loan rate within each interval implies a first-order dominant distribution. Algebraically, $\frac{dz}{dr} > 0 \Leftrightarrow \theta < 1$ in each of the two intervals from equation (10). Moreover, $\frac{ds^*}{dz} > 0 \Leftrightarrow \theta < 1$ from equation (8). Thus, savings increase in each of the two intervals if $\theta < 1$.

The discontinuous downward jump in savings follows from the fact that there must be a deposit rate combination at some loan rate $r > r_S$ which

[14] In our case, this distribution is a binary one which changes its occurrences, but not its probabilities, depending on r.

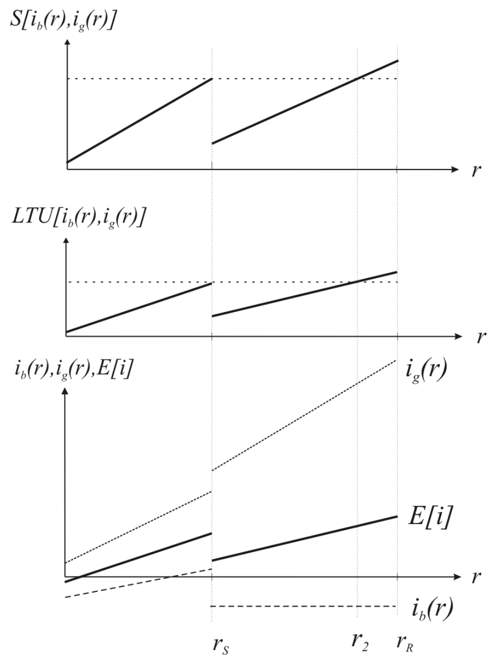

Figure 3: Capital supply, LTU and deposit rates.

constitutes a mean preserving spread (MPS) of the deposit rate combination at r_S.[15] Rothschild and Stiglitz (1971) showed that an MPS leads to a decrease in savings if $\theta < 1$. Since savings increase in the second interval, savings must decrease discontinuously at r_S. Algebraically, it immediately follows from $\frac{\Delta z}{\Delta r}|_{r=r_S} < 0 \Leftrightarrow \theta < 1$ and $\frac{ds^*}{dz} > 0 \Leftrightarrow \theta < 1$.
q.e.d.

Lemma 2: *Irrespective of θ, LTU increases monotonically in r in each of the intervals $[0, r_S]$ and $(r_S, r_R]$ with a discontinuous downward jump at r_S.*

Proof:
Differentiating equation (11) w.r.t. z and simplifying yields

$$\frac{dLTU}{dz} = \frac{Y^{1-\theta}}{1-\theta}\left[(\delta z)^{\frac{1}{\theta}} + 1\right]^{\theta-1} \delta^{\frac{1}{\theta}} z^{\frac{1-\theta}{\theta}} \gtreqless 0 \Leftrightarrow \theta \lesseqgtr 1, \tag{12}$$

[15]If the probabilities of a binary distribution do not change, an increase in variance at a constant mean is equivalent to an MPS. For general distributions, this is not true: Any MPS implies a higher variance, but the reverse is not necessarily true.

since Y, θ, δ and z are always positive. From equation (10), we know $\frac{dz}{dr}$ in each of the two intervals: $\frac{dz}{dr} \gtreqless 0 \Leftrightarrow \theta \lesseqgtr 1$. Therefore, $\frac{dLTU}{dr} > 0 \; \forall \theta$ in each of the two intervals.

LTU has a discontinuous downward jump at r_S since $\frac{\Delta z}{\Delta r}|_{r=r_S} \gtreqless 0 \Leftrightarrow \theta \gtreqless 1$ such that, for $\theta < 1$, $\frac{dLTU}{dz} > 0$ and $\frac{\Delta z}{\Delta r}|_{r=r_S} < 0$ and, for $\theta > 1$, $\frac{dLTU}{dz} < 0$ and $\frac{\Delta z}{\Delta r}|_{r=r_S} > 0$.
q.e.d.[16]

Proposition 1*: For $\theta < 1$, capital supply and LTU have their global maximum at the same loan rate, viz. either r_S or r_R.*

Proof:
From Lemmas 1 and 2, we know that both capital supply and LTU are increasing in each of the two intervals. Therefore, the global maximum of the functions can only be at r_S or at r_R. For $\theta < 1$, $\frac{ds^*}{dz} > 0$ and $\frac{dLTU}{dz} > 0$. Suppose that the maximum of capital supply occurs at r_S. Since $\frac{ds^*}{dz} > 0$, we must have $z(r_S) > z(r_R)$. Since $\frac{dLTU}{dz} > 0$, too, we must have $LTU(r_S) > LTU(r_R)$, i.e., the maximum of LTU occurs at r_S, too. If capital supply is maximum at r_R, $z(r_R) > z(r_S)$ and $LTU(r_R) > LTU(r_S)$, i.e., the maximum of LTU occurs at r_R, too.
q.e.d.

Corollary*: If s^* is the same at two loan rates, LTU must be the same at these two rates, too.*

Proof:
If s^* is the same at two loan rates, z must also be the same. Since LTU (cf. equation (11)) only depends on z and parameters, it must be the same at these two loan rates, too.
q.e.d.

2.4 Equilibrium

As outlined above, any possible equilibrium entails zero-profits for banks. Therefore, we define two different types of equilibrium with a single interest

[16] Increasing LTU within each interval is also consistent with another, verbal proof: Since an increase in r leads to a state-by-state dominant deposit rate combination, a change in s^* implies that households are better off, by revealed preferences.

rate as follows. First, an **equilibrium** is characterized as a loan rate r such that there is no r' with

$$LTU\left[i_b(r'), i_g(r')\right] > LTU\left[i_b(r), i_g(r)\right]$$

that attracts borrowers. We distinguish two types of equilibrium. A **market clearing equilibrium** has capital supply at r equal to credit demand at r. A **credit rationing equilibrium** has capital demand at r exceeding capital supply at r.

With regard to our requirement for both types of equilibrium, there is always a profitable deviation for banks if the inequality holds. If such a state-contingent deposit contract exists, households prefer that contract and a deviating bank can thus offer that contract making zero-profits, or, offering a similar contract where households' utility is still higher than with the original contract, banks can make a profit.

Due to Lemmas 1 and 2 and Proposition 1, we get six possible cases in equilibrium. Since households have no income in period 2 (apart from savings), $\lim_{c\to 0} u'(c) = \infty$ implies that savings are positive.

A graphical exposition of each case can be seen in Figure 4 where the upper graph is a capital supply and demand diagram and the lower part displays LTU, both as functions of the loan rate. Let us start with three cases in which capital supply at r_S is smaller than $N_R B$.

Case I:

One possibility is to have capital supply and LTU maximum at r_S. From our equilibrium definitions, it is clear that there is no market clearing equilibrium in this case since demand exceeds supply at any loan rate r. Instead, there is a credit rationing equilibrium at the loan rate r_S. No other loan rate can be a credit rationing equilibrium. All $r \in (r_S, r_R]$ are impossible since r_S yields a higher LTU for depositors and attracts borrowers. All $r < r_S$ do not yield a higher LTU than r_S either.

Case II:

Another possibility arises if the maxima are at r_R, but still below demand at that rate. The unique equilibrium is then to lend out all capital that can be raised at $(i_b(r_R), i_g(r_R))$ at the loan rate r_R. It is a case of credit rationing since demand exceeds supply and there is no other loan rate with corresponding deposit rates which yield a higher LTU.

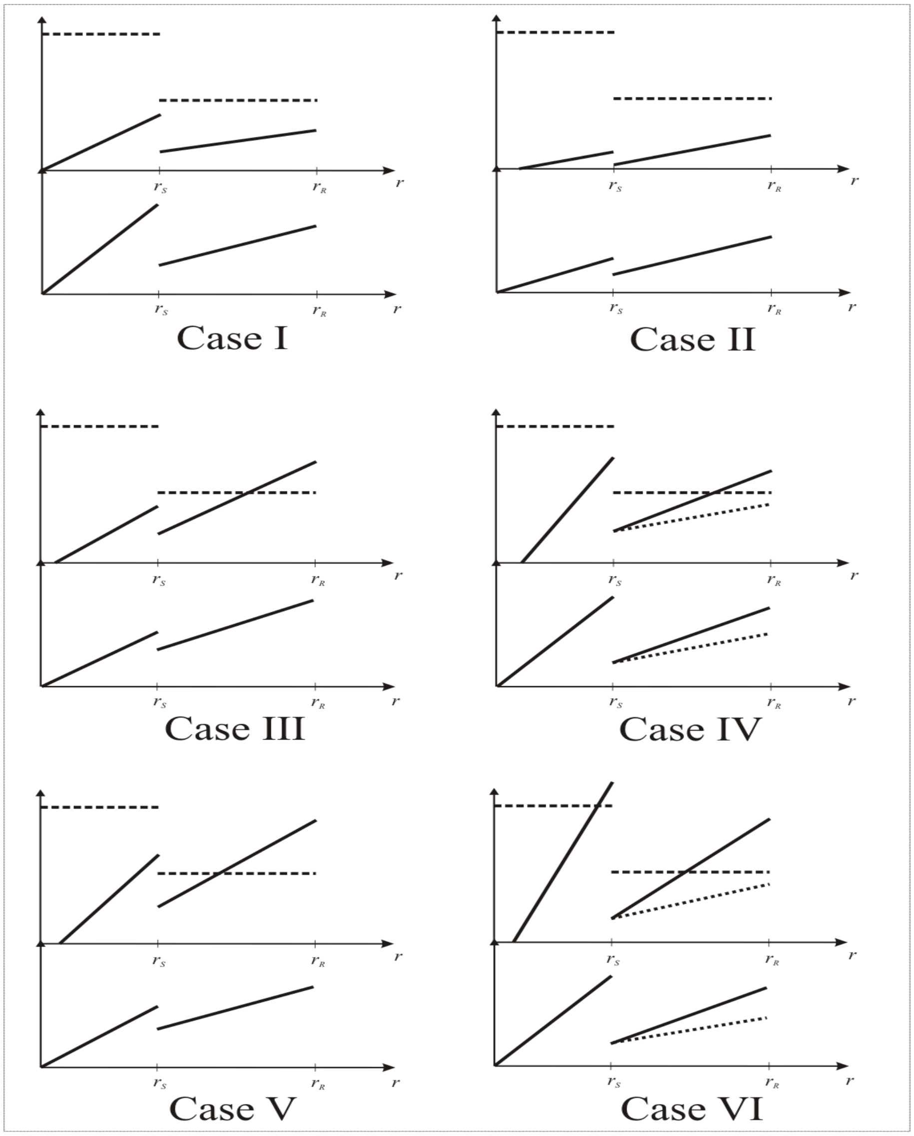

Figure 4: Capital supply and LTU: six equilibrium cases.

Case III:
If capital supply and LTU are below $N_R B$ at r_S, but above it (and thus maximum) at r_R, the unique equilibrium entails market clearing at a loan rate $\breve{r} \in (r_S, r_R]$ with $S(i_b(\breve{r}), i_g(\breve{r})) = D(\breve{r})$. Clearly, the conditions from our definition hold. Supply equals demand and the only loan rates $r > \breve{r}$ whose corresponding deposit rates yield a higher LTU do not attract borrowers.

Next, consider $N_R B < S(i_b(r_S), i_g(r_S)) < (N_S + N_R)B$. There are two qualitatively different subcases.

Case IV:
In the first one, capital supply (and LTU) are maximum at r_S. In this case, there is a credit rationing equilibrium at the loan rate r_S. Demand exceeds supply and there is no other loan rate with corresponding deposit rates which yield a higher LTU. It is irrelevant whether there is a higher, market clearing loan rate or not (cf. the dotted line in Figure 4).

Case V:
Now suppose that the maxima are at r_R. The reader can check that there is neither a market clearing nor a credit rationing equilibrium. In this case, the natural outcome of the model is a two-price equilibrium, as in SW and AR. To see this, recall our corollary and apply it to a loan rate like r_2 in Figure 3: $S(i_b(r_S), i_g(r_S)) = S(i_b(r_2), i_g(r_2)) \Leftrightarrow LTU(i_b(r_S), i_g(r_S)) = LTU(i_b(r_2), i_g(r_2))$. Since households can only invest in one contract, a simultaneous deposit rate offer of $(i_b(r_S), i_g(r_S))$ and $(i_b(r_2), i_g(r_2))$ leads to a capital supply $S(i_b(r_S), i_g(r_S)) = S(i_b(r_2), i_g(r_2))$. In analogy to SW and AR, an equilibrium situation with two prices arises if banks lend out at both loan rates r_S and r_2. The amounts must be such that $S(i_b(r_S), i_g(r_S)) = S(i_b(r_2), i_g(r_2))$ will be lent out in aggregate. Furthermore, the amount of funds lent out at the higher rate r_2 must be equal to residual demand after credit has been given at the lower rate r_S. The idea is a sequential one in that banks first lend out some funds at r_S, where all firms apply, and subsequently lend out some funds at r_2, where only the risky firms ask for capital. In this situation, all risky firms get capital. Some safe firms are rationed without missing positive expected profits, but no bank has an incentive to serve them since it cannot get additional funds to do so. Ex ante, it does not matter for households in which contract they invest their share of income (which is the same at both offers since LTU is the same). It does, however, matter for the

existence of a two-price equilibrium.[17]

Case VI:
If $S(i_b(r_S), i_g(r_S)) > (N_S + N_R)B$, there is a market clearing equilibrium at a loan rate at or below r_S, irrespective of the shape of capital supply and LTU at higher loan rates.

Up to now, we have only considered capital supply and credit demand, measuring credit rationing as the difference between what firms want and what households are willing to supply, given the information asymmetries.

However, credit rationing caused by asymmetric information is not necessarily connected to under-investment from society's point of view. Therefore, we need to find the socially optimal level of investment. A well-established notion of a social optimum is the level of investment under perfect information. The idea is that households own (the same share of all) firms so that they also know about their risk characteristics.

First, when project revenues are independent (as in SW and AR), we know from section 2.2 that risky firms have zero expected profits at r_R so that all economic rents at the corresponding deposit rate $\frac{E(\tilde{\mathcal{R}})}{B} - 1$ go to households. Since safe and risky projects have the same expected project revenue, $S(r_R)$ is an adequate measure of the socially optimal level of investment. If $S(r_R) > (N_S + N_R)B$, it is socially optimal to fund all projects.

When it comes to dependent project revenues among risky firms, the same shortcut is not available since projects differ fundamentally in the aggregate: The risky firms' market risk cannot be diversified. Clearly, risk-averse households prefer safe projects to risky ones since all project revenues have the same expectation.[18]

Thus, a social optimum either consists of only safe projects, or of all safe and some (or all) risky projects. Technically, we first optimize utility assuming that households do only safe projects.[19] If the optimal amount of

[17] Formally, every single bank has to make sure that its amount of credit given at each loan rate equals its amount of deposits collected at the respective deposit rate combinations. Otherwise, a bank might not meet its liabilities and, thus, incur an expected utility of minus infinity. Since households are indifferent between deposit contracts ex ante, banks are able to match loan rates with deposit rates.

[18] Formally, the variance of a risky project is higher than the variance of a safe project if and only if $\mathcal{R}_R > \mathcal{R}_S$, which is true by assumption.

[19] Note that we optimize utility as opposed to expected utility. If there are only safe

the safe projects, m, is in the interval $[0, N_S]$, m is the socially optimal level of investment. However, if the solution m is larger than N_S, the optimal amount of safe projects is N_S.[20] Then, we find the amount of risky projects in a social optimum, $n \in [0, N_R]$, by maximizing expected utility over n, using $m = N_S$. The optimization problem in step one is

$$\max_{s'_m} U = u(Y - s'_m) + \delta u(s'_m(1 + i')), \tag{13}$$

where $i' = \frac{E(\tilde{\mathcal{R}})}{B} - 1$ is the expected revenue rate from a safe project which results in a safe payoff rate in aggregate. Using CRRA utility and solving the FOC for $(s'_m)^*$, the optimal amount of savings with only safe firms, yields

$$(s'_m)^* = \frac{Y}{1 + (\delta(1 + i')^{1-\theta})^{-\frac{1}{\theta}}}. \tag{14}$$

This implies

$$m = \frac{H(s'_m)^*}{B} \tag{15}$$

safe projects. If $m > N_S$, the optimal amount of savings with both firm types, $(s'_{mn})^*$, solves

$$\max_{s'_{mn}} EU = u(Y - s'_{mn}) + \delta \left[p_R u(s'_{mn}(1 + i'_g)) + (1 - p_R) u(s'_{mn}(1 + i'_b)) \right]. \tag{16}$$

The payoff rates depend on $n = \frac{Hs'_{mn}}{B} - N_S$. Using this, we get

$$i'_g = \frac{N_S E(\tilde{\mathcal{R}}) + n\mathcal{R}_R}{(N_S + n)B} - 1 = \frac{N_S E(\tilde{\mathcal{R}}) + (\frac{Hs'_{mn}}{B} - N_S)\mathcal{R}_R}{Hs'_{mn}} - 1, \tag{17}$$

---

projects in a social optimum, the payoff rate is a safe one.

[20] A negative second derivative of the objective function w.r.t. savings is sufficient to guarantee that doing N_S safe projects is better than doing any other number of safe projects less than N_S if $m > N_S$. We have $\frac{d^2EU}{ds^2} = u''(Y - s) + \delta E[u''(s\tilde{R}')\tilde{R}'^2] < 0$ since $u''(c) = \frac{c^{-\theta-1}}{-\theta} < 0$ due to risk aversion. Clearly, $\tilde{R}'$ (the gross payoff rate) is only random if the social optimum entails some risky projects. We use primes to label variables in a social optimum so that i is a deposit rate offered to households by banks, whereas i' is the payoff rate in a social optimum without asymmetric information.

$$i_b' = \frac{N_S E(\tilde{\mathcal{R}})}{(N_S + n)B} - 1 = \frac{N_S E(\tilde{\mathcal{R}})}{H s_{mn}'} - 1. \tag{18}$$

Next, using CRRA utility, we derive the FOC which we solve for $(s_{mn}')^*$

$$(s_{mn}')^* = \frac{HY - \left(\frac{\delta E(\tilde{\mathcal{R}})}{B}\right)^{-\frac{1}{\theta}} N_S(E(\tilde{\mathcal{R}}) - \mathcal{R}_R)}{\left(\frac{\delta E(\tilde{\mathcal{R}})}{B}\right)^{-\frac{1}{\theta}} \frac{H\mathcal{R}_R}{B} + H}. \tag{19}$$

This uniquely determines n, i_g' and i_b'. They all depend on the number of households since an increase in the number of households results in higher savings which have to be invested in risky firms. Each household has an externality on all other households by making payoff rates riskier in that case. If $n > N_R$, the social optimum consists of doing all projects available.[21]

We can now come to a central point in our paper, namely the influence of project dependency on equilibrium outcomes. The benchmark are the results of SW and AR. For now, to get strong points, we focus on cases where it is socially optimal to fund all projects - both with independent and dependent project revenues. Then, from the stylized equilibrium cases in Figure 4, cases I, II and IV[22] are impossible when revenues are independent since $S(r_R) < (N_R + N_S)B$ such that the social optimum would not consist of all projects.

It is easy to see that case I is not fundamentally different from case IV and case II is similar to case III.[23] Therefore, we concentrate on cases III to VI when project revenues are dependent. Since project dependency introduces risk without changing the expectation of the deposit rate, our consideration of $\theta < 1$ implies that savings will decrease at any loan rate r.[24]

[21] Again, the negative second derivative of the objective function w.r.t. savings is sufficient to guarantee this. This procedure is equivalent to constrained optimization with the constraint $n \leq N_R$.

[22] These are the cases with credit rationing either at r_S (cases I and IV) or at r_R (case II).

[23] The difference between cases II and III is the number of risky firms in equilibrium and the loan rate charged. However, they are similar in that only risky firms get credit.

[24] Formally, risk leads to an MPS at any loan rate r such that the result of Rothschild and Stiglitz (1971) applies.

Transition	Equilibrium revenues indep.	Equilibrium revenues dep.	Δ # total	Δ # safe	Δ # risky
1	III	III	no	no	no
2	V	V	less	less	no
3	V	IV	less	unclear	less
4	V	III	less	less (all to zero)	no
5	VI	VI	no	no	no
6	VI	V	less	less	no
7	VI	IV	less	less	less
8	VI	III	less	less (all to zero)	no

Table 1: Equilibrium transitions and amount of projects.

Thus, we are left with eight possible transitions, where a transition is the change of the equilibrium case arising from the (perfect) dependency of project revenues. We list them in Table 1.

Transition 1 is not very interesting since the introduction of dependency does not change much. The allocation is the same in that only risky projects are funded. The difference is that the loan rate increases (in order to make households up for the overtaking of risk). Also, transition 5 is trivial: Both allocations (with and without dependency) are socially optimal in their respective environment, the difference again being a higher loan rate.

Proposition 2: *Dependency of project revenues might significantly reduce the equilibrium number of projects in a socially harmful way: While leaving unaffected the number of risky projects, (some) safe projects might not be funded.*

Proof:
Transitions 2, 4, 6 and 8 are such cases. The reduction is harmful in two ways: The overall level of investment is too small and the projects being funded are of the wrong type. The following parameter constellations prove that the transitions can occur.

- transition 2: $p_S = 0.7, p_R = 0.5, \mathcal{R}_S = \frac{6}{0.7}, \mathcal{R}_R = 12, N_S = 100, N_R = 200, C = 0.2, B = 5, Y = 3, \delta = 0.9, H = 1200$ and $\gamma = 0.2$.

- transition 4: $p_S = 0.8, p_R = 0.2, \mathcal{R}_S = 5, \mathcal{R}_R = 20, N_S = 25, N_R = 275, C = 0.1, B = 3, Y = 2, \delta = 0.9, H = 2000$ and $\gamma = 0.45$.

- transition 6: $p_S = 0.8, p_R = 0.2, \mathcal{R}_S = 8, \mathcal{R}_R = 32, N_S = 25, N_R = 275, C = 0.1, B = 3, Y = 2, \delta = 0.9, H = 1700$ and $\gamma = 0.45$.
- transition 8: $p_S = 0.8, p_R = 0.2, \mathcal{R}_S = 8, \mathcal{R}_R = 32, N_S = 10, N_R = 290, C = 0.1, B = 3, Y = 2, \delta = 0.9, H = 1800$ and $\gamma = 0.45$.

Using these parameters and then plotting capital supply and LTU with and without dependency shows that the respective transitions occur. For the case of independency, simply use $E(i)$ instead of i_b and i_g in all we have done so far. Note that all parameter constellations lead to social optima with all projects funded.
q.e.d.

The most extreme case is transition 8 where all safe projects are funded when revenues are independent but none when revenues are dependent. Intuitively, households face a trade-off: loan rate vs. firm type. A loan rate above r_S discourages safe firms from lending and makes the deposit rate riskier such that risk preferences are crucial to determine equilibrium. With few safe firms present in the economy and/or low risk aversion, the tendency is to neglect the safe firms and increase the loan rate. This is what we can observe in the parameter constellations of the proof.

Proposition 3: *Dependency of project revenues might lead to an equilibrium with credit rationing.*

Proof:
Transitions 3 and 7 are such cases. They arise at several parameter specifications, as for instance:

- transition 3: $p_S = 0.8, p_R = 0.2, \mathcal{R}_S = 10, \mathcal{R}_R = 40, N_S = 200, N_R = 100, C = 0.2, B = 5, Y = 2, \delta = 0.9, H = 1250$ and $\gamma = 0.2$.
- transition 7: $p_R = 0.2, p_S = 0.99, \mathcal{R}_S = \frac{6.4}{0.99}, \mathcal{R}_R = \frac{6.4}{0.2}, N_S = 100, N_R = 200, C = 0.1, B = 5.5, Y = 2, \delta = 0.9, H = 4000$ and $\gamma = 0.37$.

q.e.d.

The intuition is similar, now with risk-aversion rather high and/or number of safe firms rather high.

Transition 3 is the case where a two-price equilibrium becomes an equilibrium with credit rationing, transition 7 has a market clearing equilibrium turned into a credit rationing equilibrium. This is particularly interesting in light of the recent result of AR: Credit rationing as in cases I and IV is impossible in the SW setup.[25] In Proposition 3, we have shown that the introduction of project dependency might make credit rationing possible again. This happens in spite of it being socially optimal to fund all projects such that every unit of rationing is socially inefficient.[26]

Up to now, we focussed on cases where the social optimum consists of all projects. We then compared the situation with and without project dependency. Now, we look at cases where the social optimum does not necessarily consist of all projects any more.[27] Thus, rationing of firms needs not be socially harmful under-investment.

There are two kinds of inefficiencies: the amount of projects done and the type of the projects done. From these two, the total number of projects done cannot be used alone to determine inefficiencies. After all, if a social optimum requires, say, 20 safe projects and no risky one but the market equilibrium entails only risky projects, how should we judge the inefficiency from the total number of projects? This is the story of apples and oranges. We simply cannot compare them. However, we can make type-specific statements.[28]

Proposition 4: *The number of safe projects in equilibrium cannot be higher than in a social optimum.*

Proof:
We prove this for each equilibrium case separately. For cases II and III it is true since the equilibrium in both cases consists of only risky projects and the social optimum always has some safe projects funded.

[25]Recall from the introduction that a two-price equilibrium has rationing, too. However, only safe firms are rationed, whereas in a credit rationing equilibrium (at a single interest rate), both firm types are rationed, and the risky miss a strictly positive expected profit.

[26]Clearly, the harm (as measured by LTU) from not doing a safe project outweighs the harm from not doing a risky one, all other things equal.

[27]This can be considered the more natural case since parameter constellations which lead to a social optimum with funding for all projects in our six cases are rather extreme, especially collateral C is very small.

[28]For all equilibrium cases, we give some numerical examples together with the respective amounts of safe and risky projects both in equilibrium and in the social optimum in appendix 7.2.

For cases I, IV and VI, we only have to look at social optima with $m < N_S$ safe projects. Clearly, case VI is impossible then. We show that $m < N_S$ implies that the total number of projects in equilibrium and, thus, the number of safe projects in particular, is less than the number of safe projects in the social optimum.

To see this, note that the payoff rate in a social optimum is a safe one (and equal to $\frac{E[\tilde{\mathcal{R}}]}{B} - 1$), whereas the deposit rate in equilibrium is always risky with a mean below the payoff rate (from Figure 1, recall that $\frac{E[\tilde{\mathcal{R}}]}{B} - 1$ is the highest deposit rate mean possible and it only occurs in equilibrium case II). Since we focus on $\theta < 1$, the substitution effect outweighs the income effect. Therefore, if the equilibrium expected deposit rate were riskless, savings in equilibrium would be lower than in a social optimum. The additional effect of the equilibrium risk reduces savings even further, a result well-known from Rothschild and Stiglitz (1971).[29]

The same logic is true for case V, the two-price equilibrium: Having a social optimum with $m < N_S$, the deposit rate combination at either equilibrium loan rate has a mean which is lower than the riskless payoff rate in optimum ($\frac{E[\tilde{\mathcal{R}}]}{B} - 1$ for $m < N_S$), and a strictly positive variance. Thus, total savings in equilibrium are lower and, in particular, the number of safe projects must be smaller, too.
q.e.d.

However, it is interesting to see that this does not imply that the total number of projects in equilibrium is always lower than in a social optimum.

Proposition 5: *The equilibrium level of investment in each of the six cases might be higher than the respective socially optimal level.*

Proof:
As can be seen from Table 4 in appendix 7.2, there are parameter constellations which yield a social optimum with a lower level of investment than in the respective equilibrium for each of the six cases.
q.e.d.

[29]Therefore, instead of repeating this chain of arguments each time we compare two different deposit (or payoff) rate combinations, we can use mean-variance argumentation as a shortcut in our setup with a binary distribution of the deposit (or payoff) rate and CRRA utility.

If the social optimum consists of $m < N_S$ firms, this cannot happen (cf. the argument from the proof of Proposition 4). If $n > 0$, the mean of the payoff rates in the social optimum is always (equal to $\frac{E[\tilde{\mathcal{R}}]}{B} - 1$ and thus) higher than (or, in case II, equal to) the mean of any equilibrium deposit rate combination. The variance, however, can also be higher. There are two driving forces to determine differences in variance: the relationship between safe and risky and the amount of collateral. First, the relationship between safe and risky firms is always better (decreases variance) in a social optimum since $\frac{m}{n} \geq \frac{N_S}{N_R}$ (in case V, this fraction is even worse). There are several parameters influencing this relationship, as for instance N_S, N_R and H.[30] Second, it can be shown that an increase in collateral leaves the variance (and the mean anyway) of the payoff rates unaffected but decreases the variance of the deposit rate combinations. In sum, the effect of the higher risk in a social optimum can outweigh the effect of the higher mean such that, with $\theta < 1$, savings in equilibrium can be higher than in a social optimum.[31]

Another natural question arises: Can the equilibrium allocation be the one of a social optimum so that number and type of projects coincide, i.e., is there an efficient equilibrium?

Proposition 6: *Market clearing at a loan rate $r \leq r_S$ is a necessary (but not sufficient) condition for a socially efficient equilibrium.*

Proof:
In case VI, all firms are funded in equilibrium. If the social optimum consists of all projects, the equilibrium allocation is efficient. Otherwise, it is not. The former occurs when parameters are as in line 11 in Table 3 in appendix 7.2, the latter occurs for parameters as in line 12 (the allocations can be found in the respective lines in Table 4 in the appendix).

Evidently, cases II and III cannot be socially efficient since only risky firms are funded. The equilibrium in cases I, IV and V can not be efficient if $m = N_S$ since there is always rationing of safe firms. If $m < N_S$, the

[30] An increase in H, the number of households, decreases the safe-risky relationship in a social optimum with $n > 0$, but does not necessarily change this relationship in equilibrium (in case V it does).

[31] This happens, e.g., when the difference in the safe-risky relationship is small and/or C high. There are other such parameter variations, as can be seen from the parameter constellations with the respective allocations in Tables 3 and 4 in appendix 7.2.

arguments of the proof of Proposition 4 can be used to see that the number of safe firms in equilibrium must be lower than the number of safe firms in a social optimum.
q.e.d.

We close this section with a short summary. We developped the concept of social desirability for the case of dependent project revenues. Assuming that it is socially optimal to do all projects, we found that dependency of project revenues can have two strong effects: First, it might reduce the number of safe firms in equilibrium and, second, it can lead to credit rationing in equilibrium. We found and stressed the importance of a closer examination by number and type of projects in a social optimum. The interplay between asymmetric information and dependent project revenues can have serious consequences: The equilibrium amount of safe firms is never higher than the socially optimal amount. However, it is possible to have an equilibrium with more projects than in a social optimum. The only efficient equilibrium is a market clearing one with all firms active.

2.5 Comparative statics

We have made two important assumptions in our model: dependent revenues and risk aversion. We have analyzed the influence of perfectly dependent revenues at a given degree of risk aversion (and intertemporal substitution preferences at the same time). The aim of this section is to see how changes in θ influence the equilibrium given perfect dependency of project revenues (of the risky firms). An increase in θ has two economic effects: higher risk aversion and stronger preferences for smooth consumption.[32] As a consequence, changes in capital supply and LTU might cause changes in equilibrium. We have seen in Figure 4 that whether the maximum of capital supply (and LTU) occurs at r_S or r_R is of utmost importance for the equilibrium. If $S(r_R) - S(r_S)$ is positive ($\Leftrightarrow s^*(r_R) - s^*(r_S) > 0$), the maxima occur at r_R, and vice versa.

At $\theta = 0$, the difference $s^*(r_R) - s^*(r_S)$ can be either 0 or Y. $\theta = 0$ means risk neutrality so that only the expected deposit rate matters for the

[32]With CRRA utility, the relationship is an inverse one: θ is the Arrow-Pratt measure of relative risk aversion and the inverse of the elasticity of intertemporal substitution at the same time.

consumption-savings decision. Furthermore, the marginal utility of consumption is finite and equal to 1 for all consumption levels, especially $u'(0) = 1$ in both periods. Therefore, the FOC for $\theta = 0$ becomes $1 = \delta(1 + E(i))$ so that there is a critical $\overline{E(i)}$ from which on (discounted) marginal utility in period two will be higher than in period 1 so that households will save Y for $E(i) > \overline{E(i)}$ and nothing otherwise. Since $E(i(r_S)) < E(i(r_R))$, $s^*(r_R) - s^*(r_S)$ can start at either 0 or Y.[33] The former occurs if $\overline{E(i)} < E(i(r_S))$ or $\overline{E(i)} > E(i(r_R))$, the latter if $E(i(r_S)) < \overline{E(i)} < E(i(r_R))$. We get four different shapes for $s^*(r_R) - s^*(r_S)$ (as a function of θ) which we plot in Figure 5.

1. The curve starts at 0, increases until a local maximum, decreases to a negative local minimum and finally becomes zero at $\theta = 1$.

2. It starts at 0, increases until a local maximum, decreases to a root at $\theta = 1$ so that there is no intersection with the abscissa for $\theta < 1$.

3. It starts at Y, decreases to a (negative) local minimum and finally becomes zero at $\theta = 1$.

4. It starts at Y and decreases to a root at $\theta = 1$ so that there is no intersection with the abscissa for $\theta < 1$.

The root at $\theta = 1$ always occurs since $\theta = 1$ means log utility with the well-known property of a constant amount of savings. With most parameter constellations, we get shape 1 or 3, i.e., there is a critical $\theta < 1$ above which the maximum of capital supply (and LTU) occurs at r_S. However, shapes 2 and 4 show that we do not necessarily have such a critical θ.[34] If it were possible to vary risk aversion alone, an educated guess would be to always have a critical value from which on the high risk at r_R leads to $S(r_R) < S(r_S)$. This is the most important drawback of the preceding analysis: We cannot vary risk aversion keeping all other things equal. Therefore, we present a non-expected utility setup (non-EU) in the next section.

[33] In a generic case, namely $\overline{E(i)} = E(i(r_S))$, the difference at $\theta = 0$ can be anything between 0 and Y.

[34] An exemplary parameter constellation which leads to shape 2 is: $p_S = 0.9, p_R = 0.1, \mathcal{R}_S = 10, \mathcal{R}_R = 90, N_S = 200, N_R = 280, C = 2, B = 5, Y = 2, \delta = 0.5$ and arbitrary H. For shape 4: $p_S = 0.9, p_R = 0.1, \mathcal{R}_S = 10, \mathcal{R}_R = 90, N_S = 200, N_R = 280, C = 2, B = 5, Y = 2, \delta = 0.9$ and arbitrary H.

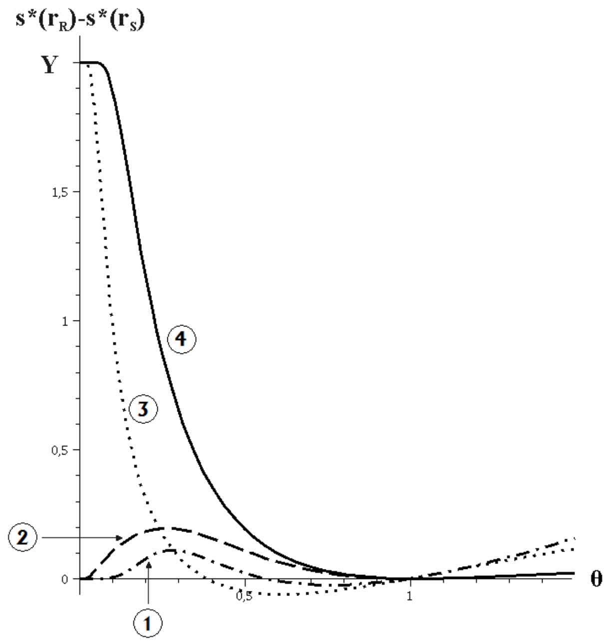

Figure 5: Four different shapes of $s^*(r_R) - s^*(r_S)$.

3 Extension I: Non-expected utility

Luckily, there is a way to separate risk aversion and intertemporal preferences. In a setup equivalent to ours, Selden (1978, 1979) implemented the consumption-savings decision with two parameters, each measuring one of the two preference components.[35]

3.1 Capital supply, LTU and equilibrium

Selden suggests to use the certainty equivalent of uncertain consumption in the second period to maximize overall utility. He calls his approach the ordinary certainty equivalent representation of preferences. Note that the optimization problem as a whole is a non-expected utility approach since the

[35] Among others, Selden's work prompted the often-quoted paper by Epstein and Zin (1989), which might be more familiar to readers, but works with an infinite horizon. This additional complication is not necessary to make our main points.

objective function is in general not linear in probabilities.[36] Agents solve

$$\max_s U = \{u(Y - s) + \delta u(\hat{c}_2)\}, \tag{20}$$

where $\hat{c}_2$ is the certainty equivalent defined by $v(\hat{c}_2) = E[v(s\tilde{R})]$, so that $\hat{c}_2 = v^{-1}(E[v(s\tilde{R})])$. As before, $s\tilde{R}$ is the uncertain income (and consumption) flow in the second period. The function $v(\cdot)$ determines the certainty equivalent for random consumption in the second period and is assumed to be of the CRRA type: $v(c) = \frac{c^{1-\gamma}}{1-\gamma}$. Instantaneous utility in the respective period is $u(c) = \frac{c^{1-\alpha}}{1-\alpha}$, which implies constant elasticity of substitution (CES) time preferences.[37] Both γ and α are positive (which represents risk aversion and a desire for smooth consumption). High γ indicates high risk aversion, whereas high α indicates a low intertemporal elasticity of substitution, i.e., there is a strong desire for a smooth consumption path.[38] Using the above functional forms in $\hat{c}_2$ and the FOC of equation (20), we get the latter as

$$(Y - s)^{-\alpha} = \delta E[(s\tilde{R})^{-\gamma}]E[\tilde{R}]\left[s(E[\tilde{R}^{1-\gamma}])^{\frac{1}{1-\gamma}}\right]^{\gamma-\alpha}. \tag{21}$$

Solving for optimal savings s^* yields

$$s^* = \frac{Y}{1 + \delta^{-\frac{1}{\alpha}}\hat{R}^{\frac{\alpha-1}{\alpha}}}, \tag{22}$$

where

$$\hat{R} = \left(E[\tilde{R}^{1-\gamma}]\right)^{\frac{1}{1-\gamma}} \tag{23}$$

is the certainty equivalent interest rate. This is the riskless interest rate which an agent with CRRA requires to be as well off as with the uncertain payoff $\tilde{R}$. Plugging s^* into the maximization problem and using $\hat{c}_2 = \hat{R}s^*$ yields LTU as a function of $\hat{R}$,

[36]Setting $\gamma = \alpha = \theta$, we are back in the expected utility case as in section 2. Note that the definition of the certainty equivalent follows pure expected utility theory.

[37]The same mathematical function displays both CRRA and CES. We omit the special cases of $u(c) = \ln c$ for $\alpha = 1$ since we restrict our attention to $\alpha < 1$ later on, and do not look at $v(c) = \ln c$ for $\gamma = 1$ either, unless necessary in mathematical proofs.

[38]The elasticity of intertemporal substitution turns out to be α^{-1}.

$$LTU = \frac{\delta}{1-\alpha}\left[\frac{\left(Y - \frac{Y}{1+\delta^{-\frac{1}{\alpha}}\hat{R}^{\frac{\alpha-1}{\alpha}}}\right)^{1-\alpha}}{\delta} + \left(\frac{Y\hat{R}}{1+\delta^{-\frac{1}{\alpha}}\hat{R}^{\frac{\alpha-1}{\alpha}}}\right)^{1-\alpha}\right]. \quad (24)$$

Lemma 3: *Using non-expected utility, the main properties of capital supply and LTU stay the same.*

Specifically, first, if $\alpha < 1$ (as opposed to the condition $\theta < 1$), capital supply increases monotonically in r in each of the intervals $[0, r_S]$ and $(r_S, r_R]$ with a discontinuous downward jump at r_S (cf. Lemma 1). Second, irrespective of α and γ, LTU increases monotonically in r in each of the intervals $[0, r_S]$ and $(r_S, r_R]$ with a discontinuous downward jump at r_S (cf. Lemma 2). Third, for $\alpha < 1$ (not $\theta < 1$), capital supply and LTU have their global maximum at the same loan rate, viz. either r_S or r_R (cf. Proposition 1). Fourth, if s^* is the same at two loan rates, LTU must be the same at these two rates, too (cf. corollary). In appendix 7.3, we prove all these properties.

Up to now, the crucial insight from introducing non-expected utility is that the parameter for intertemporal substitution (α) alone is responsible for the slope of capital supply. As long as $\alpha < 1$, capital supply is increasing in r in each of the two intervals with a downward jump at r_S, irrespectively of γ, the parameter capturing risk aversion.

Lemma 4: *Whether the maximum of capital supply and LTU occurs at r_S or r_R does not depend on the intertemporal substitution parameter α.*

The proof of this lemma is also delegated to appendix 7.3. We again focus on an increasing capital supply. Thus, we only look at $\alpha < 1$. In consequence, we get the same six equilibrium cases as in Figure 4 in the expected utility setup. We could do an analogous analysis regarding social optimum and equilibrium inefficiencies. However, we abstain from it since it yields no further insight.

3.2 Comparative statics

The main reason to present the non-EU setup is our interest in how changes in risk aversion (measured by γ) influence equilibrium. For that purpose, we analyze the influence of γ on optimal savings (and thus also on capital supply).

Proposition 7: *If $\alpha < 1$, an increase in households' risk aversion decreases optimal savings at any combination of i_b and i_g.*

Proof:
First, from Basu and Ghosh (1993), we know that a decrease in $\hat{R}$ implies a reduction in savings if $\alpha < 1$. This can be seen from differentiating optimal savings in (22) w.r.t. $\hat{R}$,

$$\frac{ds^*}{d\hat{R}} = Y \left[\frac{\delta^{\frac{1}{\alpha}} \hat{R}^{\frac{1-2\alpha}{\alpha}}}{\left(1 + \delta^{\frac{1}{\alpha}} \hat{R}^{\frac{1-\alpha}{\alpha}}\right)^2} \frac{1-\alpha}{\alpha} \right]. \tag{25}$$

Therefore, it is sufficient to show that $\frac{d\hat{R}}{d\gamma} < 0$ for $\alpha < 1$. Let $R_1 \equiv 1 + i_b$ and $R_2 \equiv 1 + i_g$ (with probabilities $(1-p)$ and p, respectively. We omit the index R of the probabilities). We distinguish three different cases: 1. $R_1 < 1$ and $R_2 > 1$, 2. $R_1 < 1$ and $R_2 < 1$ and 3. $R_1 > 1$ and $R_2 > 1$.

<u>**Case 1**, $R_1 < 1$ and $R_2 > 1$:</u>

The derivative of $\hat{R}$ in equation (23) w.r.t. γ is

$$\frac{d\hat{R}}{d\gamma} = \left(E[\tilde{R}^{1-\gamma}]\right)^{\frac{1}{1-\gamma}} \left(\ln(E[\tilde{R}^{1-\gamma}]) \frac{1}{(1-\gamma)^2} + \frac{1}{1-\gamma} \frac{1}{E[\tilde{R}^{1-\gamma}]} \frac{dE[\tilde{R}^{1-\gamma}]}{d\gamma} \right). \tag{26}$$

We show that it is negative for all admissible parameter values. We can substitute

- $\frac{dE[\tilde{R}^{1-\gamma}]}{d\gamma} = p\left(R_1^{1-\gamma}[-\ln(R_1)]\right) + (1-p)\left(R_2^{1-\gamma}[-\ln(R_2)]\right)$ and
- $\ln(E[\tilde{R}^{1-\gamma}]) = \ln\left(pR_1^{1-\gamma} + (1-p)R_2^{1-\gamma}\right)$.

Multiply equation (26) by $E[\tilde{R}^{1-\gamma}]$ and $(1-\gamma)^2$ and let $\overline{\frac{d\hat{R}}{d\gamma}}$ be the resulting expression,

$$
\begin{aligned}
\overline{\frac{d\hat{R}}{d\gamma}}(\gamma, p, R_1, R_2) \;&=\; \left[pR_1^{1-\gamma} + (1-p)R_2^{1-\gamma}\right] \ln\left(pR_1^{1-\gamma} + (1-p)R_2^{1-\gamma}\right) \\
&\;-\; (1-\gamma)\left[pR_1^{1-\gamma}\ln(R_1) + (1-p)R_2^{1-\gamma}\ln(R_2)\right].
\end{aligned} \tag{27}
$$

Since multiplication with positive values does not change inequalities, it has the same sign as the original derivative in equation (26).

We undertake the following steps: First, we show that the limit of $\overline{\frac{d\hat{R}}{d\gamma}}$ is zero as γ approaches 1 (from both left and right).[39] Second, we show that the derivative of $\overline{\frac{d\hat{R}}{d\gamma}}$ w.r.t. γ is negative for $\gamma > 1$ and positive for $\gamma < 1$. Third, we show continuity of $\hat{R}(\gamma)$. Taken together, this means that $\frac{d\hat{R}}{d\gamma}$ is negative for all $\gamma \neq 1$.

First step:

$$
\begin{aligned}
\lim_{\gamma\to 1}\left(\overline{\frac{d\hat{R}}{d\gamma}}\right) \;&=\; \lim_{\gamma\to 1}\Bigg\{ \left[pR_1^{1-\gamma} + (1-p)R_2^{1-\gamma}\right] \ln\left(pR_1^{1-\gamma} + (1-p)R_2^{1-\gamma}\right) \\
&\;-\; (1-\gamma)\left[pR_1^{1-\gamma}\ln(R_1) + (1-p)R_2^{1-\gamma}\ln(R_2)\right] \Bigg\} = 0.
\end{aligned} \tag{28}
$$

The argument of the logarithm in the first term approaches one such that the logarithm approaches zero. The second term is a product where the first factor goes to zero whereas the second factor approaches some finite real number such that the whole limit is zero.

Second step:

[39] Recall that our formulas are not defined for $\gamma = 1$.

$$
\begin{aligned}
\frac{d(\overline{\frac{d\hat{R}}{d\gamma}})}{d\gamma} &= \left[-pR_1^{1-\gamma}\ln(R_1) - (1-p)R_2^{1-\gamma}\ln(R_2)\right]\ln\left(pR_1^{1-\gamma} + (1-p)R_2^{1-\gamma}\right) \\
&- (1-\gamma)\left[-pR_1^{1-\gamma}[\ln(R_1)]^2 - (1-p)R_2^{1-\gamma}[\ln(R_2)]^2\right] \\
&= (-1)\Bigg\{pR_1^{1-\gamma}\ln(R_1)\ln\left(p + (1-p)(R_2/R_1)^{1-\gamma}\right) \\
&+ (1-p)R_2^{1-\gamma}\ln(R_2)\ln\left((1-p) + p(R_1/R_2)^{1-\gamma}\right)\Bigg\},
\end{aligned}
\tag{29}
$$

where the second equality follows from $\ln(x+y) = \ln(x) + \ln(1+y/x)$. It is the sum of two products of three factors each and is of type $(-1)\{F1 \cdot F2 \cdot F3 + F4 \cdot F5 \cdot F6\}$, where $F2, F3, F5$ and $F6$ are the logarithms.

For $\gamma > 1$, $R_1 < 1$ and $R_2 > 1$, we can see that $F1 > 0, F2 < 0, F3 < 0, F4 > 0, F5 > 0, F6 > 0$. Thus, $\frac{d(\overline{\frac{d\hat{R}}{d\gamma}})}{d\gamma} < 0$ for $\gamma > 1$.

For $\gamma < 1$, $R_1 < 1$ and $R_2 > 1$, we have $F1 > 0, F2 < 0, F3 > 0, F4 > 0, F5 > 0, F6 < 0$. Thus, $\frac{d(\overline{\frac{d\hat{R}}{d\gamma}})}{d\gamma} > 0$ for $\gamma < 1$.

Third step:

Concerning continuity, the only critical value of γ is 1. The left and right limit of $\hat{R}$ as γ approaches 1 equals the certainty equivalent interest rate from using $v(c) = \ln(c)$ (the case of $\gamma = 1$), namely $R_1^p R_2^{1-p}$.

Taken together, $\hat{R}$ is a continuous, monotonically decreasing function in γ in case 1.

Case 2 and 3, $R_1, R_2 < 1$ and $R_1, R_2 > 1$:

The same proof as in case 1 does not work if both R are either smaller or larger than one. But there is a shorter version. Let R be the tuple (R_1, R_2) with $R_1 < R_2$ and R' another tuple (R_1', R_2') with $R_1' < R_2'$. We claim that, for any $R > 0$, there are $\kappa > 0$ and $R' > 0$ such that $R = \kappa R'$ with $R_1' < 1$ and $R_2' > 1$. To see this, taking $\kappa = R_1 + \epsilon$ with $\epsilon > 0$ yields

$$R_1' = \frac{R_1}{\kappa} = \frac{R_1}{R_1 + \epsilon}, \qquad R_2' = \frac{R_2}{\kappa} = \frac{R_2}{R_1 + \epsilon}.$$

Clearly, $R_1' < 1$. Since $R_1 < R_2$, there must be an ϵ small enough to get $R_2' > 1$. Therefore

$$\hat{R}(R) = \hat{R}(\kappa R') = \kappa \hat{R}(R'), \tag{30}$$

where the second equality follows from first-degree homogeneity of $\hat{R}$ w.r.t. R.[40] From case 1, we know that $\frac{d\hat{R}}{d\gamma} < 0$ for all $R_1 < 1$ and $R_2 > 1$.[41] Therefore, equation (30) implies that $\hat{R}$ monotonically decreases in γ not only in case 1, but also in cases 2 and 3.
q.e.d.

So if risk aversion increases, savings decrease at any (combination of deposit rates corresponding to a) loan rate r. Since the shape of capital supply and LTU determine the equilibrium outcome, we can say something about the influence of risk aversion on equilibrium.

Proposition 8*: Capital supply and LTU are maximum at r_S for risk aversion high enough, they are maximum at r_R otherwise.*

Proof:
Let $f(\gamma) \equiv s^*(\gamma)|_{r=r_R} - s^*(\gamma)|_{r=r_S}$. The proposition is equivalent to saying that $f(\gamma)$ has a single root at some $\gamma > 0$. Rigorously, we can only show that there is at least one root. However, we have not found evidence for the fact that there should be more than one root in a large set of numerical examples.

From AR, we know that the expected return function attains its global maximum at r_R. This implies that, given $\alpha < 1$, capital supply must be maximum at r_R for risk-neutral households ($\gamma = 0$) so that $f(0) > 0$. The other extreme is an infinitely high aversion to risk. In this case, we can see that savings are higher at r_S by looking at the limit of the difference in savings.

[40] Due to CRRA utility, $\hat{R}$ is independent of the absolute level of both income and savings.

[41] We do not repeat the discussion about the special case of $\gamma = 1$.

$$\lim_{\gamma\to\infty}(f(\gamma)) = Y\left[\frac{1}{1+\delta^{-1/\alpha}(\hat{R}(r_R))^{\frac{\alpha-1}{\alpha}}} - \frac{1}{1+\delta^{-1/\alpha}(\hat{R}(r_S))^{\frac{\alpha-1}{\alpha}}}\right],$$

where $\hat{R}(r_i)$ is the certainty equivalent interest rate for our state-contingent deposit rates $i_g(r_i)$ and $i_b(r_i)$ ($i \in \{S, R\}$) with their corresponding probabilities p_R and $(1-p_R)$. Since γ only appears in $\hat{R}$, we only have to look at the limits of $\hat{R}$ at r_R and r_S. Since $\hat{R}$ decreases in γ (cf. proof of Proposition 7), the smallest riskless deposit rate which can make a maximum risk-averse household as well off as with a Bernoulli lottery is the worse of the two lottery outcomes, i.e., $(1+i_b(r_R))$ for r_R and $(1+i_b(r_S))$ for r_S.[42] Since $i_b(r_R) < i_b(r_S)$ (cf. property ii) in section 2.2), we get

$$(1+i_b(r_R))^{\frac{\alpha-1}{\alpha}} > (1+i_b(r_S))^{\frac{\alpha-1}{\alpha}},$$

given that $\alpha < 1$. Therefore, the above limit is negative, and its algebraic form can be obtained by substituting $\hat{R}(r_i)$ for $(1+i_b(r_i))$.

Thus, $f(\gamma)$ starts at a positive value for $\gamma = 0$ and asymptotically approaches the negative limit for $\gamma \to \infty$, either from above or from below. Different possible shapes of $f(\gamma)$ can be seen in Figure 6. Note that we show two exemplary graphs without being exhaustive.[43] Since $f(\gamma)$ is a continuous function, there is at least one root. From these figures[44], it seems that there is only one critical value of γ from which on the maximum of capital supply occurs at r_S. For a rigorous proof, we would have to preclude the possibility of more than one root, which is not straightforward.
q.e.d.

We illustrate this proposition in Figure 7. We plotted capital supply and LTU for three different values of γ, as indicated in that figure. The upper part is capital supply (and demand), the lower part is LTU, both depending

[42]The opposite is also true: For maximum risk lovers, i.e., $\gamma \to -\infty$, the largest riskless deposit rate which can make a maximum risk-loving household as well off as with a lottery is the better of the two outcomes: $(1+i_g(r_S))$ and $(1+i_g(r_R))$ at the respective loan rates.

[43]For extreme parameter constellations, we found cases where $f'(0)$ is positive, i.e., the maximum of the difference in savings might occur at a positive value of γ.

[44]Apart, we tried a large amount of numerical examples. In none of these, there was more than one root.

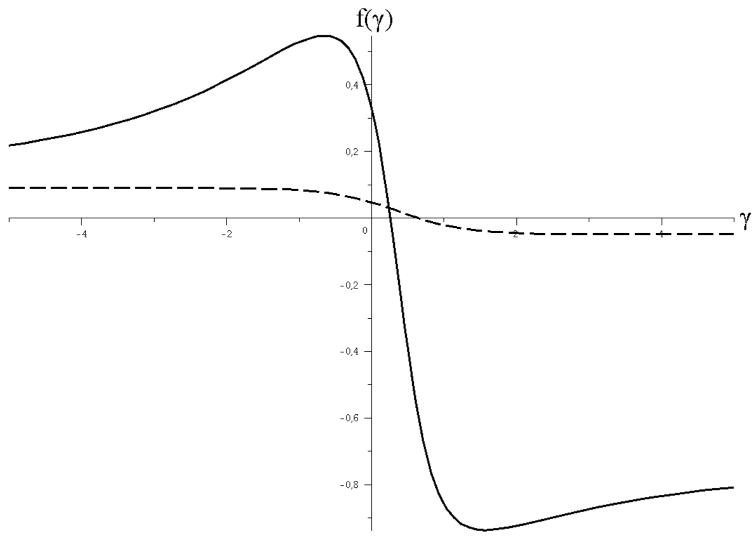

Figure 6: Two exemplary shapes of $f(\gamma)$.

on the loan rate r. For a high value of γ, both capital supply and LTU have their maximum at r_S (the dashed curves). For low γ, both maxima occur at r_R (the solid curves). At $\gamma = 0.39$, capital supply (and LTU) are the same at r_S and r_R (the dotted curves).[45]

Proposition 8 confirms the guess we were making in section 2.5: There is always a critical value for the risk-aversion parameter from which on the maximum occurs at r_S. Apart from this, the non-expected utility setup has further convenient features: On the one hand, it is a genuine generalization since setting $\alpha = \gamma = \theta$ yields the expected utility setup. This added flexibility allows to vary two preference components separately. In particular, we are able to vary risk aversion to arbitrarily high degrees of risk aversion without changing the sign of the slope of capital supply. On the other hand, the empirical literature shows that there is no unanimous relationship between risk aversion and the intertemporal elasticity of substitution. In particular, the hypothesis of an inverse relationship as implied by the expected utility setup is rejected. However, there is dissent regarding the plausible magnitudes of α and γ. In an empirical study, Attanasio and Weber (1989) get $\alpha < 1$ and

[45] The remaining parameters used for the plots are: $p_S = 0.8$, $p_R = 0.2$, $\mathcal{R}_S = 10$, $\mathcal{R}_R = 40$, $N_S = 100$, $N_R = 100$, $C = 2$, $B = 5$, $Y = 2$, $H = 500$, $\alpha = 0.5$ and $\delta = 0.9$.

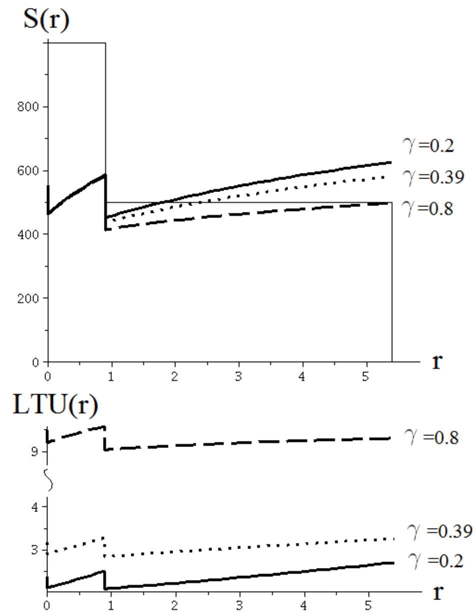

Figure 7: Capital supply and LTU for different values of γ.

$\gamma > 1$, which could not be considered using an expected utility setup.[46]

4 Extension II: Imperfect dependency

In all the above, we considered perfect dependency in that all risky firms either succeed or fail. This is the most extreme sort of dependency. To be closer to reality, we introduce a random variable $\tilde{q}$ with support $[0, 1]$ to capture the degree of dependency. Let $f(\tilde{q})$ be its density. Varying $\tilde{q}$ (or $f(\tilde{q})$) can be seen as comparative statics w.r.t. the degree of project dependency, one of our two important assumptions. We can interpret $\tilde{q}$ as an aggregate shock which determines capital risk.[47] The extreme realization $q = 0$ means that all risky (and, thus, all) firms' revenues are independent, such that we get the SW setup with a riskless deposit rate. The other extreme $q = 1$

[46]We do not want to omit contrary studies. Hall (1987) and Epstein and Zin (1991) estimate values of $\alpha > 1$. If $\alpha > 1$, the whole SW and AR analysis which is based on an increasing capital supply is not valid.

[47]The project revenue is another sort of random variable which can be interpreted as a shock to each individual firm.

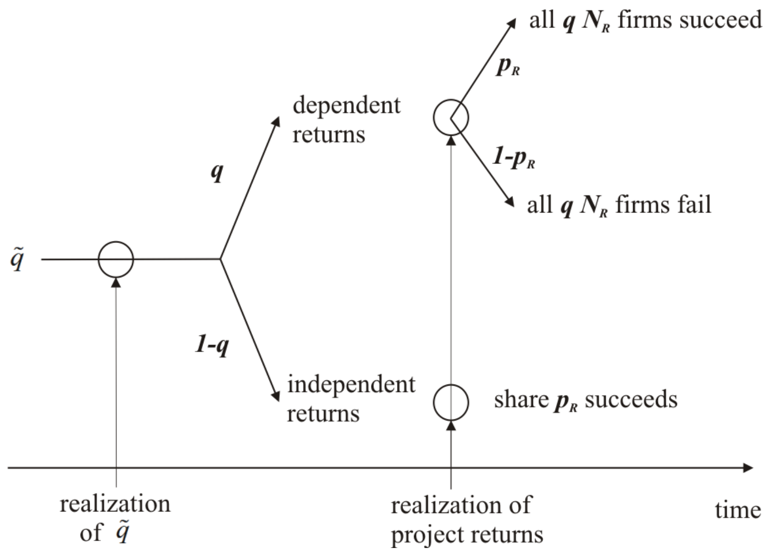

Figure 8: Timing of the new model structure.

means that all risky firms have returns perfectly depending on each other (as in the above section). Intermediate values of q yield imperfect dependency among the risky firms. For example, if $q = 0.5$, half of the risky firms have independent revenues, but the other half will either succeed or fail as a whole. The timing of this new model structure is visualized in Figure 8.

4.1 Deterministic degree of dependency

We assume that households take the consumption-savings decision after $\tilde{q}$ has realized. Evidently, credit demand of firms is unaffected by this new modelling approach. The realization of $\tilde{q}$ only changes aggregate risk and, thus, the deposit rates. Banks act in the same way as before, i.e., they pass through risk and make zero profits in any potential equilibrium. Thus, the good and bad state deposit rates (which still occur with probabilities p_R and $(1 - p_R)$) become

$$i_b(q) = (1-q)E[i] + qi_b, \qquad i_g(q) = (1-q)E[i] + qi_g. \tag{31}$$

As in section 2.3, i_b and i_g are the deposit rates for perfectly dependent revenues and $E[i]$ is the expected deposit rate. Also, all three rates depend on r which we again omit, unless stated otherwise. Again using the expected utility maximization, the optimal amount of savings becomes a function of q.

$$s^*(z(q)) = \frac{Y}{1 + (\delta z(q))^{-\frac{1}{\theta}}}, \tag{32}$$

where $z(q) \equiv p_R(1+(1-q)E[i]+qi_g)^{1-\theta}+(1-p_R)(1+(1-q)E[i]+qi_b)^{1-\theta}$, in analogy to the definition of z in section 2.3. Instead of being a function of z, optimal savings are a function of $z(q)$. The same is true for indirect lifetime utility: Take equation (11) and write $z(q)$ instead of z to get $LTU(z(q))$.

For the equilibrium analysis, we need to know the shapes of both capital supply and LTU as functions of r. They depend on the shape of the deposit rate combinations (as functions of r) as in Figure 1. For that purpose, let us first look at the influence of a change in q at a given r. A decrease in q does not change the expectation of the deposit rate, but decreases its variance.

$$E[i(q)] = p_R\left((1-q)E[i] + qi_g\right) + (1-p_R)\left((1-q)E[i] + qi_b\right) = E[i], \tag{33}$$

$$\begin{aligned} Var[i(q)] &= p_R\left((1-q)E[i] + qi_g - E[i]\right)^2 \\ &+ (1-p_R)\left((1-q)E[i] + qi_b - E[i]\right)^2 \\ &= p_R\left(q(i_g - E[i])\right)^2 + (1-p_R)\left(q(i_b - E[i])\right)^2 = q^2 Var(i), \end{aligned} \tag{34}$$

where $Var(i)$ is the variance of the deposit rate if project revenues are perfectly dependent. This is illustrated in Figure 9, where we show density functions of the deposit rates, contingent on the realization of q.[48]

Given our binary distributed random variable, we have already shown that an increase in variance at a constant mean is an MPS. Therefore, comparing $i_b(q), i_g(q)$ for $q < 1$ with i_b and i_g (i.e., $q = 1$), the latter is an MPS of

[48] Note that we used $p_R = \frac{2}{3}$ so that $E(i)$ is closer to i_g than to i_b. The three vertical dashed lines indicate the worst possible bad deposit rate, the expected deposit rate and the best possible deposit rate, respectively. The actual deposit rates after $\tilde{q}$ has realized are functions of q and occur where the thick bars are drawn.

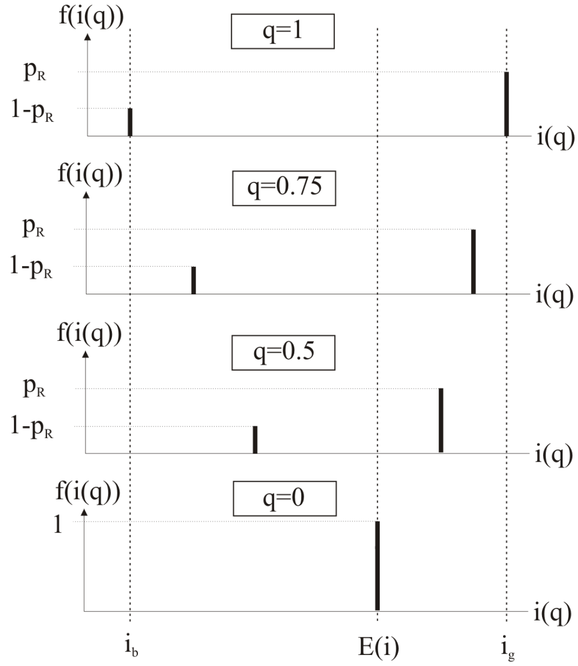

Figure 9: Density functions of deposit rates depending on q.

the former at any loan rate r.[49] Thus, Rothschild and Stiglitz (1971) applies: Savings decrease in q at any loan rate r.

Since the above relationship is valid for all r, the deposit rates (as functions of r as in Figure 1) change as follows. Not all of the general properties i) to v) necessarily apply. The jump of $i_g(q)$ at r_S (property i)) does not need to be upward. For q sufficiently close to 0, it is a downward jump. The bad-state deposit rate $i_b(q)$ is monotonically increasing in r in each of the two intervals for $q < 1$ (property ii)). Properties iii), iv) and v) stay the same. In particular, the expected deposit rate attains its global maximum at r_R and the variance of the deposit rate is monotonically increasing in r. The latter fact can be seen from equation (34): $Var(i)$ increases monotonically so that $q^2 Var(i)$ does so, too.

The reader can check that the arguments in the proofs of Lemmas 1 and 2, as well as the ones of Proposition 1 and its corollary stay the same, so that we get the same six equilibrium cases as shown in Figure 4. Therefore, we conclude:

Proposition 9: *Sign of slope, direction of the discontinuity at r_S and the fact that capital supply and LTU both have their maximum at either r_S or r_R do not depend on q. However, a higher degree of dependency amongst risky firms decreases households' savings at any loan rate r.*

Proposition 10: *The degree of project dependency might crucially influence equilibrium outcomes in our model.*

Proof:
Proposition 10 follows from Propositions 2 and 3, where we have seen and proven that the two extreme cases of $q = 0$ and $q = 1$ (all other parameters equal) can lead to different equilibrium outcomes.
q.e.d.

For a proof of the proposition, it is sufficient to look at the extreme cases. Of course, the same effect can happen with only small variations in q. Using $p_S = 0.8$, $p_R = 0.2$, $\mathcal{R}_S = 10$, $\mathcal{R}_R = 40$, $N_S = 200$, $N_R = 200$, $C = 2$, $B = 5$, $Y = 2$, $\delta = 0.9$, $H = 1030$, $\theta = 0.40$ and changing q from $q = 0.48$ to $q = 0.49$ decreases capital supply at any given r such that there is a case V equilibrium

[49] Equivalently, we could say that $i_b(q), i_g(q)$ is a mean preserving contraction of i_b and i_g for $q \in [0, 1)$.

(with two loan rates) for $q = 0.48$ but a case III market clearing equilibrium for $q = 0.49$.[50] This numerical example proves the next proposition:

Proposition 11*: There is financial fragility. A small change in a parameter can have a significant influence on the type of equilibrium.*

In our setup, not only a change in q, but also small changes in other parameters, e.g., the number of households H, can influence the type of equilibrium.

4.2 Stochastic degree of dependency

Instead of assuming that $\tilde{q}$ has realized when households decide about consumption and savings, as we did in the last section, we now assume that $\tilde{q}$ has not realized yet when they decide. We assume that households know the distribution of $\tilde{q}$, i.e., there is risk in the degree of dependency, but no uncertainty. In contrast to section 4.1, the two state-contingent deposit rates are not known any longer since they depend on the realization of $\tilde{q}$. Note that their distributions are the same as the one of $\tilde{q}$ since they are linear combinations of $\tilde{q}$. Clearly, the parameters and support of the distribution will change. Therefore, the deposit rate is a two-step random variable and the deposit rate distribution is not a binary one before $\tilde{q}$ has realized. In Figure 10, the reader can find an exemplary distribution of $\tilde{q}$ and the implied distribution of the deposit rate $i(\tilde{q})$. The upper part is a discrete uniform distribution with five occurrences. The lower part is the continuous uniform distribution of $\tilde{q}$. We used $p_R = \frac{2}{3}$ both times.

For the consumption-savings decision, we must know the expectation $E_0[i(\tilde{q})]$ and variance $Var_0(i(\tilde{q}))$ of i before $\tilde{q}$ has realized.[51] The index 0 at the expectation and variance operator indicates that we take expectations before any random variable has realized. We get

[50] Although such a marginal change in a parameter changes the equilibrium case, the allocation is not dramatically different. In the given example, the two-price equilibrium has almost no safe firm funded and most of the risky firms get credit at a high loan rate anyway. Thus, the market clearing equilibrium with only risky firms funded at a slightly higher loan rate is not very different.

[51] This does not mean that we are conducting mean-variance analysis. However, looking at the mean and the variance is sufficient for our purpose as can be seen later on.

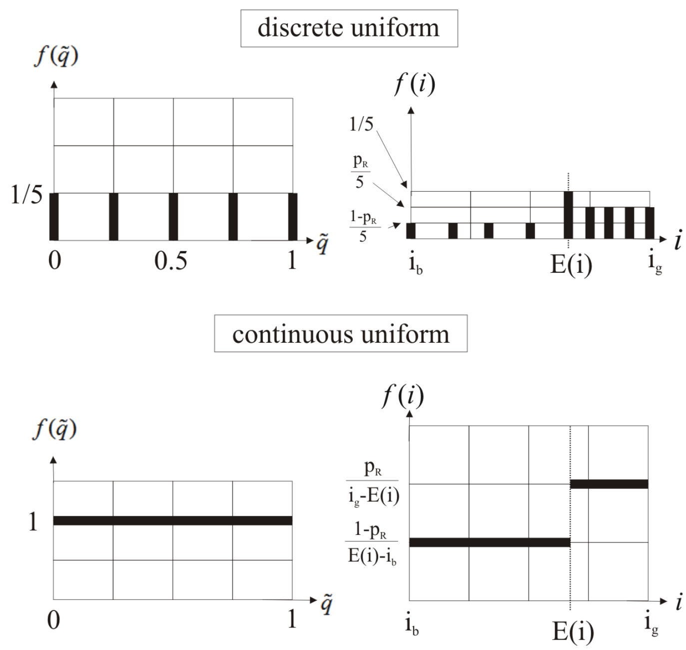

Figure 10: Density functions of $\tilde{q}$ and the implied densities of i ($p_R = \frac{2}{3}$).

$$\begin{aligned} E_0[i(\tilde{q})] &= p_R E_0\left[(1-\tilde{q})E[i] + \tilde{q} i_g\right] + (1-p_R)E_0\left[(1-\tilde{q})E[i] + \tilde{q} i_b\right] \\ &= E[i] + E_0[\tilde{q}]\left(E[i] - E[i]\right) = E[i]. \end{aligned} \tag{35}$$

Since $Var(X) = E[X^2] - E[X]^2$, we get the variance by finding the expectation of the squared deposit rate,

$$\begin{aligned} E_0[i(\tilde{q})^2] &= p_R E_0\left[((1-\tilde{q})E[i] + \tilde{q} i_g)^2\right] + (1-p_R)E_0\left[((1-\tilde{q})E[i] + \tilde{q} i_b)^2\right] \\ &= E[i]^2 + 2E[i]E_0[\tilde{q}]\left(E[i] - E[i]\right) + E_0[\tilde{q}^2]Var(i) \\ &= E[i]^2 + E_0[\tilde{q}^2]Var(i). \end{aligned} \tag{36}$$

As $E_0[\tilde{q}^2] = Var_0(\tilde{q}) + E_0[\tilde{q}]^2$, we get

$$
\begin{aligned}
Var_0[i(\tilde{q})] &= E_0[i(\tilde{q})^2] - E_0[i(\tilde{q})]^2 = E_0[\tilde{q}^2]Var(i) \\
&= \left(Var_0(\tilde{q}) + E_0[\tilde{q}]^2\right) Var(i).
\end{aligned}
\tag{37}
$$

The variance of the deposit rates only depends on the variance and expectation of any distribution of $\tilde{q}$.

Lemma 5: *Irrespective of $f(\tilde{q})$, the distribution of the dependency of project revenues, the mean of the distribution of deposit rates is the same and the variance is less than with perfectly dependent project revenues.*

Proof:
The first part of the lemma follows immediately from equation (35). From equation (37), $E_0[\tilde{q}^2] < 1$ since $\tilde{q}$ has support $[0, 1]$, irrespectively of the distribution. Therefore, $Var_0[i(\tilde{q})] < Var(i)$.
q.e.d.

Lemma 6: *A change in the distribution of $\tilde{q}$ influences savings at a given loan rate r. An increase in either expectation or variance (or both, or any combination of changes such that $Var_0(\tilde{q}) + [E_0(\tilde{q})]^2$ increases) of the distribution of project dependency decreases savings, and vice versa.*

Proof:
Equation (37) implies that the variance of the deposit rate increases from the changes in the distribution of $\tilde{q}$ indicated in the proposition. Since the mean stays the same, such changes constitute an MPS of the distribution of the deposit rate. Furthermore, since we assumed $\theta < 1$, we know from Rothschild and Stiglitz (1971) that an MPS decreases savings in an expected utility setup with CRRA utility.
q.e.d.

We should now go through all the lemmas and propositions in section 2 to check if they still hold. This task is more complex but results in the same consequences for the crucial functions, capital supply and LTU: If $\theta < 1$, savings increase with the well-known discontinuous downward jump, and so does LTU. Again, their maxima occur at the same loan rate, r_S or r_R (just define $z(\tilde{q})$ in an analogous way and apply the same arguments).

distribution	q_j	$p(q_j)$	$E_0(\tilde{q})$	$Var_0(\tilde{q})$	$Var_0(\tilde{q}) + E_0(\tilde{q})^2$
a)	0	1	0	0	0
b)	0.5	1	0.5	0	0.25
c)	0.75 0.25	0.5 0.5	0.5	$\frac{1}{16}$	0.3125
d)	0.9 0.5	0.5 0.5	0.7	$\frac{1}{25}$	0.53
e)	1	1	1	0	1

Table 2: Various (discrete) distributions of $\tilde{q}$ and some respective characteristics.

Proposition 12*: A change in the distribution of $\tilde{q}$ might crucially influence equilibrium outcomes in our model by changing whether the maximum of capital supply and LTU occurs at r_S or at r_R.*

Proof:
We only have to find one such case to prove the proposition. In analogy to the proof of Proposition 10, we can look at the two degenerate distributions $\tilde{q} = 0$ and $\tilde{q} = 1$ with certainty (all other parameters equal). The proposition then follows from Propositions 2 and 3.
q.e.d.

Again, the proposition holds for many other distributions. Consider the (simple and discrete[52]) distributions in Table 2.

The graphs in Figure 11 are based on the distributions of $\tilde{q}$ in Table 2. The other parameters are: $p_S = 0.6$, $p_R = 0.4$, $\mathcal{R}_S = \frac{40}{3}$, $\mathcal{R}_R = 20$, $N_S = 100$, $N_R = 100$, $C = 2$, $B = 5$, $Y = 2$, $\gamma = 0.8$ and $\delta = 0.9$. On the abscissa, we have the loan rate r, with breakeven loan rates of 1.4 and 2.4 under the given parameter constellation. The ordinate displays the optimal amount of savings. Going from distribution a) to e) in Figure 11, we can observe what Lemma 6 predicts. Any change in expectation and/or

[52]It would also be possible to use continuous distributions of $\tilde{q}$. However, there is no additional insight from the increasing complexity. Results do not change qualitatively, but the cost increases manyfold. From the formula of optimal savings, we must find the distribution of some power of the deposit rate, which is not necessarily possible. Literature on the search of distributions for (non-linearly) transformed random variables will help, as for instance Krishnan (2005).

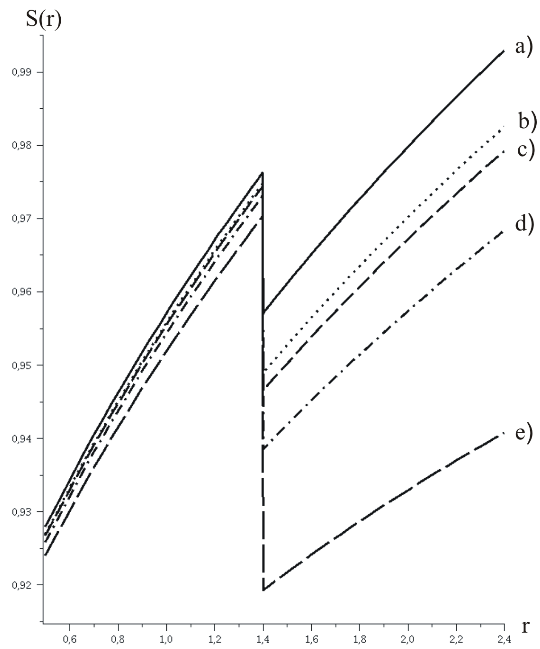

Figure 11: Capital supply for distinct distributions of $\tilde{q}$ in one graph.

variance of the distribution of $\tilde{q}$ that increases the sum of the variance and the squared expectation decreases savings. Also, for distributions a), b) and c), the maximum of capital supply occurs at r_R, whereas distributions d) and e) yield maxima at r_S (Proposition 12). Note that LTU will also have its maximum at the respective loan rate such that we have different equilibrium outcomes.

The reader has certainly observed the similarity of Propositions 10 and 12. Only making $\tilde{q}$ stochastic does not change much. However, it is a useful tool which enables us to go one step further, namely to introduce uncertainty.

4.3 Stochastic and uncertain degree of dependency - self-fulfilling expectations

We now assume that households have a prior about the distribution of the degree of dependency, which might or might not be correct. The correct distribution is unknown, i.e., there is uncertainty. As in section 4.2, households take the consumption-savings decision before $\tilde{q}$ has realized. Since we assumed homogeneous households all over the paper, we go on to assume that the prior is the same for all households.

Proposition 13: *There can be self-fulfilling expectations: If households expect a high degree of dependency among the risky firms, the equilibrium might be characterized by a high degree of dependency.*

Proof:
If the households' prior on $\tilde{q}$ has a very high mean and a very low variance, i.e., they are pretty sure that there will be a high degree of dependency, savings will be quite low. For a prior with a very low mean and a very low variance, savings could be much higher. As a proof to the proposition, assume the two most extreme cases: a prior of a degenerate distribution of $\tilde{q} = 1$ with probability 1 vs. a prior of $\tilde{q} = 0$ with probability 1. Then, for suitable parameter constellations, transition 8 in Proposition 2 can occur, i.e., if households expect a low degree of dependency, the equilibrium could be a market clearing one with all projects funded (say, equilibrium 1). On the contrary, if they expect a high degree of dependency, the equilibrium can become a market clearing one with all risky projects funded, but no safe ones (say, equilibrium 2).

Clearly, the degree of dependency as measured by the realization of $\tilde{q}$ is exogenous. However, measuring the degree of dependency with regard to all firms active in equilibrium, there is a share $q(1-\beta)$ of firms with (perfectly) dependent revenues in equilibrium 1, but a share q of firms with (perfectly) dependent revenues in equilibrium 2. Since $\beta = \frac{N_S}{N_S+N_R}$, $(1-\beta)q < q$.
q.e.d.

5 Extension III: Intra and inter-type dependency

The reader might have wondered from the very beginning about the specific form of dependency we assumed. Even though we have argued in favor of the assumption that only the risky firms' project revenues are dependent, this is not a necessary assumption of the model. In this section, we use another sort of dependency. First, we add (perfect) dependency among the safe firms, i.e., in the aggregate, all safe firms will either succeed or fail. Second, we consider inter-type dependency: We assume that risky firms can only succeed if the safe firms succeed. Therefore, the risky firms' success probability is a conditional one. This section can be considered a robustness test since we show that the main results obtain with these assumptions, too.

The individual success probability of a safe firm does not change, it is still p_S. For the risky firms, we define p'_R as the success probability conditional on the safe firms' success. Let S_i be a Bernoulli random variable which takes on the value "1" if all firms of type i succeed, and "0" otherwise ($i \in \{R, S\}$).[53] The four conditional probabilities for the risky firms are: $P(S_R = 1|S_S = 1) = p'_R$, $P(S_R = 1|S_S = 0) = 0$, $P(S_R = 0|S_S = 1) = 1 - p'_R$ and $P(S_R = 0|S_S = 0) = 1$.
The four states of the world[54] occur with the following probabilities:

- $P(S_R = 0 \cap S_S = 0) = P(S_R = 0|S_S = 0) \cdot P(S_S = 0) = 1 \cdot (1 - p_S) = 1 - p_S$,
- $P(S_R = 1 \cap S_S = 0) = P(S_R = 1|S_S = 0) \cdot P(S_S = 0) = 0 \cdot (1 - p_S) = 0$,
- $P(S_R = 0 \cap S_S = 1) = P(S_R = 0|S_S = 1) \cdot P(S_S = 1) = (1 - p'_R) \cdot p_S$,
- $P(S_R = 1 \cap S_S = 1) = P(S_R = 1|S_S = 1) \cdot P(S_S = 1) = p'_R \cdot p_S$.

Omitting the zero probability case and defining $p_R \equiv p'_R \cdot p_S$, we have three remaining states.

1. All firms fail, with probability $(1 - p_S)$. Returns in that case are $i_1 = \frac{C}{B} - 1$ for all $r \leq r_R$.
2. The safe firms succeed but the risky fail, with probability $(p_S - p_R)$. Returns are $i_2 = \beta r + (1 - \beta)\left(\frac{C}{B} - 1\right)$ for $r \leq r_S$ and $i_2 = \frac{C}{B} - 1$ for $r_S < r \leq r_R$.
3. All firms succeed, with probability p_R. Returns are $i_3 = r$ for all $r \leq r_R$.

Note that p_R is both the probability that all firms succeed and the unconditional success probability of the risky firms. A stylized graph of the return function can be found in Figure 12. In analogy to properties i) and ii) in section 2, we consider the properties of the state-contingent deposit rates. The worst deposit rate (i_1) is negative and equal to $\frac{C}{B} - 1$, irrespective of

[53] We write S_i instead of the formally preferable $\tilde{S}_i$.

[54] This time, we use the term "state" in its genuine meaning.

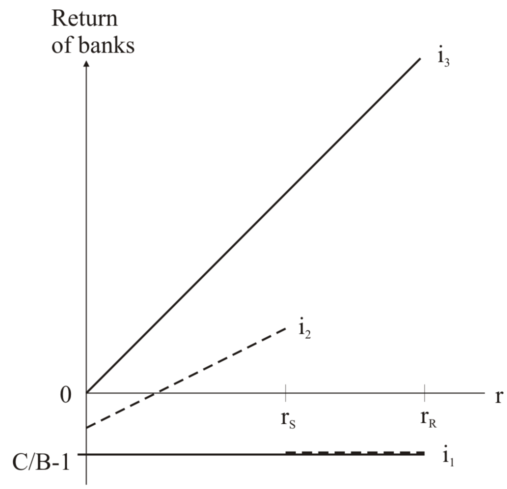

Figure 12: State-contingent return function.

the loan rate. The mediocre deposit rate is strictly increasing up to r_S, but equal to the worst deposit rate for higher loan rates (since there are no safe firms in the market for loan rates beyond r_S, the rate in the "safe succeed but risky fail" state equals the rate in the "all fail" state). The best deposit rate is continuous and monotonically increasing in the loan rate with a slope of one. From these properties, it is clear that savings must increase in each of the two intervals for $\theta < 1$.[55] The state-contingent returns lead to an expected return function as in Figure 13.

In analogy to properties iii) and iv), the expected return function is strictly increasing in each of the two intervals with a discontinuous downward jump at r_S, and the maximum occurs at r_R. We prove this property in detail in appendix 7.4. The intuitive explanation is identical as in section 2, the AR result.[56]

Property v) from section 2 does not hold any more. Even though the variance increases monotonically within each of the intervals, it need not increase at r_S. Intuition might question whether savings still have to decrease

[55]From equation (7), we know that capital supply must be increasing in each of the two intervals. This is because the derivative of optimal savings w.r.t. $E(\tilde{R}^{1-\theta})$ is positive, and because $E(\tilde{R}^{1-\theta})$ increases in each of the intervals. After all, the deposit rates weakly increase for all states of the world and taking them to some positive power is a monotonic transformation.

[56]Moreover, the formula for the expected return is exactly the same as in section 2.

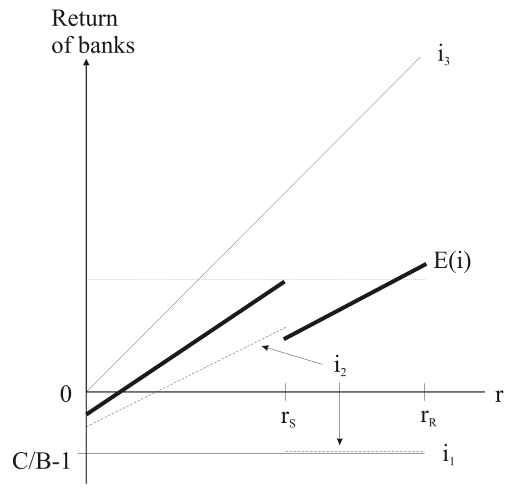

Figure 13: Expected return function.

discontinuously since the lower mean and the lower variance point to opposite directions. However, we argue that savings must decrease at r_S.

Since $\tilde{R}$ changes, the formula of z as in equation (9) changes, too: $z \equiv E(\tilde{R}^{1-\theta}) = (1-p_S)(1+i_1)^{1-\theta} + (p_S - p_R)(1+i_2)^{1-\theta} + p_R(1+i_3)^{1-\theta}$. The behavior of z as the loan rate r changes is the same as with the original formula of z (with $\tilde{R}$ as a binary random variable). Luckily, with the new definition of z, the formulas for capital supply and LTU do not change. Therefore, the proofs regarding shape and maxima of capital supply and LTU are still valid.

Proposition 14*: With intra and inter-type dependency, all six equilibrium cases from Figure 4 are possible. In particular, credit rationing might occur.*

Proof:
Plugging the new formula for $z = (1-p_S)(1+i_1)^{1-\theta} + (p_S - p_R)(1+i_2)^{1-\theta} + p_R(1+i_3)^{1-\theta}$ into optimal savings, the parameter constellations in Table 3 in appendix 7.2 again lead to the same six equilibrium cases.
q.e.d.

To get the analogue of Table 4 in appendix 7.2, we would have to set up a definition of social optimum. This is not the aim of this section. We rather look at the possible equilibrium cases. For that purpose, we compare the new savings function with two benchmarks: independent revenues as in SW and the dependency structure in section 2 with only the revenues of the risky dependent. First, in comparison to independent revenues, savings must decrease due to a well-known argument: The new deposit rate has the same mean but is now risky. Second, comparing the new setup to the original dependency structure, optimal savings only change in the first interval, where both firm types are active. The higher aggregate risk leads to more risky deposit rate combinations such that capital supply decreases at any loan rate r (the Rothschild and Stiglitz (1971) MPS argument). In the second interval, there are only risky firms active so that aggregate risk and, thus, the deposit rate combination at any r are the same.

Therefore, the same parameter constellations do not lead to the same equilibrium under the two different dependency structures. To see this numerically, using $\theta = 0.45$ instead of $\theta = 0.47$ in the first two lines in Table 3 in appendix 7.2 leads to different equilibrium cases with the two sorts of dependency. With intra and inter-type dependency, these are of type II, whereas with only the revenues of the risky firms dependent, the equilibrium cases are of type I.

6 Conclusion

Much of the literature on financial markets has not dealt with dependency of project revenues. This is surprising to us since its consideration can have far-reaching consequences both in a theoretical model and in reality. In a setup similar to the seminal SW model, we have shown that the type of equilibrium can crucially depend on the degree of project dependency. By making aggregate payoffs risky, households face capital risk in our setup. Therefore, risk aversion of households becomes a parameter of utmost importance since it influences households' consumption-savings decision. Thus, although the expected deposit rate is maximum at the highest loan rate accepted by borrowers (the AR result), savings and LTU do not have to be maximum at that rate. Capital risk deters households from saving so that there might be a credit rationing equilibrium at a lower loan rate.

To make this point stronger, we have shown that credit rationing can occur even if a social optimum required funding of all projects. The definition and analysis of the social optimum significantly differs from a situation with independent project revenues. The new social optimum is characterized both by the number and the type of the projects funded. We distinguished six different equilibrium cases. We found out that project dependency might reduce the number of safe projects in equilibrium in a socially harmful way. Thus, project dependency can aggravate adverse selection. Allowing for less than all projects in the social optimum, we attained another, unobvious result: The interplay of asymmetric information and project dependency can lead to an equilibrium with more projects than in the social optimum.

Using a non-expected utility setup to separate aversion against risk per se and aversion against differences over time, we were able to show that the degree of risk aversion alone is responsible for whether the maximum of capital supply and LTU occur at a lower or higher loan rate. Thus, the equilibrium outcome crucially depends on risk aversion on the one hand and the degree of project dependency on the other. The latter was made even more clear by showing that leaving aside the restrictive assumption of perfect dependency does not change the conclusion.

As a last robustness check, we showed that our results are not an artifact of our chosen way to model project dependency. In addition to dependency of risky firms' projects, having safe firms' project revenues depending on each other and assuming inter-type dependency at the same time does not change the results.

Our analysis points out that project dependency is an important factor in the determination of credit market outcomes. We suggest further research on project dependency in other theoretical models, especially in the realm of credit markets. One particular model to analyze is De Meza and Webb (1987), where expected project revenues of safe and risky firms are not the same.

Also, as shown by the current financial crisis, dependency appears not only between project revenues. It has often been neglected or not fully understood in many areas of the theoretical literature in finance. Therefore, we strongly suggest to consider dependencies in areas adjacent to credit markets, too.

7 Technical Appendix

7.1 Proof: Shape of return function

We prove properties i) to v) from section 2.2 one after another.

i) In the good state, all risky firms and, if they ask for capital in the first place, a fraction p_S of the safe firms succeed. Within both intervals, the composition of the firms doing projects is unaffected by a change in r. At r_S, safe firms stop asking for capital such that only risky firms are left in the market in the second interval.

In the first interval, $E[p|good] = \beta p_S + (1-\beta)$ from a bank's point of view. In the second interval, $E[p|good] = 1$. From equation (4), an increase in r will increase the expected good state return of a bank by $E[p|good]$ which is strictly positive (and smaller than one) in the first interval and equal to one in the second. Therefore, the good state rate is monotonically increasing within both intervals.

To see that there must be a discontinuous upward jump in the good state rate at r_S, note that, in the good state, banks' returns jump from equation (4) with $E[p|good] = \beta p_S + (1-\beta)(< 1)$ to r. Equation (4) can be seen to be smaller than r for any $E[p|good] < 1$ since $C < (1+r)B$ for any $r > \frac{C}{B} - 1$, and, thus, in particular for $r = r_S$.

ii) In the bad state, none of the risky firms, but a fraction p_S of the safe firms, succeed. In the first interval, $E[p|bad]$ equals $\beta p_S(> 0)$, whereas $E[p|bad] = 0$ in the second interval. Again, the composition of borrowers does not change within the respective intervals, so that an increase in r will increase expected returns in the bad state by $E[p|bad]$. This expression is positive in the first interval, but equal to zero in the second.

To see that the bad state rate is at its global minimum in the second interval, note that banks' returns from equation (4) become $\frac{C}{B} - 1$ in the second interval. In the first, at $r = 0$, it must be larger than that since $B > C$, and, thus, any weighted average of C and B (in equation (4)) must be larger than C.

iii) Since the expected rate is a probability weighted average of the good and bad state rates, the fact that it must be increasing within both intervals follows from i) and ii).

To see that there must be a discontinuous downward jump at r_S, look at an infinitesimal increase in r from r_S to $r_S + \epsilon$. All other things equal, this change in r marginally increases the banks' returns. However, since all the safe firms drop out of the market, the average success probability (also taking expectations on the state of the world) varies drastically. While it is equal to $\beta p_S + (1-\beta)p_R$ in the first interval, it equals p_R in the second. Since $p_S > p_R$, the latter expression, p_R, is smaller than the former, which proves that expected returns decrease discontinuously at r_S.

iv) Simply calculate the respective expected rates of return depending on all parameters. They are

$$E[i(r_S)] = \frac{(1-\beta)[p_R E[\tilde{\mathcal{R}}] + C(p_S - p_R)] + \beta E[\tilde{\mathcal{R}}]p_S}{Bp_S} - 1,$$

$$E[i(r_R)] = \frac{E[\tilde{\mathcal{R}}]}{B} - 1.$$

Doing some algebra on these two expressions shows that the former is smaller than the latter iff $E[\tilde{\mathcal{R}}] > C$, which is true by assumption of the model.

v) We prove the increase of the variance in each of the two intervals by showing that $(i_g - i_b)$ increases in r. This is sufficient for the proof since

$$\begin{aligned} Var(i) &= p_R(i_g - E(i))^2 + (1-p_R)(i_b - E(i))^2 \\ &= p_R(i_g - (p_R i_g + (1-p_R)i_b))^2 + (1-p_R)(i_b - (p_R i_g + (1-p_R)i_b))^2 \\ &= p_R(1-p_R)(i_g - i_b)^2. \end{aligned} \tag{38}$$

Note that these probability weights p_R and $(1-p_R)$ are the same in both intervals. From properties i) and ii), we know that $(i_g - i_b)$ increases in r both within the intervals and at r_S.

7.2 Some numerical results for social optima

In Table 3, we give some parameter specifications which we use to generate Table 4. In this latter table, we give the socially optimal levels of investment

constellation	p_S	$\mathcal{R}_S$	p_R	$\mathcal{R}_R$	N_S	N_R	C	B	Y	H	δ	θ
1	0.8	8	0.2	32	80	220	0.2	6	2	1500	0.9	0.47
2	0.8	8	0.2	32	80	220	0.2	6	2	2600	0.9	0.47
3	0.8	8	0.2	32	80	220	0.2	6	2	2600	0.9	0.40
4	0.8	8	0.2	32	80	220	0.5	6	2	2600	0.9	0.40
5	0.8	10	0.2	40	100	200	2	5	2	1225	0.9	0.30
6	0.8	10	0.2	40	50	250	2	5	2	1600	0.9	0.30
7	0.8	10	0.2	40	200	100	2	5	2	1000	0.9	0.30
8	0.8	10	0.2	40	200	100	2	5	2	1200	0.9	0.30
9	0.8	10	0.2	40	200	100	2	5	2	1000	0.9	0.20
10	0.8	10	0.2	40	120	180	4.5	5	2	1000	0.9	0.25
11	0.8	10	0.2	40	200	100	2	5	2	1500	0.9	0.20
12	0.8	10	0.2	40	200	100	2	5	2	1500	0.9	0.30

Table 3: List of parameters.

in both safe and risky firms, as well as the respective equilibrium values for each of the parameter constellations. We chose parameters in Table 3 to get all six equilibrium cases twice: once with the amount of projects done in equilibrium less than in a social optimum and the other way round.

There are ten columns in Table 4. The first one indicates the type of equilibrium (cf. Figure 4). The second (fifth, eighth) column gives the total number of projects (number of safe, number of risky) to be done in a social optimum, whereas the third (sixth, ninth) column gives the total number (safe, risky) in the respective equilibrium. Column four (seven, ten) is the difference between the former two.

7.3 Proof: Lemmas 3 and 4

The structure of the proofs is identical to the expected utility setup, only using $\hat{R}$ instead of z. Using γ instead of θ in the definition of z, we have $z^{\frac{1}{1-\gamma}} = \hat{R}$.

<u>Proof of Lemma 3</u>:

We prove these properties one after another.

1. If $\alpha < 1$, capital supply increases monotonically in r in each of the intervals $[0, r_S]$ and $(r_S, r_R]$ with a discontinuous downward jump at r_S.

case	#all SO	# all E	diff	# safe SO	# safe E	diff	# risky SO	# risky E	diff
I	$127,96$	$122,57$	$-5,38$	$80,00$	$32,69$	$-47,31$	$47,96$	$89,89$	$41,93$
I	$181,74$	$212,46$	$30,72$	$80,00$	$56,65$	$-23,35$	$101,74$	$155,80$	$54,06$
II	$180,22$	$175,29$	$-4,93$	$80,00$	$0,00$	$-80,00$	$100,22$	$175,29$	$75,07$
II	$180,22$	$210,69$	$30,47$	$80,00$	$0,00$	$-80,00$	$100,22$	$210,69$	$110,47$
III	$201,57$	$200,00$	$-1,57$	$100,00$	$0,00$	$-100,00$	$101,57$	$200,00$	$98,43$
III	$217,91$	$250,00$	$32,09$	$50,00$	$0,00$	$-50,00$	$167,91$	$250,00$	$82,09$
IV	$231,16$	$225,20$	$-5,96$	$200,00$	$150,14$	$-49,86$	$31,16$	$75,07$	$43,91$
IV	$254,88$	$270,24$	$15,36$	$200,00$	$180,16$	$-19,84$	$54,88$	$90,08$	$35,20$
V	$264,71$	$250,29$	$-14,42$	$200,00$	$150,29$	$-49,71$	$64,71$	$100,00$	$35,29$
V	$202,27$	$231,89$	$29,62$	$120,00$	$51,89$	$-68,11$	$82,27$	$180,00$	$97,73$
VI	$300,00$	$300,00$	$0,00$	$200,00$	$200,00$	$0,00$	$100,00$	$100,00$	$0,00$
VI	$290,47$	$300,00$	$9,53$	$200,00$	$200,00$	$0,00$	$90,47$	$100,00$	$9,53$

Table 4: Inefficiency results.

2. Irrespective of α and γ, LTU increases monotonically in r in each of the intervals $[0, r_S]$ and $(r_S, r_R]$ with a discontinuous downward jump at r_S.

3. For $\alpha < 1$, capital supply and LTU have their global maximum at the same loan rate, viz. either r_S or r_R.

4. If s^* is the same at two loan rates, LTU must be the same at these two rates, too.

<u>Proof of 1.</u>

We only give a proof for $\alpha < 1$, the same technique can be applied to $\alpha > 1$. The fact that capital supply increases in each of the two intervals directly follows from Basu and Ghosh (1993, p. 124) (Proposition 2). It can also be seen from differentiation of equation (22) w.r.t. $\hat{R}$,

$$\frac{ds^*}{d\hat{R}} \gtrless 0 \Leftrightarrow 1 - \alpha \gtrless 0. \tag{39}$$

Next, the derivative of $\hat{R}$ w.r.t. r is

$$\frac{d\hat{R}}{dr} = \frac{(E[\tilde{R}^{1-\gamma}])^{\frac{-\gamma}{1-\gamma}}}{1-\gamma} \left(p_R \frac{1-\gamma}{(1+i_g)^\gamma} \frac{di_g}{dr} + (1-p_R) \frac{1-\gamma}{(1+i_b)^\gamma} \frac{di_b}{dr} \right), \tag{40}$$

which is positive for all $\gamma > 0$ (there is no α in it). Therefore, for $\alpha < 1$, we have $\frac{ds^*}{d\hat{R}} > 0$ and $\frac{d\hat{R}}{dr} > 0$ so that $\frac{ds^*}{dr} > 0$, too.

To prove the discontinuous downward jump at r_S, we apply the same argument as in the expected utility case in Lemma 1. The proof was based on Rothschild and Stiglitz (1971). The corresponding argument in the non-EU case was made by Selden (1979): An MPS decreases savings in a non-expected utility setup iff $\alpha < 1$. The existence of an MPS at $r > r_S$ together with an increasing capital supply in the second interval completes the proof.

<u>Proof of 2.</u>

Differentiating equation (24) w.r.t. $\hat{R}$ and simplifying yields

$$\frac{dLTU}{d\hat{R}} = \frac{Y^{1-\alpha}}{\left(1 + \delta^{\frac{1}{\alpha}} \hat{R}^{\frac{1-\alpha}{\alpha}}\right)^{3-\alpha} \alpha \hat{R}} \left[\delta^{\frac{1}{\alpha}} \hat{R}^{\frac{1-\alpha}{\alpha}} \alpha + \delta^{\frac{2}{\alpha}} \hat{R}^{\frac{2-2\alpha}{\alpha}} 2\alpha + \delta^{\frac{3}{\alpha}} \hat{R}^{\frac{3-3\alpha}{\alpha}} \alpha\right]. \tag{41}$$

Since Y, α, δ and $\hat{R}$ are always positive, the derivative itself is always positive. Since $\frac{d\hat{R}}{dr} > 0$ in each of the two intervals, $\frac{dLTU}{dr} > 0$ in each of the two intervals. Both signs do not depend on α or γ. Since $\frac{\Delta\hat{R}}{\Delta r} < 0$ at r_S,[57] $\frac{\Delta LTU}{\Delta r} < 0$ at r_S. Again, the signs do not depend on α or γ. Therefore, LTU monotonically increases in each of the two intervals with a discontinuous downward jump at r_S, irrespective of γ and α.

<u>Proof of 3.</u>

The structure of the proof is as with expected utility: From 1. and 2., the global maximum of LTU and capital supply can only be at r_S or r_R for $\alpha < 1$. If savings are higher at, say, r_S, $\hat{R}$ must also be higher at r_S since $\frac{ds^*}{d\hat{R}} > 0$ (for $\alpha < 1$), from equation (39). Since, from equation (41), we also have $\frac{dLTU}{d\hat{R}} > 0$, $LTU(r_S) > LTU(r_R)$. An analogous argument applies if the maximum occurs at r_R.

[57] The behavior of $\hat{R}$ at r_S follows from the fact that capital supply decreases discontinuously at r_S and $\frac{ds^*}{d\hat{R}} > 0$ (for $\alpha < 1$).

Proof of 4.

The same argument (as for expected utility) applies: If savings are the same at two loan rates, $\hat{R}$ must also be the same. Since LTU in equation (24) only depends on s^*, $\hat{R}$ and parameters, LTU must be the same at these two loan rates, too.

Proof of Lemma 4:

From equation (39), we know that $\frac{ds^*}{d\hat{R}} > 0$ for $\alpha < 1$. Therefore, the maximum of capital supply occurs where $\hat{R}$ is maximum. From the definition of $\hat{R}(= (E[\tilde{R}^{1-\gamma}])^{\frac{1}{1-\gamma}})$, we know that the maximum value of $\hat{R}$ (and, thus, whether it occurs at r_S or r_R) does not depend on α. A change in α clearly influences the absolute amount of savings (as can be seen from its appearance in equation (22)). However, whether the maximum occurs at r_S or at r_R is independent of α.

7.4 Proof: Maximum of expected returns for extension III

For the first interval, expected returns are

$$\begin{aligned} E\pi^{bank}_{r \leq r_S}(r) &= (1-p_S)\left(\frac{C}{B}-1\right) + (p_S - p_R)\left[\beta r + (1-\beta)\left(\frac{C}{B}-1\right)\right] + p_R r \\ &= (1-x)\left(\frac{C}{B}-1\right) + xr, \end{aligned} \tag{42}$$

where the last line uses the definition $x \equiv (1-\beta)p_R + \beta p_S$, $x \in [p_R, p_S]$. In the second interval, expected returns do not depend on β,

$$E\pi^{bank}_{r_S < r \leq r_R}(r) = (1-p_R)\left(\frac{C}{B}-1\right) + p_R r. \tag{43}$$

Setting $\beta = 0$, we have

$$
\begin{aligned}
E\pi^{bank}(r_S) &= (1-p_R)\left(\frac{C}{B}-1\right)+p_R r_S \\
&< (1-p_R)\left(\frac{C}{B}-1\right)+p_R r_R = E\pi^{bank}(r_R),
\end{aligned}
\tag{44}
$$

since $r_S < r_R$.[58] Setting $\beta = 1$, we need to plug r_S and r_R from equations (2) and (3), as well as $x = p_S$ into equations (42) and (43) to see that they are equal.[59] Since expected returns at r_R do not depend on β, the proof is complete if the derivative of expected returns w.r.t. β is positive at r_S. From equation (42), we get

$$
\frac{dE\pi^{bank}_{r\leq r_S}(r)}{d\beta} = -\left(\frac{C}{B}-1\right)\frac{dx}{d\beta}+r\frac{dx}{d\beta} = (p_S - p_R)\left(r+1-\frac{C}{B}\right),
$$

which is positive for all positive r since $C < B$.

[58]This makes perfect sense since $\beta = 0$ means that there are only risky firms in the market. Since returns equal project revenue less firm profits, they must be maximum where expected firm profits are minimum, i.e., at r_R.

[59]Which makes sense since $\beta = 1$ means that there are only safe firms in the market such that a loan rate of r_S also extracts all rents from projects.

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